Abstract
Lucier, Gregory W. Development of a Rational Design Methodology for Precast Concrete Slender L-Shaped Spandrel Beams. (Under the direction of Dr. Sami Rizkalla.)

Precast concrete L-shaped beams are traditionally reinforced with closed stirrups for shear and torsion, as required by current design practice. Existing design methods predict that L-shaped beams loaded in torsion will develop a series of cracks that spiral down the length of the member, resulting in spalling of the face-shell concrete. Closed stirrups and longitudinal torsion steel are well-suited to resist this type of behavior, however, spiral cracking and face-shell spalling have not been observed in practice for slender L-shaped beams. Rather, the behavior of slender L-spandrels is dominated by plate bending in the end regions. As such, the value of using heavy and congested closed reinforcement in the end regions of slender L-shaped beams is questionable. Cracks develop in a distinctive pattern on the inner web face with diagonal cracks extending upwards from the support and gradually flattening to horizontal at the midspan. Cracks on the outer web face are mainly flexural in nature with vertical cracks extending upwards from the bottom edge of the beam due to in-plane and out-of-plane flexure.

This dissertation presents an extensive experimental and analytical research program documenting the development of a rational design procedure for precast L-shaped spandrel beams that reflects the observed behavior. The research included an experimental phase where full-scale slender precast concrete spandrel beams were tested to failure. Sixteen slender beams and four compact beams were tested. In addition, a detailed analytical study included three-dimensional linear and non-linear finite element models that were calibrated to the experimental data.

A rational design methodology was developed that considers torsion in two orthogonal components: bending and twist. Expressions developed for bending and twist resistance were summarized in a simple design procedure for slender precast spandrels. Data show that the proposed procedure significantly reduces reinforcement congestion while maintaining the necessary level of safety. The research demonstrated that open web reinforcement (L-shaped bars, C-shaped bars, and welded-wire reinforcement) is a safe, effective, and efficient alternative to traditional closed stirrups for precast slender spandrels. In addition, the research demonstrated that current design methods for beam ledges may over-estimate punching shear capacity. Additional research on ledge punching resistance was recommended and is currently underway.
Development of a Rational Design Methodology for Precast Concrete L-Shaped Spandrel Beams

by
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A dissertation submitted to the Graduate Faculty of North Carolina State University in partial fulfillment of the requirements for the Degree of Doctor of Philosophy

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Biography

Gregory Lucier earned his Bachelor of Science in Construction Engineering and Management in May of 2004, graduating summa cum laude and valedictorian from North Carolina State University. He then completed a thesis on the behavior of reinforced concrete bridge decks under the direction of Dr. Sami Rizkalla and graduated in May, 2006 with a Master of Science in Civil Engineering. Since 2006, Greg has been working full time on staff at the Constructed Facilities Laboratory where he has been involved in research and testing for a wide variety of structural engineering materials, components, and systems. He currently serves as manager of the laboratory and plans to continue in this capacity after completing his doctoral degree.
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I would like to thank my colleagues and friends on staff at the Constructed Facilities Laboratory. Jerry Atkinson and Johnathan McEntire both provided seemingly endless technical support to this research, and the laboratory experiments could not have been completed without them.

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Table of Contents

List of Tables ...................................................................................................................... v
List of Figures....................................................................................................................... vi

1. Introduction ..................................................................................................................... 1
   1.1. Research Purpose ..................................................................................................... 1
   1.2. Objectives ................................................................................................................ 1
   1.3. Scope ....................................................................................................................... 1
   1.4. Research Phases ...................................................................................................... 1

2. Background and Literature Review .............................................................................. 2
   2.1. Previous Studies ...................................................................................................... 4
   2.2. Characteristics of Slender Spandrels ..................................................................... 6
   2.3. Current Design and Detailing Practices .................................................................. 18

3. Experimental Program – Slender Spandrels ............................................................... 27
   3.4. Introduction ............................................................................................................. 27
   3.5. Test Specimens – Slender Spandrels .................................................................... 27
   3.6. Test Parameters ...................................................................................................... 29
   3.7. Reinforcement Details ........................................................................................... 30
   3.8. Production ............................................................................................................... 33
   3.9. Test Setup ............................................................................................................... 39
   3.10. Instrumentation .................................................................................................... 41
   3.11. Loading ................................................................................................................ 42

4. Analytical Study – Slender Spandrels ......................................................................... 44

5. Findings and Results – Slender Spandrels ................................................................. 51
   5.1. Published Results from Phase I .............................................................................. 51
   5.2. Published Results from Phase II ........................................................................... 82
   5.3. Results from Material Testing .............................................................................. 136

6. Compact Beams ............................................................................................................ 138

7. Summary and Conclusions ......................................................................................... 166

8. References ..................................................................................................................... 168

Appendix ........................................................................................................................... 170
   Appendix A: Industry Contributors ............................................................................. 171
   Appendix B: Detailed Drawings .................................................................................... 172
List of Tables

Table 3-1: Slender Test Specimens........................................................................................................28
Table 3-2: Experimental Test Matrix ......................................................................................................29
Table 3-3: Vertical Web Reinforcement ..................................................................................................31
Table 3-4: Horizontal Web Reinforcement ............................................................................................32
Table 3-5: Design Loads for Experimental Specimens ............................................................................42
Table 3-6: Load Levels for Testing .........................................................................................................43
Table 5-1: Concrete Compressive Strengths ..........................................................................................136
Table 5-2: Steel Material Sample Results .............................................................................................137
List of Figures

Figure 2-1: Typical L-Shaped Slender Spandrel with .......................................................... 3
Figure 2-2: Typical Corbelled Slender Spandrel with .......................................................... 3
Figure 2-3: Principal Axes of an L-shaped Spandrel Beam .................................................... 7
Figure 2-4: Eccentricity Contributing to Torsion ................................................................. 9
Figure 2-5: Torsional Shear Stresses Acting on a Rectangular Section ................................. 12
Figure 2-6: Stresses Acting on Elements with Different Orientations ................................. 13
Figure 2-7: Stresses Due to Torsion Acting on a 45-degree Plane .................................... 14
Figure 2-8: Torsion-Induced Plate Bending Failure of a Rectangular Section .................... 15
Figure 2-9: Directions of Shear and Torsion Stresses ......................................................... 16
Figure 2-10: Flexure, Shear, and Torsion Stresses Acting on a 45-degree Plane .................... 17
Figure 2-11: Flexure, Shear, and Torsion Stresses Acting on a 45-degree Plane .................... 17
Figure 2-12: Forces Acting on a Beam Ledge ....................................................................... 24
Figure 2-13: Punching Shear Design of a Beam Ledge ....................................................... 26
Figure 3-1: Typical L-shaped Specimen with 60 Inch Deep Web ...................................... 28
Figure 3-2: Inner-face Elevation of Typical End Region with Open Reinforcement ............. 33
Figure 3-3: Outer-face Elevation of Typical End Region with Open Reinforcement ............. 33
Figure 3-4: Typical Spandrel Specimens Prior to Casting Concrete ................................. 34
Figure 3-5: Production of an Open Reinforcement Cage .................................................... 35
Figure 3-6: A Nearly Completed Open Reinforcement Cage ............................................. 36
Figure 3-7: Production of a Closed Reinforcement Cage .................................................... 37
Figure 3-8: A Nearly Completed L-Spandrel Closed Reinforcement Cage ............................ 38
Figure 3-9: Completed Corbelled Spandrel Reinforcement Cages ...................................... 38
Figure 3-10: Photograph of Test Setup (45 foot) .................................................................. 40
Figure 3-11: The Test Setup Modification Used for Specimens SP20 and SP21 .................... 41
Figure 4-1: Mesh Configuration for Non-Linear FE Model (L-shaped Spandrel) .................. 46
Figure 4-2: Mesh Configuration for Non-Linear FE Model (Corbelled Spandrel) ............... 47
Figure 4-3: Analytical Modeling of Prestressing Strand Transfer Length ............................ 48

Figure B-1: Typical Open-Reinforcing Ledge Detail ............................................................. 172
Figure B-2: Typical Closed-Reinforcing Ledge Detail .......................................................... 173
Figure B-3: Typical Deck to Spandrel Connection Detail .................................................... 174
Figure B-4: Shop Ticket for Double-Tee Deck Sections ....................................................... 175
Figure B-5: Shop Ticket for SP1 ......................................................................................... 176
Figure B-6: Shop Ticket for SP2 ......................................................................................... 177
Figure B-7: Shop Ticket for SP3 ......................................................................................... 178
Figure B-8: Shop Ticket for SP4 ......................................................................................... 179
Figure B-9: Shop Ticket for SP10 ....................................................................................... 180
Figure B-10: Shop Ticket for SP11 ..................................................................................... 181
Figure B-11: Shop Ticket for SP12 ..................................................................................... 182
Figure B-12: Shop Ticket for SP13 ...................................................................................... 183
Figure B-13: Shop Ticket for SP14 and SP15 ..................................................................... 184
Figure B-14: Shop Ticket for SP16 ...................................................................................... 185
Figure B-15: Corbel Reinforcement Detail Used with Specimens SP17 and SP18 ............. 186
Figure B-16: Shop Ticket for SP17 (1 of 2) ......................................................................... 187
Figure B-16: Shop Ticket for SP17 (1 of 2) ......................................................................... 188
Figure B-19: Shop Ticket for SP18 (1 of 2) ................................................................. 189
Figure B-20: Shop Ticket for SP18 (2 of 2) ................................................................. 190
Figure B-21: Bent Mesh Detail Used on Inner Web Face of SP18 ......................... 191
Figure B-22: Shop Ticket for SP19 (1 of 2) ................................................................. 192
Figure B-23: Shop Ticket for SP19 (2 of 2) ................................................................. 193
Figure B-24: Shop Ticket for SP20 ........................................................................... 194
Figure B-25: Shop Ticket for SP21 ........................................................................... 195
Figure B-26: Shop Ticket for LG1 ............................................................................ 196
Figure B-27: Shop Ticket for LG2 ............................................................................ 197
Figure B-28: Shop Ticket for LG3 ............................................................................ 198
Figure B-29: Shop Ticket for LG4 ............................................................................ 199
Figure B-30: Ledge Reinforcement Detail for LG1 ................................................. 200
Figure B-31: Ledge Reinforcement Detail for LG2-LG4 .......................................... 201
1. Introduction

1.1. Research Purpose
This dissertation documents a substantial research effort sponsored by the Precast and Prestressed Concrete Institute (PCI) of Chicago, IL to develop rational design guidelines for slender precast spandrel beams. The research was commissioned by the PCI Research and Development Committee and several private PCI Producer members in response to a desire within the precast concrete industry to simplify detailing practices in the end regions of slender spandrel beams.

1.2. Objectives
The main objective of the research was to develop rational guidelines for the design of precast slender spandrel beams. These guidelines were intended to simplify the detailing requirements for slender spandrels, especially in the end regions. Specifically, the research focused on investigating whether traditional closed ties were required for the slender cross-sections of typical precast L- and corbelled spandrels. The use of open reinforcement details in lieu of longitudinal bars and closed stirrups would greatly simplify the fabrication process and reduce the cost of production.

1.3. Scope
This dissertation documents extensive experimental and analytical research programs. In total, 20 full-scale precast spandrel beams were tested to failure. This dissertation also documents extensive analyses conducted using three-dimensional non-linear finite element models, linear finite element models, and traditional methods based on equilibrium of forces. Design guidelines were developed, and are presented with conclusions.

1.4. Research Phases
The research presented in this dissertation was conducted in three phases. Phase I consisted of four experimental tests and a preliminary finite element analysis to demonstrate the validity of removing closed stirrups from slender L-shaped spandrels. The successful completion of Phase I allowed for a detailed second phase of research with a larger scope. Phase II comprised an additional 12 full-scale experimental tests and a substantial finite element and rational modeling program, culminating in the development and validation of a rational model and simple design procedure. With the outline of the rational procedure in place, the third phase of research was launched to explore the relevance of the proposed design approach to compact cross-sections.
2. Background and Literature Review

Detailed background information on the design and analysis of precast slender L-shaped beams is presented in Technical Report IS-09-10 *Development of a Rational Design Methodology for Precast Slender Spandrel Beams* by Lucier, Walter, Rizkalla, Zia, and Klein. This chapter summarizes the information presented in this dissertation.

Precast, prestressed concrete spandrel beams are commonly used in parking structures. The primary purpose of these structural members is to transfer vertical loads from deck sections to columns. In addition, spandrel beams often serve as a railing or barrier around the exterior edge of the parking structure. Large single-tees or double-tees are used as deck sections that typically span 40 to 63 feet.

Typical spandrel beams are between 5 and 7 feet deep with spans ranging from 30 to 50 feet. These beams usually have at least an 8-inch thick web. In many cases, a continuous ledge runs along the bottom edge on one side of the beam, resulting in what is known as an L-shaped spandrel. Since the ledge is used to provide bearing for the deck sections, the L-shaped spandrel beam is therefore subjected to a series of discrete eccentric loadings. A common alternative configuration is a corbelled spandrel where the continuous ledge is replaced by a series of discrete haunches.

Slender precast spandrel beams are usually simply supported at the columns with end connections to prevent torsional rotation. In addition, discrete connections between deck sections and the web of the spandrel provide more restraint along the length of the spandrel. Sketches of typical slender precast spandrels are shown in Figure 2-1 for an L-shaped spandrel and Figure 2-2 for a corbelled spandrel.
Figure 2-1: Typical L-Shaped Slender Spandrel with Applied Loads (blue), Reactions (Red), and Deck Connection Forces (yellow)

Figure 2-2: Typical Corbelled Slender Spandrel with Applied Loads (blue), Reactions (Red), and Deck Connection Forces (yellow)
The slenderness of a precast spandrel is determined by the dimensions of its cross section. Spandrels having a large aspect ratio (defined as the spandrel height divided by the thickness of the web) are considered slender members. It is not uncommon to find precast concrete spandrel beams with aspect ratios of 10 or more.

2.1. Previous Studies

A substantial body of literature exists relevant to the design of reinforced and prestressed concrete members for torsion, and for the interaction of torsion, shear and flexure. A complete review of literature relevant to concrete torsional design is not presented here, but a discussion of classical torsion theories are presented in the section below. For a thorough discussion of the subject, from classical theories to practical design examples, the reader may refer to the book by Hsu (Hsu, 1984). Other relatively recent contributions to the body of literature relevant to the design of reinforced concrete for torsion have been made by Collins, Hsu, MacGregor, Mitchell, Zia, and others. Relevant works are cited in the References section.

Several works are of particular interest to the study of slender precast concrete spandrel beams. While the first design provisions for torsion in reinforced concrete were published in the 1971 ACI 318 Building Code (ACI, 1971), they were not applicable for prestressed concrete. Zia and McGee introduced the first design methodology for beams subjected to bending, shear and torsion in 1974 (Zia, McGee, 1974). The proposed method provided equations that were used to determine the induced nominal shear and torsional stresses for a given cross-section and reinforcing material properties. The method also provided an evaluation of the concrete contribution to the member’s overall capacity, in addition to web reinforcement required for shear and torsion resistance. In a later study (Zia, McGee, 1976), the researchers determined that the minimum reinforcement required by ACI 318-71 for flexural shear was inadequate for a prestressed member subjected to combined loading conditions.

In 1977, the ACI Building Code requirements expressed in terms of stresses were changed to be in terms of forces and moments. In 1978, Zia and Hsu updated the original Zia and McGee paper to reflect this change, and presented the results at the convention of the
American Society of Civil Engineers (ASCE) (Zia, Hsu, 1978). This research developed recommendations for the minimum torsional web reinforcement for prestressed concrete. The recommendations in Zia and Hsu’s 1978 paper formed the basis for the current PCI guidelines for torsion design of reinforced and prestressed concrete members.

The procedure recommended by Zia and Hsu was not validated by full-scale tests of slender spandrels, as used in the field, until the investigations of Raths (1984) and the experimental work of Klein (1986). The findings of both Raths and Klein confirmed the findings published in Cleland’s 1984 dissertation at the University of Virginia. Cleland discovered through experimental tests and analysis that the dominant behavior of eccentrically-loaded slender spandrels was horizontal displacement. Horizontal displacements were sometimes found to be several times the magnitude of associated vertical displacements.

Raths’ 1984 article in the PCI Journal documented the results from his extensive field investigations and design experience. Raths article was a comprehensive collection of observations of precast slender spandrel behavior and of recommendations for effective design. The 1986 study by Klein was sponsored by the Precast / Prestressed Concrete Institute (PCI), and included three full-scale experimental tests on precast slender spandrels and finite element modeling. Klein developed several design recommendations for slender spandrels, including those relevant to beam ledge behavior, which are still used in practice today.

The investigations of both Raths and Klein revealed that the behavior of the spandrel was greatly affected by its support condition. Significant plate-bending effect was observed in the web of the slender spandrel.

Several analytical studies on precast slender spandrels have been published in recent years. Lini and Ramirez (2004) compiled and compared available torsion design procedures for precast spandrel girders. One key finding was that several classical design assumptions, including face-shell spalling, appeared to be too conservative. Yasdani and Ach (2004) also investigated the problem of efficiently designing precast slender spandrel beams for torsion. They noted that approaches specified in the ACI 318-99 building code
often resulted in heavily congested reinforcement when applied to slender members, and suggested that the design assumptions related to concrete contribution to torsional strength seemed far too conservative.

Logan’s 2007 paper, published in the PCI Journal, called into question the need for heavy closed reinforcement in the end regions of slender precast spandrel beams. Logan documented the results of an in-house load test conducted on a precast L-shaped spandrel beam in 1961. The test demonstrated that eccentric vertical loading was unable to generate either torsional rotation or torsional distress in a precast L-shaped spandrel beam. Rather, substantial lateral deflection effects were observed. Logan also highlighted a long-standing desire within the precast industry to simplify the reinforcing schemes for precast slender spandrel beams, suggesting that traditional closed reinforcement could perhaps be replaced with much more production-friendly open reinforcing schemes.

The author of this dissertation also published the results of four full-scale experimental tests on precast slender spandrel beams fabricated entirely with open web reinforcement in 2007 (Lucier et. al., 2007 and Walter, 2008). These tests were sponsored by several individual precast concrete producers, and demonstrated the validity of replacing traditional closed stirrups with open web reinforcement. A finite element study (Hassan et. al., 2007) confirmed the experimental results.

2.2. Characteristics of Slender Spandrels

The following sections describe the characteristics of slender spandrels based on established theories of structural behavior. Gary Klein of Wiss, Janney, Elstner, Associates is credited for developing the figures and inclined elastic finite element model presented in this section.

The eccentrically applied loading on the unsymmetrical slender spandrel beam causes not only vertical displacement, but also lateral displacement as well as rotation of the spandrel. Maximum torsional and shear effects occur near the end of the spandrel. This complex
structural behavior, coupled with the heavy loading, often results in spandrel designs requiring conservative reinforcement details.

**Flexure**

L-shaped spandrel beams are not symmetrical about either axis. The principal axes are rotated counterclockwise from the vertical and horizontal axes, as shown in Figure 2-3. The axis of rotation has little influence on the in-plane flexural strength. On the other hand, the rotation of the principal axes influences the horizontal displacement of L-shaped spandrels. As shown in Figure 2-3, a component of the vertical load acts along the weak axis, inducing an outward horizontal displacement at the bottom (outward motion is considered as motion away from ledge). Loads acting on the spandrel beams before they are connected to the deck sections can also cause inward rotation in the midspan region. For this reason, erectors commonly connect or brace double tee flanges to the spandrel beams as the tees are erected, often by driving wedges between the tees and inner face of the spandrel beam as the tees are set.

![Figure 2-3: Principal Axes of an L-shaped Spandrel Beam](image)
Cleland (1984) found that out-of-plane displacement was a dominant behavior of long slender spandrels and suggested a principal axis analysis when the span length is 40 to 50 times the web width, depending on intermediate support conditions.

**Strain distribution:** Although most spandrel beams are relatively “deep,” they are not considered deep beams according to the definition provided in ACI 318-08, Section 10.7. To be considered deep beams, the beam must be loaded at the top and supported at the bottom; typically, spandrel beams are loaded near the bottom of the section. Furthermore, the span-to-depth ratio generally exceeds four, even for the 5 to 7 foot deep spandrels used in parking structures. As such, the flexural strain distribution is linear in the midspan region. However, like all beams, flexural strain is not linear in the so-called “D-region” near the concentrated support reaction.

**Shear**

**Critical section:** Unlike typical beams, spandrel beams are indirectly loaded; that is, the loads are applied near the bottom of the beam. In accordance with ACI 318-08, Section 11.1.3, the critical section for shear occurs at the face of the support rather than a distance “d” from the support, as permitted for conventional beams.

**Shear stress distribution:** Before cracking, the distribution of shear stresses through the depth of the cross section is parabolic. The shear stress varies from zero at the top and bottom of the section to a maximum of $\tau_{\text{max}}$ at the mid-height of the section.

$$\tau_{\text{max}} = \frac{3V}{2bh}$$  \hspace{1cm} \text{Equation 2-1}

Where V is the shear force applied on the section, b is the width of the web, and h is the section height.

The principal tensile and compressive stresses occur at 45 degree angles to the section. After cracking, the shear stress distribution changes. Research on shear stress distribution
after cracking indicates that shear is resisted through a combination of aggregate interlock, compression zone shear, and dowel action of the longitudinal reinforcement.

**Torsion**
The load eccentricity contributing to torsion is of critical importance to the design of a precast slender spandrel beam. Typically, the ledge loads are positioned at the centerline of bearing (allowing for fabrication and erection tolerances) or at a point 2/3 of the distance from the face of the web to the front edge of the ledge. The eccentricity contributing to torsion is taken as the distance from the centerline of the web to the applied load, as shown in Figure 2-4. Theoretically, the eccentricity should be measured relative to the shear center, which, for an uncracked L-beam section, is slightly inside the centerline of the web. However, this difference is negligible in deep spandrels. For spandrels with corbels, the shear center is at the centerline of the web.

![Figure 2-4: Eccentricity Contributing to Torsion](image-url)
Torsional behavior -- precast versus cast-in-place: Like precast spandrels, cast-in-place edge beams are subject to torsion. However, the torsional behavior of precast spandrel beams differs in several important respects.

Most importantly, the inherent differences in the end support details change the behavior in three ways: 1) as a result of the simple support conditions, there is no bending moment at the support of the precast spandrels, whereas continuity of cast-in-place construction results in negative bending that is coincident with maximum torsion; 2) warping of precast spandrels is not restrained at the support and the longitudinal forces due to warping restraint cannot develop; and 3) precast spandrels employ mechanical connections to equilibrate torsion at the support, which requires special attention to detail in the connection region.

The nature of torsional demand is also fundamentally different. Torsion in typical cast-in-place edge beams is indeterminate and cracking due to rotation generally reduces torsional demand. But structural compatibility must be maintained. So the torsion in cast-in-place edge beams is often referred to as compatibility torsion. On the other hand, torsion of precast spandrels is determinate and force equilibrium must be maintained, so it is often referred to as equilibrium torsion. If friction at double-tee bearings is ignored (which is consistent with conventional design assumptions), torsional demand increases in proportion to increasing ledge load, regardless of cracking and rotation.

These fundamental differences in torsional behavior are recognized in the research plan and design recommendations described in this dissertation are primarily for slender precast spandrels.
Brief review of classical torsion theory for rectangular members:

In 1853, Saint-Venant developed the solution of torsion of a member of rectangular cross section. Saint-Venant showed that torsional stresses flow in a circulatory pattern, as can be seen in Figure 2-5.a. The maximum torsional shear stresses occur along the middle regions of the long sides. This stress is given by Equation 2-2.

\[
\tau = \frac{T}{\alpha \frac{x}{y}}
\]

Equation 2-2

Where \( x \) and \( y \) are the smaller and larger dimensions of the rectangular section, respectively, and \( \alpha \) is a coefficient that varies between 0.208 (for \( \frac{y}{x} = 1 \)) and 0.333 (for \( \frac{y}{x} = \infty \)).

In 1903, Prandtl discovered an analogy between torsional stresses and the deflection of an elastic membrane under uniform loading. The direction and magnitude of shear stresses can be visualized by the contour lines of the deflected surface of the membrane, such as a soap bubble, within a boundary having a shape identical to that of the cross section under tension. Shear stresses are proportional to the slope of the membrane and the total torque is proportional to the volume under the membrane. The shear stress contour lines and "soap bubble" membrane are depicted in Figure 2-5.b.
a) **Torsion on Rectangular Section.**
Arrows around the perimeter represent the “flow” of torsional shear stress around the section.

b) **“Soap bubble” contour lines.**
Circulatory shear stress is proportional to the slope of the soap bubble

**Figure 2-5: Torsional Shear Stresses Acting on a Rectangular Section**
It can be seen in Figure 2-6 (a) that an element on the front face of the rectangular member under torsion $T$ – an element near mid-depth with its principal axes parallel to those of the rectangular member – is under a set of pure shear stresses. If the axes of the element are rotated 45-degrees to the axes of the rectangular member, the element is then subjected to principal tensions and principal compressions as shown in Figure 2-6 (b). Accordingly, if the rectangular member is cut by a plane inclined at 45-degrees with respect to the longitudinal axis of the member, only normal principal tensile stresses will exist on the front half of the cut plane and normal principal compressive stresses on the back half of the cut plane. The top and bottom regions of the member will be subjected to only shear stresses.

**Figure 2-6: Stresses Acting on Elements with Different Orientations**

This distribution of stresses was confirmed by a preliminary linear finite element analysis using 3-dimensional brick elements oriented at 45-degrees, as shown in Figure 2-7. Results of the analysis indicate tensile and compressive stresses along the sides of the section form a bending couple induced by the bending component of torsion. For slender spandrels, this mechanism is referred to as plate bending. On the other hand, shear stresses acting on the 45-degree inclined section are concentrated at the top and bottom of the section, and produce a twisting couple.
Referring to the sketch shown in Figure 2-8, the section modulus about Line a-a is given by Equation 2-3.

\[ S_{a-a} = \frac{\sqrt{2} \cdot y \cdot x^2}{6} \]  

Equation 2-3

The tensile stress, \( f_r \), due to bending about a-a is given by:

\[ f_r = \frac{M_{a-a}}{S_{a-a}} = \frac{\frac{T}{\sqrt{2}}}{\frac{\sqrt{2} \cdot y \cdot x^2}{6}} = \frac{3T}{x^2 \cdot y} \]  

Equation 2-4
Rearranging this result, the torsional resistance of plain concrete is given by:

\[ T = \frac{x^2 y}{3} f_t, \]

Equation 2-5

where \( f_t \) is the tensile strength of plain concrete. Note that this torque is exactly equal to that determined from Saint-Venant’s equations for very slender sections (those having \( \frac{y}{x} \approx \infty \)). This equality is not coincidental; rather, it indicates that for slender sections, the torsional resistance of a plane concrete section is dependent on the plate bending resistance across an inclined failure plane. Out-of-plane shear stresses resist the twist component of torsion, but transverse shear does not control in an unreinforced rectangular section.

Figure 2-8: Torsion-Induced Plate Bending Failure of a Rectangular Concrete Section
**Combined torsion and shear:** Shear and torsion stresses act in the same direction on the inner web face, but oppose one another on the outer web face, as illustrated in Figure 2-8. As such, the inner face is much more vulnerable to diagonal cracking due to combine shear and torsion.

![Figure 2-9: Directions of Shear and Torsion Stresses](image)

**Combined torsion, shear, and flexure:** It is useful to consider how torsional stresses combine with shear and flexural stresses in a slender section. The 3-dimensional, linear finite element model described above was loaded with a moment along the longitudinal axis to produce torsional stresses, and a point load at mid-span to produce shear stresses. Flexural stresses are present in the model, although they are kept to a minimum by simply supporting the ends of the member.

The results of combined elastic shear and tensile stresses across a 45 degree inclined section are plotted in Figure 2-10. On the inner web face, shear stress is additive with the plate bending tensile stress. On the outer web face, shear stress counteracts the plate bending compressive stress. The flexural stresses in the member skew the line of zero stress counter-clockwise at the 45 degree inclined section.
Figure 2-10: Combined Flexure, Shear, and Torsion Stresses Acting on a 45-degree Plane

Figure 2-11 shows the combined elastic stresses due to shear, torsion and flexure of a typical slender spandrel. These combined stresses were produced by applying vertical loads at discrete points along a ledge running along the bottom edge of one web face. The ends of the model are simply supported and restrained against rotation. The 45-degree section is cut just inside the vertical bearing, where flexural stresses are minimal and shear/torsion stresses are at their greatest. At this section near the support, the flexural stresses are non-linear, and flexural tension near the concentrated reaction adds to the tensile stress from shear and torsion near the bottom of the inside face of the spandrel.

Figure 2-11: Combined Flexure, Shear, and Torsion Stresses Acting on a 45-degree Plane
Cut Just Inside the Bearing of a Typical Spandrel
2.3. Current Design and Detailing Practices

**Flexure**
In general, detailing of slender precast spandrels for flexure follows the ACI code. One noteworthy exception pertains to Section 10.6.7 of ACI 318-08, which requires “skin reinforcement” in the bottom half of non-prestressed beams. Designers do not often check this provision; instead longitudinal reinforcement is determined based on plate bending, temperature and shrinkage, or handling stresses.

**Shear and Torsion**
The current practice recommended by the American Concrete Institute for proportioning reinforcement to resist combined shear and torsion within a concrete member is based on a space truss analogy (ACI 318-08). Longitudinal steel and closed stirrups are provided to resist torsional-shear stresses which are assumed to develop and spiral along the length of a member. Well distributed longitudinal steel and closed ties serve to maintain the integrity of the concrete core enclosed within the stirrups, allowing inclined compression struts to develop and resist the applied forces. The approach recommended by ACI 318-08 assumes that later stage member response will be characterized by spalling of the concrete shell outside of the stirrups. Researchers have recommended detailing, such as 135-degree stirrup hooks, to maintain the integrity of the concrete core after face-shell spalling (Mitchell and Collins, 1976). Frequently, steel is heavily congested in critical zones such as the end regions where prestressing strands and reinforcing bars must weave through numerous closed stirrups that are closely spaced as required by the ACI Code (2008).

It is important to note that Section 11.5.7 of ACI 318-08 (2008) allows for alternative approaches to be used for the torsion design for solid sections having an aspect ratio of 3 or greater. Alternative approaches must be shown as adequate by analysis and comprehensive testing. For many years, spandrel beams have been designed by the precast and prestressed concrete industry following one such alternative procedure, originally proposed by Zia and McGee (1974), and later modified by Zia and Hsu (1978, 2004). Their design procedure was developed based on the results of laboratory testing of small symmetrical flanged sections under controlled loadings to produce various combinations of torsion, shear, and bending. The current version of the PCI Design
Handbook (version 6, 2004), recommends the guidelines proposed by Zia and Hsu for the design of precast and prestressed spandrel beams.

Both the ACI and PCI approaches to shear and torsion design (and their associated detailing requirements) result in safe designs, but often require highly congested, interwoven reinforcement, especially in the end regions of slender members. Such congested reinforcement is difficult to place, and leads to inefficiency in production.

The current procedure for proportioning shear and torsion reinforcement for slender spandrel beams (PCI Handbook, 6th Edition) is presented below.

1. Determine the design ultimate shear, $V_u$, and the design ultimate torsional moment, $T_u$, at the critical section for shear and torsion. The critical section, determined per ACI 318, is “d” from the face of the support for non-prestressed members and “h/2” for prestressed members; “d” is to be taken from the point of load application for a spandrel beam loaded along the ledge.

2. Determine if torsion can be neglected based on specimen cross section and concrete and prestressing material properties.

$$ T_u (\text{min}) = \phi (0.5 \lambda \sqrt{f'_c \sum x^2 y}) \gamma $$

Equation 2-6

Where:

- $T_u$ = Factored torsional moment, lb-in
- $\phi$ = 0.75
- $\lambda$ = Conversion factor for lightweight concrete
- $f'_c$ = Concrete compressive strength, psi
- $x, y$ = Short side and long side, respectively, of a component rectangle, inch
- $\gamma$ = A factor dependent on the level of prestress
  $$ \gamma = \sqrt{1 + 10 \frac{f_{pc}}{f'_c}} $$
  = 1.0 for non-prestressed sections
- $f_{pc}$ = Average prestress after losses
If $T_u \leq T_u(\text{min})$ no torsion reinforcement is needed and design is complete.

3. If torsion cannot be neglected, check that required nominal torsional moment and shear strengths are at appropriate limits so that potential compression failures, due to over-reinforcing, do not occur.

$$T_{u(\text{max})} = \frac{\left\{ \frac{1}{3} K_t \lambda \sqrt{f_c} \sum x^2 y \right\}}{1 + \left[ \frac{K_t V_u}{30 C_t T_u} \right]^2} \geq \frac{T_u}{\phi}$$  \hspace{1cm} \text{Equation 2-7}

$$V_{u(\text{max})} = \frac{10 \lambda \sqrt{f_c b_w d}}{1 + \left[ \frac{30 C_t T_u}{K_t V_u} \right]^2} \geq \frac{V_u}{\phi}$$  \hspace{1cm} \text{Equation 2-8}

Where:

$$K_t = \gamma \left( 12 - 10 \frac{f_{pc}}{f_c} \right)$$

$$V_u = \text{Factored shear force, lb}$$

$$C_t = \frac{b_w d}{\sum x^2 y}$$

$$b_w = \text{Web width of member, inch}$$

$$d = \text{Effective depth of member, inch}$$
4. The shear and torsion interaction has long been represented by a circular curve. When the requirements in Step 3 are met, calculate the nominal torsional moment and shear strength provided by the concrete.

\[
T_c = \frac{T'_{c}}{\sqrt{1 + \left( \frac{T'_{c}}{V'_{c}} \right)^2}}
\]

Equation 2-9

\[
V_c = \frac{V'_{c}}{\sqrt{1 + \left( \frac{V'_{c}}{V'_{u}} \right)^2}}
\]

Equation 2-10

Where:

- \( T_c \) = Nominal torsional moment strength of concrete under combined shear and torsion
- \( V_c \) = Nominal shear strength of concrete under combined shear and torsion
- \( V'_{c} \) = Nominal shear strength of concrete under pure torsion
- \( T'_{c} \) = Nominal torsional moment strength of concrete under pure torsion

5. Provide stirrups if the design torsional moment is greater than that carried by the concrete. These stirrups are in addition to those required for shear.

\[
A_s = \frac{\left( \frac{T_c}{\phi} - T_s \right)x}{\alpha_s x_s y_s f_y}
\]

Equation 2-11
To ensure reasonable member ductility, a minimum area of closed stirrups should be determined:

\[
(A_t + 2A_v)_{\text{min}} = 50 \frac{b_s x}{f_y} (\gamma)^2 \leq 200 \frac{b_s x}{f_y}
\]  
Equation 2-12

Where:

- \(A_t\) = Required area of one leg of closed tie, in\(^2\)
- \(x_1\) = Short side of closed tie, inch
- \(y_1\) = Long side of closed tie, inch
- \(s\) = \((x_1+y_1)/4\) or 12 = tie spacing, inch
- \(\alpha_t\) = \([0.66+0.33 y_1/x_1]<1.5\) = torsion coefficient
- \(f_y\) = Yield strength of closed tie, psi
- \(A_v\) = Area of shear reinforcement, inch
- \(\gamma\) = A factor dependent on the level of prestress

6. Provide longitudinal reinforcement to resist the longitudinal component of the diagonal tension induced by torsion. This longitudinal steel is in addition to that calculated for flexure.

\[
A_i = \frac{2A_{ti}(x_1 + y_1)}{s}
\]  
Equation 2-13

Or,

\[
A_i = \left[\frac{400x}{f_y} \left(\frac{T_u}{T_u + V_u/3C_{1}}\right) - \frac{2A_{vi}}{s}\right](x_1 + y_1)
\]  
Equation 2-14
The value of $A_l$, should not exceed that obtained when substituting:

$$\frac{50 \, b_w}{f_y} \left(1 + \frac{12 \, f_y}{f_c}\right) \leq \frac{200 \, b_w}{f_y} \quad \text{for} \quad \frac{2A_i}{s}$$

Equation 2-15

**Out-of-Plane Bending**

The current PCI handbook addresses out-of-plane bending in L-shaped spandrel beam end regions. An equation is given for determining the amount of vertical ($A_{wv}$) and longitudinal ($A_{wl}$) reinforcement on the inner face. It is recommended that this be distributed across a height and width equal to the distance between the two lateral equilibrium reactions.

$$A_{wv} = A_{wl} = \frac{V_u e}{2 \varphi f_y d_w}$$

Equation 2-16

Where:

- $V_u$ = Factored shear force at critical section
- $e$ = Eccentricity, distance between ledge load and main vertical reaction
- $\varphi$ = 0.75
- $f_y$ = Yield strength of reinforcement
- $d_w$ = Depth of $A_{wv}$ and $A_{wl}$ reinforcement from outside face of beam

**Ledge and Corbel Design**

Ledges and corbels must be checked for several possible failure modes:

- Bending failure of the ledge (or corbel) acting as a cantilever, extending outward from the inside face of the web
- Separation of the ledge from the web
- Punching shear failure at each concentrated double-tee stem load
These failure mechanisms for beam ledges and corbels are addressed in the *PCI Handbook* and are discussed in the following paragraphs.

