ABSTRACT

TUNC, CENGIZ. Essays on the Life-cycle Portfolio Allocation in the Presence of Housing Investment. (Under the direction of Denis Pelletier.)

This dissertation consists of three essays that examine the effects of housing investment on households life-cycle portfolio allocation decisions. The main objective of these essays is to examine the role of housing investment in addressing the gap between the empirical evidence and the standard theoretical model predictions on both households stock market participation rate and their share of financial wealth invested in the stock market. The empirical evidence from both the Survey of Consumer Finances (SCF) and the Panel Study of Income Dynamics (PSID) show that households have a low stock market participation rate as well as a moderate levels of equity holdings. On the other hand, theoretical models with the assumption of the historically prevailing equity premium suggest that almost all households should participate in the stock market and invest most of their financial wealth in this market.

The first essay introduces a basic housing investment structure into a standard life-cycle portfolio allocation model to investigate possible effects of housing investment on households life-cycle portfolio allocation decisions. The results of this first essay show that housing investment has a sizable crowding out effect on households’ risky asset investment. Since housing investment is both a durable consumption good from which households derive utility and an investment asset from which households get returns, housing investment becomes a partial substitute for risky asset investment.

The second essay introduces a comprehensive housing investment into a life-cycle model in order to have a more realistic model. Among the features of this model are an endogenous decision to be either homeowner or renter in each period, a liquidation cost of selling a house, mortgage payments in each period, and Epstein-Zin recursive preferences. The results of this comprehensive model also show the strong crowding out effect of housing investment on households risky asset investment. This effect is larger for young and middle-aged households. Early
in life, households are willing to be homeowners by paying a downpayment and annual mortgage payments. This keeps their liquid wealth at low levels so that they refrain from paying the entry cost to invest in risky assets. Hence, the owner occupied housing is a substitute for investment in risky assets. Furthermore, due to the return on housing investment and the positive correlation with return on risky asset, households prefer to invest some of their wealth in housing instead of risky assets.

In the first two essays, households can invest in only one risky and one riskless asset. However, a better approach is to distinguish between taxable accounts (TAs) and tax-deferred accounts (TDAs) in life-cycle portfolio allocation analysis. The third essay extends the model introduced in the second essay by distinguishing between TAs and TDAs. In this model, households make constant mandatory contributions to TDAs while the investment in TAs is voluntarily. This model takes into account the tax advantages of both TDAs and mortgage interest payments. The results of this essay show that both housing investment and TDAs have a crowding out effects on TAs risky asset investment as well as the participation rate to this account. The negative effect of housing investment is, however, stronger than the negative effect of TDAs.
Essays on the Life-cycle Portfolio Allocation in the Presence of Housing Investment

by

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DEDICATION

To my parents and my wife.
Cengiz Tunc was born in Van, Turkey. After finishing high school in the same city in 2001, he won a full scholarship from Fatih University to pursue his undergraduate studies in the Department of Management. In June 2006, he graduated from Fatih University, receiving his Bachelors of Arts degree and minor in Development Economics with High Honors. Upon receiving his bachelor degree, with the encouragement of Dr. M. Nihat Solakoglu, he decided to continue his academic career at North Carolina State University (NCSU) in the Department of Economics. In August 2006, he enrolled at NCSU to begin work on his Ph.D degree in Economics. While working for his Ph.D., Cengiz received his Master of Economics degree in 2008. His research interests concentrate on portfolio allocation, housing economics, asset pricing, and time series econometrics. . .
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Introduction

Empirical evidence has shown low stock market participation rates as well as moderate levels of equity in the portfolios of US households. Contrary to this empirical evidence, popular theoretical models with the historically prevailing equity premium suggest that households should invest most of their financial wealth in the risky assets. A reason for this gap is the lack of housing investment in the theoretical models. Housing investment cannot be disregarded in the portfolio allocation models for at least two reasons. First, housing investment is both a durable consumption good from which households derive utility and an investment asset from which households receive capital gains. Second, empirical evidence shows that housing investment constitutes a significant portion of households’ total wealth. By including housing investment into life-cycle portfolio allocation models, this dissertation shows that housing investment has considerable impact on a households’ portfolio allocation decisions throughout the life-cycle.

The presence of the housing investment crowds out risky asset investment through at least two channels. First, households save some of their income to make a downpayment on a house. They, then, make mortgage payments for a long period of time. As a result of this investment, households have relatively low levels of wealth to invest in financial assets. Second, risky housing investment is a partial substitute for risky financial investment.

This dissertation consists of three essays, investigating the effects of housing investment on portfolio allocation decisions of households. The first essay introduces a basic housing investment structure into a life-cycle model, where all households endogenously decide to become permanent homeowners starting from the second period. The results of this model shows some crowding out effects of housing investment on the risky asset investment. Housing investment plays a partial substitution role to risky asset investment. Introduction of even a simple housing investment into the model shows the role of housing investment on households’ behavior toward risky assets.

In the second essay, housing investment enters the model in a more comprehensive and
realistic form. Households can endogenously decide to be either homeowners or renters in each period. Homeowners make a downpayment and keep paying mortgage payments for 25 years. In each period, households decide their consumption expenditure, housing investment, and the allocation of financial investment between risky and riskless assets. Using the indirect inference estimation technique, I estimated the size of relative risk aversion (RRA) and elasticity of intertemporal substitution (EIS). The results of the model show that households reduce their risky asset investment in the presence of the housing investment. The negative effect of housing investment on households’ risky asset investment is stronger for young and middle-aged households than for older households. Furthermore, house price risk and the size of the downpayment have some sizable negative effects on households’ risky asset investment. However, the effects of both a liquidation cost of selling a house and the size of a stock market entry cost have negligible effects on households’ risky asset investment decisions.

The third essay examines the effects of both housing investment and tax-deferred accounts (TDA) on households’ life-cycle portfolio allocation decisions. In this model, households decide in each period their consumption expenditure, housing investment, the distribution of financial wealth between taxable accounts and tax-deferred accounts (i.e. asset location decision), and the allocation of financial wealth in each account between risky and riskless assets (i.e. asset allocation decision). Households make constant contributions to TDAs during the working period to accumulate funds to be used for the retirement period. Households withdraw funds from TDAs during retirement in order to keep their working period living standards during retirement period. The results of this model show that both housing investment and TDAs have sizable crowding out effects on households’ risky asset investment as well as risky asset market participation rates over the life-cycle. Because of their tax-exempt status, households prefer to invest more in TDAs than taxable accounts. The results shows that the crowding out effect of housing investment on households’ risky asset investment is stronger than the effects of TDAs. Furthermore, in the absence of randomness in the evolution of house prices, households tend to invest more in the risky asset in both types of accounts. Last, while changes in the size of
contribution rate to TDAs do not change households’ asset location decision, the share of risky assets in TAs over all risky asset investments decreases as the contribution rate increases.
Chapter 1

Incorporating Basic Housing Investment into a Life-Cycle Portfolio Allocation

1.1 Introduction

Low stock market participation rates and moderate equity holdings for stock market participants are two important empirical observations observed in US data. For instance, the 2007 Survey of Consumer Finance (SCF) data show only 55.3% of US households hold direct and indirect holdings of risky assets.\(^1\) Furthermore, 1968 - 2007 data from the Panel Study Income Dynamics (PSID) show that the median households direct risky asset holdings and indirect risky asset holdings are zero. Despite this fact, theoretical models with the assumption of the historically prevailing equity premium predict that almost 100% of households should hold risky assets as part of their financial portfolio. This gap between theoretical predictions and empirical

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\(^1\)Risky assets include tax-deferred account, directly held stock, directly held pooled investment fund, bonds, and managed investment account or an equity in a trust or annuity. Tax-deferred retirement account consists of both personally established individual retirement accounts (IRAs) and job-based 401(k) accounts. For detail information see Bucks et al. (2009).
observation still poses a great challenge to life-cycle models.

While the primary investment asset for US households is investment in owner-occupied housing, it is generally ignored in portfolio allocation models. The 2003-2007 PSID data show that about two-thirds of US households are homeowners. It is quite logical that a typical household has a higher priority to invest in housing in order to have an owner occupied house than investing in the stock market. This fact is probably the main reason for the low stock market participation observed in the data. Cocco (2004) showed that due to the investment in housing, younger and poorer households have less wealth to invest in the stock market. In other words, housing investment crowds out stockholding for young and low income people.

The focus of my research is to incorporate housing investment into a life-cycle asset allocation model to provide an explanation for these two empirical observations: low stock market participation rates and moderate equity holdings for stock market participants. Specifically, I investigate the effects of housing investment on portfolio allocation of households in a life-cycle model through constructing a structural model which involves solving and simulating the life-cycle asset allocation model and estimating some crucial parameters using the indirect inference estimation method.

Housing is different from other financial assets because it serves dual benefits. First, it is a durable consumption good from which owners obtain utility. Second, it also serves as an investment asset that enables owners to hold home equity. Contrary to liquid financial assets such as bonds and stocks, housing investment is illiquid and often highly leveraged. While housing plays an important role in portfolio allocation, it is largely unexplored because of the difficulty of dealing with various frictions associated with the housing market, such as homeowner/renter distinction, mortgage payment, liquidation cost, moving decision, etc.

Beside housing, my model incorporates some key features in order to better explain the asset allocation profiles of households. The first feature is the use of Epstein-Zin (EZ) type preferences, [Epstein and Zin (1989)], where the relative risk aversion (RRA) is disentangled from the elasticity of intertemporal substitution (EIS). The main drawback of the commonly used
constant relative risk aversion (CRRA) utility function is that RRA, which gives information about how agents deal with uncertainty across possible states of the world, is the inverse of the EIS which is mere time preference. In other words, CRRA imposes two different roles on the same parameter. However, EZ type preferences provide the flexibility to disentangle RRA from the EIS. The second feature is that households need to pay a fixed stock market entry cost the first time they decide to invest in the stock market. Stock market entry cost is widely accepted in the literature even though too little investigation has been done on its magnitude. An easy and computationally cheap way of introducing an entry cost is considering it as a fixed proportion of annual labor income as in Gomes and Michaelides (2007), Guvenen (2009b), Guo (2004), and Alan (2006). This entry cost can be considered as the cost of opening a brokerage account, understanding how the market works, and acquiring and evaluating information about the stock market plus the opportunity cost of time. The third feature is that labor supply is inelastic and households receive uninsured labor income in each period. I use PSID data from 2003 - 2007 to realistically calibrate the life-cycle labor income process of households. Finally, I incorporate a bequest motive in my model which assumes that households consume all their cash-on-hand in the terminal period and bequest the investment made in housing to their inheritors.

1.2 Literature Review

Households wealth allocation throughout the life-cycle has been analyzed in the portfolio allocation literature since the pioneering studies of Merton (1969) and Samuelson (1969). Although it is ignored by a huge body of the portfolio allocation literature over a very long period of time, there is a gradually growing literature that treats housing as an important determinant of portfolio allocation. Leung (2004) provides a comprehensive literature review of housing and asset pricing. Grossman and Laroque (1990) develop an asset allocation model where infinitely lived households derive utility from a single indivisible durable consumption good. They argue that an adjustment cost for illiquid assets could answer the equity premium puzzle. Flavin and

\footnote{For example see Alan (2006) for a detailed analysis of the entry cost.}
Nakagawa (2008) extend the Grossman and Laroque (1990) model by including both durable and nondurable consumption goods into the utility function. Using a continuous time framework, the paper compares a housing model to a habit persistence model and finds that while both deliver many of the same implications, empirical tests using household level data strongly favor the housing model. Longstaff (2009) studies the implications of illiquid assets in a continuous time asset pricing exchange economy with heterogeneous agents. Villaverde and Krueger (2010) present a general equilibrium model of life cycle asset allocation to demonstrate the effects of consumer durable goods on consumption and asset allocations. Piazzesi et al. (2007) consider a consumption based asset pricing model where housing is explicitly incorporated into the model both as an asset and as a consumption good. Their paper focuses on the effects of housing-consumption asset pricing models on the predictability of the return on stocks.

However the closest articles to my research are Hu (2005), Cocco (2004), and Yao and Zhang (2005). Hu (2005) develops a standard life cycle portfolio allocation model that provides the flexibility for households to endogenously decide whether to be a renter or a homeowner. The model employs a CRRA utility function and only five time periods where each period corresponds to either 10 or 15 years. Although the paper obtains low levels of investment in the stock market relative to standard models with no housing, the investment in the stock market is still significantly higher than the empirically observed values. Furthermore, the results of this paper shows that renters hold more risky assets in their portfolio than homeowners. This result actually is in contradiction with the data because homeowners are, by and large, wealthier than renters and people usually want to become homeowners before investing in risky assets. Cocco (2004) analyzes portfolio choice in the presence of housing in a standard life cycle model with a CRRA utility function. Each time period in this model corresponds to five years. The model assumes that all agents are homeowners. There is no endogenous decision to be a renter or homeowner. This paper concludes that (1) house price risk crowds out stockholding, (2) households have a relatively low stock market participation rate compared to standard models with no housing, and (3) younger and poorer investors have limited financial wealth to invest
in the stock market. However it is less successful at matching stock shares conditional on participation with predicted values much higher than those observed in the data. The last paper similar to mine is Yao and Zhang (2005). The model of this paper is similar to Cocco (2004) except it allows households to decide between renting or owning a house. Results show that renters invest a higher portion of their financial wealth in the stock market than owners. Furthermore, the share of wealth invested in the stock market is greatly higher than empirical observations.

My research differs from these studies in several dimensions. First, I develop a comprehensive life-cycle portfolio allocation model that incorporates all features introduced by different studies in the portfolio allocation literature. Among these are housing as an investment and a durable consumption good, an endogenous decision on being a renter or homeowner, a stock market entry cost and an endogenous decision on stock market investment, EZ type preferences, and a bequest motive. Second, the life cycle portfolio allocation papers that incorporate housing into the model generally calibrate the parameters of the model. Instead, I estimate two crucial parameters, (relative risk aversion (RRA) and elasticity of intertemporal substitution (EIS)) with indirect inference. To my knowledge, this paper is the first one in the life cycle housing-portfolio allocation literature that estimates parameters instead of calibration.

In this chapter I use an intermediate version of my model which doesn’t include all features mentioned above. However, in the second chapter, I will include the remaining features of the comprehensive model and use this comprehensive model to analyze the life cycle asset allocation of households. The intermediate model, for example, assumes that (1) households stay in their initially bought-house permanently, (2) labor income has only one idiosyncratic shock. However the results of the intermediate level model are striking. Indirect inference estimation gives estimates of 0.6 for RRA and 0.5 for the EIS. The results of the model with the estimated parameters show that housing investment has a crowding out effect on the portfolio allocation of households. For example the model with housing predicts that between their 20s and 30s people invest less than 1% of their cash-on-hand, but between their 30s and 40s that fraction
increases to 24% in the risky asset. The same model with no housing, on the other hand, predicts that between their 20s and 30s people invest only 3% of their cash-on-hand, but between their 30s and 40s that fraction jumps to 30%. The model is partially successful in obtaining moderate levels of risky assets holdings while it is not successful in obtaining low participation rates for the risky asset market. The model also shows that the crowding out effect of housing is bigger than the crowding out effect of stock market entry cost.

The rest of the chapter is organized as follows. Section 1.3 presents the model and its assumptions. A detailed explanation of the parameterization and the solution technique is explained in Section 1.4. The indirect inference estimation procedure for RRA and the EIS is described in Section 1.5. Section 1.6 presents the results and Section 1.7 draws conclusions and provides some suggestions for future research.

1.3 Model

The specifications of this intermediate level model are presented in this section.

1.3.1 Household Preferences

I use discrete time periods, where each period corresponds to one year. As a general convention in the life-cycle literature each year is actually the real age of a household minus 19. I assume that all households live for \( T \) periods. In each period, a household derives utility from a constant elasticity of substitution (CES) utility function with nondurable consumption goods, \( C \), and housing consumption (or investment), \( H \). Preferences are in the form of Epstein-Zin, where the relative risk aversion (RRA) is disentangled from the elasticity of intertemporal substitution (EIS):

\[
U_t = \left\{ u(C_t, H_t) \frac{1}{\theta} + \beta \left( E_t \left[ U_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right\}^{\frac{1}{1-\gamma}} \tag{1.1}
\]

\[
\theta = \frac{1 - \gamma}{1 - 1/\psi} \tag{1.2}
\]
\[ u(C_t, H_t) = \left[ C_t^\psi + \lambda H_t^\psi \right]^{\frac{1}{\psi}}, \quad (1.3) \]

where \( \beta \) is the time discount factor, \( \gamma \) is the RRA parameter, and \( \psi \) is the EIS parameter. Because homeowners derive more utility from the same house than renters, the parameter \( \lambda \) determines the level of utility renters and homeowners obtain from housing investment. The intratemporal elasticity of substitution between nondurable consumption goods and housing investment is \( 1/(1 - v) \).

### 1.3.2 Labor Income

In this current version of my model I follow the standard specifications in the life cycle literature. Labor income depends on the age-specific profile of households with an idiosyncratic random shock. Households supply labor inelastically in each period and receive stochastic labor income \( Y_{it} \).

\[
\log(Y_{it}) = f(t, Z_{it}) + \varepsilon_{it}^l \quad (1.4)
\]

\[
f(t, Z_{it}) = \beta_1 \text{age} + \beta_2 \text{age}^2 + \beta_3 \text{gender} + \beta_4 \text{marital status} + \beta_5 \text{education}, \quad (1.5)
\]

where \( Y_{it} \) is labor income received at time \( t \), \( f(t, Z_{it}) \) is a deterministic function of age \( (t) \), and household characteristics \( (Z_{it}) \) such as education, marital status and gender. \( \varepsilon_{it}^l \) is the random shock to log labor income distributed as i.i.d. \( N(0, \sigma_{\varepsilon_l}^2) \).

### 1.3.3 Housing Investment

Households enter the market as renter in the first period. From the second period on, they endogenously decide to buy a house and become a homeowner or stay in their current house as a renter. In this current version of my model, once a household buys a house it stays in that house permanently. I will relax this assumption in Chapter 2 by allowing households to endogenously decide to be renters or homeowners every period. However I will maintain this
assumption for the moment. Buying a house requires paying a proportion $d$ of the house value as a downpayment and financing the rest through a mortgage. Homeowners pay for house maintenance and depreciation expenses, an amount equal to a proportion $\delta$ of the house value in each period. Renters, on the other hand, don’t pay maintenance and depreciation costs, they pay only annual rent which is equal to a proportion $\alpha$ of the house value. House-related expenses are calculated as certain proportions of the house value in the literature because in this way it is easy to incorporate them into the model and reduce the computational burden.

1.3.4 Financial Assets and Wealth Accumulation

Households can invest in only two financial markets: a risky asset market and a riskless asset market. Return on risky assets follows the following stochastic process as in Campbell et al. (2001) and Cocco et al. (2005):

\[ R^s_{t+1} - R^b = \mu + \varepsilon^s_{t+1}, \]

where $R^s_{t+1}$ is the gross return on risky assets at time $t+1$, $R^b$ is the constant return on the riskless assets, $\mu$ is the excess return from the risky assets over the riskless one (equity premium), and $\varepsilon^s_{t+1}$ are i.i.d. random shocks with $N(0, \sigma^2_{\varepsilon^s})$.

In order to invest in the risky asset market, investors are required to pay a one-time fixed risky asset market entry cost. This entry cost can be considered as the cost of opening a brokerage account, understanding how the market works and acquiring and evaluating information about the risky assets market plus the opportunity cost of time. Alan (2006) estimates this cost as approximately 2 percent of annualized labor income, whereas Gomes and Michaelides (2007) assume that this fixed cost is equal to 6 percent. There is no cost for buying the riskless assets.

In this model there are two types of households: renters and homeowners. Depending on the homeownership status, households have different budget constraints and cash-on-hand structures. Renters’ liquid wealth is the sum of the returns on investments made in the previous period in both the risky and the riskless assets. Cash-on-hand is the sum of liquid wealth and stochastic labor income. This cash-on-hand is then used in the current period for consumption.
expenditure, the annual rental payment, and investments in both the risky and the riskless assets. Total wealth of renters includes the same cash-on-hand:

\[
LW_t = R^b B_{t-1} + R^s S_{t-1} \quad (1.7)
\]

\[
X_t \equiv LW_t + Y_t \quad (1.8)
\]

\[
X_t = S_t + B_t + FIX_t FY_t + \alpha P_t H_t + C_t \quad (1.9)
\]

\[
TW_t = X_t, \quad (1.10)
\]

where \(LW_t\) is the liquid wealth at time \(t\), \(B_{t-1}\) is total investment of a household in the riskless assets at time \(t-1\), \(S_{t-1}\) is the total investment of a household in the risky assets at time \(t-1\), \(X_t\) is cash-on-hand at time \(t\) and \(\alpha\) is the fraction of the house value renters pay as annual rent. \(P_t\) is the price of per unit housing such that a house of size \(H_t\) has price \(P_t H_t\) at date \(t\). \(FIX_t\) is a dummy variable that takes a value of 0 if the risky asset market entry cost is paid and 1 if it isn’t paid yet. \(F\) is the share of labor income to be paid to invest in the risky asset market. Finally \(TW_t\) denotes total wealth of a household at time \(t\).

The wealth accumulation and budget constraints of homeowners are a little different than that of renters. Similar to renters, homeowners receive stochastic labor income and return on investments made in previous periods in the risky and the riskless assets. However, they use some of this wealth for current period consumption, annual interest payments, and house depreciation and maintenance expenses. The remaining part of this wealth is used for investments in both risky and riskless assets. To keep the model computationally feasible, I assume that people pay only the interest on their mortgage debt during their life time. They don’t pay the principal loan except for the downpayment. In this way, I don’t need to keep track of time when a house is bought and when the mortgage debt vanishes. I am aware of the fact that the most common way to repay the mortgage loan is to pay regular payments of the principal and interest over a set of terms. However, there is a big computational burden if I keep track of the remaining mortgage payments. Thus, I assume that people pay only the interest payment accrued from
the mortgage debt. Then the wealth accumulation and budget constraint of homeowners are

\[
LW_t = R^b B_{t-1} + R^s S_{t-1} 
\]

(1.11)

\[
X_t \equiv LW_t + Y_t 
\]

(1.12)

\[
X_t = S_t + B_t + FIX_t F Y_t + r^m P_t H_t + \delta P_t H_t + C_t 
\]

(1.13)

\[
TW_t = X_t + dP_t H_t. 
\]

(1.14)

where \(r^m P_t H_t\) is the annual interest payment on the mortgage loan with \(r^m\) denoting the interest rate on the mortgage, \(\delta P_t H_t\) is the annual maintenance and depreciation expenses as proportion \(\delta\) of the house value, and \(dP_t H_t\) is the amount of downpayment. Total wealth is the sum of cash-on-hand and the downpayment paid when the house is bought.

