ABSTRACT

LI, WEIHUA. Uncertainty Reduction in Hydrologic Modeling and Regional Water Management Utilizing Inter-basin Transfer. (Under the direction of Sankar Arumugam.)

Surface water is the major source for supporting various demands, such as domestic, agricultural and industrial consumption. Apart from these consumption demands, ecosystem also relies on surface water flow to sustain its healthy function. However, the distribution and availability of surface water resources is largely uneven at various spatial and temporal scales. Thus, it is very important that water management should maximize the benefit of surface water usage to sustain the economic and environmental development. There are two aspects of modeling work need to be conducted in order to efficiently manage the limited water in a region. One is the modeling of surface water availability, another one is the allocation modeling of available surface water in the region. Both require appropriate tools to carry out a study of uncertainty issues relevant to the process. In this research, we systematically reduce uncertainty in streamflow predictions from daily to seasonal time scales for improving regional water management.

To evaluate the uncertainty in streamflow prediction, we investigate the utility of various algorithms at multiple time scales. These include multi-model combination algorithm, primarily deals with model uncertainty, which are relevant at monthly time scale; The data assimilation algorithm mainly focuses on input and initial model states uncertainty, thereby is relevant in a shorter time scale, e.g., daily scale; The hybrid method, which combines the strength of global optimization and data assimilation, can explicitly reduce uncertainty from parameter estimation. These three algorithms are evaluated based on an experimental design
as well as through watershed applications at daily to monthly time scales. Results show that uncertainty in hydrologic modeling can be reduced using these algorithms. Further, our analysis demonstrates that at monthly time scale, model uncertainty is the dominant source in hydrologic modeling whereas at daily time scale, reducing initial states uncertainty plays an important role in reducing uncertainty in streamflow prediction.

Another aspect of this study is to utilize the reduced uncertainty for improving regional water allocation. The study proposes an integrated water management framework based on inter-basin transfer from multiple reservoirs. The framework is implemented for the North Carolina Triangle area between Neuse River basin and Cape Fear River basin. Results show that inter-basin transfer based on climate forecast could greatly improve the water supply sustainability for the triangle area. To further investigate the role of inter-basin transfer in reducing uncertainty in regional water management, a synthetic experiment is designed to test the potential benefit in promoting inter-basin transfer considering the forecast uncertainty and spatial correlation between the inflows or initial storages. Findings from the study show that forecasts are very useful in promoting inter-basin transfer. Further, the utility of inter-basin transfer is larger among basins exhibiting large difference in inflows and initial storages.
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Uncertainty Reduction in Hydrologic Modeling and Regional Water Management Utilizing Inter-basin Transfer

by
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DEDICATION

This work is dedicated to my parents and my sister.

I thank you all for the continuous love and support in my life.
BIOGRAPHY

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CHAPTER 1: INTRODUCTION

1.1 Background and motivation

Runoff from the watershed is the most important attribute for water management. But the distribution of runoff is uneven over various spatial and temporal scales. For instance, we often hear about droughts and floods at the same time over various regions within the country. Such spatial-temporal variability forces the water managers to better manage the regional/national water supply systems. Progressive regional water supply plans could be developed to ensure competing water demands for users including domestic, agriculture, industrial and ecosystem services. The key to such regional water planning and management lies in quantifying the uncertainty in inflow as well as in translating them into appropriate water management attributes.

Typically, water managers employ hydrologic models to simulate and predict runoff from the watershed based on the inputs—precipitation and temperature. However, hydrologic models are imperfect abstractions of various hydrological processes, whose improper simplification could bring a large degree of uncertainty in prediction. These uncertainties may arise from different sources, such as model structural deficiency, input data errors, imprecise estimation of initial conditions and model parameters. Such uncertainties in hydrologic modeling could significantly impact decision analysis related to regional water management. On the other hand, given the streamflow prediction, water managers allocate the water appropriately to various competing demands. However, uncertainties presented in the inflow forecasts and
demand variability can greatly influence the operational policies and guidance for system management.

The main focus of this dissertation is to systematically assess the uncertainties presented in hydrologic modeling and effectively utilize them for improving regional water management. First, we focus on the evaluation of the algorithms that are capable of reducing uncertainties related to model structure, input, initial conditions and parameters of the hydrologic model. The algorithms investigated with regard to uncertainty reduction are multimodel combination and data assimilation. Next, uncertainty in regional water management is investigated through inter-basin transfer among multiple reservoir exhibiting different spatial correlations between inflows and initial storage conditions. The framework also investigates the role of forecast uncertainty in affecting the inter-basin transfer.

1.2 Literature review

Hydrologic models predict streamflow from the watershed based on the input data, initial states and estimated model parameters. All these information inevitably contain errors. Therefore, many research efforts [Ajami et al., 2004; Hogue et al., 2000; Vrugt et al., 2003; Montanari and Brath, 2004] have focused on reducing the uncertainty in these sources so that the predictions from the hydrologic models could be improved. One approach that reduces the model uncertainty is multimodel combination, which aims to optimally combine multiple hydrologic models to develop multimodel prediction. Devineni et al., (2008) has shown that multimodel combination can reduce the overconfidence of individual model forecasts. Another approach that is currently gaining attention is data assimilation, which
basically updates the initial states of the model based on recent observations. Many studies [Moradkhani et al., 2004; Pauwels and De Lannoy, 2006; Vrugt et al., 2006] have shown that data assimilation is effective in reducing uncertainties in streamflow predictions. Apart from uncertainty related to model, input and initial model states, reducing uncertainty in parameter estimation also received lot of attention. Duan et al., (1992) shows the improperness of using a single set of model parameters in hydrologic modeling; Moradkhani et al., (2004) investigates the role of data assimilation in updating both the initial conditions and parameters. Vrugt et al., (2005) demonstrate the utility of combining the strength of global optimization and data assimilation using the SODA (Simultaneous global Optimization and Data Assimilation) algorithm. In this research, we evaluate the above algorithms and method in reducing uncertainty related to streamflow predictions over multiple time scales.

Regional water management involves allocation of the available water among multiple users from one or multiple sources. In this context, uncertainty in reservoir inflow forecasts could substantially influence reservoir operation. Hence, many studies [Brekke et al., 2009; Georgakakos and Graham 2008; Sankarasubramanian et al., 2009; You and Cai., 2008] focused on relating streamflow forecast uncertainty to reservoir operation and decision making. Sankarasubramanian et al., (2009) have shown that the utility of inflow forecast decreases as the storage/demand ratio of the reservoir increases. Georgakakos and Graham (2008) demonstrated that reservoir operations are most sensitive if the reservoir capacity is at most 5 times the uncertainty range of the predicted seasonal inflow. Kasprzyk et al., (2009)
have investigated the uncertainty and tradeoffs in six conflicting water demands based on a case study. However, most of these studies have focused on a single reservoir operation with a specified inflow forecast uncertainty. Inter-basin transfer, which tries to balance the uneven distribution of water availability and demand in the region, is an effective measure to reduce the uncertainty associated with multiple system operation [Gupta et al., 2007]. In this study, we investigate the utility of inflow forecasts in promoting inter-basin transfers for improving regional water management.

1.3 Outline of the dissertation

The dissertation is organized as follows. Chapter 2 evaluates the role of multimodel combination in improving the hydrologic prediction through model uncertainty reduction. Chapter 3 mainly evaluates the role of data assimilation and multimodel combination in improving streamflow predictions at daily and monthly time scales. Chapter 4 presents the application of the multimodel combination and data assimilation for streamflow predictions for various river basins and demonstrates the importance of reducing parameter uncertainty in improving streamflow prediction. Chapter 5 presents an inter-basin transfer framework for improving regional water management and applies it for the North Carolina triangle area based on the inflow forecasts. Finally, chapter 6 summarizes the major findings and conclusions of this research.
CHAPTER 2: REDUCING HYDROLOGIC MODEL UNCERTAINTY IN STREAMFLOW PREDICTIONS USING MULTIMODEL COMBINATION

2.1 Introduction

Rainfall-runoff models are simplistic representation of actual physical processes that occur in a watershed in transforming the primary inputs – precipitation and potential evapotranspiration – into runoff. Most watershed models typically include conceptual parameters that are usually obtained by calibrating the model-simulated flows against the observed flows [Yapo et al. 1998; Vogel and Sankarasubramanian, 2003]. Numerous studies have been focused on parameter estimation [Hogue et al., 2000; Ajami et al., 2004 Carpenter et al. 2004; Vrugt et al., 2005] and developing calibration algorithm [Duan et al., 1992; Ajami et al., 2004; Moradkhani et al. 2004]. However, the major weakness of this parameter-calibration approach is rooted in its underlying assumption that all sources of uncertainties in the model could be expressed in the parameter error/uncertainty [Kavetski et al., 2006]. In fact, there are other sources of errors that may arise from the modeling process that include uncertainty in the initial conditions, input variables and the model itself [Renard et al., 2010]. Thus, it is impossible to account for all these uncertainties purely by adjusting the model parameters. Input and parameter uncertainties could be addressed using techniques such as Bayesian modeling and generalized approaches for uncertainty estimation [Beven and Binley, 1992; Kuczera and Parent, 1998; Vrugt et al., 2003; Montanari and Brath, 2004] for a given hydrological model. However, addressing these errors alone in a given model do not address model errors resulting from the structural deficiencies of the specific model [Ajami et al., 2007].
Recently a new approach, as suggested by Distributed Model Inter-comparison Project (DMIP) [Smith et al., 2004], aims at improving hydrologic model predictions through a combination of multiple model predictions so that one model’s deficiency could be compensated by others. Many studies have shown that that multimodel ensemble averages perform better than the best-calibrated single-model simulations [Ajami et al., 2007]. Georgakakos et al. [2004] attributed the superior skill of the multimodel ensembles to the fact that model structural uncertainty is properly accounted in the multimodel approach. Recently, Devineni et al., [2008] showed that the reliability of multimodel streamflow forecasts is superior to the reliability of seasonal streamflow forecasts with individual models.

Compared with the traditional way of building a perfect single model, the multimodel combination approach essentially seeks a different paradigm in which the modeler aims to extract as much information as possible from a pool of existing models. The rationale behind multimodel combination lies in the fact that predictions from individual models invariably contain errors from different sources that include input data, initial states of the model, uncertainty in parameter estimation and model structural deficiencies [Feyen et al., 2001]. With independently constructed models, these errors tend to be mutually independent. Thus, with multimodel combination, these errors tend to cancel each other resulting in improved predictions [Ajami et al., 2007]. Weigel et al., [2008] showed that the optimal combination of climate forecasts from multiple models reduce the overconfidence of individual model resulting in a multimodel forecast that is superior to the best single model. Many studies have
shown that combining different competing single models result in improved predictions [Georgakakos et al. 2004; Ajami et al., 2007; Devineni et al., 2008; Devineni and Sankarasubramanian, 2010]. However, most of the studies have focused on demonstrating the superiority of the multimodel algorithm by applying the proposed algorithm for a given basin. In this study, we systematically address the issue why combining multiple hydrologic models result in improved predictions based on a synthetic setup and then apply the candidate algorithms for four selected basins from different hydroclimatic regimes. Further, we also propose a unique way to combine multiple hydrologic models by modifying the multimodel combination algorithm of Devineni et al., [2008] which primarily focused on combining multiple seasonal streamflow forecasting models.

This Chapter is organized as follows: Section 2.2 illustrates the methodology of multimodel combination techniques, which is customized from Devineni et al., [2008] for watershed model combination. Following that, we describe an experimental design and discuss the results from the synthetic study. Finally, we summarize the salient features and conclusions resulting from the study.

2.2 Multimodel Combination Methodology

Given the uncertainties present in the modeling process, it is unlikely a single hydrologic model to give consistently skillful predictions during the entire evaluation period as well as over different flow values (e.g., during extremes). Multimodel prediction, basically a combination of predictions from multiple single models, can capture the strength of these single models resulting in improved predictability [Ajami et al., 2007; Duan et al., 2007;
Devineni et al., 2008]. Multimodel predictions are usually obtained by taking a weighted average of the predictions from the single models. The weights, which sum up to 1, are obtained based on the performance of the single model over the calibration/verification period. There are several established multimodel combination methods, which may include simple or weighted average of single model predictions [Georgakakos et al., 2004; Shamseldin et al., 1997; Xiong et al., 2001] or using statistical techniques such as multiple linear regression [Krishnamurthi et al., 1999] and Bayesian model averaging [Duan, et al., 2007]. Recently, Devineni et al., (2008) showed the importance of combining multiple models by evaluating the individual models’ performance conditioned on the relevant/dominant predictor state(s). The main advantage of such approach is in giving higher weights for models that perform better under a given predictor(s) condition. In this study, we modify the algorithm of Devineni et al., [2008] for combining multiple hydrologic models. Apart from this multimodel combination algorithm, we also consider optimal model combination approach in which weights are obtained purely based on optimization [Rajagopalan et al., 2002].

2.2.1 Multimodel combination: weight conditioned on predictor state

The multimodel combination algorithm presented in Figure 2.1 evaluates the performance of the model over the calibration period $t=1,2,3,...n_c$ and obtains multimodel prediction over the validation period $t=n_c+1,n_c+2,...,n$. Assume that streamflow predictions, $Q_t^m$, are available from $m=1,2,3,...M$ calibrated hydrologic models over a common verification period $t=1,2,3,...n_v$. Based on this, we calculate the errors between the observed streamflow
and model predictions as $\varepsilon_t^m = |Q_t^m - Q_t|$ for every time step during the calibration period.

