ABSTRACT

SAMAYOA SANZ-AGERO, MARTIN JOSE. Electromagnetic Membrane-based Aeroelastic Energy Harvesting. (Under the direction of Dr. Paul H. Cohen and Dr. Jingyan Dong).

The reduction of power consumption in modern wireless systems provides the basis for replacing batteries with energy harvesters. Energy harvesters offer promise in reconciling the fundamental paradox of wireless systems, which regards power supply. This investigation describes a novel small-scale energy harvesting system for extracting and converting kinetic energy in moving fluids, in particular low-speed airflow. The extraction approach consists on exploiting a nonlinear aeroelastic phenomenon known as dynamic stall. To this end, this investigation proposes a novel elastic membrane design in stimulating self-sustained aeroelastic oscillations. Electromagnetic transducers are then employed in converting the resulting mechanical energy into the electrical domain. In optimizing the proposed energy harvesting system, mathematical, computational and empirical models are constructed. The resulting power density and efficiency of an optimized system is then evaluated against relevant benchmarks.
Electromagnetic Membrane-based Aeroelastic Energy Harvesting

by
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DEDICATION

To my parents, Clara Isabel Sanz-Agero Nanne and Luis Fernando Samayoa Delgado.
Martin Samayoa was born in Guatemala City, Guatemala on January 15th, 1984. He attended the American School of Guatemala (CAG) for his primary and secondary schooling. After graduating with a Bachelors of Science degree in Mechanical Engineering from Purdue University in 2005, the author enrolled at Penn State University in pursuit of a Master of Science degree in Industrial Engineering. At Penn State, Martin began his research trajectory in the areas micro-fabrication and nanoscale material testing under the supervision of Dr. Paul H. Cohen. Upon graduating in 2007, he joined Dr. Cohen at North Carolina State University’s department of Industrial and Systems Engineering as a Doctoral student. At NC State, Martin engaged in research activities related to bio-manufacturing and energy harvesting systems. His work has been both published and presented. As a graduate student, Martin was inducted into Tau Beta Pi, Alpha Pi Mu, Phi Kappa Phi and the Golden Key International Honor Society.
ACKNOWLEDGMENTS

This work would not have been possible without the guidance, patience and generous support of my advisor, Dr. Paul H. Cohen. It is hard to find the appropriate words to thank him enough for making this journey possible. His leadership, attention to detail and hard work have set an example I hope to match someday.

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I would like to thank Öykü Aşıkoğlu for inspiring me every step of the way. Without her love and encouragement, I would not have the fortitude to come this far. We walked along this path together and while it feels like the end, it’s just the beginning.

It has been a pleasure and privilege to have as my companions Tim Horn, Guha Manogharan, Chuang Wei, Li Zhang and Rodrigo de la Fuente. Your friendship and kindness made me feel at home every day. Thanks for being there on the good days and the bad.

Most importantly, I would like to acknowledge my loving parents, Clara Isabel Sanz-Agero and Luis Fernando Samayoa, who from a young age instilled in me the conviction to choose my own path. I am forever grateful to them for providing me with the best of opportunities and unconditional support.
# TABLE OF CONTENTS

List of Tables ........................................................................................................ viii

List of Figures ......................................................................................................... ix

Chapter 1: Introduction .......................................................................................... 1

Chapter 2: Literature Review ................................................................................. 4
  2.1 Energy Harvesting Systems ........................................................................ 4
  2.2 Vibration-Based Extraction ........................................................................ 6
    2.2.1 Cantilever Extraction ......................................................................... 7
    2.2.2 Atmospheric Wind Energy and Aeroelastic Extraction ................. 10

Chapter 3: Aeroelastic Energy Harvesting .......................................................... 18
  3.1 Designs for Aeroelastic Extraction .............................................................. 19
  3.2 Designs for Electromagnetic Conversion ............................................... 20
  3.3 Power Density and Economic Analysis ................................................... 22

Chapter 4: Mathematical Modeling ..................................................................... 24
  4.1 Solid Modeling ............................................................................................ 24
  4.2 Electromagnetic Modeling ......................................................................... 31
  4.3 Aerodynamic Modeling ............................................................................. 37

Chapter 5: Computational Modeling.................................................................... 46
  5.1 Model Setup ............................................................................................... 46
  5.2 Overall Model Behavior ............................................................................ 58

Chapter 6: Design and Fabrication ...................................................................... 69
  6.1 Thin Membrane .......................................................................................... 70
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.2</td>
<td>Coil Stator</td>
<td>72</td>
</tr>
<tr>
<td>6.3</td>
<td>Thin Airfoil</td>
<td>75</td>
</tr>
<tr>
<td>6.4</td>
<td>Permanent Magnets</td>
<td>77</td>
</tr>
<tr>
<td>6.5</td>
<td>Rigid Fixture</td>
<td>79</td>
</tr>
<tr>
<td>6.6</td>
<td>Coil Fixture</td>
<td>80</td>
</tr>
<tr>
<td>7.1</td>
<td>Airfoil Planform Study</td>
<td>84</td>
</tr>
<tr>
<td>7.2</td>
<td>Coil Position Study</td>
<td>88</td>
</tr>
<tr>
<td>7.3</td>
<td>Airfoil Shape Study</td>
<td>93</td>
</tr>
<tr>
<td>7.4</td>
<td>Angle of Attack and Airspeed Study</td>
<td>96</td>
</tr>
<tr>
<td>7.5</td>
<td>Electromagnetic Damping Study</td>
<td>101</td>
</tr>
<tr>
<td>7.6</td>
<td>Moment if Inertia Study</td>
<td>105</td>
</tr>
<tr>
<td>8.1</td>
<td>Independent Design Guidelines</td>
<td>112</td>
</tr>
<tr>
<td>8.2</td>
<td>Dependent Design Guidelines</td>
<td>114</td>
</tr>
<tr>
<td>8.3</td>
<td>Design Process and Power Results</td>
<td>116</td>
</tr>
<tr>
<td>8.4</td>
<td>Cost of Power</td>
<td>122</td>
</tr>
<tr>
<td>8.5</td>
<td>Scaling Effect</td>
<td>124</td>
</tr>
<tr>
<td>9.1</td>
<td>Summary</td>
<td>126</td>
</tr>
<tr>
<td>9.2</td>
<td>Research Contributions</td>
<td>128</td>
</tr>
<tr>
<td>9.2.1</td>
<td>Torsional Electromagnetic Aeroelastic Energy Harvesting</td>
<td>129</td>
</tr>
</tbody>
</table>
9.2.2 Aeroelastic Modeling................................................................. 129
9.2.3 Design Methodology ............................................................. 129
9.2.4 Device Extensions ................................................................. 130
9.2.5 Broader Contributions ........................................................... 130

9.3 Future Work .............................................................................. 131

References....................................................................................... 133
LIST OF TABLES

Table 2.1 Cantilever Extraction of Mechanical Energy .............................................. 7

Table 2.2: Vibration Sources for Cantilever Extraction.............................................. 8

Table 2.3: Cantilever based Extraction .................................................................... 10

Table 7.1: Constant Planform Airfoil Geometries .................................................... 85
LIST OF FIGURES

Figure 2.1: Basic Subsystems of an Energy Harvesting System ........................................ 5
Figure 2.2: Generic Vibration based Extraction Linear Model ....................................... 8
Figure 2.3: Unsteady Aerodynamics ........................................................................ 12
Figure 3.1: Proposed Design .................................................................................... 19
Figure 3.2: Representation of Simplified System ...................................................... 20
Figure 3.3: Proposed Configuration for Electromagnetic Transducer ....................... 21
Figure 3.4: Energy Harvesting Design and Evaluation ............................................. 22
Figure 4.1: Simple Fiber Subject to a Transverse Force ........................................... 25
Figure 4.2: Membrane Mid-Point Section Subject to Torsional Deformation .......... 27
Figure 4.3: Nonlinear Spring Constant of Torsional Membrane ............................... 31
Figure 4.4: Two Dimensional Induction Coils ......................................................... 33
Figure 4.5: Three Dimensional Induction Coils ....................................................... 36
Figure 4.6: Typical Airfoil Section ........................................................................... 38
Figure 4.7: Phase Space Response of Aeroelastic Oscillator ................................. 44
Figure 4.8: Phase Space Plots of LCO’s ................................................................. 45
Figure 5.1: NACA 0012 in Fluid Domain ................................................................. 48
Figure 5.2: Bias Meshing ......................................................................................... 49
Figure 5.3: Bias Meshing Specification .................................................................... 50
Figure 5.4: Airfoil Element Sizing .......................................................................... 51
Figure 5.5: Decoupled Mesh Interface for Fluid Domains ....................................... 52
Figure 5.6: Symmetrical Boundary Conditions on Fluid Domain ............................ 53
Figure 8.5: Optimized Membrane-Based Aeroelastic Energy Harvester ................. 120
Figure 8.6: Rotary Devices with Similar Sweep Areas ........................................... 121
Figure 8.7: Rotary Device with Similar Sweep Area at 5 m/s ...................................... 122
Figure 8.8: Scaled Down System with Planform Area of 0.0013 m² (2 in.²) ............. 124
Chapter 1: Introduction

The Information Age may be characterized by limitations of information flow. The ongoing achievements in both computer hardware and software engineering over the past decades have broadened the information pipeline to what it is today. This has physically manifested itself in the development of countless novel computer systems and broader communication networks. The emergence of wireless technologies further relaxed traditional limitations by enabling computer systems to interact within networks remotely. This transition has prompted interest in enabling additional systems to participate in wireless networks. In order to achieve this objective, the power paradox of wireless systems must be resolved. Energy harvesters have been proposed in attempt to reconcile this paradox.

Energy harvesting is the century old practice of extracting and converting raw energy from the environment for the purpose of consumption or storage. In contrast to traditional power generation, energy harvesting focuses on accessing the virtually inexhaustible sources (e.g. vibrational, thermal, photonic, etc.) with minimal adverse environmental effects as a consequence. This practice has recently been extended to encompass numerous energy sources at multiple scales. Small-scale energy harvesting has particularly become relevant due to the low power consumption of miniaturized electronics. The objective is to deliver economically feasible energy systems in order to reduce and replace battery dependency. To accomplish this, energy harvesting systems have been proposed.

For instance, energy harvesting eels [1] have been developed for the remote extraction and conversion of kinetic energy found in flowing streams and rivers into electrical energy. These simple yet novel devices, which are fabricated from inexpensive
piezoelectric polymers, are subject to time-varying mechanical stresses within flowing water that arise due to von Karman vortices forming behind bluff bodies. The result is a long piezoelectric membrane which oscillates in the same way an eel swims or a flag flutters behind a flagpole. This simplistic approach illustrates the role of innovation and creativity in the search for feasible energy harvesting solutions.