Renters who are buying a house at time \(\tau\), have different budget constraints and cash-on-hand structures than renters and homeowners. They pay the downpayment and begin to pay annual interest payments on the mortgage loan. Their cash-on-hand, total wealth and budget constraints at the time the house is bought are as follows.

\[
LW_{\tau} = R^b B_{\tau-1} + R^s S_{\tau-1} 
\]

(1.15)

\[
X_{\tau} \equiv LW_{\tau} + Y_{\tau} 
\]

(1.16)

\[
X_{\tau} = S_{\tau} + B_{\tau} + FIX_{\tau} F Y_{\tau} + r^m P_{\tau} H_{\tau} + dP_{\tau} H_{\tau} + C_{\tau} 
\]

(1.17)

\[
TW_{\tau} = X_{\tau} + dP_{\tau} H_{\tau}. 
\]

(1.18)

Households are assumed to satisfy the non-negativity constraints on consumption and housing investment and short-sale constraints on the risky and the riskless assets. They cannot have negative consumption and housing investment and cannot short-sale risky assets on borrowing at the riskless rate:

\[
C_t \geq 0, H_t \geq 0, B_t \geq 0, S_t \geq 0, \quad \forall t. 
\]

(1.19)
It is common in recent life-cycle portfolio allocation models to incorporate a bequest motive in order to match the skewness of the wealth distribution. For example see Laitner (2002), Gomes and Michaelides (2005), and Gourinchas and Parker (2002). I assume a simple bequest motive where households consume all their cash-on-hand and bequest their housing investment to the inheritors as bequests in the terminal period. The bequest function is

\[ B(H_T, P_T) = \lambda dP_T H_T, \]  

(1.20)

where \( dP_T H_T \) is the total housing investment households make during their life cycle. Since I assume that people pay only the downpayment and interest payment on the mortgage, they actually don’t pay the remaining principal of the mortgage, \( (1-d)P_T H_T \). I made this assumption for the sake of computational feasibility. In order to bequest the whole house to inheritors at the terminal period, I set \( \lambda = 1/d \).

### 1.3.5 Optimization Problem

Before defining the value function, I need to list the state and control variables. The state variables are liquid wealth \((LW_t)\), risky assets market participation status \((IFI_X_t)\) house ownership status \((O_t)\), the size of house owned \((H_t)\), and age. I denote the state variables by \( \Omega \) where \( \Omega_t = \{LW_t, IFIX_t, O_t, H_t, t\} \). Household control variables are consumption \((C_t)\), house buying decision \((H_t)\), the risky asset market participation decisions \((FIX_t)\), and the share of financial investment made in the risky assets \((s_t)\). I denote control variables by \( \Psi \) where \( \Psi_t = \{C_t, H_t, FIX_t, s_t\} \) is the function of control variables. Households’ optimization problem then consists in maximizing

\[
V_t(\Omega_t) = \max_{\{C_t, H_t\}} \left\{ u(C_t, H_t)^{1-\gamma} + \beta (E_t V_{t+1}(\Omega_{t+1})^{1-\gamma} \right\} \right)^{\frac{\theta}{1-\gamma}} 
\]

\[ V_{T+1} = B(H_T, P_T), \]

subject to Equations 1.1 through 1.20.
1.4 Parameterization

Each period in the model corresponds to one year. Agents are assumed to enter the market at age of 20 and live for 80 years. The age-dependent labor income process is based on PSID data from 2003-2007. In order to obtain a random sample of US households, I dropped families that were part of the Survey of Economic Opportunities\(^2\). Previous studies in the life cycle portfolio allocation models treat retirement period labor income as a constant fraction of labor income of the last working period as in Gomes and Michaelides (2005) and Cocco (2004). However, data show that households’ labor income isn’t constant after retirement, it increases until their 60s and then decreases. Hence I assume that households supply labor inelastically and receive stochastic labor income during their life time. The permanent component of labor income is a second order regression function of households’ age and dummy variables for education, marital status, and gender (i.e. Equation 1.5). The dummy variable for education is 1 for households with 12 or more years of education and 0 otherwise. I define labor income widely enough to account for endogenous ways of self insurance against labor income shocks. Labor income includes total labor income, unemployment compensation, social security, total transfers, child support, and other welfare received by both head of the family and his wife. I consider only idiosyncratic transitory shocks to labor income. Similar to Gourinchas and Parker (2002) and Gomes and Michaelides (2005), I set the standard deviation of this shock at 15% per year.

I need to calibrate several parameters related to housing. Yao and Zhang (2005) and Hu (2005) set the required downpayment rate at 20% of the house price while Cocco (2004) set it at 15%. I set the downpayment rate as 20% of the house value. To be consistent with previous studies, I set the annual rental rate of the house at 7% of the market value of the rental property and annual maintenance and depreciation cost at 1% of the market value of house. I set \(\nu = 0.2\) so that the intratemporal elasticity of substitution between nondurable consumption

\(^2\)The original PSID sample was drawn from two independent sources: an over-sample of roughly 2000 poor families selected from the Survey of Economic Opportunities (SEO), and a nationally-representative sample of roughly 3000 families from all states. Since the SEO over-samples of low income families, it isn’t a random sample of the US population.
and housing, \(1/(1 - v)\), is 1.25 as in Piazzesi et al. (2007). I assume there is no real return on housing and set house price \(P_t\) to 1. Since for the same size of a house renters receive relatively less utility than homeowners as suggested by Hu (2005), I set \(\lambda\) to 0.85.

The risk free rate \(R_b\) is 3%. I use annual mean return on risky assets as 9% which means an equity premium equal to 6%, \(\mu\). The standard deviation of shocks to risky assets is 20%. I assume no correlation between return on risky assets and labor income. In general, the mortgage rate is higher than the constant return on the riskless asset because a mortgage bears a long-term interest rate risk and a default risk. Since my model doesn’t have interest risk and default risk I set the interest rate on mortgage loan at 3%. Relying on the estimation made by Alan (2006), I use risky asset market entry cost as 2% of annual labor income. Similar to Viceira (2001), I define two different time discount factors for young and elderly people. For people between age of 20 to 65, the time discount factor is 0.95 and it is 0.85 for people who are older than 65.

An analytical solution of this problem doesn’t exist. Therefore, we rely on numerical approximation methods (the details are provided in Appendix B).

### 1.5 Estimation

Because my structural model is time consuming to solve, I estimate only two parameters: the elasticity of intertemporal substitution (EIS) and the relative risk aversion (RRA).\(^2\) These two parameters play an important role in obtaining life cycle asset allocation profiles. Depending on the values we assign to these parameters we can obtain different life cycle asset allocations. Furthermore, there is no clear consensus in the literature on the exact values of these parameters. For instance, Vissing-Jorgensen (2002) suggests that limited asset market participation is important for the estimation of the elasticity of intertemporal substitution. She estimates the EIS for stockholders to be around 0.3-0.4 and for bond holders around 0.8-1. On the other hand, Guvenen (2003) uses an EIS for stockholders close to 1 while for nonstockholders close

\(^2\)The details of numerical solution of the model is explained in Appendix B.
to 0.1. Hall (1988) finds that the EIS is unlikely to be much higher than 0.1. However, Hansen and Singleton (1982) and Vissing-Jorgensen and Attanasio (2003) estimate the EIS as greater than 1. They also estimate relative risk aversion between 5-10. In a life cycle portfolio allocation model, Gomes and Michaelides (2005) uses different values of RRA and EIS and come up with the conclusion that life cycle portfolio allocations are very sensitive to the values of RRA and EIS. Because these two parameters play critical roles in obtaining life cycle asset allocation and because there is no consensus in the literature on the exact values for them, it is important to estimate these parameters in order to reach more accurate life cycle portfolio allocation profiles.

I estimate my model using the indirect inference estimation procedure introduced by Gourieroux et al. (1993). Given the parameters defined above, I solve the optimization problem of households numerically and obtain the life-cycle optimal consumption, housing, and portfolio allocation rules. From these optimal rules, I can numerically simulate the corresponding life-cycle consumption, housing decision, and portfolio allocation profiles of households. Then I use indirect inference to minimize the distance between the parameter of the auxiliary model obtained from both the observed and the simulated data.

Being in the class of Simulated Method of Moments (SMM), indirect inference is used when a model is too complicated and its likelihood function is untractable. Indirect inference requires simulated data from the complicated model for different parameter values. Unlike other SMM methods, indirect inference uses an auxiliary model to construct a criterion function. The goal of this method is to choose the parameters of the underlying structural model in a way such that the auxiliary model parameters of the simulated data are as close as possible to the auxiliary model parameters of the observed data.

I use a vector autoregression model of order one, VAR(1), as the auxiliary model of the indirect inference estimation to obtain parameters to be used in the minimization. Data used for the estimation are both the observed and the simulated data on investment in the risky assets and investment in the risky assets as a share of cash-on-hand. The VAR model for the
observed data is

\[ y_t^a = \Phi^a_0 + \Phi^a_1 y_{t-1}^a + \varepsilon_t^a, \]  

(1.23)

where \( y_t^a \) is time \( t \) investment in risky assets and the share of risky assets in cash-on-hand of the observed data. Denote \( \Theta^a \) by \( \left[ \Phi^a_0 \quad vec(\Phi^a_1) \right]' \) the vectorized form of parameters of the auxiliary VAR model. The VAR of the simulated data is similar to the VAR of the observed data:

\[ y_t^{s,i} = \Phi^{s,i}_0(\theta) + \Phi^{s,i}_1(\theta)y_{t-1}^{s,i} + \varepsilon_t^{s,i}(\theta), \]  

(1.24)

where \( y_t^{s,i} \) denotes the \( i^{th} \) simulation data used in VAR model. Then the average of the parameters estimated in VAR will be used for the minimization process. Define \( \Theta^{s,i}(\theta) = \left[ \Phi^{s,i}_0(\theta)' \quad vec\left(\Phi^{s,i}_1(\theta)\right) \right]' \) where \( \theta \) denotes the parameters of interest in the underlying structural model. The parameters of the simulation used in the minimization process are obtained by averaging out parameters of the VAR models obtained from all simulations:

\[ \Theta^s(\theta) = \frac{1}{S} \sum_{s=1}^{S} \Theta^{s,i}(\theta), \]  

(1.25)

where \( S \) is the number of simulations.

The indirect inference estimator of \( \theta \) is then defined by choosing the optimum value of \( \theta^s \) in such a way that the distance between \( \Theta^a \) and \( \Theta^s \) is minimized:

\[ \theta^s = \min_{\theta} \left[ \Theta^a - \Theta^s(\theta) \right]' \Pi \left[ \Theta^a - \Theta^s(\theta) \right], \]  

(1.26)

where the weighting matrix \( \Pi \) is the identity matrix which is symmetric and nonnegative definite.
1.5.1 Data for Indirect Inference Estimation

I use PSID data from 2003 to 2007 for the estimation as in the calibration of the labor income process. Similar to the parameterization part, I exclude the Survey of Economic Opportunity (SEO) sample from the data to have a random sample of the US population. Total investment in the risky assets in PSID data refers to the sum of shares of stocks in publicly or privately held corporations including those held in mutual funds, investment trusts and Individual Retirement Accounts (IRAs). Total financial assets on the other hand refers to total risky assets defined above plus money in saving and checking accounts, money market funds, certificates of deposits, government saving bonds, Treasury bills (not including assets held employer-based pensions or IRAs), other savings or assets, such as bond funds, cash value in a life insurance policy, and a valuable collection for investment purposes. I define cash-on-hand as the sum of the total financial assets and the labor income.

Data used for the estimation include only households that report all demographic information (i.e. age, gender, marital status, education) and necessary data for the estimation. The sample includes 19801 households. I average out the data on investment in the risky assets and the share of the risky assets in cash-on-hand for each age. Then I use kernel regression to smooth out this averaged data. Kernel regression, as a non-parametric regression method, is appropriate when the relation between the dependent and the independent variables is clearly not linear. Define $\bar{y}_a^t$ as the age-dependent averaged data on investment in risky assets and the share of the risky assets in cash-on-hand for each age, then the goal of kernel regression is to estimate the following functional form where $m(x_t)$ isn’t specified.

$$\bar{y}_a^t = m(x_t) + \varepsilon^t_k.$$ (1.27)

Kernel regression finds the average of the $\bar{y}_t$ using all observations. The bandwidth parameter $(h)$ rescales the spread of the kernel. I use $0.2\sigma_x$ as the bandwidth parameter. It is preferable to use a kernel weighting function, $K_h(x)$, in order to give the greatest weight to $x_0$ and decrease
the weight as we move away from \(x_0\). \(K_h(x)\) in this model is constructed from the normal distribution merely because the normal distribution is convenient to compute the weighted averages:

\[
K_h(x) = \frac{1}{h\sqrt{2\pi}} e^{-\frac{x^2}{2h^2}}.
\] (1.28)

This yields the kernel regression estimator

\[
\hat{m}_h(x_0) = \frac{\sum_{t=1}^{T} K_h(x_t - x_0) \bar{y}_t}{\sum_{t=1}^{T} K_h(x_t - x_0)}.
\] (1.29)

The resulting income from the kernel regression is used in Equation 1.23 to obtain the parameters of the auxiliary VAR from the observed data. These parameters are then used as the inputs to the minimization problem.

### 1.5.2 The Estimation Procedure

The structural estimation involves a global search for the optimum parameters for the RRA and EIS. The other parameters are calibrated and displayed in Table 1.1. The estimation procedure is composed of the following steps:

1. Use Equation (1.23) to obtain auxiliary parameters from the data, \(\Theta^a\),
2. Given the initial parameters of RRA and EIS and the calibrated values for other parameters, solve the structural model numerically and obtain life-cycle optimal consumption, housing investment, and portfolio allocation rules,
3. Simulate corresponding life-cycle consumption, housing investment, and portfolio allocation profiles of households from these rules,
4. Use Equation (1.24) and Equation (1.25) to obtain auxiliary VAR model parameters of the simulated data, \(\Theta^s(\theta)\),
5. Minimize the distance between the auxiliary parameters obtained in Step 1,(\(\Theta^a\)), and Step 4, (\(\Theta^s(\theta)\)), in Equation 1.26,
Table 1.1: Parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intratemporal elasticity of substitution</td>
<td>1/(1 − v)</td>
<td>1.25</td>
</tr>
<tr>
<td>Std of shocks to labor income</td>
<td>σ_e^l</td>
<td>0.15</td>
</tr>
<tr>
<td>Downpayment</td>
<td>d</td>
<td>0.20</td>
</tr>
<tr>
<td>Rent</td>
<td>rent</td>
<td>0.07</td>
</tr>
<tr>
<td>Maintenance and depreciation exp.</td>
<td>δ</td>
<td>0.01</td>
</tr>
<tr>
<td>Real return on housing</td>
<td>P_h</td>
<td>1</td>
</tr>
<tr>
<td>Mortgage rate</td>
<td>r^m</td>
<td>0.03</td>
</tr>
<tr>
<td>Risk free rate</td>
<td>R^b</td>
<td>0.03</td>
</tr>
<tr>
<td>Equity premium</td>
<td>μ</td>
<td>0.06</td>
</tr>
<tr>
<td>Time discount factor</td>
<td>β</td>
<td>0.95</td>
</tr>
<tr>
<td>Std of shocks to return on risky assets</td>
<td>σ_s^e</td>
<td>0.20</td>
</tr>
<tr>
<td>Risky asset market entry cost</td>
<td>FIX</td>
<td>0.02</td>
</tr>
</tbody>
</table>

(6) Keep updating the parameters of interest and redo steps 2-5 until the change in the value of the objective function is less than the termination tolerance on function value.

1.6 Results

Using the estimation procedure explained above, the RRA and EIS estimates through indirect inference estimation are 0.6 and 0.5 respectively. The estimated RRA is quite low compared to the calibrated values used by previous studies in the life-cycle portfolio allocation literature such as 5 in Hu (2005), 2 in Villaverde and Krueger (2010), and 5 in Campbell et al. (2001). Using a relatively high RRA value may be one reason for the failure to obtain life-cycle portfolio allocation profiles similar to the observed counterparts. EIS, on the other hand, is within the generally accepted interval.

Based on the estimated parameters, I solved the optimization problem once more and obtain life cycle consumption, housing investment, and portfolio allocation decision rules. Then, using these optimal rules, I simulate corresponding life-cycle consumption, housing investment, and portfolio allocation profiles of households. Figure 1.1 shows the life cycle profiles for con-
sumption, cash-on-hand, and investments in financial markets. Results show that consumption, which constitutes the largest fraction of cash-on-hand, is not as smooth as proposed in life-cycle models with complete financial markets. Villaverde and Krueger (2010) also point out that consumption expenditure follows a hump shape contrary to the basic prediction of life-cycle models with complete financial markets that consumption profiles are smooth. Consumption expenditures increase rapidly between the age of 20 to 40, increase relatively slowly for next 30 years, and then decrease gradually.

Figure 1.1: Life Cycle Asset Allocation

Figure 1.2 shows the life-cycle homeownership rate. The motivation to have their own house leads people to buy a house early in life. By their mid 20s, everyone becomes a homeowner. Due to the downpayment when a house is bought and the annual mortgage payments, people don’t have enough cash-on-hand to invest in financial assets. Cocco (2004), Fratantoni (1998), Hu (2005), and Yao and Zhang (2005) find that housing investment crowds out investment in risky asset market. As Figure 1.1 shows, people don’t have enough investment in the risky asset market early in their lives, so housing has crowding out effects on investment in the risky asset
market. This effect will be clear when I compare the model with housing investment to the same model with no housing investment. For the first 8 years there is almost no investment in the risky asset market. Between the late 20s until early 40s the fraction of cash-on-hand invested in the risky asset market increases, it then decreases gradually until the late 60s. After that period the investment in this market is almost zero. Cocco (2004) and Hu (2005) find that as people get older the share of the risky assets investment continues to increase until their mid 60s. Yao and Zhang (2005), on the other hand, claim that as people get older this share decreases.

![Figure 1.2: Life Cycle Homeownership Rate](image)

Figure 1.2: Life Cycle Homeownership Rate

Table 1.2 shows the life cycle allocation of wealth and other variables for different age groups. Results in the first 8 rows are actually the same as in the Figures 1.1 and 1.2. The last two rows give information about the share of risky and riskless assets in cash-on-hand. Investment in the risky asset market shows a hump-shape pattern. In the first ten years, people don’t invest in the risky asset market. This is because they need to pay the downpayment and begin paying the annual mortgage payments, so not enough wealth is left to pay the fixed entry cost to make investments in this market. The ratio of risky asset to cash-on-hand increases to 24% in the next ten years and it is 32% between the 40s-50s. However, as people get closer to the terminal period they begin to invest less and less in this market. During the last ten years
they have 1% of cash-on-hand in the risky asset market. The second row of Table 1.3 shows
the share of investment in the risky assets in cash-on-hand for the PSID data between 2003 to
2007. Contrary to what the model predicts, the data show that people don’t invest more than
10% of cash-on-hand in the risky asset market until their 50s. However, the model is partially
successful in that the predictions of the model for the share of the risky assets investment is
almost always less than 30%. Hence the model is partially successful in matching the moderate
equity holding observed in the data.
<table>
<thead>
<tr>
<th>Age</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>70-80</th>
<th>80-90</th>
<th>90-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption&lt;sup&gt;a&lt;/sup&gt;</td>
<td>0.524</td>
<td>1.030</td>
<td>1.090</td>
<td>1.174</td>
<td>1.162</td>
<td>0.911</td>
<td>0.596</td>
<td>0.287</td>
</tr>
<tr>
<td>Labor Income</td>
<td>0.744</td>
<td>1.246</td>
<td>1.263</td>
<td>1.327</td>
<td>1.298</td>
<td>1.050</td>
<td>0.733</td>
<td>0.416</td>
</tr>
<tr>
<td>Risky Assets</td>
<td>0.008</td>
<td>0.375</td>
<td>0.567</td>
<td>0.370</td>
<td>0.109</td>
<td>0.017</td>
<td>0.011</td>
<td>0.000</td>
</tr>
<tr>
<td>Riskless Assets</td>
<td>0.061</td>
<td>0.022</td>
<td>0.008</td>
<td>0.014</td>
<td>0.041</td>
<td>0.076</td>
<td>0.081</td>
<td>0.009</td>
</tr>
<tr>
<td>Cash-on-hand</td>
<td>0.592</td>
<td>1.398</td>
<td>1.624</td>
<td>1.522</td>
<td>1.311</td>
<td>1.003</td>
<td>0.687</td>
<td>0.296</td>
</tr>
<tr>
<td>Risky asset Market Part. Rate</td>
<td>0.080</td>
<td>0.936</td>
<td>0.999</td>
<td>0.998</td>
<td>0.947</td>
<td>0.612</td>
<td>0.324</td>
<td>0.008</td>
</tr>
<tr>
<td>House Size</td>
<td>2.248</td>
<td>2.018</td>
<td>2.018</td>
<td>2.018</td>
<td>2.018</td>
<td>2.018</td>
<td>2.018</td>
<td>2.018</td>
</tr>
<tr>
<td>House Ownership Rate</td>
<td>0.724</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Risky Assets/Cash-on-Hand</td>
<td>0.007</td>
<td>0.242</td>
<td>0.318</td>
<td>0.219</td>
<td>0.073</td>
<td>0.015</td>
<td>0.014</td>
<td>0.001</td>
</tr>
<tr>
<td>Riskless Assets/Cash-on-Hand</td>
<td>0.075</td>
<td>0.020</td>
<td>0.006</td>
<td>0.010</td>
<td>0.033</td>
<td>0.078</td>
<td>0.121</td>
<td>0.018</td>
</tr>
</tbody>
</table>

<sup>a</sup> Levels are normalized by life-cycle average labor income. For example, Consumption represent the level of consumption over the life-cycle divided by average labor income over the life-cycle.
Table 1.3: Life Cycle Asset Allocation by Age in PSID Data between 2003 - 2007

<table>
<thead>
<tr>
<th>Age</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>70-80</th>
<th>80-90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risky Assets</td>
<td>0.041</td>
<td>0.167</td>
<td>0.443</td>
<td>0.857</td>
<td>1.162</td>
<td>1.159</td>
<td>0.958</td>
</tr>
<tr>
<td>Risky Assets/Cash-on-Hand</td>
<td>0.016</td>
<td>0.043</td>
<td>0.080</td>
<td>0.129</td>
<td>0.176</td>
<td>0.182</td>
<td>0.158</td>
</tr>
<tr>
<td>Riskless Assets</td>
<td>0.097</td>
<td>0.230</td>
<td>0.353</td>
<td>0.487</td>
<td>0.646</td>
<td>0.757</td>
<td>0.779</td>
</tr>
<tr>
<td>Riskless Assets/Cash-on-Hand</td>
<td>0.057</td>
<td>0.073</td>
<td>0.085</td>
<td>0.101</td>
<td>0.134</td>
<td>0.177</td>
<td>0.211</td>
</tr>
<tr>
<td>Risky Asset Market Part. Rate</td>
<td>0.112</td>
<td>0.247</td>
<td>0.307</td>
<td>0.381</td>
<td>0.436</td>
<td>0.391</td>
<td>0.331</td>
</tr>
<tr>
<td>House Value</td>
<td>0.722</td>
<td>2.050</td>
<td>2.834</td>
<td>3.340</td>
<td>3.426</td>
<td>2.986</td>
<td>2.423</td>
</tr>
<tr>
<td>House Ownership Rate</td>
<td>0.281</td>
<td>0.569</td>
<td>0.693</td>
<td>0.784</td>
<td>0.838</td>
<td>0.816</td>
<td>0.717</td>
</tr>
</tbody>
</table>
The model predicts that investment in the riskless asset market, which seems to be negatively correlated with the investment in the risky asset market, follows a different path. People begin to invest in the riskless assets market in their 20s until accumulating enough wealth to pay the fixed entry cost to the risky asset market. Once they pay the entry cost, then they don't make investment in the riskless asset market anymore. Hence the investment in this market decreases due to the investments in the risky assets market. After 60s, as people retire and as they put less weight on future utilities, they begin to invest again in the riskless asset market. Hence as the share of the riskless assets in cash-on-hand increases, the share of the risky assets decreases.