Store $\varepsilon_t^m$ from all the models and $P_t$ in a matrix of size $n_c \times (m+1)$, with $P_t$ in the very last column of the matrix $\Sigma$. Given the precipitation (predictor), $P_t$, $t = n_c + 1, n_c + 2, ..., n$ over the validation period, we identify ‘$K$’ nearest neighbors ($K=20$) for $P_t$ from the calibration period. For the identified neighbors, we can find the corresponding error $\varepsilon^m_{(j)}$, where $j = 1, 2, 3...K$ for this ‘$K$’ neighbors from matrix $\Sigma$ and obtain the average error over ‘$K$’ neighbors as $\lambda^m_{t,K} = \frac{1}{K} \sum_{j=1}^{K} \varepsilon^m_{(j)}$ for the conditioning variable $P_t$; Based on $\lambda^m_{t,k}$, the weight for a specific model $m$, for the given time step in the validation period can be determined by

$$w^m_{t,k} = \frac{1}{\sum_{m=1}^{M} 1/ \lambda^m_{t,k}}.$$ The underlying basis behind this weight estimation is to give higher (lower) weights for the models that perform better (poorer) under similar predictor conditions. Based on the estimated weights, we combine the predictions from different single models and obtain the multimodel prediction (MM-1) as $Q_t^{MM-1} = \sum_{m=1}^{M} w^m_{t,k} Q_t^m$.
Given streamflow prediction, $Q^m_t$, available from $m=1,2...,M$ models, with $t=1,2...n_c$ represents the number of months available from calibration.

Obtain the absolute error in prediction, $\varepsilon^m_t = |Q^m_t - Q_t|$ for each time step for each model.

Compute the distance between the current validation point, $P_t$, and the rest of the points $P_l$ where $l=1,2...,n_c$ and order neighbors on precipitation based on the distance.

Choose the number of neighbors, $K$, from the ordered precipitation and find the mean absolute error over ‘$K$’ neighbors:

$$\lambda^m_{r,K} = \frac{1}{K} \sum_{j=1}^{K} \varepsilon^m_{(j)}$$

Compute weights for each model for each time step based on the average error over ‘$K$’ neighbor

$$w^m_{r,K} = \frac{1/\lambda^m_{r,K}}{\sum_{m=1}^{M} 1/\lambda^m_{r,K}}$$

Using the weights, $w^m_{r,K}$, compute the multimodel streamflow prediction by weighting the calibrated flow obtained from each model:

$$Q_t^{MM-1} = \sum_{m=1}^{M} w^m_{r,K} \ast Q^m_t$$

Figure 2.1: Multimodel combination based on the performance of individual model under a given predictor(s) state.
2.2.2 Optimal Combination of Individual Models

For comparing the proposed algorithm $MM-1$, we obtain weights for individual models purely based on optimization, which is the baseline multimodel procedure that is considered for comparing the performance of $MM-1$. We obtain weights based on (1) that minimize the squared error between the multimodel predictions and the observations over the calibration period.

$$\text{Min} \sum_{t=1}^{n} (w^m Q^m_t - Q_t)^2$$

… (1)

Where $w^m$ are weights (decision variables) for model $m$, $Q^m_t$ is the streamflow prediction at time step $t$ from model $m$, $Q_t$ is the streamflow observation at time step $t$. It is obvious that the weights have to sum up to $1 (\sum_{m=1}^{M} w^m = 1)$, which is included as a constraint for minimizing the objective function (1). The weights are obtained using the optimization algorithm available in Matlab optimization toolbox [Byrd et al., 1999]. Once the weight for each model is determined during the calibration period, we employ these optimal weights in the validation period to combine the predictions from multiple single models and obtain the multimodel prediction ($MM-O$), $Q_t^{MM-O} = \sum_{m=1}^{M} w^m Q^m_t$, based on optimal combination. Thus, in $MM-O$, the weight for a given model does not vary for each time step ‘$t$’ for obtaining the multimodel combination. The next section details a synthetic streamflow generation scheme for comparing the performance of four single models and the two multimodel combinations ($MM-1$ and $MM-O$) proposed in this section.
2.3 Multimodel Performance Evaluation – Experimental Design

An experimental design for understanding why multimodel combination techniques perform better than individual models is presented. The experimental design is based on a synthetic streamflow generation scheme, which assumes that the ‘true’ parameters of a given hydrologic model are known. Given a hydrologic model, \( f_m(X_t^m, I_t^m, \theta^m) \) where \( X_t^m, I_t^m, \theta^m \) denote the inputs, initial conditions corresponding to time step ‘\( t \)’, and true model parameters respectively, one can obtain the true flows, \( \tilde{Q}_t^m \), from the model \( m \). Basically, the function, \( f_m(.) \), is a set of nonlinear storage equations that convert the inputs and initial conditions into a set of left over storage and outflows (i.e., streamflow and evapotranspiration) from the watershed. Since we employ prescribed inputs, \( X_t^m \), across all the models, it is reasonable to assume the input uncertainty to be zero. We add noise to the true flows \( Q_t^m \) to develop corrupted flows which will be employed for testing the performance of different candidate watershed models and multimodel combination techniques in predicting the true flows from a given model. The exact steps of the experimental design are discussed below:

a) Generate corrupted streamflow \( Q_t = \tilde{Q}_t^m + \varepsilon_t \), from true flows \( \tilde{Q}_t^m \) with \( \varepsilon_t \sim N(0, \sigma_{\varepsilon_t}^2) \) denoting the model error term having zero mean and variance \( \sigma_{\varepsilon_t}^2 = f_2 \times \sigma_t^2 \), and \( f_2 \in 0.1-1.5 \) represents variance inflation factor [Cureton et al., 1983] and \( \sigma_t^2 \) represents the variance of monthly flows in a given month with \( t=1,2...n \). This indicates that the model errors are heteroscedastic and the error variances vary depending on the month.
We split the corrupted flows $Q_t$ into two parts $t=1,2..n_c$ and $t=n_c+1,n_c+2....n$ with one for calibration and another for validation respectively.

b) Calibrate the candidate model $m$ (discussed in the next section) using $Q_t$ and $X_t^m$ and obtain the model parameters $\hat{\theta}^m$ using the data available for the calibration period (\(t=1,2..n_c\)). The parameters of the watershed models are obtained by minimizing the chosen objective function (discussed in the next section) by using the optimization algorithm in the Matlab optimization toolbox [Byrd et al., 1999].

c) Validate the model using $\hat{\theta}^m$ and $X_t^m$ available, obtain model predicted flows, $\hat{Q}_t^m$ over the validation period $t= n_c+1,n_c+2....n$ and compute the root mean square error (RMSE) between the model predicted flows, $\hat{Q}_t^m$ and the corrupted flows, $Q_t$ over the validation period.

d) Combine the flows obtained from individual models, $\hat{Q}_t^m$ using the multimodel combination techniques (discussed in Sections 2.1 and 2.2) and obtain RMSE between the multimodel predicted flows, $\hat{Q}_{t}^{MM-1}, \hat{Q}_{t}^{MM-O}$ and the true flows,$\hat{Q}_t^m$, for the two combination techniques.

e) Repeat the above procedures a) ~ d) 100 times and record the RMSE for individual models and multimodels for further analysis.

2.3.1 Candidate Models and Data sources

We consider two hydrological models, ‘abcd’ model [Thomas, 1981] and ‘VIC’ model [Liang et. al., 1996], for developing true flows $\hat{Q}_t^m$ as well as for estimating the predicted
flows, $\hat{Q}_t^m$. The inputs, $X_t^m$, monthly time series of precipitation and potential evapotranspiration available for the Tar River at Tarboro are obtained from the monthly climate database (water year; 1951-1987) for the Hydroclimatic Data Network (HCDN) sites [Vogel and Sankarasubramanian, 1999]. Using the true parameters obtained for each model, we generate the true flows, $Q_t^m$, from the two models and then the respective monthly variances and $f_2$ are used to develop flows with model errors. Both models are calibrated against the synthetically generated streamflow, $Q_t$, based on two objective functions (step (b) in the experimental design) – sum of squares of errors ($\sum_{t=1}^{n_t} (Q_t - \hat{Q}_t^m)^2$ (SSE)) and sum of absolute deviation (SAE) $\sum_{t=1}^{n_t} |Q_t - \hat{Q}_t^m|$ – to estimate the flows $\hat{Q}_t^m$ under validation using the calibrated model parameters $\hat{\theta}^m$. For instance, it has been shown in the watershed model calibration literature that models calibrated using ordinary least squares objective function perform well in predicting extreme conditions, whereas models calibrated with heteroscedastic maximum likelihood estimator (HMLE) perform well in predicting normal flow conditions [Yapo et al., 1998]. By calibrating two hydrologic models using two different objective functions, we obtain a total of four streamflow predictions, $\hat{Q}_t^m$, over the validation period with $m=1(2)$ representing ‘abcd’ model predicted flows using SSE (SAE) parameter estimates and $m=3(4)$ representing ‘VIC’ model predicted flows using SSE (SAE) parameter estimates. Using these four streamflow predictions, we develop two different
multimodel predictions, $\hat{Q}_{t}^{\text{MM}-1}$, $\hat{Q}_{t}^{\text{MM}-O}$, whose performances will be compared against the individual model predictions over the validation period.

### 2.3.2 Results and Analysis

Following the steps described in Section 2.3.1, RMSE of two multimodel predictions ($\hat{Q}_{t}^{\text{MM}-1}$, $\hat{Q}_{t}^{\text{MM}-O}$) and four individual model predictions ($m=1, 2, 3, 4$) are obtained by evaluating the model over the validation period. The corresponding weights for each individual model under the two multimodel combination algorithms are also considered for analysis. We consider the median of the RMSE as well as the entire probability distribution of RMSE for future analysis.
Figure 2.2: Comparison of the performance of multimodels with the performance of individual models under different heteroscedastic error variance – (a) Median RMSE of four candidate models and two multimodels (MM-I and MM-O) with ‘abcd’ as true model, (b) Median RMSE of four candidate models and two multimodels with ‘VIC’ as true model, (c) Probability of multimodel (MM-I) RMSE being smaller than the minimum RMSE of individual models/MM-O with ‘abcd’ as true model (d) Probability of multimodel (MM-I) RMSE being smaller than the minimum RMSE of individual models/MM-O with ‘VIC’ as true model.
Figure 2.2 shows the overall performance of different models given ‘abcd’ model as ‘true’ model (2a & 2c) and ‘VIC’ model as ‘true’ model (2b & 2d) with model errors being heteroscedastic. From Figures 2a and 2b, we can infer that for very small model errors (e.g., \( f_2 = 0 \) or 0.1), the median RMSEs of \( MM-I \) and \( MM-O \) are larger than the median RMSE of the ‘true’ model (i.e., ‘abcd’ model or ‘VIC’ model). Given that the true flows are from ‘abcd’ model, it is to be expected that the ‘VIC’ model would not be able to outperform the ‘abcd’ model. As the model errors (\( f_2 \)) increase, the multimodel \( MM-I \) outperforms all the other candidate models them including the predictions from the ‘true’ model. In these situations (\( f_2 > 0.5 \)), the median RMSE of \( MM-I \) is much lower than the median RMSE of the individual models as well as the optimal multimodel combination algorithm \( MM-O \). It is important to note that all the RMSE is obtained only over the validation period indicating the model performance outside the calibration period. For the case with true model being the ‘VIC’ model (Figure 2b), as expected, ‘VIC’ model has the lowest RMSE among all the models for small model error variances. However, as model error variance increases, both \( MM-I \) and \( MM-O \) perform better than individual models.