Additional novel solutions examine the use of human energy as a power source. To be precise, researchers [2] have developed shoe inserts which employ piezoelectric materials for harvesting human biomechanical energy. These simple devices extract and convert time-varying mechanical stresses found on ordinary shoes during walking and/or running. Despite the basic mechanisms, the literature suggests that energy harvesting research lies at the intersection between several disciplines, mainly industrial, materials, electrical and mechanical engineering.

These novel techniques which exploit complex natural phenomena in extracting energy from available resources highlight the vast solution space corresponding to the energy harvesting feasibility problem. Ironically, current research has primarily focused on exploring very particular areas of this solution space. As such, only a limited number of methods have been examined. Some investigations however, decide to venture into new domains. For example, vibration based energy harvesters that provide both nonlinear reaction forces while minimizing structural damping by use of diamagnetic levitation have been proposed [3]. These systems demonstrate that complex phenomena may be successfully harnessed by novel energy harvesting solutions.
This investigation presents a novel membrane-based torsional aeroelastic energy harvesting approach for harvesting low-speed airflow. The solution consists of extracting kinetic energy from atmospheric air by use of nonlinear aeroelastic phenomena and converting it into the electrical domain by employing electromagnetic transduction. This work was motivated by the desire to improve efficiency in harvesting kinetic energy from low-speed airflow. To overcome standing limitations in existing approaches, this investigation set forward to examine a novel vibrations-based torsional solution. The proposed solution is modeled, optimized and a functional prototype tested.
Chapter 2: Literature Review

This chapter aims at examining the literature within the energy harvesting field. In order to do so, the chapter has been divided into multiple sections for examining both the breadth and depth within the field in relation to the proposed solution. The review begins by considering energy harvesting from a general standpoint provided that it is a broad field. As such, the first section examines the building blocks of existing energy harvesting systems. The goal of this section is to examine the breadth within the energy harvesting field.

By first identified the building blocks of energy harvesters, it was then possible to examine how designs may vary. Provided that a large number of energy harvesting systems have been proposed and examined, it was unfeasible to review all devices in detail. For that reason, this chapter aims at examining relevant accomplishments in energy harvesting in relation to the proposed solution. In other words, the literature was reviewed in depth for particular subsystems that are related to the proposed solution.

2.1 Energy Harvesting Systems

Energy harvesting systems comprise of at minimum four basic subsystems. These include the following: extraction, conversion, management and storage, as shown in Figure 2.1. Over the last century, several extraction and conversion subsystems have been proposed to interact with numerous energy sources at multiple scales. Some of the most well-known subsystems include photovoltaics (solar cells) and windmills which aim at exploiting solar energy and atmospheric wind, respectively.
More recently, additional novel subsystems have been investigated within energy harvesting. This includes micro-thermoelectric generators [4] that harvest temperature gradients, micro-cantilevers [5] that harvest ambient vibrations by use of electromagnetism and more. Essentially, advances in materials, computer modeling and fabrication have provided vast opportunity for developing energy harvesting solutions that take advantage of multiple energy sources available within our environment. Novel energy harvesting systems are developed by investigating specific subsystems or a combination thereof.

Another alternative for examining energy harvesting systems is by dichotomizing overall systems into energy extraction-conversion modules and energy management-storage modules. Although equally important, each module may be regarded as a sub-problem of the overall energy harvesting problem. In practice, an arbitrary output, provided by the energy extraction-conversion module, is supplied to the energy management-storage module by conductors. It is the objective of the energy management-storage module to retain and store with minimal losses the highly volatile output power provided by the energy extraction-conversion module. Thus, while the energy extraction-conversion module aims at
maximizing the energy density output, the energy management-storage module seeks to maximize the energy retention and storage efficiency.

The following sections detail major research accomplishments in relation to energy extraction-conversion modules in energy harvesting. The energy extraction-conversion module is considered the critical component in energy harvesting systems provided it sets an upper limit in power density. That is, a downstream system may only incur power losses due to inefficiencies in the energy management-storage module. Hence, without loss of generality, it is the objective of this investigation to mainly focus on energy extraction-conversion modules. It should be emphasized that significant achievements have been accomplished in power management strategies, particularly in the area of active power management [6-10]. Active power management strategies employ synchronized switching circuitry for maximizing the flow of energy into storage sub systems.

2.2 Vibration-Based Extraction

Vibration based energy harvesting focuses on the extraction and conversion of raw kinetic energy from the environment. The term vibration is generally employed in describing periodic or random oscillations of a system about its equilibrium point. The vibrations arise as a result of single or continuous mechanical interferences to a system. In vibration based energy harvesting applications, the interference typically arises from kinetic energy in a surrounding environment. Previous investigations primarily focus on kinetic energy found in commercial and industrial environments, transportation mediums (automobile, aircraft, etc.) and fluidic systems (wind and water flow).
The followings sections review major topics of interest related to the extraction of usable energy by means of vibrations. The first section corresponds to the well-studied use of cantilevers for the extraction of mechanical energy from the environment. This has been the predominant area of research within the energy harvesting field, as shown in Table 2.1. Alternately, the second section introduces aeroelasticity and its potential for extracting kinetic energy from moving fluids. Even though both extraction mechanisms are very different, they both use mechanical vibrations as an intermediary for converting energy into the electrical domain.

Table 2.1: Cantilever Extraction of Mechanical Energy

<table>
<thead>
<tr>
<th>Papers</th>
<th>References</th>
<th>Conversion</th>
</tr>
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<tbody>
<tr>
<td>3</td>
<td>[36,43,46]</td>
<td>Piezoelectric/Electromagnetic</td>
</tr>
<tr>
<td>1</td>
<td>[32]</td>
<td>Electromagnetic</td>
</tr>
</tbody>
</table>

2.2.1 Cantilever Extraction

The extraction of mechanical energy by means of cantilevers is well known both for its modest modeling requirements and fabrication. Traditionally, a beam supported at only one end is regarded as a cantilever. In contrast to other structures, a cantilever transfers all loading to its supported end which resolves into moments and shear stresses. Vibrations in cantilevers result from interactions that arise between inertial, damping, elastic and external forces applied to the system. The approach is typically simplified into the classic forced mass-spring-damper system. By doing so, researchers [12] have been able estimate the
theoretical power availability from known external forces (vibration sources). Several common vibration sources are quantified in Table 2.2.

Table 2.2: Vibration Sources for Cantilever Extraction [11]

<table>
<thead>
<tr>
<th>Vibration Source</th>
<th>Peak Acceleration (g)</th>
<th>Frequency at Peak (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base of a 5 HP 3-axis machine tool.</td>
<td>1.0194</td>
<td>70</td>
</tr>
<tr>
<td>Kitchen blender casing.</td>
<td>0.6524</td>
<td>120</td>
</tr>
<tr>
<td>Clothes dryer.</td>
<td>0.3568</td>
<td>120</td>
</tr>
<tr>
<td>Door frame after closing door.</td>
<td>0.3058</td>
<td>125</td>
</tr>
<tr>
<td>Small microwave oven.</td>
<td>0.2294</td>
<td>120</td>
</tr>
<tr>
<td>HVAC vents in office building.</td>
<td>0.1784</td>
<td>60</td>
</tr>
<tr>
<td>Wooden deck with people walking.</td>
<td>0.1325</td>
<td>385</td>
</tr>
<tr>
<td>Windows (0.6m²) next to busy street.</td>
<td>0.0714</td>
<td>100</td>
</tr>
<tr>
<td>Notebook computer while walking.</td>
<td>0.0612</td>
<td>75</td>
</tr>
<tr>
<td>Washing machine.</td>
<td>0.0510</td>
<td>109</td>
</tr>
<tr>
<td>Second story floor of a wood frame office building.</td>
<td>0.0204</td>
<td>100</td>
</tr>
<tr>
<td>Refrigerator.</td>
<td>0.0102</td>
<td>240</td>
</tr>
</tbody>
</table>

A typical extraction by means of a cantilever results in a vibrating mass exchanging kinetic energy to potential energy and vice versa, with a local spring. In energy harvesting applications [13-14] the vibrating mass is typically regarded as an end-mass on the cantilever beam and the restoring force (local spring) is provided by the cantilever structure, as depicted in Figure 2.2.

![Figure 2.2: Generic Vibration based Extraction Linear Model [10]](image-url)
Subject to external vibrations, both the motion of the end-mass and/or the elastic deformation of the cantilever structure forms the basis for discerning amongst the different energy conversion approaches. The major conversion approaches implement at least one of the following transducers [15]: electromagnetic, electrostatic and/or piezoelectric. From these basic fundamentals, several cantilever based energy harvesting systems, both at the micro-scale [16-33] and macro-scale [34-52] have been constructed.

The generalized models serve as a useful tool in estimating the power output of energy harvesting systems. The simplest configuration, which is shown in Figure 2.2, consists of at least mass, a spring and two dampers (one damper represents the mechanical resistance against motion \((b_m)\) and the other the mechanical resistance against energy conversion \((b_e)\)). The theoretical power output is simply resolved as the power dissipated by the conversion damper \((b_e)\) during motion. By resolving the equation of motion and assuming the system resonates at the unique frequency of the input vibrations, a theoretical expression for the power output is attained [11]:

\[
P = F_e \int_0^z \dot{z} dz = \frac{1}{2} b_e \dot{z}^2 = \frac{m \xi_e A^2}{4 \omega (\xi_e + \xi_m)^2}
\]  

(2.1)

This basic expression provides insight into the effects of input acceleration \(A\), frequency \(\omega\), mass \(m\), mechanical damping ratio \(\xi_m\) and electrical damping ratio \(\xi_e\) on power conversion. In maximizing power output, the end mass should increase as much as possible without violating volumetric constraints, the vibration source should encompass high
accelerations at low frequencies and the mechanical damping should be diminished to its limit while matching the electrical damping.

These results are verified in Table 2.3 by comparing two micro PZT piezoelectric energy harvesters. The sensitivity to input acceleration is such that an increment from 0.75 g to 2 g amplifies the power output of the system by a factor of 7. These results emphasize the importance of achieving high accelerations in working with energy harvesting systems. Both high input frequencies and large amplitudes are desirable.

Table 2.3: Cantilever based Extraction

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>PZT Piezoelectric [27]</td>
<td>0.00169 g*</td>
<td>0.75 g</td>
<td>183.8 Hz</td>
<td>0.32 µW</td>
<td>416 µW/cm²</td>
</tr>
<tr>
<td>PZT Piezoelectric [24]</td>
<td>0.00135 g*</td>
<td>2 g</td>
<td>461.15 Hz</td>
<td>2.15 µW</td>
<td>3272 µW/cm²</td>
</tr>
</tbody>
</table>

*Estimated from data in reference.