The fourth row of Table 1.3 shows the share of investment in the riskless assets market to total cash-on-hand for the PSID data. Data show that the US households invest less than 10% of their cash-on-hand in the riskless assets until 50s. The model also predicts similar results. However, while the data show that people increase this investment gradually until their 90s, the model predicts that they invest relatively little in this market after age 50.

The predictions of the model aren't good enough in matching the risky asset market participation rates. This is actually one of the biggest problems faced in the portfolio allocation literature. Campbell et al. (2001), Gomes and Michaelides (2005), and Cocco et al. (2005) face the same issue. While the model predicts that more than 90% of households invest in the risky assets market between their 30s to 70s, the data show that less than half of the population invest in this market. Furthermore, the model also predicts that about 60% of the people invest during their 70s while the data show that this participation ratio is about 40% during their 70s.

In this intermediate model, I won't compare the results for housing investment and house ownership rate because of the assumption that people stay in their initially bought house permanently. I will compare the predictions of the model to the data when I solve the comprehensive model in Chapter 2, where people are allowed to move endogenously when it is optimal to do so.

Next, I will compare the results of the model to the same model that ignores housing (no-housing model) but with entry cost and the same model that ignores entry cost to the risky
asset market (no-entry-cost model) but has housing investment. Table 1.4 presents the life cycle asset allocations for the no-housing model. The last two rows are interesting for comparison to the baseline model. The ratio of the risky assets to the cash-on-hand is always higher in the no-housing model than the housing model. During the first ten years in the baseline model, the ratio of risky assets to cash-on-hand is less than 1%, between age of 30 to 40 this ratio increases to 24% and in the next ten years it is 31%. For the no-housing model the same ratios are 2.5%, 30%, and 44% respectively. Figure 1.3 also depicts the share of risky assets in cash-on-hand for the baseline model, no-housing model, and the no-entry-cost model. This share is always greater in the no-housing model than the baseline model. This shows very clearly the crowding out effect of housing investment on the investment in risky assets as claimed by Cocco (2004), Hu (2005), Yao and Zhang (2005), and Fratantoni (1998). In the no-housing model, more investment is done in the risky assets in the early periods which leads to higher levels of cash-on-hand in these periods. These cash-on-hands are then used for consumption and more investment in the risky assets in the subsequent periods. Therefore, due to the investment in the risky asset market in the early periods, people in the no-housing model always have higher levels of consumption, cash-on-hand, and investment in the risky asset market during their life cycle.
Table 1.4: Life Cycle Asset Allocation of Households with no Housing Investment

<table>
<thead>
<tr>
<th>Age</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>70-80</th>
<th>80-90</th>
<th>90-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>0.700</td>
<td>1.124</td>
<td>1.208</td>
<td>1.288</td>
<td>1.319</td>
<td>1.011</td>
<td>0.696</td>
<td>0.387</td>
</tr>
<tr>
<td>Labor Income</td>
<td>0.744</td>
<td>1.246</td>
<td>1.263</td>
<td>1.327</td>
<td>1.298</td>
<td>1.050</td>
<td>0.733</td>
<td>0.416</td>
</tr>
<tr>
<td>Risky Assets</td>
<td>0.028</td>
<td>0.552</td>
<td>1.007</td>
<td>0.917</td>
<td>0.434</td>
<td>0.054</td>
<td>0.063</td>
<td>0.001</td>
</tr>
<tr>
<td>Riskless Assets</td>
<td>0.022</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.011</td>
<td>0.001</td>
</tr>
<tr>
<td>Cash-on-Hand</td>
<td>0.748</td>
<td>1.637</td>
<td>2.158</td>
<td>2.133</td>
<td>1.741</td>
<td>1.065</td>
<td>0.767</td>
<td>0.389</td>
</tr>
<tr>
<td>Risky Asset Market Part. Rate</td>
<td>0.284</td>
<td>0.999</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.990</td>
<td>0.115</td>
</tr>
<tr>
<td>Risky Assets/Cash-on-Hand</td>
<td>0.026</td>
<td>0.306</td>
<td>0.441</td>
<td>0.404</td>
<td>0.201</td>
<td>0.049</td>
<td>0.077</td>
<td>0.001</td>
</tr>
<tr>
<td>Riskless Assets/Cash-on-Hand</td>
<td>0.027</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
<td>0.016</td>
<td>0.002</td>
</tr>
</tbody>
</table>
One drawback of this model is that it assumes that once people buy a house they stay in that house permanently. In Chapter 2, I will incorporate people’s decision to buy or move to a new house in each period. Then, once people accumulate enough wealth from their financial investments, they can buy or move to a bigger house. As a result, this will have a negative effect on the level of investment in the risky asset market. Therefore, if all features of the housing investment are included, the effects of housing on the investment in financial assets, especially the risky assets, should be greater.

Last, I compare the effects of the fixed entry cost to the risky asset market in this life cycle asset allocation model. If there is no entry cost (i.e. no-entry-cost model), people invest more than 3% of their cash-on-hand in the risky asset market during the first ten years whereas this is less than 1% if entry cost is present. The investment in the risky assets in other periods is just slightly higher in the no-entry-cost-model than the baseline model. So the crowding out effect of housing on the investment in risky asset market is stronger than the crowding out effect of the entry cost. Since I used 2% of annual labor income as the entry cost, as estimated by Alan (2006), I suspected this small entry cost can be the reason of the minimal effects of the
entry cost on the investment in the risky assets. Then I increased this entry cost to 5 percent of annual labor income which is similar to the calibrated value in Gomes and Michaelides (2005) and Gomes and Michaelides (2007). As in the previous case, during the first ten years people don’t invest in the risky asset market due to the entry cost. Surprisingly, contrary to the low entry cost case (i.e. 2 percent of labor income) people make more investment in risky assets in the presence of 5% entry cost than they do in the no-entry-cost-model (see Table 1.6). This surprising result shows that the results of the model are sensitive to the value of entry cost.
<table>
<thead>
<tr>
<th>Age</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>70-80</th>
<th>80-90</th>
<th>90-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>0.537</td>
<td>1.036</td>
<td>1.100</td>
<td>1.187</td>
<td>1.182</td>
<td>0.924</td>
<td>0.608</td>
<td>0.299</td>
</tr>
<tr>
<td>Labor Income</td>
<td>0.744</td>
<td>1.246</td>
<td>1.263</td>
<td>1.327</td>
<td>1.298</td>
<td>1.050</td>
<td>0.733</td>
<td>0.416</td>
</tr>
<tr>
<td>Risky Assets</td>
<td>0.028</td>
<td>0.406</td>
<td>0.632</td>
<td>0.377</td>
<td>0.125</td>
<td>0.017</td>
<td>0.016</td>
<td>0.000</td>
</tr>
<tr>
<td>Riskless Assets</td>
<td>0.049</td>
<td>0.014</td>
<td>0.003</td>
<td>0.013</td>
<td>0.039</td>
<td>0.077</td>
<td>0.080</td>
<td>0.011</td>
</tr>
<tr>
<td>Cash-on-Hand</td>
<td>0.614</td>
<td>1.456</td>
<td>1.736</td>
<td>1.576</td>
<td>1.346</td>
<td>1.018</td>
<td>0.704</td>
<td>0.310</td>
</tr>
<tr>
<td>Risky Asset Market Part. Rate</td>
<td>0.492</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.954</td>
<td>0.620</td>
<td>0.473</td>
<td>0.001</td>
</tr>
<tr>
<td>House Size</td>
<td>2.246</td>
<td>2.041</td>
<td>2.041</td>
<td>2.041</td>
<td>2.041</td>
<td>2.041</td>
<td>2.041</td>
<td>2.041</td>
</tr>
<tr>
<td>House Ownership Rate</td>
<td>0.743</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Risky Assets/Cash-on-Hand</td>
<td>0.033</td>
<td>0.257</td>
<td>0.342</td>
<td>0.221</td>
<td>0.084</td>
<td>0.016</td>
<td>0.021</td>
<td>0.000</td>
</tr>
<tr>
<td>Riskless Assets/Cash-on-Hand</td>
<td>0.064</td>
<td>0.012</td>
<td>0.003</td>
<td>0.010</td>
<td>0.031</td>
<td>0.077</td>
<td>0.118</td>
<td>0.023</td>
</tr>
</tbody>
</table>
Table 1.6: Life Cycle Asset Allocation of Households with Housing (fixed 0.05)

<table>
<thead>
<tr>
<th>Age</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>70-80</th>
<th>80-90</th>
<th>90-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>0.524</td>
<td>1.037</td>
<td>1.116</td>
<td>1.202</td>
<td>1.167</td>
<td>0.911</td>
<td>0.596</td>
<td>0.287</td>
</tr>
<tr>
<td>Labor Income</td>
<td>0.744</td>
<td>1.246</td>
<td>1.263</td>
<td>1.327</td>
<td>1.298</td>
<td>1.050</td>
<td>0.733</td>
<td>0.416</td>
</tr>
<tr>
<td>Risky Assets</td>
<td>0.007</td>
<td>0.507</td>
<td>0.897</td>
<td>0.640</td>
<td>0.125</td>
<td>0.017</td>
<td>0.011</td>
<td>0.000</td>
</tr>
<tr>
<td>Riskless Assets</td>
<td>0.059</td>
<td>0.022</td>
<td>0.003</td>
<td>0.010</td>
<td>0.041</td>
<td>0.076</td>
<td>0.080</td>
<td>0.009</td>
</tr>
<tr>
<td>Cash-on-Hand</td>
<td>0.590</td>
<td>1.479</td>
<td>1.884</td>
<td>1.763</td>
<td>1.332</td>
<td>1.004</td>
<td>0.688</td>
<td>0.296</td>
</tr>
<tr>
<td>Risky Asset Market Part. Rate</td>
<td>0.065</td>
<td>0.892</td>
<td>0.999</td>
<td>0.998</td>
<td>0.946</td>
<td>0.613</td>
<td>0.330</td>
<td>0.008</td>
</tr>
<tr>
<td>House Size</td>
<td>2.247</td>
<td>2.017</td>
<td>2.017</td>
<td>2.017</td>
<td>2.017</td>
<td>2.017</td>
<td>2.017</td>
<td>2.017</td>
</tr>
<tr>
<td>House Ownership Rate</td>
<td>0.725</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Risky Assets/Cash-on-Hand</td>
<td>0.006</td>
<td>0.299</td>
<td>0.435</td>
<td>0.314</td>
<td>0.079</td>
<td>0.015</td>
<td>0.014</td>
<td>0.001</td>
</tr>
<tr>
<td>Riskless Assets/Cash-on-Hand</td>
<td>0.074</td>
<td>0.020</td>
<td>0.003</td>
<td>0.007</td>
<td>0.034</td>
<td>0.078</td>
<td>0.121</td>
<td>0.018</td>
</tr>
</tbody>
</table>
1.7 Summary

Housing investment plays a key role in portfolio allocation in life cycle models. While it doesn’t incorporate all features of housing, the purpose of this chapter is to build an intermediate level housing asset allocation model to analyze the effects of housing investment on the decision of households on their portfolio allocations during their life cycle. Housing in this model is both a durable consumption good from which households derive utility and an investment asset from which they get returns. The model shows that housing investment does affect households’ portfolio allocation decision.

Due to the importance of relative risk aversion and elasticity of intertemporal substitution, I estimated these parameters through indirect inference. The estimated parameter for EIS is within the generally accepted interval, whereas the estimated parameter for RRA is lower than the usually accepted range. Using these parameters, and calibrating the others, I solved and simulated the model. The results showed that housing has a crowding out effect on investment in the risky asset market. The strong desire to become homeowners leads agents to buy a house in the first five years. Because they pay the downpayment and begin to pay interest payments on mortgage debt, they don’t leave enough wealth to pay the fixed cost and to have investments in the risky asset market during first ten years. Once they accumulate enough wealth, then they pay this entry cost and invest in this market. The risky assets investment follows a hump-shape pattern. Between their 30s to early 40s people invest heavily in the risky asset market, from mid 40s to late 60s we see a decline in the amount of this investment, and after late 60s it closes to zero. Although the riskless assets investment is always at low levels, it is high only when the risky assets investment is at low levels.

My model is partially successful in matching the asset allocation profiles in the simulated data to the observed values. The investment in the risky assets market during the life cycle is almost always less than 40% of cash-on-hand. This ratio is always less than 20% in the data. The model captures the fact that the investment in the riskless assets is less than 10% of total cash-on-hand until their 50s. One major problem with the model is the risky asset market
participation rate. The predictions of the model are too high compared to the data.

The effects of housing on the portfolio allocation decision of households is stronger than the effects of the risky asset market entry cost. Housing investment has a clear crowding out effect on the investment in the risky assets whereas the crowding out effect of the entry cost is relatively small. Increasing the size of the entry cost from 2% to 5% of annual labor income has a surprisingly opposite effect. Except for the first 10 years, people invest more in the risky asset market in the case of a high entry cost relative to no-entry-cost model. Because of the ambiguous results stemming from the entry cost, it is worth estimating the size of the entry cost in future work.

The model is an intermediate level model. To be able to show the ultimate effects of housing investment on the portfolio allocation, future work should incorporate a comprehensive housing investment into life-cycle models. Among these features are (1) allowing people to buy or move to a house at any time when it is optimal (2) introducing a transaction cost of selling their current house and buying a new one (3) specifying a stochastic return on housing, and (4) allowing for the correlation between house price and the risky asset market return and labor income. The most important effect will come from the first feature because, due to the liquidity constraint, people prefer to have a small house in early periods. Using the accumulated wealth from the investments made in both risky and riskless assets in subsequent periods, people can move to larger houses. This, then, causes a further reduction in the amount of wealth in the risky asset market.

The other features that should be included to life-cycle models are as follows. First, labor income should have both temporary and permanent shocks as in Guvenen (2009a), Cocco et al. (2005) as well as many life cycle portfolio allocation models. Furthermore, labor income shocks has correlation between shocks to return on housing market and shocks to return on the risky asset markets. Second, since different values of the entry cost give different portfolio allocation profiles, an estimation for the entry cost is important in order to obtain reliable effects of the entry cost to investment in the risky assets market. The fact that not everyone lives for 100
years needs to be included into the model by including survival probabilities of households.
Chapter 2

Housing Investment and Portfolio Allocation

2.1 Introduction

The first chapter has introduced housing investment into a life-cycle model with some limitations. In this chapter, however, I relax some of these limitations in order to have a more realistic life cycle portfolio allocation model. Households are no longer forced to stay in an initially-bought house permanently. They endogenously make homeownership decision in each period. Homeowners make principal and interest mortgage payments until their loans vanishes. Contrary to the previous model, households exogeneously retire at age 65. They receive a constant fraction of their last working year labor income (i.e. replacement ratio).

This comprehensive model is able to explain the aforementioned empirical observations stated in the introduction of the first chapter. We obtain a low stock market participation rate as well as moderate equity holding among the stock market participants. Hence, housing investment, incorporated into a life-cycle asset allocation model has strong crowding out effects on investment in the risky asset. The effects are more significant for young and middle-aged households. The main contribution of the model for successfully addressing the empirical ob-
servations comes from the existence of housing investment. House price risk, liquidation cost of selling a house, and a high down-payment rate also have some sizable effects on households' portfolio allocation. One can argue that it would be optimal to invest remaining wealth in the risky assets after households make housing investment and consumption decisions. However, there are some reasons that keep households away from investing all remaining wealth only in the risky asset. One reason is precautionary saving which leads young households to invest in the riskless asset in order to hedge against both labor income shocks and shocks to the return on the risky asset. This is especially true when a positive correlation exists between labor income shocks and shocks to the return on the risky asset. Another reason stems from using EZ type preferences and estimated high values of $\text{RRA}(\gamma)$ and $\text{EIS}(\psi)$. The estimated parameters are 8.9 for RRA and 1.1 for EIS. A high value of RRA brings about highly risk-averse households that prefer to have some investment in the riskless asset. Furthermore households prefer early resolution of uncertainty since the estimated value of $\text{RRA} \gamma$ is greater than the inverse of the EIS. Finally, a positive correlation between shocks to the return on the risky asset and shocks to the return on housing investment encourages households to have some investment in the riskless asset as a hedge against negative shocks to the returns on these investments.

The other results are as follows. The model over-estimates the life-cycle homeownership rates to some extent. While PSID data show that about three-quarters of US households are homeowners after age 40, the model predictions estimate this rate above 80% and close to 90% after age 30. The model is also able to explain that homeowners, by and large, have more investment in the risky asset than renters. Similarly, the stock market participation rates among homeowners are significantly higher than the participation rates among renters. We find that the entry cost has insignificant effects on households’ portfolio allocation decision.

The rest of the paper is organized as follows. Section 2.2 presents the model with all features and defines the optimization problem. Parameters of the model and the solution technique constitute Section 2.3. I present the indirect inference estimation and the required data for the estimation in Section 2.4. Section 2.5 presents the results of the model, comparative static
analysis, and comparison of the model results to the data. Section 2.6 provides concluding
remarks.

2.2 Model

2.2.1 Household Preferences

This model is a discrete time life-cycle model where each period corresponds to one year. As a
general convention in the life-cycle literature each year is actually the real age of a household
minus 19. I assume that households live for at most $T$ periods. The probability that a household
is alive at time $t$ conditional on being alive at time $t-1$ is $q_t$.

In each period, a household derives utility from a constant elasticity of substitution (CES)
utility function with nondurable consumption goods, $C$, and housing investment (or consump-
tion), $H$. Preferences are in the form of Epstein-Zin, where the relative risk aversion (RRA) is
disentangled from the elasticity of intertemporal substitution (EIS):

$$V_t = \left\{ u(C_t, H_t)^{\frac{1-\gamma}{\psi}} + \beta \left( E_t \left[ q_t V_{t+1}^{1-\gamma} + (1-q_t) W_{t+1}^{1-\gamma} \right] \right) \right\}^\frac{1}{1-\gamma}$$

$$\theta = \frac{1-\gamma}{1-1/\psi}$$

$$u(C_t, H_t) = [C_t^\psi + H_t^\psi]^{\frac{1}{\psi}},$$

where $\beta$ is the time discount factor, $\gamma$ is the RRA parameter, and $\psi$ is the EIS parameter. The
intratemporal elasticity of substitution between nondurable consumption goods and housing
investment is $1/(1-\nu)$. $W_{t+1}$ is the total wealth that a household would bequest if he passes
away at time $t$ with probability $1-q_t$.

2.2.2 Labor Income Process

Following the standard specifications in the life-cycle literature, labor income depends on the
age-specific profiles of households with an idiosyncratic random shock. During the working, life
households supply labor inelastically in each period and receive stochastic labor income $Y_{it}$. Define $y_{it} = \log(Y_{it})$ where $y_{it}$ has the following process:

$$
y_{it} = f(t, Z_{it}) + \varepsilon_{it}^l$$

(2.4)

$$
f(t, Z_{it}) = \beta_1 age + \beta_2 age^2 + \beta_3 gender + \beta_4 marital\_status + \beta_5 educ,$$

(2.5)

where $Y_{it}$ is labor income received by household $i$ at time $t$, $f(t, Z_{it})$ is a deterministic function of age, $t$ and household characteristics, $Z_{it}$ (education, marital status, and gender). The random shock to log labor income $\varepsilon_{it}^l$ are i.i.d. random shocks with $N(0, \sigma_{it}^2)$. It is quite common in the life-cycle literature to decompose the shock to labor income into an aggregate and an idiosyncratic part. However due to the computational burden of having one more state variable, I assume only one idiosyncratic shock. Since the focus of the paper is on the effects of housing investment on portfolio allocation, I prefer to have a more realistic housing notion in my model than having a more sophisticated labor income process.

For simplicity, I assume that the retirement age is deterministic. Households work until period $K$ where $K$ corresponds to an age of 65 ($K = 46$). During the retirement periods ($t > K$), households receive a constant and deterministic labor income $Y_{it} = \xi Y_{iK}$ where $0 < \xi < 1$ (a fraction of their income during the last year before retiring).

### 2.2.3 Housing Investment

Households enter the market as renters in the first period. From the second period on, they endogenously decide to buy a house and become homeowners or stay in their current house as renters. A typical homeowner at any time can endogenously decide either (1) to stay in his current house, or (2) to sell his current house and buy a new one, or (3) to sell his house and rent a new one. Similarly, a renter can decide either (1) to stay in current rented house, or (2) to rent a new house, or (3) to buy a new house and become a homeowner. Buying a house requires paying proportion $d$ of the house market value as down payment and financing the rest through
a mortgage. To capture the illiquidity of housing investment, households incur a liquidation cost equal to a fraction $\kappa$ of the house market value. Homeowners pay an amount equal to a proportion $\delta$ of the house market value for house maintenance and depreciation expenses in each period. Renters, on the other hand, don’t pay for maintenance and depreciation costs; they only pay annual rent which is equal to a proportion $\alpha$ of the house market value. House-related expenses are calculated as certain proportions of house market value in the literature because it is easy in this way to incorporate them into the model and reduces the computational burden.