**Cantilever bending of ledge or corbel:** The effective cantilever distance is the distance from the ledge load to the hanger reinforcement near the inside face of the spandrel beam. Horizontal reinforcement at the top of the beam ledge is provided to resist the resulting bending force. In addition, transverse reinforcement for beam ledges and corbels is added to resist an inward frictional force taken as at least 0.2 times the factored dead load portion of the ledge reaction.

For continuous ledges, the required cantilever reinforcement may be uniformly spaced over a width equal to six times the height of the ledge on either side of the T-stem bearing, but not to exceed half the distance to the next load. Of course, for corbels, the cantilever reinforcement must be provided within the width of the corbel.

**Ledge-to-web attachment:** The requirements for attachment of the ledge to the web are given in PCI 4.5.4 and are based on a study of ledge-to-web attachment by Klein (1986). The transverse forces acting on a free body of a beam ledge are shown in Figure 2-12. The PCI procedure recognizes that not all of the load acting on the ledge is suspended from the web, and the effective eccentricity of the ledge load is significantly reduced due to torsion within the ledge itself. The PCI design procedure is based on these principles.

![Figure 2-12: Forces Acting on a Beam Ledge (Image from Klein, 1986)](image-url)
Hanger reinforcement for ledge-to-web attachment may be distributed as described above for cantilever reinforcement. PCI 4.5.3 includes provisions for longitudinal reinforcement in the ledge that helps distribute the concentrated T-stem reaction along the length of the ledge.

These same principles can be applied to design of spandrel beam corbels, although it is common practice to conservatively neglect the shear and torsion in the beam web below the corbel reaction. Accordingly, hanger reinforcement in corbels is proportion for the entire ledge load and full eccentricity.

**Ledge punching:** Section 4.5.1 in the PCI Handbook gives the design equation for the punching shear strength of beam ledges. Handbook equation 4.5.1.1 applies to interior beam reactions and allows a shear stress of $3 \lambda \sqrt{f_c}$ along a 3-sided shear perimeter with a depth equal to the ledge height. As can be seen in Figure 2-13, the 3-sided perimeter runs along the inside face of the web for a length equal to the width of the bearing plus the height of the ledge, and then transversely across the width of the ledge. For ledge reactions close to the end of the beam, handbook equation 4.5.1.2 governs. This equation allows a shear stress of $2 \lambda \sqrt{f_c}$ along a 2-sided perimeter extending from the end of the ledge along the inside face of the web and then transversely across the width of the ledge (see Figure 2-13). Additional equations are provided for closely spaced and continuous ledge reactions.
These design equations have been in the PCI Handbook since the Second Edition (1978), and beam ledges have performed well in service. However, several researchers have found that these design equations are unconservative (Mirza, 1983; Raths 1984; Krauklis, 1985). The PCI procedure does not fully account for the eccentricity between the ledge reaction and the centroid of the resisting section, and the design equations do not account for reduction in strength due to interaction with global tensile and shear stresses in the beam ledge. For these reasons, special reinforcement brackets were provided in many of the spandrel beams tested as part of this research.

The PCI procedures for ledge punching resistance do not apply to corbelled spandrels because hanger reinforcement is used to “hang up” the entire corbel reaction. However, the transverse cantilever produces an outward horizontal reaction at the bottom of the corbel that must be resisted by the concrete section. Regardless of the amount of hanger and cantilever reinforcement used, the strength of the corbel may be governed by the out-of-plane punching resistance at the bottom of the ledge. Furthermore, this resistance is reduced by global tensile and shear stresses at the bottom of the web. No design equations have been developed to check this resistance.
3. Experimental Program – Slender Spandrels

3.4. Introduction

The experimental program undertaken at the Constructed Facilities Laboratory at North Carolina State University consisted of 16 tests of full-scale slender spandrel beams tested in two phases. A third phase of research included an additional 4 tests were conducted on compact L-shaped beams and are described in that chapter. This chapter summarizes the slender test specimens and experimental program. Additional information on the slender spandrel experimental program is presented in two published papers included in Chapter 5.

3.5. Test Specimens – Slender Spandrels

A total of sixteen full-scale precast slender spandrel beams were tested in the research, as shown in Table 3-1. Phase I of the experimental program included 4 specimens designed to demonstrate the validity of the proposed open web reinforcement scheme. Successful completion of Phase I led to a second phase of research that included 12 additional full-scale tests. Most beams tested in Phases I and II had the same general configuration and cross-section, as shown in Figure 3-1. Fourteen of the 16 specimens had a height of 60 inches, and a web thickness of 8 inches. All L-shaped spandrel beams had an 8 inch x 8 inch continuous ledge running along the bottom of the inner web face, while all corbelled spandrels had discrete haunches at the double tee stem load located at 5 feet on center in lieu of the continuous ledge. In the case of the L-spandrels, the ledge was cut back 12 inches from the end of the beam at each end to replicate a typical field detail which allows for the spandrel to be bolted squarely to the column.

Two holes through the web thickness were provided at each end of all specimens. For beams with a 60 inch deep web, these holes were set 6 inches in from the ends and 12 inches in from the top and bottom of the spandrel. For the two beams with a 46 inch deep web, the holes were set 6 inches in from the ends of the beams and 6 inches in from the top and bottom. The holes were sized to accommodate rods which were used to bolt the spandrels to a test frame in a manner that mimics field conditions. Test beams were fabricated and cast at the precast plants of several different PCI Producer Members. Test specimens were delivered to the laboratory as they were needed in the testing program.
### Table 3-1: Slender Test Specimens

<table>
<thead>
<tr>
<th>Research Phase</th>
<th>Test Specimen</th>
<th>Nominal Span</th>
<th>Specimen Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase I</td>
<td>SP1</td>
<td>30'</td>
<td>8&quot;x60&quot; L-Spandrel</td>
</tr>
<tr>
<td></td>
<td>SP2</td>
<td>30'</td>
<td>8&quot;x60&quot; L-Spandrel</td>
</tr>
<tr>
<td></td>
<td>SP3</td>
<td>45'</td>
<td>8&quot;x60&quot; L-Spandrel</td>
</tr>
<tr>
<td></td>
<td>SP4</td>
<td>45'</td>
<td>8&quot;x60&quot; L-Spandrel</td>
</tr>
<tr>
<td></td>
<td>SP10</td>
<td>45'</td>
<td>8&quot;x60&quot; L-Spandrel</td>
</tr>
<tr>
<td></td>
<td>SP11</td>
<td>45'</td>
<td>8&quot;x60&quot; L-Spandrel</td>
</tr>
<tr>
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<td>SP12</td>
<td>45'</td>
<td>8&quot;x60&quot; L-Spandrel</td>
</tr>
<tr>
<td></td>
<td>SP13</td>
<td>45'</td>
<td>8&quot;x60&quot; L-Spandrel</td>
</tr>
<tr>
<td></td>
<td>SP14</td>
<td>45'</td>
<td>8&quot;x60&quot; L-Spandrel</td>
</tr>
<tr>
<td></td>
<td>SP15</td>
<td>45'</td>
<td>8&quot;x60&quot; L-Spandrel</td>
</tr>
<tr>
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<td>SP16</td>
<td>45'</td>
<td>8&quot;x60&quot; L-Spandrel</td>
</tr>
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<td>SP17</td>
<td>45'</td>
<td>8&quot;x60&quot; Corbelled Spandrel</td>
</tr>
<tr>
<td></td>
<td>SP18</td>
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<td>SP19</td>
<td>45'</td>
<td>8&quot;x60&quot; Corbelled Spandrel</td>
</tr>
<tr>
<td></td>
<td>SP20</td>
<td>45'</td>
<td>10&quot;x46&quot; L-Spandrel</td>
</tr>
<tr>
<td></td>
<td>SP21</td>
<td>45'</td>
<td>10&quot;x46&quot; L-Spandrel</td>
</tr>
</tbody>
</table>

**Figure 3-1: Typical L-shaped Specimen with 60 Inch Deep Web**
3.6. Test Parameters

The parameters considered in Phases I and II of the experimental program are shown in Table 3-2. These parameters and their effects on behavior are discussed in greater detail in Chapter 5.

<table>
<thead>
<tr>
<th>Depth (in)</th>
<th>Span (feet)</th>
<th>Aspect Ratio (h/b)</th>
<th>Designation</th>
<th>Configuration</th>
<th>Concrete</th>
<th>Reinforcement</th>
<th>Detailing</th>
<th>Bearing</th>
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</thead>
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<td>*SP2 8L60.30.P.O.E</td>
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</tbody>
</table>

* Specimens were tested as part of prior research sponsored by PCI producer members
† Specimen SP18 was constructed with special closed reinforcement in a hooked-C shape

The parameters in the test matrix are described below, along with the labeling convention used to identify each specimen.

**Designation**

Test beams were designated by the following labeling format: \text{SP}_{i}.\text{#L##.45.P.O.E}

Where:

\text{SP}_{i} - unique number for each specimen: SP10, SP11, SP12, etc.

\#L## or \#CB## – L-spandrel or Corbelled spandrel with # inch web and ## inch depth. For example, “8L60” is an L-spandrel with an 8 inch by 60 inch web.

45 (or 30) – nominal span in feet

P (or R) – Prestressed or conventionally Reinforced

O (or C or S) – Open or Closed web reinforcement or Special closed reinforcement

E (or T) – Extra detailing or Typical Beam
Specimens were supported using either Teflon-coated bearing pads or typical Masticord rubber pads, as noted in Table 3-2.

3.7. Reinforcement Details
The steel reinforcement used for the spandrels consisted of prestressing strands, welded-wire reinforcement (WWR) and conventional deformed reinforcing bars. Detailed sketches of the reinforcement provided in each specimen are provided in Appendix B: Detailed Drawings. A summary of the reinforcement provided in the end regions of each specimen is provided in the following sections.
Web Reinforcement

The vertical reinforcement in the web of each experimental spandrel is summarized in Table 3-3. The table lists the vertical web reinforcement in the end region of each of the beams on the inner and outer faces.

Table 3-3: Vertical Web Reinforcement

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Vertical Steel – Inner Web Face</th>
<th>Vertical Steel – Outer Web Face</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>End Regions</td>
<td>End Regions</td>
</tr>
<tr>
<td>SP1.8L60.30.P.O.E</td>
<td>4”x4” W4.0xW4.0 WWR (4) #4 L-bars and (5) #4 C-bars</td>
<td>4”x4” W4.0xW4.0 WWR (5) #4 C-bars</td>
</tr>
<tr>
<td>SP2.8L60.30.P.O.E</td>
<td>4”x4” W4.0xW4.0 WWR (6) #4 L-bars (5) #4 C-bars</td>
<td>4”x4” W4.0xW4.0 WWR (5) #4 C-bars</td>
</tr>
<tr>
<td>SP3.8L60.45.P.O.E</td>
<td>(10) #4 L-bars (2) #4 C-bars</td>
<td>6”x6” W4.0xW4.0 WWR (2) #4 C-bars</td>
</tr>
<tr>
<td>SP4.8L60.45.P.O.E</td>
<td>6”x6” W4.0xW4.0 WWR (10) #3 L-bars (10) #4 L-bars (2) #4 C-bars</td>
<td>6”x6” W4.0xW4.0 WWR (2) #3 C-bars</td>
</tr>
<tr>
<td>SP10.8L60.45.R.O.E</td>
<td>(10) #4 L-bars (2) #4 C-bars</td>
<td>(5) #4 L-bars (2) #4 C-bars</td>
</tr>
<tr>
<td>SP11.8L60.45.R.C.E</td>
<td>(19) legs of #4 closed stirrup</td>
<td>(19) legs of #4 closed stirrup</td>
</tr>
<tr>
<td>SP12.8L60.45.P.O.E</td>
<td>(10) #4 L-bars (2) #4 C-bars</td>
<td>6”x6” W4.0xW4.0 WWR (2) #4 C-bars</td>
</tr>
<tr>
<td>SP13.8L60.45.P.C.E</td>
<td>(15) legs of #4 closed stirrup</td>
<td>(15) legs of #4 closed stirrup</td>
</tr>
<tr>
<td>SP14.8L60.45.P.O.T</td>
<td>(10) #4 L-bars (2) #4 C-bars</td>
<td>6”x6” W2.5xW2.5 WWR (2) #4 C-bars</td>
</tr>
<tr>
<td>SP15.8L60.45.P.O.T</td>
<td>(10) #4 L-bars (2) #4 C-bars</td>
<td>6”x6” W2.5xW2.5 WWR (2) #4 C-bars</td>
</tr>
<tr>
<td>SP16.8L60.45.R.O.T</td>
<td>(11) #4 L-bars (2) #4 C-bars</td>
<td>(11) #4 L-bars (2) #4 C-bars</td>
</tr>
<tr>
<td>SP17.8CB60.45.P.O.E</td>
<td>(10) #4 L-bars (2) #4 C-bars</td>
<td>6”x6” W4.0xW4.0 WWR (2) #4 C-bars</td>
</tr>
<tr>
<td>SP18.8CB60.45.P.S.E</td>
<td>(12) #4 Hooked C-bars</td>
<td>6”x6” W4.0xW4.0 WWR (2) #4 C-bars</td>
</tr>
<tr>
<td>SP19.8CB60.45.P.O.T</td>
<td>(7) #4 L-bars (2) #4 C-bars</td>
<td>6”x6” W4.0xW4.0 WWR (2) #4 C-bars</td>
</tr>
<tr>
<td>SP20.10L46.45.P.O.E</td>
<td>(9) #4 L-bars (2) #4 C-bars</td>
<td>6”x6” W2.5xW2.5 WWR (2) #4 C-bars</td>
</tr>
<tr>
<td>SP21.10L46.45.P.O.T</td>
<td>(9) #4 L-bars (2) #4 C-bars</td>
<td>6”x6” W2.5xW2.5 WWR (2) #4 C-bars</td>
</tr>
</tbody>
</table>

The end region of each beam is defined as the initial portion of the beam up to a distance ‘h’ from the face of the support, where ‘h’ is the overall height of the section. In the case of spandrels with closed reinforcement (SP11 and SP13), the reinforcement on the inner and outer web faces is the same. For the other spandrels, the use of open reinforcement allowed for optimizing the steel quantity required on each face.
The horizontal reinforcement in the web of each spandrel is summarized in Table 3-4. Horizontal web reinforcement reported in the table is the longitudinal reinforcement located above the level of the lower lateral reaction. Flexural reinforcements (such as prestressing strands) are not shown in the table.

### Table 3-4: Horizontal Web Reinforcement

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Horizontal Steel – Inner Web Face</th>
<th>Horizontal Steel – Outer Web Face</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>End Regions</td>
<td>End Regions</td>
</tr>
<tr>
<td>SP1 8L60.30.P.O.E</td>
<td>(2) legs of #5 U-bar 4&quot;x4&quot; W4.0xW4.0 WWR</td>
<td>(2) legs of #5 U-bar 4&quot;x4&quot; W4.0xW4.0 WWR</td>
</tr>
<tr>
<td>SP2 8L60.30.P.O.E</td>
<td>(2) legs of #5 U-bar 4&quot;x4&quot; W4.0xW4.0 WWR</td>
<td>(2) legs of #5 U-bar 4&quot;x4&quot; W4.0xW4.0 WWR</td>
</tr>
<tr>
<td>SP3 8L60.45.P.O.E</td>
<td>(4) legs of #4 U-bar</td>
<td>(4) legs of #4 U-bar</td>
</tr>
<tr>
<td>SP4 8L60.45.P.O.E</td>
<td>(1) legs of #4 U-bar  (4) straight #4 U-bar</td>
<td>(1) legs of #4 U-bar</td>
</tr>
<tr>
<td>SP10 8L60.45.R.O.E</td>
<td>(7) legs of #4 U-bar</td>
<td>(7) legs of #4 U-bar</td>
</tr>
<tr>
<td>SP11 8L60.45.R.C.E</td>
<td>(7) legs of #4 U-bar  (3) #5 straight bars</td>
<td>(7) legs of #4 U-bar  (3) #5 straight bars</td>
</tr>
<tr>
<td>SP12 8L60.45.P.O.E</td>
<td>(7) legs of #4 U-bar</td>
<td>(7) legs of #4 U-bar</td>
</tr>
<tr>
<td>SP13 8L60.45.P.C.E</td>
<td>(7) legs of #4 U-bar  (5) #5 straight bars</td>
<td>(7) legs of #4 U-bar  (5) #5 straight bars</td>
</tr>
<tr>
<td>SP14 8L60.45.P.O.T</td>
<td>(4) legs of #5 U-bar</td>
<td>(4) legs of #5 U-bar</td>
</tr>
<tr>
<td>SP15 8L60.45.P.O.T</td>
<td>(4) legs of #5 U-bar</td>
<td>(4) legs of #5 U-bar</td>
</tr>
<tr>
<td>SP16 8L60.45.R.O.T</td>
<td>(5) legs of #5 U-bar</td>
<td>(5) legs of #5 U-bar</td>
</tr>
<tr>
<td>SP17 8CB60.45.P.O.E</td>
<td>(7) legs of #4 U-bar</td>
<td>(7) legs of #4 U-bar</td>
</tr>
<tr>
<td>SP18 8CB60.45.P.S.E</td>
<td>(7) legs of #4 U-bar</td>
<td>(7) legs of #4 U-bar</td>
</tr>
<tr>
<td>SP19 8CB60.45.P.O.T</td>
<td>(7) legs of #4 U-bar</td>
<td>(7) legs of #4 U-bar</td>
</tr>
<tr>
<td>SP20 10L46.45.P.O.E</td>
<td>(5) legs of #5 U-bar</td>
<td>(5) legs of #5 U-bar</td>
</tr>
<tr>
<td>SP21 10L46.45.P.O.T</td>
<td>(5) legs of #5 U-bar</td>
<td>(5) legs of #5 U-bar</td>
</tr>
</tbody>
</table>

A sketch of a typical open-reinforcement configuration in the end region of a slender spandrel is shown in Figure 3-2 for the inner web face, and in Figure 3-3 for the outer web face. Typically, the required vertical web steel was provided with a single sheet of welded-wire reinforcement on the outer face. Vertical steel was provided on the inner face by L-shaped deformed reinforcing bars. In the end regions, longitudinal steel was provided on both faces with horizontal U-shaped bars.
3.8. Production

Test specimens were produced by three separate companies at five separate plants. All spandrels were cast on long-line flat table forms, lying on their outer face, as shown in Figure 3-4, with the exception of non-prestressed SP16 which was cast in an architectural plant on a wooden table form. Usually two or more specimens were cast at a time to gain efficiency and to ensure similar material properties within pairs of specimens. The author
inspected the steel cages, and observed the casting of every specimen. Samples of the concrete and steel used to produce the specimens were taken and tested for each cast.

A significant advantage in using open web reinforcement was the efficiency gained in production. Observations of the production of the experimental beams in this program indicate that assembling an open reinforcing cage took roughly 30-50% less time than assembling a closed reinforcing cage. The gains in efficiency were especially obvious when an open cage was produced on the same form line adjacent to a closed cage, as was the case for specimens SP12 and SP13. In producing the open cage (with the spandrel lying outer-face down on the form), the outer-face web reinforcement (often WWR) was placed in the empty form. The strands were then pulled and stressed without obstructions, as shown in Figure 3-5. After stressing the strands, any required longitudinal steel bars (for example, horizontal U-bars in the end regions) were simply placed in the form near their final locations.
With the strands stressed, the other components of the open reinforcing cage were dropped into the form at the correct locations and tied into place. L-shaped or C-shaped bars on the inner spandrel face were placed so that they rested on the stressed strands. C-shaped ledge bars or corbel assemblies were hooked around the longitudinal steel and secured to strand or bars. With the web steel in place, the additional longitudinal steel (U-bars) were secured. The flexibility of the open reinforcement allowed for spacing of bars to be easily adjusted as the cage was finalized. If a bar was misplaced, it could be removed and replaced without disrupting any other components of the cage. A nearly completed open reinforcement cage is shown in Figure 3-6.
In the case of the closed reinforcing cages, the stirrups (both web and ledge) had to be placed in the empty form. During this step, it was important to verify that the sequence of the stirrups corresponded to their final locations in the beam. With the stirrups in the form, the prestressing strands and any other required longitudinal steel were threaded through the stirrups, taking care not to disrupt the stirrup order. The strands were then prestressed. A typical slender spandrel beam (SP13) with a closed cage and longitudinal steel in place is shown in Figure 3-7 prior to stressing of the strands.
After stressing the strands, the stirrups and additional longitudinal bars were spaced out and secured in place at their final locations, as shown in Figure 3-8. If errors were made in placing the stirrups in a closed cage, few options were available to correct the mistakes, short of detensioning the strands. Misplaced stirrups could be cut and removed from the cage, but inserting additional stirrups was a challenge. In some cases, the side-rails of the form could be removed and any missing stirrups bent into place around the already-stressed strands, but this procedure required significant effort. Of course, careful planning and layout at the start of a closed-cage assembly will minimize mistakes, but even rare assembly errors are costly with a closed reinforcement cage.
Reinforcement cages for corbelled spandrels were produced in a similar fashion, as shown in Figure 3-9.
In addition to gains in production efficiency, the use of an open reinforcement cage offered a significant savings in steel compared to traditional designs using closed stirrups. In examining the test specimens in this program, an open reinforcement cage required up to 50% less shear and torsion steel than a comparable closed cage.

The difference in required steel can be highlighted for specimens SP10 and SP11. Recall that both of these specimens are slender spandrel beams with 60 inch by 8 inch webs and 45 foot spans. Both were over-reinforced for flexure to ensure end region failures. Specimen SP10 was designed with open reinforcement, specimen SP11 with traditional closed stirrups. Both were designed for the same applied loads. Flexural reinforcement was the same for both specimens, and is excluded from the calculated steel quantities. The total quantity of steel used to produce specimen SP10 was 715 pounds compared to the 1396 pounds required to produce specimen SP11 (both weights excluding the common flexural steel). The 681 pound difference is equivalent to a 48% reduction in web steel.

A similar analysis can be performed on prestressed beams SP12 and SP13. Neglecting flexural steel common to both beams, the steel required for the open cage of specimen SP12 is 778 pounds compared to 1251 pounds for closed cage of SP13. The 473 pound difference is equivalent to a 37% reduction in web steel.

3.9. Test Setup

A summary of the experimental test setup is presented in this section including instrumentation and loading sequence. Additional details are provided in two published papers included in Chapter 5.

All tested spandrels were simply supported at their ends. The web was laterally restrained at two points at each end to resist torsion. Loads were applied through short full-scale double-tee deck sections bearing on the ledge or corbels. The double-tee deck sections were attached to the spandrel using typical welded field connections. Hydraulic jacks and spreader beams were used to apply loads to the top of the short double-tee sections which in turn applied load to the spandrel ledge. As the applied loads were eccentric with respect
to the center of support, the beams were subjected to a combination of in plane bending, out of plane bending, shear, and torsion. A photograph of the slender spandrel test setup is given in Figure 3-10.

![Figure 3-10: Photograph of Test Setup (45 foot)](image)

**Test Setup Modification for Beams SP20 and SP21**
The setup described above was used for all slender spandrel tests, but was modified for specimens SP20 and SP21. These two specimens were designed for the same applied loads as previous specimens, but had shorter webs and a lower aspect ratio. Higher predicted lateral reactions required that the lateral supports be modified for tests of SP20 and S21. Instead of connecting the spandrel directly to the flanges of the steel supporting frames, a pair of stiff back-to-back channels were oriented vertically and attached to the connections at each end of the web. The ends of these channels extended above and below the spandrel web, where they were attached to the steel support frame and laboratory strong floor, as shown in Figure 3-11. Lateral loads were measured at the attachment points, where the magnitudes were lower than the loads acting on the spandrel web itself.
The dimensions of the channels were known so that the reactions actually acting on the spandrel could be easily calculated from the measured values.

3.10. Instrumentation

Approximately 40 instruments were used to record data during each test. The main vertical reaction and both lateral reactions were measured with loadcells at each end of each spandrel. In addition, the applied loads were measured with additional loadcells. Deformations in the vertical and lateral direction were measured with linear potentiometers, and rotations were measured with inclinometers. Concrete strains were measured with
wire-arch clip gauges (or PI gauges). All instruments were connected to an electronic data acquisition system which captured data at a rate of 1 Hz.

### 3.11. Loading

The loads used to design all of the tested spandrels are summarized in Table 3-5.

<table>
<thead>
<tr>
<th>Load</th>
<th>Value</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead Load</td>
<td>71.6 lbs./ft²</td>
<td>Weight of a 10DT26 deck</td>
</tr>
<tr>
<td>Dead Load</td>
<td>567 lbs./ft²</td>
<td>Self weight of 8” x 60” L-spandrel</td>
</tr>
<tr>
<td></td>
<td>500 lbs./ft²</td>
<td>Self weight of 8” x 60” Corbelled-spandrel</td>
</tr>
<tr>
<td></td>
<td>546 lbs./ft²</td>
<td>Self weight of 10” x 46” L-spandrel</td>
</tr>
<tr>
<td>Live Load</td>
<td>40 lbs./ft²</td>
<td>Live load acting on a 60’ long deck</td>
</tr>
<tr>
<td>Snow Load</td>
<td>30 lbs./ft²</td>
<td>Snow load acting on a 60’ long deck</td>
</tr>
</tbody>
</table>

For design, all loads other than the self-weight were transferred to the spandrel through the stem reactions of the 10DT26 deck sections, spaced evenly at 5 feet on center. For calculations involving eccentricity, loads were assumed to act two inches back from the edge of the ledge or corbel.

Given the design loads in Table 3-5, the vertical and lateral reactions for several desired load combinations were determined. The vertical end reaction of each simply supported spandrel was monitored throughout testing, and served as the basis for controlling a loading system of hydraulic jacks during the test. Thus, all discussion of load levels references the main vertical reaction for a given beam.

The following load combinations were considered in the test program: service load without snow (DL+LL), a reduced service load with snow (DL+0.75LL+0.75SL), the unfactored service load (1.0DL+1.0LL+1.0SL), and the factored design load (1.2DL+1.6LL+0.5SL). These load combinations and their values are summarized in the following tables. Note that three spandrel types are represented under the column for 45 foot spandrels. These three types include the 8 inch x 60 inch L-spandrel, 8 inch x 60 inch corbelled spandrel, and 10
The design loads for all spandrel types were the same. Differences in self-weight between these three types were around 1% of the design loads. Thus, to facilitate comparison of the results, all spandrels for a given span were tested to the same load levels, as shown in Table 3-6.

<table>
<thead>
<tr>
<th>Designation</th>
<th>Load</th>
<th>Spandrel Reaction (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>30' Span Spandrels</td>
</tr>
<tr>
<td>Service</td>
<td>DL+LL</td>
<td>58.7</td>
</tr>
<tr>
<td>ASCE7 Service with Snow</td>
<td>1.0DL+0.75LL+0.75SL</td>
<td>64.4</td>
</tr>
<tr>
<td>Service with Snow</td>
<td>1.0DL+1.0LL+1.0SL</td>
<td>72.2</td>
</tr>
<tr>
<td>Factored</td>
<td>1.2DL+1.6LL+0.5SL</td>
<td>84.8</td>
</tr>
</tbody>
</table>

It should be noted that references to ‘dead load’ within this testing program assume that the spandrel is supporting the reaction of a 60-foot span double-tee deck. However, due to space limitations, a 12 foot span double-tee deck was used in the test setup. The design “dead load” was equal to the self-weight of the test spandrel, the associated 12’ deck sections, and an additional load applied by the jacking system to represent the full reaction of a 60-foot span double-tee deck.

Load was applied to each spandrel in increments following the levels shown in Table 3-6. Each spandrel was unloaded from each increment and then reloaded to the next higher increment. The load was held at each level to allow for observations and marking of cracks. The factored load was held on each beam for 24-hours prior to unloading at the end of the factored load cycle and monitoring the recovery for 1 hour. After the factored load cycle, a spandrel was loaded in incremental cycles to failure.
4. **Analytical Study – Slender Spandrels**

This chapter presents the analytical study undertaken to investigate the behavior of slender spandrel beams. The analytical study included three components, linear finite element analysis, non-linear finite element analysis, and rational modeling. Experimental data were used to validate, refine, and calibrate the various analytical models. The three-dimensional finite element model was developed in Phase I, but was greatly expanded in Phase II. The rational model was developed at the end of Phase II. Additional details regarding the analytical study are included the published papers presented in Chapter 5.

**Linear Finite Element Analysis Program**

Initially, a linear finite element model was performed using the SAP2000 finite element program to examine the behavior of a typical slender spandrel end-region. The purpose of the linear FEM model was to characterize the stress distribution across planes cut through a slender rectangular cross-section at various angles. A unique feature of the model was the implementation of three-dimensional brick elements turned at 45-degrees from the longitudinal axis of the beam. This model employing angled elements facilitated examination of developed stresses on a 45-degree plane. Results of the linear finite element analysis were described in Chapter 2.

**Non-linear Finite Element Analysis**

A three-dimensional nonlinear finite element model (FEM) was developed and calibrated with the measured results from the experimental program to study various parameters that were identified to influence the behavior of slender L-shaped spandrel beams. Results of the FEM were used in conjunction with the experimental results to develop a rational model for design.

**Background on the Non-Linear Finite Element Model**

The finite element analysis selected for this study is based on the ANATECH Concrete Analysis Program (ANACAP). The program is capable of analyzing plain, reinforced, or prestressed concrete structures, in either two or three dimensions. The program code has extensive nonlinear capabilities and includes advanced concrete material modeling. The built-in material models for concrete and steel require the user to input several key properties including elastic modulus, ultimate strength, and fracture strain.
The FEM includes material modeling that accounts for uniaxial and multiaxial stress/strain states under the framework of isotropic hardening plasticity formulation. The concrete material model is assumed to be linear when the compressive stresses are less than one-half of the compressive strength and follows a strain hardening model until the concrete compressive strength limitations are reached. In order to effectively model the shear performance of concrete, ANACAP reduces the shear modulus of the concrete at cracking and accounts for further reductions as the cracks continue to open and propagate. The concrete model relies on the smeared cracking methodology to predict the propagation of cracks, assuming that all the cracks form perpendicular to the direction of the largest tensile strains. When a crack forms, the normal stresses across the crack reduce, and the forces and stresses around the crack are redistributed to the surrounding concrete and reinforcing elements. While the direction of initiated cracks cannot change, cracks may close to resist compression, and then reopen during cyclic loading. Reinforcing bars are modeled individually as discrete sub-elements within the concrete elements. The stress and stiffness of the reinforcing sub-elements are superimposed on the concrete element in which the reinforcing bar resides.

The finite element code is able to capture crack propagation and widespread damage prior to structural failure. The analysis terminates when the displacement convergence at any node in a given solution step fails to reach a selected tolerance. To ensure that analysis termination corresponds with a real structural failure, post-processing allows for examination of specific failure criteria. Selected criteria include concrete compressive strain limits of 0.002 in shear regions and 0.003 in flexural regions. Graphical output such as deformed shape, cracking patterns, and strain contours can also be used to verify failure, to determine failure mode, and to examine behavior. Additional details describing the ANACAP finite element code can be found elsewhere (Anacap 2003).

**Development of the Non-Linear FEM Model**

Considering symmetry, the nonlinear FEM analysis was based on modeling one-half of a typical slender spandrel beam using approximately 4,700 20-node brick elements. The exact number of elements varied depending on the specifics of the case under study. The
A large number of elements was necessary to maintain a sufficiently fine mesh around all of the loading and boundary conditions as well as in the end region where failure was expected to occur. In addition, since out-of-plane behavior was a dominant response for the spandrels, four elements were used through the thickness of the web to capture potential out-of-plane failure modes. The modeled portion of a typical 45 foot long beam has 5 ledge loads, 3 tieback connections, 2 lateral end restraints, 1 vertical end reaction, and the symmetry condition at midspan. The finite element mesh and boundary conditions used are depicted in Figure 4-1 for a typical L-shaped spandrel and in Figure 4-2 for a typical corbelled spandrel.

Figure 4-1: Typical Mesh Configuration for Non-Linear FE Analytical Model (L-shaped Spandrel)
The boundary conditions used in the model were chosen to simulate the connections typically used to support spandrel beams in the field. The spandrel-to-column tiebacks were simulated by restraining movement in all lateral direction at those two locations in the model. Vertical movement was restrained at the end to simulate the main vertical reaction. Symmetry boundary conditions were applied at mid-span.

The measured ultimate concrete compressive strength at the time of testing was specified in the material models, and was used to determine the elastic modulus, $E_c$, and fracture strain, $\varepsilon_{tu}$, as recommended by the PCI Design Handbook (2004). The density, fracture strain and Poisson’s Ratio for the concrete material were included in the material input as well as the elastic modulus, yield stress and Poisson’s Ratio for the Grade 60 deformed bars, Grade 75 welded wire reinforcement, and the prestressing strands, where applicable. The initial
prestressing force was also a requirement for the tendon material model; a requirement which included estimated loss of prestress.

Since the behavior of the end-region is of particular interest in this study, care was taken to simulate the transfer length of each prestressing strand in the analytical model. Each strand was split into 10 smaller strands, each strand has one-tenth the area of the original strand and all occupy the same location in the model. The first of the 10 partial strands stretched the entire length of the spandrel and the tenth partial strand started at a distance equal to the transfer length away from the end. The remaining eight partial strands started at equal, incremental distances between the first and tenth partial strand. A sketch of the transfer length model is shown in Figure 4-3.

![Figure 4-3: Analytical Modeling of Prestressing Strand Transfer Length](image)

Each of the 9 applied vertical loads on the spandrel ledge from the deck sections were modeled as uniform pressure acting on an area equal to the size of the bearing pads used in the experimental program. The load was increased incrementally, at 5 psi per load step, until failure.

Several key parameters and their effects on the spandrel behavior were examined using the nonlinear finite element model. These parameters included closed versus open reinforcement, web reinforcement type and ratio, cross-section dimensions, concrete strength, and the influence of boundary conditions such as bearing friction and deck connections.
Boundary Conditions

Accurate simulation of the boundary conditions is essential to predicting realistic behavior. Two boundary conditions are particularly challenging in the analysis of slender precast spandrel beam model: the bearing reactions between the deck stems and the ledge, and the welded connections between the deck members and the web inner face. Prior research and initial analysis indicated that as a spandrel beam deformed under load, the ledge tended to rotate outward away from the deck while the top of the web tended to rotate inward towards the deck. This rotation caused the ledge to slip out from underneath the deck stems as the load increased. It was expected that significant friction forces developed between each stem of the supported deck sections and the ledge of the spandrel beam which restrained this motion. In addition, the welded connections between deck sections and the inner web face also served to exert additional lateral forces to the spandrel. It was important to capture these forces in the analytical model.