2.2.2 Atmospheric Wind Energy and Aeroelastic Extraction

The harvesting of kinetic energy from atmospheric wind traces back centuries. The use of rotational sails or blades was, and still remains, the predominant extraction mechanism in the field. Well established for its simplicity and potential applications, wind mills have been constructed at multiple scales around the world. In these systems, the kinetic energy is typically extracted as rotational energy and is further converted to manageable kinetic or potential energy. The kinetic energy in atmospheric wind is easily derived as:
\[ P = \frac{1}{2} A_{\text{swept}} \rho_{\text{air}} V^3 \quad (2.2) \]

where \( A_{\text{swept}} \) represents the swept cross sections of the flow, \( \rho_{\text{air}} \) is the local density of the air and \( V \) is the velocity of the flow. Common models for modern wind turbines seek to optimize the size of windmills (\( A_{\text{swept}} \)) subject to economic (cost) objective functions.

Another important expression, which regards the kinetic energy density of a flowing fluid, e.g. atmospheric wind, is the following:

\[ P_{\text{density}} = \frac{1}{2} \rho_{\text{air}} V^3 \quad (2.3) \]

It is easily shown that the relative power density of a flowing fluid is extremely sensitive to its velocity and linearly related to density. There is, however, considerable kinetic energy in atmospheric wind. For example, the kinetic energy density of a 4.47 m/s (10 mph) wind at standard temperature and pressure (\( \rho = 1.29 \text{ kg/m}^3 \)) is 57.62 W/m². The efficiency at which this kinetic energy may be converted into potential energy is dependent on two major design factors, mainly the energy extraction approach (activation phenomenon) and the energy conversion (transducer) method.

Aeroelasticity is a field in science that studies the interaction between inertial, elastic and aerodynamic forces on structural elements. In reality, any system that is subject to aerodynamic forces may be considered an aeroelastic system provided that by necessity it must comprise of a mass and supporting structure. The field was established by aerospace engineers concerned with minimizing dynamic stability problems associated with aircraft.
But its potential applications do not end with aircraft design. This proposal extends the aeroelastic science into the energy harvesting domain.

Both rigid aeroelastic energy harvesters (wind mills and wind turbines) and low stiffness aeroelastic systems have the potential to result in self-sustained oscillations. One important characteristic of aeroelastic systems regards their susceptibility to positive feedback dynamics. That is, aerodynamic loading can increases with structural deformation. For example, the angle of attack of an airfoil increases with aeroelastic deformation, which in turn increases the original aerodynamic loading. Depending on the interaction between the inertial, elastic and aerodynamic forces the result can be negligible or catastrophic.

It should be emphasized that aeroelastic systems may result in self-sustained oscillations for different reasons. The most important and commons sources of instability, encountered on rigid aeroelastic structures such as aircraft, are collectively studied under incompressible unsteady aerodynamics, which includes both quasi-steady and wake vortex effects. Quasi-steady aerodynamics studies the simplest model, which consist of an airfoil in dynamic pitching and plunging motion. Furthermore, unsteady aerodynamics seeks to generalize the quasi-steady model by including the instantaneous effects of a vortex street behind the airfoil, as shown in Figure 2.3

![Figure 2.3: Unsteady Aerodynamics](image-url)
In symmetrical airfoils subject to aerodynamic loading, the center of pressure usually lies about 25%, or a quarter, behind the leading airfoil. This is where the aerodynamic forces are usually resolved. In contrast, the aerodynamic moment is resolved at the flexural, or structural, axis $x_f$. By use of the Euler-Lagrange equation, the coupled equations of motion which describe the behavior of the basic structural model are derived as follows:

$$\begin{bmatrix} m & m(b - x_f) \\ m(b - x_f) & I_\alpha \end{bmatrix} \begin{bmatrix} \ddot{y} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} K_y & 0 \\ 0 & K_\alpha \end{bmatrix} \begin{bmatrix} y \\ \alpha \end{bmatrix} = \begin{bmatrix} -L \\ M_{x_f} \end{bmatrix}$$  \hspace{1cm} (2.4)

where $\alpha$ and $y$ are the torsional and vertical displacements, respectively, $m$ is the mass of the airfoil, $I_\alpha$ is the moment of inertia at the flexural axis $x_f$, $b$ is the half-chord distance, $K_y$ is the vertical structural stiffness, $K_\alpha$ is the torsional structural stiffness, $L$ is the aerodynamic lift and $M$ is the aerodynamic moment at the flexural axis $x_f$.

This problem if commonly examined by use of the now classical approximation proposed by Theodorsen [53]. The approach rests mainly on four important assumptions: (1) sinusoidal pitching and heaving only, (2) fully attached flow (low amplitude vibrations), (3) wing is a rectangular plate and the (4) wake is flat. The main significance of this model regards its ability to correlate wing design and critical flutter speed, which marks the starting point for self-excited sinusoidal oscillations. This result is of particular significance for aerospace engineers who design aircraft to operate well beyond critical points. By assuming sinusoidal motion (e.g. $\alpha = \alpha_0 e^{j\omega t}$ and $h = h_0 e^{j\omega t}$), the Theodorsen unsteady lift and moment are classically resolved as follows:
\[ L(t) = \left[ \pi \rho b^2 U_\infty \alpha_0 j \omega + \pi p c U_\infty C(k) \left( U_\infty \alpha_0 + h_0 j \omega + \left( \frac{3c}{4} - x_f \right) \alpha_0 j \omega \right) + \pi \rho b^2 \left( -\omega^2 h_0 + \left( x_f - \frac{c}{2} \right) \omega^2 \alpha_0 \right) \right] e^{j\omega t} \] (2.5)

\[ M_{x_f}(t) = -\pi \rho b^2 U_\infty \alpha_0 j \omega \left( \frac{3c}{4} - x_f \right) + \pi p e c U_\infty C(k) \left( U_\infty \alpha_0 + h_0 j \omega + \left( \frac{3c}{4} - x_f \right) \alpha_0 j \omega \right) + \pi \rho b^2 \left( x_f - \frac{c}{2} \right) \left( -\omega^2 h_0 + \left( x_f - \frac{c}{2} \right) \omega^2 \alpha_0 \right) + \frac{\pi \rho b^4}{8} \omega^2 \alpha_0 \right] e^{j\omega t} \] (2.6)

\[ C(k) = 1 - 0.165 (1 - 0.0455 k^{-1})^{-1} - 0.335 (1 - 0.3 j k^{-1}) \] (2.7)

\[ k = \omega b U_\infty^{-1} \] (2.8)

where \( \alpha_0 \) is the pitching amplitude, \( h_0 \) is the heaving amplitude, \( \omega \) is the angular frequency for both, \( U_\infty \) is the free stream velocity, \( c \) is the chord length, \( b \) is the half the chord length \( b = c/2 \) and \( e \) is simplifies to \( e = x_f / c - 1/4 \). For practical considerations, the Theodorsen function \( C(k) \) is typically approximated by Equation 2.7. The marginal effects of aggregating wake vorticity on quasi-steady aerodynamics are significant. In general, overall forces are attenuated by this generalization.

The flutter phenomenon is potentially destructive for stiff aeroelastic systems, such as aircraft wings. This can be due to both plastic deformation and/or mechanical fatigue. On the other hand, aeroelastic instability is not only limited to stiff structures. Self-excited oscillations may also arise in low stiffness aeroelastic systems for different reasons. In contrast to unsteady aerodynamics, research in this domain is not as widespread due to its limited applications. That is, aerodynamic systems are not typically designed to be inherently unstable.
The low stiffness aeroelasticity problem may be investigated by performing a static equilibrium analysis. The objective is to examine the airfoil at zero velocity and acceleration subject to a free stream. The equations of motion, from Equation 2.4-2.8, are therefore reduced to the following:

\[
\begin{bmatrix}
K_y & \pi \rho c U_\infty^2 \\
0 & K_\alpha - \pi \rho c^2 U_\infty^2
\end{bmatrix}
\begin{bmatrix}
y \\
\alpha
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]  \hspace{1cm} (2.13)

This basic analysis demonstrates that torsional equilibrium results from either a zero angle of attack \(\alpha = 0\) and/or a moment balance \(K_\alpha = \pi \rho c^2 U_\infty^2\). Additionally, if the flexural axis is positioned at the quarter chord point, i.e. \(x_f = c/4\), the aerodynamic moment is eliminated. Now, if a positive external moment \(M\) is applied on a static system with \(K_\alpha - \pi \rho c^2 U_\infty^2 > 0\), the angle of attack is expected to increase. However, if \(K_\alpha - \pi \rho c^2 U_\infty^2 < 0\) the equations predict erroneous results, such that applying a moment to the system results in deflections along the opposite direction, which is physically impossible.

This forms the foundation for static divergence in aeroelastic systems. If the flexural axis of an airfoil is not at the center of pressure and \(K_\alpha - \pi \rho c^2 U_\infty^2 < 0\), the system is unstable in pitch. Instability is therefore accessible by ultra-low structural stiffness, flexural axes positioned in the aft of the airfoil, long airfoils and high air speeds. A system that statically diverges will do so by positive feedback dynamics. As such, an airfoil with an initial angle of attack greater than zero \((\alpha > 0)\) and \(K_\alpha - \pi \rho c^2 U_\infty^2 < 0\), will pitch
backwards in search of torsional equilibrium. This equilibrium may be found by instating nonlinear structural stiffness in the system, such that $K_\alpha = f(\alpha)$.

This aeroelastic instability may further be coupled with other nonlinear aerodynamic phenomenon, such as dynamic stall, to achieve self-sustained aeroelastic oscillations. Dynamic stall is simply manifested as loading hysteresis on a pitching airfoil. As an unstable pitching airfoil searches for torsional equilibrium at high angles of attack, the aerodynamic forces temporarily jump and are then deactivated due to loss of suction (stall) on the airfoil. The temporary jump in aerodynamic forces is related to the formation and travel of a strong vortex along the topside of the airfoil. From this principle of dual nonlinearity, aeroelastic systems that achieve self-sustained oscillations have been proposed [54] for energy harvesting.

In general, researchers [55] have reported that nonlinearity in aeroelastic systems promotes self-sustained oscillations. The earliest attempt [56] at extracting kinetic energy from atmospheric wind energy by use of an aeroelastic system consisted of an H-section wing and a pendulum spring. This particular result highlighted the potential for aeroelastic systems to function efficiently at low speeds, in contrast to conventional rotary systems. Additional energy aeroelastic harvesting applications have been proposed, such as heaving systems [57] for linear electromagnetic transduction. This system is activated by a combination of aeroelastic phenomena, such as vortex induced vibrations, buffeting divergent oscillations, rain vibration and wake galloping.