Per unit price of housing is denoted by $P^h_t$, such that a house of size $H_i$ has price $P^h_t H_i$ at time $t$. Define $\mu_h$ as mean log of house price growth and define $p^h_t$ as log house price $P^h_t$, then the log house price evolves over time with the following stochastic process:

$$ \Delta p^h_t = \mu_h + \varepsilon^h_t, \quad \varepsilon^h_t \sim N(0, \sigma^2_{\varepsilon^h}). $$

(2.6)

When a household buys a house of size $H_i$ at time $t$, he pays the down payment which is equal to $dP^h_t H_i$ and finances the rest through a mortgage with a fixed rate $R^m$. Denote $RM_t$ as the amount of mortgage debt a household has at time $t$. Hence households may borrow up to the house value minus the down payment:

$$ RM_t \leq (1 - d) P^h_t H_i, $$

(2.7)

2.2.4 Financial Assets and Wealth Accumulation

There are only two financial assets that households can invest in: a risky asset and a riskless asset. Return on investment in the risky asset follows the following stochastic process as in Campbell et al. (2001), Cocco (2004), and others:

$$ R^e_{t+1} - R^b = \mu_s + \varepsilon^s_{t+1}, $$

(2.8)
where $R_{t+1}^s$ is the gross return on the risky asset at time $t+1$, $R^b$ is the constant gross return on the riskless asset, $\mu_s$ is the excess return from the risky asset over the riskless one (equity premium), and $\varepsilon^s_{t+1}$ is i.i.d. random shock with $N(0, \sigma^2_{\varepsilon^s})$. In order to invest in the risky asset, households are required to pay a one-time fixed entry cost for the investment in the risky asset. This entry cost can be considered as the cost of opening a brokerage account, understanding how the market works, the cost of acquiring and evaluating information about the risky asset, and the opportunity cost of time. Alan (2006) estimates this cost as approximately 2 percent of annualized labor income, whereas Gomes and Michaelides (2007) calibrate that this fixed cost is about 6 percent of annualized labor income. Unlike the risky asset, there is no cost for investing in the riskless asset.

**Renter’s Wealth Accumulation**

In this model, there are two types of households: renters and homeowners. Depending on the homeownership status households have different budget constraints and cash-on-hand structures. The liquid wealth of a typical household who was a renter at time $t-1$ is the sum of investments made at time $t-1$ in both risky and riskless assets.

$$LW_t = R^b B_{t-1} + R^s S_{t-1}.$$  \hspace{1cm} (2.9)

I refer to the sum of liquid wealth and labor income at time $t$ as cash-on-hand. At time $t$, cash-on-hand is $X_t \equiv LW_t + Y_t$. A typical renter uses this cash-on-hand to decide whether to buy a house, pay the down payment, and begin to pay annual mortgage payments or just pay annual rent and stay in a rental property. He then decides on consumption expenditure, whether to pay the fixed entry cost for investment in the risky asset (if it has not been paid yet), and decides on portfolio composition among different financial assets:

$$X_t = C_t + S_t + B_t + FIX_t FY_t + (1 - HR_t) \left[ \alpha P^b_t H_t \right] + HR_t \left[ M_t + dP^b_t H_t \right].$$  \hspace{1cm} (2.10)
where \( S_t \) is the total investment made in the risky asset and \( B_t \) is the total investment made in the riskless asset at time \( t \). \( FIX_t \) is a dummy variable that takes the value of 1 if a renter pays the fixed entry cost at time \( t \), and 0 otherwise. \( F_t \) is the fixed entry cost which is a constant proportion of annualized labor income at time \( t \). \( HR_t \) is a dummy variable that takes the value of 1 if a renter decides to buy a house and become a homeowner and it is 0 otherwise. \( M_t \) is households’ annual mortgage payment at time \( t \).

**Homeowner’s Wealth Accumulation**

The liquid wealth structure of a typical homeowner at time \( t \) is the same as the liquid wealth structure of a renter at time \( t \):

\[
LW_t = R^b B_{t-1} + R^s S_{t-1}.
\]

(2.11)

Similarly, the cash-on-hand of homeowners at time \( t \) is the sum of liquid wealth and labor income \( X_t = LW_t + Y_t \). A typical homeowner decides in every period whether to stay in his current house or sell the current house and buy a new one or sell the current house and move to a rental property. Once he sells his house, his total cash-on-hand would be the sum of the amount received from the house sale minus the liquidation cost plus his cash-on-hand. He then would use this cash-on-hand either to buy a new house (upgrade or downgrade) by paying the down payment or move to a rental property by only paying rent. He would then decide on consumption expenditure and investment in financial assets using the remaining cash-on-hand. On the other hand, if he decides to stay in the current house, then he would pay the annual mortgage payment, and use the remaining cash-on-hand for consumption expenditure and investments in financial assets. Furthermore, households pay for the maintenance and depreciation expense which is equal to proportion \( \delta \) of house market value. Then the budget constraint of households takes the following form:

\[
X_t = C_t + S_t + B_t + FIX_t F_t + HR_t (1 - MS_t) \left[ M_t + \delta P^h_t H_{t-1} \right] +
\]

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$$HR_t MS_t \left[ dP^h_t H_t + M_t + \delta P^h_t H_{t-1} - \left( (1 - \kappa) P^h_t H_{t-1} - RM_t \right) \right] +$$

$$(1 - HR_t) MS_t \left[ \alpha P^h_t H_t - \left( (1 - \kappa) P^h_t H_{t-1} - RM_t \right) \right], \quad (2.12)$$

where $MS_t$ is a dummy variable that takes the value of 1 if a homeowner moves to a new house or rental property and 0 if he stays in his current house. The term $(1 - \kappa) P^h_t H_{t-1}$ denotes the amount a homeowner receives if he sells his house, adjusted for the liquidation cost. $RM_t$ is the total mortgage debt a homeowner has at time $t$ and $M_t$ is the annual mortgage payment, which consists of both the annual interest payment $MI_t$ and the annual principal payment $MP_t$. The mortgage debt and annual mortgage payments have the following processes.

$$M_t = MP_t + MI_t \quad (2.13)$$

$$RM_t = RM_{t-1} - MP_t. \quad (2.14)$$

Households are assumed to satisfy the non-negativity constraints on consumption and housing investment and short-sale constraints on the risky and the riskless assets. They cannot have negative amounts of consumption and housing investment. Furthermore, households cannot borrow at the riskless rate in order to invest in the risky asset.

$$C_t \geq 0, \ S_t \geq 0, \ B_t \geq 0, \ H_t \geq 0, \ \forall t \quad (2.15)$$

Furthermore, I define the correlations between the shocks to returns on investment in the risky asset and the shocks to labor income as $\rho_{sl}$, the shocks to returns on investment in the risky asset and the shocks to returns on housing investment as $\rho_{sh}$, and finally the shocks to returns on housing investment and the shocks to labor income as $\rho_{hl}$.

It is common in the recent life cycle portfolio allocation models to incorporate a bequest motive in order to match the skewness of the wealth distribution.\footnote{For example see Laitner (2002), Gomes and Michaelides (2005), Gourinchas and Parker (2002), and Yao and Zhang (2005).} Conditional on being alive
at time \( t - 1 \), a household could pass away with probability \( 1 - q_t \). I assume that households bequest their total liquid wealth and all investment made in housing to their inheritors at any period they pass away.

\[
W_t = R^b_t B_{t-1} + R^s_t S_{t-1} + P^b_t H_{t-1} - RM_{t-1} \tag{2.16}
\]

### 2.2.5 Optimization Problem

Before defining the optimization problem and the value function, I list the state and the control variables in a compact form. The state variables are age \( (t) \), liquid wealth \( (LW_t) \), risky asset participation status \( (IFIX_t) \), homeownership status \( (O_t) \), the size of house owned from previous period \( (H_{t-1}) \). I denote the state variables by \( \Omega \), where \( \Omega_t = \{t, LW_t, IFIX_t, O_t, H_{t-1}\} \). The control variables are consumption \( (C_t) \), the size of house households choose at time \( t \) \( (H_t) \), the risky asset participation decision \( (FIX_t) \), and the share of financial investment in the risky asset \( (s_t = \{C_t, H_t, FIX_t, s_t\}) \). The household’s optimization problem then is maximizing

\[
V_t(\Omega_t) = \max_{\Psi_t} \left\{ u(C_t, H_t) \frac{1}{\delta} + \beta \left( E_t \left[ q_t V_{t+1} (\Omega_{t+1})^{1-\gamma} + (1 - q_t) W_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\gamma}} \right\}^{\frac{\delta}{1-\gamma}}, \tag{2.17}
\]

subject to Equations 2.1 to 2.16.

### 2.3 Parameterization

Tables 2.1 and 2.2 display the list of parameters that are calibrated and their values. Each period in the model corresponds to one year. Households are assumed to enter the market at age of 20 and live for at most 80 years. The age-dependent labor income process is based on PSID data from 1968-2007. In order to obtain a random sample of US households, I dropped families that were part of the Survey of Economic Opportunities\(^1\). I treat retirement period

---

\(^1\)The original PSID sample was drawn from two independent sources: an over-sample of roughly 2000 poor families selected from the Survey of Economic Opportunities (SEO), and a nationally-representative sample of roughly 3000 families from all states. Since the SEO is over-sample of low income families, it isn’t a random
Table 2.1: Correlations Between Shocks from Different Datasets

<table>
<thead>
<tr>
<th></th>
<th>PSID&lt;sup&gt;a&lt;/sup&gt;</th>
<th>NIPA&lt;sup&gt;b&lt;/sup&gt;</th>
<th>Case-Shiller&lt;sup&gt;c&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{sh}$ (risky asset and housing)</td>
<td>0.019</td>
<td>0.2385</td>
<td>0.0419</td>
</tr>
<tr>
<td>$\rho_{sl}$ (risky asset and labor income)</td>
<td>0.0241</td>
<td>0.1425</td>
<td></td>
</tr>
<tr>
<td>$\rho_{hl}$ (housing and labor income)</td>
<td>0.0587</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup> I use PSID data from 1999 to 2007 for all correlations in this column.

<sup>b</sup> I obtained labor income from National Income and Products Accounts’s (NIPA) Personal Income and Outlays. I choose S&P500 for return on investment in the risky assets. Finally, I use Census Bureau’s Average Price of Houses Sold for housing investment. All datasets cover the period from 1968 to 2007.

<sup>c</sup> I obtained housing related data from Case-Shiller Index and risky assets investment data from S&P500 Index.

labor income as a constant fraction of labor income of the last working period as in Gomes and Michaelides (2005) and Cocco (2004). The permanent component of labor income is a function of households’ age and age squared, and dummy variables for education (1 if the head of the household has at least 12 years of education, 0 if not), marital status (1 if married and 0 if not), and gender (1 if male and zero if female), (see Equation 2.5). The dummy variable for education is 1 for households with 12 or more years of education and 0 otherwise. I define labor income widely enough to account for endogenous ways of self insurance against labor income shocks. Labor income includes total labor income, unemployment compensation, social security, total transfers, child support, and other welfare of both the head of the family and his wife if present. Due to the computational burden I assume only one idiosyncratic transitory shock to labor income. Similar to Gourinchas and Parker (2002) and Gomes and Michaelides (2005), I set the standard deviation of this shock at 15% per year.

I calibrate several parameters related to housing. Yao and Zhang (2005) and Hu (2005) set the required down payment rate at 20% of the house value while Cocco (2004) sets it at 15%. I set the down payment rate as 20% of the house market value. To be consistent with previous studies, I set the annual rental rate of houses at 7% of the market value of the rental property,
Table 2.2: Baseline Parameter Values

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time discount factor</td>
<td>$\beta$</td>
<td>0.95</td>
</tr>
<tr>
<td>Gross return on riskless asset inv.</td>
<td>$R^b$</td>
<td>1.03</td>
</tr>
<tr>
<td>Equity premium</td>
<td>$\mu_s$</td>
<td>0.045</td>
</tr>
<tr>
<td>Liquidation cost</td>
<td>$\kappa$</td>
<td>0.10</td>
</tr>
<tr>
<td>Intratemporal elas. of subs. parameter</td>
<td>$\nu$</td>
<td>0.2</td>
</tr>
<tr>
<td>Rent</td>
<td>$\alpha$</td>
<td>0.07</td>
</tr>
<tr>
<td>Mortgage rate</td>
<td>$R^m$</td>
<td>0.04</td>
</tr>
<tr>
<td>Fixed entry cost</td>
<td>$F$</td>
<td>0.05</td>
</tr>
<tr>
<td>Down payment</td>
<td>$d$</td>
<td>0.20</td>
</tr>
<tr>
<td>Depr. maintenance</td>
<td>$\delta$</td>
<td>0.01</td>
</tr>
<tr>
<td>Mean return on housing inv.</td>
<td>$\mu_h$</td>
<td>0.019</td>
</tr>
<tr>
<td>Std. of shocks to labor income</td>
<td>$\sigma_{\varepsilon l}$</td>
<td>0.15</td>
</tr>
<tr>
<td>Std. of shocks to return on housing inv.</td>
<td>$\sigma_{\varepsilon h}$</td>
<td>0.057</td>
</tr>
<tr>
<td>Std. of shocks to return on risky asset inv.</td>
<td>$\sigma_{\varepsilon s}$</td>
<td>0.20</td>
</tr>
<tr>
<td>Retirement income factor</td>
<td>$\xi$</td>
<td>0.66</td>
</tr>
</tbody>
</table>

and annual maintenance and depreciation cost at 1% of the house market value. I set $v = 0.2$ so that the intratemporal elasticity of substitution between nondurable consumption and housing, $1/(1 - v)$, is 1.25 as in Piazzesi et al. (2007).

The log real return on housing investment has a constant mean and a stochastic process. Cocco (2004) uses self-assessed house values from the PSID data from 1970-1992 to construct a house price index. He calculates an average annual return of 1.6% on housing investment. However, one should be sceptical about the house price in the PSID data for at least two reasons. First, house prices in PSID data are self-assessed values. Second, while earlier PSID surveys were conducted every year, later surveys are conducted every two years. Thus, a house price index obtained from PSID data may not give sufficient information about most recent year(s). In order to address this issue I follow the same method that Cocco (2004) employed and I calculated a house price index from PSID data from all waves (i.e. 1968-2007). Then I compared this house price index to the commonly used Case-Shiller index. Figure 2.1 shows
both house price indices. The index obtained from PSID falls behind the Case-Shiller index. It also shows sharp fluctuations after the mid 1980s. Annual average real return on housing investment is 1.9% for the Case-Shiller index with a standard deviation of 5.72. However the PSID data produce annual real housing returns of 1.6% with a standard deviation of 8.33%. In the baseline case, I use the Case-Shiller index with annual mean return on housing investment $\mu_h = 0.019$ and $\sigma_{e,h} = 0.0572$.

![Figure 2.1: Growth Rate of House Price Indices](image)

I set the time discount factor $\beta$ to 0.95. The real risk free rate $R^b$ is 3%. I set the annual mean return on the risky asset at 7.5% which means a 4.5% equity premium, $\mu_s$. The standard deviation of shocks to the return on the risky asset is 20%. There is no consensus in the literature on the size of correlations between shocks to the returns on the investment in the risky asset, shocks to labor income, and shocks to the return on housing investment. Flavin and Yamashita (2002) use PSID data and find that there is almost no correlation between return on investment in the risky asset and the return on housing investment $\rho_{sh}$. Although the authors find that the city-level Case-Shiller index conflicts with PSID results, they claim that the PSID data are
nationwide while city level data aren’t. Cocco (2004), Hu (2005), and Yao and Zhang (2005) assume \( \rho_{sh} \) is 0. Furthermore, the correlation between shocks to labor income and shocks to the return on investment in the risky asset \( \rho_{sl} \), is set to zero in Cocco (2004), Hu (2005), and Yao and Zhang (2005). However, Cocco (2004) sets the correlation between shocks to labor income and shocks to the return on housing investment (\( \rho_{hl} \)) to 0.553, Hu (2005) sets it to 0, and finally Yao and Zhang (2005) use 0.2. I use different datasets in order to find good and reliable values for these correlations. Table 2.1 displays these correlations for each datasets. From these different results, I set \( \rho_{sh} \) to 0.2385, \( \rho_{sl} \) to 0.20, and \( \rho_{hl} \) to 0.1425 in the baseline model based on my judgement that they are the most trustworthy estimates. I obtained these correlations using data from National Income and Product Account (NIPA) to obtain annualized labor income, S&P500 for return on investment in the risky asset, and data from the Census Bureau’s Average Price of House Sold to obtain average annual return on housing investment.

In general, the mortgage rate is higher than the constant return on the riskless asset because mortgages bear a long-term interest rate risk and a default risk. Since my model doesn’t have interest rate risk and default risk I set the interest rate on mortgage loans at 4%. Households pay their mortgage debt over 25 years. The one-time fixed-entry cost for investment in the risky asset is 5% of annual labor income. Finally, I parameterized the conditional survival probabilities from the mortality tables produced by the National Center for Health Statistics.\(^2\)

There is no analytical solution to this problem. Therefore I use numerical approximation based on the value function iteration. The details are given in Appendix B.

### 2.4 Estimation

Similar to the previous chapter, I estimated only RRA and EIS because the structural model is time consuming to solve. The detail of the estimation procedure are discussed in the Estimation section the the previous chapter. I use a vector autoregression model of order one, VAR(1), as my auxiliary model of the indirect inference estimation to obtain parameters to be used in the

\(^2\)http://www.cdc.gov/nchs/data/nvsr/nvsr58/nvsr58_21.pdf Table 1.
minimization process. Data used for the estimation are both the observed and the simulated data on life-cycle consumption, investment in risky asset, investment in the risky asset as a share of total financial assets, and homeownership rate.

2.4.1 Data for Indirect Inference Estimation

The VAR(1) model of indirect inference estimation requires life-cycle data for consumption, investment in the risky asset, investment in the risky asset as a share of total financial assets, and homeownership status. The PSID data are too disaggregated to obtain good life-cycle consumption data. Hence, I use the Consumer Expenditure Survey (CEX) of 2008 to get life-cycle consumption data for an average US household. For other variables, I use the waves of PSID data that contain information on the financial wealth of households (i.e. the 1984, 1989, 1994, 1999, 2001, 2003, 2005, 2007 waves).

I exclude the Survey of Economic Opportunity (SEO) sample from the data to have a random sample of the US population. Total investment in risky assets in the PSID data refers to the sum of shares of stocks in publicly or privately held corporations including those held in mutual funds, investment trusts and Individual Retirement Accounts (IRAs). Total financial assets refers to total risky assets plus bonds in publicly or privately held corporations, money in saving and checking accounts, money market funds, certificates of deposits, government saving bonds, Treasury bills, other savings or assets, such as cash value in a life insurance policy and valuable collections for investment purposes. All values are adjusted to their current corresponding values using the CPI.

Data used for the estimation include only households that report all demographic information (i.e. age, gender, marital status, education background) and necessary data for the estimation. I average out the data for each age and use kernel regression to smooth out these averaged data. The details of the kernel regression are discussed in the previous chapter.
2.5 Results

The estimated values for RRA ($\gamma$) and EIS($\psi$) are 8.9 and 1.1 respectively. Although the estimated value of RRA is within the generally accepted interval, we find that households are more risk averse than what many previous studies have found. For example, the estimated RRA parameter is around 2-4 in Villaverde and Krueger (2010), Gomes and Michaelides (2005), Guo (2004) among many. The higher RRA value is an important factor for matching model-predicted life-cycle portfolio allocation profiles to their empirical counterparts. The estimated value of EIS is greater than 1, which combining with the estimated value of RRA, implies that households prefer an earlier resolution of economic uncertainty. The value of EIS in Hansen and Singleton (1982), Vissing-Jorgensen and Attanasio (2003) and Zhou (2010) is also greater than 1.

Having estimated these parameters within acceptable intervals, I solve the optimization problem once more with these parameters to obtain portfolio allocation rules. Then, using these optimal rules, I simulate corresponding life-cycle consumption, housing investment, and portfolio allocation profiles of households. Table 2.3 shows life-cycle portfolio allocation and homeownership status of households for the model with baseline parameters. The fixed cost payment rate refers to the proportion of the population that has paid the fixed entry cost for the investment in the risky asset. The participation rate refers to the share of population having investment in the risky asset.
<table>
<thead>
<tr>
<th>Age</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>70-80</th>
<th>80-90</th>
<th>90-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riskless Asset / Financial Asset</td>
<td>0.770</td>
<td>0.607</td>
<td>0.397</td>
<td>0.275</td>
<td>0.283</td>
<td>0.339</td>
<td>0.318</td>
<td>0.349</td>
</tr>
<tr>
<td>Risky Asset / Financial Asset</td>
<td>0.230</td>
<td>0.393</td>
<td>0.603</td>
<td>0.725</td>
<td>0.717</td>
<td>0.661</td>
<td>0.682</td>
<td>0.651</td>
</tr>
<tr>
<td>Fixed Cost Payment Rate</td>
<td>0.009</td>
<td>0.268</td>
<td>0.632</td>
<td>0.823</td>
<td>0.911</td>
<td>0.929</td>
<td>0.932</td>
<td>0.934</td>
</tr>
<tr>
<td>Risky Asset Participation Rate</td>
<td>0.007</td>
<td>0.235</td>
<td>0.582</td>
<td>0.766</td>
<td>0.762</td>
<td>0.497</td>
<td>0.467</td>
<td>0.472</td>
</tr>
<tr>
<td>Homeownership Rate</td>
<td>0.205</td>
<td>0.819</td>
<td>0.872</td>
<td>0.906</td>
<td>0.887</td>
<td>0.826</td>
<td>0.792</td>
<td>0.770</td>
</tr>
</tbody>
</table>
The first row of Table 2.3 shows that households hold more than half of their financial investment in the riskless asset during their first 20 years. It then decreases and stays below 40%. On the other hand, investment in the risky asset increases over time and stays above 60% of total financial assets during the rest of the life-cycle. This result is striking in the sense that the portfolio allocation literature in incomplete markets has shown that given the historically prevailing equity premium, households always invest nearly all of their financial assets in the risky asset. For example see Cocco (2004), Hu (2005), Cocco et al. (2005), and Heaton and Lucas (2000). Hence the model is successful in obtaining moderate shares of risky asset holding for US households. Furthermore, the predictions of the model are closer to the empirical evidence presented in the next section.\(^1\)

The theoretical literature on portfolio allocation composition in the presence of nontradable labor income predicts that the portfolio shares invested in the risky asset decreases over life. On the other hand, the empirical literature has found that the portfolio shares invested in risky asset increase over the lifetime. There is also some mixed evidence that the shares decrease slightly in later life. The second row of Table 2.3 shows that the share of the portfolio invested in the risky asset rapidly increases until the 60s and slightly decreases for the rest of the lifetime.