Under Figures 2c and 2d, we compare the performance of individual models and multimodels based on their respective distribution of RMSE obtained from the 100 validation trials. Figures 2c and 2d plot the probability of multimodel \( MM-I \) being the best model having the minimum RMSE in comparison to the rest of the individual models (‘abcd’ model or ‘VIC’ model) and \( MM-O \). From these two figures, we can clearly see that for lower model errors,
the probability of RMSE of \textit{MM-1} being lower than the minimum RMSE of the ‘true’ models and multimodel \textit{MM-O} is very small (around 5-10\%). ‘\textit{VIC}’ model (‘\textit{abcd}’ model) is not seen in Figure 2c (2d), since the true model is ‘\textit{abcd}’ model (‘\textit{VIC}’ model), ‘\textit{VIC}’ model (‘\textit{abcd}’ model) did not result with minimum RMSE in comparison to the rest of the candidate models even once out of the 100 validation trials. As the model error increases, the probability of \textit{MM-1} being the best model increases and approaches towards 1. Comparing the performance of \textit{MM-1} with \textit{MM-O} (Figures 2c and 2d), we infer that \textit{MM-O} performs better than \textit{MM-1} for smaller values ($f < 0.4$). However, as $f$ increases further, we clearly see that \textit{MM-1} outperforms \textit{MM-O} with the RMSE from \textit{MM-1} being lower than \textit{MM-O} with higher probability (>65\%). This is primarily due to the nature of combination methods employed in \textit{MM-1} and \textit{MM-O}. Under \textit{MM-O}, the weights are obtained optimally based on the model performance over the calibration period, whereas under \textit{MM-1}, weights for multimodel combination vary depending on the predictor state and they are estimated statistically based on mean absolute error over ‘\textit{K}’ neighbors. Thus, under large model error variance, analyzing the model performance conditioned on similar predictor (s) state provides better estimates of weights for multimodel combination.
Figure 2.3: Median RMSE of four candidate models and two multimodels (MM-1 and MM-O) under six different flow conditions with true model being ‘abcd’ having heteroscedastic errors – (a) <10th percentile of flow (b) 10th - 25th percentile (c) 25th - 50th percentile (d) 50th - 75th percentile (e) 75th - 90th percentile (f) >90th percentile of flow.
Figure 2.4: Median RMSE of four candidate models and two multimodels (MM-1 and MM-O) under six different flow conditions with true model being ‘VIC’ having heteroscedastic errors – (a) <10th percentile of flow (b) 10th-25th percentile (c) 25th-50th percentile (d) 50th-75th percentile (e) 75th-90th percentile (f) >90th percentile of flow.
Given the overall reduction in RMSE from multimodel combination methods, it is instinctive to look at the performance under different flow conditions. Figure 2.3 (2.4) presents the result of ‘abcd’ model (‘VIC’ model) as ‘true’ model by comparing their performance (median RMSE) under six different monthly flow conditions: a) $<10^{th}$ percentile; b) $10^{th} \sim 25^{th}$ percentile; c) $25^{th} \sim 50^{th}$ percentile; d) $50^{th} \sim 75^{th}$ percentile; e) $75^{th} \sim 90^{th}$ percentile; f) $>90^{th}$ percentile. From both figures, apart from the previously discussed findings, we infer that the improvement of multimodel MM-I under extreme conditions ($10^{th}$ percentile and $90^{th}$ percentile) is larger than normal conditions ($10^{th}$ percentile to $90^{th}$ percentile). On the other hand, MM-O performs as good as the flows predicted by the best individual model. One reason for improved performance of MM-I compared to MM-O under high flow and low flow conditions stems from the fact that MM-O evaluates the performance of individual model during the entire calibration period. On the other hand, MM-I evaluates the performance of individual models based on the mean absolute error around the conditioning state. Thus, MM-I outperforms the best performing individual model and MM-O not just over the considered validation period, but also under different flow conditions. The only exception is under $25^{th} - 50^{th}$ percentile for large $f_2$ (Figures 2.3c and 2.4c), under which MM-I performs poorly in comparison to MM-O and abcd model. This increased RMSE for MM-I could be due to ignoring additional predictors that could relate to proper identification of neighbors ($K$). For instance, during normal flow conditions, it is reasonable to expect that errors in predicting the flows could depend on both precipitation and potential evapotranspiration. Similarly, in Figure 2.4 which has true flows from ‘VIC’ model, we
observe similar behavior with ‘VIC’ model outperforming abcd model and $MM-I$ outperforming all the individual models and $MM-O$ during all the flow conditions except during 25th-50th percentile.

Figure 2.5: Median weight of each candidate models for different heteroscedastic errors under the $MM-I$ scheme – (a) ‘abcd’ as true model, (b) ‘VIC’ as true model.
Figure 2.6: Optimized weights of each candidate models over the 100 calibration runs for different heteroscedastic errors under the MM-O scheme – (a) ‘abcd’ as true model, (b) ‘VIC’ as true model.
To understand why multimodel $MM-1$ performs better in comparison to individual models and $MM-O$, we also show the box plots of weights obtained for multimodel schemes $MM-1$ (Figure 2.5a & 2.5b) and $MM-O$ (Figures 2.6a and 2.6b) with the true model being ‘abcd’ model (Figures 2.5a and 2.6a) and ‘VIC’ model (Figures 2.5b and 2.6b). Each box in Figure 2.5 represents the various percentiles of the median weights for the four individual models from the 100 validation runs under $MM-1$ scheme. Similarly, boxes in Figure 2.6 represent various percentiles of optimized weights ($MM-O$) obtained from the 100 validation trials. It is important to note that we employ median weights under a given validation run for $MM-1$, since $MM-1$ weights vary with respect to time. On the other hand, $MM-O$ scheme employs the same weight for each model in a given validation trial. From Figure 2.5, we can clearly see that when model error ($f_2$) is very small, $MM-1$ draws weights close to 1 from the true model and weights close to zero from other candidate model. However, as model errors ($f_2$) increase, $MM-1$ draws weights equally from all the models since the flows from the true model are corrupted. On the other hand, comparing these weights with optimized weights obtained from $MM-O$ scheme (Figure 2.6), we see that the weights are drawn mostly from ‘abcd-OLS’ model alone during most of the situations (Figure 2.6a), although others models are assigned slightly higher weights (0 ~ 0.2) as model errors ($f_2$) increase. With ‘VIC’ model as true model (Figure 2.6b), we can see that both ‘VIC-OLS’ and ‘VIC-ABS’ obtain higher weights whereas the ‘abcd’ models gain little weights under large model error variance. This implies that $MM-O$ mostly draws weights from the best performing individual models over the calibration period regardless of the model error ($f_2$) condition. The above
discussion underscores the importance of combining multiple models conditioning on the predictor state as well as employing a proper weighting scheme for multimodel combination.

**Homoscedastic errors**

Results discussed in Figures 2.2-2.6 are obtained with synthetic streamflow obtained with the true flows being corrupted with heteroscedastic variance. Though heteroscedastic variance is more realistic, we briefly present results with synthetic streamflows exhibiting homoscedastic variance. For instance, humid basins that receive uniform precipitation all through the years with limited/no seasonality in streamflow usually exhibit homoscedastic error variance structure ($\sigma^2 = f_2 \sigma^2$), where $\sigma^2$ denotes the average variance of the monthly flows. Thus, the noise variance did not vary for each month.
Figure 2.7: Comparison of the performance of multimodels with the performance of individual models under different homoscedastic error variance – (a) Median RMSE of four candidate models and two multimodels ($MM-1$ and $MM-O$) with ‘abcd’ as true model, (b) Median RMSE of four candidate models and two multimodels with ‘VIC’ as true model, (c) Probability of multimodel ($MM-1$) RMSE being smaller than the minimum RMSE of individual models/MM-O with ‘abcd’ as true model (d) Probability of multimodel ($MM-1$) RMSE being smaller than the minimum RMSE of individual models/MM-O with ‘VIC’ as true model.
Figure 2.7 shows the median of RMSE with true flows generated from ‘abcd’ model and ‘VIC’ model under homoscedastic variance. Multimodel combinations based on conditioning dominant predictors (MM-I) perform better than the individual models and multimodel combinations based on overall skill (MM-O). In addition, we find that the median of RMSE of both candidate models and the multimodels are smaller in comparison to the heteroscedastic error case. This is mainly due to smaller error variance under the homoscedastic analysis, which helps us to estimate the true flows, $\tilde{Q}_m^w$, with less error. Analysis on the performance of all the candidate models under different flow conditions also revealed similar conclusions with MM-I performing better as $f_2$ increases. To summarize, the synthetic study under homoscedastic and heteroscedastic error variance reveal that multimodel combination performs better than the individual models particularly under large model errors. Further, combining multiple models based on their performance under similar input conditions (i.e., MM-I) result in improved performance compared to combining multiple models purely based on their performance during the calibration period.

2.4 Summary and Conclusions

A systematic analysis on how multimodel combination reduces model uncertainty is presented. The study also proposes a new approach (MM-I) which combines multiple hydrologic model predictions by assigning higher weights for a model that performs well under a given predictor(s) state. The performance of MM-I is compared with optimized multimodel combination (MM-O) which obtains weights for individual models purely based
on optimization by minimizing the errors over the calibration period. Thus, under $MM-1$, the model weights vary with time, whereas under $MM-O$, the weights are purely dependent on the model performance over calibration period. The study compares the performance of two single models with parameters obtained from two different objective functions with these two multimodel combination schemes with flows generated from a known hydrologic model as well as by predicting observed flows over four different watersheds.

Results from the synthetic study shows that under increased model errors, both multimodels ($MM-1$ and $MM-O$) perform better than the candidate single models even if the flows are generated from one of them. Overall, $MM-1$ performs better than $MM-O$ in predicting the monthly flow values as well as in predicting extreme monthly flows ($<10^{th}$ percentile and $>90^{th}$ percentile). Comparing the weights obtained from each candidate models reveals that as model errors increase, $MM-1$ assigns weights equally for all the models, whereas $MM-O$ assigns higher weights for always the best-performing model under the calibration period. Given that the flows are corrupted under large model errors, MM-1 provides higher weights for the best-performing single model under a given predictor state resulting in reduced RMSE over the entire validation period as well as under extreme flow conditions.
CHAPTER 3: ROLE OF MULTIMODEL COMBINATION AND DATA ASSIMILATION IN IMPROVING THE HYDROLOGICAL MODEL PREDICTION AT MULTIPLE TIME SCALES

3.1 Introduction

Hydrological models are usually simplified, conceptual representation of the complex rainfall-runoff process. Uncertainty from various sources may arise in the modeling process resulting in poor prediction or lower confidence in model estimates. These uncertainties typically arise from different aspects, such as model structural deficiency, imprecisely estimated model parameters, inaccurate estimates of initial model states and input/output data errors. Traditional hydrologic modeling assumes that all the uncertainties arise from the model parameters [Beven and Binley, 1992; Kuczera and Parent, 1998; Vrugt et al., 2003; Pappenberger et al., 2005; Wilby, 2005]. Hence, most of the modeling efforts have focused on searching for the global parameters (model calibration) that can provide the best fit between model predictions and observations [Ajami et al., 2004; Hogue et al., 2000]. Since hydrologic models are abstraction of highly nonlinear systems, estimation of global parameters is difficult. Even if global parameters are obtained using innovative calibration algorithms such as the shuffled complex evolution (SCE-UA) global optimization algorithm [Duan et al., 1992], estimated parameters may not have physical meaning due to model structural deficiency. Hence, it is not only sufficient to reduce the uncertainty in parameter estimation but also other sources of uncertainties related to input/output data, initial states of the model and even the model itself.
Apart from parameter uncertainty reduction techniques, efforts to reduce uncertainties in the input, initial condition and model structure were explored in the early 1980s [Young and Beck, 1974; Kitanidis and Bras, 1980; Hebason and Wood, 1985; Young, 1986]. Among these three sources of uncertainty, reducing model and initial conditions uncertainty have resulted in significant improvement in streamflow predictions. One approach to reduce model uncertainty is the multimodel combination algorithm, which is aimed to optimally combine multiple model predictions so that developed multimodel prediction will have improved predictability. The rationale behind multimodel combination lies in the fact that predictions from individual models invariably contain errors from various sources that include input data, initial states of the model, uncertainty in parameter estimation and model structural deficiencies [Beven and Freer, 2001]. With independently constructed models, these errors tend to be mutually independent and these errors would act to cancel each other out under the multimodel combination resulting in better overall predictions [Ajami et al., 2007]. Through an experimental design, Li and Sankarasubramanian, (2011) demonstrates how the multimodel combination can produce improved predictability especially when the uncertainty of the candidate models are large.

The other approach, data assimilation, is also gaining attention since it explicitly considers uncertainties in the initial model state. The objective is to update the model state by minimizing the error covariance matrix between the model predictions and observations. It was initially widely used in oceanic and atmospheric scientific studies to improve the estimation of the state variables for General Circulation Models (GCMs) [Annan et al., 2005;
One of the simplest forms of sequential data assimilation is the Kalman Filter [Kalman, 1960], a recursive data processing technique that produce optimal estimation for linear dynamic models [Moradkhani et al., 2004]. However, most of the hydrologic models are complicated nonlinear system, which restricts the applicability of the linear Kalman Filter in these situations. For nonlinear system, the extended Kalman filter (EKF), which relies on linearization of model using first order approximation of the Taylor series, could be employed [Moradkhani et al., 2004]. However, the EKF has many well-known drawbacks such as demanding computational cost due to the calculation of error covariance and approximation by ignoring high-order derivatives of the model [Moradkhani et al., 2004] resulting in unbounded linear instabilities for error evolution [Evensen, 1992]. The Ensemble Kalman Filter (EnKF) proposed by Evensen [1994] successfully eliminates the computational inefficiency and approximation issues by using an ensemble representation of the model state. The propagation of each ensemble member is independent and the error covariance matrix can be easily calculated through the ensemble members before and after the update, thereby making it easier to implement for a nonlinear system with relatively low computational cost. Many studies [Moradkhani et al., 2004; Pauwels and De Lannoy, 2006; Vrugt et al., 2006; Weerts and El Serafy, 2006] have shown that EnKF is an efficient method in reducing uncertainties in hydrologic modeling practices. Vrugt et al. (2004) compared the performance of data assimilation and Bayesian multimodel combination in reducing the uncertainty primarily related to daily streamflow predictions. In this study, we systematically compare the role of multimodel combination and
data assimilation in reducing uncertainties at multiple scales based on a detailed synthetic study and the implementation for the Tar River at Tarboro.

The goal of this chapter is to investigate the performance of the EnKF algorithm in terms of improving hydrologic prediction on multiple time scales and compare its performance with the multimodel combination. The organization of this chapter is as follows: Section 3.2 introduces data assimilation methodology and provides detailed EnKF application on a lumped watershed model; Section 3.3 presents the monthly and daily data as well as the study basin; Section 3.4 proposes an experimental design to compare the multimodel approach and EnKF algorithm and evaluates their ability in improving hydrologic predictions at multiple time scale; Section 3.5 demonstrates the utility of both approaches by applying them at the Tar River basin; Section 3.6 summarizes the salient findings and conclusions from the study.

3.2 Ensemble Kalman Filter: Methodology

From model uncertainty reduction perspective, Li and Sankarasubramanian, (2011) have demonstrated that multimodel combination algorithm can result in improved predictions in the case of large model errors. Li and Sankarasubramanian, (2011) did not compare the proposed multimodel combination with data assimilation for improving monthly streamflow predictions. Vrugt et al. (2005) compared the bayesian multimodel algorithm and data assimilation for improving streamflow at daily time scale. The basic objective of data assimilation is to update the model states at each prediction time based on the observation available up to that time [Moradkhani et al., 2004]. Hence, with improved estimation of the
initial states of the model, subsequent predictions from the model could be increased. We briefly introduce the general framework of sequential Data Assimilation and apply Ensemble Kalman Filter (EnKF) for a lumped watershed model.