Additional novel aeroelastic energy harvesters have been proposed. Dynamic stall-based systems that employ piezoelectric cantilevers for structural support have also been
developed [58]. The main caveat associated with this design is the significant mechanical damping inherent to cantilever vibrations. The results also demonstrated that stable limit cycle oscillations are attainable from various initial conditions and that significance of break-in wind speeds.
Chapter 3: Aeroelastic Energy Harvesting: Problem Specification

This investigation presents a novel approach for extracting and converting kinetic energy in moving fluids, in particular atmospheric air. The proposed approach consists of a simple oscillating system that is driven by nonlinear forces. These forces driving the system arise as result of nonlinear phenomena occurring both in the fluid and solid domains. It is the nonlinear nature of these forces which provides the necessary conditions for driving the proposed system about its equilibrium point. The targeted dynamics resemble that of child in a swing, where the system is subject to pulsating/pumping forces. The key hypothesis of this research is that vibrations can be efficiently and economically harvested from a moving fluid. Toward this end, we approach the problem by examining the extraction and conversion design components, discussed in Section 3.1 and 3.2, respectively. Additionally, the overall approach for constructing and testing the device is discussed in Section 3.3.

The objective is to extract the kinetic energy in moving fluids by generating stable oscillations. In contrast to traditional rotary systems, the stable oscillations act as an intermediate state for converting the kinetic energy into the electrical domain. The conversion of energy from the mechanical domain into the electrical domain may be achieved by a variety of conversion strategies. The common denominator among all conversion strategies within oscillating systems is the obstructive electrical damping. This investigation proposes to convert the mechanical energy in the oscillating system into the electrical domain by use of electromagnetic transduction.
3.1 Designs for Aeroelastic Extraction

The proposed solution consists of a system which is periodically activated by nonlinear aerodynamic phenomena. The novel system may be achieved by constraining a rigid airfoil to an ultra-low stiffness nonlinear torsional spring. This investigation proposes a novel elastic membrane design for achieving the ultra-low stiffness nonlinear torsional reaction forces. An illustration of the proposed design concept is shown in Figure 3.1. The proposed torsional membrane consists of a long strip of thin elastic material attached to two rigid anchors. The anchors are then axially displaced to achieve a target stiffness response in the membrane.

The targeted aerodynamic nonlinearities in the system arise from a phenomenon known as dynamic stall. Simply put, the dynamic stall phenomenon endows rapidly moving airfoils with the ability to provide pulsating lift and drag forces. These nonlinear, or pulsating, forces result from vortices being shed from the leading and trailing edge of the
airfoil at high angles of attack. Each vortex is shed as a result of complete and sudden flow separation above the airfoil. It is the progression of these vortices on the airfoil which gives rise to torque hysteresis on every cycle. In general, the fluid does work on the system while the flow is attached to the airfoil.

The nonlinear aerodynamic forces promote self-sustained oscillations when coupled with nonlinear reaction forces (torsional springs). For every cycle, the flow is initially attached and later separated, initiating two vortices; both on the leading and trailing edge of the airfoil. As this vortex exits the airfoil the aerodynamic forces decrease drastically (following a traditional stall) and the nonlinear torsional spring restores the system back to initial position. Intuitively, the proposed system may be represented by an arbitrary mass subject to both external pumping (aerodynamic) and restoring (membrane torsional spring) forces, as shown in Figure 3.2.

![Figure 3.2: Representation of Simplified System](image)

### 3.2 Designs for Electromagnetic Conversion

Provided that the proposed design achieves self-sustained aeroelastic oscillations, the next task aims at addressing the possibility of converting the mechanical energy into electrical power. The objective of such a challenge is to design a transducer that maximizes
electrical power output subject to input constraints, in other words, maximizing power density. In traditional wind mills/turbines, power density is characterized by the electrical power that is generated per cross-section of wind exposure. Improving the performance of a transducer within a mechanical system is a multivariate nonlinear optimization problem. High sensitivity is expected to result due to interaction/coupling effects of the transducer with the system and overall system scale.

This investigation proposes to employ electromagnetic transduction for converting torsional mechanical vibrations into electrical power. The proposed design aims at fixing a permanent magnet within the rotational axis of the rigid airfoil. Important considerations include maximizing the magnetic flux density while minimizing the additive moment of inertia. With the oscillating permanent magnet in place, the varying magnetic flux can therefore be converted into electrical power by means of electromagnetic induction. This investigation aims at determining the optimal transducer design for maximizing power density. A conceptual representation of a potential design is shown in Figure 3.3, where the magnetic poles \{N,S\} of the permanent magnet are displaced with the aeroelastic system.

![Figure 3.3: Proposed Configuration for Electromagnetic Transducer](image-url)
3.3 Power Density and Economic Analysis

This investigation hypothesizes that membrane-based aeroelastic energy harvesting is superior over existing approaches in terms of efficiency and feasibility. Feasibility in energy harvesting may be represented in several ways. One method for characterizing feasibility is by cost analysis, i.e., the cost of power. The goal is to obtain values that may be used for comparison purposes amongst energy harvesting classes. The problem with existing scoring methods is that superiority in power density alone is not sufficient to make a device feasible given that its materials and construction might be overwhelmed with capital cost.

Economic considerations lie at the core of the energy harvesting field. Unfortunately, the inherent design complexity in energy harvesting makes it very difficult to implement concurrent engineering practices, such as embedding cost considerations into the design process. For this reason the energy harvesting problem is commonly split into two independent problems, mainly the (1) design problem and the (2) manufacturing problem, as depicted in Figure 3.4. This traditional design methodology is in effect a heuristic which is very suitable for complex design problems.

Figure 3.4: Energy Harvesting Design and Evaluation
This investigation aims at employing the shown methodology in exploring the feasibility of the proposed energy solution. As such, the investigation initially focuses on maximizing power density by design optimization. To achieve this goal, both mathematical and computer modeling is employed in identifying design guidelines. The models were constructed cumulatively by adding/including components in order of complexity. The objective was to successfully model the self-sustained aeroelastic oscillations for power optimization.

Some of the model components studied include the airfoil (angle of attack, size, mass and position), torsional membrane (geometry, elasticity and tension), permanent magnet (geometry, position, mass, flux density and pole orientation) and stator coil (position, geometry, conductance, number of windings and wire gauge). The objective was to determine the combination of variables which maximized power density. By understanding the proposed solution, it is then possible to develop a conceptual design prior to fabricating physical prototypes.

The manufacturing phase of this investigation aims to focus on constructing physical prototypes for testing both power density and cost minimization. It was anticipated that further power density improvements would be realized during the fabrication stage due to model aberrations, resulting from incorrect model assumptions. After implementing these design changes, the investigation focused on realizing cost effective prototypes. This consisted of establishing the best materials suitable for the proposed system. The goal was to provide working and cost effective prototypes for benchmarking. The final stage of this investigation consisted on performing an overview of the results.
Chapter 4 Mathematical Modeling

This chapter focuses on constructing mathematical models as tools for understanding and optimizing the proposed energy harvester. The overall goal of constructing mathematical models is to study system sensitivity subject to design variables. The major challenge associated with constructing a mathematical model for this system includes modeling the individual components dominated by different phenomena. The major components of the system are dominated by behavior in solid mechanics, electromagnetism and aerodynamics. Of the three major groups, the aerodynamic phenomenon is the most challenging to model provided it’s both chaotic and stochastic in nature.

4.1 Solid Modeling

Constructing mathematical models for the mechanical components is permissible provided that macro-property variations are deterministic. The critical mechanical component to model for the energy harvester was the torsional membrane because it provides the structural stability in the system. This torsional membrane itself is fundamentally a nonlinear spring. The nonlinearity found in this component was purely geometrical in nature and not related to the material properties. That is, the large deflections were mainly related to rigid-body dynamics in contrast to elastic deformation. The applied stresses were relatively small such that the mechanical strain remained well within the elastic region.

The analysis for the torsional membrane began by conducting a simple static calibration on the component. This was accomplished by loading the system incrementally with magnets with equal and known mass. After each loading step the displacement was
measured. As the torsional spring deflected is was necessary to rotate the structure such that gravity was always tangent to rotating system. This simple experiment elucidated the nonlinear behavior of the torsional membrane. The system exhibited significant nonlinear behavior as the membrane rotates to approximately 90 degrees.

Accurate modeling of the torsional membrane was extremely important as it played a critical role in the overall system dynamics. The mathematical modeling for the membrane initially analyzed a simple fiber under tension subject to transverse force and deformation, as shown in Figure 4.1. The objective was to model a smaller problem which could be extended to the membrane case.

![Figure 4.1: Simple Fiber Subject to a Transverse Force](image)

The figure shows a transverse force $F$ deflecting a fiber at its midpoint by a distance $x$. The fiber is constrained at both ends such that applying a transverse force results in symmetrical deformation on both sides. The transverse force increases the total tension in the
fiber to $T'$. That is, the fiber is under tension before any deformation. The other model variables include the deformed leg length $L'$ and corresponding angles $\theta$ for the resulting isosceles triangle. A simple balance of forces along the force axis yields the following relationship:

$$F = 2T'\sin(\theta) = 2T'\left(\frac{x}{L'}\right)$$  \hspace{1cm} (4.1)$$

The initial tension of the fiber $T$ may therefore be expressed as the difference between the deformed tension $T'$ and the elastic reaction force according to Hooke’s Law, were $k$ is the material’s spring constant and $L$ its original leg length. That is, the new tension is the original tension plus the reaction forces resulting from the external stress. The resulting force balance relationship is as follows:

$$T' = T + k(L' - L)$$  \hspace{1cm} (4.2)$$

This expression may be further simplified to include system parameters. The objective was to determine an expression which related strain in the membrane to original tension. The final relationship for modeling the fiber displacements $x$ and $L'$ subject to the material’s spring constant $k$, initial tension $T$ and input force $F$ is attained by substituting Equation 4.1 into 4.2 for $T'$. The final result is the following:

$$T = \frac{FL'}{2x} - k(L' - L)$$  \hspace{1cm} (4.3)$$
The next model is intended to generalize the results obtained for the single fiber. To achieve this, the rectangular membrane was approximated as a set of infinitesimal fibers in parallel. The underlying assumption in doing so is that all deformation is uniaxial along the fibers length. This assumption is reasonable provided that the rectangular membrane is guided by the airfoil mount when subject to torsional deformation. In other words, the airfoil mount inhibits the membrane from compressing transversely due to the Poisson’s effect. Each fiber is elastically deformed by a distance $x$ from its initial position by an external torque $\tau$, as shown in Figure 4.2.

Figure 4.2: Membrane Mid-Point Section Subject to Torsional Deformation

Figure 4.2 shows half the membrane at an arbitrary deformed state. The applied torque $\tau$ results in an angular deflection angle $\vartheta$ around the elastic axis of the membrane.
Each fiber within the membrane is displaced from its resting state yielding both radial and tangential reaction forces. The airfoil mount is sufficiently rigid that it opposes the radial forces statically, requiring no further analysis. In contrast, the tangential component of the reaction forces balances the applied torque. As the deflection angle $\theta$ increases the reaction torque is distributed between both the radial and tangential directions.