Including the influence of stem-to-ledge bearing friction and deck to spandrel tieback forces was critical in obtaining an accurate analytical model. In both cases, spring elements were used to emulate these connections in the model. For the deck connections, the stiffness of the spring elements was estimated based on the material and cross-sectional properties of the weld plate. Each typical welded connection was 6 inches by 3 inches and 3/8 inch thick. In developing and refining the finite element model, the welded connections were considered to be flexible in the vertical plane since they could rotate easily about the relatively thin welds. However, the connections were considered quite stiff in the horizontal plane since they were placed in direct tension or compression. The overall horizontal stiffness of each connection, considered an upper-bound limit, was determined using the physical dimensions and material properties of the standard connection plates as follows:

\[ k = \frac{EA}{L} = \frac{29 \times 3 \times \left(\frac{3}{8} \times 6\right)}{3} = 21750 \text{ k/in} \]  

Equation 4-1
Where $k$ is the stiffness, $E$ is the elastic modulus for steel, $A$ is the cross-section area of the connection plate, and $L$ is the length of plate under axial compression of tension. The FEM model was used to evaluate several values for connection stiffness, starting with the upper bound estimate given by Equation 4-1.

In most models, lateral springs were also applied at each stem-to-ledge bearing reaction to simulate the friction at these locations. The spring constant was determined based on the coefficient of friction for the chosen bearing pads and the measured lateral deflections at the factored load level. The coefficient of friction for a given bearing pad was assumed to remain constant at all levels of applied load. These assumptions probably tended to slightly over-estimate the friction force at very high load levels where out-of-plane load-deflection behavior was significantly non-linear, but remained a good estimate overall.

**Rational Model**

In addition to the linear and non-linear finite element models, a rational model was developed to describe the behavior of a slender spandrel. This rational model was validated with the experimental and FE data, and forms the basis of the final recommended design method. The development of this model and the final proposed design method are presented in the following chapter.
5. Findings and Results – Slender Spandrels

The findings and results from Phases I and II, covering slender precast spandrel beams, are presented in this chapter.

5.1. Published Results from Phase I

Results from the first phase of research are presented in two papers published in the PCI Journal and reprinted here. The first paper, Precast Concrete, L-Shaped Spandrels Revisited: Full-Scale Tests by Lucier, Rizkalla, Zia, and Klein, documents the behavior of specimens SP1 through SP4. This paper was published early in the research effort and demonstrates that the end region behavior of slender L-shaped spandrels is dominated by plate-bending and controlled by a skewed diagonal failure plane. No spandrels exhibited the spiral cracking and face-shell spalling predicted by traditional design approaches. The paper demonstrated the potential for designing slender L-shaped spandrels with open web reinforcement and led to an expansion of the research program (specimens SP10-SP21). It should be noted that the paper received both the 2007 Martin P. Korn and the George D. Nassar awards from the PCI Journal Editorial Board. These awards recognize the best paper in research and design, and the most outstanding paper by a young author, respectively.

The second paper, Modeling of L-Shaped, Precast, Prestressed Concrete Spandrels by Hassan, Lucier, Rizkalla, and Zia presents initial results from the non-linear, three-dimensional finite element model and compares the output of the model to measured data. The ability of the model to accurately capture and predict the behavior of slender L-shaped spandrel beams is confirmed.

Both papers are included in this dissertation with permission from the PCI Journal.
This paper presents the results from full-scale testing conducted at North Carolina State University on four precast, prestressed concrete L-shaped spandrels. The four L-shaped spandrels were each loaded through 12-ft-long (3.7 m) prestressed double tees that rested on the spandrel ledge at one end and on an independent support at the other. None of the beams were constructed with closed stirrups of mild-steel reinforcement. Rather, different arrangements of transverse L-shaped bars, welded-wire reinforcement, and longitudinal bars were provided to resist the shear and torsion induced in the spandrels. Shear and torsion forces were created by the double-tee reaction forces that were loaded eccentrically to the spandrels. The transverse and longitudinal reinforcement resisted the combined effects of vertical shear and out-of-plane bending of the web and satisfied the minimum vertical hanger reinforcement requirement for ledge-to-web attachment. All beams sustained loads well in excess of their factored design loads. Eliminating the need for closed reinforcement in slender spandrels would be of significant benefit to the precast concrete industry. This design approach would enhance the constructability of slender members, which could increase plant productivity and reduce overall costs. The paper presents the behavior of all four spandrels at various limit states, including their crack patterns and modes of failure. Researchers used these test results to better understand the fundamental mechanism developed in the L-shaped spandrels to resist shear and torsion.
Precast, prestressed concrete spandrels are commonly used in parking structures. The primary purpose of these structural members is to transfer vertical loads from deck beams to columns. In addition, spandrels often serve the dual purpose of railings or barriers around the exterior edge of such structures.

L-shaped spandrels are typically 5 ft to 7 ft deep (1.5 m to 2.1 m) with spans ranging from 30 ft to 50 ft (9.1 m to 15.2 m). They frequently have an 8-in.-thick (203 mm) web. An L-shaped spandrel is denoted by a continuous ledge that runs along the bottom of one face of the web. Because an L-shaped spandrel’s ledge provides the bearing surface for the deck beams, the L-shaped spandrel is subjected to a series of concentrated, eccentric loads along its span. Large single tees or double tees are frequently used as deck beams. They typically span 40 ft to 60 ft (12.2 m to 18.3 m).

Spandrels are usually simply supported at the columns, and their ends are connected to the columns to prevent torsional rotation. In addition, the deck beams are connected to the spandrel web to provide lateral restraint to the spandrel along its length.

Applying the loads at an eccentric location with respect to the unsymmetrical, L-shaped spandrel’s cross section causes vertical displacement in addition to significant lateral displacement and rotation in the spandrel. The greatest effects of torsion and shear occur near the end of the spandrel. By following current code requirements, this complex structural behavior, coupled with the heavy loadings, often requires the designer to use tightly spaced, closed stirrups at the ends of the spandrel. By locating numerous closed stirrups in the end regions of the spandrel, the placement of prestressing strands and reinforcing bars is complicated and these zones become even more congested.

For many years, spandrels have been designed according to the general procedure originally proposed by Zia and McGee (1974) and later by the modified method introduced by Zia and Hsu (1978 and 2004). The design procedure was developed based on the results of laboratory testing of small, symmetrical-flanged sections under controlled loadings designed to produce various combinations of torsion, shear, and bending. The procedure was not validated for slender spandrels, such as those used in practice, until investigations by Raths (1984) and Klein (1986) revealed that the behavior of spandrels is significantly affected by their support conditions. A significant plate-bending effect was observed in the web of the slender spandrel.

Because of this observed behavior,
the need for closed stirrups in slender, noncompact sections, such as the L-shaped spandrel, has been questioned. In addition, it has been suggested that welded-wire reinforcement (WWR) could be used effectively to reinforce the web of slender spandrels instead of using closed stirrups. Using flat sheets of WWR would greatly simplify the fabrication process, thereby reducing the cost of production.

THE NEED FOR CLOSED TIES

Simply stated, the fundamental concept of providing reinforcement in a concrete member is to position the reinforcement to cross planes of cracking within the concrete. For flexural and shear reinforcement, this concept is easy to visualize: longitudinal bars are provided to cross vertical crack planes for flexure, and vertical bars are provided to cross inclined shear crack planes. The concept of using closed ties for torsional reinforcement is exactly the same; reinforcement is provided to cross planes of cracking caused by an externally applied torque. Frequently, however, the case of torsion can be more difficult to visualize.

It is well established that the failure of a reinforced concrete section subjected to flexure and torsion will be in the form of skew bending.6-9 This skew plane may be idealized by three inclined edges forming a distorted surface. In compact, rectangular sections, this failure plane is crossed effectively on all four faces by closed ties, as shown in Fig. 1. For slender sections, however, the value of the ties’ shorter legs is questionable because the projection of the failure plane crossing the narrow face of the member is often less than the longitudinal spacing of such ties (Fig. 1).

It has been suggested, considering a slender spandrel as an example, that the legs of the closed ties that cross the top or bottom edge of the web probably have minimal effect on the overall torsional resistance of the member. Rather, it is the vertical legs of such closed ties that provide the majority of the torsional resistance because these legs cross the failure plane along the much longer inner and outer faces of the web.

For a slender spandrel, assuming a failure angle of 45 degrees, the short legs of any closed ties would effectively contribute to the torsional resistance of the member only if the tie spacing s were at least equal to half of the web thickness b. A tie spacing s greater than \(\frac{1}{2}b\) creates a situation where the top edge of the failure surface will likely pass between adjacent ties. Thus, for the typical web thickness of a slender spandrel (Fig. 2), the contribution of the short legs of stirrups spaced farther than 4 in. (102 mm) apart is questionable. Such tight stirrup spacing (less than 4 in. [102 mm]) is uncommon, however, for long slender members. Thus, from the geometry of the skewed failure surface, the short legs of closed ties do not seem to significantly contribute to the torsional resistance of slender spandrels.

If the short legs of closed ties are indeed insignificant to the torsional resistance of slender members, then the proposal to eliminate closed ties from slender spandrels has merit. Accordingly, closed ties could be replaced with a combination of straight bars, L-shaped bars, and sheets of WWR. Given the potential value of reducing the extensive production difficulties associated with closed reinforcement, it is worth investigating whether providing closed ties in slender members is necessary.

OBJECTIVE AND SCOPE

This study’s main objective was to validate the concept of eliminating closed stirrups in slender spandrels. In the experimental program, four full-scale, prestressed concrete, L-shaped spandrels were fabricated and tested to failure.
During design and detailing of the four test spandrels, no closed stirrups were used to resist shear or torsion forces in the members (as is common in current practice). Instead, open reinforcement was provided to resist out-of-plane flexure, web plate bending, and shear.

Special reinforcement details were developed to address localized issues, such as punching shear failure in the web, web-to-ledge attachment, and ledge bending. This approach resulted in prestressed concrete spandrels that were reinforced with prestressing strands; straight mild-steel reinforcing bars; open, bent mild-steel reinforcing bars including L, U, and C shapes; and flat sheets of WWR. Closed stirrups were virtually eliminated from the four tested beams.

TEST SPECIMENS

Figure 2 shows all of the tested spandrels, which had the same general configuration and cross section. Two spandrels measured 45 ft 6 in. (13.9 m) long from end to end, and two spandrels measured 30 ft 6 in. (9.3 m) from end to end. All spandrels were 5 ft (1.5 m) deep and had a web thickness of 8 in. (203 mm) and an 8-in-square continuous ledge running along the bottom of the inner web face. The ledge was cut back 12 in. (305 mm) from each end in all spandrels to replicate a typical field detail, where a spandrel is supported only at its web, bolted squarely to the precast, prestressed columns.

In addition, two through-holes were provided at each end of each specimen. These holes were set 6 in. (152 mm) from the top and bottom, as shown in Fig. 2. The holes were sized to accommodate the high-strength threaded rods used to bolt the spandrels to the test frame in a way that mimicked field conditions. Embedded steel plates were provided at the inner face of each spandrel and were welded to similar plates embedded in the flanges of the double tees. The embedded plates were centered between the double-tee stems and were located approximately 26 in. (660 mm) from the top edge of each spandrel.

All specimens in this study were fabricated by a PCI Producer Member. The two spandrels of a given length were cast together in pairs using the same strand pattern and concrete to ensure that each pair would have similar prestressing forces and concrete properties. The spandrels were delivered to the Constructed Facilities Laboratory at North Carolina State University for testing and evaluation as each was needed during the testing program.

Concrete strengths for each spandrel were based on the average strength of three 4 in. × 8 in. cylinders (102 mm × 204 mm) tested in compliance with ASTM C 3910 (Table 1).

The entire concrete stress-strain curve was evaluated based on a pair of concrete cylinders cast with the 45-ft-long (13.7 m) spandrels to determine the elastic modulus of the concrete for use in future analysis. The measured elastic modulus of 2570 ksi (17.7 GPa) was found to be lower than the value based on 57,000 psi (391 MPa). However, this low value was consistent with data provided by the precast concrete producer, and is attributed to the characteristics of the coarse aggregate used in this concrete.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Nominal Length, ft (m)</th>
<th>Date Cast</th>
<th>Dates of Testing</th>
<th>Specified Concrete Strength, psi (MPa)</th>
<th>Cylinder Strength, psi (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP1</td>
<td>30 (9.1)</td>
<td>September 7, 2005</td>
<td>October 26–27, 2005</td>
<td>5000 (34.5)</td>
<td>7450 (51.4)</td>
</tr>
<tr>
<td>SP2</td>
<td>30 (9.1)</td>
<td>September 7, 2005</td>
<td>September 29–30, 2005</td>
<td>5000 (34.5)</td>
<td>6940 (47.8)</td>
</tr>
<tr>
<td>SP3</td>
<td>45 (13.7)</td>
<td>November 9, 2005</td>
<td>November 28–29, 2005</td>
<td>6000 (41.4)</td>
<td>5790 (39.9)</td>
</tr>
<tr>
<td>SP4</td>
<td>45 (13.7)</td>
<td>November 9, 2005</td>
<td>December 13–14, 2005</td>
<td>6000 (41.4)</td>
<td>7190 (49.6)</td>
</tr>
</tbody>
</table>

DESIGN LOADS

In addition to their own weight, the tested spandrels were designed to support one end of a 60-ft-long (18.3 m) 10DT26 double tee weighing 718 lb/ft (10.5 kN/m). The applied live load was 40 psf (1.9 kN/m²) and the snow load was 30 psf (1.4 kN/m²), both of which were considered to cover the entire deck surface. The American Concrete Institute’s ACI 31811 load factors were used. All loads applied on the spandrels acted eccentrically because they were transferred (via bearing) from the double-tee stems to the spandrel ledge.

REINFORCEMENT DETAILS

Prestressed and mild-steel reinforcement was used along with WWR to resist the combination of shear, bending, and torsion that was induced in each spandrel. Because researchers wanted to evaluate the feasibility of eliminating closed stirrups in spandrel end regions, the four test spandrels were carefully designed and detailed to prevent unwanted localized failures, such as ledge punching under the double-tee stems or ledge-to-web attachment failure.

In addition, the spandrels were over-reinforced in flexure to ensure that their failures were governed by end-region behavior. Over-reinforcement of the flexural capacity was accomplished by increasing the number of prestressing strands in the bottom of the spandrels.

In the cross sections of the 30-ft-long (9.1 m) spandrels, SP1 and SP2, nine low-relaxation, 270 ksi (1860 MPa), ½-in.-diameter (12.7 mm) special strands were placed. Of these, five
were fully tensioned at the bottom of the section. A unique strand pattern was employed to accommodate 17 strands within the cross sections of the 45-ft-long (13.7 m) spandrels (SP3 and SP4). Of the 17 strands, 15 were in the bottom of the section and 8 were partially tensioned. This unique strand pattern was employed to provide sufficient longitudinal reinforcement to prevent premature flexural failure during testing while keeping the prestress level realistic for a given span.

Prestressing was the same for both spandrels of a given length, and all strands were fully bonded in all four members. Figure 3 shows the strand patterns and initial prestressing tension.

In addition to the prestressing strands, mild-steel reinforcement was used to provide ledge reinforcement. Figure 4 shows how the same configuration was used in the ledges of all four spandrels. Continuous, No. 6 (19M), mild-steel reinforcing bars ran along the outer edge of the ledge farthest from the spandrel face, providing reinforcement. C-shaped, mild-steel reinforcing bars were also placed at 8 in. (203 mm) centers along the entire length of the ledge and served to tie the ledge back to the spandrel web.

These C-shaped, mild-steel reinforcing bars were not provided directly underneath the double-tee stems, however. A weld detail was developed for these locations to provide closer bar spacing, enhanced web-to-ledge attachment, and greater resistance to punching shear. The detail consisted of short pieces of angle welded to closely spaced, bent, mild-steel reinforcing bars. The bars hooked around the strand in a similar manner to the C-shaped bars, allowing the attached steel angles to remain flush with the inner face of the ledge. Tables 2 and 3 present the reinforcement configuration within each spandrel web. Because general guidelines for designing concrete members without closed ties do not exist, the reinforcing schemes were developed using the design approach briefly summarized in the following section.

In all spandrels, flat sheets of WWR were provided on the front and back
faces of the webs. The WWR was supplemented by L-shaped bars that ran along the front face and hooked underneath the prestressing strands. In spandrels SP1 and SP2, two different sizes of WWR were used on the front face of the beam. A more tightly spaced WWR was used at the ends of these beams, but this was replaced by one with a wider spacing outside of the end regions.

In addition, five closed-loop, mild-steel stirrups were provided at each end of SP1 and SP2 to provide additional confinement. For the 45-ft-long (13.7 m) spandrels, these stirrups were eliminated in favor of two pairs of C-shaped, mild-steel reinforcing bars at each end, which resulted in beams completely devoid of closed reinforcement.

**DESIGN APPROACH**

Because no formal approach currently exists for the design of slender spandrels without closed mild-steel reinforcement, the following guidelines were used to create the spandrels tested in this program. It was also hoped that this approach would demonstrate the potential for designing slender spandrels without closed ties.

The general approach followed in the design of the test spandrels was to reinforce the spandrel webs for out-of-plane web bending in lieu of designing directly for torsion. Some brief details of the design concept are outlined using Fig. 5:

- Traditional torsion provisions are neglected.
- Torque acting on the spandrel cross section was determined from the applied loads and the eccentricity of the ledge reaction. The torque generated due to the eccentric dead load from the ledge itself was neglected.
- The applied torque was used to calculate the lateral reactions at the connections to the columns.
- The failure plane was assumed to be a 45-degree line extending upward from the center of the bottom connection.
- The perpendicular distance between the center of the top horizontal connection and the assumed 45-degree failure plane $R$ is the moment arm acting on that failure plane.
- Requirements for hanger reinforcement and vertical shear reinforcement are calculated using conventional methods.
- Reinforcement was distributed along the inner face of the spandrel (ledge side) so that requirements for hanger reinforcement were met. The inner face reinforcement also had to satisfy half of the vertical-shear reinforcement requirement in addition to meeting the out-of-plane bending requirement along the assumed 45-degree failure plane. A combination of WWR and L-shaped mild-steel reinforcing bars was used to meet these requirements. At the ends of the members, where out-of-plane plate-bending effects were most demanding, the L-shaped mild-steel reinforcing bars and WWR were supplemented with longitudinal mild-steel reinforcing bars, either straight or U-shaped, to resist the tensile forces across the assumed failure plane. The way in which reinforcement is distributed within the inner face

Table 3. Web Reinforcement of Spandrels SP3 and SP4

<table>
<thead>
<tr>
<th>Detail:</th>
<th>(2) #4 C-bar each location</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP3:</td>
<td>W4.0 by W4.0 mesh 6&quot; x 6&quot;</td>
</tr>
<tr>
<td></td>
<td>SP3 - No mesh first 7' 6&quot; x 6&quot; for balance</td>
</tr>
<tr>
<td></td>
<td>SP4 - 6&quot; x 6&quot; entire face</td>
</tr>
<tr>
<td></td>
<td>#4 U-bar 6' long (Typ. of 2 ea. end)</td>
</tr>
<tr>
<td></td>
<td>#4 U-bar 2' long (Typ. of 2 ea. end)</td>
</tr>
<tr>
<td></td>
<td>#4 U-bar 2' long (Typ. of 2 ea. end)</td>
</tr>
<tr>
<td></td>
<td>#4 U-bar 6' long (Typ. of 2 ea. end)</td>
</tr>
</tbody>
</table>

| SP4:   | First bar 15” from ends. |
|        | 15 bars in next 94”. |
|        | A bar per stem thereafter. |
|        | Mesh as shown. |
|        | U-bars as shown. |
|        | 2' long bars confine connections. |

Table Note: ‘ = ft; " = in.; 1 ft = 0.3048 m; 1 in. = 25.4 mm.
of the two spandrels varies somewhat because different sizes and arrangements of the WWR, L-shaped mild-steel reinforcing bars, and longitudinal mild-steel reinforcing bars are used in each spandrel.

- Reinforcement was provided on the outer face of the web (the face without a ledge) to satisfy half of the total vertical shear reinforcement requirement. In the case of both tested spandrels, this reinforcement was provided by a continuous sheet of WWR.
- Web bending is checked about a second 45-degree plane, drawn to include the effect of the lower lateral reaction. Because the lateral reactions are assumed to be equal, the magnitude of the out-of-plane bending moment will be the same at the secondary plane as it is at the assumed failure plane.

**TEST SETUP**

The general setup used to test both lengths of spandrels was the same (Fig. 6). Spandrels were simply supported and loaded with simply supported double tees. One end of each double tee rested on the spandrel ledge, while the other was supported on a system of large precast concrete blocks and steel channels.

Each double tee was connected to the inner spandrel face via a single deck tie centered between stems. This tie was welded to plates embedded in the spandrel web. The double tees were not connected to one another to prevent unintended lateral load transfer from one double tee to adjacent double tees as the spandrels deflected during testing.

The test setup consisted of several primary components, including the following:

- A system of columns, beams, and supports was designed to transfer the end reactions of a spandrel to the strong floor with minimal support deflections. A spandrel end rested on a rigid support base, which provided the vertical reaction. Each spandrel end was also restrained by heavy threaded rods to a rigid steel column, providing lateral restraint.
- A system of spreader beams, tie-down rods, hydraulic jacks, and a control system was designed to produce the required applied load.
that was evenly transferred to the appropriate points on the test specimens.

- A system of concrete support blocks, steel channels, and tie-down rods supported the end of the double tee opposite the spandrel.
- An array of load cells and other instrumentation monitored spandrel behavior.
- Figure 6 shows the profile of a typical test setup. The primary difference between the setup for the 30-ft-long (9.1 m) and the 45-ft-long (13.7 m) specimens was the extra double tees needed for the longer spandrels. For the tests of spandrels SP1 and SP2, three 10-ft-wide (3 m) double tees were used to load the spandrel ledge. However, for spandrels SP3 and SP4, four 10-ft-wide double tees were used in conjunction with a 5-ft-wide (1.5 m) single tee to cover the 45 ft (13.7 m) length. Because one hydraulic jack was used to load each double-tee section, a half-capacity hydraulic jack had to be used to load the lone stem of the single tee.

This configuration ensured that at a given pressure, the load produced by the smaller jack on one stem was half of that produced by the larger jack on two stems. In addition, the deck tie for the single tee had to be centered directly over this tee’s stem. Figures 7 and 8 show photographs of the completed test setups for a typical 30-ft-long (9.1 m) and 45-ft-long (13.7 m) specimen, respectively.

**LOADING SEQUENCE**

Based on the design loads discussed previously, the end reactions for several desired load combinations were determined. These reactions were monitored throughout all of the tests and served as the basis for controlling the hydraulic-jack loading system during each test. The spandrel end reaction is considered the total vertical force necessary to support one end of a simply supported test spandrel.

The following load combinations were used in the test program: service load, DL+LL; reduced service load with snow (specified by the American Soci-
As the load was held at these marked at all pertinent load levels for observations were made, and cracks were marked at all pertinent load levels for each cycle. The load was held at these various levels during all cycles to provide time for marking of cracks and making observations.

The 45-ft-long (13.7 m) spandrels were tested in a similar fashion, except that they were loaded with one additional loading cycle. These spandrels were loaded to service, unloaded, and then reloaded to service plus snow. The loading program from that point was identical to that just described for the 30-ft-long (9.1 m) beams, including the 24-hour load test. **Bearing Pads**

In discussing the setups for all four spandrel tests, it is important to mention the bearing pads used in the various tests. Because boundary connections are critical to slender spandrel behavior, bearing pads can have a large influence on spandrel response.

In the first test (spandrel SP2), flexible neoprene pads were used between the double-tee stems and the spandrel ledge. These pads deformed severely, even under the self-weight of the deck alone. In this first test, no bearing pads were used between the main spandrel reactions and the steel plates resting on the main load cells. All spandrels were cast against steel forms and, thus, these bearing surfaces were originally deemed suitable for direct bearing against the steel plates with a thin layer of grout. This configuration was initially chosen to minimize support deflections.

While there were no particular problems related to any of the bearing locations during the testing of spandrel SP2, preliminary data indicated that the measured lateral reactions were not as high as they were theoretically expected to be. It was concluded that rotational resistance at the main supports, along with friction at the double-tee bearing locations, reduced the reactions at the lateral tiebacks. While bearing pad friction certainly exists in reality, serving to lessen the out-of-plane demands placed on a slender spandrel, by no means should this friction be relied on in design.

Because it was desired to subject the test spandrels to the most severe loading case possible, the setup was modified for the remaining three tests (spandrels SP1, SP3, and SP4) in an attempt to reduce friction at all bearing locations. Teflon-coated bearing pads and stainless-steel plates were introduced in an attempt to increase the lateral reactions and to ensure that the remaining spandrels were being subjected to significant out-of-plane behavior.

In each of the three remaining tests, slide bearings were used to limit friction between all double-tee stems and the spandrel ledge and also at both vertical spandrel bearing reactions. The slide bearings proved effective, and measured lateral reactions increased significantly.

**INSTRUMENTATION**

Four basic types of instrumentation were used to monitor spandrel behavior during testing. All instruments were wired into a computerized data acquisition system, and data were recorded electronically.

Six load cells were used to measure the vertical and lateral spandrel reactions at both ends of each spandrel. Two additional load cells were used to monitor the load being applied by the jacks. These additional load cells monitored reactions from the spreader system on top of the double tees and were used to ensure that load was being distributed evenly across the spandrel. In addition, a pressure transducer was used to monitor the pressure being applied by the hydraulic pump. Potentiometers were used to measure the vertical displacements of each spandrel. Vertical displacement measurements were taken along the longitudinal centerline of the spandrel...
webs at their midpoints and at both quarterspans. In addition, vertical measurements were taken at the innermost edge of each spandrel ledge at the midpoint and at both quarterspans. A final vertical measurement was taken at one of the main vertical reactions for each spandrel to monitor potential support displacement.

Potentiometers were also used to measure lateral displacements of each spandrel. Lateral displacement at midpoint was monitored at the top and bottom edges of the web. Lateral displacement was also monitored at both lateral supports on one end to record any support displacements.

Inclinometers were used to measure the lateral rotation of each spandrel web at both quarter-points.

Twenty-two wire-arch gauges or PI gauges were used to measure concrete strains on the top, bottom, and inner faces of each spandrel. Three gauges were used to measure the compressive flexural strains at the midpoint and both quarter-points on the top surface of each member. Likewise, three gauges were used along the bottoms of each member to monitor the tensile flexural strains at midpoint and both quarterspans. The remaining sixteen gauges were arranged in pairs on the inner surface (ledge side) at both ends of each spandrel.

**TEST RESULTS**

All four specimens sustained their ACI 318 factored design loads for 24 hours, and all demonstrated ultimate capacities far in excess of those factored loads. In addition, all four specimens passed the 1-hour ACI recovery criterion for 24-hour load tests for both the midpoint vertical and lateral deflections.

Tests of both of the 30-ft-long (9.1 m) spandrels were ultimately terminated due to localized failures, but both 45-ft-long (13.7 m) beams failed in their end regions due to a global skew-bending mechanism. Table 5 shows the ultimate failure loads and corresponding deflections. It should be noted that the reactions given in the table represent the total force needed to support one end of a simply supported spandrel. These values include the self-weight of the spandrel, double tees, and loading system.

**Cracking Patterns**

The cracking patterns observed in all four test specimens were fairly consistent with one another and were in agreement with what would typically be expected for a slender spandrel subjected to eccentric ledge loading. Inner face cracking patterns were classically diagonal, reflecting the additive nature of the principal tensile stresses caused by shear and torsion on the ledge side of the L-shaped spandrel. On the outer spandrel faces, where principal tensile stresses from shear and torsion tend to negate one another, flexural cracking was primarily observed. Figure 9 shows the cracking patterns observed on the inner faces during testing of the 30-ft-long (9.1 m) spandrels. In both cases, diagonal cracking initiated at the outer corners of the ledge and extended upward along a diagonal at an angle of roughly 45 degrees. Moving away from the ends, cracks extended...
from the ledge diagonally upward toward the center of the beam, but the angles of these cracks grew progressively flatter. This gradual reduction in crack angle, culminating in nearly horizontal cracks at the center of each beam, results in the rainbow-shaped cracking pattern observed on the spandrel faces. As mentioned previously, the cracking observed in spandrel SP1 was more extensive than that observed in spandrel SP2, even though spandrel SP2 was subjected to higher loads. This discrepancy is almost certainly due to the inclusion of slide bearings in the test of spandrel SP1.

Cracking in the inner faces of spandrels SP3 and SP4 was of a similar pattern to that observed in the tests of the shorter spandrels. The ends of each member were marked by extensive diagonal cracking, which primarily extended from the corners of the ledge at an angle of roughly 45 degrees. As in the previous tests, the angle of the diagonal cracks tended to flatten out toward the center of each member, ultimately resulting in a sort of rainbow-shaped cracking pattern with near-horizontal cracks on the inner face at midpoint. **Figure 10** shows the cracking observed on the inner faces of spandrels SP3 and SP4.

A few vertical flexural cracks can also be seen extending upward from the bottom of the ledge. These cracks wrap under the bottom of the spandrels and connect to the flexural cracks on their outer faces.

Cracking patterns on the outer faces of all four spandrels were also similar, as all exhibited flexural cracking on
the outer face, initiating at the bottom edges of their webs and extending vertically upward. As would be expected, flexural cracks were most prominent near the midpoints of the members, with the heights of the flexural cracks decreasing toward the spandrels’ ends.

Figure 11 shows this outer-face cracking pattern for spandrel SP2, which is also typical of the results for spandrel SP1. Figure 12 shows cracking on the outer face of spandrel SP3, which is also typical of the cracking observed on the outer face of spandrel SP4. It can easily be seen from the figures that, as with the inner-face cracking, the outer-face cracking was much more severe in spandrels SP3 and SP4 than it was in spandrels SP1 and SP2.

An interesting feature of the outer-face cracking pattern seen in spandrels SP3 and SP4 is the presence of cracks extending diagonally downward at an angle of roughly 45 degrees from the top lateral tiebacks. These outer-face cracks run perpendicular to the diagonal cracks on the inner surface of the same upper corners (see Fig. 10).

This result is explained by the fact that the top lateral tieback has the tendency to bend back the corners of the spandrel as the inner face of the spandrel is pulled away from the supports during loading. Diagonal cracks develop along the inner surface parallel to this bend. A parallel compression zone exists on the outer surface. The principal tensile direction of this compression zone is perpendicular to the bend, which is reflected in the cracking pattern.

Failure Modes

For spandrel SP1, loading was terminated due to a localized ledge-to-web attachment failure that developed at one end of the beam. This occurred at a load far greater than the factored design load, however, and the spandrel showed extensive diagonal and rainbow cracking in its inner face, along with extensive flexural cracking on its outer face, prior to this localized failure. Spandrel SP2 also exhibited a localized failure, but the failure occurred at the location of one of the double tees used to load the beam.

Similar to spandrel SP1, spandrel SP2 reached a failure load much greater than the factored design load. However, spandrel SP2 only showed moderate diagonal and rainbow cracking on the inner face and moderate flexural cracking on the outer face, even though the ultimate loads sustained by spandrel SP2 were in excess of those sustained by spandrel SP1. Recall that spandrel SP2 did not have the slide bearings that were included in the testing setup of spandrel SP1.

Based on the experience gained during testing of the 30-ft-long (9.1 m) spandrels, localized failures did not occur during testing of the 45-ft-long (13.7 m) specimens. Both of the 45-ft-long specimens ultimately failed in their end regions due to...
a global skew-bending mechanism in combination with vertical shear. Spandrels SP3 and SP4 failed in virtually identical modes at nearly identical end reactions. The only difference in the failures of these two specimens was that spandrel SP3 failed in the right end region, while spandrel SP4 failed in the left end region. Both of these 45-ft-long (13.7 m) spandrels failed along a skewed-diagonal crack and showed extensive diagonal and rainbow cracking on their inner faces along with extensive flexural cracking on their outer faces.

The skew-crack plane intersected the bottom edge of a given member approximately 4 ft (1.2 m) from the respective end of that member and intersected the top edge of that member approximately 8 ft (2.4 m) from the respective end. Figures 13 through 15 show this failure mechanism for the 45-ft-long (13.7 m) specimens.

### Deflections

Because the cracking patterns were similar for all four spandrels, it is logical that the deflection patterns also correlate. In general, vertical and lateral deflections were significant features in the overall behavior of the spandrels. Measured data demonstrate that at midpoint, points at the bottom of a spandrel moved downward and outward (away from the ledge face). Points at the top of a spandrel moved downward and inward (toward the ledge face). As such, each spandrel face was substantially warped toward the end of each test.

Vertical-load-deflection curves plotted based on values at midpoint of the spandrels illustrate the various loading cycles endured by each. Residual deflections can be seen at the end of each of these cycles, with subsequent cycles continuing from the residual values. Horizontal plateaus are also present in the load-deflection data, which indicate increasing deflections at a constant load for several instances throughout the testing. These plateaus are a byproduct of holding the applied load at several intervals to make observations and mark cracks. In most cases, these plateaus represent pauses of 5 to 10 minutes during the testing, with the exception of the large plateau corresponding to the 24-hour load test. Similar plateaus are seen in most of the recorded data. Figure 16 shows a vertical-load-deflection curve measured at the midpoint of spandrel SP3. This curve is also similar to the curve obtained for spandrel SP4. The curves obtained for spandrels SP1 and SP2 show similar trends but have different magnitudes of load and deflection.

In examining Fig. 16, note that the zero deflection reading occurs at a spandrel reaction of roughly 22 kip (98 kN). This reaction corresponds to the self-weight of the system, including the spandrel, the double tees, and the loading system. Deflections were assumed to be zero after each spandrel was completely assembled.

While vertical deflection data are certainly valuable, it is the out-of-plane behavior of the test spandrels that was of real interest to the researchers. Figure 17 shows a typical load-deflection curve representing the top and bottom lateral deflections as recorded at midpoint of spandrel SP4. As with the vertical-load-deflection curve, this plot is also similar to that of spandrel SP3. In addition, it represents the general trend seen in spandrels SP1 and SP2, though magnitudes of the data were different for the shorter beams.

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**Fig. 16.** Typical load-vertical deflection curve from testing of the 45-ft-long spandrel (data from spandrel SP3). Note: DL = dead load; LL = live load; SL = snow load; 1 in. = 25.4 mm; 1 kip = 4.448 kN.
As mentioned previously, the top of the spandrel web had a tendency to move inward, which is reflected as a positive deflection in this graph. Outward motion of the lower points on the spandrel web is reflected as negative. In every case, the maximum magnitude of measured lateral deflection occurred at the bottom edge of each beam.

Again, the initial spandrel reaction of roughly 22 kip (98 kN) represents the self-weight of the spandrel, the double tees, and loading equipment. What can be seen from this figure is that the magnitudes of the lateral deflections were similar to the vertical deflections. In fact, in Table 5, one can see that in each of the four tests, magnitudes of maximum lateral deflections were far greater than the magnitudes of maximum vertical deflections. This result further highlights the significance of out-of-plane behavior in overall spandrel response.

**DISCUSSION OF TEST RESULTS**

In considering the test results, it is important to reiterate that the design of the spandrels ignored the currently accepted torsion design equations and, further, did not include any of the reinforcing details typically associated with current shear/torsion design practice. All four spandrels were fabricated without any reinforcement crossing the top of the web outside of the first 14 in. (356 mm) at each end. All sheets of WWR were unbent, and no provisions were made to simulate closed ties in any way.

In addition, reinforcement was detailed such that it could be placed after all of the strands were stressed, allowing for strands to be tensioned in a totally unobstructed casting bed. Detailing the spandrels by this method dramatically increased their constructability; thus, this design approach has the potential to greatly enhance plant productivity.

All four spandrels in the test program behaved satisfactorily at all loading conditions up to and including the factored load condition. In addition, all four spandrels easily passed the ACI 318 recovery criteria for a 24-hour load test and recovered to acceptable levels within 1 hour of releasing that load. It is important to note that this load test was conducted at the load level of 1.2DL + 1.6LL + 0.5SL, which is well in excess of the 0.85(1.4DL + 1.7LL) minimum loading level prescribed by Chapter 20 of ACI 318 for the 24-hour test.

What is further demonstrated by the test results is that lateral bending, torsion, and web-plate bending all greatly affected the responses of the spandrels. Even in the cases of spandrels SP1 and SP2, in which failures were localized, significant out-of-plane deformations were recorded, as reflected by the rainbow cracking patterns.