### 3.2.1 General framework of Sequential Data Assimilation

Typically, EnKF is trying to addresses the general problem that is governed by the following nonlinear system of equations:

\[ \psi_t = f(\theta, \psi_{t-1}, x_t) \quad \ldots \quad (1) \]

with a measurement equation represented by

\[ y_t = H(\psi_t) + \epsilon_t \quad \epsilon_t \sim N(0, \sigma_t) \quad \ldots \quad (2) \]

where, the function \( f(\bullet) \) relates the parameter \( \theta \), initial model state \( \psi_{t-1} \) and stochastic perturbation input \( x_t \) to the current model state \( \psi_t \); \( H(\bullet) \) is a function that relates the current model state \( \psi_t \) to the measurement \( y_t \); \( \epsilon_t \) is a random measurement error which follows normal distribution with mean equal to zero and variance being the observational variance.

For each time step from \( t = 1, 2, ..., T \), when the measurement \( y_t \) is available, the forecast error between the measurement \( y_t \) and model output \( H(\psi_t^-) \) can be computed by

\[ z_t = y_t - H(\psi_t^-) \quad \ldots \quad (3) \]

and the updated model state \( \psi_t^+ \) can be computed using the standard analysis equation

\[ \psi_t^+ = \psi_t^- + K_t[y_t - H(\psi_t^-)] \quad \ldots \quad (4) \]

Where \( K_t \) denotes the weights (Kalman gain) that correct the prediction depending on its accuracy to the measurement, it can be computed as
\[ K_t = \sum_{i}^m H^T (H \sum_{i}^m H^T + \sum_{i}^o)^{-1} \] 

......(5)

where \(\sum_{i}^m\) and \(\sum_{i}^o\) denotes the covariance matrix of the model prediction error and observation error.

The updated model state \(\psi_t^+\) then recursively propagates through the model as the initial condition for next step:

\[ \psi_{t+1}^- = f(\theta, \psi_t^+, x_{t+1}) \] 

......(6)

With improved estimation of the initial model state, the model output \(\tilde{y}_t = H(\psi_t^+)\) can be more accurate.

In the above scheme, uncertainty in the initial model state can be reduced through better estimation by the update equation, while uncertainty in the input is reflected by the stochastic perturbation term \(x_{t+1}\). Therefore, EnKF approach offers a general scheme that can explicitly deal with initial model state uncertainty and input uncertainty.

In the next section we will illustrate how EnKF algorithm can be adopted for a conceptual hydrologic model ‘abcd’ model, which is developed by [Thomas, 1981] and widely used in various research studies [Sankarasubramanian et al., 2001].

### 3.2.2 EnKF application on hydrologic model: methodology

We denote ‘abcd’ model as a nonlinear system \(f(\theta, \psi_{t-1}, x_t')\), in which \(\theta\) denotes the model parameter set, \(\psi_{t-1} \in [S_{t-1}, G_{t-1}]\) denotes the initial model states and \(x_t' \in [P_t', PE_t']\) denotes the stochastic perturbed input for the model. In this nonlinear system, soil moisture storage
$S_i$ and groundwater storage $G_i$ are the two model states. During the updating process, for each realization of the ensemble member, at each time step $t$ the model states $S_i$ and $G_i$ will be updated using the streamflow observation $y_{t-1}$ from the previous time step $t - 1$. The process of implementing the EnKF method is described in the following paragraph.

Step a. Initial estimation of the two model states. The initial estimate of soil moisture $S_i^0 \in 1 \times N$ and groundwater storage $G_i^0 \in 1 \times N$ is generated using their corresponding mean and variance for the first month (Sept) based on the calibration period. Both $S^0$ and $G^0$ follow multivariate normal distribution with the covariance estimated from the ensemble members with $i = 1, 2, ..., N$ denoting the ensemble members. The ensemble members of both initial states are stored in a $2 \times N$ matrix $A$.

$$A = (\psi_{t-1}^1, ..., \psi_{t-1}^N)$$

Step b. Model prediction step. Let each pair of the ensemble member $\psi^i$ from matrix $A$ propagate through the nonlinear ‘abcd’ model $f(\theta, \psi_{t-1}, x_i^t)$ given by equation from ‘abcd’ model with a set of model parameters $\theta \in (a, b, c, d)$ and perturbed input $x_i^t = x_i + \eta_t \sim N(0, \sigma_t)$ for time step $t$, get the ensemble model prediction output corresponding to the current time step, which include the streamflow forecast $Q_i^t$ and the model state forecast $\psi_i^{t-}$, the minus sign denotes the pre-update condition.

$$[Q_i^t, \psi_i^{t-}] = f(\theta, \psi_{t-1}, x_i^t)$$
Step c. Compute the covariance matrix of the predicted model states. At each time step $t$, the covariance matrix of the predicted states is calculated as the following:

$$
\sum_i = \frac{A'(A)'}{N-1}, \text{ where } A = A - \bar{A} \text{ and } \bar{A} \text{ denotes the ensemble mean of the forecasted states.}
$$

Step d. Get vector of ‘noisy’ observations. At each time step $t$, generate an ensemble of $N$ streamflow measurements with mean equal to the true observation $y_t$ and variance equal to $\sum_{i}^{\sigma_i}$:

$$
\sum_{i}^{\sigma_i} : y_i^t = y_t + \varepsilon^i, \text{ where } \varepsilon^i \in N(0, \sigma_i^t) \text{ and } \sum_{i}^{\sigma_i} = \varepsilon^i \varepsilon'^i. \text{ Then store the measurement in a matrix } D \text{ which } D = (y_{i1}^t, y_{i2}^t, ..., y_{iN}^t).
$$

Step e. Update model state. At each time step $t$, use the standard update equation to update each pair of ensemble state member $\psi_i^t$ from matrix $A$ and get the updated model states $\psi_i^{t+1}$.

$$
A = A + \sum_i^{m} H^T (H \sum_i^{m} H^T + \sum_{i}^{\sigma_i})^{-1} [D - H(A)].
$$

Step f. Recursively update and check for stop. Bring the updated state variables $\psi_i^{t+1}$ as the initial conditions for next time step $t + 1$ and repeat steps b-e until we get predictions for the last time step $t$.

The entire process of EnKF implementation for ‘abcd’ model has also been summarized in Figure 3.1.
Calibrate the ‘abcd’ model and obtain calibrated parameter set \( \theta \in (a,b,c,d) \) based on \( P_t, PE_t \) and \( Q_t \) for \( t = 1,2,...,n \) where \( n \) denotes the number of months of records available for calibration.

Select "N", the ensemble size and set initial ensemble for model states \( S_0 \) \& \( G_0 \), set \( t = 1 \)

Store soil moisture \( S_0 \in 1 \times N \) and ground water storage \( G_0 \in 1 \times N \) in a \( 2 \times N \) matrix \( A \)

At time \( t \), let each ensemble member from \( A \) propagate through the ‘abcd’ model \( f(\theta, y_{t-1}, x_t) \), then store the model state prediction \( S_t^i \) and \( G_t^i \) into matrix \( A \)

Compute the covariance matrix of the forecasted states as \( \sum_i = \frac{A(A)^T}{N-1} \), where \( A = A - \bar{A} \) and \( \bar{A} \) denotes the ensemble mean of the forecasted states at time \( t \)

At time \( t \), when an observation \( y_t \) becomes available, sample \( N \) measurement \( y_t^i \) using \( y_t^i = y_t + \epsilon^i, \epsilon^i \sim N(0,\sigma^2) \), where \( \sigma^2 \) denotes the standard deviation of the measurement error, store them in the matrix \( D = (y_1^i, y_2^i, ..., y_N^i) \)

At time \( t \), update the model states from matrix \( A \) using the analysis equation: \( A^o = A + \sum_i H^T (H \sum_i H^T + \sum_o)^{-1} [D - H(A)] \), where \( H \) is the ‘abcd’ model function that maps the model states to the model output.

Compute the ensemble mean of streamflow prediction \( Q_t^\mu \) from kalman update \( H(A) \)

Return the ensemble mean prediction \( Q_t^\mu; t = 1,2,......,n \)

Figure 3.1: Flow chart of EnKF algorithm application on ‘abcd’ hydrological model.
3.3 An experimental design

3.3.1 Methodology

Given a hydrologic model, \( f_m(X^m_t, \psi^m_t, \theta^m) \) where \( X^m_t, \psi^m_t, \theta^m \) denote the inputs, initial condition and parameter corresponding to time step ‘\( t \)’, true model parameters respectively with \( m=1,2..., M \) denoting the model index, one can obtain the true flows, \( \tilde{Q}^m_t \), from the model \( m \). Basically, the function \( f_m(.,) \), is a set of nonlinear storage equations that convert the inputs and initial conditions into a set of left over storage and outflows (i.e., streamflow and evapotranspiration) from the watershed. The underlying assumption is that, with absolutely no error from the input and the initial condition, if we know the true parameters representing the watershed process, we can predict the true response from the model, in another word, the ‘true’ streamflow. Based on this assumption, we use \( \tilde{Q}^m_t \), from two different models to synthetically generate streamflows with different magnitude of model errors, so that corrupted flows with model error will be employed for testing the performance of hydrologic models as well as using the Multimodel algorithm and EnKF algorithm. The experimental design for this test is detailed as follows:

a) Generate streamflows, \( Q_t = \tilde{Q}^m_t + \epsilon_t \) where \( \epsilon_t \sim N(0, \sigma^2_\epsilon) \), from true flows with model error variance \( \sigma^2_\epsilon = f_2 * \sigma^2_t \) with \( f_2 \in 0.1 \sim 1.5 \) representing inflation factor and \( \sigma^2_t \) representing the variance of monthly flows in a given month \( t=1,2...n \) and split the generated flows \( Q_t \) two parts \( t=1,2..n_c \) and \( t=n_c+1,n_c+2...n \) for calibration and validation respectively.
b) Calibrate the $m^{th}$ candidate model (discussed in the next section) using $Q_t$ and $X_t^m$ and obtain the model parameters $\hat{\theta}^m$.

c) Validate the model using $\hat{\theta}^m$ and $X_t^m$ available over the remaining period $t = n_c + 1, n_c + 2, \ldots , n$ and obtain model predicted flows, $\hat{Q}_t^m$ and compute root mean square error (RMSE) between the model predicted flows, $\hat{Q}_t^m$ and the synthetically generated flows, $Q_t$.

d) Combine the flows obtained from individual models, $\hat{Q}_t^m$ using the multimodel algorithm and obtain RMSE between the multimodel predicted flows, $\hat{Q}_t^{MM-1}$ and the synthetically generated flows, $Q_t$.

e) Use the calibrated parameter $\hat{\theta}^m$ and input $X_t^m$ available over the validation period, obtain RMSE between the EnKF predicted flows $\hat{Q}_t^{EnKF}$, and the synthetically generated flows, $Q_t$.

f) Repeat 100 times the procedures a)-e) and record the RMSE for individual models, multimodel and EnKF for further analysis.

3.3.2 Site Description and Data set

The experimental design is performed at the Tar River basin, North Carolina, which is a humid basin with annual precipitation amount of 44 inches and the drainage area is 2183 square miles. Monthly time series of the data for the gauge is obtained through the Hydro-Climatic Data Network (HCDN), the ID is 02083500. Among the data, monthly time series of Precipitation (P), Potential Evaportranspiration (PET) and streamflow (Q) observations for
this gauge during the period Oct 1952~ Sept 1988 are obtained for the experimental design. The available data is split into two data sets, with one for calibration purpose (Oct 1952~Sept 1977) and the remaining for validation purpose (Oct 1977~Sept 1988).

At daily time scales, observations for daily Precipitation (P) and daily Potential Evapotranspiration (PET) data for this basin were obtained from the NC CRONOS Database. Observations of daily streamflow (Q) were obtained from USGS daily streamflow record. The daily data is also divided into two periods, with calibration and validation period spanning from 10/1/1983~9/30/1993 and 10/1/1993~12/31/1998 respectively.

3.4 Results and discussion

Based on the experimental design presented in section 3.3, the hydrologic models are calibrated against synthetically generated streamflows to get the optimal model parameters, which will be used in the validation period (Oct 1977~Sept 1988) and streamflow was estimated for the validation period from the individual model predictions, multimodel prediction \( MM-1 \), as well as using EnKF algorithm. Each of these predictions was evaluated using different verification measures (RMSE and Correlation) for further analysis. The same procedure of the experimental design was implemented at daily scale using daily data, and RMSE and Correlation were calculated for individual model predictions, multimodel combination (\( MM-1 \)) and EnKF algorithm.

We repeat the experimental design for 100 such validation trials which results in the 100 values of RMSE and Correlation. For the entire analysis, we calculate the median of the
RMSE to show the importance of multimodel combination and EnKF algorithm. It is clearly shown in Chapter 2 that ‘abcd-OLS’ performs better than the rest of individual models under most of the conditions, thus we only apply EnKF algorithm on ‘abcd-OLS’ in the experimental design for ‘abcd’ model as the ‘true’ model.

3.4.1 Uncertainty reduction at monthly time scale

In order to get the synthetic streamflow, we use a initial set of ‘known’ model parameters \( \theta \in (a_0, b_0, c_0, d_0) \). Random noise which follows normal distribution is added to the synthetic flow estimated by the true model based on the parameter set ‘\( \theta \)’. The random noise with zero mean and variance equal to \( f_2\sigma \) is generated, with the inflation factor \( f_2 \) corresponding to different level of model uncertainty. For model calibration, the sum of squared errors (SSE) as defined in Chapter 2 is used to calibrate the model for each given scenario of \( f_2 \).