When the airfoil mount is at its resting position ($\theta = 0$), a small positive torsional increment ($+\delta\theta$) results in fully tangential reaction forces. As the deflection angle increases the load is increasingly distributed in the radial direction, which results in compression of the airfoil mount. When the system achieves a 90 degree deflection, the forces are distributed equally (50-50) in both directions. This effect is purely geometrical and results in overall nonlinear reaction forces within the system because the radial forces are statically eliminated by the airfoil mount. Without an airfoil mount guiding the fibers along a circular path during deformation, the membrane would bend and curl due to the radial compressive reaction forces at the midpoint.

The results in Equation 4.3 apply for each individual fiber in the approximate membrane system. The next step is to develop an expression that relates the geometrical and material properties of the membrane to its final state following an externally applied torque. This is accomplished by resolving an expression for the membrane’s torsional spring constant $K_{\theta}$ as a function of its torsional deflection, material properties, geometry and pre-loaded tension. This derivation begins by resolving the forces acting on the airfoil mount, which is balanced by the restoring tangential load $R'(y)$ and the applied torque $\tau$ as follows:
\[
\tau = 2 \int_0^{w/2} R'(y)y \, dy \quad (4.4)
\]

The tangential load \( R'(y) \) may be further expressed in terms of the restoring forces from individual fibers along the direction of deformation \( x \), where \( w \) represents the width of the membrane. This realignment is necessary in order to correct the direction of the applied torque and the reaction forces. As shown in Figure 4.2, the direction of the reaction forces from the fibers is offset \( \vartheta/2 \) radians from the tangential direction, following an angular displacement in the system of \( \vartheta \). The following relationship aims to balance the tangential forces acting on the system by correcting the direction. The transformation takes place as follows:

\[
R'(y) = F'(y) \cos \left( \frac{\vartheta}{2} \right) \quad (4.5)
\]

This relationship shows that the applied torque is balanced by \( \cos(\vartheta/2) \) when subject to angular deformation \( \vartheta \). As stated previously, an applied torque resulting in total angular deflection of \( \vartheta = \pi/2 \) would balance the forces on the fibers at \( \cos(\vartheta/4) = \sqrt{2}/2 \) in both directions. The nonlinear relationship \( \cos(\vartheta/2) \) is responsible for the increasing stiffness in the system because it converts external torques to compressive forces on a rigid element. In achieving a mathematical expression, Equations 4.3, 4.4 and 4.5 may be further combined with the identity \( L' = \sqrt{L^2 + x^2} \) and the spring constant \( k = \frac{twe}{L} \), where \( t \) and \( E \) represent the
thickness and modulus of Elasticity of the membrane, respectively, to yield the final expressions for torque $\tau$ and spring constant $K_\theta$:

$$
\tau = \frac{(\sin \varphi)(T-twE)\sqrt{1+\left(\frac{w}{L} \sin \frac{\varphi}{2}\right)^2}}{L\left(\frac{2}{L} \sin \frac{\varphi}{2}\right)^2} \frac{\sin \varphi}{wL\left(\frac{2}{L} \sin \frac{w}{2}\right)^3} - \frac{2}{L} \frac{(\sin \varphi)(T-twE)\sinh^{-1}\left(\frac{w}{L} \sin \frac{\varphi}{2}\right)}{wL\left(\frac{2}{L} \sin \frac{w}{2}\right)^3} + \frac{(\sin \varphi)Etw^3}{6L} \quad (4.6)
$$

$$
K_\theta = \frac{(\sin \varphi)(T-twE)\sqrt{1+\left(\frac{w}{L} \sin \frac{\varphi}{2}\right)^2}}{L\left(\frac{2}{L} \sin \frac{\varphi}{2}\right)^2} \frac{\sin \varphi}{wL\left(\frac{2}{L} \sin \frac{w}{2}\right)^3} - \frac{2}{L} \frac{(\sin \varphi)(T-twE)\sinh^{-1}\left(\frac{w}{L} \sin \frac{\varphi}{2}\right)}{wL\left(\frac{2}{L} \sin \frac{w}{2}\right)^3} + \frac{(\sin \varphi)Etw^3}{6L\varphi} \quad (4.7)
$$

The nonlinear response of the torsional membrane, modeled by use of Equation 4.7 is shown in Figure 4.3. A membrane of half-length $L=10$ cm (total length is 20 cm), width of $w=5$ cm, modulus of elasticity of $E=3$ GPa, tension of $T=5$ N and thickness of $t=52$ μm was employed in modeling the response of the system. The results illustrate that the torsional spring has an initial non-zero stiffness which sets the minimal natural frequency of the system. For this particular design, as the angular deflection increases beyond 10 degrees, the nonlinearity in the spring stiffness becomes significant. The output from the mathematical model confirmed prior results obtained from the static calibration on the torsional model.

The results illustrate that torsional deformation along the mid-point of a rectangular membrane results in significant nonlinear reaction forces. A tenfold increase in stiffness was observed as the torsional membrane deflected above 40 degrees. The model demonstrates a high sensitivity in relation to the geometry of the membrane, in particular to its width. This result was expected provided that fibers on the edge have a greater travel distance during
deformation in comparison to fibers near the elastic axis. It was also expected that a torsional membrane too wide and short in length would inhibit torsional deflections within the elastic region of the material. That is, variations in the length of the membrane have the inverse effect whereas the spring constant decreases when elongated.

![Graph showing the nonlinear spring constant of a torsional membrane](image)

**Figure 4.3: Nonlinear Spring Constant of Torsional Membrane**

### 4.2 Electromagnetic Modeling

This investigation employed the use of electromagnetic transducers for converting the vibrational mechanical energy into the electrical domain. The major components for electromagnetic induction include the permanent magnets and inductor coil. In optimizing the system, magnets do not require mathematical modeling provided that increasing magnetic
flux density is always desirable. The only limitation for the permanent magnets was the added moment of inertia to the system. As such, this investigation focused on procuring magnets with the highest magnetic field strength subject to weight and volume constraints.

On the other hand, the inductor coil parameters were not unbounded as the permanent magnets in design optimization. As it will be shown within this section, the inductor coil parameters dictated the energy conversion efficiency for the proposed energy harvester. The analysis begins by modeling an electromagnetic harvester with fixed magnetic properties, similar to that of the proposed solution. A typical coil within an electromagnetic transducer is typically composed of low resistance copper wire wound around non-magnetic materials. According to Lenz’s law, the voltage output of a coil is:

\[
V = -N \left( \frac{d\Phi}{dt} \right)
\]

where N represent the number of turns in the coil and \((d\Phi/dt)\) is the rate of change in magnetic flux \((d\Phi/dt)\). For a fixed volume, the number of turns N in a coil is linearly proportional to the thickness of a conductor. A coil with thicker wire allows for a larger number of turns which in turn results in higher voltages for a fixed magnetic flux rate.

Basic electromagnetic theory also dictates that the electrical resistance of a conductor if proportional to its cross-sectional area. That is, the resistance of conductors may be expressed as follows:

\[
R = \frac{\rho L}{\pi r^2}
\]
where $\rho$ is the materials resistivity, $L$ is the length of the conductor and $r$ is the radius of the circular cross sections (wire). This expression may be further expanded to determine the total resistance based on the geometry of the inductor coil. For a rectangular inductor coil with length $l$ and width $w$, corresponding to that employed in this investigation, the overall resistance may also be expressed as follows:

$$R = \frac{\rho(2w+2l)N}{\pi r^2}$$  \hspace{1cm} (4.10)

where $N$ corresponds to the number of turns in the coil. Additionally, it may also be deducted that for a 2D limited surface, the total number of turns is inversely proportional to the area of the conductors such that $N \propto 1/r$, where $r$ is the radius of the conducting wire. This effect is demonstrated in Figure 4.4, were two different wires (one is half in diameter with respect to the other) allow for different winding capacity. As such, if the thickness of a conducting wire within an inductor coil is reduced by a factor of two, there would be sufficient volume within the cavity to increase the number of turns within the coil by a factor of 2.

![Figure 4.4: Two-dimensional Induction Coils](image)

$N=n$ (5)
$R=r$
$N=2n$ (10)
$R=r/2$
The average power generated by an energy harvesting system may be determined by measuring the average voltage driving a load. Researchers typically vary the load resistance in order to maximize the transfer of power from the mechanical to electrical domain. In impedance matching, the load impedance is set to match the operating output impedance of the transducer. The average power generated by the system is therefore expressed as follows:

\[
P_{\text{Average}} = \frac{V_{\text{RMS}}^2}{R_{\text{Load}}} \quad (4.11)
\]

where \( V_{\text{RMS}} \) represents the root mean square voltage and \( R_{\text{Load}} \) corresponds to the resistance of the load. By combining Equations 4.8 to 4.11 it is then possible to determine an expression that correlates power generation to the thickness of a conductor (number of turns). This is achieved by substituting the voltage generation expression (Equation 4.8) and resistance expression (Equation 4.10) into the power expression (Equation 4.11) as shown below:

\[
P_{\text{Average}} = \frac{N^2 \left(\frac{d\Phi}{dt}\right)^2}{\frac{1}{n(4w+2l)}N} \propto N r^2 \quad (4.12)
\]

This substitution assumes that the DC resistance is matched to that of the coil. That is, the load resistance is assumed to be equal to the DC resistance of the inductor coil. By eliminating the constants from the expression and substituting the volumetric relationship \( (N \propto 1/r) \) it is then possible to obtain a final relationship that models the power output in
relation to the magnetic wire gauge, or conversely, number of turns in the coil. The final expression that captures the targeted relationship for a 2D coil is:

\[ P_{2D,Average} \propto \frac{1}{N} \propto r \]  (4.13)

This simple relationship demonstrates that power generation is inversely proportional to the number of turns for a 2D inductor coil, and thus, linearly proportional to the thickness of the conductor. It should also be noted that the voltage output is linearly proportional to the number of turns in the coil. Provided that it is very difficult and inefficient to work with low voltages, researchers first examine the workable volume for an inductor coil in order to determine the required wire gauge for achieving a target voltage (number of turns).

Equation 4.13 for a 2D inductor coil may also be understood by examining the power losses associated with the inductor coil itself. Provided that the current driving the load must travel along the coil, its overall resistance dictates the amount of electrical energy that is converted to heat in the process. This particular energy is lost in the process provided it’s not delivered to the load. Both the length (number of turns) and diameter (resistance) of the wire in the coil determines the overall efficiency of the energy harvester. To summarize, the losses in the 2D inductor coil may be minimized by decreasing the DC resistance of the inductor coil. This is achieved by implementing a thicker wire (lower resistance) or decreasing the number of turns (length of wire) in the coil. Careful consideration must be exercised such that workable voltages are achieved.
The previous analysis may also be extended for three-dimensional coils, which were employed in this investigation. For a 3D limited volume, the total number of turns is inversely proportional to the area of the conductors such that $N \propto 1/r^2$, where $r$ is the radius of the conducting wire. This effect is demonstrated in Figure 4.5, where two different wires (one is half in diameter with respect to the other) allow for different winding capacity. As such, if the thickness of a conducting wire within an inductor coil is reduced by a factor of two, there would be sufficient volume within the cavity to increase the number of turns within the coil by a factor of 4.