The third row of the table shows the proportion of the population that has paid the fixed entry cost. We assume that this is a one-time fixed cost. Once households pay this cost, then they can freely make investments in the risky asset without any other costs. During the first 10 years, almost no one has paid the fixed cost. The rate monotonically increases over the lifetime but never reaches 1, which means that a certain proportion of the population never pays the fixed cost and never invests in the risky asset. The fourth row shows the proportion of the population investing in the risky asset. Even though households have paid the entry cost, they can decide not to invest in the risky asset. Hence the risky asset participation rate is always below the fixed cost payment rate. The relevant indicator for our analysis is the risky asset participation rate.

\(^1\)The possibility of the household borrowing for a mortgage while still holding positive amounts of bonds could seem counterintuitive. However, we see in real world simultaneous debt and positive amount of bond holding. Furthermore, Cocco (2004), Hu (2005), and Hurst and Willen (2007) find similar results.
rate, which is the proportion of the population investing in the risky asset. The fourth row shows that about 1% of the population has investment in the risky asset during the first 10 years. This rate steadily increases during the subsequent periods and reaches 77% for people in their 60’s. It then decreases to about 47% for the rest of their lifetime. Contrary to previous life-cycle portfolio allocation studies such as Campbell et al. (2001), Gomes and Michaelides (2005), Polkovnichenko (2007), Cocco (2004), the model is able to generate a moderate level of risky asset market participation rate and moderate level of risky asset holding.

The baseline model finally shows that households have a strong preference for homeownership. While during the first 10 years, about 20% of the population own a house, the homeownership rate jumps to 82% during the next 10 years. This rate further increases to 90% in their 50’s and then begins to decrease slightly. For the age of 90 to 100, the model predicts that about 77% of the population are homeowners. As we will see in the subsequent sections, this rate is always less than 80% in the PSID data. Hence the model slightly over-estimates the homeownership rate relative to the empirical evidence.\(^1\)

Next, I compare the aforementioned model to the same model with no housing investment (NHI) in order to analyze and measure the effects of housing investment on portfolio allocation. First, it is worth making clear the main features of the NHI model. The NHI model includes all features of the main model except those related to housing investment. In the NHI model, there is no housing investment, no house price risk, no mortgage payments, no homeownership status, etc. Households pay a constant annual rent for renting. They make decisions on consumption expenditure and on investment in financial markets (riskless and risky assets). Table 2.4 shows the life-cycle portfolio allocation profiles of households for the NHI model.

\(^1\)This model doesn’t include the case of older people selling their house and moving because of health reasons.
<table>
<thead>
<tr>
<th>Age</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>70-80</th>
<th>80-90</th>
<th>90-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riskless Asset / Financial Asset</td>
<td>0.392</td>
<td>0.177</td>
<td>0.071</td>
<td>0.057</td>
<td>0.086</td>
<td>0.147</td>
<td>0.200</td>
<td>0.270</td>
</tr>
<tr>
<td>Risky Asset / Financial Asset</td>
<td>0.608</td>
<td>0.823</td>
<td>0.929</td>
<td>0.943</td>
<td>0.914</td>
<td>0.853</td>
<td>0.800</td>
<td>0.730</td>
</tr>
<tr>
<td>Fixed Cost Payment Rate</td>
<td>0.489</td>
<td>0.985</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>Risky Asset Participation Rate</td>
<td>0.405</td>
<td>0.960</td>
<td>0.999</td>
<td>0.998</td>
<td>0.979</td>
<td>0.887</td>
<td>0.816</td>
<td>0.792</td>
</tr>
</tbody>
</table>
There are significant changes in life cycle wealth allocation profiles between these two models. Housing investment has a strong crowding out effect on investment in the risky asset during the life-cycle. For instance, during the first 10 years, due to the housing investment, the portfolio share of the risky asset is only 23% while the same portfolio share is about 61% in the absence of housing investment. Similarly, the NHI model predicts that for households in their 50's about 95% of financial investment is in the risky asset while the baseline model predicts that the risky asset constitutes only 72% of financial investment. While the gap between the prediction of the models on the share of the risky asset decreases from 37% to 8% over time, the effects of housing investment never vanishes.

A similar crowding out effect is observed in the risky asset participation rate. The participation rate in the NHI model shows that between the ages of 40-70 almost all households have investment in the risky asset whereas the highest participation rate in the baseline model reaches 76% at age of 60. Figure 2.2 visually depicts the crowding out effect of housing investment on risky asset investment during the life cycle. Although the crowding out effects are observed over the life cycle, the effects are distinctive for especially young and middle-aged households.

Figure 2.2: Life-Cycle Risky Asset Investment and Participation Rate

One main reason for observing a long lasting crowding out effect is the duration of mortgage
debt. Most of the households become homeowners and begin to pay their mortgage debt between the ages of 20-30. They pay back the mortgage over 25 years. When reaching ages around 50, many households, if not all, are homeowners. Then, they can make investments in financial assets without much concern about their housing investment. However, we also should take into account the fact that households can sell their houses at any time and buy a new one or become renters. Nevertheless, a high liquidation cost deters households from frequent housing transactions.

2.5.1 Comparative Statics

In this section I discuss the effects of certain parameters of my model on the life cycle portfolio allocations. Specifically, I investigate the effects of (1) house price risk, (2) the size of down payment, (3) the size of the liquidation cost. I also compare the life-cycle wealth allocation profiles of homeowners and renters in order to see if these profiles are different.

The baseline model assumes that house prices are affected by stochastic shocks. In order to analyze the effects of these shocks on households’ portfolio allocation decisions, I set the variance of these shocks to zero. Thus, the growth rate of the house price is constant and housing investment has a 1.9% annual real return. Table 2.5 shows the life cycle portfolio allocation and homeownership rate of households when there is no house price risk. Comparing Table 2.3 to Table 2.5, we see that a riskless constant return on housing investment leads households to invest more in housing. Since households also obtain utility from housing, they reduce investment in the risky asset and switch to housing investment. Furthermore, in the absence of the correlation between the returns from these investments due to zero variance in housing investment return, households prefer to invest more in the safer asset.
Table 2.5: Life-Cycle Profiles - No House Price Risk

<table>
<thead>
<tr>
<th>Age</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>70-80</th>
<th>80-90</th>
<th>90-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riskless Asset / Financial Asset</td>
<td>0.664</td>
<td>0.624</td>
<td>0.607</td>
<td>0.371</td>
<td>0.356</td>
<td>0.395</td>
<td>0.411</td>
<td>0.467</td>
</tr>
<tr>
<td>Risky Asset / Financial Asset</td>
<td>0.336</td>
<td>0.376</td>
<td>0.393</td>
<td>0.629</td>
<td>0.644</td>
<td>0.605</td>
<td>0.589</td>
<td>0.533</td>
</tr>
<tr>
<td>Fixed Cost Payment Rate</td>
<td>0.013</td>
<td>0.037</td>
<td>0.495</td>
<td>0.929</td>
<td>0.993</td>
<td>0.994</td>
<td>0.994</td>
<td>0.994</td>
</tr>
<tr>
<td>Risky Asset Participation Rate</td>
<td>0.006</td>
<td>0.281</td>
<td>0.487</td>
<td>0.924</td>
<td>0.940</td>
<td>0.891</td>
<td>0.877</td>
<td>0.793</td>
</tr>
<tr>
<td>Homeownership Rate</td>
<td>0.248</td>
<td>0.693</td>
<td>0.899</td>
<td>0.994</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.999</td>
</tr>
</tbody>
</table>
Finally, not surprisingly the homeownership rates are generally lower in the baseline case then the in case with no house price risk. Housing is a more appealing asset because households receive the same expected return without any uncertainty. The homeownership decision is made in a more predictable way because there is no house price risk. Hence homeownership rates are higher when house price is riskless than when it is risky.

Next, I investigate the effects of the down-payment rate on the life-cycle wealth allocation profiles of households. Table 2.6 displays life-cycle profiles of households for different rates of down-payment. Households postpone the homeownership decision when the down-payment rate is large. For example when the rate is 20% of house value, only 21% of households buy a house during the first 10 years. However, when the rate is down to 10% of house value, the homeownership rate jumps to 38% during the same period. It jumps to 43% when the down-payment rate is reduced further to 5% of house value. The homeownership rates don’t change significantly for the rest of the life-cycle for the down-payment rates of 10% and 20%. However, the homeownership rate decreases significantly after the 70s when the down-payment rate is 5% of house value.
### Table 2.6: Life-Cycle Profiles for Different Down-payment Rates

<table>
<thead>
<tr>
<th>Age</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>70-80</th>
<th>80-90</th>
<th>90-100</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Riskless Asset / Financial Asset</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.20</td>
<td>0.770</td>
<td>0.607</td>
<td>0.397</td>
<td>0.275</td>
<td>0.283</td>
<td>0.339</td>
<td>0.318</td>
<td>0.349</td>
</tr>
<tr>
<td>d=0.10</td>
<td>0.634</td>
<td>0.372</td>
<td>0.184</td>
<td>0.128</td>
<td>0.217</td>
<td>0.301</td>
<td>0.283</td>
<td>0.314</td>
</tr>
<tr>
<td>d=0.05</td>
<td>0.576</td>
<td>0.390</td>
<td>0.197</td>
<td>0.148</td>
<td>0.279</td>
<td>0.286</td>
<td>0.274</td>
<td>0.281</td>
</tr>
<tr>
<td><strong>Risky Asset / Financial Asset</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.20</td>
<td>0.230</td>
<td>0.393</td>
<td>0.603</td>
<td>0.725</td>
<td>0.717</td>
<td>0.661</td>
<td>0.682</td>
<td>0.651</td>
</tr>
<tr>
<td>d=0.10</td>
<td>0.366</td>
<td>0.628</td>
<td>0.816</td>
<td>0.872</td>
<td>0.783</td>
<td>0.699</td>
<td>0.717</td>
<td>0.686</td>
</tr>
<tr>
<td>d=0.05</td>
<td>0.424</td>
<td>0.610</td>
<td>0.803</td>
<td>0.852</td>
<td>0.721</td>
<td>0.714</td>
<td>0.726</td>
<td>0.719</td>
</tr>
<tr>
<td><strong>Risky Asset Participation Rate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.20</td>
<td>0.007</td>
<td>0.235</td>
<td>0.582</td>
<td>0.766</td>
<td>0.762</td>
<td>0.497</td>
<td>0.467</td>
<td>0.472</td>
</tr>
<tr>
<td>d=0.10</td>
<td>0.066</td>
<td>0.513</td>
<td>0.859</td>
<td>0.938</td>
<td>0.825</td>
<td>0.444</td>
<td>0.418</td>
<td>0.388</td>
</tr>
<tr>
<td>d=0.05</td>
<td>0.088</td>
<td>0.514</td>
<td>0.842</td>
<td>0.913</td>
<td>0.689</td>
<td>0.316</td>
<td>0.290</td>
<td>0.270</td>
</tr>
<tr>
<td><strong>Homeownership Rate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.20</td>
<td>0.205</td>
<td>0.819</td>
<td>0.872</td>
<td>0.906</td>
<td>0.887</td>
<td>0.826</td>
<td>0.792</td>
<td>0.770</td>
</tr>
<tr>
<td>d=0.10</td>
<td>0.383</td>
<td>0.914</td>
<td>0.936</td>
<td>0.934</td>
<td>0.930</td>
<td>0.887</td>
<td>0.841</td>
<td>0.779</td>
</tr>
<tr>
<td>d=0.05</td>
<td>0.435</td>
<td>0.798</td>
<td>0.865</td>
<td>0.844</td>
<td>0.731</td>
<td>0.630</td>
<td>0.549</td>
<td>0.496</td>
</tr>
</tbody>
</table>
When the down-payment is low, people can invest more in the risky asset. The second part of Table 2.6 shows when the down-payment rate decreases, the risky asset share increases over the life cycle. The difference in the risky asset shares are more observable when the down-payment rate decreases from 20% to 10% than when it decreases from 10% to 5%. A large down-payment rate has a negative impact on the risky asset participation rate as well. For instance, during the first 10 years, reducing the down-payment from 20% to 10% leads to a 5% increase in the participation rate. The increase in the participation rate is about 28% for people in their 30s. The negative effects of the down-payment on the risky asset participation rate continue until age 60. The results show that a large down-payment rate has some crowding out effect on investment in the risky asset, the risky asset participation rate, and on the homeownership rate. The effects are more prominent for young and middle-aged households. Furthermore, the effects are more prominent when the down-payment rate is high. The last three rows of Table 2.6 show the homeownership rates for different downpayments. As we reduce the downpayment from 20% to 10%, homeownership rates increase because households are able to pay relatively lower downpayments. However, if we keep reducing the down-payment to 5%, the homeownership rate decreases. This is because households change their houses so frequently that they end up being renters for some periods.

The model assumes that when households sell their house they are required to pay a certain proportion of the house value as the liquidation cost. Table 2.7 shows the life cycle profiles for different values of the liquidation cost. Since households bear a certain proportion of their house value as the liquidation cost when they sell their house, the size of this liquidation cost can have some influence on the life-cycle profiles of households. The second part of the table shows no significant changes on the share of the risky asset during the life-cycle. Hence the liquidation cost doesn’t have observable influence on the allocation of financial assets.
Table 2.7: Life-Cycle Profiles - The Effects of the Liquidation Cost

<table>
<thead>
<tr>
<th>Age</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>70-80</th>
<th>80-90</th>
<th>90-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riskless Asset / Financial Asset</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa = 0.10$</td>
<td>0.10</td>
<td>0.770</td>
<td>0.607</td>
<td>0.397</td>
<td>0.275</td>
<td>0.283</td>
<td>0.339</td>
<td>0.318</td>
</tr>
<tr>
<td>$\kappa = 0.05$</td>
<td>0.05</td>
<td>0.790</td>
<td>0.683</td>
<td>0.419</td>
<td>0.303</td>
<td>0.305</td>
<td>0.341</td>
<td>0.315</td>
</tr>
<tr>
<td>$\kappa = 0.00$</td>
<td>0.00</td>
<td>0.808</td>
<td>0.638</td>
<td>0.409</td>
<td>0.319</td>
<td>0.313</td>
<td>0.319</td>
<td>0.306</td>
</tr>
<tr>
<td>Risky Asset / Financial Asset</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa = 0.10$</td>
<td>0.10</td>
<td>0.230</td>
<td>0.393</td>
<td>0.603</td>
<td>0.725</td>
<td>0.717</td>
<td>0.661</td>
<td>0.682</td>
</tr>
<tr>
<td>$\kappa = 0.05$</td>
<td>0.05</td>
<td>0.210</td>
<td>0.317</td>
<td>0.581</td>
<td>0.697</td>
<td>0.695</td>
<td>0.659</td>
<td>0.685</td>
</tr>
<tr>
<td>$\kappa = 0.00$</td>
<td>0.00</td>
<td>0.192</td>
<td>0.362</td>
<td>0.591</td>
<td>0.681</td>
<td>0.687</td>
<td>0.681</td>
<td>0.694</td>
</tr>
<tr>
<td>Risky Asset Participation Rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa = 0.10$</td>
<td>0.10</td>
<td>0.007</td>
<td>0.235</td>
<td>0.582</td>
<td>0.766</td>
<td>0.762</td>
<td>0.497</td>
<td>0.467</td>
</tr>
<tr>
<td>$\kappa = 0.05$</td>
<td>0.05</td>
<td>0.009</td>
<td>0.266</td>
<td>0.585</td>
<td>0.753</td>
<td>0.748</td>
<td>0.504</td>
<td>0.469</td>
</tr>
<tr>
<td>$\kappa = 0.00$</td>
<td>0.00</td>
<td>0.022</td>
<td>0.308</td>
<td>0.528</td>
<td>0.666</td>
<td>0.608</td>
<td>0.365</td>
<td>0.339</td>
</tr>
<tr>
<td>Homeownership Rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa = 0.10$</td>
<td>0.10</td>
<td>0.205</td>
<td>0.819</td>
<td>0.872</td>
<td>0.906</td>
<td>0.887</td>
<td>0.826</td>
<td>0.792</td>
</tr>
<tr>
<td>$\kappa = 0.05$</td>
<td>0.05</td>
<td>0.193</td>
<td>0.723</td>
<td>0.805</td>
<td>0.876</td>
<td>0.868</td>
<td>0.797</td>
<td>0.768</td>
</tr>
<tr>
<td>$\kappa = 0.00$</td>
<td>0.00</td>
<td>0.195</td>
<td>0.620</td>
<td>0.620</td>
<td>0.650</td>
<td>0.601</td>
<td>0.488</td>
<td>0.483</td>
</tr>
</tbody>
</table>
The results are slightly different for the participation rate. When the liquidation cost decreases from 10% to 5% of house value, we don’t observe a sizable difference in the participation rate during the life-cycle. However, the participation rate is lower after age 40 when there is no liquidation cost. Surprisingly, the homeownership rate is also low when there is no liquidation cost. In this case, people have more willingness to sell their house which leads to frequent house sales. More frequent house sales lead households to be renters for some periods. This can be a reason for having a lower homeownership rate. However, analyzing the effects of the liquidation cost in more detail is beyond the scope of this study.

In this part, I investigate the life-cycle profiles of homeowners and renters. The model assumes that all households are renters in the initial period. From the second period on in each period households endogeneously make a homeownership decision. For instance, a homeowner at time $t$ has three options regarding homeownership: (1) stay in the current house, (2) move to a new house, (3) move to a rental property. Similarly, a renter decides to either stay in the current rental property or buy a house and become a homeowner. Table 2.8 presents the life-cycle profiles for both homeowners and renters. Note that a typical household can be a renter for some periods and can be a homeowner for some other periods. The homeowners part of the table shows the life cycle profiles for households when they are homeowners. On the other hand, the renter part of the table shows the life-cycle profiles for households when they are renters. For example, assume that a household was a renter for the first 15 years, and became a homeowner for the next 50 years, and then again became a renter for the rest of his life. Then the life cycle profile for the first 15 years will be used for obtaining renters’ life cycle profiles, the next 50 years will be used to for obtaining an average homeowner’s life cycle profile, and finally remaining life-cycle profiles will again be used for obtaining an average renter’s life cycle profile.

Table 2.8 shows that homeowners hold significantly more risky asset shares than renters. The gap between homeowners’ risky asset shares and renters’ risky asset shares decreases over time. We observe a similar outcome for the fixed entry cost payment rate and the risky asset
Table 2.8: Life-Cycle Profiles - Homeowners vs Renters

<table>
<thead>
<tr>
<th>Age</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>70-80</th>
<th>80-90</th>
<th>90-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homeowners</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Riskless A / Financial A</td>
<td>0.608</td>
<td>0.679</td>
<td>0.400</td>
<td>0.274</td>
<td>0.276</td>
<td>0.325</td>
<td>0.303</td>
<td>0.338</td>
</tr>
<tr>
<td>Risky A. / Financial A</td>
<td>0.392</td>
<td>0.321</td>
<td>0.600</td>
<td>0.726</td>
<td>0.724</td>
<td>0.675</td>
<td>0.697</td>
<td>0.662</td>
</tr>
<tr>
<td>Fixed Cost Payment Rate</td>
<td>0.015</td>
<td>0.351</td>
<td>0.719</td>
<td>0.854</td>
<td>0.919</td>
<td>0.952</td>
<td>0.961</td>
<td>0.963</td>
</tr>
<tr>
<td>Risky Asset Participation Rate</td>
<td>0.014</td>
<td>0.327</td>
<td>0.685</td>
<td>0.826</td>
<td>0.822</td>
<td>0.578</td>
<td>0.555</td>
<td>0.574</td>
</tr>
<tr>
<td>Renters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Riskless A / Financial A</td>
<td>0.818</td>
<td>0.695</td>
<td>0.500</td>
<td>0.506</td>
<td>0.506</td>
<td>0.404</td>
<td>0.356</td>
<td>0.346</td>
</tr>
<tr>
<td>Risky A. / Financial A</td>
<td>0.182</td>
<td>0.305</td>
<td>0.500</td>
<td>0.494</td>
<td>0.494</td>
<td>0.596</td>
<td>0.644</td>
<td>0.654</td>
</tr>
<tr>
<td>Fixed Cost Payment Rate</td>
<td>0.010</td>
<td>0.320</td>
<td>0.552</td>
<td>0.657</td>
<td>0.777</td>
<td>0.831</td>
<td>0.857</td>
<td>0.869</td>
</tr>
<tr>
<td>Risky Asset Participation Rate</td>
<td>0.006</td>
<td>0.097</td>
<td>0.160</td>
<td>0.239</td>
<td>0.251</td>
<td>0.210</td>
<td>0.185</td>
<td>0.181</td>
</tr>
</tbody>
</table>

participation rate. The proportion of homeowners who have paid the fixed cost and the proportion of homeowners who participate in risky asset investment are significantly higher than those for renters. Furthermore, the gap between the homeowners’ and renters’ profiles for these two variables don’t vanish over time. The overall results show that the model is able to explain the life-cycle portfolio allocation differences between renters and homeowners.

2.5.2 Comparison of the Model with the Data

In this part of the paper, I investigate the extent to which the portfolio allocation profiles in the model match the data. I use all PSID waves that contain information about households financial wealth. These waves are 1984, 1989, 1994, 1999, 2001, 2003, 2005, and 2007. The data collection process is explained in detail in the discussion of the estimation part of the model. I define risky assets in the PSID data as the sum of shares of stocks in publicly or privately held corporations including those held in mutual funds, investment trusts and Individual Retirement Accounts (IRAs). Total financial assets is the sum of the risky asset defined above plus money in saving and checking accounts, money market funds, certificates of deposits, government saving bonds, Treasury bills, bond funds, other savings or assets, such as cash value in a life insurance
policy, and a valuable collection for investment purposes. All values are adjusted to their current corresponding values using the CPI.

Table 2.9 displays the life-cycle portfolio shares in the risky asset, the risky asset participation rate, and the homeownership rate obtained from the PSID data. Figure 2.3 depicts these life-cycle profiles for the real data, the baseline model, and the NHI model. The model slightly underestimates the risky asset share for young households when compared to the data. However, the model is successful in matching the risky asset share for the subsequent periods. Hence, the model is able to predicts moderate levels of risky asset holdings. In many previous studies of life cycle portfolio allocation models, the portfolio shares invested in the risky asset were always significantly higher than the empirical evidence. For example, see Campbell et al. (2001), Cocco (2004), Cocco et al. (2005), Gomes and Michaelides (2007), Hu (2005), Yao and Zhang (2005) among many. The gap between the predictions of life-cycle models and the empirical evidence is called the “portfolio allocation puzzle” which is a kind of flip side of the “equity premium puzzle” of Mehra and Prescott (1985).