The multimodel prediction MM-1 is developed following the algorithm described in Chapter 2. The EnKF is implemented for ‘abcd’ model according to the procedure described in Figure 3.1 and we denote this new model prediction as ‘EnKF-abcd’. Following the procedures described in the experimental design, a total of 100 sets of model calibration and validation are performed and the results for the validation period are evaluated. From the performance measures available from 100 validation runs, the Median Root Mean Square Error (RMSE) for individual models, MM-1 and EnKF-abcd were calculated (Figure 3.2).

From Figure 3.2, it is obvious that, as the error \( (f_2) \) increases, the EnKF algorithm is able to improve the streamflow prediction from the true model ‘abcd-OLS’ model. The Root Mean
Square Error (RMSE) for ‘EnKF-abcd’ is reduced and is smaller than the ‘true’ model and the rest of the single models. This indicates that by assimilating the available observation information, the initial model state can be estimated more precisely resulting in improved predictions. However, when compared with mutlimodel MM-1, the RMSE of ‘EnKF-abcd’ is higher than ‘MM-1’ because ‘MM-1’ has the advantage of optimally combining all the participating single models (‘abcd-OLS’, ‘abcd-ABS’, ‘VIC-OLS’ and ‘VIC-ABS’) such that attributes of different single models can be optimally extracted and evaluated, as we see in the median weight plot in Chapter 2.

![Figure 3.2: Comparison of the performance of different models under different heteroscedastic error variance – Median RMSE of four candidate models, multimodel (MM-1) and EnKF model (EnKF-abcd) with ‘abcd’ as true model at monthly scale.](image)

Figure 3.2: Comparison of the performance of different models under different heteroscedastic error variance – Median RMSE of four candidate models, multimodel (MM-1) and EnKF model (EnKF-abcd) with ‘abcd’ as true model at monthly scale.
In order to quantify the performance of the single models, ‘MM-1’ and ‘EnKF-abcd’ under different flow conditions, median RMSE under six flow profiles were computed: <10\textsuperscript{th} percentile, 10\textsuperscript{th} ~< 25\textsuperscript{th} percentile, 25\textsuperscript{th} ~< 50\textsuperscript{th} percentile, 50\textsuperscript{th} ~< 75\textsuperscript{th} percentile, 75\textsuperscript{th} ~< 90\textsuperscript{th} percentile, >90\textsuperscript{th} percentile.
Figure 3.3: Median RMSE of four candidate models, multimodel (MM-1) and EnKF model (EnKF-abcd) under six different flow conditions with true model being ‘abcd’ having heteroscedastic errors – (a) <10th percentile of flow (b) 10th -25th percentile (c) 25th-50th percentile (d) 50th -75th percentile (e) 75th-90th percentile (f) >90th percentile of flow at monthly scale.
Figure 3.3 shows the result of the model performance under these flow profiles. It indicates that the multimodel ‘MM-1’ performs better than all individual models under low (<10\textsuperscript{th} percentile) and high flows (>90\textsuperscript{th} percentile). Moreover, the EnKF model ‘EnKF-abcd’ obtained from the ‘true’ model ‘abcd-OLS’ clearly improves the skill of predictions, thereby having smaller median RMSE. However, when comparing ‘MM-1’ and ‘EnKF-abcd’ in Figure 3.3, again, we see that the multimodel ‘MM-1’ outperforms ‘EnKF-abcd’ over almost all of the flow profiles and error variance scenarios defined, which indicates that at monthly time scale, the Multimodel algorithm has better capability to reduce uncertainty in streamflow predictions in comparison to the EnKF algorithm. This is primarily due to the reason that at monthly time scale, model uncertainty reduction is more critical in comparison to the estimation of initial conditions. Therefore, the multimodel algorithm performs better than the EnKF algorithm since it reduces the model uncertainty substantially by combining all participating models under large model error variance.

### 3.4.2 Uncertainty reduction at daily time scale

The synthetic streamflow was also generated using the experimental design at daily time scale using the time series of P and PET. Results from the 100 simulations of the experimental design for mutlimodel algorithm and EnKF algorithm are presented in Figures 3.4 and 3.5.
Figure 3.4: Comparison of the performance of different models under different heteroscedastic error variance – Median RMSE of four candidate models, multimodel (MM-1) and EnKF model (EnKF-abcd) with ‘abcd’ as the true model at daily time scale.

Figure 3.4 shows that at daily time scale, ‘EnKF-abcd’ performs better than the ‘true’ model ‘abcd-OLS’ model and two other single models in terms of RMSE under almost all the error variance scenarios considered. The comparison between multimodel ‘MM-1’ and single models is similar, but the improvement from ‘MM-1’ over the single model is not as much as we reported from the monthly model. Moreover, comparing ‘EnKF-abcd’ with ‘MM-1’, the ‘EnKF-abcd’ outperforms the multimodel ‘MM-1’ in terms of median RMSE. Since ‘EnKF-abcd’ is able to better estimate the initial states, which plays an important role at daily time scales, we see EnKF-abcd performing better under all model error variance. In other words,
updating the model states at daily time scale using observed information is more important in comparison to reducing model uncertainty, since the flows are primarily determined by storage and initial state conditions. However, this does not imply that model uncertainty is not important at all at daily time scale. If the model uncertainty is really large, no matter how good the estimation of the initial state conditions are, the model predictions will not estimate the observation well. This is why we choose the model ‘abcd-OLS’ (since the flaws are known to be from ‘abcd’ model) to update its initial state using EnKF algorithm. We can conclude that by recursively updating the state variable of the best performing individual model, EnKF reduces the uncertainty better at daily time scale compared to the uncertainty reduction that could be obtained with the multimodel algorithm.

The performance of these models under various flow profiles has also been investigated based on the median RMSE of each model under daily time scale (Figure 3.5). It is shown that in general, the model ‘EnKF-abcd’ is superior to the individual models and the multimodel ‘MM-1’ under six different flow conditions. While in the meantime the multi-model ‘MM-1’ is better than the individual models in the case of large error variance \( f_2\sigma \) but cannot perform better than the ‘EnKF-abcd’ model.
Figure 3.5: Median RMSE of four candidate models, multimodel (MM-1) and EnKF model (EnKF-abcd) under six different flow conditions with true model being ‘abcd’ having heteroscedastic errors – (a) <10th percentile of flow (b) 10th-25th percentile (c) 25th-50th percentile (d) 50th-75th percentile (e) 75th-90th percentile (f) >90th percentile of flow at daily scale.
3.4.3 Discussion on model uncertainty and initial condition uncertainty

As demonstrated in section 3.4.1 and 3.4.2, at monthly time scale, the multimodel algorithm performs better than the EnKF algorithm with a smaller median RMSE under various flow conditions. This implies that addressing model uncertainty is more critical at monthly time scales. Multimodel algorithm can reduce the model uncertainty by optimally assigning a higher weight to other models that have small model uncertainty so that the adverse effect of this poorly performing model will be compensated by better performing models. In the meantime, EnKF algorithm is mainly aimed to reduce the uncertainty arising from initial conditions of the model. However, at monthly time scale, the effect of initial condition uncertainty will be dampened by the variability in forcing and by the differences across the models. This emphasizes the importance of addressing model uncertainty at monthly time scale.

On the other hand, we infer that at daily time scale, the EnKF algorithm is outperforming multimodel algorithm, its median RMSE is smaller than the multimodel MM-1 under all the flow profiles. This is mainly due to the ability of EnKF algorithm in updating the initial model states using recent observations. Since all models perform poorly under large model errors, applying EnKF to the best performing individual model results in reduced uncertainty in comparison to the uncertainty reduction that could be obtained with the multimodel combination. This implies that uncertainty from the initial state condition is dominant at the daily time scale. Thus, we can conclude that model structure uncertainty is more important
for hydrologic modeling at monthly time scales, while uncertainty in initial state condition plays a dominant role for hydrologic modeling at daily time scales.

3.5 Summary

Estimating streamflow using a hydrologic model includes many uncertainties related to model structure, initial states, input and output. Many modeling efforts have focused on how to reduce or eliminate these uncertainty sources so that better predictions can be made. Among those various research efforts, two algorithms that are distinct but mutually different are multimodel combination and ensemble kalman filter. Multimodel combination algorithm tries to address model structural uncertainty through optimally combining individual models. Data assimilation aims to update the initial states of the model by utilizing the available observation information, thus better prediction from the model can be developed.

In this Chapter, we implemented the multimodel algorithm, and EnKF algorithm through an experimental design to reduce the uncertainty in streamflow prediction at different temporal scales. The setup of this experimental design is based on a synthetic streamflow generation scheme, where the model structural uncertainty and ‘true’ parameters are assumed to be ‘known’. Both multimodel algorithm and EnKF algorithm were compared in improving streamflow prediction under monthly and daily time scale. The comparison reveals that, on a monthly time scale, the multimodel algorithm can give better performance than the EnKF algorithm while on a daily time scale, the EnKF algorithm is better in predicting ‘true’ streamflow than multimodel algorithm. The underlying rational of this performance difference is that different source of uncertainty are dominating the modeling at different
time scales. Under monthly time scale, the model structural uncertainty plays a dominant role, which could be addressed by employing multimodel algorithm. While under daily time scale, the initial state condition uncertainty is playing an important role, and EnKF algorithm greatly improves the estimation of the initial states of the model, thus resulting in improved predictions.

From the uncertainty reduction point of view, this study addresses the issues related to several aspects:

1. It investigates the role of multimodel combination and data assimilation (primarily EnKF algorithm) in improving streamflow prediction at monthly and daily time scale.

2. It addresses the uncertainty issues related to input, model structural and initial model state in hydrological modeling.

3. It demonstrates that role of different uncertainty through an experimental design and a watershed application.
CHAPTER 4: WATERSHED APPLICATIONS FOR MULTIMODEL COMBINATION ALGORITHM AND ENSEMBLE KALMAN FILTER ALGORITHM

4.1 Introduction

In Chapter 2 and 3, we evaluated the performance of multimodel combination and EnKF in uncertainty reduction related to model structural limitations and initial conditions through a synthetic experiment. Results from Chapter 2 show that multimodel $MM-1$ improves streamflow prediction especially under large model uncertainty. In Chapter 3, EnKF algorithm has been demonstrated to be useful in reducing uncertainties related to initial states of the model. Furthermore, by comparing multimodel combination and EnKF algorithm, Chapter 3 demonstrated that reducing model uncertainty is more important at monthly time scale whereas uncertainty reduction related to initial states is critical at daily time scale. In this Chapter, we will implement multimodel combination and EnKF over one watershed at monthly and daily time scales to verify their effectiveness in improving streamflow prediction. In addition, an approach that combines EnKF and global optimization for parameter estimation is evaluated for better representing the uncertainties related to parameter estimation and initial conditions.

4.2 Application to watersheds: Multimodel combination algorithm

For application of the proposed multimodel combination techniques, we consider four basins with two humid basins from North Carolina and two semi-arid basins from Arizona as our study region. Selected four basins are: Tar River at Tarboro (HCDN ID: 02083500), NC and
Haw River near Benaja, NC (HCDN ID: 02093500), San Pedro River at Charleston, AZ (HCDN ID: 09471000) and Salt River near Roosevelt, AZ (HCDN ID: 09498500). All these basins are from HCDN database. Detailed physical characteristics of these four basins as well as the length of the study period are summarized in Table 4.1. Monthly time series of precipitation, PET and streamflow are obtained from the national climatic database developed by Sankarasubramanian and Vogel [2005].

Table 4.1: Physical characteristics of different selected basins

<table>
<thead>
<tr>
<th>Basin Characteristics</th>
<th>Basin names</th>
<th>Tar river</th>
<th>Haw river</th>
<th>San Pedro river</th>
<th>Salt river</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>Tarboro, NC</td>
<td>Near Benaja, NC</td>
<td>Charleston, AZ</td>
<td>Near Roosevelt, AZ</td>
<td></td>
</tr>
<tr>
<td>Area (Km²)</td>
<td>5588.5</td>
<td>430.08</td>
<td>3120.64</td>
<td>11023.36</td>
<td></td>
</tr>
<tr>
<td>Annual Precipitation (mm)</td>
<td>1117.6</td>
<td>1066.8</td>
<td>419.1</td>
<td>558.8</td>
<td></td>
</tr>
</tbody>
</table>

We calibrate ‘abcd’ model and ‘VIC’ model based on two objective functions that minimize SAE and SSE over the calibration period. Based on the calibrated parameters, we obtain individual model predictions over the validation period. Following that, we apply two multimodel combination algorithms (MM-1 & MM-O) discussed in Section 2.3 for these four basins and compare their performance with single model predictions developed for these basins. Under MM-1, given the precipitation for a month in the validation period, MM-1 obtains for ‘K’ neighbors for the precipitation and obtains mean absolute error (MAE) from
‘k’ neighbors for each model over the calibration period. The MAE ($\epsilon_i^m$) for each model is converted into weights $w_{i,k}^m$ (see Figure 2.1) to obtain the multimodel predictions, $MM-I$, over the validation period. For $MM-O$, the model weights, $w_i^m$, are obtained based on the model performance over the entire calibration period. We calculate correlation and RMSE (Figure 4.2) from all the four single model prediction $Q_i^m$ and two multimodel predictions ($Q_i^{MM-I}$, $Q_i^{MM-O}$) over the validation period.
Figure 4.1: Performance of multimodel schemes (MM-I and MM-O), in terms of (a) correlation and (b) RMSE, in comparison to individual models under validation.