![Fixed Length vs Fixed Width](image)

*Figure 4.5: Three-dimensional Induction Coils*

The implication of the 3D case is much different than the 2D provided the number of turns is inversely related to the square of the wire radius. By substituting the relationship into the power expression we show that:

$$P_{3D,Average} \propto N r^2 = \left(\frac{1}{r^2}\right) r^2 = 1$$  \hspace{1cm} (4.14)
This relationship demonstrates that the average power generated by a fixed 3D volume coil within an energy harvester is independent of wire gauge or number of turns. In contrast to the 2D case, there is no limitation for using magnetic wire with smaller diameter yielding a higher number of turns. Rather, the wire gauge and number of turns which is employed should be determined based on the application. High voltage applications will require thinner wire and larger number of turns, whereas high current applications will have the opposite effect.

It should be noted that accurate analysis of the 3D coil requires careful examination of the flux density for each loop (turn). The previous results assume that the magnetic flux lines are cut uniformly when varying the wire gauges/number of turns in the coil. In applications with high flux gradients, a reduction in wire thickness may prove advantageous provided stronger fluxes may be captured.

### 4.3 Aerodynamic Modeling

The aerodynamic modeling begins by reducing the system to a simple mass-spring-damper system. This type of steady state analysis is common provided aircraft designer typically intend to achieve stable flight conditions. If the system is assumed to behave quasi-statically such that any dynamic effects are fully damped a stable equilibrium point may be determined for each angle of attack. It is in this fashion that the lift and drag curves are experimentally evaluated for airfoil designs. Once the forces are evaluated they can be employed in reverse mode for modeling novel system configurations.
The proposed energy harvesting system is evaluated as a mass-spring-damper in torsional mode. The equation of motion that describes the behavior of the airfoil is as follows:

\[
I\ddot{\theta} + C\dot{\theta} + K\theta = M + eL + fD
\]  

(4.15)

where \(I\) is the mass moment of inertia about the elastic axis and \(C\) is the structural damping coefficient. The elastic axis location for the airfoil is conventionally defined by the dimensionless parameter \(a\) such that \(e = b(1/2 + a) \cos \theta\) and \(f = b(1/2 + a) \sin \theta\), where \(b\) corresponds to half the chord length, as depicted in Figure 4.6. This structural model is sufficient for modeling quasi-static problems below the critical stall angles. In evaluating a design for a particular airfoil, the moment M, lift L and drag D may also be determined from experimental tables.

Figure 4.6: Typical Airfoil Section
The major challenge associated with implementing the proposed quasi-static mathematical model is that the lift, drag and moment forces are time dependent under dynamic conditions. The departure from quasi-static experimental data may be significant even at low angles of attack due to turbulence. Vortex formation behind the airfoil is a common source for aeroelastic vibrations. The vortices can develop behind the leading and trailing edge of the airfoil. In general, the instantaneous effect of a vortex is to temporarily induced lift and drag. For an airfoil that is fixed behind the quarter-chord point, the temporarily increased forces result in an amplified moment. This effect is maximized by placing the elastic axis of the aeroelastic system at the trailing edge of the airfoil.

The mathematical modeling of turbulent flow is particularly challenging because it’s both chaotic and stochastic in nature. This limitation inhibits the use of experimental data even in constructing quasi-static models within the stall envelope. Thus far, mathematical expressions have only been developed to approximate the effect of trailing vortices at small angles of attack [53]. These approaches are particularly applicable to small amplitude vibrations which arise in long slender wings. In contrast, these results are not applicable to the proposed system because it encompasses a rigid airfoil which does not flex structurally but rather displaces as a rigid body.

Several approaches have been proposed for modeling high angle of attack aeroelastic systems. The most well-known method is the *Office National d’Etudes et de Recherches Aérospatiales* (ONERA) dynamic stall model [59]. This is a semi-empirical model which incorporates both linear unsteady effects and nonlinear contributions due to dynamic stall. A semi-empirical model is one which relies to some extent on data derived from
experimentation and observation. The model consists of generalizing the traditional lift, drag and moment curves by adding a second order delayed effect. According to the mode, the differential equations that describe the behavior of the aerodynamic coefficients ($M$, $L$ and $D$) are as follows:

\[
C = C_1 + C_2 
\]

\[
C_1 = a_1 \dot{\alpha} + a_2 \ddot{\alpha} + C_\gamma 
\]

\[
\ddot{C}_\gamma + a_3 C_\gamma = a_4 \dot{\alpha} + a_5 \ddot{\alpha} + a_6 \alpha 
\]

\[
\dddot{C}_2 + a_7 \dot{C}_2 + a_9 C_2 = -a_8 \Delta \dot{C}(\alpha) - a_9 \Delta \ddot{C}(\alpha) 
\]

where $\alpha$ is the instantaneous angle of attack ($\dot{\alpha} = \dot{\theta}$ for pitching airfoils), the function $C_Z$ represents any of the dimensionless aerodynamic coefficients ($M$, $L$ and $D$), the function $C_1$ and $C_2$ characterize unstalled and stalled aerodynamics, respectively, $C_\gamma$ represent an arbitrary first order function, the function $\Delta \dot{C}(\alpha)$ represents the nonlinear deviation from extended linear aerodynamics and $a_i$ are empirically determined coefficients. The hysteresis attributed to dynamic stall is governed by Equation 4.19.

The major caveats associated with the ONERA model regard its semi-empirical nature. That is, nine coefficients ($a_i$) must be evaluated from experimentation alone. Additionally, the function $\Delta \dot{C}(\alpha)$, which models the nonlinear inputs, must be approximated. Although the ONERA model may be calibrated to represent the underlying phenomena, it is of little use in optimization. This is because the model requires that physical prototypes be
built ahead of time. The ONERA model exemplifies the difficulty associated with analyzing high angle of attack aeroelastic systems.

Despite the fact that the ONERA model may not be implemented in power density optimization, it does in fact accurately model the force hysteresis in dynamic stall. A numerical solution that illustrates the nonlinear behavior of the model is presented next. In order to solve the problem numerically it must be formulated in first-order differential equations. This may be accomplished by defining a state vector $x$ as follows:

$$x = (x_1, \ldots, x_8)^T = (\alpha, \dot{\alpha}, C_{L\gamma}, C_{M\gamma}, C_{L2}, \dot{C}_{L2}, C_{M2}, \dot{C}_{M2})^T$$  \hspace{1cm} (4.20)$$

where all the system variables are reduced from second to first order. The state vector may then be linearized with the model constants in order to arrive to complete state variable representation for the structural system (Equation 4.15) and the ONERA aerodynamic forces (Equations 4.16-4.19). The nonlinear second order model is expressed as follows:

$$\{\dot{x}\} + [A]\{x\} = \{0\}$$  \hspace{1cm} (4.21)$$

where $[A]$ is a coefficient matrix that encompasses all aeroelastic constants. This model configuration may then be solved by use of conventional differential equation solvers. The present work employed a differential equation solver (ODE45) within MATLAB. The ODE45 algorithm employs a time-marching routine which utilizes variable time steps for efficient computation. As such, the algorithm slows down (smaller time steps) when the
model is highly nonlinear for improving resolution and speeds-up (increase time steps) when not.

The simulated system consisted of a small National Advisory Committee for Aeronautics (NACA) 0012 airfoil with the following dimensions: chord length of \( c = 0.03 \, m \), span of \( s_p = 0.038 \, m \) and maximum thickness of \( t = 0.0015 \, m \). The dimensionless distance from the mid-chord to the elastic axis is defined as \( a = 0.8 \), such that the displaced mass moment of inertia is \( I_\theta = 4 e - 7 \, kg \cdot m^2 \). A simple approximation of Equation 4.7 was employed in modeling the spring constant \( K_\theta \) as follows:

\[
K_\theta = \sum_{i=1}^5 A_i \left( e^\theta + e^{-\theta} \right) - B_i
\]  

(4.22)

where the following 10 non-negative coefficients: \( A_1 = 0.0027 \), \( B_1 = 0.0008 \), \( A_{2-5} = 0.003 \) and \( B_{2-5} = 0.006 \). The coefficients were obtained by fitting a static calibration curve obtained by deforming a torsional membrane \( (L = 10 \, cm, w = 1 \, cm \text{ and } t = 52 \, \mu m) \) at a tension of 10N. The main advantage of implementing a torsional spring approximation was that the exponential function fit the data well. In contrast, measurement errors in the torsional membrane’s material properties and geometry made the theoretical multivariate expression (Equation 4.7) difficult to calibrate. In regards to quantifying the damping in the system, the method of logarithmic decrement was implemented in estimating a total damping ratio of \( \zeta = 0.0175 \).

The total damping calculation for the unloaded system (not driving the electric load) included both structural (mechanical) and aerodynamic forces. The aerodynamic component
was exceptionally large provided the airfoil is practically perpendicular to the flow while oscillating. This results in significant drag forces which in turn drastically slow down the airfoil during motion. The only approach for experimentally characterizing the mechanical damping is by conducting a free-response analysis in vacuum. For the ONERA model simulation, the total damping ratio was used which overestimates system damping.

The ONERA dynamic stall model was fitted with the NACA 0012 airfoil aerodynamic characteristics found in static [60] and dynamic [61] empirical studies. Three break points were employed in describing the nonlinear deviations from extended linear aerodynamics $\Delta C_Z(\alpha)$, starting with the static stall angle $\alpha_{ss} = \alpha_1 = 12^\circ$ and respectively followed by $\alpha_2 = 20^\circ$ and $\alpha_3 = 40^\circ$. It was assumed that there is no hysteresis due to dynamic stall in the drag coefficient. The low aspect ratio characteristic of the airfoil was also taken into consideration for linear aerodynamics. The airfoil was set to a neutral angle of attack of 20 degrees such that the instantaneous angle of attack was $\alpha = \theta + 20^\circ$. The model was then subject to a free stream with velocity $U_\infty = 3.8 \ m/s$ and a starting position of 9 degrees, as shown in Figure 4.7.
The limit cycle oscillations (LCO) shown in Figure 4.7 originate due to the combination of structural and aerodynamic nonlinearities present in the system. The oscillations grow as a result of the net positive work provided by nonlinear aerodynamics and settle as the effective stiffness of the torsional spring penetrates its highly nonlinear region.