For the 45-ft-long (13.7 m) specimens, the interaction between flexure, shear, and torsion was evident in the dramatic skew bending failure mode observed during both tests. In the case of these 45-ft-long members, the ledge detailing, along with the extra flexural reinforcement, proved satisfactory at forcing the skew-bending failure mode and demonstrating the capability of the uniquely reinforced end regions to resist combined shear, torsion, and plate bending.

An additional point that should be raised in regard to this testing is the influence of bearing friction. Friction at bearing connections can play a major

![Typical load-lateral deflection curve from testing of the 45-ft-long spandrel (data from spandrel SP4). Note: DL = dead load; LL = live load; SL = snow load; 1 in. = 25.4 mm; 1 kip = 4.4kN.](image-url)
role in determining the out-of-plane behavior of a spandrel, as was highlighted in the experimental differences between spandrels SP2 and SP1. Incorporating slide bearings in the testing program were welded in this testing program were welded in service, further demonstrating the capability of open reinforcement to resist out-of-plane deformations in slender members.

A final note on the welded connections between double tees and spandrel face is that this connection configuration was a critical factor in determining the out-of-plane movement of the slender spandrels. Forces transferred through this connection have the potential to greatly influence behavior. All double-tee-to-spandrel connections in this testing program were welded to represent the reality of a parking structure, but the impact of such connections should be considered in any future studies.

CONCLUSIONS

The potential benefits of eliminating closed stirrups from slender precast concrete spandrels are immense. Open reinforcement provides greatly enhanced constructability over traditional closed stirrups and presents significant potential for increasing plant productivity. Unfortunately, the concept of constructing slender spandrels without closed stirrups is not yet a reality because practical guidelines for implementation do not exist. The relatively small testing program outlined here demonstrated the feasibility of this concept for one specific set of conditions.

While the test results of the four spandrels presented are encouraging, a large number of factors need to be examined, both experimentally and analytically, before the precast, prestressed concrete industry can fully adopt a new design approach for proportioning slender spandrels. However, with significant and appropriate study, it appears that a design approach could be developed for proportioning slender precast concrete spandrels without closed reinforcement while still maintaining the safety, serviceability, and reliability expected from precast concrete members.

ACKNOWLEDGMENTS

This project was sponsored by the combined efforts of Harry Gleich of Metromont Corp. in Greensville, S.C.; Don Logan of Stresscon Corp. in Colorado Springs, Colo.; and Ken Baur of High Concrete Products in Denver, Pa. The authors are grateful for the support and guidance provided by all three of these producer members throughout the duration of the project.

REFERENCES

This paper presents the results of nonlinear finite element analyses conducted to model the behavior of L-shaped, precast, prestressed concrete spandrels constructed with open web reinforcement. The finite element model was calibrated using experimental results from recent tests of slender, L-shaped, precast, prestressed concrete spandrels. Detailed correlative studies between analytical and experimental results are presented, demonstrating the capability of the finite element program to describe the observed experimental behavior.

The feasibility of using open web reinforcement in compact, L-shaped, precast, prestressed concrete spandrels to achieve a more construction-friendly reinforcement scheme is also examined. Five different web reinforcement configurations for the compact spandrels were studied in order to evaluate the contribution of closed stirrups to the spandrels’ shear-torsion behavior.

The behavior, ultimate load-carrying capacity, and mode of failure of both the slender and compact L-shaped precast, prestressed concrete spandrels are presented. For loading values near the ultimate, the out-of-plane bending behavior of compact, L-shaped, precast, prestressed concrete spandrels is strongly influenced by the web-reinforcement configuration. Results from the analysis show that for long-span, compact spandrels, open web reinforcement can be used effectively to resist torsional forces throughout the member.
Despite past research, there still exists a need to study the behavior of L-shaped, precast, prestressed concrete spandrels when subjected to different combinations of torsional, flexural, and shear loads. Industry methods and published procedures vary significantly with respect to several fundamental aspects of the design and detailing of such members. Current U.S. and Canadian provisions for the design of members for compatibility torsion are simple to use and conservative for design, but they often result in areas of heavily congested reinforcement within a beam.

Significant potential exists for reducing the complexity of L-shaped, precast, prestressed concrete spandrel designs by removing closed ties from slender members. Limited tests on full-scale L-shaped spandrels revealed the possibility of reducing the transverse reinforcement at their end regions. Elastic theory (assuming an uncracked section) is a necessary tool for proportioning the member. However, an analysis of the post-elastic behavior—including stiffness, deformation, and cracking patterns—is essential for evaluating the complete response of the member to different loading conditions.

Knowledge of the complete response of an L-shaped spandrel to different loading conditions is critical for assessing the amount of the transverse reinforcement needed at the member ends. Test results have shown that the torsional stiffness of a member is greatly affected by cracking and by the interaction among torsional, flexural, and shear loads.6 Figure 1 shows a typical L-shaped spandrel that is used in parking structures.

A unified procedure for the design of prestressed concrete members for shear and torsion was originally developed by Zia and McGee in 1974.4 Their design procedures were derived from a comprehensive set of test data and were coordinated with existing design practice. Further refinement of these procedures was subsequently proposed by Zia and Hsu.5

Although these procedures are commonly used, research data have never validated them for slender spandrels, which are typically used in practice.

Recent efforts to classify spandrel behavior include a study by Rahal and Collins,7 which describes a procedure to calculate compatibility torsion in spandrels. Their procedure relies on modified compression field theory to calculate the cracked torsional and flexural stiffnesses for sections subjected to various combinations of stress resultants. Rahal and Collins’ procedure was capable of predicting the response of concrete members where the effect of compatibility torsion is dominant.

The American Concrete Institute’s ACI 318-051 requires closed stirrups to be placed throughout a concrete member subjected to combined shear and torsion. According to this document, closed stirrups are mandatory to avoid spalling of the concrete cover. Test results by several researchers showed that this type of behavior is unlikely to occur in deep spandrels.

Recently, the Precast/Prestressed Concrete Institute (PCI), and many PCI Producer Members, have questioned the need for closed stirrups along the entire length of a slender spandrel. It should be noted that in the precast concrete industry, common detailing practices for torsional reinforcement in deep spandrels do not usually follow the ACI requirements. Transverse reinforcement is often provided in L-shaped spandrels with pairs of lap-spliced, mild-steel, U-shaped stirrups.5

Unfortunately, widespread, full-scale experimental testing to examine the influence of various web reinforcement configurations in L-shaped spandrels is prohibitively expensive.

Therefore, the use of nonlinear finite element analysis coupled with limited experimental studies is a powerful tool for predicting the behavior and failure modes of L-shaped, precast, prestressed concrete spandrels. The complex combination of stress resultants that develop in the member due to bending, shear, and torsion, as well as the size effect of the L-shaped spandrel’s slender web, dictate the intricacy of such analyses.

This paper presents the results of nonlinear finite element analyses conducted to simulate the behavior of L-shaped, precast, prestressed concrete spandrels. The main objective of the current study was to develop reliable and computationally efficient finite element models (FEMs) to analyze L-shaped, precast, prestressed concrete spandrels subjected to combined bending, shear, and torsion. Results from previous testing were used to calibrate the FEM. Once a model was validated, it was used to investigate the response of compact, L-shaped, precast, prestressed concrete spandrels designed with open web reinforcement.

The behavior, ultimate load-carrying capacity, and failure mode of both slender and compact, L-shaped, precast, prestressed concrete spandrels are presented. The influence of the lateral deck ties and several different
Fig. 2. Reinforcement details of spandrels SP3 and SP4. Note: ' = ft; " = in.; 1 ft = 304.8 mm; 1 in. = 25.4 mm; 1 lb = 0.00448 kN; 24 = 12M; 25 = 16M; 26 = 19M.

Fig. 3. Mesh dimensions used in the finite element model. Note: ' = ft; " = in.; 1 ft = 304.8 mm; 1 in. = 25.4 mm.
web reinforcement configurations on the out-of-plane behavior of compact, L-shaped, precast, prestressed concrete spandrels is also discussed.

VALIDATION OF THE FEM

The first reinforced concrete FEM that included the effects of cracking was developed in 1967. Cracks were modeled by separating the nodal points of the finite-element mesh, thus creating a discrete crack model. With the change of topology and the redefinition of nodal points, the narrow bandwidth of the stiffness matrix was destroyed, resulting in increased computational effort. Moreover, the lack of generality in crack orientation has made the discrete crack model unpopular. The need for a crack model offering automatic generation of cracks and complete generality in crack orientation, without the need for redefining the finite element topology, has led the majority of investigators to adopt other crack models.

In the current study, the ANATECH Concrete Analysis Program (ANACAP) was used to model the behavior of the L-shaped, precast, prestressed concrete spandrels. The concrete material model in ANACAP has evolved over the past 30 years and is based on smeared cracking methodology for the treatment of concrete tensile cracking. Modeling of the compressive behavior of the concrete follows the generally accepted principles of computational plasticity, though these principles are modified for the unique and computationally demanding aspects of concrete response.

Cracks are assumed to form perpendicular to the directions of the largest tensile strains. Multiple cracks are allowed to form at each material point, but they are constrained to be mutually orthogonal. At the onset of cracking, the normal stress across the crack is reduced, and the distribution of stresses around the crack is recalculated through iteration of equilibrium equations. This recalculation allows stress redistribution and load transfer to the reinforcement. Once a crack forms in the model, the direction of the crack remains fixed and it can never heal. However, a crack may close to resist compression and then reopen.

The smeared-crack model represents an engineering approximation to the concrete’s actual behavior and permits the analysis of concrete structures up to and during failure. In the smeared-crack approach, the modulus and strength of the concrete in the direction normal to an open-crack surface is zero, but the shear modulus and shear strength remain intact. The shear modulus is gradually reduced, however, as crack widths increase. This gradually reducing shear resistance is critical to the continued load resistance of the structure.

Several attempts have been made in the past few years to model the behavior of L-shaped, precast, prestressed concrete spandrels using finite element analysis. Nevertheless, the complex behavior of these spandrels under combined bending, shear, and torsion limited the previous analyses to modeling only linear-elastic behavior.

Two L-shaped, precast, prestressed concrete spandrels, denoted spandrels SP3 and SP4, were selected from the literature to validate the

<table>
<thead>
<tr>
<th>Property</th>
<th>SP3</th>
<th>SP4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete compressive strength, psi</td>
<td>5790</td>
<td>7190</td>
</tr>
<tr>
<td>Modulus of rupture of concrete, psi</td>
<td>456</td>
<td>509</td>
</tr>
<tr>
<td>Yield strength of welded wire reinforce, psi</td>
<td>98,000</td>
<td>98,000</td>
</tr>
<tr>
<td>Yield strength of conventional mild-steel reinforcing bars, psi</td>
<td>64,500</td>
<td>64,500</td>
</tr>
<tr>
<td>Yield strength of prestressing strands, psi</td>
<td>243,000</td>
<td>243,000</td>
</tr>
<tr>
<td>Prestressing losses, %</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>

Note: Modulus of elasticity of all conventional and prestressing steel is 29,000 ksi. 1 psi = 0.006895 MPa., 1 ksi = 6.895 MPa.
Modeling the Concrete Spandrels

Because geometry and loading of the members were symmetrical about their midspans, half of each spandrel was modeled using 20-node brick elements, each node having three translational degrees of freedom. The finite-element mesh was chosen so that elements would maintain acceptable aspect ratios while accurately representing geometry, loading conditions, and support conditions. Figure 3 shows the finite-element mesh dimensions used in the FEM.

Modeling the Prestressing and Mild-Steel Reinforcement

The prestressing force in each member was applied gradually to the spandrel ends in the model to replicate the transfer length of the strands. This was accomplished by splitting each strand into 10 small strands. Each small strand has one-tenth the area of the original strand, but all occupy virtually the same location in the spandrel.

The first of the 10 strands started at the spandrel end, and the 10th started at a distance equal to the transfer length. The remaining eight strands started at equal, incremental distances between the spandrel end and the transfer length, as shown in Fig. 4. The reinforcement was modeled as individual subelements within the concrete elements. The stress and stiffness of the mild-steel reinforcing bar subelements were superimposed on the concrete element in which the reinforcing bar resided. The analytical model accounted for every mild-steel reinforcing bar used in each of the spandrels.

Simulation of the Applied Load

Load was applied to the spandrel ledge at each double-tee stem as a uniform pressure acting over the stem bearing area. The analysis was conducted using an incremental-iterative solution procedure, in which the applied load was incrementally increased. The loading increment was set to 1 kip (4.448 kN) per step. Within each step, equilibrium was achieved and iteration was repeated until internal equilibrium conditions were sufficiently fulfilled and convergence was obtained. At the end of each step, the program adjusted the stiffness matrix to reflect any nonlinear changes in the spandrel’s stiffness.

The self-weight of the spandrel, loading jacks, and spreader beams, along with the weight of the double tees, were introduced at the first load-
March–April 2007

from the figures, it is observed that the predicted post-cracking stiffness is slightly lower than the measured values, especially for spandrel SP4. A significant portion of this error can possibly be attributed to the instruments used to obtain the vertical deflection measurements. As the spandrel rotates and deflects vertically, a component of the lateral deflections is included in the vertical measurements. This error, inherent to obtaining vertical
deflections were then increased to failure.

**Materials and Boundary Conditions**

Table 1 summarizes the material properties used in the FEM for spandrels SP3 and SP4. The spandrel model employed the same boundary conditions as those implemented in the laboratory tests. In the model, the spandrel was restrained vertically throughout its width for the first 12 in. (305 mm) along both ends to simulate the bearing pads used at the laboratory spandrels’ ends. Lateral restraint was provided throughout the width of the spandrel, 6 in. (152 mm) from each end and 12 in. (305 mm) from the top and bottom of the spandrel. This lateral restraint simulates the tiebacks provided by the threaded rods during laboratory testing of the actual spandrels. A symmetry boundary condition was applied at midspan for each analysis because only half of each spandrel was modeled.

**RESULTS AND DISCUSSION**

**Deflections**

Figures 5 and 6 plot the predicted and measured vertical end reactions versus midspan deflections for spandrels SP3 and SP4, respectively. It should be noted that the load was held during testing for several relatively long periods of time, including a 24-hour period, causing a small amount of creep, which is reflected by the progressive increase in residual deflections upon each unloading cycle. This short-term creep behavior was not simulated in the ANACAP program and, thus, the increases in deflection at various load levels are not seen in the FEM-predicted behavior. It should also be noted that the end reactions plotted for both spandrels represent the externally applied loads and do not include the dead load of the system. Linear behavior was predicted for both specimens up to the initiation of the first crack at a load level of 95 kip (423 kN). Predictably, this initial behavior was followed by a nonlinear behavior up to failure. In general, the FEM-predicted behavior is in good agreement with the measured values, with the exception of the effect of creep as discussed previously.

From the figures, it is observed that the predicted post-cracking stiffness is slightly lower than the measured values, especially for spandrel SP4. A significant portion of this error can possibly be attributed to the instruments used to obtain the vertical deflection measurements. As the spandrel rotates and deflects vertically, a component of the lateral deflections is included in the vertical measurements. This error, inherent to obtaining vertical

![Fig. 7. Cracking potential of spandrel SP3 with an end reaction of 60 kip (267 kN) (above) and 100 kip (445 kN) (below).](image)

![Fig. 8. Predicted crack pattern at different loading stages.](image)
measurements from a rotating cross-section that is moving both vertically and laterally, is discussed elsewhere. Contributions of the double tees at greater load levels could also result in the higher spandrel stiffness values than the predicted values.

**Crack Pattern**

Cracking potential is defined as the ratio of the principal concrete tensile stress to the tensile strength of the concrete at any given point in the analysis (expressed in terms of percentage). Concrete cracking will occur when the cracking potential reaches a value of 100%. At this stage, the principal tensile stress at a given location is equal to the tensile strength of the concrete. After cracking, the cracking potential will drop to zero in the vicinity of the crack. Figure 7 depicts the cracking potential for spandrel SP3 with an end reaction of 60 kip (267 kN). The figure clearly shows the tendency of the concrete to crack along a diagonal near the end of the spandrel. Figure 7 also shows the cracking potential of spandrel SP3 with an end reaction of 100 kip (445 kN). At an end reaction of 100 kip, the shear crack has already developed because the cracking potential in the marked area has been reduced to zero.

Although these figures are shown for spandrel SP3 only, spandrel SP4 had a nearly identical cracking pattern. Figure 8 shows the predicted cracking patterns for the spandrel at various loading stages. The FEM effectively captures the observed deflection behavior. In the model, the top of the spandrel rotates forward at midspan, the ledge rotates back, and the entire cross section deflects downward.

**Rotation**

Figures 9 and 10 show the predicted rotations of spandrels SP3 and SP4 at their quarter spans, respectively. FEM-predicted rotations compare well with the measured values up to failure. The figures clearly illustrate the capability of the FEM to reasonably predict the out-of-plane deflections of the spandrels.

**Shear Stresses**

Figure 11 illustrates the predicted shear stresses for spandrels SP3 and SP4 along the front face of the spandrels. High shear stresses were observed at the junction of the ledge and the spandrel web. Spandrel SP4 experienced slightly higher shear stresses than spandrel SP3 did at different loading stages. This increase could be attributed to the distribution of the web reinforcement at the ends of the spandrel. Spandrel SP4 had relatively uniform web reinforcement, whereas in spandrel SP3, the web reinforcement was more concentrated at the ends.

**Failure Mode**

In the laboratory, both spandrels SP3 and SP4 failed along a skewed-diagonal crack and experienced a horizontal separation across the diagonal crack extending across the top of the web. Compression shear failure at the end
regions of the spandrels was the governing mode of failure for both specimens.

Failure in the FEM ultimately occurred in both spandrels due to crushing of the concrete along the primary compressive strut, as shown in Fig. 12 for spandrel SP3 (spandrel SP4 was virtually identical). Analysis was terminated when the principal compressive strains along the compressive strut reached a value of 0.002, as recommended by modified compression field theory. The predicted failure loads for spandrels SP3 and SP4 are within 3% of the measured values. Table 2 summarizes the predicted ultimate loads and deflections for both specimens.

Influence of Deck Ties

Deck ties consisting of steel plates of dimensions 3 in. × 6 in. × ¼ in. (76 mm × 152 mm × 0.5 mm) were used to connect the double tees to the spandrel webs in the actual specimens. To investigate the influence of the lateral restraint provided by deck ties on the predicted behavior of the spandrel, the FEM incorporated lateral springs at the spandrel front face at the center of these plates. The stiffness of the springs was set to 21,750 kip/in. (3809 kN/m), which is equivalent to $EA/L$ of a given steel plate, in which $E$ is the elastic modulus of the steel, $A$ is the cross-sectional area of the plate, and $L$ is the length of the plate.

It should be noted that using spring supports simulates an upper boundary condition for the lateral stiffness provided in the actual test. Figure 13 shows the predicted load-deflection behaviors with and without deck ties for spandrel SP3. The finite-element analysis demonstrated the lateral restraint provided by the deck ties had a minor effect on the stiffness of the spandrel.

This discrepancy could be attributed to the fact that the location of the deck ties within the spandrel web nearly coincides with the center of rotation of the web. Figure 14 shows the FEM-predicted lateral displacements at midspan at the bottom of spandrel SP3. The lateral restraint provided by the deck ties reduces the post-cracking behavior of the spandrel.

Table 2. Results of the Finite Element Analysis for Specimens SP3 and SP4

<table>
<thead>
<tr>
<th></th>
<th>SP3</th>
<th></th>
<th>SP4</th>
</tr>
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<tbody>
<tr>
<td>$R_u$ kip</td>
<td>174</td>
<td>Experimental</td>
<td>177</td>
</tr>
<tr>
<td>$\Delta v_e$ in.</td>
<td>1.98</td>
<td>2.22</td>
<td>1.66</td>
</tr>
</tbody>
</table>

Note: $R_u$ = the end reaction of the spandrel at ultimate; $\Delta v_e$ = the vertical deflection at midspan at ultimate; 1 kip = 4.448 kN; 1 in. = 25.4 mm.
As expected, the actual behavior of the spandrel falls between the two extreme cases considered in the analysis. Such a phenomenon indicates that the assumed spring stiffness was much higher than the actual stiffness provided by the deck ties.

Lateral displacements 45% to 65%, depending on the load level. The finite-element analysis indicates that the only significant effect of the deck ties is the restraint of lateral displacements induced by bending about the weak axis of the spandrel.

Compact Sections

While the previous analysis focused on slender, L-shaped spandrel cross sections (d/b of 7.5), the following analysis is related to compact, L-shaped cross sections (d/b of 1.75), in which d and b are the depth and the width of the spandrel web, respectively. This study relies on the validated analytical model discussed previously to investigate the influence of various shear and torsion reinforcement schemes on the behavior of compact spandrels.

Five different reinforcement schemes were considered. Because the researchers desired to compare the transverse reinforcing schemes in the slender and compact L-shaped spandrels to one another, the cross-sectional dimensions and prestressing levels were kept constant for all five cases. All analyses were conducted using a 45 ft (13.7 m) span.

The compact section geometry and reinforcement layouts were proposed, designed, and detailed by the PCI Producer Members sponsoring the study. Longitudinal reinforcement complied with ACI 318-05 requirements. Shear and torsion design of the first reinforcement case (utilizing closed stirrups) followed the procedure recommended by Zia and Hsu.5 The remaining four reinforcement configurations are variations of the first.

Case 1 and 2 are included to demonstrate the efficiency of open vertical stirrups with 90-degree hooks at the top and bottom. Case 1 also serves as a basis for comparison with the other four cases because it is the only case currently accepted in common practice. The influence of hooking the vertical web reinforcement at the front face of the spandrel is investigated by comparison of cases 3 and 4.

In these cases, welded-wire rein-

![Fig. 13. Predicted load-deflection behavior with and without the deck ties for spandrel SP3.](image1)

![Fig. 14. Predicted lateral displacements at midspan for spandrel SP3.](image2)
Vertical Deflections

Figure 18 shows the vertical-load-deflection behaviors of the five compact, L-shaped spandrels for the different reinforcement configurations. Identical precracking and postcracking stiffnesses were predicted, regardless of the web reinforcement configuration.

All five load-deflection curves demonstrate a typical flexural response for the respective precast, prestressed concrete spandrel. Linear behavior was predicted up to the initiation of the first flexural crack at an end reaction of 45 kip (200 kN), followed by a nonlinear behavior to failure. All five cases demonstrate substantial ductility prior to failure. While the deflection behavior of the spandrel certainly does not provide great insight into the effectiveness of a particular shear and torsion reinforcement configuration.

Half of the compact L-shaped, precast, prestressed concrete spandrel was modeled using 1472 twenty-node brick elements, as shown in Fig. 17. The spandrel web was divided into four equal layers within its thickness to accurately model the shear-torsional stress distribution within the width of the spandrel. For all cases, the design concrete compressive strength and modulus of elasticity were taken as 6000 psi (41 MPa) and 4200 ksi (29 GPa), respectively. Grade 60 mild-steel reinforcement with a yield strength and modulus of elasticity of 60 ksi (414 MPa) and 29,000 ksi (200 GPa), respectively, was utilized as the non-prestressed reinforcement.

Seventeen 0.5-in.-diameter (13 mm) low-relaxation strands with a nominal cross-sectional area of 0.167 in.² (107 mm²) were used within the spandrel. Prestressing strands were modeled using the same approach as described for spandrels SP3 and SP4. Prestressing losses of 15% were assumed in the analysis.

Two prestressing strands were debonded for the first 4 ft (1219 mm) at each end of the spandrel to avoid crushing of the concrete in the end region. The spandrel was restrained vertically throughout the width of the web for the first 12 in. (305 mm) along the ends. Lateral restraints were provided 6 in. (152 mm) from each end at the top and bottom of the spandrel.

Nine spring supports were provided along the length of the spandrel to simulate deck ties. The springs were positioned at the top front face of the spandrel with an axial stiffness of 21,750 kip/in. (3809 kN/m), as discussed. Load was applied gradually using a step-by-step analysis, as described for spandrels SP3 and SP4.
load, while cases 1, 2, and 5 sustained a slightly higher end reaction of 105 kip (467 kN). Ultimate vertical deflections for the five cases ranged from 5.5 in. to 6.8 in. (140 mm to 173 mm), with cases 1, 2, and 5 outperforming cases 3 and 4.

The analysis indicates that all five reinforcement cases were sufficient for preventing premature end-region failures.

Cases 3 and 4 sustained an ultimate applied end reaction of approximately 100 kip (445 kN), not including dead load, while cases 1, 2, and 5 sustained a slightly higher end reaction of 105 kip (467 kN). Ultimate vertical deflections for the five cases ranged from 5.5 in. to 6.8 in. (140 mm to 173 mm), with cases 1, 2, and 5 outperforming cases 3 and 4.

Lateral Displacements

When lateral displacements at mid-span at ultimate load are considered, the influence of the five reinforcement configurations becomes much more pronounced, as shown in Fig. 19.

In the FEM, lateral displacements are predicted at the bottom edge of the web on the back face of the spandrel. Displacements toward the ledge side are considered positive, while those away from the ledge side are negative. While the ultimate end reactions sustained by the five cases are all similar, the lateral displacements predicted for each case vary substantially.

Case 1 (using closed stirrups) demonstrates the least lateral displacement of all cases. The maximum predicted lateral displacement at mid-span was about 0.8 in. (20 mm). Absence of the hooks on the front vertical web reinforcement (case 4) resulted in larger lateral deformations of the spandrel than in other cases. The maximum lateral displacement in this case was nearly three times that predicted using closed stirrups.

This behavior demonstrates that the lateral and torsional stiffness of the member is significantly influenced by the amount of reinforcement crossing the top and bottom faces of the web.
Interestingly, the lateral displacement results from case 5 are nearly identical to those from case 1. Therefore, the reinforcement crossing the top web face is more significant than that crossing the bottom web face. On the other hand, under service load, the lateral displacement of case 4 is about 0.4 in. (10 mm), almost twice that of the other four cases.

Crack Pattern

A similar crack pattern was predicted for all five cases, regardless of the web reinforcement configuration. Flexural cracks were initiated at an end reaction of 45 kip (200 kN), as shown in Fig. 20. These cracks were first initiated at the back face of the spandrel as a result of the out-of-plane bending behavior of the spandrel. The cracks started to propagate toward the ledge of the spandrel as the applied load was increased.

Localized cracks around the spring supports were also observed as the result of stress concentrations at these locations. Diagonal cracks at the spandrels’ ends started to appear shortly after the initiation of the flexural cracks at an end reaction of 55 kip (245 kN). As the load was increased, the cracks were further extended and diagonal tension cracks developed farther from the support. In general, extensive diagonal and rainbow cracking was predicted by the FEMs along the front faces of the spandrels due to the combined torsional and shear stresses. The back faces of the spandrels showed rather evenly spaced vertical cracking, mostly due to the flexural effect (because the stresses due to torsion and shear counteracted each other). The vertical cracks were tallest toward the center and gradually decreased in height toward the end of the spandrel. Minor diagonal cracks were also predicted by the FEM at the back faces of the spandrels toward their ends.

Shear Stresses

Figure 21 shows the ultimate shear stress distributions at the ends of each spandrel for the different reinforcement configurations. The use of open vertical stirrups with 90-degree hooks at the top and bottom did not have any detrimental effect on the induced shear stresses in the spandrels (compared with the case with closed stirrups). The FEM predicted the same level of stress for both cases 1 and 2.

A direct comparison between cases 4 and 5 indicates that absence of the horizontal top web reinforcement increases the concrete shear stress 20%. It was also observed, by comparing the induced shear stresses in cases 3 and 4, that the presence of hooks enhances the
behavior and reduces the shear stresses 20%. Obviously, this is because the hooks provided more anchorage for the web reinforcement.

**Failure Mode**

Flexural failure due to crushing of the concrete at the midspan section of the spandrel was predicted by the FEM for all five cases. Failure loads were nearly identical for all specimens. Cases 1 and 4 exhibited the highest and lowest ultimate load-carrying capacity, respectively. Nevertheless, the variation of the ultimate load between these two extreme cases was less than 12 kip (54 kN), which corresponds to approximately 6% of the capacity of the spandrel. Finite-element analysis was terminated when the principal compressive strains exceeded 0.003 according to ACI 318-05.

It was observed that the principal compressive strains were much higher at the front face of the spandrel than at the back face due to out-of-plane bending behavior of the spandrel. Such behavior was highly pronounced for the spandrels analyzed without deck ties. At the onset of flexural failure, the maximum principal compressive strains along the diagonal compression strut were less than 0.002, which is recommended by other researchers for shear compression failure.

**Forced Shear Failure Mode**

To further examine the influence of the different web reinforcement configurations on the shear-torsional behavior, the authors conducted a series of experiments. The results are tabulated in Table 3.

**Table 3. Results of the Finite Element Analysis for Cases 1, 2, and 4 for Compact Sections**

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Flexural Reinforcement</th>
<th>$R_u$, kip</th>
<th>Failure Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Normal</td>
<td>104</td>
<td>Flexural failure</td>
</tr>
<tr>
<td>1</td>
<td>Nine #11 bars were added at midspan</td>
<td>133</td>
<td>Shear-compression failure</td>
</tr>
<tr>
<td>2</td>
<td>Normal</td>
<td>104</td>
<td>Flexural</td>
</tr>
<tr>
<td>2</td>
<td>Nine #11 bars were added at midspan</td>
<td>123</td>
<td>Shear-compression failure</td>
</tr>
<tr>
<td>4</td>
<td>Normal</td>
<td>99</td>
<td>Flexural</td>
</tr>
<tr>
<td>4</td>
<td>Nine #11 bars were added at midspan</td>
<td>110</td>
<td>Shear-compression failure</td>
</tr>
</tbody>
</table>

Note: $R_u$ = the end reaction of the spandrel at ultimate; 1 kip = 4.448 kN.
strength of compact, L-shaped spandrels, additional top and bottom flexural reinforcement was provided at midspan. Placement of this additional reinforcement was limited to between the quarter points \( L/4 \) and the midpoint \( L/2 \) to eliminate the possibility of affecting the shear-torsion strength of the spandrels at their end regions (\( L \) is the span of the spandrel).

It was intended that this additional reinforcement would prevent the flexural failure mode observed previously, allowing a mode governed by shear and torsion to develop. Cases 1, 2, and 4 were all reanalyzed with the additional flexural reinforcement, and Table 3 summarizes the results of the analysis.

In all three of these cases, failures occurred in the end regions and were due to crushing of the concrete along the primary compressive strut, as shown in Fig. 22. Finite-element analysis was terminated when the principal compressive strains along the compressive strut reached a value of 0.002. Figure 23 shows the predicted lateral displacements at midspan. The maximum predicted end reaction for the case with closed stirrups (case 1) was 133 kip (592 kN), which did not include dead load.

Finite-element analysis indicated that using open vertical stirrups with 90-degree hooks instead of closed stirrups did not have a dramatic effect on the strength of L-shaped spandrels. For case 2, the FEM predicted a reduction of 8% in the ultimate load-carrying capacity of the spandrel. Using open, unhooked web reinforcement (case 4) reduced the shear capacity of the spandrel 17% compared with case 1. Based on these results, the analysis indicates that it is possible to use open web reinforcement effectively in compact L-shaped spandrels, provided that the designer accounts for reductions in the shear-torsion strength of the spandrel.

CONCLUSIONS

Based on the results of this investigation, the following conclusions are drawn:

- FEM is capable of accurately predicting the response, up to failure, of L-shaped, precast, prestressed concrete spandrels subjected to combined shear, bending, and torsion.
- For the compact, L-shaped spandrels spanning 45 ft (13.7 m), typically used by the precast/prestressed concrete industry, flexural failure controls design.

In this case, web reinforcement configurations have a trivial effect on serviceability as well as on the spandrel’s ultimate load-carrying capacity.

- The out-of-plane bending behavior of compact, L-shaped spandrels is highly dependent...
on the configuration of the web reinforcement. The absence of hooks in the front vertical web reinforcement (as in case 4) may result in larger lateral deformations of the spandrel compared with spandrels using closed stirrups, without reductions in load-carrying capacity.

- Deck ties reduce the lateral displacements induced in L-shaped spandrels typically caused by bending about the weak axis of the spandrel. The presence of ties does not have any significant effect on a spandrel’s ultimate load-carrying capacity or its failure mode.
- The use of open vertical stirrups with 90-degree hooks at the top and bottom did not have any detrimental effect on the induced shear stresses at spandrel ends (compared with closed stirrups).
- The absence of horizontal top web reinforcement increases the shear stress in the spandrel 20%. Conversely, the presence of hooks in the web reinforcement at the front face enhanced the spandrel’s behavior and reduced its shear stresses 20%.
- Using additional reinforcement to prevent flexural failure led to compression shear failure at the end regions of the compact L-shaped spandrels. Finite-element analysis indicated that the use of open, unhooked web reinforcement reduces the spandrel’s shear strength 17% compared with a closed-stirrups configuration. The spandrel’s shear strength reduction is about half as much when open vertical stirrups with 90-degree hooks replace closed stirrups.

ACKNOWLEDGMENTS

This project was conducted while Tarek Hassan was a visiting scholar at North Carolina State University. The project was jointly sponsored by Harry Gleich of Metromont Corp. in Greenville, S.C., and Don Logan of Stresscon Corp. in Colorado Springs, Colo. The authors are grateful for the support and guidance provided by all of the PCI Producer Members throughout the duration of the project. In addition, the authors acknowledge the efforts of Gary Klein of Wiss, Janney, Elstner Associates Inc. for his valuable comments during the research program.

REFERENCES

10. ANATECH Corp. 2003. ANATECH Concrete Analysis Program (ANACAP) Version 2.2.3 Reference Manuals.
5.2. Published Results from Phase II

Results from the second phase of research are presented below in two papers published in the *PCI Journal*. Both papers are included in this dissertation with permission from the *PCI Journal*.

The first paper is titled *Development of a Rational Design Methodology for Precast Concrete Slender Spandrel Beams: Part 1, Experimental Results* by Lucier, Walter, Rizkalla, Zia, and Klein. This paper documents the experimental results from all 16 of the slender spandrels tested in Phases I and II and compares the effect of relevant parameters on behavior.

Detailed analytical results are presented in a second paper, *Development of a Rational Design Methodology for Precast Concrete Slender Spandrel Beams: Part 2, Analysis and Design Guidelines* by Lucier, Walter, Rizkalla, Zia, and Klein. The paper documents expanded finite element results and presents a rational model to explain the measured behavior. The rational model is calibrated to experimental data, and is demonstrated to be safe and effective for slender L-shaped beams. Finally, a simple, rational procedure is presented for the design of precast concrete slender L-shaped beams.
Editor's quick points

- This article is the first part of a two-part paper that studies detailing requirements for the end regions of precast concrete slender spandrel beams.

- An extensive experimental program was undertaken to develop a rational design procedure for precast concrete slender spandrel beams.

- The experimental results, combined with the analytical results and rational modeling in the companion paper, demonstrate that properly designed open web reinforcement is a safe, effective, and efficient alternative to traditional closed stirrups for precast concrete slender spandrel beams that have an aspect ratio of 4.6 or greater.

Development of a rational design methodology for precast concrete slender spandrel beams: Part 1, experimental results

Gregory Lucier, Catrina Walter, Sami Rizkalla, Paul Zia, and Gary Klein

Precast concrete slender spandrel beams are commonly used in parking structures to transfer vertical loads from deck sections to columns. In addition, slender spandrels often serve as a railing or barrier around the exterior edge of the parking structure. Large single-tees or double-tees frequently serve as deck sections and generally span 40 ft to 65 ft (12 m to 20 m).

Typical slender spandrel beams are from 5 ft to 7 ft (1.5 m and 2.1 m) deep with spans ranging from 30 ft to 50 ft (9.1 m to 15 m). These beams usually have a web thickness of at least 8 in. (200 mm). Spandrels that have a large aspect ratio (defined as the spandrel height divided by the thickness of the web) are commonly considered slender members. In this investigation, spandrels with aspect ratios of 4.6 and 7.5 were tested.

In many cases, a continuous ledge runs along the bottom edge of the web on one side of the beam, resulting in what is known as an L-shaped spandrel. The ledge is used to provide bearing for the deck sections, so the L-shaped slen-
to maintain the integrity of the core after spalling. These detailing requirements often result in tightly congested, interwoven reinforcement, especially in the end regions.