From Figure 4.1, we can see that the correlation between the multimodel (MM-I) predicted streamflow and observed streamflow shows significant improvement from all the single model predictions across all the four basins. On the other hand, correlation of MM-O is just
around the same level with the best single model. This is probably due to MM-O’s weighting scheme, which gives higher weights for the best single model over the entire calibration period. Further, the RMSE of MM-I is also lower than four single models and MM-O for all the four basins during validation period. The primary reason behind MM-I’s better performance is due to its weighting scheme which evaluates the performance of candidate models conditioned on the input state, thereby giving higher weights for the model that performs well under similar input conditions.

We also evaluate the ability of individual models and multimodels (MM-I and MM-O) in predicting the seasonality of streamflow (Figure 4.2). To compare across months, we consider the Relative-RMSE, which is calculated as

\[
\text{Relative RMSE}_j^m = \sqrt{\frac{1}{n_y} \sum_{i=1}^{n_y} \left( \frac{\hat{Q}_{j,i}^m - Q_{j,i}}{Q_{j,i}} \right)^2},
\]

where \( \hat{Q}_{j,i}^m \) and \( Q_{j,i} \) indicate the single model ‘m’ predicted flows and observed flows for month ‘j’ in year \( i \). Thus, the squared relative errors in predicting the monthly flows are averaged over the respective month in the validation period to compute the relative-RMSE. Similarly, relative-RMSE for the multimodels, MM-I and MM-O, are also computed.
Figure 4.2: Performance of multimodel schemes (MM-1 and MM-O), expressed in terms of Relative-RMSE, in comparison to individual models for different months: (a) Tar river; b) Haw river; c) San Pedro river; d) Salt river.

The primary Y-axis in Figure 4.2 shows the relative-RMSE for the four single models and multimodels and the secondary Y-axis indicates the mean monthly streamflow. From Figure 4.2, we can clearly see that the multimodel MM-1 performs better than the individual models and multimodel MM-O for high flow months for each station. However, during the respective low flow months, multimodel MM-1 does not perform better than the candidate single
models or $MM-O$. This could be due to inaccurate identification of neighbors ($K$) for the given conditioning state. During low flow months, the precipitation is low and the runoff from the watershed could be dependent more on the available energy. Thus, it may be appropriate to identify neighbors based on potential evapotranspiration (PET) as well. The higher relative-RMSE is also occurring due to very small values of flows under which a small difference in the prediction could be amplified due to the lower values of the flows. But, it is very clear from Figures 4.1 and 4.2, $MM-I$ performs better than the individual models and $MM-I$ over the entire validation period (Figure 4.1) as well as in predicting the high flow months (Figure 4.2). In our future studies, we plan to investigate the role of PET as a conditioning variable in our future studies by considering basins exclusively from the arid regions. Though the proposed algorithm was demonstrated for lumped models, we expect similar behavior upon application of the model for distributed models. We intend to investigate this as part of future study.

4.3 Application to watershed: Ensemble Kalman Filter

The experimental design in section 3.4 demonstrates that multimodel algorithm and EnKF algorithm have their own strength in reducing uncertainty at different time scales. In this section, we investigate their ability to improve streamflow prediction at Tar River basin, North Carolina. For multimodel algorithm, we develop multimodel ‘MM-1’ prediction from ‘abcd’ model and ‘VIC’ model following the procedure detailed in Chapter 2. The EnKF prediction ‘EnKF-abcd’ is developed using the calibrated parameters by minimizing the Sum Square Errors (SSE) over the calibration period. Figure 4.3 compares the performance of ‘abcd’ model, EnKF-abcd and MM-1 in predicting streamflow for the Tar River basin during
the validation period. We can clearly see that both ‘MM-1’ and ‘EnKF-abcd’ are performing better than the best individual model ‘abcd-OLS’. In addition, when comparing the performance between ‘MM-1’ and ‘EnKF-abcd’, Figure 4.3 indicates that multimodel ‘MM-1’ is better at monthly time scale while EnKF model ‘EnKF-abcd’ is better at daily time scale, which is consistent with the synthetic streamflow analysis presented in chapter 2.

Figure 4.3: Performance of best single model ‘abcd-OLS’, multimodel ‘MM-1’ and EnKF model ‘EnKF-abcd’ at tar river basin at Tarboro: (a) RMSE and (b) Correlation at monthly time scale; (c) RMSE and (d) Correlation at daily time scale.
Figure 4.4 further examines the performance of these models in predicting various flow profiles for Tar River at Tarboro. We can see that, overall, ‘MM-1’ and ‘EnKF-abcd’ can improve the streamflow predictions with the RMSE being reduced for the validation period both at monthly and daily time scales. However, ‘MM-1’ and ‘EnKF-abcd’ perform better at monthly and daily time scale respectively.
Figure 4.4: RMSE of the best single model ‘abcd-OLS’, multimodel ‘MM-1’ and EnKF model ‘EnKF-abcd’ at tar river basin at tarboro under six different flow conditions: <10th percentile of flow; 10th-25th percentile; 25th-50th percentile; 50th-75th percentile; 75th-90th percentile; >90th percentile of flow at (a) monthly and (b) daily scale.
Based on the synthetic experiment and the application to Tar River basin, we find that multimodel combination and data assimilation have their own respective time scales in reducing uncertainties related to model structure and initial model state. At monthly time scale, reducing model structural uncertainty is more important, while reducing uncertainty in initial state conditions is more significant at daily time scale.

### 4.4 Application to watershed: Ensemble Kalman Filter with global parameter estimation

Traditionally, hydrologic modelling assumes that the parameters of the model are fixed and usually obtained by calibration. However, this approach fails to consider the uncertainty related to parameter estimation. In this section, we incorporate parameter uncertainty within EnKF for improving streamflow prediction at daily time scale. To reduce parameter uncertainty, a global optimum of the parameter set or its posterior distribution should be indentified properly. Shuffled Complex Evolution Metropolis (SCEM-UA) global optimization algorithm is computational simple and efficient to obtain the posterior distribution of the parameter set. In addition, Ensemble Kalman Filter (EnKF) algorithm is also is included to address the uncertainty in model initial condition. In this application, we consider both EnKF algorithm and SCEM-UA for improving streamflow prediction for Tar River basin at Tarboro at monthly and daily time scale.
4.4.1 Results and discussion

Figure 4.5 shows the evolution of the parameter sets within different complex, which is denoted by different colors. We can see that at the parameter sets tend to converge with iteration goes on and they are finally converge to a small region.

![Figure 4.5](image)

Figure 4.5: Parameter evolution of EnKF combined with global parameter estimation at (a) monthly and (b) daily evaluation for Tar River at Tarboro, North Carolina.

Figure 4.6 gives the Gelman-Rubin convergence test statistic for all the iterations. The dash line in the figure is the threshold of the test statistic, which indicates that any number smaller than the threshold means convergence to a posterior distribution. From this figure, we can clearly see that at the very first several iterations, since the parameters are sampled from a uniform distribution, the test statistics are smaller than the threshold. As the evolution of the
parameters proceed, the test statistics finally drop below the threshold after many number of iterations for all the four parameters, indicating convergence to their posterior distribution.

Figure 4.6: Convergence test statistic evolution for the parameters at (a) monthly and (b) daily evaluation for the ‘abcd’ model.
Figure 4.7: Posterior distribution of the parameters at (a) monthly and (b) daily evaluation for the ‘abcd’ model.
Figure 4.7 shows the posterior distribution for the parameters after they converge at (a) monthly and (b) daily time scale. In Figure 4.7(a), it is clear that the parameter ‘a’ converges to a relatively small range, while the other three parameters ‘b’, ‘c’ and ‘d’ have a relatively large range. In Figure 4.7(b), we can see that except for parameter ‘a’, the rest three parameters ‘b’, ‘c’ and ‘d’ converge to a very different region at daily time scale compared to the monthly time scale.

Figure 4.8: RMSE and Correlation in predicting streamflow at daily time scale for Tar River at Tarboro over the validation period (Oct 1\textsuperscript{st} 1993~Dec 31\textsuperscript{st} 1998).

From Figure 4.8, we can understand that EnKF algorithm improves streamflow prediction with initial condition uncertainty reduced from ‘abcd’ model at daily time scale. Furthermore, it is clear that the approach combining EnKF with global parameter estimation
can result in improved streamflow prediction. This strengthens that reducing parameter uncertainty improves streamflow prediction.

4.5 Summary

Hydrologic modelling is associated with many sources of uncertainty that arises from model and their parameters as well as the initial conditions of the model. Hence, different algorithms that can reduce these uncertainty sources at various time scales started to gain attention. Among those, in Chapters 2 and 3, we have demonstrated the utility of multimodel combination and EnKF in improving hydrologic prediction by reducing uncertainty related to model and initial conditions through an experimental design.

In this chapter, these two algorithms are implemented over four river basins with varied hydroclimatic settings. Results from the application confirm the findings from Chapters 2 and 3. In addition, to incorporate uncertainty arising from parameter estimation, we implemented the EnKF combined with global parameter estimation for streamflow prediction at Tar River basin at monthly and daily time scale. Results show that the combined approach is able to further improve the streamflow prediction by reducing uncertainty related to parameters and initial conditions. Further, it also provides a confidence interval of the streamflow prediction.
CHAPTER 5: UTILITY OF CLIMATE FORECASTS IN PROMOTING INTER-BASIN TRANSFER IN THE NORTH CAROLINA TRIANGLE AREA

5.1 Introduction

Regional water management essentially depends on water availability (supply) and water usage (demand) over the region. Typically, over seasonal to inter-annual time scales, the water demand does not vary. Thus, uncertainty in regional water supply system essentially depends on inflow potential and initial storage conditions for the prescribed seasonal demand. It is well known that seasonal inflow over a region is partly influenced by variations in climatic conditions such as sea surface temperatures (SST) [Sankarasubramanian et al., 2009; Golembesky et al., 2009]. Hence, considerable efforts have focused on improving the skill of seasonal climate forecast [Maurer and Lettenmaier, 2004; Piechota et al., 2001; Sicard et al., 2002]. Recent studies [Yao and Georgakakos, 2001; Hamlet et al., 2002; Georgakakos and Graham, 2008; Golembesky et al., 2009] have also shown that seasonal streamflow forecasts could be effectively utilized for improving regional water management.

Typically, reservoir management follows an operational rule curve that determines the releases to multiple water users based on the initial storage and the expected inflows. Though, seasonal ahead inflow forecasts derived from climate information have been proven to be useful [Sankarasubramanian, et al., 2009; Golembesky et al., 2009] to improve adaptive water management. As opposed to use expected inflows (climatology), the utility of climate forecast in reservoir management is very sensitive to prediction skill levels, system characteristics and management objectives [Georgakakos and Graham, 2008]. For instance,
for a large reservoir system, forecasts are not beneficial since the demand could be met purely based on the initial storage. Thus, within-year reservoirs, which are typically smaller storage systems, depend heavily on uncertain information (inflow) and the initial storage.

In the reservoir management literature, most of the studies [VanRheenen et al., 2004; Tanaka et al., 2006; Draper and Lund, 2004] examined the release policy of a single reservoir under perfect inflow and climate/population change scenarios using simulation or optimization models. However, assuming perfect inflow information fails to incorporate the uncertainty in the inflow information. Another way to improve regional water management is to use inter-basin transfer. In this chapter, we investigate the role of climate forecasts in promoting inter-basin transfer over the triangle area of North Carolina. This chapter focuses on three aspects related to inter-basin transfer (IBT): (a) Role of inflow forecasts skill in promoting IBT; (b) Importance of spatial correlation between inflows and (c) Role of spatial correlation between the initial storages of regional systems.

This chapter is organized as follows: Section 5.2 introduces the study area in North Carolina Triangle area and investigates its current condition in water management. Section 5.3 proposes an IBT optimization model and demonstrates the utility of climate forecast in promoting IBT in the study area. Section 5.4 investigates the role of inter-basin transfer in reducing the uncertainty in meeting target storage conditional on synthetic streamflow forecasts. Section 5.5 discusses the results obtained from the synthetic study to understand the role of IBT under various settings. Section 5.6 summarizes the findings from this study.
5.2 Study Region and Data Sources

5.2.1 Basin information

The focus area of the study is the research triangle region of North Carolina. The municipalities are primarily served by two major water supply systems: Falls Lake reservoir in the Neuse River basin, is the primary source for city of Raleigh, with a consumption of 70 MGD (million gallon per day); Jordan lake reservoir, in the Cape Fear River basin, supplies water for Cary/Apex, with a consumption of 16 MGD. Apart from these two systems, Lake Mitchile and Little River Reservoir in the Neuse River basin also have a small portion of water allocated to Durham. Figure 5.1 summarizes the water supply and demand nodes in the triangle area.

Figure 5.1: Water availability and demand for multiple users in the area. Note: WS in this figure means water supply release and WQ means water quality release.
Apart from supplying water for the municipalities, both reservoirs also have to maintain certain amount of water quality releases to sustain the health of downstream ecology. The mandated water quality release during the summer (July-August-September) season is 255 cfs (cubic feet per second) for Falls Lake and 600 cfs for Jordan Lake. In order to manage the reservoir for serving water supply and water quality purposes, the US Army Crop of Engineers have two storage accounting units within the conservation pool of the two reservoirs (Table 5.1). Both the inflows and evaporation of the lakes are also apportioned in the same ratio as specified for water supply and water quality pools in both systems (Table 5.1).