Furthermore, the model predicts moderate levels of the participation rate during the life-cycle. The participation rate is lower than the empirical counterpart during the early periods. It is slightly higher than the empirical counterpart after age 50 and reaches it after age 60. Figure 2.3 shows the contribution of housing investment in obtaining moderate levels of the risky asset participation rate as well as moderate shares of the risky asset. In the NHI model, these two are significantly higher than their empirical counterpart.

<table>
<thead>
<tr>
<th>Age</th>
<th>20-30</th>
<th>30-40</th>
<th>40-50</th>
<th>50-60</th>
<th>60-70</th>
<th>70-80</th>
<th>80-90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riskless A / Financial A</td>
<td>0.483</td>
<td>0.434</td>
<td>0.393</td>
<td>0.388</td>
<td>0.432</td>
<td>0.502</td>
<td>0.576</td>
</tr>
<tr>
<td>Risky A. / Financial A</td>
<td>0.517</td>
<td>0.566</td>
<td>0.607</td>
<td>0.612</td>
<td>0.568</td>
<td>0.498</td>
<td>0.424</td>
</tr>
<tr>
<td>Risky A. Particip. Rate</td>
<td>0.218</td>
<td>0.363</td>
<td>0.413</td>
<td>0.477</td>
<td>0.447</td>
<td>0.440</td>
<td>0.454</td>
</tr>
<tr>
<td>Homeownership Rate</td>
<td>0.292</td>
<td>0.543</td>
<td>0.667</td>
<td>0.724</td>
<td>0.737</td>
<td>0.725</td>
<td>0.678</td>
</tr>
</tbody>
</table>
When compared to the PSID data, the model slightly overestimates life cycle homeownership rates. The homeownership rate during the first 10 years is 21% in the baseline model while it is about 29% in the data. This rate increases dramatically for the next 10 years to 82% in the model. The gap between the model predictions and the empirical evidence on homeownership rate decreases from 28% to 11% during the life cycle.

### 2.6 Conclusion

In this study, I focus on the effects of housing investment on portfolio allocation in a life-cycle model. The importance of housing investment for households is high since housing constitutes an important part of households’ portfolios. In order to fully understand the effects of housing investment on portfolio allocation, I developed a fairly comprehensive life cycle asset allocation model where many important features of housing investment are taken into account.

Housing is both an investment asset which enables owners to hold home equity, and a durable consumption good from which households derive utility. I show that housing investment has a
strong crowding out effect on investment in risky asset and this effect is larger for young and middle-aged households. Early in life, households are willing to be homeowners by paying a down payment and annual mortgage payments. This keeps their liquid wealth at low levels so that they refrain from paying the entry cost to invest in the risky asset. Hence, owner-occupied housing is a substitute for investment in the risky asset. Even after accumulating enough wealth to pay the fixed cost and begin to invest in risky assets, the portfolio share of risky assets is still at lower levels than predicted by the model with no housing investment.

There are many features of housing investment that one can analyze. I investigate the effects of some of these features that deserve some focus. I found that, in the absence of house price risk, households invest more in housing, the homeownership rates go up and share of investment in the risky assets goes down. The riskiness of housing investment leads households to make more investment in the risky asset. The results also show that a large down-payment rate has some crowding out effect on investment in the risky asset, on the risky asset participation rate, and on the homeownership rate. The effects are more prominent for young and middle-aged households. However, liquidation costs of selling houses have insignificant effects on the life-cycle asset allocation of households.

I also analyze the differences in life-cycle asset allocation between homeowners and renters. The results show that homeowners hold a significantly larger share of their financial wealth in the risky asset than renters. Furthermore, they have higher risky asset participation rates than renters.

The model is able to addresses the two empirical observations mentioned earlier. That is, it is able to explain that about half of the population invests in the risky asset and those who invest in the risky asset hold moderate equity in the risky asset.

Some extensions of the model for future research include focusing more on the real estate side of the model by analyzing the size and the effects of the liquidation cost, introducing an exogeneous mandatory moving and selling of houses (job relocation, old age, health issues, etc.), and the possibility of default on mortgage. Furthermore, estimating the size of the fixed entry
cost, and introducing more realistic house price dynamics are left for future research.
Chapter 3

Optimal Portfolio Allocation in the Presence of Tax-Deferred Accounts and Housing Investment

3.1 Introduction

Tax-deferred retirement accounts (TDAs) have shown considerable growth for more than two decades\(^1\). These accounts provide households with the opportunity of investing in tax-exempt assets to help accumulate enough wealth for their retirement. Taxes on these accounts are only effective upon withdrawals of funds. As more households rely on TDAs (i.e. defined contributions), they face both an optimal an asset allocation problem (i.e. deciding how much of each asset to hold) and an asset location problem (i.e. deciding which asset to hold in taxable and tax-deferred accounts). Hence understanding the effects of the presence of TDAs on households’ life cycle wealth allocation decisions becomes increasingly important.

\(^1\)Assets in employer-sponsored defined-contribution (DC) plans have grown more rapidly than assets in other types of employer-sponsored retirement plans over the past quarter century, increasing from 27 percent of employer plan assets in 1985 to 40 percent at year-end 2010. At the end of 2010, employer-sponsored DC plans—which include 401(k) plans, 403(b) plans, 457 plans, Keoghs, and other DC plans accounted for $4.5 trillion dollars (Investment Companies Institute Factbook, http://www.icifactbook.org/ )
Similar to TDAs, housing investment plays a critical role on household life-cycle wealth allocation decisions because housing investment constitutes a significant share of homeowners’ wealth. For instance, data from the Panel Study Income of Dynamics (PSID) for the period 1999-2007 show that about two-thirds of US households are homeowners. Furthermore, the renter-homeowner distinction is necessary in analyzing life-cycle portfolio allocation profiles since homeowners, in general, follow a different portfolio allocation pattern than renters. For example, the PSID data for the same period shows that the mean and median of TDAs wealth for renters is $14,967 and $0 while for homeowners it is $148,352 and $95,000. Hence it is important to take housing investment into account in addition to TDAs investment when analyzing life-cycle portfolio allocation profiles.

The contribution of this chapter to the portfolio allocation literature is to combine these two important components of wealth into a life-cycle model in order to analyze the portfolio allocation of households in a more realistic framework. So far the portfolio allocation literature deals with these two components separately. This paper employs a fairly comprehensive life-cycle portfolio allocation model and investigates the effects of TDAs and housing investment on households life-cycle consumption, housing investment, and asset location and allocation decisions. The standard life-cycle models are not sufficiently comprehensive in the sense that they exclude either (1) TDAs or (2) a comprehensive housing investment or (3) both and therefore can’t fully explain life-cycle portfolio allocation profiles of households.

In order to capture the effects of TDAs and housing investment on households life-cycle wealth and portfolio allocation decisions more precisely, I build a fairly rich structural life-cycle model parameterized to be consistent with empirical data. In this model, I consider finitely living households facing mortality risk, borrowing and short-sale constraints, and receiving stochastic labor income. In every period, households make consumption, housing investment, and asset location and allocation decisions. Regarding financial assets, households can invest in both TDAs and TAs (asset location) and decide the allocation of wealth between the risky and the riskless assets for each account (asset allocation). Both households and employers make
contributions to TDAs. TDAs provide higher returns than taxable accounts since taxes are paid only upon withdrawal of funds whereas returns on taxable assets are taxed once these returns accrued. Hence households can accumulate wealth in TDAs faster than in taxable accounts.

Besides the distinction between TDAs and TAs, I have modeled housing investment in a fairly comprehensive way. Housing investment is different from other financial investments because it is not only an asset but also a durable consumption good from which households derive utility. It is also, contrary to liquid financial assets, illiquid and often highly leveraged. Furthermore, the wealth allocation of households depends on their homeownership status. While homeowners are committed to pay a flow of payments so as to pay back their mortgage they also enjoy homeownership. Renters, on the other hand, have no commitments and only pay rent but they don’t get the benefits of being homeowners.

This model is also specified using Epstein-Zin type recursive preferences [Epstein and Zin (1989)] where the relative risk aversion (RRA) is disentangled from the elasticity of intertemporal substitution (EIS). While RRA gives information about how agents deal with uncertainty across possible states of the world, (EIS) is only a time preference parameter. The problem with the commonly used constant relative risk aversion (CRRA) utility function is that RRA is the inverse of EIS. In other words, CRRA imposes two different roles on the same parameter while EZ provides the flexibility to relax this constraint and have two separate values for these two parameters.

The results of my model suggest the presence of TDAs and housing investment have significant effects on consumption, asset location and asset allocation decisions of households. The share of the risky asset in TAs decreases once households begin to invest in their TDAs. For instance, during the working period, the share of risky assets invested in TAs is about 10% less in the presence of TDAs. This shows that TDAs partially crowd out risky asset investment in TAs. Because of its tax-exempt status, households prefer to invest heavily in their TDAs. As funds in the TDAs can’t be withdrawn before retirement in the model, households also make investments in TAs in part due to the motivation for precautionary saving. The results of the
model also suggest that housing investment has a crowding out effect on risky asset investment in both TDAs and TAs. In the presence of housing investment, the share of risky assets decreases in both TAs and TDAs by more than 15% and 20%, respectively. These crowding out effects come from the (1) motivation to hedge against risky assets investment and (2) liquidity constraints caused from making a downpayment and regular mortgage payments.

The main purpose of TDAs is to provide households with an income during their retirement so that they can sustain their working period living standards in retirement. The model shows that household consumption increases by about 35% during the retirement period if they contribute to their TDAs during the working life.

3.2 Literature Review

This paper is related to two separate strands of the literature on portfolio allocation, analyzing either (1) the effects of TDAs or (2) housing investment on household portfolio allocation profiles. In the first strand, Gomes et al. (2009) use a life-cycle model with both TDAs and TAs to analyze the portfolio allocation of both direct stockholders (those investing in both TDAs and TAs) and indirect stockholders (those investing in only TDAs). They find that TDAs increase wealth accumulation which is generally transformed into consumption for retirement. Furthermore, households with lower saving incentives respond less to TDAs. Zhou (2011) finds that differential costs of stock market participation in TDAs and TAs can explain the higher participation rate in TDAs early in life relative to TAs. He also shows that the decline in the participation rate in TDAs late in life is due to the differential tax treatments between these two accounts. Bergstresser and Poterba (2004) study household behavior from the Survey of Consumer Finances (SCF). The findings of their paper suggest that about one-third of households with both TAs and TDAs are following asset location strategies that are tax inefficient\(^2\). They conclude that non-tax considerations may play an important part in households’ portfo-

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\(^2\) Tax-efficiency means putting the least tax efficient funds (funds that have higher tax-rate) in TDAs and most tax-efficient in TAs.
lio allocation decisions. Dammon et al. (2004) show that taxes play an important role in asset location and allocation decisions. They find that households prefer to hold bonds in TDAs and equities in TAs. However, households’ decisions also depend on other factors such as liquidity. The predictions of their model substantially deviate from the empirical observations that households hold a mix of both stocks and bonds in both their TDAs and TAs.

Since housing investment in portfolio allocation literature is discussed in the Literature section of the first chapter, I skip it in this chapter.

Unlike these studies, this paper combines these two strands of the literature in order to analyze life cycle portfolio allocation decisions of households in the presence of both housing investment and TDAs. By joining these two separate lines of research, the results show that disregarding either one distorts households’ optimal portfolio allocation decisions.

The reminder of the paper proceeds as follows. Section 3.3 discusses the realistic life-cycle model and its assumptions. Section 3.4 presents the parameterization and calibration. The numerical solution method of the model in terms of the optimal portfolio allocation of households is discussed in Section 3.5. The simulation results of the model with the baseline parameters, various extensions of the baseline model, and a comparison of the model to the data are given in Section 3.6. Finally, Section 3.7 gives concluding remarks and possible extensions of the model for future work.

3.3 Model Setup

3.3.1 Household Preferences

The model is a discrete time life-cycle portfolio allocation model. Each period in the model corresponds to one year and as a general convention in the life-cycle literature each year is actually the real age of a household minus 19. Households live for a maximum of $T$ years. The probability that a household is alive at time $t + 1$ conditional on being alive at time $t$ is equal to $q_t$. Households derive utility from a constant elasticity of substitution (CES) utility function
with non-durable consumption goods (C), and housing investment (H). Preferences are in the form of Epstein-Zin where the RRA is disentangled from the EIS:

\[ V_t = \left\{ u(C_t, H_t) \frac{1}{1-\gamma} + \beta \left( E_t \left[ q_t V_{t+1}^{1-\gamma} + (1-q_t)W_{t+1}^{1-\gamma} \right] \right) \frac{1}{\theta} \right\}^{\frac{\theta}{1-\gamma}} \] (3.1)

\[ \theta = \frac{1-\gamma}{1-1/\psi} \] (3.2)

\[ u(C_t, H_t) = \left[ C_t^\rho + H_t^\rho \right]^{\frac{1}{\rho}}, \] (3.3)

where \( \beta \) is the time discount factor, \( \gamma \) is the RRA parameter, and \( \psi \) is the EIS parameter. The intratemporal elasticity of substitution between nondurable consumption goods and housing investment is \( 1/(1-\nu) \). \( W_{t+1} \) is the households’ total wealth at time \( t+1 \) that would be bequested in the case households pass away.

### 3.3.2 Labor Income Process

Following the standard specifications in the life cycle literature (Campbell et al. (2001), Guvenen (2009a), and Gomes and Michaelides (2005) among many), labor income depends on households’ age, education, marital status, and gender with an idiosyncratic random shock. Households supply labor inelastically during working life and receive a stochastic labor income \( Y_{it} \). Retirement period labor income is a constant fraction of labor income received during the last working life period. Define \( y_{it} = \log(Y_{it}) \) where \( y_{it} \) has the following process:

\[ y_{it} = f(t, Z_{it}) + \epsilon_{it}^l \quad t \leq K \] (3.4)

\[ y_{it} = \xi y_{iK} \quad t > K \] (3.5)

\[ f(t, Z_{it}) = \beta_1 age + \beta_2 age^2 + \beta_3 gender + \beta_4 marital\_status + \beta_5 educ, \] (3.6)

where \( Y_{it} \) is the labor income received by household \( i \) at time \( t \), \( f(t, Z_{it}) \) is a deterministic function of age and household characteristics such as education, marital status, and gender. The variable \( \epsilon_{it}^l \) is the random shock to log-labor income and is distributed as \( i.i.d.N(0,\sigma_{it}^2) \).
Although it is very common in the life-cycle literature to decompose the shocks to labor income into an aggregate and an idiosyncratic part, the complexity of the model doesn’t allow us to have this decomposition.

Households retire in period \( K \) where \( K \) corresponds to a real age of 65. During the retirement period, households receive a constant and deterministic labor income, which is a constant fraction \( \xi \) of the last working period labor income (i.e. replacement rate).

### 3.3.3 Housing Investment

All households are renters in the first period. They endogeneously decide to be homeowners or stay as renter in each of the following periods. A typical homeowner at any time can endogeneously decide either (1) to stay in his current house, or (2) to sell his current house and buy a new one, or (3) to sell his house and become a renter. Similarly, a typical renter can decide either (1) to stay as a renter or (2) to buy a house and become a homeowner. Households are required to pay a fraction \( d \) of the market value of the house as downpayment and to finance the rest through a mortgage. In order to capture the illiquidity of housing investment, when households decide to sell their house, they incur a liquidation cost equal to a fraction \( \kappa \) of the house’s market value. Homeowners also pay in each period for the maintenance and depreciation expenses an amount equal to a proportion \( \delta \) of the house’s market value. However, renters only pay annual rent which is equal to a proportion \( \alpha \) of the house’s market value. I assume that house-related expenses are a certain proportion of the market value of houses in order to both reduce the computational burden of the model and incorporate easily these expenses into the model.

The per unit price of housing is \( P^h_t \). The price of a house of size \( H_i \) is \( P^h_t H_i \). Define \( p^h_t = \log(P^h_t) \), and assume that \( p^h_t \) has the following dynamic:

\[
\Delta p^h_t = \mu_h + \epsilon^h_t, \quad \epsilon^h_t \sim N(0, \sigma^2_{\epsilon^h}),
\]

where \( \mu_h \) as the average growth rate of house prices. The downpayment of a house size \( H_i \) at
time $t$ is $dP_t^h H_t$. Hence the mortgage loan $RM_t$ that a typical household can take against their house is $RM_t \leq (1 - d)P_t^h H_t$.

### 3.3.4 Financial Assets and Wealth Accumulation

Households can invest in only two assets: a risky asset and a riskless asset. Investment in TAs and TDAs have different returns due to the tax treatments. Return on investment in TAs are taxed once these returns accrued. However, investment on TDAs are taxed when households withdraw funds from them. Throughout their working life, households contribute to TDAs a fraction $k_l$ of before-tax earnings. Employers also contribute a fraction $k_e$ of before-tax earnings to households’ TDAs. Households cannot withdraw funds from TDAs before retirement. Once households retire in period $K$, they face a withdrawal rate equal to the inverse of their life expectancy. The tax rate on these withdrawals is the same as the income tax rate, $\tau_y$. We assume that there is no entry cost for investing in the risky asset in either account because we found in Chapter 2 that this fixed entry cost doesn’t have any sizable effects on households’ life-cycle portfolio allocation decisions.

Real gross return on investment in the riskless asset is $R^b$. After-tax gross return is

$$\tilde{R}^b = \frac{1 + [R^b (1 + \pi) - 1] (1 - \tau_b)}{1 + \pi}, \tag{3.8}$$

where $\pi$ is a constant inflation rate and $\tau_b$ is the tax-rate on nominal return for riskless asset investments. Gross real return on the risky asset is

$$R^s_t = R^b_t + \mu_s + \varepsilon^s_{t+1}, \tag{3.9}$$

where $\mu_s$ is the real, before-tax equity premium and $\varepsilon^s_{t+1}$ is a random shock that is $i.i.d. N(0, \sigma^2_{\varepsilon^s})$. The gross return on the risky asset at time $t$ is comprised of a constant nominal dividend yield $d_y$ and a stochastic nominal capital gain $c_y$. Dividend yields are taxed at the rate $\tau_{dy}$ and capital gains are taxed at the rate $\tau_{cg}$. The before and after-tax real gross return on the risky
asset investment takes the following forms:

\[ R^s_{t+1} = \frac{1 + cg_t + dy}{1 + \pi}, \]  

(3.10)

\[ \tilde{R}^s_{t+1} = \frac{1 + cg_t (1 - \tau y) + dy (1 - \tau dy)}{1 + \pi}. \]

(3.11)

**Renter’s Wealth Accumulation**

There are two types of households in this model. I start first by considering a typical renter’s wealth accumulation and budget constraint. The labor income of renters is taxed after the contribution to TDAs is made. Once a renter receives his taxed labor income and taxed return on investment made in TAs at time \( t - 1 \), he allocates the total cash-on-hand \( (X_t) \) to (1) consumption expenditure, (2) housing investment, and finally (3) investment in the TAs and the allocation of funds within this account to risky and riskless assets. Regarding housing investment, renters decide to pay the down payment and be homeowners if it is optimal or stay as renters and wait until the next period. Wealth accumulation and budget constraints of renters have the following form:

\[ W_{t}^{TD} = S_{t-1}^{TD}R^s_t + B_{t-1}^{TD}R^b_t + k^*Y_t \]

(3.12)

\[ LW_t = S_{t-1}^{TD}R^s_t + B_{t-1}^{TD}R^b_t \]

(3.13)

\[ X_t \equiv LW_t + (1 - \tau_y) [Y_t - k_t Y_t] \]

(3.14)

\[ k^* = k_t + k_e \]

(3.15)

\[ X_t = S_t + B_t + C_t + (1 - HR_t) \left[ \alpha P^b_t H_t \right] + HR_t \left[ M_t + dP^b_t H_t \right] \]

(3.16)

\[ M_t = MP_t + MI_t \]

(3.17)

\[ RM_t = RM_{t-1} - MP_t, \]

(3.18)
where $W_{t}^{TD}$ is the total funds of a typical households in TDAs at time $t$. The investment made in the risky (riskless) asset in TDAs at time $t - 1$ is denoted by $S_{t-1}^{TD} (B_{t-1}^{TD})$. Likewise, $S_{t-1} (B_{t-1})$ denotes the investment made in the risky (riskless) asset in TAs at time $t - 1$. $M_{t}$ is the annual mortgage payment that homeowners pay. Mortgage payments consist of both principal payments ($MP_{t}$) and interest payments ($MI_{t}$). The remaining balance of the mortgage loan at time $t$ is $RM_{t}$. $HR_{t}$ is a dummy variable that equals 1 if a household is a homeowner at time $t$, and 0 otherwise.

The wealth allocation and budget constraints of renters during the retirement period is different than the working period since households don’t make contributions to TDAs but withdraw funds from it. During the retirement period, renters have the following form for wealth allocation and budget constraint:

\begin{align*}
W_{t}^{TD} &= S_{t-1}^{TD} R_{t}^{s} + B_{t-1}^{TD} R^{b} - Q_{t} \quad (3.19) \\
Q_{t} &= \varsigma_{t} W_{t}^{TD} \quad (3.20) \\
LW_{t} &= S_{t-1} \tilde{R}_{t}^{s} + B_{t-1} \tilde{R}^{b} \quad (3.21) \\
X_{t} &= LW_{t} + (1 - \tau_{y}) Y_{t} + (1 - \tau_{y}) Q_{t} \\
X_{t} &= S_{t} + B_{t} + C_{t} + (1 - HR_{t}) \left[ \alpha P_{t}^{h} H_{t} \right] + HR_{t} \left[ M_{t} + dP_{t}^{h} H_{t} \right] \quad (3.22) \\
M_{t} &= MP_{t} + MI_{t} \quad (3.23) \\
RM_{t} &= RM_{t-1} - MP_{t} \quad (3.24)
\end{align*}

where $\varsigma_{t}$ is the withdrawal rate of funds from TDAs at time $t$ and $Q_{t}$ is the amount of funds households withdraw from TDAs at this time.