The precast concrete industry currently designs slender spandrel beams subjected to combined loading using an approach developed by Zia and Hsu,4 which is outlined in the sixth edition of the PCI Design Handbook: Precast and Prestressed Concrete.7 The PCI Design Handbook method is based on assumptions similar to those made by the ACI 318-08 approach, including behavior marked by spiral cracking in the concrete and face shell spalling. Section 11.5.7 of ACI 318-08 references the Zia-Hsu procedure as a permissible alternative method for torsion design of cross sections having an aspect ratio of 3 or greater. The design method can be used for prestressed or conventionally reinforced concrete beams and is commonly used to proportion shear and torsion reinforcement in the webs of slender spandrel beams. Zia and Hsu developed their design method based on tests of the sectional torsional strength of compact rectangular specimens that have aspect ratios of 3 or less, and the procedure was never intended to be used for slender cross sections. Although the Zia-Hsu approach has proved safe and reliable for slender spandrels, it typically requires large quantities of severely congested vertical and lateral supports.

Objective

The main objective of this research program was to develop rational design guidelines for precast concrete slender spandrel beams. These design guidelines, presented in a companion paper,4 are expected to simplify the reinforcement detailing required for slender spandrels, especially in...
End regions. Specifically, the research focused on investigating whether traditional closed stirrups were required for the slender cross sections of typical precast concrete L-shaped and corbelled spandrels. The use of open reinforcement in lieu of closed stirrups would greatly simplify fabrication and reduce the cost of production.

**Experimental program**

The experimental program was part of a larger research effort, which also included analytical studies based on three-dimensional nonlinear finite-element models, rational analyses, and the development of a simple rational design procedure.

In total, 16 precast concrete spandrel beams were tested to failure. All specimens were full-scale beams, most spanning 45 ft (13.7 m). Two beams were tested at a 30 ft (9.1 m) span. Each specimen was loaded through associated full-scale double-tee deck sections to mimic typical field conditions. Prior to final failure testing, all spandrels were loaded to several stages of interest, including the full service load and the factored design load, and measurements and observations were made at each stage. In the case of the factored design load, the load was held on each beam for 24 hours to evaluate the performance under sustained load.

Two test specimens were designed and detailed with closed stirrups, according to current practice, to serve as controls. The remaining specimens were designed with various configurations of open web reinforcement. The open transverse reinforcement was proportioned in the test specimens using ACI 318-08 procedures without considering torsion. Additional transverse reinforcement was provided on the inner web face based on plate bending about a 45 deg inclined crack. In both cases, ACI 318-08 strength reduction factors were considered. These procedures are explained in detail in the companion paper.8

Table 1 shows other parameters included in the test matrix.

---

**Table 1. Test matrix**

<table>
<thead>
<tr>
<th>Depth, in.</th>
<th>Span, ft</th>
<th>Aspect ratio, h/b</th>
<th>Designation</th>
<th>Configuration</th>
</tr>
</thead>
<tbody>
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<td>60</td>
<td>30</td>
<td>7.5</td>
<td>SP1.8L60.30.P.O.E</td>
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</tr>
<tr>
<td>60</td>
<td>45</td>
<td>7.5</td>
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<td>X</td>
</tr>
<tr>
<td>60</td>
<td>45</td>
<td>7.5</td>
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<td>X</td>
</tr>
<tr>
<td>60</td>
<td>45</td>
<td>7.5</td>
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<td>X</td>
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<tr>
<td>60</td>
<td>45</td>
<td>7.5</td>
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<td>X</td>
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<tr>
<td>60</td>
<td>45</td>
<td>7.5</td>
<td>SP11.8L60.45.R.C.E</td>
<td>X</td>
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<tr>
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<td>45</td>
<td>7.5</td>
<td>SP12.8L60.45.P.O.E</td>
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<tr>
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<td>45</td>
<td>7.5</td>
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<td>60</td>
<td>45</td>
<td>7.5</td>
<td>SP15.8L60.45.P.O.T</td>
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<td>60</td>
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<td>7.5</td>
<td>SP16.8L60.45.R.O.T</td>
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<tr>
<td>60</td>
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<td>7.5</td>
<td>SP17.8CB60.45.P.O.E</td>
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<td>60</td>
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<td>7.5</td>
<td>SP18.8CB60.45.P.S.E</td>
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<td>SP19.8CB60.45.P.O.T</td>
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<td>45</td>
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<td>46</td>
<td>45</td>
<td>4.6</td>
<td>SP21.10L46.45.P.O.T</td>
<td>X</td>
</tr>
</tbody>
</table>

*Specimens were sponsored privately by individual PCI Producer Members.
†SP18 was constructed with special closed reinforcement in a hooked-C shape.

Note: $b$ = web thickness; $h$ = beam height. 1 in. = 25.4 mm; 1 ft = 0.305 m.
The scheme used in several specimens was the combination of WWR on the outer spandrel face and L-shaped bars on the inner spandrel face. As discussed, the primary advantage of using open reinforcement compared with traditional closed stirrups is the ease of fabrication.

**Production of open versus closed reinforcement**

A significant advantage in using open web reinforcement was the efficiency gained in production. Observations made during the production of the experimental beams indicate that assembling an open reinforcing cage took 30% to 50% less time than assembling a traditional closed reinforcing cage. The gains in efficiency were especially obvious when an open cage was produced on the same form line adjacent to a closed cage, as was the case for SP12 and SP13. In producing the open cage (with the spandrel lying outer-face down on the form), the outer-face web reinforcement (often WWR) was placed in the empty form first. The strands were then pulled and stressed without obstructions. After stressing the strands, any required longitudinal steel bars, such as U-bars in the end regions, were simply placed in the form near their final locations.

**Parameters**

**Open versus closed reinforcement**

The primary variable considered in this experimental program was the use of open web reinforcement. Thirteen of the sixteen experimental specimens were designed and fabricated with open web reinforcement. The two control specimens were reinforced with traditional closed stirrups. Another specimen, SP18, was fabricated with partially closed reinforcement including inner-face vertical steel hooked over the top and bottom of the beam. This specimen was not intended to be a practical design option but was included to serve as a direct comparison between a companion beam reinforced with an identical amount of web steel without the hooks. Figure 2 shows sketches of a typical L-shaped spandrel cross section with typical open and closed web reinforcement schemes. In addition, the same figure shows a sketch of the special partially closed reinforcement scheme used for SP18.

In this research program, open reinforcement included flat sheets of welded-wire reinforcement (WWR); conventional deformed reinforcing bars bent into L, C, or U shapes; and straight bars or tendons. An open web reinforcement scheme used in several specimens was the combination of WWR on the outer spandrel face and L-shaped bars on the inner spandrel face. As discussed, the primary advantage of using open reinforcement compared with traditional closed stirrups is the ease of fabrication.

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<table>
<thead>
<tr>
<th>Concrete</th>
<th>Reinforcement</th>
<th>Detailing</th>
<th>Bearing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prestressed</td>
<td>Reinforced</td>
<td>Open</td>
<td>Closed</td>
</tr>
<tr>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
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</table>
With the strands stressed, the other components of the open reinforcing cage were dropped into the form at the correct locations and tied into place. L-shaped or C-shaped bars on the inner spandrel face were placed so that they rested on the stressed strands. C-shaped ledge bars or corbel assemblies were hooked around the longitudinal steel and secured to strands or bars. With the web steel in place, the additional longitudinal U-bars were secured. The flexibility of the open reinforcement allowed for spacing of the bars to be easily adjusted as the cage was finalized. If a bar was misplaced, it could be removed and replaced without disrupting any other components of the cage.

The stirrups (both web and ledge) had to be placed in the empty form for the closed reinforcing cages. During this step, it was important to verify that the sequence of the stirrups corresponded to their final locations in the beam. With the stirrups in the form, the prestressing strands, along with any other required longitudinal steel, were threaded through the stirrups, taking care not to disrupt the stirrup order. The strands were then prestressed.

After stressing the strands, the stirrups and additional longitudinal bars were spaced and secured in place at their final locations. If errors were made in placing the stirrups in a closed cage, few options were available to correct them, short of detensioning the strands. Misplaced stirrups could be cut and removed from the cage, but inserting replacement or additional stirrups was a challenge. In some cases, the side rails of the form could be removed and any missing stirrups bent into place around the already-stressed strands, but this procedure required significant effort. Careful planning and layout at the start of a closed-cage assembly can minimize mistakes, but even infrequent assembly errors can be costly with a closed reinforcement cage.

In addition to gains in production efficiency, the use of an open reinforcement cage offered significant savings in steel compared with traditional designs using closed stirrups. In examining the test specimens in this program, an open reinforcement cage required up to 50% less shear and torsion steel than a comparable closed cage.

The difference in required steel can be highlighted for SP10 and SP11. Both specimens were slender spandrels with 60 in. × 8 in. (1500 mm × 200 mm) webs and 45 ft (13.7 m) spans. Both contained extra reinforcement for flexure to ensure end-region failures. SP10 was designed with open web reinforcement, while SP11 was designed with traditional closed stirrups. Both were designed for the same applied loads. Flexural reinforcement was the same for both specimens and is excluded from the calculated steel quantities. The total quantity of steel used to produce SP10 was 715 lb (3180 N), compared with 1396 lb (6209 N) required to produce SP11 (both weights exclude the common flexural steel). The 681 lb (3030 N) difference is equivalent to a 48% reduction in web steel.

A similar analysis was performed on prestressed beams SP12 and SP13. Neglecting the flexural steel common to both beams, the steel required for the open cage of SP12 was 778 lb (3460 N) compared with 1251 lb (5564 N) for the closed cage of SP13. The 473 lb (2100 N) difference is equivalent to a 37% reduction in web steel. Additional details of the reinforcement for these beams are reported elsewhere.

Continuous ledge versus corbels The two types of precast concrete slender spandrel beams considered in this experimental program were L-shaped and corbelled spandrels. Both types are commonly used. Figure 1 shows the two types of spandrels.

Span Specimens with two span lengths, 30 ft (9.1 m) and 45 ft (13.7 m), were tested.
Aspect ratio

Aspect ratio is defined as beam height divided by web thickness \((\frac{h}{b})\). The two aspect ratios studied in this experimental program were 4.6 and 7.5. Fourteen L-shaped and corbelled spandrels were tested with 8-in.-thick \(\times\) 60-in.-deep \((200 \text{ mm} \times 1500 \text{ mm})\) webs, giving an aspect ratio of 7.5. The lateral tiebacks at the support for these beams were spaced 36 in. \((910 \text{ mm})\) apart, centered in the height of the web. In addition, two L-shaped spandrels were tested with a web thickness of 10 in. \((250 \text{ mm})\) and a web depth of 46 in. \((1200 \text{ mm})\) for an aspect ratio of 4.6. The lateral tiebacks at the support for these beams were spaced 32 in. \((810 \text{ mm})\) apart and centered in the height of the web. Figure 3 shows the three spandrel cross sections considered in the experimental program.

Prestressed versus reinforced concrete

Thirteen of the sixteen specimens were designed with prestressed tendons as the primary flexural reinforcement. The remaining three specimens were reinforced with conventional mild-steel deformed bars as the only flexural reinforcement.

Typical versus enhanced reinforcement

A slender precast concrete spandrel beam would typically fail in flexure if it were loaded beyond the factored design load. For the purposes of investigating end-region behavior, however, it was necessary to force failures to take place in the end regions. Therefore, a number of selected beams were designed to fail in their end regions by using extra reinforcement for flexure. In addition, the ledges or corbels of these selected beams were also strengthened to prevent punching shear or other localized failure modes.

In strengthening the selected test specimens against possible failure modes outside their end regions, care was taken to avoid enhancing the end-region shear and torsion strength. The reserve flexural strength was provided by adding mild steel bars at the midspan of specimens otherwise designed with normal levels of flexural reinforcement. When provided, the additional bars were terminated well outside the end regions. In addition, steel angle details were provided in the ledges of selected L-shaped spandrel specimens in the localized area underneath each double-tee stem. These welded details enhanced the ability of the ledge to resist punching shear without altering the shear and torsion strength of the cross section.

Extra hanger reinforcement was provided in the middle region of some beams to prevent separation of the ledge or corbel from the web. While 11 of the 16 test beams were designed to induce end-region failures, the remaining 5 specimens were designed with typical amounts of flexural, ledge or corbel, and hanger reinforcement at all locations, as recommended by the PCI Design Handbook. These specimens were included in the test matrix to examine the behavior and to determine the failure modes of specimens reinforced with open web reinforcement but otherwise detailed according to current practice.

Beams designated in the test matrix as having enhanced
detailing included the additional reinforcement described. Beams designated as having typical detailing were designed according to PCI Design Handbook recommendations. The typical specimens are representative of beams that would be designed for an actual project, while the enhanced specimens were included to allow detailed study of end-region behavior and failure modes.

**Bearing pads** Two different types of bearing pads were used in the experimental tests: common randomly oriented fiber and rubber composition pads and Teflon-coated preformed fabric pads. Bearing pads were located between each double-tee stem and the ledge or corbel. Bearing pads were also located between the spandrel and the support at each end of the beam. Although randomly oriented fiber and rubber composition pads are commonly used by the industry in parking garages, these pads have a relatively low stiffness and a relatively high coefficient of friction. Bearing-pad friction helps to decrease the out-of-plane movement of a slender spandrel by providing a horizontal stabilizing reaction at every double-tee stem; however, the benefit of this bearing friction should not be relied on in design. Thus, Teflon-coated, preformed fabric bearing pads and polished stainless steel plates were used in the majority of the tests to eliminate bearing-pad friction as much as possible, thereby creating a test condition more severe than is likely to occur in the field. Conventional randomly oriented fiber and rubber composition bearing pads were used only for two of the sixteen tests to investigate the effect of bearing pad friction on slender spandrel behavior.

**General test setup**

The framework used to test all spandrels was designed around a strong floor in the testing laboratory. The test setup consisted of the following primary components:

- a system of columns, beams, and stands designed to transfer the vertical and horizontal reactions of the spandrels to the strong floor with minimal support deflections
- a system of spreader beams, tie-down rods, and hydraulic jacks designed to produce the required load and transfer it evenly to the appropriate points on the
Loading

All loads other than the spandrel self-weight were transferred to the spandrel through the stem reactions of 10DT26 deck sections. The stems were spaced evenly along the ledge or corbels of each spandrel at 5 ft (1.5 m) on center. The double-tee bearing pads were centered 2 in. (50 mm) back from the edge of the ledge and 6 in. (150 mm) from the inner face of the web.

Designs of the tested specimens were based on a live load LL of 40 lb/ft² (1.92 kN/m²) and a snow load SL of 30 lb/ft² (1.44 kN/m²). The dead load DL included the self-weight of the spandrel beam and the 71.6 lb/ft² (3.43 kN/m²) weight of the double-tee decks. The controlling factored load case of 1.2DL + 1.6LL + 0.5SL was considered. The vertical end reaction of each simply supported spandrel was monitored throughout testing and served as the basis for controlling a loading system of hydraulic jacks during the test. Thus, all discussion of load levels refers to the main vertical reaction for a given beam.

Load combinations other than the factored load were also important for the tests. Load levels considered during testing included service load without snow (DL + LL), the reduced service load with snow specified by the American Society of Civil Engineers’ (ASCE’s) Minimum Design Loads for Buildings and Other Structures (7-10) (DL + 0.75LL + 0.75SL), service load with full snow load (1.0DL + 1.0LL + 1.0SL), and the ACI 318-08/ASCE 7-10 factored design load (1.2DL + 1.6LL + 0.5SL). Three types of spandrels were tested with a 45 ft (13.7 m) nominal span. These three types include the 8 in. × 60 in. L-shaped spandrel, 8 in. × 60 in. corbelled spandrel, and 10 in. × 60 in. (250 mm × 1500 mm) L-shaped spandrel. To simplify testing and comparison, the design loads for all spandrel types were considered to be the same because the differences in self-weight among the
test specimens

- a system of concrete support blocks, steel channels, and tie-down rods to support the end of the double-tee deck opposite the spandrel
- an array of load cells and other instrumentation used to measure data loads, deformations, and strains

Figure 4 shows a sketch of the test configuration. Fig. 5 shows a plan view of the test setup for the 45 ft (13.7 m) spandrels. Labeling conventions for inside, outside, left, and right are also established in these figures and will be used throughout this paper. Further information on the test setup is reported elsewhere.

Instrumentation

About 40 instruments recorded data during each test. Four basic types of instrumentation were used. All instruments were connected to an electronic data acquisition system. Additional details of the instrumentation used are reported elsewhere.

- Load cells were used to measure the vertical and lateral spandrel reactions and to measure the load applied by the jacks. In addition, a pressure transducer was used to record the pressure applied by the hydraulic pump.
- String and linear potentiometers (pots) were used to measure vertical and lateral displacements of the spandrel.
- Inclinometers were used to measure the rotation of each spandrel at the quarter points.
- Pi gauges were used to measure concrete strains on the top, bottom, and inside face of each spandrel.

Figure 5. Top view of a typical 45 ft (13.7 m) test setup. Note: 1 ft = 0.305 m; 1 kip = 4.448 kN.
The observed cracking patterns had similar characteristics for all spandrels, regardless of configuration, reinforcement, or aspect ratio. The inner-face cracking pattern was the tied-arch type, previously documented by other researchers. The observed behavior of the slender spandrel beams also indicates that the effects of shear and torsion dominate in the disturbed end region. This end region is followed by a transition region where the effects of shear and torsion gradually decrease along with increasing effects of flexure. Beyond the transition region, flexural effects dominate slender spandrel behavior.

During each test, cracks were marked on the surface of each spandrel at several load levels of interest. Spandrels were whitewashed prior to testing to make cracks more visible. In all tests, inner-face cracking initiated near the support and extended upward toward midspan from each end of the beam at an angle of about 45 deg. These cracks gradually flattened out and arched toward the center of the beam. Vertical cracks initiating from the bottom of the beam were observed on the inner web face near midspan for all tests. Localized cracking was observed around the concentrated loads in the ledge or corbels. Figure 7 shows the inner-face cracking pattern typical of all tests for a representative continuous L-shaped spandrel beam, and Fig. 8 shows that of a corbelled spandrel.

### Cracking pattern on the outer web face

The observed outer-face cracking patterns were also similar for all tested beams. Figure 9 shows the observed outer-face cracking pattern for a typical beam with an aspect ratio of 7.5. As with the inner-face cracking patterns,
Table 2. Summary of failure load and failure mode for all tested beams

<table>
<thead>
<tr>
<th>Spandrel</th>
<th>Spandrel reaction</th>
<th>Lateral reactions at failure, kip</th>
<th>Description of failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Failure, kip</td>
<td>Factored, kip</td>
<td>Top left</td>
</tr>
<tr>
<td>SP1.8L60.30.P.O.E</td>
<td>135</td>
<td>84.8</td>
<td>23.3</td>
</tr>
<tr>
<td>SP2.8L60.30.P.O.E common randomly oriented fiber and rubber composition pads</td>
<td>150</td>
<td>84.8</td>
<td>14.2</td>
</tr>
<tr>
<td>SP3.8L60.45.P.O.E</td>
<td>196</td>
<td>126.6</td>
<td>32.4</td>
</tr>
<tr>
<td>SP4.8L60.45.P.O.E</td>
<td>200</td>
<td>126.6</td>
<td>34.5</td>
</tr>
<tr>
<td>SP10.8L60.45.R.O.E</td>
<td>208</td>
<td>126.6</td>
<td>27.7</td>
</tr>
<tr>
<td>SP11.8L60.45.R.C.E</td>
<td>251</td>
<td>126.6</td>
<td>35.8</td>
</tr>
<tr>
<td>SP12.8L60.45.P.O.E</td>
<td>185</td>
<td>126.6</td>
<td>23.7</td>
</tr>
<tr>
<td>SP13.8L60.45.P.C.E</td>
<td>240</td>
<td>126.6</td>
<td>41.0</td>
</tr>
<tr>
<td>SP14.8L60.45.P.O.T</td>
<td>160</td>
<td>126.6</td>
<td>22.6</td>
</tr>
<tr>
<td>SP15.8L60.45.P.O.T common randomly oriented fiber and rubber composition pads</td>
<td>140</td>
<td>126.6</td>
<td>19.2</td>
</tr>
<tr>
<td>SP16.8L60.45.R.O.T</td>
<td>127</td>
<td>126.6</td>
<td>16.9</td>
</tr>
<tr>
<td>SP17.8CB60.45.P.O.E</td>
<td>200</td>
<td>126.6</td>
<td>30.3</td>
</tr>
<tr>
<td>SP18.8CB60.45.P.S.E</td>
<td>220</td>
<td>126.6</td>
<td>27.3</td>
</tr>
<tr>
<td>SP19.8CB60.45.P.O.T</td>
<td>173</td>
<td>126.6</td>
<td>18.5</td>
</tr>
<tr>
<td>SP20.10L46.45.P.O.E</td>
<td>171</td>
<td>126.6</td>
<td>27.4</td>
</tr>
<tr>
<td>SP21.10L46.45.P.O.T</td>
<td>127</td>
<td>126.6</td>
<td>24.4</td>
</tr>
</tbody>
</table>

Note: 1 ft = 0.305 m; 1 kip = 4.448 kN.
the outer-face patterns were symmetrical about midspan. Initial cracks on the outer face were usually observed near midspan, where a region with only vertical cracks initiated from the bottom of the beam. Near the ends of a beam, cracks were observed extending downward from the top lateral reaction. These cracks developed during later stages of loading in all tests due to the effect of the lower lateral reaction on the outer face. The outer-face cracking pattern seems to indicate that the disturbed end region extends for a distance equal to approximately 1.5 times the height of a spandrel. The area on the outer web face between the disturbed end region and the midspan flexural region also exhibited significant vertical flexural cracking. However, in most beams, inclined shear cracks were also observed in this region on the outer face. Note that the diagonal cracks in the end region are oriented orthogonally with respect
to the shear and torsion cracks on the inside face. These cracks indicate that at high overloads, the plate bending stresses due to torsion exceed the diagonal compressive stresses due to shear. The three regions suggested by the cracking pattern are further defined and discussed in the companion paper.

**Failure modes**

Seven of the sixteen slender spandrels tested failed at their end regions along a skewed diagonal crack plane extending upward from the support. These seven spandrels were reinforced with excess flexural steel and were designed to prevent possible localized failure modes. End-region failures were observed in L-shaped spandrels and corbelled spandrels and in beams having aspect ratios of 7.5 and 4.6 (Fig. 10).

End-region failure modes were only observed in beams specially designed to force failures in the end region. These beams were particularly important to the research effort because they demonstrate the mechanism by which the end region of a slender spandrel beam could fail when overloaded in combined shear, flexure, and torsion, even if such failures only occurred at extreme overload in specially configured test specimens. The observed end-region failure mode forms the basis of the rational design approach introduced in the companion paper.

The typical end-region failure mode is shown in more detail in Fig. 11 and 12. The primary diagonal crack initiates at the face of the support and extends upward at an angle of approximately 45 deg. The crack crosses over the top of the web surface at a skewed angle of about 45 deg (Fig. 11). Finally, the crack returns along the same 45 deg angle to the face of the support on the outer face (Fig. 12). A key feature of the skewed end-region failure mode is the significant displacement of the failure surface out of plane. The failure suggests that the top lateral reaction causes the top corner of the web to bend outward, opening the diagonal crack. Simultaneously, the top edge of the web tends to twist out of plane at failure.

**Effect of parameters**

The effects of selected key parameters on spandrel behavior are described in the next sections. A detailed description of the influence of parameters on behavior is presented elsewhere for all parameters listed in Table 1.
Figure 10. End-region failure modes in beams with extra local and flexural reinforcement. Note: Each beam’s main vertical reaction at failure is shown in parentheses.
Measured deflection data demonstrate the similar behavior of beams with open and closed reinforcement. The measured load-deflection data at midspan are plotted in Fig. 14 for SP12 (open reinforcement) and SP13 (closed reinforcement). Plots of the load versus vertical deflection were nearly identical for both beams through the factored load level. Both behaviors were linear to the service load.

The plotted experimental data show that the series of loading and unloading cycles is evident and that the residual deflection at zero applied load increases after each cycle. The horizontal segments in the load-deflection curves represent the effects of creep, where a beam continued to deflect slightly under constant applied load. The effects of creep are evident where the load was held for short observation periods during testing but are most notable during the 24 hr sustained loading at the factored load level.

The measured vertical end reaction is the entire reaction supported by a given beam, including the self-weight. Thus, plots of end reaction data do not start from the origin. The offset of approximately 22 kip (98 kN) represents the self-weight end reaction of the spandrel beam plus the short double-tee decks and loading system.

In this research, lateral deflections are highly significant due to the eccentrically applied loads. Lateral deflections were recorded for each spandrel at several locations,

**Open versus closed reinforcement**

The most significant parameter examined by the research is the use of open web reinforcement compared with traditional closed stirrups. Three pairs of test specimens were included in the test matrix to highlight the differences between open and closed web reinforcement (Fig. 13). SP11 and SP13 were designed for torsion following current practice. Thus, the quantity of longitudinal and closed transverse reinforcement provided in these two beams was significantly more than that provided in companion beams with open reinforcement.

In general, service-level behavior was virtually identical when specimens with open web reinforcement are compared with those with closed reinforcement. The shear and torsion strength of end regions with closed reinforcement (designed using the procedure in the PCI Design Handbook) is greater than the strength of end regions designed with open reinforcement according to the procedure proposed in the companion paper. However, the strengths of all end regions reinforced with open or closed stirrups were significantly higher than the factored design loads and were sufficient to ensure that failure modes outside the end region would always control. The factored design load \((1.2DL + 1.0LL + 0.5SL)\) was 126.6 kip (563.1 kN) for all six of the beams in Fig. 13. End-region failures occurred only when a given beam was specially reinforced to prevent other potentially controlling failure modes from occurring.
including the top and bottom edges of the web at midspan. Figure 15 shows measured lateral deflection data at these locations for SP12 and SP13 to further compare the effects of open reinforcement on behavior.

As with measured vertical deflections, the measured lateral deflections did not indicate any substantial difference in behavior due to open reinforcement through the factored load. At high levels of overload, the out-of-plane stiffness of SP13 (closed) was greater than that of SP12 (open), and the failure load was higher. SP13 (closed) contained 37% more web steel than SP12 (open).

The lateral deflection data plotted in Fig. 15 is typical for all spandrels with an aspect ratio of 7.5; that is, with increasing applied load, the upper edge of the web tended to move inward toward the double-tee decks at midspan, while the bottom edge of the web tended to move outward.

Figure 16 compares the measured rotation of the web at midspan for the same two spandrels. Both spandrels exhibited the same type of out-of-plane behavior, and there was virtually no difference in measured rotations under service load. At higher loads, the beam with closed stirrups (SP13) had a higher rotational stiffness, but through the factored load level differences in measured rotations were minimal.

The performance advantage of closed stirrups compared with open reinforcement appears less significant when corbelled spandrel specimen SP17, reinforced with open web steel, is compared with corbelled spandrel specimen SP18, reinforced with special hooked stirrups. SP18 was an exact copy of SP17 except that all vertical web steel on the inner face was hooked over the top and bottom web (Fig. 2). Hooked web steel is not practical from the standpoint of production, but it is relevant for research purposes.

Comparison of SP17 and SP18 is useful because these two
<table>
<thead>
<tr>
<th>Reinforced concrete L-spandrels</th>
<th>Open web reinforcement</th>
<th>Closed web reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP10.8L60.45.R.O.E. (208 kip [925 kN])</td>
<td>SP11.8L60.45.R.C.E. (251 kip [1120 kN])</td>
<td></td>
</tr>
<tr>
<td>Prestressed concrete L-spandrels</td>
<td>SP12.8L60.45.P.O.E. (185 kip [823 kN])</td>
<td>SP13.8L60.45.P.C.E. (240 kip [1070 kN])</td>
</tr>
<tr>
<td>Prestressed concrete corbelled spandrels</td>
<td>SP17.8CB60.45.P.O.E. (200 kip [890 kN])</td>
<td>SP18.8CB60.45.P.S.E. (220 kip [979 kN])</td>
</tr>
</tbody>
</table>

*Figure 13.* Matrix of failure modes for beams with open versus closed reinforcing schemes. Note: Failure loads are shown in parentheses for each beam.
Figure 14. Measured load–vertical deflection response of two selected spandrels. Note: 1 kip = 4.448 kN.

Figure 15. Measured lateral deflections at midspan for beams SP12 and SP13. Note: 1 kip = 4.448 kN.
beams had the same quantities of vertical and longitudinal steel. Thus, any difference in performance is directly attributable to the steel crossing the top and bottom web faces. In other comparisons between open and closed reinforcement (SP10 versus SP11 and SP12 versus SP13), the beam with closed reinforcement was designed and detailed according to current practice. Thus, the beam with closed reinforcement had significantly more vertical and longitudinal web reinforcement (nearly twice the amount), in addition to having reinforcement in the form of closed ties.

The failure mode for SP17 was identical to that of SP18 (Fig. 17 and 18). Both beams failed along a skewed diagonal crack plane in their end regions. The vertical load-deflection behaviors at midspan (Fig. 19) were virtually identical to the load level of 200 kip (890 kN). The ultimate load of SP18 (special closed) was about 10% greater than the ultimate load of SP17 (open), suggesting that the steel on the top and bottom of the web face likely contributed to torsional resistance in the slender spandrel.

Continuous ledges versus corbels

It was important for the research to examine both L-shaped spandrels and corbelled spandrels because both types of beams are commonly used by the industry. Test results indicate that the end-region failure mode takes the form of a skewed diagonal crack in both L-shaped spandrels and corbelled spandrels. SP12 and SP17 were both reinforced with excess flexural and local reinforcement to allow the end-region behavior to be examined.

A general comparison between L-shaped spandrel deflection data and comparable corbelled spandrel deflection data indicates little difference in vertical load-deflection response for similarly reinforced beams. However, the outward lateral deflection at the bottom of the L-shaped spandrel is somewhat greater than that of the corbelled spandrel because the principal axes of the L-shaped spandrel are inclined. The effects of inclined axes and the differences between L-shaped spandrels and corbelled spandrels have been discussed elsewhere.5,10

Typical beams without extra and special reinforcements

End-region failure modes were observed only in specimens designed with extra flexural steel and special ledge or hanger reinforcements. All beams reinforced at typical levels, as specified by ACI 318-08,4 failed outside their end regions. Figure 20 presents a direct comparison between the failure modes of enhanced and typical specimens for three pairs of specimens. The observed failure modes for specimens designed with typical levels of reinforcement included localized ledge or corbel failures and flexural failure at midspan.

The only difference between the typical beam and the
Conclusion

The experimental program presented in this paper is one component of a larger research effort sponsored by PCI. The research effort also included significant finite-element and rational analysis and resulted in the development of a proposed rational design procedure. The companion paper presents the analytical work and design procedure. Tests conducted on 16 full-scale beams revealed that the end-region failures of slender precast concrete spandrel beams develop because of combined shear and torsion in the end region. The tests showed that spiral cracking and face-shell spalling did not develop. Rather, slender spandrel beams develop a tied-arch cracking pattern and if other failure modes are intentionally precluded, will ultimately fail along a skewed diagonal crack extending upward from the support.

Several conclusions are drawn based on the results of the experimental program. These conclusions apply to slender enhanced beam in each pair of specimens was that the enhanced beam had added partial-length mild steel and welded ledge reinforcements. The web reinforcement was identical for beams in a given pair.

Figure 21 compares the vertical load-deflection behaviors of L-shaped spandrels SP12 (enhanced) and SP14 (typical), and Fig. 22 compares the lateral-load-deflection behaviors. The flexural stiffness of SP14 (typical beam) is less than that of SP12 (enhanced beam), as would be expected. However, the lateral deflections measured at midspan for SP12 and SP14 are nearly identical, indicating that the excess flexural steel and local reinforcements did not alter behavior in the end region. The extra partial-length mild steel provided in the enhanced beams was held short of the end regions to avoid unintentionally improving end-region performance.

Figure 17. Inner-face view of SP17 and SP18 after testing. Note: SP17 has open reinforcement, and SP18 has special closed reinforcement.
Figure 18. Outer-face view of SP17 and SP18 after testing. Note: SP17 has open reinforcement, and SP18 has special closed reinforcement.

Figure 19. Measured vertical deflections at midspan for SP17 (open) and SP18 (special closed). Note: 1 kip = 4.448 kN.
<table>
<thead>
<tr>
<th>Prestressed L-spandrels</th>
<th>Enhanced flexural and local reinforcement</th>
<th>Typical flexural and local reinforcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP12.8L60.45.P.O.E.</td>
<td>SP14.8L60.45.P.O.T.</td>
<td></td>
</tr>
<tr>
<td>Prestressed L-spandrels aspect ratio 4.6</td>
<td>SP20.10L46.45.P.O.E.</td>
<td>SP21.10L60.45.P.O.T.</td>
</tr>
<tr>
<td>Corbelled spandrels</td>
<td>SP17.8CB60.45.P.O.E.</td>
<td>SP14.8L60.45.P.O.T.</td>
</tr>
</tbody>
</table>

Figure 20: Comparisons of failure modes for enhanced versus typical reinforcement detailing.
Figure 21. Measured vertical deflections from SP12 (enhanced) and SP14 (typical). Note: 1 kip = 4.448 kN.

Figure 22. Measured lateral deflections from SP12 (enhanced) and SP14 (typical). Note: 1 kip = 4.448 kN.
Distinct differences in the cracking patterns of slender spandrel beams occur in three regions: the end region, the transition region, and the flexure region at mid-span.

The shear and torsion resistance of tested slender spandrel beams with traditional closed stirrups was almost twice the factored load demand when reinforcement was proportioned according to the practice outlined in the PCI Design Handbook.

The shear and torsion resistance of tested slender spandrel beams with open web reinforcement exceeded the factored load demand by 35% to 74% when reinforcement was proportioned according to the procedure described in the companion paper.

The additional end-region strength provided by traditional closed stirrups compared with open web reinforcement is evident only when beams are subjected to extreme overload conditions by intentionally overreinforcing against other failure modes such as flexure or ledge failures.

Open web reinforcement can provide virtually the same performance as traditional closed web reinforcement under factored design loads when slender spandrel beams are designed with typical levels of flexural and hanger reinforcement.

The ledge and corbel failures observed in the typical specimens demonstrate that failure modes outside the end region will control the strength of properly detailed slender spandrel beams, regardless of whether open or closed web reinforcement is used.

Several ledge punching failures occurred at loads below those predicted by the PCI Design Handbook. Other researchers have observed similar results. The interaction between ledge punching behavior and global flexure and shear appears to significantly reduce punching shear capacity. Although beyond the scope of this investigation, further study of ledge punching capacity is strongly recommended.

Acknowledgments

This research was sponsored by the PCI Research and Development Committee. The work was overseen by an L-spandrel advisory group chaired by Donald Logan. The authors are extremely grateful for the support and guidance provided by this group throughout all phases of the research. In addition, the authors would like to thank the numerous PCI producer members who donated test specimens, materials, and expertise in support of the experimental program.

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**Notation**

\( b \)  = web thickness

\( \text{DL} \) = dead load

\( h \) = beam height

\( \text{LL} \) = live load

\( \text{SL} \) = snow load
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Synopsis

This paper summarizes test results of an extensive experimental program undertaken to develop a rational design procedure for precast concrete slender spandrel beams. Experimental research findings presented in this paper are used to propose a rational design procedure that will be presented in a forthcoming companion paper. The research introduced significantly simplified detailing requirements for the end regions of precast concrete slender spandrel beams. Such regions are often congested with heavy reinforcing cages when designed according to current procedures.

In total, 16 full-scale precast concrete spandrel beams were tested to failure to study the limit state behavior. Each specimen was loaded through full-scale double-tee deck sections to mimic typical field conditions. Three of the specimens were designed and detailed with closed stirrups, according to current practice, and served as controls for the experimental program. The remaining thirteen specimens were designed with various configurations of open web reinforcement. Several specimens were specially configured with flexural, ledge/corbel, and hanger reinforcement in excess of what would be provided in a normal design. The enhanced reinforcement helped to delay typical midspan and local failure modes and allowed for observation and study of failure modes in the end region.

The experimental results, combined with the analytical results and rational modeling in the companion paper, demonstrate that properly designed open web reinforcement is a safe, effective, and efficient alternative to traditional closed stirrups for precast concrete slender spandrel beams that have an aspect ratio of 4.6 or greater.