Table 5.1: Water supply and water quality ratio within conservation storage of Falls Lake and Jordan Lake reservoir

<table>
<thead>
<tr>
<th></th>
<th>Storages (acre feet)</th>
<th>sedimentation storage</th>
<th>Conservation storage</th>
<th>Controlled flood storage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Water supply</td>
<td>Water quality</td>
</tr>
<tr>
<td>Falls Lake</td>
<td>25073</td>
<td>45000 (42.3%)</td>
<td>61322 (57.7%)</td>
<td>221182</td>
</tr>
<tr>
<td>Jordan Lake</td>
<td>74730</td>
<td>45800 (32.62%)</td>
<td>94600 (67.38%)</td>
<td>538400</td>
</tr>
</tbody>
</table>

5.2.2 Potential for IBT

Given that both reservoirs are within year systems, the storage in each pool needs to be at the maximum conservation storage by July 1st. Any deviation above that storage will increase the flood risk and deviations below than the target storage will increase the failure to supply the summer demand. Historical storages during JAS for both systems indicate that water supply pool within Falls Lake is always stressed in meeting the target demand (Figure 5.2). Figure 5.2 shows the water availability to demand ratio as $\psi = \frac{S_0 + I_t}{O_t}$, where $S_0$ denotes the
fraction of storage for water supply/quality in the beginning season, $I$, denotes the fraction of inflow that comes into the reservoir for water supply/quality purpose at current season $t$, $O_t$ denotes the water supply/quality release at current season $t$.

Figure 5.2: Water supply and water quality ratio for Falls Lake and Jordan Lake reservoir.
Figure 5.2 compares the water supply/quality availability to demand ratio for the two reservoirs in the summer season. From Figure 5.2, we can infer that water availability for water supply purpose in Jordan Lake is abundant compared to that of Falls Lake, indicating a potential for inter-basin transfer. On the other hand, water availability for water quality purpose is similar for both systems. The intent of this research is to initiate IBT across the four pools so that the total deviation from the end of the season target storage across the four systems could be minimized conditional on the climate information based seasonal forecasts.

5.2.3 Data Sources and inflow forecast development

Historical inflow, outflow, storage information of the reservoirs from the US Army Corps of Engineers, Wilmington district were collected for Falls Lake and Jordan Lake reservoir. Precipitation forecasts from ECHAM4.5 General Circulation Model (GCM) forced with constructed analogue SSTs (http://iridl.ldeo.columbia.edu/SOURCES/.IRI/.FD/.ECHAM4p5/.Forecast/.ca_sst/) were obtained from the IRI data library. Principal component of the JAS inflow into both lakes explained 86% of the variance in inflows. The first principal component was correlated with JFM precipitation forecasts (issued in July 1st). We downscaled precipitation forecasts from selected grid points to develop streamflow forecasts for both lakes using Model Output Statistics (MOS). It is otherwise known as Principal Component Regression (PCR), which is a statistical downscaling technique, is applied to develop principal component regression relationships between the gridded GCM precipitation and observed streamflow. Using the point forecast error and the conditional mean obtained from the regression, we develop ensembles of streamflow forecasts based on leave-one-out cross validation. As shown in
Figure 5.3, the correlation between the conditional mean of 1000 ensemble streamflow forecasts and the historical observation in the summer season was found to be statistically significant with 0.55 and 0.54 for the Falls Lake (Figure 5.3a) and Jordan Lake (Figure 5.3b) respectively. We employed these streamflow/inflow forecasts and forced with inter-basin transfer model to estimate the water allocation.

![Figure 5.3a: Falls Lake JAS inflow forecast](image)

**Figure 5.3a:** Falls Lake JAS inflow forecast with observed data and forecasted inflows for the years 1991 to 2008. The correlation between the conditional mean and observed data is 0.55.

![Figure 5.3b: Jordan Lake JAS inflow forecast](image)

**Figure 5.3b:** Jordan Lake JAS inflow forecast with observed data and forecasted inflows for the years 1991 to 2008. The correlation between the conditional mean and observed data is 0.54.

Figure 5.3: ECHAM 4.5 inflow forecasts for Falls Lake and Jordan Lake reservoir in summer season during 1991–2008.
5.3 Inter-basin transfer and water allocation model: formulation

Typically, the distribution of water availability and demand is uneven in space and time [Gupta et al., 2007] due to climatic variations as well as due to the intended purpose of the system. In this study, we present an inter-basin transfer model for regional water management contingent on climate information based streamflow forecasts.

5.3.1 Inter-basin transfer: Optimization model

Basically, for a single reservoir, the mass balance equation for simulation can be described as equation (1):

\[ S_t = S_{t-1} + I_t - R_t - E_t - SP_t + Def_t \] ...........(1)

where, \( S_t \) is the end of the time step storage we are interested in; \( S_{t-1} \) is beginning of the time step storage, which is usually known at the time of allocation. \( I_t \) is the total inflow into the reservoir during time step t; \( R_t \) is the required release for all the users; \( E_t \) is the evaporation; \( SP_t \) is the spill during t if \( S_t > S_{max} \); \( Def_t \) is the deficit of water if \( S_t < S_{min} \). All the variables in equation (1) represent volume of water (acre feet).

The proposed inter-basin water allocation model considers a total of ‘n’ pools represented by storage over various basins. Figure 5.4 shows the structure of the proposed inter-basin transfer within four pools in a given region.
The objective is to minimize the expected deviation between forecasted end of season storage ($S_i$) and target storage ($S_i^*$) across ‘n’ reservoir/pools, as equation (2) shows.

$$E[\sum_{i=1}^{n} |S_i - S_i^* / S_i^*|]$$

..........(2)

$$S_{i,j} = S_i^0 + I_{i,j} - E_{ij} - (R_i + \sum_{k=1}^{n} \sum_{j=k+1}^{n} x_{jk})$$

..........(3)

$$S_{i}^{\text{min}} \leq S_{i,j} \leq S_{i}^{\text{max}}, \quad i = 1, 2, \ldots, n$$

..........(4)

In which

$i = 1, 2, \ldots, n$ denotes the corresponding single reservoir/pool participating IBT.

$j = 1, 2, \ldots, m$ denotes the forecast inflow ensembles.
\[ k = 1, 2, ..., f(n) \] denotes the transfer variables in Figure 5.1 among the four pools, it is a function of \( n \).

\( S_{i,j} \) is the end of season storage at reservoir \( i \) for ensemble \( j \).

\( S_i^0 \) is the beginning of season storage at reservoir \( i \).

\( I_{i,j} \) denotes the ensemble inflow \( j \) for reservoir \( i \).

\( E_i \) is the evaporation from reservoir.

\( R_i \) is the required water supply release from reservoir \( i \).

\( S_i^{\text{min}} \) is the minimum storage for reservoir \( i \).

\( S_i^{\text{max}} \) is the maximum storage for reservoir \( i \).

\( x_{ik} \) is the transfer water between user \( k \) and rest of the users.

Equation (3) shows that each of the reservoirs has to meet the water balance constraint. It is also constrained that the end of season storage for each of the reservoirs should be between its minimum and maximum level, expressed as equation (4).

All the constraints in the above model are linear, except for the objective function, by introducing two surrogate variables \( S_{i,j}^+ \) and \( S_{i,j}^- \), the above model can be easily linearized so that the model becomes a linear programming (LP) model. Reformulating the above model, we get:

Objective function:
Min \[ E \left[ \sum_{i=1}^{n} \left( \frac{S_{i,j}^+ + S_{i,j}^-}{S_i^+} \right) \right] \] ............(5)

Subject to: \[ S_{i,j}^+ - S_{i,j}^- = S_{i,j} - S_i^+ \] ............(6)
\[ S_{i,j}^+ \geq 0, \ S_{i,j}^- \geq 0 \] ............(7)

and constraints (2)~(4) from the original model.

The two new surrogate variables \( S_i^+ \) and \( S_i^- \) are defined as follows:

\( S_{i,j}^+ \) denotes the excess storage above target storage \( (S_i^+) \) for reservoir \( i \) corresponding to ensemble \( j \);

\( S_{i,j}^- \) denotes the shortfall storage below target storage \( (S_i^+) \) for reservoir \( i \) corresponding to ensemble \( j \);

The above LP model was solved in the IBM ILOG CPLEX optimization studio to obtain different transfer scenarios.

5.3.2 Inter-basin transfer based on climate information: application

Based on the framework presented in this section, the inter-basin transfer scheme for the study area is evaluated. From figure 5.4, six potential transfers are possible among the four pools (FL-WS: Falls Lake water supply; FL-WQ: Falls Lake water quality; JL-WS: Jordan Lake water supply; JL-WQ: Jordan Lake water quality). For each of the participating pool, reservoir model discussed in Section 5.3 is developed for promoting inter-basin transfer. The goal is to minimize the expected end of season storage deviation across the four pools, as show in equation (2), subject to constraints equation (3) and (4).
Historical years from 1991 to 2008 were used to perform retrospective analysis using the IBT optimization model. We consider three different inflow scenarios (Climatology, ECHAM 4.5 forecast and observed inflow) for obtaining the transfer:

*Perfect forecast:* Force the IBT model with observed inflow, and obtain the optimal transfer.

*Retrospective ECHAM 4.5 forecast:* Force the IBT model with downscaled inflow from ECHAM 4.5, and obtain the optimal transfers which denotes the transfer based on existing skill in predicting streamflow for the region.

*No forecast (climatology):* Force the model with climatological inflows, and obtain the optimal transfer.

We evaluate the optimal transfers under these three inflow scenarios by combing the transfer and releases with observed inflows to obtain the end of season storages for each pool over the period 1991–2008. The result is calculated as the deviation index $\lambda_{i,j} = \frac{S_{i,j} - \hat{S}_i}{\hat{S}_i}$ for each pool $i$. 

Figure 5.5: Deviation index for individual pools in Falls Lake and Jordan Lake reservoir for inter-basin transfer promoted by three inflow forecasting schemes (b~d). For comparison, we provide the deviation index under no transfer scheme (a). Note: the plus (+) sign in the figure is counted as an outlier. It actually represents the deviation index for a particular year, since the study period is relatively short (1991~2008), we also include these years for analysis.

Figure 5.5 shows the boxplot of the target storage deviation index for each pool during the period 1991~2008 under three inflow information. Each box represents the 10th, 25th, 50th, 75th and 90th percentile of the deviation index. We also include a baseline scenario (Figure 5.5a), which indicates the deviation index obtained through observed information without any transfer. Comparing figure 5.5(a) with the rest, we can see clearly that the end of season storage deviation is decreased under IBT. Furthermore, looking at figures 5.5(b), 5.5(c) and
5.5(d), it is obvious that uncertainty in the inflow information plays an important role in determining the utility of optimal transfer. For optimal transfers obtained using climatological inflow, which has no skill, the target storage deviation is reduced for the water supply pools from the no transfer scenario, but not significant. With ECHAM 4.5 forecast, the resulting deviation index is further reduced compared with the climatology scenario and the no transfer scenario. Finally, the deviation index is the least across the four pools under perfect inflow information, indicating all the pools approach closer to their target storage.

Figure 5.6: Reduction of total deviation over the no transfer scenario for inter-basin promoted by different inflow forecasts.

To quantify the improvement over the no-transfer scheme, we computed the difference in the sum of absolute deviations across the four pools for a given inflow scenario to the sum of
absolute deviations across the four pools for the no-transfer scenario. From Figure 5.6, we can see that the total deviation is reduced through IBT and forecasts facilitate IBT further by reducing the deviation. The above discussion related to Figures 5.5 and 5.6 implies that (1) IBT could be beneficial for regional water management; (2) Climate-information based forecasts are more beneficial in improving IBT.

5.4 IBT among systems with different spatial correlations: synthetic experiment

In section 5.3, the IBT was investigated using climate forecasts based on the objective to minimize the deviation from the target storage for the four pools in two reservoirs, which are adjacent to each other. Under such conditions, the inflows into the systems could be highly correlated. On the other hand, even if the reservoirs are next to each other, the initial storage in both systems at the beginning of the season need not be highly correlated. Initial storage conditions, which play a significant role in seasonal water allocation, could be completely independent since they depend on the demand patterns in both systems. Thus, the utility of IBT contingent on forecasts depends on the spatial correlation between the initial storage conditions as well as the spatial correlation between inflows. To understand how these spatial correlations could impact IBT under various forecasting skill, we conduct a synthetic experiment that will systematically analyze under various spatial correlation patterns. For this purpose, we will consider the IBT to occur between two river basins with four pools (similar to Figure 5.1), we also utilize the inflow and demand characteristics of Falls Lake and Jordan Lake to generate different spatial correlation scenarios.
5.4.1 Synthetic inflow and initial storage generation scheme

In order to generate inflows or initial storages with a particular spatial correlation, we considered bi-variate lognormal distribution to synthetically generate 100 years of records (inflow and initial storage) for the summer season. The following procedure outlines the detailed steps in generating the spatial correlated variable (inflow or initial storage) for the two systems.

a) Log-transform the observed historical inflows or initial storages for Falls Lake and Jordan Lake, $Y = \log X$, where $X$ denotes observed inflows or initial storages.

b) The mean and variance of these log-transformed variables for both reservoirs will be represented as $\mu^j_Y$ and $\sigma^j_Y$ where $j$ denotes the system.

c) Given the desired spatial correlation $\gamma$, the covariance matrix for generating log-normal variates could be obtained as

\[
\Sigma = \begin{bmatrix}
\sigma_{y^1} & \gamma \cdot \sigma_{y^1} \cdot \sigma_{y^2} \\
\gamma \cdot \sigma_{y^1} \cdot \sigma_{y^2} & \sigma_{y^2}
\end{bmatrix}
\]

d) Generate 100 bi-variate lognormal random values using the mean and covariance matrix estimated in step (b) and (c).

e) Transform the log values to the original space (inflows or initial storages) for each system.