**Homeowner’s Wealth Accumulation**

Homeowners’ wealth in TDAs ($W_{t}^{TD}$), the contributions to this account, and the form of liquid wealth ($LW_{t}$) are the same as for renters. However, the amount of cash-on-hand, $X_{t}$, is
different since homeowners make mortgage payments. The interest component of this payment is tax-exempt. Labor income of homeowners is taxed after the TDAs contribution and interest component of mortgage payments are deducted. Combining this deducted and taxed labor income with taxed returns on investment made in TAs at time \( t-1 \), homeowners allocate this total cash-on-hand \( (X_t) \) to (1) consumption expenditures, (2) housing investment and expenditures, and finally (3) investment in TAs. Furthermore they allocate their funds between risky and riskless assets within each account. Homeowners wealth accumulation and budget constraints before retirement take the following structure:

\[
W_{t}^{TD} = S_{t-1}^{TD} R_t^s + B_{t-1}^{TD} R^b + k_t^* Y_t \tag{3.25}
\]

\[
LW_t = S_{t-1} \tilde{R}_t^s + B_{t-1} \tilde{R}^b \tag{3.26}
\]

\[
X_t \equiv LW_t + (1 - \tau_y) [Y_t - k_t Y_t - MI_t] \tag{3.27}
\]

\[
X_t = C_t + S_t + B_t + \tau_h P_t^h H_{t-1} + \delta P_t^h H_{t-1} + H R_t (1 - MS_t) [M_t] + HR_t MS_t \left[ d P_t^h H_t + M_t - ((1 - \kappa) P_t H_{t-1} - RM_t) \right] + (1 - HR_t) MS_t \left[ \alpha P_t^h H_t - \left( (1 - \kappa) P_t^h H_{t-1} - RM_t \right) \right], \tag{3.28}
\]

where \( \tau_h \) denotes the property tax for housing, \( \delta \) denotes the rate for maintenance and depreciation expenses of houses, \( \kappa \) is the rate of liquidation cost in the case of selling the house, and \( MS_t \) is a dummy variable which is 1 if a homeowner decides to move from the current house and is 0 otherwise. The after-retirement wealth accumulation and budget constraints, however, are slightly different than the before-retirement case because homeowners are withdrawing funds from their TDAs and making no contribution to this account. Furthermore, these withdrawals are subject to income tax.

\[
W_{t}^{TD} = S_{t-1}^{TD} R_t^s + B_{t-1}^{TD} R^b - Q_t \tag{3.29}
\]

\[
Q_t = \varsigma W_{k}^{TD} \tag{3.30}
\]

\[
LW_t = S_{t-1} \tilde{R}_t^s + B_{t-1} \tilde{R}^b \tag{3.31}
\]
\[ X_t \equiv LW_t + (1 - \tau_y) [Y_t - MI_t] + (1 - \tau_y) Q_t \]  
(3.32)

\[ X_t = C_t + S_t + B_t + \tau_h P^h_{t-1} H_{t-1} + \delta P^h_{t-1} H_{t-1} + HR_t (1 - MS_t) [M_t] + 
\]

\[ HR_t MS_t \left[ dP^h_t H_t + M_t - \left( (1 - \kappa) P^h_{t-1} H_{t-1} - RM_t \right) \right] + 
\]

\[ (1 - HR_t) MS_t \left[ \alpha P^h_t H_t - \left( (1 - \kappa) P^h_{t-1} H_{t-1} - RM_t \right) \right]. \]  
(3.33)

Household total wealth at time \( t \) (\( W_t \)) is the sum of investment made in both TDAs and TAs at time \( t - 1 \) plus the house market value minus the remaining mortgage debt. Households bequest this total wealth to inheritors if they pass away before the terminal period \( T \). If a household lives up to the final period \( T \) then he bequests \( W_{T+1} \). I also impose short-sale and non-negativity constraints on consumption, housing investment, and financial asset investments:

\[ C_t \geq 0, S_t \geq 0, B_t^{TD} \geq 0, S_t^{TD} \geq 0, B_t \geq 0, H_t \geq 0, \forall t, \]  
(3.34)

\[ W_t = R^h B_{t-1} + R^h s_{t-1} + P^h_{t-1} H_{t-1} - RM_{t-1} + W_{T+1}. \]  
(3.35)

### 3.3.5 Optimization Problem

Before defining the optimization problem in a more concrete form, I list the state and control variables. The state variables of this problem are age, \( t \), liquid wealth (\( LW_t \)), wealth in TDAs (\( W_{t}^{TD} \)), homeownership status at time \( t - 1 \) (\( O_{t-1} \)), the size of house owners have at time \( t - 1 \) (\( H_{t-1} \)), and mortgage debt at time \( t - 1 \) (\( RM_{t-1} \)). I denote the state variables by \( \Omega \) where \( \Omega_t = (-t, LW_t, W_{t}^{TD}, O_{t-1}, H_{t-1}, RM_{t-1}). \) The control variables are consumption (\( C_t \)), the size of house to be chosen at time \( t \) (\( H_t \)), the homeownership decision at time \( t \) (\( O_t \)), asset allocation decision within TDAs (\( s_{t,TDAs} \)), and asset allocation decision in TAs (\( s_{t,TAs} \)). I define the control variables by \( \Psi \) where \( \Psi_t = (C_t, H_t, O_t, s_{t,TDAs}, s_{t,TAs}). \)

After defining the state and control variables in a more compact form, the household opti-
mization problem is given by

\[ V_t(\Omega_t) = \max_{\Psi_t} \left\{ u(C_t, H_t)^{\frac{1-\gamma}{\psi}} + \beta \left( E_t \left[ q_{t+1} (\Omega_{t+1})^{1-\gamma} + (1-q_t)W_{t+1}^{1-\gamma} \right] \right)^{\theta} \right\}^{\frac{\theta}{1-\gamma}}, \quad (3.36) \]

subject to Equations 3.1 to 3.35.

This model has no analytical solution. Hence we use a numerical approximation method and solve the model using backward induction starting from time \( T \) to \( t=1 \). Please see the details of the solution method in Appendix B.

### 3.4 Parameterization

#### 3.4.1 Preference Parameters

The baseline parameters of the model are presented in Table 3.1. The annual discount factor \( \beta \) is 0.95. Thanks to Epstein and Zin (1989) recursive preferences I am able to disentangle relative risk aversion from the elasticity of intertemporal substitution\(^3\). The estimated values for both relative risk aversion, \( \gamma \), and the elasticity of intertemporal substitution, \( \psi \), from Chapter 2 are 8.9 and 1.1, respectively. I parameterized the conditional survival probabilities from the mortality tables produced by the National Center for Health Statistics (NCHS).\(^4\)

#### 3.4.2 Labor Income Process

Each period in the model corresponds to one year. Households enter the market at the real age of 20 and stay in the market for up to 80 years. The retirement age \( K \) is 46 (65-19). The age-dependent labor income process is based on the PSID data from 1999-2007. In order to obtain a random sample of US households, I exclude families that were part of the Survey of Economic Opportunities\(^5\). Similar to previous studies in life-cycle portfolio allocation models,\(^6\)

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\(^3\)Gomes and Michaelides (2005) has shown that this feature matches wealth allocation over the life-cycle.


\(^5\)The original PSID sample was drawn from two independent sources: an over-sample of roughly 2000 poor families selected from the Survey of Economic Opportunities (SEO) and a nationally-representative sample of roughly 3000 families from all states. Since the SEO oversamples low income families, it isn’t a random sample.
### Table 3.1: Parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time discount factor</td>
<td>$\beta$</td>
<td>0.95</td>
</tr>
<tr>
<td>Relative risk aversion</td>
<td>$\gamma$</td>
<td>8.9</td>
</tr>
<tr>
<td>Elasticity of intertemporal substitution</td>
<td>$\psi$</td>
<td>1.1</td>
</tr>
<tr>
<td>Elasticity of intratemporal substitution parameter</td>
<td>$\upsilon$</td>
<td>0.2</td>
</tr>
<tr>
<td>Retirement labor income factor</td>
<td>$\lambda$</td>
<td>0.60</td>
</tr>
<tr>
<td>Std of labor income shocks</td>
<td>$\sigma_{\varepsilon_{l}}$</td>
<td>0.15</td>
</tr>
<tr>
<td>Mean return on housing investment</td>
<td>$\mu_{h}$</td>
<td>1.9%</td>
</tr>
<tr>
<td>Std of shocks to housing investment</td>
<td>$\sigma_{\varepsilon_{h}}$</td>
<td>5.7%</td>
</tr>
<tr>
<td>Down-payment rate</td>
<td>$D$</td>
<td>20%</td>
</tr>
<tr>
<td>Before-tax gross return on riskless asset investment</td>
<td>$R^b$</td>
<td>1.03</td>
</tr>
<tr>
<td>Inflation rate</td>
<td>$\pi$</td>
<td>3.15%</td>
</tr>
<tr>
<td>Tax rate on riskless asset investment</td>
<td>$\tau_b$</td>
<td>25%</td>
</tr>
<tr>
<td>Equity premium</td>
<td>$\mu_s$</td>
<td>0.04</td>
</tr>
<tr>
<td>Std of shocks to risky asset investment</td>
<td>$\sigma_{\varepsilon_s}$</td>
<td>20%</td>
</tr>
<tr>
<td>Nominal dividend yield</td>
<td>$d_i$</td>
<td>3.05%</td>
</tr>
<tr>
<td>Tax rate on capital gains</td>
<td>$\tau_{cg}$</td>
<td>20%</td>
</tr>
<tr>
<td>Tax rate on dividend yields</td>
<td>$\tau_{dy}$</td>
<td>25%</td>
</tr>
<tr>
<td>Employer’s contribution to retirement account</td>
<td>$k_e$</td>
<td>3%</td>
</tr>
<tr>
<td>Employee’s contribution to retirement account</td>
<td>$k_l$</td>
<td>6%</td>
</tr>
<tr>
<td>Tax rate on labor income</td>
<td>$\tau_d$</td>
<td>25%</td>
</tr>
<tr>
<td>Annual Rent</td>
<td>$\alpha$</td>
<td>7%</td>
</tr>
<tr>
<td>Property tax (housing)</td>
<td>$\tau_h$</td>
<td>1%</td>
</tr>
<tr>
<td>House maintenance and depreciation expenses</td>
<td>$\delta$</td>
<td>1%</td>
</tr>
<tr>
<td>Liquidation cost of selling house</td>
<td>$\kappa$</td>
<td>10%</td>
</tr>
<tr>
<td>Corr. bet. shocks to housing inv &amp; shocks to risky assets inv</td>
<td>$\rho_{sh}$</td>
<td>0.2385</td>
</tr>
<tr>
<td>Corr. bet. shocks to risky assets inv &amp; shocks to labor inc.</td>
<td>$\rho_{st}$</td>
<td>0.20</td>
</tr>
<tr>
<td>Corr. bet. shocks to housing inv &amp; shocks to labor inc.</td>
<td>$\rho_{hl}$</td>
<td>0.1425</td>
</tr>
</tbody>
</table>
labor income is a function of age and other individual characteristics. I define labor income widely enough to account for endogenous ways of self insurance against labor income shocks. Labor income includes total labor income, unemployment compensations, social security, total transfers, child supports, and other welfare of both head of the family and his wife if present. I estimate Equation 3.6 by ols with the PSID data. The deterministic labor income component \( f(t, Z_{i,t}) \) captures the hump shape of earnings during the working period of the life-cycle. Retired households receive a constant fraction of their last working period labor income. I set this constant rate at 0.60, which is very close to the estimated value by Cocco (2004), 0.66. The standard deviation of the idiosyncratic shocks to labor income (\( \sigma_{\varepsilon} \)) is set at 15\% as in Gourinchas and Parker (2002) and Gomes and Michaelides (2005).

### 3.4.3 Housing Investment

Yao and Zhang (2005) and Hu (2005) set the downpayment rate (\( d \)) to 20\% of the house’s market value while Cocco (2004) set it to 15\%. I set the down-payment rate at 20\% . Although the down-payment could be as low as 5\% in real life, not all households qualify for it. Renters pay 7\% of the house’s market value as annual rent\(^6\). Homeowners pay 1\% of house’s market value as the cost for maintenance and depreciation expenses and 1\% of house’s value as property tax. Similar to Cocco (2004) Hu (2005) and Yao and Zhang (2005), homeowners bear a liquidation cost (\( \kappa \)) of 10\% of house value when they sell their house. I set \( \nu \) to 0.20 so that the intratemporal substitution between housing and consumption, \( 1 / (1 - \nu) \), is 1.25 as estimated by Piazzesi et al. (2007).

The mean real log-return on housing investment (\( \mu_h \)) calculated from the Case-Shiller index is 1.9\% with a standard deviation (\( \sigma_{\varepsilon h} \)) equal to 5.7\%. By using self assessed house prices from the PSID data from 1970-1992, Cocco (2004) estimates a mean real log-return of 1.6\% with a standard deviation of 8.3\%. However, I prefer to use the Case-Shiller index because self-assessed

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\(^6\)According to Housing Statistics of the United States, in 2000 the median price specified housing units in the United States is $119,600 and monthly median rent is $602. Then a rough calculation shows an annual rent of 6.1\% of house value, close to 7\% used in this paper.
house prices might not reflect the true prices. I calibrated the probability of involuntarily moving for households from the PSID data from 1999 to 2007. The involuntarily moving rate looks like a wide U-shape. When households are very young and very old they face a higher probability of being forced to move than when they are in the middle-ages.

3.4.4 Asset Returns, Taxes, and Correlations

The before-tax real gross return on riskless assets \( R^b \) is 1.03. For the risky assets, I set the equity premium \( \mu_s \) to 4% contrary to the historical 6% to account for transaction costs. An equity premium equal to 4% instead of 6% is common in the literature (e.g. Gomes and Michaelides (2005), Campbell et al. (2001), and Yao and Zhang (2005)). Return on the risky assets is decomposed into a constant dividend yield and a stochastic capital gains. The constant nominal dividend yield \( d_i \) is 3.05% calculated from S&P500 between 1968-2010. The standard deviation of returns on the risky asset \( \sigma_{\varepsilon_s} \) is 20%. I set the tax on dividend yields \( \tau_{dy} \) at 25% and the tax on capital gains \( \tau_{cg} \) at 20% as in Gomes et al. (2009). In order to reflect the average income tax of a typical household, I set the proportional tax on labor income \( \tau_y \) at 25%. The inflation rate \( \pi \) is 3.15% obtained from the Bureau of Labor Statistics from 1926 to 2010. According to Joulfaian and Richardson (2001) the average contribution rate to TDAs is 5.9%. I assume that households make a constant contribution \( k_l \) to their TDAs which is equal to 6% of their annual labor income while employer contribution is 3%. During their retirement period, households withdraw funds from their TDAs based on the withdrawal rate. The withdrawal rate is the inverse of the maximum remaining years a household can live.

There is no consensus in the literature on the existence and the size of correlations between shocks to the returns on the investment in risky asset, shocks to labor income, and shocks.

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7 Please see Chapter 2 for a detailed comparison between the Case-Shiller index and PSID data regarding house price.

8 In the PSID surveys, households are asked the reason of moving from their previous house if they had moved since the previous year. If the reason of moving is exogeneous (i.e. to get nearer to work, response to outside events, health reasons), then I consider this one as an involuntarily moving.
Table 3.2: Correlations Between Shocks from Different Datasets

<table>
<thead>
<tr>
<th></th>
<th>PSID(^a)</th>
<th>NIPA(^b)</th>
<th>Case-Shiller(^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho_{sh}) (risky asset and housing)</td>
<td>0.019</td>
<td>0.2385</td>
<td>0.0419</td>
</tr>
<tr>
<td>(\rho_{sl}) (risky asset and labor income)</td>
<td>0.0241</td>
<td>0.1425</td>
<td></td>
</tr>
<tr>
<td>(\rho_{hl}) (housing and labor income)</td>
<td>0.0587</td>
<td>0.20</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) I use PSID data from 1999 to 2007 for all correlations in this column.
\(^b\) I obtained labor income from National Income and Products Accounts’s (NIPA) Personal Income and Outlays. I choose S&P500 for return on investment in the risky assets. Finally, I use Census Bureau’s Average Price of Houses Sold for housing investment. All datasets cover the period from 1968 to 2007.
\(^c\) I obtained housing related data from Case-Shiller Index and risky assets investment data from S&P500 Index.

to returns on housing investment. Flavin and Yamashita (2002) use the PSID data and find that there is almost no correlation between returns on the risky asset and returns on housing investment \(\rho_{sh}\). Although the authors find that the city-level Case-Shiller index conflicts with the PSID results, they claim that PSID data are nationwide while city level data aren’t. Cocco (2004), Hu (2005), and Yao and Zhang (2005) assume \(\rho_{sh}\) is 0. Furthermore, the correlation between shocks to labor income and shocks to the returns on investment in the risky asset \(\rho_{sl}\) is set to zero in Cocco (2004), Hu (2005), and Yao and Zhang (2005). However, Cocco (2004) sets the correlation between shocks to labor income and shocks to the return on housing investment \(\rho_{hl}\) to 0.553, Hu (2005) sets it to 0, and finally Yao and Zhang (2005) uses 0.2. I use different datasets in order to find good and reliable values for these correlations. Table 3.2 displays these correlations for each datasets. From these different results, I set \(\rho_{sh}\) to 0.2385, \(\rho_{sl}\) to 0.20, and \(\rho_{hl}\) to 0.1425 in the baseline model based on my judgement that they are the most trustworthy estimates. I obtained these correlations using data from the National Income and Product Accounts (NIPA) to obtain annualized labor income, S&P500 for return on investment in the risky asset, and data from the Census Bureau’s Average Price of House Sold to obtain average annual return on housing investment.

The mortgage rate, in general, is higher than the return on the riskless asset because it bears
a long-term interest risk and a default risk. However, in this model there is no risk associated with mortgages hence I set the loans on mortgage loan at 4.5% which means 1.5% mortgage rate premium to account for a proxy for a borrow-lend spread and the cost of monitoring real-estate assets. Homeowners pay back the mortgage loan over 25 years.

3.5 Results

I begin by discussing the results of the model for the baseline case. Table 3.3 displays the life cycle asset location and allocation profiles of households, risky asset participation rates and homeownership status. Contrary to the professional financial advice that the fraction of wealth that people should hold in the risky asset should decline with age (see Ameriks and Zeldes (2004) for example), the shares of investment in the risky asset in both TAs and TDAs are increasing during the working life until retirement, albeit the shares in TDAs are slightly higher. Since working-age households receive higher labor income than their retirement income and they are able to compensate losses stemming from negative shocks to the risky asset investments, they invest more in risky asset. However, the income of retired people is relatively low and investors might not have enough time to compensate losses arising from negative shocks to risky asset investment. Hence the shares of the risky asset in both accounts during the retirement period display sharp decreases. Similar patterns in the share of risky asset over the life cycle are also found in Cocco (2004), Zhou (2011), Zhou (2010).

The third row of Table 3.3 shows the share of the risky asset in TAs over the sum of the risky assets (i.e. the risky asset in TAs plus the risky asset in TDAs). Since returns on investment in TDAs are tax-free, households prefer to have more investment in their TDAs. As they accumulate more wealth in this account, the shares of risky assets in TAs decrease during the working period. However, these shares increase when households retire as they withdraw

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9Two of famous asset allocation rules are 100 Rule and 120 Rule. 100 Rule means 100 - your current age is the percentage you should be investing in risky asset and the balance should be in the riskless asset. Furthermore, the following website from Vanguard gives similar advice for portfolio allocation during the life-cycle: https://personal.vanguard.com/us/funds/vanguard/TargetRetirementList#targetAnchor
Table 3.3: Life-Cycle Profiles for the Baseline Case

<table>
<thead>
<tr>
<th>Age</th>
<th>20-35</th>
<th>36-50</th>
<th>51-65</th>
<th>66-80</th>
<th>81+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of Risky Asset in TAs(^a)</td>
<td>0.78</td>
<td>0.81</td>
<td>0.87</td>
<td>0.61</td>
<td>0.61</td>
</tr>
<tr>
<td>Share of Risky Asset in TDAs(^b)</td>
<td>0.76</td>
<td>0.88</td>
<td>0.89</td>
<td>0.69</td>
<td>0.57</td>
</tr>
<tr>
<td>Risky Asset in TAs/All Risky Assets(^c)</td>
<td>0.43</td>
<td>0.35</td>
<td>0.29</td>
<td>0.24</td>
<td>0.66</td>
</tr>
<tr>
<td>TAs Risky Asset Participation Rate</td>
<td>0.59</td>
<td>0.93</td>
<td>0.95</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>Homeownership Rate</td>
<td>0.77</td>
<td>0.93</td>
<td>0.98</td>
<td>0.98</td>
<td>0.90</td>
</tr>
</tbody>
</table>

\(^a\) The share of risky asset in TAs is calculated as \(\frac{\text{Risky Asset Investment in TAs}}{\text{Total Investment in the TAs}}\).

\(^b\) The share of risky asset in TDAs is calculated as \(\frac{\text{Risky Asset Investment in TDAs}}{\text{Total Investment in the TDAs}}\).

\(^c\) Risky Asset in TAs/All Risky Assets = \(\frac{\text{Risky Asset Investment in TAs}}{\text{Risky Asset Investment in TAs} + \text{Risky Asset Investment in TDAs}}\).

The share of risky asset in TAs and TDAs at an increasing rate. The reason that the share of the risky asset in TAs over all the risky assets is still lower between age of 65 to age of 80 is because households keep investing a significant portion of their wealth in TDAs in the risky assets for the first couple of years of their retirement period. Starting at the age of 70, this share increases sharply when households begin to change their asset allocation decision more toward riskless asset.

There is no barrier for investing in the risky asset in either accounts. However the calibrated risky asset returns are lower than the historically prevailing values in order to account for various transaction costs that the average investor faces. Since there is no entry cost associated with the risky asset investment, about 60% of households have investment in the riskless asset in TAs during the first 15 years.\(^{10}\) This rate jumps to above 90% and stays there for the rest of their life, though we observe a slight decrease during the retirement period. The reason for a relatively low participation rate in the risky asset for young households is because they have relatively low labor income compared to middle-aged households and have higher expected future labor income, against which they cannot borrow. Hence, they spend most of their labor income on consumption, the down-payment and regular annual mortgage payments. The risky

\(^{10}\)A discussion of the size and relevancy of the entry cost for investment in the risky asset is presented in Alan (2006). However, as found in Chapter 2 and ignored by Zhou (2010), and Hu (2005), this one-time entry cost doesn’t play an important role in portfolio allocation of households. Furthermore, the dimensionality of the model doesn’t allow for another state variable.
asset participation rate in TAs is, therefore, lower than the same rate for the rest of the life. When compared with the empirical data in the subsequent sections, the participation rate in the simulated data is relatively high as in Campbell et al. (2001), Gomes and Michaelides (2005), Gomes et al. (2009).