By use of the ONERA model, it was possible to conduct a qualitative stability analysis on the system, as shown in Figure 4.8. This was accomplished by varying the initial conditions of the model to 40 degrees above and below the neutral axis of system (20 degrees). The figure demonstrates the trajectory of the system asymptotically returns to the original limit cycle. That is, moderate perturbations introduced into the system vanish with time. This result demonstrates the robustness of the model subject to perturbations (e.g. wind gusts) commonly found in typical atmospheric wind.
Although simulating the coupled structural-ONERA model did not provide the means for optimizing power density, it did verify that dynamic stall is the source of aerodynamic hysteresis. Additionally, it demonstrated that the proposed membrane-based torsional spring design provided the highly nonlinear reaction forces and minimal damping required for achieving self-sustained aeroelastic oscillations. This investigation required that additional numerical and experimental approaches be implemented in gaining insight regarding the self-sustained aeroelastic oscillations for the purpose of optimization. As depicted by the ONERA model, the main challenge associated with modeling this phenomenon is the chaotic and stochastic nature of turbulent flow characteristics.
Chapter 5: Computational Modeling

This chapter outlines the construction of the computational model as well as some basic results. The major challenges associated with constructing a fluid-structure-interaction (FSI) include 1) coupling the mechanical and fluid domains, 2) resolving large mesh deformation and 3) maximizing time-step resolution. Section 5.1 covers the different approaches which were employed in constructing a working model. The overall behavior of the model is then investigated in Section 5.2.

5.1 Model Setup

Turbulence is a standard chaotic phenomenon observed in fluid flows. Dominated by inertial forces and observed in the time domain, turbulence is well known for challenging mathematical modeling. The motion of fluid substances has long been studied by use of the Navier-Stokes equations (19th Century) which were classically derived from Newton’s Laws. In the most general case, the set equations consist of nonlinear partial differential equations. The exact solution to these equations has eluded mathematicians for over a century and is considered one of the most well-known unsolved problems.

Within steady state conditions, turbulence in a flow field is commonly quantified by the dimensionless Reynolds number, which is the ratio of inertial to viscous forces. The Reynolds number is a well-recognized measure for gauging fluid flow over static standardized objects. In contrast, the measure is not so well-suited for evaluating turbulent flow in dynamic and deformable bodies. For aeroelastic systems exhibiting large deformations, the development of a turbulent boundary layer is dependent on the dynamic
conditions of the system. As such, this measure provides no useful information in characterizing the flow regime for aeroelastic systems.

The only approach for characterizing the flow field in an aeroelastic system is by use of experimental techniques and computational models. For steady state systems, transitioning from a laminar boundary layer into the turbulent regime is characterized by localized backflow or flow circulation. For dynamic systems this condition is complex. A simple example of this behavior may be observed in a rotating flat plate. The plate might exhibit a laminar boundary layer when parallel to the flow and transition to vortex shedding in a perpendicular configuration. As such, the dependence on orientation for high-aspect ratio geometries makes characterizing flow conditions in aeroelastic systems extremely difficult.

From a practical perspective, engineers typically rely on numerical approaches for modeling turbulent three-dimensional fluid flows. There are several challenges associated with modeling aeroelastic systems, both in the elastic (mechanical) and aerodynamic (fluidic) domains. The coupling effect in fluid-structure-interactions (FSI) imposes a third challenge since both domains need to be calibrated for iterative solution. This investigation employed ANSYS 13.0 in modeling the complex behavior of the aeroelastic energy harvesters.

The geometry for the NACA 0012 airfoil was imported into ANSYS DesignModeler for accurate modeling of the airfoil. With an accurate solid representation of the airfoil system it was then possible to extend the fluid domain to 5x chord distances away from the airfoil, as shown in Figure 5.1. In examining the effects of a vortex shedding airfoil it was important to extend the fluid domain to capture the vortex street. The objective was to incorporate the generation and propagation of turbulent vortices as they travel through the
fluid field. This served a dual purpose provided that vortex streets also affect the instantaneous aerodynamic forces in dynamic airfoils.

Figure 5.1: NACA 0012 in Fluid Domain

Meshing was the following step after defining the solid and fluid domains. Meshing quality in finite element analysis (FEA) is typically dominated by the scale of interest and available computational resources. In contrast to rigid mechanics, computational fluid dynamics (CFD) typically demands higher meshing density due the chaotic nature and scale of fluidic interactions. In working with high resolution requirements, the meshing density throughout domains of interest may be varied without compromising on computation resources. This is accomplished by biased meshing strategies which employ linear models in
distributing the elements across specified regions as shown in Figure 5.2. Bias meshing provides the capability for redistributing a constant number of elements within a mesh.

![Bias Meshing](image)

**Figure 5.2: Bias Meshing**

This element distribution for the model is shown below in Figure 5.3. The total number of elements in the model was 766,625 with highest density near the airfoil. The element resolution for the airfoil surface along the longitudinal and transverse directions was set to 500 μm, as demonstrated in Figure 5.4. A bias factor of 100 was used to distribute the elements across the radius of the cylindrical fluid domain. This indicates that the ratio between the largest element length and the smallest is 100. For this aeroelastic energy harvester, it was crucial to implement the fine elements in the airfoil surfaces in order to accurately model vortex formation and propagation.
Large deformations in computational modeling add another level of complexity in maintaining mesh stability. These large distortions may easily invert mesh elements resulting in negative volume calculations. This condition will typically result in a fatal error forcing the solver to exit without writing a results file. In conducting transient analyses, this is of serious concern provided that the solver might be running for several hours prior to experiencing large deformations. In the proposed aeroelastic energy harvester, peak-to-peak amplitudes exceed 180 degrees. Preliminary experiments failed at multiple stages since the displacements were too large for the standard meshing.
Several approaches have been developed in dealing with negative elements in large mesh deformations. The first approach is to re-mesh the domains iteratively during the solving procedure. The main disadvantage with this approach is that the computational requirements increase drastically, making the model practically unsolvable. For vibration-based models there exists a second approach for dealing with large deformations. Provided that the degrees of freedom are known or may be estimated beforehand, mesh domains may be decoupled to minimize element deformations. This is of particular interest for torsional systems since they are most sensitive to negative volume errors. An example of a decoupled mesh system with its corresponding mesh interface is shown in Figure 5.5.
A rotary decoupled domain allows the immediate mesh structure to rotate with the rigid body (airfoil) in order to minimize element deformation. In this configuration, the immediate mesh structure follows the angular position of the airfoil. Hence, the motion of the rigid body dictates affine transformations for the disengaged mesh structure. During the solving procedure, the internal mesh structure has the capability of being displaced on every iterative cycle. The computational requirements for this setup are considerably lower in comparison to the re-meshing approach.
The next stage in constructing the FSI model was to define the material, boundary condition and forces in ANSYS CFX. Both fluid domains were defined as air at 25°C and 1 atm. The side boundary conditions were specified as symmetrical such that the flow on one side of the domain is a mirror image of the flow on the opposite side. In contrast, the inlet condition was defined by specifying the input flow velocity in Cartesian components. This provided full control over the angle of attack for conducting the experimental runs. The input flow was also relaxed to allow medium intensity turbulence (5%). On the other hand, the outlet condition was set to a relative pressure of 0 atm. Both the input and output flow boundaries, shown in Figure 5.7, were constrained in motion to maintain correct flow orientations.
A “general connection” interface model was used to transfer data across the fluid to fluid interface. This type of interface model is commonly employed in rotary applications provided it has the capability of connecting both matching and non-matching grids. In rotary applications such as a gas turbine stage, the rotor and stator are disengaged allowing rotation of the internal component without destroying the mesh structure. The “general connection” model employed transfers data across the interface even without the elements matching across the boundaries. A “conservative interface flux” model was used for mass and momentum conservation across the interface. It is the robustness of this data transfer model which makes it highly applicable to nonlinear transient studies.
The next stage in constructing the numerical model was to define the solid-fluid domain interaction. A wall boundary condition was used to define this interface. The wall boundaries may be configured to represent impermeable surfaces with no slip constraints. That is, the fluid particles that are in immediate contact with the wall have zero velocity. This constraint extends itself to relative displacements and deformations in the rigid body. The surface roughness of the rigid body was set to the default smooth condition. This treatment is appropriate provided the airfoil surfaces are smooth and the transition point is not critical to the oscillating system. Variations in the surface roughness parameters did not impact the model behavior as depicted in Chapter 6.

Figure 5.8: Elastic Axis of Rigid Body

The reaction forces for the rigid body were defined at the elastic axis of the system as shown in Figure 5.8. The position of elastic axis itself depended on the fixturing of airfoil to
the torsional membrane. The main advantages of implementing a mathematical reaction torque within the numerical model included a significant reduction in computational effort for solving the model and the ability to vary design parameters for optimization. The implemented mathematical models were derived in Chapter 4. The design parameters for the torsional membrane included material properties, geometry and tension. A completed model with all boundaries and constraints is shown in Figure 5.9. Gravitational effects were also incorporated into the model, improving accuracy.

![Completed Model](image)

**Figure 5.9: Completed Model**

The last stage of constructing the ANSYS numerical model included defining the solver parameters. The analysis type was set to transient provided the system is unstable and
results in self-sustained aeroelastic oscillations. The objective for simulating the system was to capture the frequency and amplitude of the energy harvesting system subject to design parameters. The total time for each simulation was set to 1 second at 0.001 second intervals. The “advection scheme”, which denoted the numerical discretization method and “turbulence numerics” were both set to “high resolution”.

For convergence control the ANSYS CFX solver was set to complete two coefficient loops for every time step. Within each coefficient loop, the convergence criterion was set to Root-Mean-Square (RMS) with a residual target of 1e-4. The two-way coupling control was set so that data was transferred across domains on every coefficient loop. That is, the coupled system was resolved twice for every time step. Angular momentum control was also included in the solver in order to increase the rigid body’s accuracy in response to variations in mass and moment of inertia.

The output control was modified in order to prevent all data from being recorded on every time step. In default mode, ANSYS records 60 data sets for each of the 766,625 elements on every time step (1000 total). The default setting was trimmed down to allow 15 data sets to be recorded on every time step. This included but was not limited to pressure, velocity, displacement and turbulence data sets. Additionally, a monitor tag was embedded in the solver for controlling angular displacements while solving. A sample of the monitoring process is shown in Figure 5.10.
5.2 General Model Behavior

The core of this investigation consisted of evaluating both computational models and prototypes for optimizing the proposed energy harvester. The limitations associated with constructing prototypes for each design parameters motivated the development of the numerical model. The goal of this model was not only to evaluate the designs but also to provide additional insight regarding the complex aeroelastic behavior driving the system. This section aims at analyzing the behavior of the aeroelastic energy harvester as depicted by Figure 5.10: Angular Displacement of Rigid Body.
the numerical model. The first step in verifying the energy harvesting model consisted of observing the targeted self-sustained aeroelastic oscillations as shown in Figure 5.11.