We generate inflows and initial storages for Falls Lake and Jordan Lake with different spatial correlations. For the inflows, we consider two spatial correlation $\gamma_i = 0.9$ and 0.1. Similarly for the initial storage, we consider $\gamma_{s_0} = 0.8$ and 0.1.
5.4.2 Synthetic inflow forecasts of different skill

As demonstrated earlier in the case study, inflow forecasts based on climate-information provide a better transfer strategy (leading to a smaller deviation) compared to the climatologic inflows. In order to further evaluate the sensitivity of the optimal solution (inter-basin transfer strategy) to the inflow forecast uncertainty, we synthetically develop inflow forecasts with different ‘known’ skill levels with the observation. Sankarasubramanian et al., (2009) generated inflow forecasts with specified skill using a parametric periodic gamma autoregressive model [Fernandez and Salas, 1986]. Given the observed inflow $Q_t$, with $t=1,2,...,n$ denotes each of the summer season for the time period concerned, we basically add gaussian noise to each observation such that the synthetic inflows preserves the mean and variance structure of the entire historical observation. For a given forecast skill, $\rho$, the noise $\varepsilon_t$ follows normal distribution with mean $\mu_s(1-\rho)$ and standard deviation $\sigma_s\sqrt{1-\rho^2}$. Thus, the synthetic inflow can be generated as the following:

$$I_{i,j} = \rho Q_{obs} + \varepsilon_t$$

Using the method discussed above, synthetic inflow forecasts are generated for three predictive skills $\rho = 0.9, 0.56$ and $0.3$. These skills along with different spatial correlations of $\gamma_i$ and $\gamma_{s_i}$ were used to evaluate IBT under different system characteristics.

5.5 Result and discussion: IBT under various system characteristics

Using the above discussed generation scheme, four different IBT scenarios were considered:

(a) High spatial correlations between inflows and initial storages (i.e., similar to the Falls
Lake and Jordan Lake system); (b) High spatial correlation between inflows and low spatial correlation between initial storages (i.e., systems having a very different demand patterns); (c) Low spatial correlation between inflows and high spatial correlation between initial storages (i.e., two systems with different spatial characteristics in precipitation due to orography); (d) Low spatial correlations between inflows and low spatial correlation between initial storages (i.e., two systems that have different inflow characteristics and demand patterns). In the above scenarios, (a), (b) and (d) are the most realistic ones. Hence, we present the results only for these three scenarios.

(a) High spatial correlations between both inflows and initial storages
This is the most commonly seen case, which could be found in two adjacent basins where both inflows and the demand pattern of both systems are very similar. Based on the generated inflow and initial storages having high spatial correlations for both systems, we develop ensemble inflow forecast with a special predictive skill levels ($p=0.3, 0.56$ and 0.9) to investigate the effect of inflow forecast uncertainty on initiating the optimal IBT. Using the optimization framework, different optimal IBT strategies corresponding to ensemble inflow forecast of different predictive skill levels and the synthetically generated inflow are obtained. To verify the benefit of IBT using inflow forecasts, the optimal IBT strategies are combined with the generated true inflow to simulate the end of season storage for the 100 year period. Based on IBT strategies obtained from different inflow forecasts, Figure 5.7 shows the boxplot of the deviation index for all the four pools. First, we can see clearly from Figure 5.7 that IBT could reduce the end of season storage deviation for the four pools.
Additionally, when the inflow forecast skill is not good (i.e., \( \rho = 0.3 \)), we can see that the end of season storage deviation for the four pools are still large. As inflow forecast skill is increasing (from \( \rho = 0.3 \) to \( \rho = 0.9 \)), the end of season storage deviation is further decreased, showing improved IBT strategies obtained from a good forecast. We can also see that the target storage deviation goes down to zero under perfect forecasts, indicating these three pools reach their target storage at the end of the season over the 100 years of simulation. Comparing across the four pools, we see that JL-WQ pool has a slightly larger deviation, since JL-WQ pool primarily transfers water to the other three pools. The above discussion underscores the importance of considering inflow forecasts when promoting IBTs.

![Box plots showing end of season storage deviation for all four pools under different forecast skills.](image)

**Figure 5.7:** End of season storage deviation for all the four pools under the scenario of high spatial correlations in inflows and initial storages for three different forecast skills.
The need for IBT primarily depends on the spatial correlations in inflows and initial storages between two systems. However, for a single season, it is the difference in the inflows or initial storages within the transferring pools determine the potential for IBTs. The potential for transfer is quantified by the absolute sum of deviations in inflows or initial storages to their respective climatological values during year $t$.

$$\omega_t = \left| \sum_{i=1}^{4} (V_i' - V_{i \text{ ave}}) \right| \tag{9}$$

In equation (9), $V_{i \text{ ave}}$ is the long term average (climatology) of the inflows or target storage for the initial storages of the four pools, $V_i'$ is the inflow or initial storage of a particular pool ($i=1,2,3,4$) for the year ‘$t$’. Through equation (9), we can make sure that the difference between the inflows or initial storage is completely represented by the following conditions for a given year. For instance, if the current season inflows or initial storages for all the four pools are above or below their climatology/target value, then the difference of the inflows or initial storages is small and the index is large. This means that all four pools are in surplus/deficit conditions, indicating the potential of initiating IBTs is not much. On the other hand, inflows or initial storages for the four pools exhibit surplus and deficit in relation to their climatology, then the $\omega_t$ is small indicating potential for IBTs. We computed $\omega_t$ for both initial storages and inflows for each year over the 100 years of synthetic simulation.
Figure 5.8: Total deviation index ($TD_t$) with regard to the difference index ($\omega_t$) of the inflow and initial storage for the scenario of high spatial correlations between inflow and initial storages for three different synthetic inflow forecast skills.

Figure 5.8 shows the end of season total storage deviation ($TD_t$) with $W_t^s$ and $W_t^I$. We also computed total deviation in storages across the four pools at the end of the season $t$ as

$$TD_t = \sum_{i=1}^{4} \left| \frac{S_t^i - S_t^*}{S_t^*} \right|,$$

with $i$ denotes a particular pool. Each circle in the figure represents the total deviation of the four pools over the 100 year period. We can clearly see the following: if $W_t^s$ and $W_t^I$ are small, then $TD_t$ is also small. This demonstrates that if surpluses and deficits exhibit across the four pools, the potential for meeting the end of season target storage is
large. Further, we also infer that as the inflow forecast predictive skill increases from 0.3 to 0.9, the total deviation decreases. This again underscores the role of inflow forecasts skill in improving IBT across the four pools.

(b) High spatial correlation between inflows and low spatial correlation between initial storages

This scenario could possibly happen between two adjacent basins which receive similar inflow patterns but the demand patterns are completely different. In this situation, the spatial correlation for IBT is high with $W_i^S$ being small in most of the situations. Based on the same procedures described in scenario (a), we calculated $TD_i$ over the 100 year simulation period as Figure 5.9. Comparing Figures 5.7a and 5.9a, we can also clearly see that the overall distribution of the deviation for the four pools are narrower especially when the inflow forecast predictive skill is not good, like $p=0.3$. This demonstrates the fact that IBTs could be effectively employed even under limited inflow forecasting skill.
Figure 5.9: End of season storage deviation for all the four pools under the scenario of high spatial correlations between inflows and low spatial correlation between initial storages for different forecast skills.

(d) Low spatial correlations between inflows and initial storages

This is the most ideal situation for promoting IBTs. We can expect this situation if two basins belong to different climatic regions so that the inflow and the demand patterns of these two basins do not show any temporal covariability. Figure 5.10 compares the total deviation $TD_t$ for all the four spatial correlation scenarios considered (scenarios: (a) High-High; (b) High-Low; (c) Low-Low; (d) Low-High). From Figure 5.10, we can see that the distribution of $TD_t$ for scenario (a) is the widest for all the inflow forecast skills. Following that, the distribution of $TD_t$ under scenario (b) is narrow. Under scenario (d), which is the ideal case for promoting
IBT, we can clearly see that the median value of $TD_i$ are the smallest compared to the rest of the scenarios under all skill level. The above discussion emphasizes that potential for IBTs are higher for river basins with limited covariablelity in inflow and demand patterns.

![Box plot showing total storage deviation index ($TD_i$) across four pools under four scenarios of spatial correlations for three different forecasting skills.](image)

Figure 5.10: Comparison of total storage deviation index ($TD_i$) across the four pools under four scenarios of spatial correlations for three different forecasting skills.

### 5.6 Summary and Conclusions

Reservoir management usually requires seasonal ahead information to provide assistance for water allocation. The information may include inflow forecasts, initial storage conditions and required demand. This study proposes a framework for regional water management by promoting an inter-basin transfer model that minimizes the deviation from the target storage across the participating pools. Using ensemble streamflow forecast, the IBT water allocation model was applied for two reservoir systems in the North Carolina Triangle area. Results
show that inter-basin transfer initiated by ensemble streamflow forecast could greatly improve the overall water supply reliability within the region. To further understand the utility of inter-basin transfer under different spatial correlation between inflows and initial storages, a synthetic experiment was designed to evaluate the framework under three forecasting skills. Several observations are derived from the synthetic experiment, they are summarized as follows:

(1) Inter-basin transfer is an effective method to alleviate the water stress caused by the imbalance in the regional water availability and demand;

(2) Better inflow forecasting skill result in reduced deviation in target storage;

(3) In a given year, the benefit from IBT is larger if $W_t^s$ and $W_t^l$ are smaller;

(4) The utility of inter-basin transfer is significant among reservoir systems exhibiting low spatial correlations between inflows and initial storages.
CHAPTER 6: SUMMARY AND CONCLUSION

6.1 Summary

The primary motivation of this dissertation research is on uncertainty reduction in hydrological modeling and in demonstrating their utility in improving inter-basin transfer. Two research problems under this context are presented:

1. Runoff from a watershed is the primary source of water supply, which is crucial for maintaining water system sustainability. However, the prediction of runoff from a watershed model has a large degree of uncertainty arising from various sources. Thus, one goal of this research is to reduce uncertainty in runoff predictions from the watershed.

2. Given the available storage and the predicted streamflow, water managers have to decide how to allocate the limited resource among multiple water users over a given region by considering inter-basin transfer (IBT) model. As part of this research, this study investigates the effectiveness of IBT under various scenarios of spatial correlations in inflows, initial storages and inflow forecasting skills.

In this dissertation, Chapter 2 primarily deals with uncertainty reduction in model structure error utilizing multimodel combination algorithm. Through a synthetic experimental design, it is demonstrated that multimodel combination algorithm is capable of improving streamflow prediction by reducing model uncertainty, especially when model errors of the candidate models are large.
In chapter 3, uncertainty reduction in streamflow is achieved by better representing initial conditions of the model utilizing Ensemble Kalman Filter algorithm. EnKF is aimed to provide better estimation for the initial states of the model. Using the same experimental design described in Chapter 2, this chapter compares multimodel combination algorithm with Ensemble Kalman Filter in improving streamflow prediction at monthly and daily time scales. Results show that multimodel combination algorithm performs better at monthly time scale, while Ensemble Kalman Filter performs better at daily time scale. This implies that reducing model uncertainty at monthly time scale is critical whereas reducing uncertainty in initial conditions plays an important role at daily time scale.

In an effort to demonstrate the findings from Chapters 2 and 3 for actual watersheds, Chapter 4 implements multimodel combination algorithm and Ensemble Kalman Filter algorithm in improving streamflow prediction at monthly and daily time scales. Results show that both algorithms are capable of improving streamflow prediction at monthly and daily time scale. To provide further understanding of the uncertainty arising from parameter estimation, this chapter also applies the Simultaneous global Optimization and Data Assimilation (SODA) algorithm for improving streamflow prediction at monthly and daily time scales. Results demonstrate that Ensemble Kalman Filter algorithm combined with enhanced parameter estimation can further improve streamflow prediction.

Chapter 5 investigates the uncertainty issue in regional water resources management by proposing an inter-basin transfer model. The inter-basin transfer model promotes water
allocation among multiple reservoirs using climate forecasts. Application of this framework in North Carolina Triangle area shows that inter-basin transfer based on climates forecast can reduce uncertainty in reservoir management resulting in improved water allocation. Through a synthetic experiment, this chapter also quantifies the utility of inter-basin transfer under different forecast skill as well as under various spatial correlations between inflows and initial storage conditions. Results show that inter-basin transfer using a skillful forecast is better than that of climatology. Furthermore, spatial correlation in inflow and initial storage of the reservoirs also influence the effectiveness of inter-basin transfer.

6.2 Conclusions

The analysis performed for the above two research problems focused on reducing uncertainty in hydrological modeling and regional water management. Major findings from this dissertation research are listed as follows:

- Multimodel combination algorithm can reduce uncertainty related to model structural deficiency, thereby improving streamflow prediction. The strength of multimodel combination lies in its optimal weight assignment scheme: models performing better during similar input conditions will obtain higher weights during the combination process.

- Ensemble Kalman Filter algorithm can reduce uncertainty related to initial conditions of the model, resulting in improved streamflow prediction at daily time scale. The strength of ensemble kalman filter lies in the initial state updating scheme, which
abstracts the recent observation to better estimate the initial states of the model over the subsequent step.

- The comparison between multimodel combination and ensemble kalman filter implies that model uncertainty reduction is more important for improving streamflow predictions at monthly time scale while reducing uncertainty related to initial conditions of the model plays a significant role for improving streamflow prediction at daily time scale;

- Inter-basin transfer is an efficient way to reduce uncertainty in water allocation and for reducing deficits at the end of the season storages. Inflow forecasts combined with an optimization model ensures improved allocation under different systems and basin hydroclimatology.

- Spatial correlation between inflows and initial storages among participating reservoirs could also influence the potential benefits that could be achieved through IBT.
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