Keywords

Beam, failure, load, reinforcement, spandrel, torsion.

Review policy

This paper was reviewed in accordance with the Precast/Prestressed Concrete Institute’s peer-review process.

Reader comments

Please address any reader comments to journal@pci.org or Precast/Prestressed Concrete Institute, c/o PCI Journal, 200 W. Adams St., Suite 2100, Chicago, IL 60606.
Editor’s quick points

■ This paper summarizes the results of an analytical research program undertaken to develop a rational design procedure for precast concrete slender spandrel beams.

■ The analytical and rational models use test results and research findings of an extensive experimental program presented in a companion paper.

■ The overall research effort demonstrated the validity of using open web reinforcement in precast concrete slender spandrel beams and proposed a simplified procedure for design.

Development of a rational design methodology for precast concrete slender spandrel beams: Part 2, analysis and design guidelines

Gregory Lucier, Catrina Walter, Sami Rizkalla, Paul Zia, and Gary Klein

Design and analysis of precast concrete slender spandrel beams is not a simple task because of eccentrically applied loads, lateral support conditions, and asymmetrical cross sections. Significant shear and torsion stresses develop in the end regions and act in combination with in-plane and out-of-plane bending stresses. When vertical eccentric loads are applied to the ledge or corbels of a typical slender spandrel beam, it will deflect downward, laterally inward at the top edge of the web, and laterally outward at the bottom edge of the web at midspan. The combination of vertical and lateral deflections results in a warped deflected shape and a tied arch cracking pattern. It is well documented that face-shell spalling and spiral cracking do not develop in slender spandrel beams at failure, though these limit-state behaviors are implicitly assumed in current practice.1–5 The design method found in the sixth edition of the PCI Design Handbook: Precast and Prestressed Concrete often results in conservative, heavy reinforcement...
When used for slender spandrel beams, which often have aspect ratios much greater than 3.0, this paper discusses the design and analysis of precast concrete slender spandrel beams. The experimental program on which it is based was reported in the companion paper.

**Objective**

The objective of this research was to develop rational design guidelines for precast concrete slender spandrel beams with an aspect ratio (height divided by width) of at least 4.6. The guidelines are expected to simplify the reinforcement detailing required for slender spandrels, especially in the end regions. Specifically, the research focused on investigating whether traditional closed stirrups are required for the slender cross sections of typical precast concrete L- and corbelled spandrels. The use of open web reinforcement in lieu of closed stirrups would greatly simplify fabrication and reduce the cost of production.

**Nonlinear finite element analysis**

A three-dimensional nonlinear finite element model (FEM) was developed and then calibrated with experimental results to study the various parameters that influence the behavior of slender L-shaped spandrel beams.

The finite element code ANACAP used in this study is capable of analyzing plain, reinforced, or prestressed concrete members in three dimensions. The program has extensive nonlinear capabilities and includes an advanced concrete material model. Additional details describing the finite element code can be found elsewhere.

**Nonlinear finite element model**

Considering symmetry, the nonlinear finite element analysis was based on modeling one-half of a typical slender spandrel beam using approximately 4,700 twenty-node brick elements. The exact number of elements varied depending on the specifics of the case under study. The large number of elements was necessary to maintain a sufficiently fine mesh around all of the loading and boundary conditions as well as in the end region, where failure was expected to occur. The modeled portion of a typical 45-ft-long (13.7 m) beam consisted of five ledge loads, three tieback connections, two lateral end restraints, one primary vertical end reaction, and a symmetry condition at mid-span. Figure 1 shows the finite element mesh and boundary conditions used for a typical L-shaped spandrel.
Several parameters were examined using the nonlinear FEM. These parameters included closed versus open web reinforcement, web reinforcement type and ratio, cross section dimensions, concrete strength, and the influence of boundary conditions such as bearing friction and deck connections.

**Boundary conditions**

The boundary conditions used in the model were chosen to simulate the connections typically used to support spandrel beams in the field. The spandrel-to-column tiebacks were simulated by restraining lateral movement at those two locations in the model. Vertical movement was restrained in the bearing area of the main vertical reaction. Symmetry boundary conditions were applied at midspan. Each vertical load applied to the spandrel ledge was modeled as uniform pressure acting on an area equal to the size of the bearing pads used in the experimental program. These applied loads were increased incrementally until failure.

Spring elements were used to simulate stem-to-ledge bearing friction and deck-to-spandrel tieback forces. For the deck connections, the stiffness of the spring elements was estimated based on the material and cross-sectional properties of the weld plate. The spring constant simulating the friction at bearing reactions was determined based on the coefficient of friction for the chosen bearing pads. The coefficient of friction for a given bearing pad was assumed to remain constant at all levels of applied load. These assumptions tended to slightly overestimate the friction force at high overload levels when the out-of-plane behavior became significantly nonlinear.

Selected results from the nonlinear finite element analysis are presented in the following sections.

**Deflected shape and cracking pattern**

Figure 2 shows the predicted and observed inner web face cracking patterns in the end region of a reinforced concrete L-spandrel, a prestressed concrete L-spandrel, and a prestressed concrete corbelled spandrel. For clarity, the observed cracking patterns were photographed after failure with the deck sections removed. The analytical cracking patterns are shown at the factored load level for clarity. Only half of each beam is shown in the figure because the analysis was symmetric about the midspan.

In general, the predicted deformed shape was a combination of vertical and lateral deflections, and matched well to the observed behavior. The midspan of the beam moved downward, and the top of the web rolled inward toward the applied loads at midspan. The degree of lateral deformation was influenced by the deck connections, as discussed in detail in the technical report.¹

Figure 2 indicates that the predicted cracking patterns matched well with those observed in experiments. Flexural cracking was observed in the midspan region, primarily on the outer web face. A primary diagonal crack initiated in the end region at the face of the support. This crack extended upward at an angle of approximately 45 deg. Away from the end region, the crack angles began to flatten and arch toward midspan.

The predicted cracking pattern for a reinforced concrete L-spandrel shows extensive flexural cracking at midspan, as expected for a nonprestressed concrete beam. In comparison, analysis of a prestressed concrete L-spandrel showed little flexural cracking at corresponding load levels. These predictions were confirmed by observed cracking patterns. The primary diagonal crack developed from the face of the support for all three selected beams. One interesting difference is that the predicted angle of this crack is almost 45 deg for the reinforced concrete beam, and as expected, it was flatter for the two prestressed concrete beams closer to 35 deg. This finding was not confirmed by the tests, as shown in the photographs in the left column of Fig. 2. While cracks on the inner face of the prestressed concrete beams flattened out more rapidly than did cracks on comparable nonprestressed concrete beams, there was minimal difference in the angle of the primary crack extending from the support of each specimen.

**Failure mode**

Figure 3 shows the failure modes predicted by the nonlinear FEM for the same three selected beams alongside the observed failure modes. The failure mode determined from analysis is illustrated by plotting the predicted shearing strain contours in the plane of the web face. All beams in Fig. 3 were reinforced with additional flexural and local reinforcement to avoid premature flexural failure and induce end-region behavior, as described in the companion paper.¹ The comparison shown in the figure indicates that the analytical model was capable of predicting the observed skewed-diagonal end-region failure mode for the L-shaped and corbelled spandrels.

**Deflections**

Figure 4 compares a representative vertical load-deflection prediction with the experimental results. The initial stiffness of the predicted load-deflection curves tended to be slightly higher than the measured values; however, the predicted stiffness tended to match closely to measured values after cracking. The analytical model closely predicted the failure load and failure deflection, with some deviation in the final stages of loading.

Figure 5 shows the predicted and measured lateral deflections for a representative spandrel beam. The comparison highlights the effect of short-term creep during the test, as evidenced by horizontal portions of the load-deflection
Based on the observed behavior and the nonlinear finite element analysis, a rational model was developed to describe the behavior of a slender spandrel beam. The rational model forms the basis of the proposed simplified design approach, presented later in this paper. The model is based on equilibrium of forces at the diagonal skewed failure plane observed in the experimental program.

The design principles are only applicable for the modeling of slender beams with an aspect ratio equal to or greater than 4.6 as this was the smallest aspect ratio tested. The beams considered had the following controls:

**Rational model**

Behavior. Short-term creep was especially prominent over the 24-hour sustained load tests. The FEMs do not account for creep. If creep effects were removed, representing an experimental test conducted without sustained loading, the experimental results would match the predicted values more closely. The predicted lateral deflections matched well the experimental values through most of the tested range. The FEM captured the spandrel behavior with the top edge of a slender spandrel beam moving inward and the bottom edge moving outward at midspan, pivoting about the deck connection. Further description of the finite element analysis is presented along with the full analytical results in the research report.\(^2\)
They were supported at each end by two horizontal reactions that form a couple to provide torsional stability.

They were simply supported for gravity load.

They had loads applied to a continuous ledge or to discrete corbels located along the bottom edge of one web face.

They had a ledge or corbels designed according to the standard procedures recommended by the PCI Design Handbook.

Inspection of the cracking pattern of all tested slender spandrel beams loaded to failure allowed for identification of the following three distinct zones (Fig. 6).

**End region**

The end region is defined as the portion of the beam from the end to a distance h from the face of the support. The test results of full-sized spandrels reported in the companion paper indicate that the angle of the critical diagonal crack that developed on the inner web face within this region is 45 deg. The observed failure mode clearly indicated that plate bending, vertical shear, and lateral shear (twist) dominate the end-region response because shear and torsion demands are highest in this region.
The transition region extends a distance $2h$ beyond the end region. Test results indicate that the primary crack angle is approximately 30 deg. Shear and torsion demands are reduced in this region while bending moment demands are increasing compared with the end region.

The flexure region is defined as the portion of the beam beyond the transition region to the midspan of the beam. The behavior in this region is marked by vertical cracking on the inner and outer web faces due to in- and out-of-plane flexure and by horizontal cracking on the inner web face due to hanger loads. Shear and torsion demands are relatively low, while moment demands are relatively high.

The loading demands considered in the rational model for each region consisted of the factored bending moment $M_r$, the factored shear force $V_r$, and the factored torque $T_r$. Eccentricity $e$ for the calculation of torsion should be considered from the point of load application to the center of the web. Flexure design should follow the recommendations in the PCI Design Handbook.

**Proposed rational model for resisting the applied torsion $T_r$**

The vertical eccentric loads acting on a slender spandrel beam produce torque acting about the longitudinal axis of the beam. The torsion demand $T_r$ is calculated by multiplying the applied factored ledge loads by the distance from the applied load to the center of the web. The $T_r$ vector at any diagonal crack can be resolved into two equivalent orthogonal vectors (Fig. 7). One component of the torque vector, defined in the analysis as $T_{rb}$, will act to bend the spandrel web out of plane about the diagonal axis defined by the diagonal crack angle $\theta$ (Eq. [1]). The second component of the torque vector, defined in the analysis as $T_{rt}$, acts to twist the web about an axis perpendicular to that diagonal crack (Eq. [2]).

\[
T_{rb} = T_r \cos \theta \quad (1)
\]

\[
T_{rt} = T_r \sin \theta \quad (2)
\]
Designing for the plate-bending component of torsion $T_{wb}$

Equation (1) provides the factored out-of-plane bending moment demand about the diagonal axis defined by angle $\theta$. The resistance to $T_{wb}$ is developed by plate bending in the spandrel web. The nominal resisting moment can be closely determined by Eq. (3).

$$T_{wb} = A' f_y d_e$$  \hspace{1cm} (3)

where

- $T_{wb}$ = plate-bending component of torsion
- $A'$ = total area of steel required on the inner web face crossing the critical diagonal crack in a direction perpendicular to the crack
- $f_y$ = yield stress of the inner-face reinforcing steel
- $d_e$ = effective depth from the outer surface of the web to the centroid of the combined horizontal and vertical steel reinforcement of the web; usually taken as web thickness less concrete cover less the diameter of the inner-face vertical steel bars

The lateral bending resistance must satisfy the lateral bending demand (Eq. [4]).

$$T_{wb} \leq \phi f_{Tnb}$$  \hspace{1cm} (4)

where

- $\phi = \text{strength reduction factor for flexure} = 0.9$
- $T_{nb}$ = nominal plate-bending resistance of the web

Combining Eq. (1), (3), and (4) into Eq. (5) results in the total required area of steel $A'_t$.

$$A'_t \geq \frac{T_{wb} \cos \theta}{\phi f_y d_e}$$  \hspace{1cm} (5)

Although $T_{wb}$ is a component of torsion, the strength reduction factor for flexure is considered appropriate because $T_{wb}$ is resisted by out-of-plane flexure of the web, which is plate bending.

Because the web steel resisting plate bending is usually...
Figure 6. Inspection of the cracking pattern of all tested slender spandrel beams loaded to failure allowed for identification of three distinct zones: end region, transition region, and flexure region. Note: $a =$ vertical distance from bottom of spandrel to center of lower tieback connection; $h =$ height of spandrel; $\theta =$ angle of critical diagonal crack for region under consideration with respect to horizontal.
orthogonal to the beam (not perpendicular to a crack), the total required area must be expressed in terms of vertical steel and longitudinal steel. The total required area of vertical steel crossing the diagonal crack $A_v$ can be determined by Eq. (6).

$$A_v = A_m \cos \theta$$

Using Eq. (5) and (6), the required $A_v$ can be determined by Eq. (7).

$$A_v \geq \frac{T_u \cos \theta \sin \phi}{f_yd_z}$$

Similarly, the total required area of longitudinal steel on the inner face to resist plate bending $A_l$ can be determined by Eq. (8).

$$A_l \geq \frac{T_u \cos \phi}{f_yd_z}$$

The area of vertical steel required to resist plate bending $A_v$ over the horizontal crack projection $l_{ch}$ can be determined by Eq. (9).

$$l_{ch} \geq \frac{h}{\cos \phi \tan \theta}$$

The required area of plate-bending reinforcement distributed over the horizontal crack projection $A_{v/s}$ can be determined by Eq. (10).

$$A_{v/s} \geq \frac{T_u \cos \phi \sin \theta}{f_yd_z h}$$

Similarly, the distributed longitudinal steel required on the inner web face for plate bending $A_{l/s}$ can be determined by Eq. (11).

$$A_{l/s} \geq \frac{T_u \cos \phi \sin \theta}{f_yd_z h}$$

Equations (10) and (11) are identical irrespective of crack angle or beam region. The required vertical plate-bending steel distributed over the horizontal crack projection equals the required horizontal plate-bending steel distributed over the vertical crack projection in a given beam region. $A_v$
Commentary (ACI 318R-08) recommended that the twisting resistance of the cracked section \( \phi T_n \) (where \( \phi \) is the strength reduction factor for shear and \( T_n \) is the nominal twist resistance of the section) exceed the twisting demand, with \( \phi \) equal to 0.75, according to the Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary (ACI 318R-08) recommendation for torsion design. The twist demand must be resisted by a cross section through a diagonal crack, normal to the \( T_a \) vector, having dimensions of \( h/\sin \theta \) and thickness \( b \) (Fig. 8). Evaluation of the twisting resistance in slender spandrel beams is based on the well-accepted mechanism used to transfer unbalanced moments from reinforced concrete slabs to columns. In the case of a column-slab interface, ACI-318-08 section 11.11.7.2 recommends that the shear stress “shall vary linearly about the centroid of the critical section.” The same concept of linear shear distribution is applied to the inclined cross section of a slender spandrel beam (Fig. 8). The linear model assumes a shear-stress distribution with a maximum value at the extreme end of the section. More complex models for twisting resistance were evaluated using rational models and FEMs. The complex models and finite element analysis closely match the linear approximation, as discussed in the technical report.2

The vertical reinforcement resisting plate bending \( A_{sv} \) is based on the assumption that the inner-face web steel yields under the effect of the applied factored plate-bending moment \( T_{ub} \). Calculation indicates that, in practical designs, the reinforcement requirements for plate bending are substantially less than the tension-controlled limit. Thus, plate-bending web steel will yield with significant ductility required for tension-controlled failure, and a value of 0.9 for \( \phi \) is justified.

### Figure 8. Linear distribution of shear stresses on inclined cross section. Note: \( b = \) thickness of web; \( h = \) height of spandrel; \( H_n \) = out-of-plane force resultants; \( L_{twist} \) = lever arm; \( T_u \) = twisting component of torsion; \( \theta \) = critical diagonal crack angle for region under consideration with respect to horizontal.

and \( A_{sv} \) should be calculated for each region at the point of maximum torsion \( T_u \) in that region. In the end region, not all possible diagonal cracks intersect the bottom of the beam. Therefore, it is appropriate to consider a diagonal crack extending upward from the lower lateral tieback, where crack projections would be calculated using the distance \( h - a \) in place of \( h \). Further details are presented in the technical report.2

The vertical reinforcement resisting plate bending \( A_{sv} \) is calculated based on the assumption that the inner-face web steel yields under the effect of the applied factored plate-bending moment \( T_{ub} \). Calculation indicates that, in practical designs, the reinforcement requirements for plate bending are substantially less than the tension-controlled limit. Thus, plate-bending web steel will yield with significant ductility required for tension-controlled failure, and a value of 0.9 for \( \phi \) is justified.

### Designing for the twisting component of torsion \( T_u \)

The proposed rational model requires that the twisting resistance of the cracked section \( \phi T_n \) (where \( \phi \) is the strength reduction factor for shear and \( T_n \) is the nominal twist resistance of the section) exceed the twisting demand, with \( \phi \) equal to 0.75, according to the Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary (ACI 318R-08) recommendation for torsion design. The twist demand must be resisted by a cross section through a diagonal crack, normal to the \( T_a \) vector, having dimensions of \( h/\sin \theta \) and thickness \( b \) (Fig. 8). Evaluation of the twisting resistance in slender spandrel beams is based on the well-accepted mechanism used to transfer unbalanced moments from reinforced concrete slabs to columns. In the case of a column-slab interface, ACI-318-08 section 11.11.7.2 recommends that the shear stress “shall vary linearly about the centroid of the critical section.” The same concept of linear shear distribution is applied to the inclined cross section of a slender spandrel beam (Fig. 8). The linear model assumes a shear-stress distribution with a maximum value at the extreme end of the section. More complex models for twisting resistance were evaluated using rational models and FEMs. The complex models and finite element analysis closely match the linear approximation, as discussed in the technical report.2 In all cases, the reduced nominal twisting resistance must exceed the factored twist demand, as shown in Eq. (12).

\[
T_u \leq \phi T_n
\]  

Based on the linear stress distribution in Fig. 8, the out-of-plane resultant forces \( H_n \) are separated by a distance \( L_{twist} \).
equal to $2/3$ the height of the diagonal crack. The magnitude of these resultant forces is determined in Eq. (13).

$$H_w = \frac{1}{2} \frac{h}{\sin \theta} \int \frac{1}{2} X f d z$$

(13)

where

$H_w$ = out-of-plane force resultants from a linear shear-stress distribution

$f^*$ = specified concrete compressive strength

$X$ = out-of-plane shear-stress coefficient (calibrated later in this paper from experimental data)

$X f^*$ = maximum shear stress of the linear distribution

(calibrated from the experimental data later in this paper)

Accordingly, the nominal twist resistance of the section $T_{nt}$ can be evaluated as the product of $H_w$ and $L_{atw}$ (Eq. [14]).

$$T_{nt} = X f^* \frac{1}{6} \frac{h^i}{\sin \theta} d_c$$

(14)

To ensure that the twisting resistance according to the linear model exceeds or equals the applied factored torque demand, Eq. (15) should be satisfied. Equation (15) is derived by substituting Eq. (14) and (2) into Eq. (12).

$$T_c \leq \phi X f^* \frac{1}{6} \frac{h^i}{\sin \theta} d_c$$

(15)

where

$\phi = 0.75$

**Calibration of the twist-resistance model**

Of the 16 tested beams, seven failed in out-of-plane modes in their end regions. For each of these seven beams, the measured lateral reactions at failure were used to calculate the out-of-plane shear stresses resisted by a given beam just before failure. It is recognized that there are interactions among the flexural, shear, and torsional stresses on the failure plane. Because the proposed twist resistance is calibrated to the measured test data, it accounts for the effects of the flexural stress and the vertical shear and torsional stresses. However, the proposed method neglects the effect of the torsional stress on the vertical shear stress based on the experimental evidence for beams tested.

The twist-resistance model was calibrated by first determining the torsion acting on the end region of a given beam at failure. For each tested beam, all lateral forces were included to determine the twisting moment at failure $T^*_c$. Lateral forces not directly measured during the tests were evaluated by considering the equilibrium of the beam as a whole about a selected origin, point O (Fig. 9).

The sum of lateral forces at the deck connections $\sum H_w$ and the friction coefficient $\mu$ were determined by considering moment equilibrium about point O and force equilibrium in the horizontal direction. The analysis was performed for each of the seven beams that failed in its end region. The range of calculated friction coefficients corresponds well to the value of 0.05 determined by the nonlinear finite element analysis described previously. In addition, the calculated values correspond well to the range published in the PCI Design Handbook for Teflon-coated bearing pads similar to those used in the tests.

With all lateral forces determined, the twisting component of torsion resisted by each beam at failure was evaluated. Figure 10 shows a free body diagram taken along a 45 deg crack plane extending upward from the face of the support (line 1-1 in the figure). All dimensions are known from the geometry of a given beam, and all forces were measured or determined from equilibrium. There are no deck connections to the left of crack plane 1-1; therefore, the deck connection forces $H_w$ do not appear in the free body diagram to the left of the crack.

The out-of-plane shear stress induced by the twisting moment at failure $T^*_c$ was assumed to be distributed linearly across the selected crack plane 1-1 (Fig. 10). The maximum values at the extremes of the distribution are identified as $X f^* \Sigma$ and $X f^*$, where $X$ is the height of the centroid and $\Sigma$ is the extreme value of the stress distribution. The location of the centroid of the twisting moment was determined by an iteration process that satisfied moment equilibrium about the selected centroid and equilibrium of the horizontal forces. The tendency of the vertical reaction to shift inward (toward the ledge) as a spandrel deforms under load was conservatively neglected. Based on the two equilibrium equations, the height of the centroid $X^\prime$ and the extreme values of the stress distribution $X f^*$ were determined for each experimental case (Table 1). $X^\prime$ and $X f^*$ are related by the linear distribution.

The calculated height of the twist axis $Z_t$ is consistently just above the midheight of all specimens. Therefore, assuming a balanced linear stress distribution for design is an acceptable approximation.

Given the results in Table 1, a conservative value of 2.4 is recommended for $X$ in Eq. (15). Because the centroid of the linear distribution assumed for design will be considered at the midheight of the cross section, it is appropriate to compare the selected value of $X$ to the values of $X_{avg}$.
Selecting a value below $X_{avg}$ is a safe choice for design, especially considering the conservative nature of the linear distribution and the assumptions made in the calibration.

Figure 11 plots the ratio of twisting moment at failure to nominal twisting capacity for all seven spandrels that failed in their end regions. These seven spandrel beams were specially configured to induce end-region failure modes by over-reinforcing other controlling failure modes. The plot shows that Eq. (15), with a value of 2.4 for $X$, conservatively predicts the twisting capacity of each tested beam. Thus, Eq. (15) is recommended for design with 2.4 for the value of $X$. This recommendation provides a conservative prediction of the twisting moment capacity of a slender spandrel and is consistent with the historical association (ACI 318-719) of 2.4 $f_c l$ with torsion. Thus, Eq. (16) gives the proposed twist resistance using the linear shear-stress distribution model.

$$T_s \leq \phi \left( \frac{2.4 \sqrt{f_y d_z h^2}}{6 \sin \theta} \right) \tag{16}$$

**Shear and torsion stresses**

Due to the loading and support conditions, shear and torsion stresses act in the same direction on the inner web face (the face on the ledge or corbel side) but oppose each other on the outer web face. The concrete on the inner face is more vulnerable to diagonal cracking. This behavior is clearly identified by experimental cracking patterns and is verified by linear finite element analysis presented in the technical research report.

**Designing for the applied shear**

As discussed previously, the rational model considers the design of shear and torsion independently. Thus, design...
The minimum requirements for area of shear reinforcement $A_v$ specified by ACI 318-08 may control over Eq. (17).

Proportioning the web reinforcement

The previously calculated quantities of steel reinforcement required to resist torsion and shear must be provided in the spandrel web according to the nature of shear and torsion stresses illustrated in Fig. 12.

The total quantity of distributed vertical steel required on the inner web face $A_v/s$ is the quantity required for the plate-bending component of torsion plus half of the total quantity required for shear (Eq. [18]).

$$A_v/s \geq \frac{V_c/\phi - V_c}{f_d}$$

where

- $\phi = 0.75$
- $V_c = \text{nominal concrete shear strength equal to the lesser of } V_{ci} \text{ or } V_{cw}$ as given by Eq. (11-11) and (11-12) of ACI 318-08
- $V_{ci} = \text{nominal shear strength provided by concrete when diagonal cracking results from combined shear and moment}$
- $V_{cw} = \text{nominal shear strength provided by concrete when diagonal cracking results from high principal tensile stress in the web}$

$$d = \text{distance from the extreme compression fiber to the centroid of the longitudinal reinforcement}$$
The remaining half of the required shear reinforcement is allocated to the outer spandrel face where shear and torsion stresses oppose each other. On the outer face, the vertical component of the plate-bending compressive force reduces the amount of required vertical reinforcement for shear (Fig. 13).

Thus, the total distributed vertical reinforcement required on the outer web face $A_{so}$ can be theoretically reduced by the component of vertical plate bending (Eq. [19]), provided that the total amount of transverse reinforcement in the beam is sufficient for shear. However, it is recommended that $A_{so}$ be provided as half of $A_v$.

### Table 1. Results from twist model calibration

<table>
<thead>
<tr>
<th>Test specimen</th>
<th>$Z_t$, in.</th>
<th>$X^*$, psi/√$f_c$</th>
<th>$X$, psi/√$f_c$</th>
<th>$X_{avg}$, psi/√$f_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP3.8L60.45.P.O.E</td>
<td>34.5</td>
<td>2.36</td>
<td>3.19</td>
<td>2.77</td>
</tr>
<tr>
<td>SP4.8L60.45.P.O.E</td>
<td>33.9</td>
<td>2.02</td>
<td>2.63</td>
<td>2.32</td>
</tr>
<tr>
<td>SP10.8L60.45.R.O.E</td>
<td>36.7</td>
<td>2.48</td>
<td>3.91</td>
<td>3.20</td>
</tr>
<tr>
<td>SP12.8L60.45.P.O.E</td>
<td>35.9</td>
<td>1.90</td>
<td>2.83</td>
<td>2.36</td>
</tr>
<tr>
<td>SP17.8C660.45.P.O.E</td>
<td>37.0</td>
<td>1.82</td>
<td>2.94</td>
<td>2.38</td>
</tr>
<tr>
<td>SP18.8CB60.45.P.S.E</td>
<td>36.8</td>
<td>2.17</td>
<td>3.45</td>
<td>2.81</td>
</tr>
<tr>
<td>SP20.10L46.45.P.O.E</td>
<td>26.4</td>
<td>2.11</td>
<td>2.83</td>
<td>2.47</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td>2.62</td>
<td>2.32</td>
</tr>
<tr>
<td>Minimum</td>
<td></td>
<td></td>
<td>2.32</td>
<td></td>
</tr>
<tr>
<td>Standard deviation</td>
<td></td>
<td></td>
<td>0.32</td>
<td></td>
</tr>
</tbody>
</table>

Note: $X^*$ = height of the centroid; $X$ = extreme values of stress distribution; $X_{avg}$ = average of $X^*$ and $X$; $Z_t$ = height of twist axis.

1 in. = 25.4 mm; 1 psi = 6.895 kPa.
Steel required for shear and torsion. Only the greater of the two should be used.

First cracking load

The anticipated load causing initial cracking in the end region of a slender spandrel beam is often of interest to the designer. While minor hairline cracking is usually not a concern from a structural or durability standpoint, it may be desirable to minimize or eliminate such cracking in cases where aesthetics are relevant. The first cracking load will be the same whether open or closed transverse web reinforcement is used.

Equation (20) gives the diagonal tensile stress due to shear of a slender spandrel $f_{cr1}$.

$$f_{cr1} = \frac{3V}{2bh}$$

where

- $f_{cr1}$ = diagonal tension stress due to shear
- $V$ = the applied shear force at the level of interest
- $b$ = width of web
- $h$ = height of the spandrel

On the outer web face, plate bending about a crack extending downward from the top lateral reaction can, in theory, control a design; however, this orthogonal plate bending is counteracted by vertical shear. Failure about this potential plane could only develop if torsional stresses, acting upward on the outer face, greatly exceeded the shear stresses, acting downward. Further details and analysis related to orthogonal plate bending are presented in the technical report.²

The longitudinal steel $A_s$, calculated by Eq. (8) is to be provided on both the inner and the outer web faces. Providing longitudinal steel on the outer face protects against possible plate bending on the outer face. In addition, longitudinal steel provides dowel forces beneficial to twist resistance, particularly in the end region.

Potential additional failure modes

Other reinforcement requirements may control the design of web reinforcement and must be checked according to procedures specified in the PCI Design Handbook. By experience, it was found that hanger steel requirements will often control the design of vertical reinforcement on the inner web face, especially outside of the end region. Hanger steel requirements need not be combined with the vertical steel required for shear and torsion. Only the greater of the two should be used.

First cracking load

The anticipated load causing initial cracking in the end region of a slender spandrel beam is often of interest to the designer. While minor hairline cracking is usually not a concern from a structural or durability standpoint, it may be desirable to minimize or eliminate such cracking in cases where aesthetics are relevant. The first cracking load will be the same whether open or closed transverse web reinforcement is used.

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- $b$ = width of web
- $h$ = height of the spandrel

Figure 12. Directions of shear and torsional stresses acting on slender spandrel cross section.
In actual structures, cracking can be influenced by handling, curing conditions, thermal exposure, and other factors. Also, concentration of stresses near the bearing area increases the likelihood of cracking just inside the support. As such, cracking may occur sooner than expected. However, if designed as recommended herein, end-region diagonal cracks will be narrow and will not adversely affect strength or durability. The aesthetic impact should also be minimal.

Proposed simplified design guidelines

To assist the designer, a step-by-step simplified design procedure was developed based on the rational model. The procedure is intended for slender precast concrete spandrel beams subject to the following restrictions:

- A simply supported precast concrete spandrel is loaded along the bottom edge of the web.
- Normalweight concrete is used.
- The web is laterally restrained at two points at each end.
- Applied loads are evenly spaced along the bottom edge of the web.
- The aspect ratio (height divided by web thickness) is equal to or more than 4.6.

Equation (21) calculates the diagonal tensile stress due to plate bending of a slender spandrel.

\[ f_{cr} = \frac{3T}{b'h} \quad (21) \]

where

- \( f_{cr} \): diagonal tension stress due to plate bending
- \( T \): applied torque
- \( e \): eccentricity contributing to torsion

Combining Eq. (20) and (21) and assuming a limiting tensile stress of concrete of \( 6f_c \), Eq. (22) can determine the shear force at cracking \( V_{cr} \).

\[ V_{cr} = \frac{4\sqrt{T}}{(1 + 2e/e)} \quad (22) \]

Comparison with experimental data indicates that Eq. (22) provides a conservative estimate of the load at which cracking is first likely to appear. First cracking loads can be increased by increasing the web thickness, increasing the concrete strength, or distributing prestressing force through the height of the web. Prestressing force concentrated only near the bottom of the section is not effective in controlling diagonal cracks near the support.
Step 1: Determine the loading demands

Construct the factored bending moment, shear, and torsion diagrams. Eccentricity for calculating torsion should be taken from the point of applied load to the center of the web.

Step 2: Divide the slender spandrel into the three regions shown in Fig. 14

Step 3: Verify that the cross section can sustain the twisting component of torsion

Verify that the maximum torque $T_u$ in the end region does not exceed the twist limit for this region by considering Eq. (23).

With $\theta$ taken as 45 deg for the end region, Eq. (16) becomes Eq. (23).

$$T_u \leq \phi \left( 1.13 \sqrt{\sigma_t d} h \right)$$

where

$\phi_s = 0.75$

Conservatively, Eq. (23) can also be used to check twist capacity in the transition region. However, twist in the transition region will not control the design for a simply supported beam with a uniform cross section. Similarly, there is no need to check the twist limit in the flexure region. Because the proposed twist resistance is calibrated to the measured test data, it accounts for the effects of the flexural stress and the vertical shear and torsional stresses.

If the lateral end reactions are spaced vertically at least 0.6h, a check for twist resistance on a secondary failure plane in the end region (line 2-2 in Fig. 14) is not required. Otherwise a check is necessary by using the height of the section above the lower connection (distance $h - a$, where $a$ is the vertical distance from the bottom of a spandrel to the center of the lower tieback connection) in Eq. (23) in lieu of $h$. Calculations involving the transition region would still use the full section height $h$. If this check for the twisting component of torsion is not satisfied, the concrete strength or cross-section dimensions would have to be increased; otherwise, closed reinforcement would have to be used.

Step 4: Design the beam for flexure

Design and proportion the longitudinal steel reinforcement (mild or prestressed) in accordance with section 4.2 of the PCI Design Handbook.

Step 5: Calculate the required shear steel at all sections along the beam

Design the beam for shear according to all provisions in section 4.3 of the PCI Design Handbook. For the vertical shear design in this step, ignore eccentricity.

Determine the amount of vertical shear reinforcement required per unit length ($A_v/s$) at all locations along the length of the beam, $A_v$ will likely change at each applied point load along the ledge and will also be influenced by the development of prestressing strands in the end region. All requirements for minimum $A_v$ still apply.

The proposed method neglects the effect of the torsional stress on the vertical shear stress based on successful performance of the test specimens, which were designed without consideration of the influence of torsion on shear strength. However, Zia and Hsu indicated that torsion reduces shear strength, especially in compact sections. As
such, this procedure should only be applied to precast concrete spandrel beams with an aspect ratio of 4.6 or more.

**Step 6: Calculate the vertical reinforcement required on the inner web face to resist the plate-bending component of torsion**

Consider plate bending in the end region and in the transition region (Fig. 14). There is no need to consider plate bending in the flexure region.

Use Eq. (24) and Eq. (25) to calculate the distributed quantity of vertical steel (in.²/in.) required to resist plate bending for the end region and transition region, respectively, using the maximum $T_u$ from each region. These equations are the same as Eq. (10) with $\theta$ taken as 45 deg (for the end region) or 30 deg (for the transition region).

For the end region:

$$A_{sv}/s \geq \frac{T_u}{2\phi_f f_y d_w h}$$

Units: $A_{sv}$ (in.²), $s$ (in.), $T_u$ (lb-in.), $h$ (in.), $d_w$ (in.), $f_y$ (psi), $\phi_f = 0.9$

For the transition region:

$$A_{sv}/s \geq \frac{T_u}{2.3\phi_f f_y d_w h}$$

Units: $A_{sv}$ (in.²), $s$ (in.), $T_u$ (lb-in.), $h$ (in.), $d_w$ (in.), $f_y$ (psi), $\phi_f = 0.9$

**Step 7: Proportion the vertical steel on the inner web face**

Provide vertical steel on the inner web face (ledge or corbel side) to satisfy Eq. (26) at all locations.

$$A_{vi}/s \geq \left(A_v + \frac{1}{2}A_{sv}\right)/s$$

Units: $A_{vi}$ (in.²), $A_v$ (in.²)

Additional requirements, such as hanger steel or impact steel, may control the design of the inner-face web reinforcement. Provide hanger steel or the steel required by Eq. (26), whichever is greater.

**Step 8: Check the plate-bending capacity about line 2-2 in the end region**

Plate bending about line 2-2 may be more critical than plate bending about line 1-1 in the end region (Fig. 14). Whether line 2-2 is critical for plate bending depends on the location of the lateral tieback reactions and on the quantity of vertical shear steel $A_v$. After proportioning vertical steel in step 7, verify that the total quantity of vertical steel crossing line 2-2 on the inner face $A_{sv2}$ exceeds Eq. (27).

$$A_{sv2} \geq \frac{T_u}{\phi_f/2 f_y d_w}$$

**Step 9: Proportion the vertical steel on the outer web face**

Provide vertical steel on the outer web face to satisfy Eq. (28) at all locations.

$$A_{vo} \geq \frac{1}{2}A_v$$

**Step 10: Calculate the longitudinal reinforcement required to resist the plate-bending component of torsion**

Calculate the total area of longitudinal steel required to resist plate bending with Eq. (29) and Eq. (30) for the end region and transition region, respectively, using the maximum $T_u$ from each region. There is no need to consider plate bending in the flexure region. These equations are the same as Eq. (8) with $\theta$ taken as 45 deg for the end region and 30 deg for the transition region.