Next, I compare the baseline case to the same model with no TDAs to invest in. Table 3.4 displays the life-cycle profiles in the absence of TDAs. A comparison between Table 3.4 and Table 3.3 shows that the presence of TDAs has a crowding out effect on investment in the risky asset in TAs. The share of risky assets in TAs decreases by 5-10% if households have the opportunity to invest in TDAs. In the absence of TDAs, households are forced to invest only in TAs to accumulate enough wealth for retirement. Since the after-tax real return on investment in the risky asset is about two times higher than the after-tax return on the riskless asset, households have strong incentives to invest in the risky asset. This is the main reason for the higher shares of the risky asset relative to the shares of the riskless asset predicted by the model. In a recent paper, Gomes et al. (2009) find that in the absence of TDAs households invest 100% of their financial wealth in the risky asset and in the presence of TDAs households invest more than 70% in the risky asset. The reason for 100% investment in the risky asset in the Gomes et al. (2009)’s model is most likely due to the lack of housing investment. Our model predicts similar rates for the risky asset market participation in the absence and presence of TDAs. Likewise, the homeownership rates follow very similar patterns over the life-cycle due to the calibrated involuntarily moving force. The main impact of introducing the TDAs is the crowding out effect on risky asset investment in TAs.

Table 3.5 displays the life-cycle wealth allocation profiles in the absence of housing investment. Housing investment shows a strong crowding out effect on risky asset investment in both TAs and TDAs. The shares of the risky asset in TAs decrease by 15% to 25%. The same shares in TDAs decrease by 20% to 30%. Contrary to the baseline case, the share of risky assets in TAs over all risky assets is significantly higher. This means that households have more risky assets investment in their TAs than TDAs. Households have constant contribution to TDAs,
so the investment in this account is limited. However, the investment in the TAs isn’t limited. Households can make as much investment in their TAs as they can afford. This is the main reason of having more risky asset investment in TAs than TDAs in the absence of housing investment.

### 3.5.1 Comparative Statics

#### Changes in Contribution Rates

The model assumes constant contributions rates to TDAs from both the employee and the employer. The annual contribution rates are 6% and 3%, respectively. Since the contribution rates are exogeneously determined, analyzing any change of the size of the contribution rates on households life-cycle portfolio allocation profiles deserves attention. Table 3.6 shows the life-cycle portfolio allocation profiles of households for different values of the contribution rates.
### Table 3.6: Portfolio Allocation Scheme with Different Contribution Rates

<table>
<thead>
<tr>
<th>Age</th>
<th>20-35</th>
<th>36-50</th>
<th>51-65</th>
<th>66-80</th>
<th>81+</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ce=0.03 cl=0.06</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of Risky Asset in TAs</td>
<td>0.76</td>
<td>0.81</td>
<td>0.87</td>
<td>0.62</td>
<td>0.61</td>
</tr>
<tr>
<td>Share of Risky Asset in TDAs</td>
<td>0.76</td>
<td>0.88</td>
<td>0.89</td>
<td>0.69</td>
<td>0.57</td>
</tr>
<tr>
<td>Risky Asset in TAs/All Risky Assets</td>
<td>0.43</td>
<td>0.35</td>
<td>0.29</td>
<td>0.24</td>
<td>0.66</td>
</tr>
<tr>
<td><strong>ce=0.015 cl=0.06</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of Risky Asset in TAs</td>
<td>0.76</td>
<td>0.83</td>
<td>0.87</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>Share of Risky Asset in TDAs</td>
<td>0.74</td>
<td>0.87</td>
<td>0.87</td>
<td>0.68</td>
<td>0.57</td>
</tr>
<tr>
<td>Risky Asset in TAs/All Risky Assets</td>
<td>0.48</td>
<td>0.41</td>
<td>0.34</td>
<td>0.27</td>
<td>0.68</td>
</tr>
<tr>
<td><strong>ce=0 cl=0.06</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of Risky Asset in TAs</td>
<td>0.77</td>
<td>0.84</td>
<td>0.87</td>
<td>0.57</td>
<td>0.59</td>
</tr>
<tr>
<td>Share of Risky Asset in TDAs</td>
<td>0.79</td>
<td>0.84</td>
<td>0.87</td>
<td>0.67</td>
<td>0.57</td>
</tr>
<tr>
<td>Risky Asset in TAs/All Risky Assets</td>
<td>0.54</td>
<td>0.49</td>
<td>0.42</td>
<td>0.31</td>
<td>0.71</td>
</tr>
<tr>
<td><strong>ce=0.015 cl=0.03</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of Risky Asset in TAs</td>
<td>0.76</td>
<td>0.84</td>
<td>0.87</td>
<td>0.58</td>
<td>0.59</td>
</tr>
<tr>
<td>Share of Risky Asset in TDAs</td>
<td>0.80</td>
<td>0.83</td>
<td>0.87</td>
<td>0.67</td>
<td>0.56</td>
</tr>
<tr>
<td>Risky Asset in TAs/All Risky Assets</td>
<td>0.55</td>
<td>0.50</td>
<td>0.42</td>
<td>0.30</td>
<td>0.70</td>
</tr>
</tbody>
</table>
No difference in the share of the risky asset in TAs and TDAs is observed across different combinations of contribution rates. The life-cycle share of the risky asset in TAs increases during the working period and decreases during the retirement period. We observe similar patterns for the same share in TDAs. Households don’t change their portfolio allocation patterns within each account even when they face different contribution rates to the TDAs. However, model predictions differ slightly with respect to the share of the risky asset in TAs over all risky asset investments. As the total contribution rate (i.e. the sum of employee and employer contribution rates) increases, households make more contributions to the TDAs and the accumulated wealth in this account increases more rapidly. Then households invest more in the risky asset in this account although the share of the risky asset in this account doesn’t change much. Hence the share of the risky asset in TAs over all risky asset investments decreases as the total contribution rate increases. This effect is more observable during the working period because wealth in TDAs accumulates in this period whereas households withdraw funds during the retirement period.

**Changes in Consumption**

In this section, I analyze the effects of TDAs on household consumption during the retirement period. By investing in TDAs, households aim to sustain their working life living standards during retirement. In order to analyze whether households' living standards change when they invest in TDAs, I compared the consumption of households in the presence and the absence of TDAs. Figure 3.1 shows the consumption patterns across the two cases for the whole life cycle. The vertical axis is the level of consumption normalized by mean labor income. The figure clearly shows the positive impact of the presence of TDAs on retirement period consumption. Thanks to the savings in TDAs, households are able to sustain their working period consumption level during the retirement period. In both models, households have almost the same level of consumption until their late 50s. In the case of no TDAs, households begin to reduce their consumption smoothly before the retirement age 65. Because households are forward-looking, they don’t want to reduce sharply their consumption at retirement. Hence, we observe a smooth
transition period for adjusting consumption from the working period to the retirement period. The hump-shaped life-cycle consumption profile is also found by Villaverde and Krueger (2010). The model with TDAs shows a fairly smooth consumption pattern over the life-cycle. In this model, households withdraw funds from TDAs to use for their consumption. Hence, they can maintain their working period consumption level during the retirement period. The gap between the two models’ consumption profiles shows that TDAs play an important role in keeping the same living standards throughout the life-cycle. However, TDAs in this model aren’t sufficiently flexible. All households are forced to make constant contributions to their accounts. If households were allowed to decide to contribute or not, and if so how much, then we could have a better understanding of the effects of TDAs on household consumption profiles.

Figure 3.1: Consumption Pattern with and without TDAs
Table 3.7: When House Price is Constant

<table>
<thead>
<tr>
<th>Age</th>
<th>20-35</th>
<th>36-50</th>
<th>51-65</th>
<th>66-80</th>
<th>81+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of Risky Asset in TAs</td>
<td>0.82</td>
<td>0.92</td>
<td>0.92</td>
<td>0.74</td>
<td>0.67</td>
</tr>
<tr>
<td>Share of Risky Asset in TDAs</td>
<td>0.83</td>
<td>0.91</td>
<td>0.91</td>
<td>0.58</td>
<td>0.47</td>
</tr>
<tr>
<td>Risky Asset in TAs/All Risky Assets</td>
<td>0.50</td>
<td>0.53</td>
<td>0.47</td>
<td>0.47</td>
<td>0.78</td>
</tr>
<tr>
<td>TAs Risky Asset Participation Rate</td>
<td>0.70</td>
<td>0.98</td>
<td>0.97</td>
<td>0.97</td>
<td>0.96</td>
</tr>
</tbody>
</table>

**House Price Risk**

The baseline model assumes that house prices are risky. The riskiness of house prices has some effects on households’ life cycle portfolio allocation decisions. In order to measure the size of these effects I consider the case where house prices are riskless with the constant real return of 1.9%. Table 3.7 presents the life cycle portfolio allocation of households for this particular case. A comparison between Table 3.7 and the baseline case in Table 3.3 shows that due to the riskiness of house prices, households invest about 5%-10% less in the risky asset in TAs during the life cycle because risky housing investment is more compatible to investment in risky financial assets. A similar crowding out effect of house price risk on the risky asset investment is found by Cocco (2004). Contrary to TAs, the effects of house price risk on TDAs investment depends on whether households are retired or not. During the working period, house price risk has some crowding out effect on risky asset investment in TDAs, albeit the size is negligible. The crowding out effect is surprisingly reversed during the retirement period. In the absence of house price risk, households increase their risky asset share in TDAs by about 10% despite the expectation that they are supposed to reduce the share of the risky asset during retirement because they might not have enough time to compensate for possible losses from negative shocks to the risky asset.

The share of the risky asset in TAs over all risky assets is always higher in the absence of house price risk than the baseline model. Households invest from 6% to 23% more of their overall risky asset share in TAs if they don’t face a risk associated with housing investment. The
crowding out effect of house price risk on risky asset investment is more observable in TAs than TDAs. Hence in the absence of house price risk, households invest relatively more in the risky asset in TAs, which leads to higher shares of the risky asset in TAs over all risky assets. Lastly, the absence of house price risk leads to about a 6% increase in the risky asset participation rate.

3.5.2 Model vs Data

In this subsection, I examine the extent to which the portfolio allocation profiles presented in the model are observable in the real data. The real data are from the PSID and include all the waves that have information about financial wealth of households (i.e. 1984, 1989, 1994, 1999, 2001, 2003, 2005, and 2007). I exclude the Survey of Economic Opportunity (SEO) sample from the data to have a random sample of the US population. For the PSID data, I define total investment in the risky asset in TAs as the sum of stocks in publicly or privately held corporations including those held in mutual funds and investment trusts. I define the total riskless asset in TAs as the sum of bond funds, money in saving and checking accounts, money market funds, certificates of deposits, government saving bonds, bond funds, Treasury bills, other savings or assets, such as cash value in a life insurance policy and a valuable collection for investment purposes. In order to obtain a good measure of risky and riskless allocation in TDAs, I use the answer that households give for the allocation of their wealth in TDAs. Depending on how households respond to the question “Are TDAs investment mostly in stocks, mostly in interest earning assets, split between the two, or what?”: (1) if investment is mostly in stocks I put all investments in the risky asset, (2) if investment is mostly in interest earning assets I put all investment in the riskless asset, and finally (3) if the household splits between the two I split between risky and riskless equally.\(^\text{11}\) All values are adjusted to their current corresponding

\(^{11}\)For a comparison, I also looked at the Survey of Consumer Finances (SCF) data for 2007. The portfolio allocation profiles between both PSID and SCF data are very similar. The only difference is in the share of risky asset in TAs over all risky assets (i.e both risky TAs and risky TDAs). The allocation of TDAs between risky and riskless assets in both datasets is equally noisy. Since except for this ratio, all other relevant profiles are quite similar across both datasets, I use PSID data because it has information for multiple waves.
Table 3.8: Life-Cycle Portfolio Allocation Profiles in PSID

<table>
<thead>
<tr>
<th>Age</th>
<th>20-35</th>
<th>36-50</th>
<th>51-65</th>
<th>66-80</th>
<th>81+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of Risky Asset in TAs</td>
<td>0.57</td>
<td>0.62</td>
<td>0.64</td>
<td>0.63</td>
<td>0.54</td>
</tr>
<tr>
<td>Share of Risky Asset in TDAs</td>
<td>0.74</td>
<td>0.76</td>
<td>0.73</td>
<td>0.70</td>
<td>0.66</td>
</tr>
<tr>
<td>Risky Asset in TAs/All Risky Assets</td>
<td>0.57</td>
<td>0.51</td>
<td>0.51</td>
<td>0.49</td>
<td>0.56</td>
</tr>
<tr>
<td>TAs Risky Asset Participation Rate</td>
<td>0.19</td>
<td>0.33</td>
<td>0.46</td>
<td>0.47</td>
<td>0.33</td>
</tr>
<tr>
<td>Homeownership Rate</td>
<td>0.37</td>
<td>0.67</td>
<td>0.80</td>
<td>0.84</td>
<td>0.56</td>
</tr>
</tbody>
</table>

values using the CPI.

Table 3.8 presents some of the life-cycle profiles of households obtained from the PSID data. The data show that the share of risky assets in TAs follow a pattern similar to the one predicted by the model. While model predictions for this share is about 20% higher than the empirical evidence for the working period life, the predictions for the retirement period are very close to the empirical evidence. Although the model predictions are higher than the empirical data, the gap is lower than in some previous studies such as Cocco (2004), Gomes and Michaelides (2005), and Zhou (2011). The model predicts very close values for the share of risky asset in TDAs for both initial working period and retirement periods. However, the model over-estimates this share for people between 36-65 by about 15%. Nevertheless, the model predictions for the risky asset share in TDAs have a very similar pattern to what we observe in the data. Households increase their risky asset investment in TDAs when they are working. Upon retirement, they reduce the investment in the risky asset by investing more in the riskless one.

One big difference between the model and the data stems from the share of risky asset in TAs over all risky assets. The model predicts a declining share for the risky asset in TAs over all risky assets during the working period. Once households retire, the amount of wealth in TDAs decreases since households withdraw funds from TDAs. Hence the share of the risky asset in TAs increases as less and less wealth remains in TDAs. On the other hand, empirical evidence from the PSID data shows an almost constant share over the life-cycle.\textsuperscript{12} Another important

\textsuperscript{12}SCFs give a very similar, almost constant share of risky asset in TAs over all risky assets. A big problem
difference between the model and the empirical data comes from the risky asset participation rates. Although the participation rates follow similar patterns on both of them, there is a significant difference in the size of the participation rate over the life cycle. The predictions of the model are about 50% higher than the empirical evidence obtained from the PSID data. The participation rate from the Survey of Consumer Finances (SCF) is much higher than the participation rate obtained from the PSID data and closer to the model predictions\textsuperscript{13}.

The last comparison between the model predictions and the empirical evidence is the life-cycle homeownership status. Homeownership rates increase when households are working and decrease when they are retired. While both the real data and the model follows the same pattern, there is a gap between these two, where the model over-estimates the homeownership rate.

### 3.6 Conclusion

I have examined a comprehensive life-cycle portfolio allocation model. The main contribution of the paper is combining both TDAs and housing investment into the life-cycle model in order to have a more realistic framework to analyze household portfolio allocation patterns. The findings of the paper show that the presence of TDAs and housing investment have some crowding out effects on investment in the risky asset in TAs. The effect of housing investment is stronger than the effect of TDAs. Consistent with the objective of TDAs, the wealth accumulated in TDAs is used for the retirement period consumption. Furthermore, in the absence of house price risk households tend to invest more in the risky asset in both accounts. While changes in the size of the contribution rate doesn’t affect the asset allocation decision of households, it does have some impact on asset location decision. The share of risky assets in TAs over all risky asset

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\textsuperscript{13}The risky asset participation rate for our 5 age-groups in SCFs are 0.57, 0.67, 0.75, 0.74, and 0.68. These rates don’t differ much from the model predictions.
investments decreases as the contribution rate to the TDAs increases.

Although the model is quite comprehensive, several extensions can be made for future work. Instead of a constant contribution rate to TDAs, a better assumption would be to allow households to make the contribution decisions endogeneously. Another extension would be to distinguish households according to their perception to risk as in Gomes et al. (2009), Attanasio et al. (2002), Guvenen (2009b) in order to address higher participation and investment rates in the risky assets. Finally, instead of applying a flat tax-rate to everyone, replacing it with a more realistic progressive tax-rate would likely increase the benefits from TDAs as retired people are subject to lower tax-rate than working people.
Conclusion

Households wealth allocation throughout the life-cycle has been analyzed in the portfolio allocation literature since the pioneering study of Merton (1969) and Samuelson (1969). An important puzzle that the literature currently has not answered yet, namely the asset or portfolio allocation puzzle, is analyzed in this dissertation. A main drawback of the literature is that it has focused only on financial assets by disregarding durable and illiquid assets for a long time. The literature began considering durable and illiquid assets by Grossman and Laroque (1990). Although there has been some studies on this topic since Grossman and Laroque (1990), the theoretical models still predicts exceptionally high stock market participation rate as well as even higher proportion of financial wealth invested in the risky assets. Contrary to these predictions, the empirical evidence shows low level of stock market participation rate and moderate level of equity holding.

This dissertation introduces a comprehensive housing investment dynamic into otherwise standard life-cycle portfolio allocation models, addresses the gap between the theoretical model predictions and the empirical evidence. Housing investment is unique in the sense that it is not only a durable and illiquid good that households derive utility from but also an investment tool that they enjoy return from. Hence, once a comprehensive housing investment is incorporated into a life-cycle model, households begin to invest relatively less in the risky asset and the participation rate to this market decrease as well. In the last chapter, this dissertation takes a step further and incorporates both housing investment and tax-deferred accounts (TDAs) into a life-cycle model. The results show that both housing investment and TDAs have crowding out effects on the risky asset investment of households. The main contribution of this dissertation is that I obtained moderate level of risky asset market participation rate as well as moderate level of equity holding. Hence, a fairly comprehensive treatment of housing investment and TDAs are crucial to address the aforementioned gap between the theoretical model predictions and empirical evidence.
Although the models presented in this dissertation are clearly comprehensive, there are some possible extensions. Instead of a constant contribution rate to TDAs, a better approach would be to allow households to make the contribution decisions endogenously. Another extension would be to provide household heterogeneity according to their risk perception. The models employed in this dissertation are in the partial equilibrium settings. One can use a similar model in a general equilibrium setting through the inclusion of labor demand market, house supply market, and government. Furthermore, including the default risk on mortgage and a detailed analysis on the possibility of simultaneous debt and positive amount of bond holding remain for future.
REFERENCES


APPENDICES
Appendix A

PSID Data

PSID is the longest running longitudinal household survey in the US. Starting in 1968 with a nationally representative sample of over 5000 families, information on these families and their descendants has been collected continuously. However, the original PSID sample isn’t exactly a nationally representative sample of US population because it contains an over-sample of roughly 2000 poor families selected from the Survey of Economic Opportunities (SEO). Hence, I first drop the SEO sample from the PSID and then use this adjusted PSID as my data source. Not all PSID waves contains information about financial wealth of households. For this reason, I only use the 1984, 1989, 1994, 1999, 2001, 2003, 2005, and 2007 waves that provides financial wealth of households.

I construct a measure for labor income that closely matches the labor income defined in the model. I define labor income from the PSID as the sum of wages and salaries (ER40903), any separate reports of bonuses (ER40905), overtime (ER40907), tips (ER40909), commissions (ER40911), professional practice or trade (ER40913), market gardening (ER40915), additional job income (ER40917), and miscellaneous labor income (ER40919). I use this definition of labor income for head of the family and if present his/her spouse. The variables in the risky asset are made up of shares of stock in publicly held corporations, mutual funds, or investment trusts (ER37566), the share of private annuities or Individual Retirement Accounts (IRAs)
(ER37587) invested in the risky asset, dividend income of head (ER37019) and his spouse (ER37352). Riskless asset, on the other hand, is composed of bond funds, cash value in a life insurance policy, a valuable collection for investment purposes, or rights in a trust or estate (ER37615), interest earnings of head (ER37035) and his/her spouse (ER37368), the share of private annuities or Individual Retirement Accounts (IRAs) (ER37587) invested in the riskless asset, money in checking or savings accounts, money market funds, certificates of deposit, government savings bonds, or Treasury bills (ER37594).

Since Chapter 3 makes a distinction between taxable and tax-deferred accounts, I construct measures for these accounts from PSID data to match them as closely as possible. Not all PSID waves has distinct information about tax-deferred accounts. Hence, I use the waves that explicitly provides this information (i.e 1999, 2001, 2003, 2005, 2007). I define risky asset in taxable account as the sum of wealth in non-IRA stocks (ER37567), dividend income of both head and his wife (ER37019 and ER37352); taxable riskless asset as the sum of bond funds (ER37615), money in saving and checking accounts, money market funds, certificates of deposit, government saving bonds, or Treasury bills (ER37594), interest earning of head and wife (ER37036 and ER37369). Wealth in tax-deferred account in PSID is ER37589. In order to obtain the share of tax-deferred account wealth in risky assets, I use the questions that asks how households invets their tax-deferred wealth (ER37588). If households invets mostly in stocks (interest earnings), then their tax-deferred wealth is treated as risky (riskless) asset. However, if household split their tax-deferred wealth between the two, then half of this weath goes to risky assets and the other half goes to the riskless asset.
Appendix B

Solution Method

I begin by discretizing the state space and variables over which the choices are made with equally spaced grids. The density functions of the random variables (i.e. shocks to labor income process, shocks to return on risky asset, and shocks to return on housing investment) are approximated by the Gaussian quadrature method of Tauchen and Hussey (1991).

In period $T+1$, the policy functions are determined by the bequest motive. The value function in this period coincides with the utility function, which is the bequest function. In every period prior to $T+1$, I obtain the utility function for different combinations of housing, consumption, and other state and choice variables. Then the value function for a typical time $t$ is equal to the utility function of that period plus the discounted expected continuation value ($E_t (V_{t+1})$). If the continuation value doesn’t lie on the state space grid, I compute the value function using cubic spline interpolation. This backward induction process is iterated from time $T$ to 1.

Once I compute the value function of all the alternatives, I choose the one that maximizes the value function over all choice variables. The optimum policy rules for consumption, housing, and investment in financial assets correspond to ones that maximize the value function. At each point in the state space, the risky asset participation decision is done by comparing the value function conditional on having paid the fixed cost with the value function conditional on having
not yet paid the fixed cost. Similarly, the homeownership decision (e.g. house buying or selling decision) is done by comparing the value function conditional on being a renter with the value function conditional on being a homeowner. In both comparisons, adjustments for the payment of the fixed cost of risky asset participation and costs accrued from buying/selling a house (e.g. downpayment, annual mortgage payment, liquidation cost etc.) are taken into account respectively.