For the end region:

$$A_{sl} \geq \frac{T_u}{2\phi_f f_y d_w h}$$

Units: $A_{sl}$ (in.²), $f_y$ (psi), $h$ (in.), $d_w$ (in.), $T_u$ (lb-in.), $f_y$ (psi), $\phi_f = 0.9$

For the transition region:

$$A_{sl} \geq \frac{T_u}{2.3\phi_f f_y d_w h}$$

Units: $A_{sl}$ (in.²), $f_y$ (psi), $h$ (in.), $d_w$ (in.), $T_u$ (lb-in.), $f_y$ (psi), $\phi_f = 0.9$

**Step 11: Proportion the longitudinal web steel for plate bending**

Provide the required longitudinal web steel (from Eq. [29] and [30]) on both the inner and outer web faces. The longitudinal bars should be developed at the start of each region.
and extend for the full length of that region. Generally, horizontal U-shaped bars are effective in the end regions because they allow for development at the end of the beam.

In the end region, longitudinal steel below the level of the lower lateral reaction should not be counted toward the plate-bending requirement.

In the transition region, fully developed excess longitudinal reinforcement not required for flexure may be used as part of the plate-bending requirement.

**Step 12: Detail the beam**

Recommendations in the PCI Design Handbook regarding ledge design, connection detailing, hanger steel, vehicular-impact steel, and reinforcement for other possible localized failure modes are to be considered. In addition, it is recommended that continuous bars or strands be placed in the longitudinal direction at all four corners of the web to control the possible cracking from lateral bending away from the ends.

**Step 13: Check service-level cracking**

If desired, the load at which diagonal cracking in the end region is first likely to appear may be conservatively estimated according to Eq. (31). In many cases, beams will be cracked at service load, regardless of end-region reinforcement. First cracking load is independent of web reinforcement configuration (that is, open or closed stirrups). The effects of prestressing are ignored in Eq. (31) because some of the prestressing strands are not fully developed across the potential crack.

\[ V_{cr} = \frac{4\sqrt{f_p}}{1 + 2e(b)} bh \]  

(31)

where

\[ V_{cr} \] = shear force at cracking

**Abstract**

A rational design method is recommended for the design of precast concrete slender spandrel beams with aspect ratios greater than or equal to 4.6. The procedure was developed using data from an extensive research program including 16 full-scale tests, extensive finite element modeling, and a rational analysis. It is recommended that the design for shear and torsion of slender precast concrete spandrel beams use the concept of resolving the applied torsion into two orthogonal vectors: a plate-bending component and a twisting component. Equations were presented for evaluating the capacity of a slender precast concrete spandrel beam to resist both components of torsion. The research demonstrates that slender precast concrete spandrel beams can be safely designed to resist the combined effects of flexure, shear, and torsion without the use of traditional closed web reinforcement. Use of open web reinforcement could greatly reduce reinforcement congestion in the end regions of spandrel beams and provide significant savings in fabrication cost.

**Acknowledgments**

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**References**


8. ACI Committee 318. 2008. *Building Requirements for Structural Concrete (ACI 318-08) and Commen-
125

Notation

- $a$ = vertical distance from the bottom of a spandrel to the center of the lower tieback connection
- $A'_d$ = total area of steel required on the inner web face crossing the critical diagonal crack in a direction perpendicular to the crack
- $A_{sh}$ = required hanger steel
- $A_{si}$ = total required area of vertical steel on inner web face
- $A_{is}$ = total quantity of distributed vertical steel required on inner web face
- $A_{is}$ = total required area of horizontal steel on inner face to resist plate bending
- $A_{is}$ = distributed longitudinal steel required on inner web face for plate bending
- $A_{so}$ = total distributed vertical reinforcement required on outer web face
- $A_{si}$ = total required area of steel crossing diagonal crack in vertical direction
- $A_{si}$ = total quantity of vertical steel crossing line 2-2 on inner face
- $A_v$ = area of shear reinforcement
- $A_{is}$ = uniformly distributed vertical shear reinforcement
- $b$ = thickness of web
- $d$ = distance from extreme compression fiber to centroid of longitudinal reinforcement
- $d_e$ = effective depth from outer surface of web to centroid of combined horizontal and vertical steel reinforcement of web; usually taken as web thickness less concrete cover less diameter of inner-face vertical steel bars
- $e$ = eccentricity
- $f'_c$ = specified concrete compressive strength
- $f_{cd}$ = diagonal tensile stress due to plate bending of a slender spandrel
- $f_y$ = yield strength of mild steel
- $h$ = height of spandrel section
- $h/b$ = aspect ratio
- $H_1$ = measured lower lateral tieback reaction
- $H_1$ = measured deck connection forces
- $H_2$ = measured upper lateral tieback reaction
- $H_{nt}$ = out-of-plane force resultants
- $l_{sh}$ = length of horizontal projection of diagonal crack
- $l_{twist}$ = lever arm
- $M_u$ = factored bending moment
- $P_s$ = factored stem load
- $s$ = spacing of vertical reinforcement
- $T$ = applied torque
- $T_{ab}$ = nominal plate-bending resistance of web
- $T_s$ = factored torque
- $T_{tw}$ = plate-bending component of torsion
- $T_{tw}$ = twisting component of torsion
- $V$ = experimentally measured main spandrel end reaction
- $V_{is}$ = experimentally applied stem reaction
- $V_c$ = nominal concrete shear strength = lesser of $V_i$ or $V_{is}$, as given by Eq. (11-11) and (11-12) of ACI 318-08
- $V_c'$ = nominal shear strength provided by concrete when diagonal cracking results from combined shear and moment
- $V_{is}$ = shear force at cracking
$V_{cw}$ = nominal shear strength provided by concrete when diagonal cracking results from high principal tensile stress in web

$V_a$ = factored shear force

$X$ = out-of-plane shear-stress coefficient

$X'$ = height of centroid

$X_\infty$ = extreme values of stress distribution

$X_{ave}$ = average of $X'$ and $X_\infty$

$X\sqrt{\overline{X'}}$ = effective out-of-plane shear stress

$X_{v}$ = distance from outer web face to center of main vertical reaction, estimated to remain at center of the web

$X_{vl}$ = distance from outer web face to center of ledge reactions, estimated to remain at center of ledge bearing pads

$X_{vs}$ = distance from outer web face to center of self-weight reaction, estimated to remain at center of gravity

$Z_{h1}$ = height of lower lateral reaction

$Z_{h2}$ = height of deck connection

$Z_{h3}$ = height of upper lateral reaction

$Z_{v}$ = height to Teflon surface of main bearing pad

$Z_{vl}$ = height to Teflon surface of ledge (or corbel) bearing pad

$Z_t$ = height of twist axis

$\theta$ = critical diagonal crack angle for region under consideration with respect to horizontal

= 45 deg for end regions

= 30 deg for transition regions

= 0 deg for flexure regions

$\mu$ = friction coefficient

$\Sigma V$ = sum of measured vertical reactions at failure

$\Sigma V_a$ = sum of loads applied to the spandrel by the double-tee decks and jacks

$\Sigma V_s$ = self-weight of the spandrel beam

$\Sigma \mu v$ = sum of lateral forces at the main vertical bearing due to friction

$\Sigma \mu vl$ = sum of lateral forces at the ledge (or corbel) due to friction

$\phi_f$ = strength reduction factor for flexure = 0.9

$\phi_s$ = strength reduction factor for shear = 0.75

$H_1$ = sum of measured lower lateral reactions at failure

$H_2$ = sum of lateral forces at the deck connections

$H_3$ = sum of measured upper lateral reactions at failure
Appendix: Design example

Problem statement

Determine the reinforcement required in the web of the simply supported, L-shaped spandrel beam (Fig. A1). The spandrel supports nine double-tee stems, each spaced 5 ft (1.5 m) apart. One stem is centered at midspan. Each stem reaction comprises a dead-load reaction of 10.74 kip (47.8 kN), a live-load reaction of 6 kip (27 kN), and a snow-load reaction of 4.5 kip (20 kN). Stem loads are centered 6 in. (150 mm) from the inside web face. \(1.2D + 1.6L + 0.5S\) is the controlling load case. The spandrel self-weight is 567 lb/ft (8.27 kN/m).

Given information

- Web thickness \(b = 8\) in. (200 mm)
- Web depth \(h = 60\) in. (1500 mm)
- Eccentricity \(e = 10\) in. (250 mm)
- Span = 44 ft 6 in. (13.6 m)
- Depth of web steel \(d_w = 6.5\) in. (165 mm)
- Concrete design strength \(f' = 6000\) psi (40 MPa)

---

**Figure A1.** L-shaped spandrel considered in design example. Note: \(a = \) vertical distance from bottom of spandrel to center of lower tie-back connection; \(b = \) thickness of web; \(d_w = \) effective depth from outer surface of web to centroid of combined horizontal and vertical steel reinforcement of web; usually taken as web thickness less concrete cover less diameter of inner-face vertical steel bars; \(e = \) eccentricity; \(h = \) height of spandrel; \(P_{u} = \) factored stem load; \(V_u = \) factored shear force. 1 in. = 25.4 mm; 1 ft = 0.305 m; 1 kip = 4.448 kN.
The maximum factored torque at the support \( T_u = 1113 \text{ kip-in.} \ (125.8 \text{ kN-m}) \)

The eccentric self-weight of the ledge is neglected.

Step 2: Divide the slender spandrel into three regions

Figure A3 shows the divisions of the slender spandrel into three regions.

Step 3: Verify that the cross section can sustain the twisting component of torsion

Verify that the maximum torque \( T_u \) in the end region does not exceed the twist limit for this region by considering Eq. (23).

Typically, a section is checked for the twisting component of torsion about the 1-1 crack.

Step 1: Determine the loading demands

The maximum factored bending moment at midspan \( M_u = 1440 \text{ kip-ft} \ (1950 \text{ kN-m}) \)

The maximum factored vertical shear at the face of the support \( V_u = 126 \text{ kip} \ (560 \text{ kN}) \)

Yield strength of mild steel \( f_y = 60,000 \text{ psi} \ (410 \text{ MPa}) \)

Aspect ratio \( h/b = 7.5 \)

Given sketches

Figure A1 and Fig. A2 provide additional information for this example.

Solution

The beam meets the following criteria, so the design approach presented in this paper may be used:

- A simply supported precast concrete spandrel is loaded along the bottom edge of the web.
- The web is laterally restrained at two points at each end.
- Applied loads are evenly spaced along the bottom edge of the web.
- The aspect ratio of 7.5 (height divided by web thickness) is greater than or equal to 4.6.

Beam and regions symmetric about midspan

Distance from left end of beam, ft

\[ \begin{array}{|c|c|c|c|c|c|c|}
\hline
& 0 & 1.0 & 2.75 & 6.0 & 12.75 & 16 & 17.75 \\
\hline
\text{Distance from left end of beam, ft} & \text{start} & \text{end} & \text{region} & \text{stem} & \text{stem} & \text{stem} & \text{stem} \\
\hline
\text{Distance from left end of beam, ft} & \text{region} & \text{region} & \text{region} & \text{region} & \text{region} & \text{midspan} \\
\hline
\text{Distance from left end of beam, ft} & \text{region} & \text{region} & \text{region} & \text{region} & \text{region} & \text{midspan} \\
\hline
\text{Distance from left end of beam, ft} & \text{region} & \text{region} & \text{region} & \text{region} & \text{region} & \text{midspan} \\
\hline
\end{array} \]

\[ h = 60 \text{ in.} \]

\[ a = 12 \text{ in.} \]

\[ \theta = 45 \text{ deg} \]

\[ \phi = 30 \text{ deg} \]

Figure A2. Divide spandrel beam into three regions (step 2). Note: \( a \) = vertical distance from bottom of spandrel to center of lower tie-back connection; \( h \) = height of spandrel; \( \theta \) = critical diagonal crack angle for region under consideration with respect to horizontal. 1 in. = 25.4 mm; 1 ft = 0.305 m.

The maximum factored torque at the support \( T_u = 1113 \text{ kip-in.} \ (125.8 \text{ kN-m}) \)

\[ T_u = \left[ 1.2 \left( 10.74 \right) + 1.6 \left( 6 \right) + 0.5 \left( 4.5 \right) \right] \left( \frac{9}{2} \right) \left( 10 \right) \]

\[ T_u = 1113 \text{ kip-in.} \ (125.8 \text{ kN-m}) \]

Figure A3 shows the divisions of the slender spandrel into three regions.

Beam and regeions symmetric about midspan

The eccentric self-weight of the ledge is neglected.

Step 2: Divide the slender spandrel into three regions.

Step 3: Verify that the cross section can sustain the twisting component of torsion.

Given sketches

Figure A1 and Fig. A2 provide additional information for this example.

Solution

The beam meets the following criteria, so the design approach presented in this paper may be used:

- A simply supported precast concrete spandrel is loaded along the bottom edge of the web.
- The web is laterally restrained at two points at each end.
- Applied loads are evenly spaced along the bottom edge of the web.
- The aspect ratio of 7.5 (height divided by web thickness) is greater than or equal to 4.6.

Step 1: Determine the loading demands

The maximum factored bending moment at midspan \( M_u = 1440 \text{ kip-ft} \ (1950 \text{ kN-m}) \)

The maximum factored vertical shear at the face of the support \( V_u = 126 \text{ kip} \ (560 \text{ kN}) \)

Yield strength of mild steel \( f_y = 60,000 \text{ psi} \ (410 \text{ MPa}) \)

Aspect ratio \( h/b = 7.5 \)
Step 6: Calculate the vertical reinforcement required on the inner web face to resist the plate-bending component of torsion

Plate bending is considered in the end region and in the transition region. There is no need to consider plate bending in the flexure region. The vertical steel required to resist plate bending is calculated using the maximum $T_u$ from each region.

For the end region (maximum $T_u$ in this region is 1113 kip-in. [125.8 kN-m]):

$$A_{sv} \geq \frac{T_u}{2f_yd_wu}$$

For the transition region (maximum $T_u$ in this region is 866 kip-in. [97.8 kN-m]):

$$A_{sv} \geq \frac{1,113,000}{2(0.9)(60,000)(6.5)(60)}$$

$A_{sv} \geq 0.0264 \text{ in.}^2/\text{in.}$ (670 mm$^2$/m)

$A_{sv} \geq 0.317 \text{ in.}^2/\text{ft.}$ (670 mm$^2$/m)

For the transition region (maximum $T_u$ in this region is 866 kip-in. [97.8 kN-m]):

$$A_{sv} \geq \frac{T_u}{2.3f_yd_wu}$$

Step 4: Design the beam for flexure

The beam is designed for flexure using the techniques in section 4.2 of the PCI Design Handbook (not shown). Thirteen longitudinal prestressing strands are selected to provide flexural resistance.

Step 5: Calculate the required shear steel at all sections along the beam

The beam is designed for vertical shear using the techniques in section 4.3 of the PCI Design Handbook (not shown). For the vertical shear design in this step, eccentricity is ignored. The vertical shear steel required ($A_{sv}$) is determined to be 0.08 in.$^2$/ft at all locations (Table A1). $V_c$ is relatively high in this example; thus, minimum steel requirements govern.

### Table A1. Vertical shear steel requirement

<table>
<thead>
<tr>
<th>Distance from left end of beam, ft</th>
<th>1</th>
<th>2.75</th>
<th>6.0</th>
<th>7.75</th>
<th>12.75</th>
<th>16.0</th>
<th>17.75</th>
<th>22.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear steel required, in.$^2$/ft</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Note: 1 in. = 25.4 mm; 1 ft = 0.305 m.
Step 8: Check the plate-bending capacity about line 2-2 in the end region

Plate bending about line 2-2 may be more critical than plate bending about line 1-1 in the end region (Fig. A1 and A2). Whether line 2-2 is critical for plate bending depends on the location of the lateral tieback reactions and on the quantity of vertical shear steel $A_{v}$. After proportioning the vertical steel in step 7, verify that the total quantity of vertical steel crossing line 2-2 on the inner face exceeds Eq. (27).

$$A_{v} / s \geq \frac{886,000}{2.3\left[0.9\left(60,000\right)\right]^{\left(6.5\right)/60}}$$

$$A_{v} / s \geq 0.0179 \text{ in.}^2/\text{in.} \left(460 \text{ mm}^2/\text{m} \right)$$

$$A_{v} / s \geq 0.215 \text{ in.}^2/\text{ft} \left(460 \text{ mm}^2/\text{m} \right)$$

**Table A2. Shear and torsion steel requirements, inner web face**

<table>
<thead>
<tr>
<th>Beam region</th>
<th>End</th>
<th>Transition</th>
<th>Flexure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance from left end of beam, ft</td>
<td>1</td>
<td>2.75</td>
<td>6.0</td>
</tr>
<tr>
<td>Shear steel $A_{v} / s$, in.$^2$/ft</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>$\sqrt{2}(A_{v} / s)$, in.$^2$/ft</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>$A_{v} / s$, in.$^2$/ft</td>
<td>0.317</td>
<td>0.317</td>
<td>0.317</td>
</tr>
<tr>
<td>Steel required on inner web face for shear and torsion, in.$^2$/ft</td>
<td>0.357</td>
<td>0.357</td>
<td>0.357</td>
</tr>
</tbody>
</table>

Other inner-face steel requirements, such as hanger steel, may still control design. Using section 4.5 of PCI Design Handbook, required hanger steel $A_{h}$ is 0.222 in.$^2$/ft (470 mm$^2$/m) (not shown). In addition, hanger steel design dictates that hanger bar spacing not exceed ledge depth of 8 in. (200 mm). Hanger steel requirement is not additive. Thus, designer should choose larger of hanger steel requirement or shear/torsion steel requirement at given location.

Note: $A_{v}$ = area of vertical plate-bending reinforcement; $A_{s}$ = area of shear reinforcement; $s$ = spacing of vertical reinforcement. 1 in. = 25.4 mm; 1 ft = 0.305 m.

Step 7: Proportion the vertical steel on the inner web face

Vertical steel is provided for shear and torsion on the inner web face (ledge or corbel side) to satisfy Eq. (26) at all locations:

$$A_{v} / s \geq \frac{886,000}{2.3\left[0.9\left(60,000\right)\right]^{\left(6.5\right)/60}}$$

$$A_{v} / s \geq \frac{1,130,000}{0.9\left[60,000\right]^{\left(6.5\right)/2/6}}$$

$$A_{v} / s \geq 1.59 \text{ in.}^2 \left(1030 \text{ mm}^2 \right)$$

**Table A2** lists the values of the shear and torsion steel requirements on the inner web face.

Vertical L-shaped bars are spaced on the inner web face (Fig. A4) to satisfy the shear/torsion and hanger steel requirements.

The no. 4 (13M) reinforcing bars spaced at 6.5 in. (165 mm) satisfy the 0.357 in.$^2$/ft (750 mm$^2$/m) required in the end region. The no. 4 reinforcing bars spaced at the hanger steel limit of 8 in. (200 mm) satisfy the 0.255 in.$^2$/ft (540 mm$^2$/m) required in the transition zone and also the 0.222 in.$^2$/ft (470 mm$^2$/m) required at all locations for hanger steel. Two C-shaped bars are provided at each end of the beam to confine the connection zone and control longitudinal splitting at the ends of the prestressing strands. The remaining bars are L-shaped bars.

**Figure A4.** Profile view of web steel (step 9). Note: no. 4 = 13M; 1 in. = 25.4 mm.
crack plane, providing 1.60 in.² (1030 mm²) of steel and satisfying the $A_{sv}$ requirement.

**Step 9: Proportion the vertical steel on the outer web face**

Vertical steel is provided on the outer web face to satisfy Eq. (28) at all locations.

$$A_{sv} \geq \frac{1}{2} A_v$$  \hspace{1cm} (28)  

$A_v$ was calculated in step 5. Thus, the vertical steel required on the outer face $A_{sv}$ for shear and torsion is 0.04 in.² (85 mm²/m) at all locations. A continuous sheet of 6 in. × 6 in. (150 mm × 150 mm), W2.1 × W2.1 welded-wire reinforcement will satisfy this requirement (Fig. A5).

**Step 10: Calculate the longitudinal reinforcement required to resist the plate-bending component of torsion**

The total quantity of longitudinal steel required to resist plate bending is determined.

For the end region (maximum $T_u$ in this region is 1113 kip-in. [125.8 kN-m]):

$$A_{sl} \geq \frac{T_u}{2\phi f_y d_s}$$  \hspace{1cm} (29)  

$$A_{sl} \geq \frac{1,113,000}{2 \left( 0.9 \right) \left( 60,000 \right) \left( 6.5 \right)}$$  

$$A_{sl} \geq 1.59 \text{ in.}^2 (1030 \text{ mm}^2)$$

For the transition region (maximum $T_u$ in this region is 866 kip-in. [97.8 kN-m]):

$$A_{sl} \geq \frac{T_u}{2.3\phi f_y d_s}$$  \hspace{1cm} (30)  

$$A_{sl} \geq \frac{866,000}{2.3 \left( 0.9 \right) \left( 60,000 \right) \left( 6.5 \right)}$$  

$$A_{sl} \geq 1.07 \text{ in.}^2 (690 \text{ mm}^2)$$

**Step 11: Proportion the longitudinal web steel for plate bending**

Six no. 5 (16M), horizontal U-bars provide the required longitudinal steel on each face in the end region. All bars are located above the lower lateral reaction.

Sufficient fully developed excess flexural reinforcement may be used in the transition zone to satisfy the longitudinal steel requirement. In this example, prestressing strands not required for flexural reinforcement are used to meet the longitudinal plate-bending steel requirement in the transition zone. The longitudinal steel required in this zone $A_{sl}$ (as previously calculated assuming 60 ksi (420 MPa)) can be reduced proportionally according to the increase in steel strength (from 60 ksi to 270 ksi (420 MPa to 1860 MPa)). Thus, 0.24 in.² (150 mm²) of 270 ksi (1860 MPa) prestressing steel would satisfy the $A_{sl}$ requirement in the transition zone for this example.

**Step 12: Detail the beam**

Recommendations in the PCI Design Handbook regarding ledge design, connection detailing, hanger steel, impact steel, and reinforcement for other possible localized failure
modes are to be considered. In addition, it is recommended that continuous bars or strands be placed in the longitudinal direction at all four corners of the web to control the possible cracking from lateral bending away from the ends.

**Step 13: Check service-level cracking**

If desired, the load at which diagonal cracking in the end region is first likely to appear may be conservatively estimated according to Eq. (31). In many cases, beams will be cracked at service load, regardless of end-region reinforcement. First cracking load is independent of web reinforcement configuration (that is, open or closed stirrups). The effects of prestressing are ignored in Eq. (31) because some of the prestressing strands are not fully developed across the potential crack.

$$V_{cr} = \frac{4 \sqrt{f_{c} b}}{1 + 2e/b} bh \quad (31)$$

where

$$V_{cr} = \text{shear force at cracking}$$
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Synopsis

This paper summarizes the results of an analytical research program undertaken to develop a rational design procedure for normalweight precast concrete slender spandrel beams. The analytical and rational models use test results and research findings of an extensive experimental program presented in the companion paper “Development of a Rational Design Methodology for Precast Concrete Slender Spandrel Beams: Part 1, Experimental Results,” which appeared in the Spring 2011 issue of PCI Journal. The overall research effort demonstrated the validity of using open web reinforcement in precast concrete slender spandrel beams and proposed a simplified procedure for design. The webs of such slender spandrels, particularly in their end regions, are often heavily congested with reinforcing cages when designed with current procedures. The experimental and analytical results demonstrate that open web reinforcement designed according to the proposed procedure is safe and effective and provides an efficient alternative to traditional closed stirrups for precast concrete slender spandrel beams.

Keywords
Open reinforcement, slender spandrel beam, torsion, twist.

Review policy

This paper was reviewed in accordance with the Precast/Prestressed Concrete Institute’s peer-review process.

Reader comments

Please address any reader comments to journal@pci.org or Precast/Prestressed Concrete Institute, c/o PCI Journal, 200 W. Adams St., Suite 2100, Chicago, IL 60606.
5.3. Results from Material Testing

Concrete cylinders (4"x8") were cast and field cured with each spandrel. Cylinders were tested in compression according to ASTM C39 Compressive Strength of Cylindrical Concrete Specimens close to the time of testing each spandrel. Test results of the cylinders were used to determine the concrete compressive strengths given in Table 5-1.

Table 5-1: Concrete Compressive Strengths

<table>
<thead>
<tr>
<th>Spandrel</th>
<th>Specified 28-Day Design Strength (psi)</th>
<th>Average Test Cylinder Compressive Strength (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP1</td>
<td>5,000</td>
<td>7,500</td>
</tr>
<tr>
<td>SP2</td>
<td>5,000</td>
<td>7,000</td>
</tr>
<tr>
<td>SP3</td>
<td>6,000</td>
<td>5,800</td>
</tr>
<tr>
<td>SP4</td>
<td>6,000</td>
<td>7,200</td>
</tr>
<tr>
<td>SP10</td>
<td>7,600</td>
<td>6,800</td>
</tr>
<tr>
<td>SP11</td>
<td>7,600</td>
<td>6,800</td>
</tr>
<tr>
<td>SP12</td>
<td>6,500</td>
<td>6,800</td>
</tr>
<tr>
<td>SP13</td>
<td>6,500</td>
<td>6,800</td>
</tr>
<tr>
<td>SP14</td>
<td>6,000</td>
<td>6,000</td>
</tr>
<tr>
<td>SP15</td>
<td>6,000</td>
<td>6,000</td>
</tr>
<tr>
<td>SP16</td>
<td>6,000</td>
<td>5,200</td>
</tr>
<tr>
<td>SP17</td>
<td>6,000</td>
<td>8,500</td>
</tr>
<tr>
<td>SP18</td>
<td>6,000</td>
<td>8,000</td>
</tr>
<tr>
<td>SP19</td>
<td>7,000</td>
<td>8,100</td>
</tr>
<tr>
<td>SP20</td>
<td>7,000</td>
<td>6,800</td>
</tr>
<tr>
<td>SP21</td>
<td>7,000</td>
<td>6,800</td>
</tr>
</tbody>
</table>

It is important to note that the measured modulus of elasticity of the concrete used to produce spandrels SP1, SP2, SP3, SP4, SP14, and SP15 was significantly lower than the predicted values of $\sqrt{f_c}$, psi. These six spandrels were produced in a precast plant.
where the elastic modulus of the concrete was consistently close to \(32.000 \sqrt{f_c}\), as confirmed by two concrete cylinders instrumented to measure axial strain and tested in compression. The remaining 10 spandrels were cast in various other plants where the concrete used was assumed to have an elastic modulus equal to \(57.000 \sqrt{f_c}\).

Samples of the steel reinforcement were also taken from each precast plant. In cases where multiple spandrels were cast together, a single set of steel material samples was taken and tested for the group. Samples were collected and tested to determine the mechanical properties of the steel for all of the reinforcing bars sizes and welded wire reinforcement types used in the webs of the spandrels. Steel samples were tested using a 220 kip capacity Universal Testing Machine to determine the material properties. Results from all tests of steel material samples are given in Table 5-2.

**Table 5-2: Steel Material Sample Results**

<table>
<thead>
<tr>
<th>Spandrel</th>
<th>Sample Description</th>
<th>Tested Yield Stress (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP1 through SP4</td>
<td>#3 bars</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>#4 bars</td>
<td>76</td>
</tr>
<tr>
<td></td>
<td>4x4 W4.0 Wire mesh</td>
<td>92</td>
</tr>
<tr>
<td></td>
<td>6x6 W4.0 Wire mesh</td>
<td>97</td>
</tr>
<tr>
<td>SP10 through SP13</td>
<td>#4 bars</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>4x4 W4.0 Wire mesh</td>
<td>85</td>
</tr>
<tr>
<td></td>
<td>6x6 W4.0 Wire mesh</td>
<td>78</td>
</tr>
<tr>
<td>SP14 and SP15</td>
<td>#4 bars</td>
<td>76</td>
</tr>
<tr>
<td></td>
<td>6x6 W2.5 Wire mesh</td>
<td>97</td>
</tr>
<tr>
<td>SP16</td>
<td>#4 bars</td>
<td>65</td>
</tr>
<tr>
<td>SP17 and SP18</td>
<td>#3 bars</td>
<td>68</td>
</tr>
<tr>
<td></td>
<td># 4 bars</td>
<td>78</td>
</tr>
<tr>
<td></td>
<td>4x4 W4.0 Mesh</td>
<td>80</td>
</tr>
<tr>
<td>SP19</td>
<td>#4 bars</td>
<td>70</td>
</tr>
<tr>
<td>SP20 and SP21</td>
<td>#4 bars</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>6x6 W2.5 Wire mesh</td>
<td>75</td>
</tr>
</tbody>
</table>
6. Compact Beams

After Phases I and II of the research demonstrated that slender spandrels could be successfully designed with open web reinforcement, the research effort was expanded to a third phase consisting of four additional tests on compact sections. The four compact L-shaped beams tested had aspect ratios of 1.2, significantly below the ratios of 4.6 and 7.5 of the slender specimens. Three of the four compact specimens were fabricated with open web reinforcement designed to resist plate bending in the end region. The fourth specimen was designed with traditional closed stirrups to serve as a control. The Phase III tests of compact L-shaped beams are documented in the following paper, *Behavior of Compact L-Shaped Spandrel Beams with Alternative Web Reinforcement* by Lucier, Hariharan, Rizkalla, Zia, and Klein. The paper has been submitted for publication in the *PCI Journal*. 
Behavior of Compact L-Shaped Spandrel Beams with Alternative Web Reinforcement
Gregory Lucier, Vivek Hariharan, Sami Rizkalla, Paul Zia, Gary Klein

Abstract
Open web reinforcement has been shown to be an effective alternative to closed stirrups in the webs of slender precast L-shaped spandrel beams subjected to combined shear and torsion. For slender beams, an open reinforcement scheme is a better alternative to the traditional closed stirrups mandated by ACI-318, primarily due to ease of production. While the behavior of slender L-shaped beams (having aspect ratios of 4.6 or greater) with open web reinforcement has been well documented, the use of open web reinforcement for compact L-shaped cross sections has not been investigated previously.

This paper presents an experimental study in which four full-scale, 46’ long, precast, compact L-shaped girders were tested to failure. One of the test specimens served as a control, and was designed with traditional closed stirrups. The remaining three beams were designed with various open reinforcement configurations.

The results of the study demonstrate the viability of replacing closed stirrups with properly designed and detailed open web reinforcement. All four girders behaved satisfactorily at all loading stages, and ultimately failed at loads much greater than the factored design loads. When failure occurred in the end region of such beams with open reinforcement, the failure modes did not include the spiral cracking and face-shell spalling commonly associated with the torsion design concept of the ACI Code. Rather, the failure planes observed in the compact L-shaped beams were similar to those observed in slender L-shaped beams, taking the form of a skewed-bending type failure about a diagonal crack extending upwards from the support. The results confirm the potential to simplify the design and detailing of compact L-shaped beams by using open web reinforcement proportioned with a failure plane based design approach.
Introduction

Precast, prestressed concrete L-shaped beams with compact cross-sections are frequently used to support double-tee deck sections in parking structures when the top of the beam cannot extend above the top surface of the deck. For example, compact beams may be used in locations where traffic must pass over the end of a double-tee, such as a ramp or a crossing point between bays. Compact L-shaped beams may also be used to support deck sections along the edges of a parking garage when a separate railing system will be installed. Thus, the primary purpose of the compact L-shaped beam is to transfer vertical loads from deck sections to columns.

Typical compact L-shaped spandrel beams are between 2 and 3 feet deep (0.7m to 1m) and can have spans as large as 50 feet (15.2m). These beams usually have a web thickness of 16-inches (0.4m) or more with a continuous ledge running along the bottom edge on one side of the beam, creating the L-shaped cross-section. The ledge provides a bearing surface for the stems of the deck sections, so the compact L-shaped beam is subjected to a series of discrete eccentric loadings. The beams are simply supported at the columns with lateral connections to resist torsion due to eccentric loading. Discrete connections between deck sections and the web of the spandrel beam provide restraint along the length of a typical compact L-shaped beam.

Compact L-shaped beams are often subjected to heavy loads applied at high eccentricities. Thus, the torsion demand on these members can be significant, and designs usually require conservative web reinforcement details, especially in the end regions. The web steel is most congested in the end regions where prestressing strands and reinforcing bars must weave through numerous closed stirrups that are closely spaced as required by the ACI Code (318-11) and the 7th Edition of the PCI Design Handbook. Precast, prestressed L-shaped spandrel beams (both slender and compact) are currently designed by the precast concrete industry according to a general procedure originally proposed by Zia and McGee (1974), and later modified by Zia and Hsu (1978, 2004).

The early years of research on torsion largely focused on the behavior of rectangular beams subjected to applied twisting moments. Empirical design equations and sophisticated rational models were devised to reflect the observed response of concrete rectangular beams subjected to torsion. The complexities of these design formulas and their intricate detailing requirements were incorporated into ACI 318 and were applied in practice to L-shaped beams. However, the need for such complex detailing was difficult to reconcile with the observed behavior of precast L-shaped spandrel beams subjected to eccentric vertical loading. Experimental testing by Logan and Klein along with field observations by Raths indicated that L-shaped spandrels did not develop the kind of internal torsional distress which would lead to face shell spalling. Logan, Raths, and Klein all observed significant plate-bending in the webs of slender spandrels.

In view of the observed behavior, questions were raised as to the need for closed stirrups in a slender (non-compact) L-shaped section. Lucier et al. and Hassan et al. demonstrated through full-scale tests and finite element analysis that open web reinforcement could be used safely and effectively in slender precast L-shaped spandrel beams. Subsequent testing and analytical study led to the development of rational design guidelines for precast slender L-shaped beams. These guidelines considered the torsion demand on an L-shaped section as two separate orthogonal components, as shown in Figure 1. One component of the torsion vector (the plate bending component) acts to bend the beam web out of plane about a diagonal line.
extending upward from the support. The second orthogonal component of torsion (the twisting component) acts to twist the cross-section about an axis perpendicular to the diagonal line. Web steel is proportioned in the cross-section to resist the plate bending component of torsion and a method for evaluating the resistance of the cross section to the twisting component is proposed. Tests confirmed the proposed guidelines to be safe and effective for slender L-shaped beams having aspect ratios (web height divided by web thickness) of 4.6 or greater. The limiting aspect ratio was selected based on experimental data available for beams having aspect ratios of 4.6 and higher. The study presented in this paper was undertaken to examine the applicability of the previously-developed design guidelines to beams with aspect ratios lower than 4.6.
Objective and Scope

The primary objective of this study was to document the behavior of full-size compact L-shaped beams designed with open web reinforcement. Special attention was focused on the type of failure mode, the end-region capacity, the cracking pattern, and the crack angles.

The experimental program included four (4) full-size compact L-shaped beams. One of the four beams served as a control specimen and was designed using closed stirrups according to the guidelines in the 7th Edition of the PCI Handbook. The remaining three beams were designed using combinations of welded wire reinforcement (WWR) and L- and C-shaped reinforcing bars as the torsion, shear, and ledge reinforcement. No closed stirrups were used for these three specimens so that the prestressing strands did not have to be threaded through the stirrups prior to stressing. The L-shaped bars, C-shaped bars, and WWR are much easier to produce and to install than a series of closed stirrups.
Test Specimens

The four L-shaped spandrels tested in this program had identical cross-sections of 28" x 34" (711mm x 864mm) with a continuous 8" x 8" (203 mm x 203 mm) ledge along the bottom edge of one web face, as shown in Figure 2. The ledge was held 12" (305 mm) back from each end of the beam, and all four spandrels were 46ft (14m) long.

Two holes were provided through the web at each end of each beam to allow the beams to be connected to the test setup. These holes were set 6 in. (152 mm) in from the beam ends and 6" away from the top and bottom surfaces as shown in Figure 2. The holes were sized to accommodate the threaded rods used to bolt the spandrels to the test frame in a way that simulated discrete field conditions. Embedded steel plates were provided along the top face of each spandrel to enable connection of the L-shaped beam to the flanges of the double-tee deck.

![Figure 2: Isometric (left) and Cross-section (right) Sketches of the Tested Compact Beams](image)

The concrete used for all four beams was a typical self-consolidating mixture of normal density (150pcf) with a measured average compressive strength (at the time of girder testing) of 8,700psi (60 MPa). All four spandrel beams were prestressed and cast together on a long-line casting bed, and 4"x8" (0.1 x 0.2 m) concrete control cylinders were prepared from the concrete used to cast each beam. The beams were delivered to the laboratory as needed for testing.

The steel reinforcement consisted of prestressing strands, WWR, and conventional deformed reinforcing bars. The prestressing strands were ½" diameter, 7-wire, 270 ksi, low-relaxation strands with a nominal cross-sectional area of 0.167 in². Conventional Grade 60 (Metric Gr. 420) #3, #4, #5 and #9 deformed bars were also used in various forms.

Welded-wire reinforcement was utilized in conjunction with open L- and C-shaped reinforcing bars as the main shear and torsion reinforcement in the front and back faces of specimens LG2, LG3 and LG4. The WWR used on the inner (ledge) face in specimens LG2 and LG3 was D4 x D10 with a 4" spacing providing a steel area of 0.1 in²/ft in the vertical direction. WWR was also used on the outer face of LG2, LG3 and LG4 (W2.5 x W2.5 with a 6" square spacing providing 0.08 in²/ft of steel in both directions).

In order to ensure end region failures in this study, additional flexural and ledge reinforcement was provided in each beam. A typical amount of prestressing steel was provided in each girder,
and then partial-length mild-steel reinforcement was added to provide additional moment capacity away from the end regions. A total of twenty prestressing strands were used for each beam; seventeen strands were laid out at the bottom of the web on a 2” grid, and the remaining three strands placed on the top of the web. All strands were prestressed to an initial tension of 31,600 lbs (70% of ultimate) and had a clear cover of 1.75”.

With the strands in place, additional flexural reinforcement was provided by six 30’ long #9 bars centered at midspan. This partial-length additional flexural steel was held back from the end regions so that it would not increase the end-region shear and torsion capacity. In addition to the additional flexural reinforcement, additional reinforcements were provided to strengthen the ledges at each loading point to prevent punching shear failure.

A different configuration of transverse web reinforcement was examined for each beam, as described in Table 1 and shown in Figure 3.

<table>
<thead>
<tr>
<th>Test Specimen</th>
<th>Web Reinforcement Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>LG1</td>
<td>Control beam with closed stirrups in the web and ledge.</td>
</tr>
<tr>
<td>LG2</td>
<td>Welded wire reinforcement (WWR) on the inner and outer web face. No transverse steel crossing the top or bottom surface of the beam. L-shaped bars provide ledge-to-web attachment.</td>
</tr>
<tr>
<td>LG3</td>
<td>WWR on the inner and outer web faces. A U-shaped bar crosses the top of the web. L-shaped bars provide ledge-to-web attachment.</td>
</tr>
<tr>
<td>LG4</td>
<td>C-shaped bars are used to provide steel on the inner web face. The shorter legs of each C-shaped bar cross the top and bottom faces of the web. WWR is used to provide steel on the outer face.</td>
</tr>
</tbody>
</table>
LG1 was designed with conventional closed stirrups to serve as a control specimen for the testing program. Closed stirrups were provided according to the approach recommended by Zia and Hsu\(^\text{10}\), as outlined in the PCI Design Handbook.\(^\text{8}\) In addition to closed stirrups, longitudinal reinforcement was provided in the end regions of LG1 to resist torsion. As required by the Zia-Hsu approach, this reinforcement was provided in addition to the reinforcement provided for flexure. A total of 116 (#3) closed stirrups were spaced along the web of LG1. Stirrups were spaced at 5" for a majority of the length and at 3" near the ends. The end regions of LG1 also had 8 (#5) bars placed longitudinally for a length of 15 feet to meet the longitudinal torsion steel requirement. These bars overlapped #5 U-shaped bars at each end of the specimen to ensure development of the longitudinal torsion steel at the ends of the beam. The ledge of specimen LG1 was reinforced with (#3) closed stirrups spaced at 5". As with all four test specimens, the ledge also included heavy welded reinforcements at each bearing point to prevent localized failure. The welded reinforcements were included only for the purposes of the tests.
Design of the web steel in specimens LG2, LG3, and LG4 was based on two components of the applied torsion: a plate-bending component and a twisting component. The applied vertical shear was considered using conventional methods. First, vertical steel was placed along the length of the web to satisfy requirements for shear steel as specified in the PCI Design Handbook. The vertical steel required for shear was distributed equally between the inner (ledge side) and outer web faces. Next, vertical steel required to resist the plate-bending component of torsion was added to the inner web face in the end-region. Additional longitudinal steel was also provided on the inner and outer web faces (in the form of U-shaped bars) for plate bending. The plate bending steel quantity was calculated using an assumed 45-degree failure plane. It was assumed that the concrete section would resist the twisting component of torsion and that failure would be controlled by twist. Additional details regarding the approach used to design the web steel in these beams are provided elsewhere.3

Details of the web steel provided in each beam are given in Table 2.

<table>
<thead>
<tr>
<th>Test Beam</th>
<th>Longitudinal Web Reinforcement for Torsion @ Beam Ends</th>
<th>Vertical Web Reinforcement</th>
<th>Reinforcement Crossing Top and Bottom Web Surfaces</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Inner Face</td>
<td>Outer Face</td>
</tr>
<tr>
<td>LG1</td>
<td>(8) #5 bars x 15” plus (7) #5 U-shaped bars x 2’6”</td>
<td>#3 closed stirrup @ 3” for first 2’0” each end, then @ 5” for balance of span. Equals 0.44 in.²/ft. for first 2’0” then 0.26 in.²/ft. for balance on each face (inner, outer, top, and bottom).</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>WWR Var.x4 D4xD10 equals 0.3 in.²/ft. plus 6”x6 W2.5xW2.5 for first 3’ each end = 0.35 in.²/ft.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>WWR 6”x6” W2.1xW2.1 equals 0.042 in.²/ft.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>None</td>
<td>False</td>
</tr>
<tr>
<td>LG2</td>
<td>(2) #5 x 4’4” U-shaped bars crossing the critical diagonal crack (3) additional #5 x 2’6” U-shaped bars did not cross the diagonal crack and served to confine splitting stresses at the ends of the strands</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>#4 C-shaped bar @6.5” to 8” spacing equals 0.37 to 0.30 in.²/ft.</td>
<td></td>
</tr>
<tr>
<td>LG3</td>
<td>Note: LG2 and LG3 also had additional longitudinal wires provided by inner face WWR not considered in design.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>None</td>
<td>False</td>
</tr>
<tr>
<td>LG4</td>
<td></td>
<td>#4 @ 12” equals 0.20 in.²/ft.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Short legs of inner-face #4 C-shaped bar @6.5” to 8” spacing equals 0.37 to 0.30 in.²/ft.</td>
<td></td>
</tr>
</tbody>
</table>

The longitudinal steel provided in the end regions of specimens LG2, LG3, and LG4 to resist plate bending was identical. Five U-shaped (#5) bars were placed at each end of the web. One of these bars was placed at each of the three layers of prestressing, and the remaining two bars were placed at 12” and at 22” down from the top surface of the beam. The top two U-shaped bars were 4’4” long, and the bottom (3) U-shaped bars were 2’6” long. A series of #3 L-shaped bars spaced at 8” provided the ledge-to-web attachment steel for specimens LG2 through LG4.

The remaining web steel was varied from beam to beam. Specimen LG2 was designed using flat sheets of welded wire reinforcement (WWR) on the inner and outer web faces. The ledge was attached to the web with L-shaped bars, as shown in Figure 3. No steel crossed the top or bottom surface of the web in LG2. WWR (6” x 6” – W2.5 x W2.5) was provided along the entire length on the outer web face (the face opposite the ledge). The inner web face was reinforced with D4.0 x D10.0 WWR with a 4” spacing between vertical wires. The continuous D4.0 x D10.0 mesh was supplemented by an additional piece of W2.5 x W2.5 6” x 6” WWR for the first three
feet at each end of the specimen. All WWR extended the full depth of the web, and no steel of any kind crossed over the top or bottom surfaces of the web in LG2. The purpose of specimen LG2 was to evaluate the ability of the compact concrete cross-section to resist the twisting component of torsion without reinforcement over the top and bottom surfaces of the web.

The reinforcement used for specimen LG3 was identical to that used in LG2 except that additional U-shaped bars were placed along the top of the web at 12” on center, as shown in Figure 3. These U-bars were placed on top of the upper prestressing strands and hooked over the vertical WWR sheets onto the faces of the web. The purpose of including this additional steel across the top surface of the web was to compare its effect on the torsional capacity and failure mode with respect to LG2.

The web of specimen LG4 was reinforced with a combination of WWR and conventional reinforcement. A layer of 6”x6” - W2.5 x W2.5 WWR was provided on the outer (non-ledge) face. On the inner web face, #4 C-shaped bars were provided at a spacing of 8”. The C-shaped bars were placed so that the shorter legs of the C-shape extended over the top and bottom web surfaces, fully developing the vertical leg of the bar.

It should be noted that while the design of LG2, LG3, and LG4 followed the same principles developed by Lucier et al.4 for slender L-spandrel beams, there are some differences in the actual reinforcement detailing, as the compact beams were designed before the guidelines for slender beams were finalized. The compact beams were fabricated with slightly more (about 10%) vertical web steel than would be required had the approach for slender spandrel beams been followed. However, the compact beams had less longitudinal steel (about 20% less) crossing the critical diagonal crack in the end-region than would be required by the slender procedure because the provided longitudinal steel did not extend the full distance of the end region.
Test Setup

The compact L-shaped beams were tested to failure by applying loads to the beam ledge through the stems of short (12' span) double-tee deck sections. Each L-shaped beam was simply supported on a 45' span by a 16" wide bearing pad centered with respect to the beam web. Teflon-coated bearing pads and stainless steel plates were used to remove as much friction as possible from the test setup. The main vertical reaction of each beam was measured at one end by two 200-kip load-cells, also centered with the web thickness. Using two load cells at this location allowed for measuring the main vertical reaction and provided insight into how the location of this reaction shifted with respect to the beam web during the test as the beam and bearing pads deformed.

Each L-shaped beam was also supported laterally at both ends. Lateral tie-backs were provided by attaching a stiff steel beam vertically to the inner face of the L-girder through holes in the web that were precisely located during casting. The stiff steel beam extended above and below the top and bottom surfaces of the L-shaped beam. Threaded rods were used to connect the steel beam to supporting columns and to the laboratory floor, providing torsional restraint to the girder web as shown in Figure 5. The forces in these threaded rods were measured using load cells.

Loads were applied to the L-shaped beam ledge by four 10’ (3.1m) wide double-tees and one 5’ (3.7m) wide single-tee, as shown in Figure 5. All tee sections were 26” deep. Together, the double-tee and single-tee sections created a 45’ wide deck with a 12’ span. The tees were supported by the beam ledge at one end and by concrete support blocks opposite to the beam ledge. All deck sections were placed along the ledge of the girder with a 1” gap between the inner face of the web and the edge of the deck. Each deck section was then connected to the L-girder with a welded flange connection at the mid-width. Deck sections were not connected to adjacent deck sections (as would commonly be done in the field) to prevent the transfer of load between decks.

Hydraulic jacks and spreader beams were used to apply loads to the top surface of the double-tee and single-tee deck sections. The stems of the decks in turn applied load to the beam ledge in a manner representative of the field condition. Teflon-coated bearing pads and stainless steel plates were used at each stem-to-ledge bearing reaction to reduce friction.
Figure 4: End region of a compact L-shaped beam in the testing setup

- Load Cells
- Inclinometer
- Hydraulics Jack and Spreader Beam
- Strongback provides lateral support
- 12' long double-tee decks
Loading

All four of the tested beams were designed for the loads shown in Table 3.

<table>
<thead>
<tr>
<th>Table 3: Design Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Load</strong></td>
</tr>
<tr>
<td>Dead Load</td>
</tr>
<tr>
<td>Dead Load</td>
</tr>
<tr>
<td>Live Load</td>
</tr>
<tr>
<td>Snow Load</td>
</tr>
</tbody>
</table>

Based on the given design loads, the L-girder reactions for any of several selected load combinations were determined, as shown in Table 4.

<table>
<thead>
<tr>
<th>Table 4: Selected Load Levels for Testing</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Designation</strong></td>
</tr>
<tr>
<td>Dead load</td>
</tr>
<tr>
<td>Service load</td>
</tr>
<tr>
<td>Reduced service load with snow</td>
</tr>
<tr>
<td>Service load with snow</td>
</tr>
<tr>
<td>Factored load</td>
</tr>
<tr>
<td>Nominal strength</td>
</tr>
</tbody>
</table>

Note: ASCE = American Society of Civil Engineers; DL = dead load; LL = live load; SL = snow load; 1 ft = 0.3048 m; 1 kip = 4.448 kN.

The beams were designed and loaded as if they were supporting one end of a 45’ wide, 60’ span 10DT26 deck. References to ‘dead load’ within this testing program assume a full 60-foot span double-tee deck. Space limitation only allowed for a 45’ wide by 12’ span double-tee deck to be used, so the hydraulic jacks were used to make up the extra dead load.

Load was applied to each beam in incremental cycles based on the load designations shown in Table 4. For each cycle, the specimen was loaded to the given level, observations were made, and the specimen was then unloaded. Load cycles were completed in this fashion up to the factored load level. Once the factored load was applied, it was then held on each beam for 24-hours. After completing the 24-hour sustained load test, a beam was unloaded and its recovery was monitored for 1 hour. At the end of the hour (provided the girder passed the ACI-318 Chapter 20 recovery criterion), each spandrel beam was then loaded to failure in incremental cycles.
Instrumentation

Four types of instrumentation were used to measure loads, strains, deflections, and rotations for each compact L-girder tested. Load cells were used to measure the main vertical and lateral reactions and to measure the loads being applied by the hydraulic jacks. Linear displacement transducers were used to measure the vertical and lateral displacements of each L-girder at several locations. Vertical displacement measurements were taken along the longitudinal centerline of the spandrel webs at their mid span and quarter span. In addition, vertical measurements were also taken at the inner most edge of each spandrel ledge at the mid span and quarter span. A final vertical measurement was taken at one of the main vertical reactions for each spandrel to monitor support displacement. Lateral displacement at mid-span and at each quarter-span was monitored at the top and bottom edges of the web. Similarly, lateral displacement was also monitored at the supports to record any support displacements.

Inclinometers were used to measure the lateral rotation of each spandrel web at the mid-span, quarter-span, and support locations. Wire-arch clip gauges (or PI gauges) were used to measure concrete strains on the top, bottom, and inner faces of each spandrel in the end regions. Gauges were also used at the mid-span and at one quarter-span to measure the flexural strains on the top and bottom surfaces of each member.

Summary of Test Results

All four specimens carried their factored design loads for 24 hours, and all demonstrated ultimate capacities far in excess of those factored loads. In addition, all four specimens passed the 1-hour ACI recovery criterion for the 24-hour load test for both the vertical and lateral deflections at the midspan. The maximum vertical reactions, measured vertical deflections at failure, and a brief description of the observed failure mode are summarized in Table 5. It should be noted that the reactions given in the table represents the total force needed to support one end of the simply supported spandrel. These values include the self-weight of the L-beam, double tees, and loading system.

The lateral reactions measured at the peak vertical reaction are presented in Table 6. Note that the lateral reactions shown in Table 6 were those determined to be acting directly on the beam web (at the through-holes). These values were determined from measurements made by the load cells included in the lateral restraint system.

It should be noted that it was not possible to produce an end-region failure in the control specimen LG1, although extensive diagonal cracking was observed when the test was terminated when the main vertical reaction reached 220 kips. The other three specimens ultimately did exhibit end-region failure modes at nearly identical vertical reactions of 220 kips.
Table 5: Summary of Test Results

<table>
<thead>
<tr>
<th>Test</th>
<th>Web Reinforcement</th>
<th>Maximum Vertical Reaction (kips)</th>
<th>Midspan Vertical Deflection (in.)</th>
<th>Description of Failure Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>LG1*</td>
<td>Closed Stirrups</td>
<td>220.5</td>
<td>4.73&quot;</td>
<td>Beam did not fail and the test was stopped to avoid damage to the loading system. Spandrel showed diagonal cracking on the inner face and flexural cracking on the outer face.</td>
</tr>
<tr>
<td>LG2</td>
<td>Flat Sheets of WWR Only</td>
<td>220.1</td>
<td>4.90&quot;</td>
<td>Failure in the end region along a diagonal plane with flatter skew. Extensive diagonal cracking was observed on the inner face and moderate flexural cracking on the outer face.</td>
</tr>
<tr>
<td>LG3</td>
<td>Flat Sheets of WWR plus U-shaped Bars</td>
<td>220.0</td>
<td>4.06&quot;</td>
<td>Failure in the end region along a diagonal plane with flatter skew. Cracking similar to LG2.</td>
</tr>
<tr>
<td>LG4</td>
<td>C-shaped Bars Inner Face, WWR Outer Face</td>
<td>220.3</td>
<td>6.19&quot;</td>
<td>Failure in the end region along a diagonal plane with steeper skew. Cracking similar to LG2.</td>
</tr>
</tbody>
</table>

*Girder did not fail. Test was stopped at 220 kips to avoid damage to the loading system.

Table 6: Test Results at Failure

<table>
<thead>
<tr>
<th>Test</th>
<th>Vertical Reactions (kips)</th>
<th>Lateral Reactions (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Bottom Left</td>
</tr>
<tr>
<td>LG1*</td>
<td>220.5</td>
<td>75.2</td>
</tr>
<tr>
<td>LG2</td>
<td>220.1</td>
<td>97.6</td>
</tr>
<tr>
<td>LG3</td>
<td>220.0</td>
<td>83.1</td>
</tr>
<tr>
<td>LG4</td>
<td>220.3</td>
<td>107.1</td>
</tr>
</tbody>
</table>

*Girder did not fail. Test was stopped at 220 kips to avoid damage to the loading system.
**Cracking Patterns**

The cracking observed at the service load was very minimal in all four beams. The inner face of all beams remained uncracked at the service level, as shown in the left portion of Figure 6. Some light flexural cracking was observed on the outer face of all beams near the midspan, as shown in the right portion of Figure 6.

![Figure 6: Typical Cracking on Inner Face (left) and Outer Face (right) under Service Load (LG3 Shown)](image)

The cracking pattern observed up to failure was consistent for all four specimens. The general cracking pattern observed was similar to the pattern observed by others in slender L-shaped spandrels (Logan, Klein, Raths, Lucier). Such a cracking pattern indicates an interaction between torsion and shear stresses in an L-shaped beam. On the inner web face, the torsion and shear stresses act in the same direction, creating a high diagonal tension demand in the end regions. On the outer web face, the vertical shear stress counteracts the stresses developed in torsion. Thus, the diagonal tension demand on the outer web face is reduced, and could even be opposite to the demand on the inner face depending on the relative magnitudes of the shear and torsional stresses.

Cracks on the inner face inclined upward from the support at an initial angle of approximately 45-degrees, as shown in Figure 7 and Figure 8. Moving away from the supports, the crack angle flattened, and the inner face cracks began to propagate towards midspan. In addition, vertical flexural cracks extended upwards from the beam in the middle portion. Cracking in the end region of the beam with traditional closed stirrups (LG1) was not as wide as in the beams with open web reinforcement, but was more numerous. However, the crack pattern observed for all beams was practically the same.
One important feature of the cracking pattern at the inner face was the tendency for a diagonal crack extending upwards from the support to cross the top web surface at a skew. This behavior was observed for all beams, as shown in Figure 9. While the inner face cracking near the support exhibited an initial angle of about 45-degrees for all beams, the angle of the critical diagonal crack and the angle at which that crack skewed across the top web surface appeared to vary according to the quantity of reinforcement crossing that surface. Beams with little or no top steel exhibited smaller skew angles than did beams with higher quantities of top steel. The critical crack angle observed for beams LG1 and LG4 was approximately 45 degrees. Both beams had relatively high quantities of steel crossing the top web surface. The critical crack and skew angles observed for beam LG2, having no top steel, were closer to 32-degrees. The skew angle observed for beam LG3, having an intermediate amount of top steel was approximately 40-degrees.
The observed patterns of outer face cracking were also similar for all four specimens. The cracking patterns also matched the general pattern observed for slender L-shaped spandrels. In general, cracking on the outer face was minimal near the supports. Vertical cracks extending up from the bottom of the beam (due to vertical and lateral flexure) were observed near the mid-span of all beams, as shown in Figure 10. In the region between the end of a beam and the middle portion of a beam, a limited number of inclined cracks were observed, indicating the effect of shear.

Figure 10: Typical Outer Face Cracking in a Compact L-shaped Beam (digitally enhanced cracks)
Failure Modes

End region failure modes were observed for three of the four tested compact L-shaped beams. Specimen LG1, the beam with closed stirrups, did not fail in the end region and the test was terminated to avoid damaging the testing system. In considering the end region failure modes observed in the other three beams (LG2, LG3, and LG4), it is important to recall that all beams in this testing program were provided with extra flexural and ledge reinforcement in order to prevent premature flexure or ledge punching failure. If any of these four beams had been reinforced with standard levels of flexural or ledge reinforcement, then flexural or ledge failure would have controlled the behavior long before end region failures could be observed.

The failure modes observed for each of the four tested beams are shown in Figure 11. In the case of LG2 and LG3, the observed failure took the form of skew bending about a critical diagonal crack extending upwards from one support. When this crack reached the top surface of the web, it continued across the top of the beam at a skew. The skewed nature of the crack plane can be seen in Figure 12, showing the separated failure surface of LG2 after testing.

The observed failure for beam LG4 also occurred in an end region, however, the skew of the LG4 failure plane was very short as compared to the skew of LG2 and LG3. A critical diagonal crack extended upwards from one support and also crossed over the top of the web. However, for specimen LG4, the skew of the failure plane was very steep, crossing the top of the web in a direction nearly normal to the web face. The difference between the LG2/LG3 failure mode and the LG4 failure mode can be seen by comparing the outer web faces of the specimens after testing in Figure 13.
Figure 12: View of LG2 Separated Failure Surface (Outer Web Face)

Figure 13: Outer Face of Views after Failure

LG3 (LG2 Similar)  LG4
Measured Deflections

The measured load-deflection behaviors at mid-span for all four specimens were very similar, as shown in Figure 14 and Figure 15. The behaviors of beams LG1, LG2, and LG3 were nearly identical through the 220 kip load level. The behavior of beam LG4 was very similar to that of the other 3 beams; however, LG4 appeared to be slightly softer at all load levels. The vertical load-deflection curves illustrate the various loading cycles that each beam was subjected to. Residual deflections can be observed at the end of each of these cycles with subsequent cycles continuing from the residual values. Horizontal plateaus are also present in the load-deflection data, and are indicative of locations where the load was held to make observations and to mark cracks. The large plateau at the factored load corresponds to the 24-hour sustained load test. It should also be noted that the zero deflection reading occurs at a vertical reaction of approximately 37 kip (165 kN), the self-weight of the system, including the spandrel, the double tees, and the loading equipment.

Figure 14: Measured Load-Deflection Data for Individual Beams
Horizontal deflections were also measured at the mid-span for each tested beam. Measurements were taken at the top and bottom edge of the web. For all beams, the measured data indicate a tendency for the compact L-shaped beams to move outward at midspan, more so at the bottom than at the top. Thus, as load is applied to the ledge, the compact beam rolls towards the applied load, but is restrained at its top surface by the welded deck connections. Figure 16 shows a typical load-lateral deflection curve representing the top and bottom lateral reactions measured at mid span of specimen LG3.
Figure 16: Typical Lateral Deflection Measurements at Mid-span (LG3)
Analysis for Twisting Resistance

The results from LG2, LG3, and LG4 can be evaluated within the context of the design procedure for slender L-shaped spandrel beams previously developed. The slender spandrel design procedure considers three actions in the end region of an L-shaped beam independently: vertical shear, the plate-bending component of torsion, and the twisting component of torsion. The procedure predicts that the strengths of specimens LG2, LG3, and LG4 are all controlled by the twisting capacity.

The resistance of the concrete cross-section to the twisting component of torsion, according to the slender spandrel beam design procedure, is given by Eq. 1 below.

\[ T_u \leq \phi_1 \left( 1.13 \sqrt{f'_c d_w h^2} \right) \]

Where:
- \( T_u \) = factored torsion demand
- \( f'_c \) = concrete compressive strength
- \( d_w \) = effective web thickness
- \( h \) = height of the web
- \( \phi_1 \) = strength reduction factor (0.75)

For the three test specimens, \( f'_c = 8700 \) psi, \( d_w = 26.25'' \), and \( h = 34'' \). Thus, the factored torsional strength, as predicted by Equation 1, is 2399 kip-in. The nominal torsional strength \( (T_u / \phi_1) \) is therefore equal to 3198 kip-in. For Equation 1 to be applicable (conservative) for these compact cross-sections, all tested beams should fail at an applied torque of at least 3198 kip-in.

All three specimens (LG2, LG3, LG4) failed at a spandrel end reaction of 220 kips. Subtracting the concentric self-weight (24.3 kips) from this reaction results in 195.7 kips of eccentrically applied ledge load at failure. Since the load eccentricity is 20'', the tested spandrels actually carried an applied torque of 3914 kip-in. at failure, which exceeds the predicted nominal torsional strength of 3198 kip-in as controlled by twist. Thus, the provisions for twist component in the slender spandrel design procedure are conservative when applied to the compact spandrel beams tested in this study.
Discussion

In evaluating the observed end region failures (LG2, LG3, LG4), it is important to point out that at the design service load level for all beams the end reaction is 100 kips. The test of LG1 was terminated at a 220 kip end reaction and the other three specimens failed in their end regions at end reactions of 220 kips. Thus, all the specimens carried well over twice the service load. Without the enhancement of flexure and ledge reinforcement, other failure modes (probably ledge punching) would have governed the beam behavior with reduced strength in all four cases.

Based on the failure modes and test data, it is clear that the performance of the end region reinforced with closed stirrups and traditional torsional detailing (LG1) was superior to the performance of the end regions of the other three specimens with open reinforcement. There is no doubt that closed stirrups and longitudinal torsion reinforcement provide excellent crack control and strong end regions, especially at higher load levels. What is debatable, however, is whether the expense and complexity of a closed reinforcement scheme can be justified given that the service load behavior was identical for specimens with open and closed reinforcement, and that the strength of all end regions far exceeded other typically controlling failure modes (flexure or ledge punching).

None of the compact beams tested exhibited behaviors that would be described as classically torsional in nature. That is to say, spalling of the concrete cover was not observed in any of the beams, nor was a distinctive spiral cracking pattern. Rather, the behavior of the compact L-shaped sections was similar to that previously observed in slender L-shaped spandrel beams. The distinctive tied-arch type cracking pattern, with torsional and shear stresses acting together on the inner web face and acting to oppose one another on the outer web face, was clearly evident in all four of the tested beams. The end region failures observed in the compact beams were in the form of skewed-bending. Comparing the failure loads to the predicted twist resistance capacity of the compact section indicates that the twist resistance model proposed for slender spandrel beams appears to also be applicable and conservative for compact sections.

The cracking patterns and failure modes observed in all four tests indicate that while web steel on the inner and outer web faces is most critical in an L-shaped section, web steel crossing the top surface of the beam does influence behavior. The skewed angle at which a crack crosses the top web surface appears to be strongly influenced by the quantity of top surface steel. The angle of the critical diagonal skewed-bending crack was observed to be approximately 45-degrees when a relatively high level of steel was provided across the top surface of the beam. The angle of this crack flattened to approximately 30-degrees when no steel crossed the top web surface. Since the design procedure for open reinforcement assumes a crack angle of 45-degrees in the end region, top steel would be required to ensure that the critical diagonal crack develops at the assumed angle. In addition, the presence of top steel provides additional benefits such as control of crack widths and enhanced aggregate interlock, so the top steel should be included in the design of a compact section. The top steel should be spaced at a distance of no more than ½ the web thickness in order to be effective. Additional study and further testing would be desirable to better understand how the beam behavior is influenced by top steel.
Conclusions
Based on the results of this investigation, the following conclusions can be drawn:

1. The behavior of the four test specimens was almost identical. Their behavior was satisfactory at all loading conditions including load levels well above the factored load condition. The specimens also easily met the recovery criteria prescribed by Chapter 20 of ACI 318-11 following a 24-hour sustained load test at the factored load level.

2. The failure load of the four test specimens exceeded the factored design load by a substantial margin.

3. The failure mode of these compact L-shaped test specimens was in combined shear and torsion along a skewed diagonal plane, just as in the case of slender L-shaped beams observed in previous investigations. There was no spiral cracking or face-shell spalling associated with the torsion design methods in the ACI Code.

4. The tests confirmed the validity of designing for torsion by considering the applied torque as two independent orthogonal components (bending and twisting), thus greatly simplifying torsion design and detailing. Analysis of the test data using the design equations developed previously for slender L-shaped beams indicated that the equations are also applicable to compact L-shaped beams.

5. Comparing the failure loads to the predicted twist resistance capacity of the compact section indicated that the twist resistance model proposed for slender spandrel beams appears also applicable and conservative for compact spandrel sections.

6. The tests also confirmed that the use of open web reinforcement as opposed to closed web reinforcement was satisfactory, as in the case of slender L-shaped spandrel beams. Using open web reinforcement greatly simplified design and detailing.

7. Since the design procedure for open reinforcement assumes a crack angle of 45-degrees in the end region, top steel would be required to ensure that the critical diagonal crack develops at the assumed angle. In addition, the presence of top steel provides additional benefits such as control of crack widths and enhanced aggregate interlock, so it should be included in the design of a compact section. The top steel should be spaced at a distance of no more than ½ the web thickness in order to be effective. However, additional study and further testing are recommended to better understand how the failure mode is influenced by top steel and to provide better guidance on the quantity of top steel required.
References


7. ACI Committee 318, "Building Code Requirements for Structural Concrete (ACI 318-11) and Commentary," American Concrete Institute, Farmington Hills, Michigan, 2011, 503 pp.


10. Zia, P. and Hsu, T. “Design for Torsion and Shear in Prestressed Concrete Flexural Members.” *PCI Journal*. 49(3): 34-42. May-June 2004. (This paper was previously presented at the American Society of Civil Engineers Convention, October 16-20, 1978, Chicago, IL, reprint #3424.)


7. Summary and Conclusions

A simple, rational procedure was developed and proposed for the design of precast slender spandrel beams with aspect ratios not less than 4.6. The procedure was developed from data from 16 full-scale experimental tests, extensive finite element modeling, and a rational analysis. The outcomes from this effort are summarized below.

1. The torsion acting on a slender precast spandrel beam can be resolved into two orthogonal components: the bending component and the twisting component. Equations were presented for evaluating the capacity of a slender precast spandrel to resist both components of torsion, and the proposed equations were shown to be safe for all of the tested beams.

2. Slender precast spandrel beams of the same general configuration as those tested can be designed and detailed appropriately and safely using the proposed procedure by considering three distinct beam regions, the end region, the transition region, and the flexure region.

3. The design of slender spandrel beams should consider flexure and shear in all regions. The design should consider the bending and twisting components of torsion in the end region and the transition region.

4. Additional end-region strength and ductility were provided by traditional closed stirrups, as compared to open web reinforcement. However, this additional strength and ductility are of no consequence unless beams are intentionally forced to fail in their end regions by excessively over-reinforcing against other controlling failure modes. Failure of typical slender spandrel beams will be controlled by flexural failure or by local failure modes in the ledge or corbels.

5. Design of slender spandrel beam end regions with traditional closed web reinforcement according to the current practice outlined in the PCI Design Handbook, 6th Edition (PCI, 2004), is an overly-conservative approach.
6. Open web reinforcement provides virtually the same performance as traditional closed web reinforcement when slender spandrel beams are designed with typical levels of flexural and hanger reinforcement under factored design loads.

7. Production of a typical spandrel beam with open web reinforcement requires 30-50% less web steel than a comparable beam with traditional closed web reinforcement. The labor required to produce an open reinforcement cage is estimated as 50% of the labor required to produce a traditional closed reinforcement cage.

8. Preliminary tests of compact L-shaped cross-sections indicate that the developed equations may also be applicable to compact beams, however, additional work is needed in this area.

9. Several ledge punching failures occurred at loads below that predicted by the PCI Design Handbook. Other researchers have observed similar results. The interaction between ledge punching behavior and global flexure and shear appears to significantly reduce capacity. Although beyond the scope of this investigation, further study of ledge punching capacity is recommended.
8. References

1. ACI Committee 318, “Building Requirements for Structural Concrete (ACI 318-11) and Commentary,” American Concrete Institute, Farmington Hills, MI, 2011, 503 pp.

2. ANATECH Corporation, 2003, ANACAP Version 2.2.3 Reference Manuals.


Appendix
Appendix A: Industry Contributors

In addition to those recognized in the Acknowledgements section, the following individuals and organizations are recognized. Their efforts and support at all stages were instrumental to the success of the research. The generous donations of time, expertise, engineering, supplies, software, and precast products made this research possible.

Sponsor of the Slender Spandrel Research
The Precast / Prestressed Concrete Institute, Chicago, IL
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PCI Producer-Members Providing Test Specimens
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Tindall Corporation, Spartanburg, SC
Finfrock Design, Incorporated, Orlando, FL
High Concrete Group, Denver, PA
Gate Precast Company, Oxford, NC

Additional Supporters
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North Carolina State University, Raleigh, NC
JVI Incorporated, Lincolnwood, IL
Leap Software Incorporated, Tampa, FL

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Stresscon Corporation, Colorado Springs, CO
High Concrete Group, Denver, PA

Sponsors of the Compact Beam Tests
Metromont Corporation, Greenville, SC
Logan Structural Research Foundation, Colorado Springs, CO
Appendix B: Detailed Drawings

Detailed production and assembly drawings for are presented in this section for each beam.

Figure B-1: Typical Open-Reinforcing Ledge Detail for Beams with "Extra" Detailing
Figure B-2: Typical Closed-Reinforcing Ledge Detail for Beams with "Extra" Detailing
Figure B-4: Shop Ticket for Double-Tee Deck Sections
Figure B-6: Shop Ticket for SP2
Figure B-7: Shop Ticket for SP3
Figure B-8: Shop Ticket for SP4
Figure B-9: Shop Ticket for SP10
Figure B-10: Shop Ticket for SP11
Figure B-11: Shop Ticket for SP12
Figure B-12: Shop Ticket for SP13
Figure B-13: Shop Ticket for SP14 and SP15
Figure B-14: Shop Ticket for SP16
Figure B-15: Corbel Reinforcement Detail Used with Specimens SP17 and SP18
Figure B-16: Shop Ticket for SP17 (1 of 2)
Figure B-17: Shop Ticket for SP17 (1 of 2)
Figure B-18: Shop Ticket for SP18 (1 of 2)
Figure B-19: Shop Ticket for SP18 (2 of 2)
Figure B-20: Bent Mesh Detail Used on Inner Web Face of SP18
Figure B-21: Shop Ticket for SP19 (1 of 2)
Figure B-22: Shop Ticket for SP19 (2 of 2)
Figure B-23: Shop Ticket for SP20
NOTES:
USE 7000 P.S.I. CONCRETE AT 28 DAYS MINIMUM
RELEASE AT 3500 P.S.I. MINIMUM
USE NORMAL WT. 150#/C.F. GRAY CONCRETE
REBAR REINFORCEMENT SHALL BE GRADE 60 ASTM A706

Figure B-24: Shop Ticket for SP21
Figure B-25: Shop Ticket for LG1
Figure B-26: Shop Ticket for LG2
Figure B-27: Shop Ticket for LG3
Figure B-28: Shop Ticket for LG4
Figure B-29: Ledge Reinforcement Detail for LG1
Figure B-30: Ledge Reinforcement Detail for LG2-LG4