![Figure 5.11: Self-Sustained Aeroelastic Oscillations](image)

Results that demonstrate self-sustained aeroelastic oscillations are extremely difficult to achieve with traditional mathematical modeling provided the system is driven by nonlinear forces resulting from turbulent aerodynamics. So far, there are no non-empirical modeling alternatives besides computational fluid dynamics (CFD) for addressing this phenomenon. The result in Figure 5.11 depicts the model subject to airflow at 3.5 m/s and a 25 degree
angle of attack. The model displayed vibrations at a frequency of 7.5 Hz and a mean amplitude of 85 degrees. This model was solved in millisecond intervals for a period of 1 second (1000 time steps) and required 10 hours of runtime by use of a 4 gigabyte 2 Quad CPU (2.4 GHz) PC in local parallel mode (2 partitions).

Aeroelastic vibrations arise as a result of rapid and repetitive transitioning between laminar and turbulent flow in the system. This phenomenon is known as dynamic stall, and occurs when the system is subjected to a rapid decrease in aerodynamic lift resulting from a loss of suction on the topside of the airfoil. The numerical model was then used to verify that the forces hysteresis driving the system was resulting from dynamic stall. This was accomplished by analyzing the numerical results from transient runs in millisecond intervals. The objective of this analysis was to characterize the flow at different stages of the oscillation cycle.

Figure 5.12 shows the rigid body (airfoil) subject to both unstalled and stalled aerodynamic flow conditions. The different colors in the figure correspond to variations in turbulence, as outlined by the eddy viscosity approximation. The maximum turbulence is depicted in color red, while the minimum is blue. Within the unstalled regime, the airflow is attached to the top and bottom surfaces of the airfoil. As the angles of attack increases, the boundary layer on the topside will also tend to increase in thickness and shift towards the fore of the airfoil. As the inertia of the flow continues to increases, the likelihood of observing separated flow within the boundary layer amplifies. At some point, the flow will completely separate above the airfoil marking a full stall. In this mode, the air circulation behind the airfoil yields vortices which are shed downstream.
Figure 5.12: Turbulence Modeling at 0.015 Second Intervals (Airfoil in White Color)

A typical cycle for the aeroelastic system consists of two (2) distinct stages. The first stage, displayed in Figure 5.13, corresponds to the upstroke (half-cycle) of the system and is defined by a strong correlation between aerodynamic forces and the angular deflection of the system. In contrast to static aerodynamics, the stalling angle of attack in the dynamic system is much greater and usually stochastic in nature. This extension in angle of attack leads to force hysteresis which is the foundation for archiving aerodynamic hysteresis on the system. It allows the energy harvester to extract kinetic energy from the moving fluid during the upstroke in a similar fashion to a piston within an internal combustion engine. The completion of the first stage is marked by zero instantaneous velocity in the airfoil.
Figure 5.13: Stage 1 – Airfoil Attains High-Angles of Attack in Delayed Aerodynamic Stall

The second stage, shown in Figure 5.14, corresponds to the downstroke of the system and is dominated by the potential energy stored within the energy harvester. This energy is extracted during the upstroke and stored within the torsional membrane. During this stage, the energy is converted back into kinetic energy following a rapid reduction in aerodynamic forces. The downstroke begins at zero instantaneous velocity and follows through the natural response of the spring-mass-damper system. As such, system dynamics are mainly dominated by structural stiffness and moment of inertia. Both aerodynamic and counter-electromotive damping is present during all stages of the cycle.

The downstroke of the airfoil is also marked by the creation and shedding of a vortex in the airfoil. The vortex formation and propagation, shown in Figure 5.14, results from the boundary layer separating on the upper surface of the airfoil. This turbulent flow pattern is characterized by its high velocity and low pressure at its center. The disturbance, which
initiates on the leading edge of the airfoil, results in backflow and is amplified as it travels downstream. A second vortex forms in the trailing edge of the airfoil following the formation of the first vortex. The flow is reversed on the topside of the airfoil following the formation of the second vortex as depicted by the red circles on Figure 5.14. This opposing flow yields an increasing pressure on the upper surface on the airfoil which results in the reduction of aerodynamic lift (stall).

![Reverse Flow](image)

Figure 5.14: Stage 2 – Airfoil Stall due to Vortex Formation and Propagation

The next stage in verifying the aeroelastic phenomenon was to quantify this force hysteresis driving the system. This was accomplished by monitoring the lift forces on the system subject to input flow conditions. The hysteresis corresponding to the lift forces,
represented in dimensionless form, is shown in Figure 5.15. The dimensionless lift coefficient ($C_L$) is derived as follows:

$$C_L = \frac{L}{\frac{1}{2} \rho V^2 A}$$  \hspace{1cm} (5.1)

where $L$ is the lift force, $\rho$ is the local air density, $V$ is the airspeed and $A$ is the planform area. The figure depicts the model subject to airflow at 4 m/s and a resting 20 degree angle of attack. The results show that the lift forces follow a path-dependent trajectory which results in the self-sustained oscillations. Conventional airfoils typically do not operate beyond a 15 degree angle of attack, which roughly corresponds to a Coefficient of Lift of 1.5 for a NACA 0012 airfoil.

![Figure 5.15: Hysteresis in Lift Coefficient](image)

Figure 5.15: Hysteresis in Lift Coefficient
The lift curve (Figure 5.15) demonstrates the remarkable ability for the dynamic system to stall at extremely high angles of attack (50 degrees). This stall angle is approximately double in comparison to a NACA 0012 quasi-static airfoil, which stalls at roughly 17 degrees. In turn, the delayed stall allowed the system to experience lift forces \( (C_{L,max}=2) \) exceeding that of the typical NACA 0012 airfoil. The lift path follows a drastic increase in forces which is then subject to rapid reductions (stall) and reverse lift conditions. The reverse lift condition arises as a result of the dual vortex formation in the system.

Figure 5.16: Hysteresis in Drag Coefficient

The drag characteristics of the aeroelastic energy harvester were also evaluated. It was expected that drag forces would also play a significant role in achieving self-sustained
aeroelastic oscillations. The hysteresis corresponding to the drag forces, represented in
dimensionless form, is shown in Figure 5.16. The dimensionless drag coefficient ($C_D$) is
derived as follows:

$$C_D = \frac{D}{\frac{1}{2} \rho V^2 A}$$  \hspace{1cm} (5.2)$$

where $D$ is the drag force, $\rho$ is the local air density, $V$ is the airspeed and $A$ is the
planform area. The figure also depicts the model subject to airflow at 4 m/s and a resting 20
degree angle of attack.

The results demonstrate that drag hysteresis is as significant as the lift hysteresis. In
contrast to conventional drag curves, the proposed energy harvester is subject to low speed
airflow, which results in increased drag. The drag coefficient is typically inversely related to
the Reynolds number provided that frictional forces are dominant in laminar flow. The drag
curve also demonstrates that the energy harvesting system loses energy to the flow above the
neutral axis. After the airfoil stalls at about 50 degrees, the vortex creates significant amount
of drag as the airfoil attempts to cycle back. This energy loss is countered by the energy
gained during the complete cycle by the lift forces.

The total forces acting on the system were also analyzed in gaining insight regarding
the aeroelastic phenomena driving the system. The complete moment reaction, in
dimensionless form, is shown in Figure 5.17. The dimensionless total moment coefficient is
derived as follows:
\[ C_{TM} = \frac{I\alpha}{2\rho V^2 A_c} \]  

(5.3)

where \( I \) is the moment of inertia, \( \alpha \) is the instantaneous angular acceleration, \( \rho \) is the local air density, \( V \) is the airspeed, \( A \) is the planform area and \( c \) is the length of the chord of the airfoil. The figure depicts the model subject to airflow at 4 m/s and a resting 20 degree angle of attack. The moment coefficient is extremely large provided that it represents the summation of all aeroelastic moments at the elastic axis. This includes forces resulting from aerodynamic lift and drag, and mechanical reactions from the torsional membrane.

The total moment coefficient is extremely large for the energy harvester due to design considerations, in particular to the position of the elastic axis. That is, by having the elastic
axis located in the back of the airfoil the large moments are achieved due to large moment arms, provided the aerodynamic forces (lift and drag) act in front of the airfoil (quarter-chord point). The combined hysteresis of both aerodynamic forces allows the system to sustain stable aeroelastic oscillations. Both mechanical and electric damping in the system allows the airfoil to smoothly transition between the dynamic forces exerted by the vortices.

The hysteresis shown in the moment diagram (Figure 5.17) also confirms that the self-sustained aeroelastic oscillations arise as a result of vortex formation and propagation. The shown trajectory for the total moment is path dependent with maximum work being realized above the neutral axis. The total work done on the system may also be characterized by the line integral on the moment curve. This corresponds to the area within the closed-loop trajectory. For this design, the energy harvesting system will also do work on the moving fluid. This is marked by the cross-over point below the neutral axis. While the upstroke is mainly dominated by nonlinear aerodynamics, the work performed by the system on the downstroke results from the potential energy stored in the torsional membrane.
Chapter 6: Design and Fabrication

Several physical prototypes were designed and fabricated throughout this investigation. This chapter examines the design and fabrication processes. The evaluation begins by analyzing the design for each of the individual component which makes up the physical system. This will cover geometric and material information for each of the components. Apart from covering the design for the components, this chapter will also focus on fabrication, assembly and calibration. The objective of this chapter is to provide all the necessary information required to reproduce the proposed energy harvesting system.

The design examination process begins by identifying the basic components of a physical prototype, shown in Figure 6.1. The major components of the prototype include the 1) thin membrane, 2) coil stator, 3) thin airfoil, 4) permanent magnets, 5) rigid fixture and 6) coil fixture. All of these components are critical for the energy harvesting system to function properly. While some components encompass single materials and basic geometries others are more complex in nature. The chapter also seeks to emphasize the simplicity behind the proposed energy harvesting mechanism.

![Figure 6.1: Basic Components of Physical Prototype](image)

(1) Thin Membrane  (2) Coil Stator  (3) Thin Airfoil  (4) Permanent Magnets (two)  (5) Rigid Fixture  (6) Coil Fixture
6.1 Thin Membrane

The first component to be examined in this section is the elastic membrane. This investigation identified polyvinylidene fluoride (PVDF) as an excellent candidate for this component due to its low elastic modulus (2.1-2.9 GPa) and significant elongation (above 50%). These two properties made the material an appropriate fit in regards to functional requirements. The key characteristic of the PVDF was that high elongation permitted the torsional membrane to be rotated 90 degrees while not exiting the elastic region. Additionally, it was possible to do so even while the material was subject to initial axial tension.

The design of the torsional membrane was relatively simple. A long and thin rectangular strip of the material was sufficient for achieving the required functionality. The critical design parameter for observing the targeted ultra-low nonlinear torsional stiffness, as depicted by the mathematical models, was thickness. The investigation identified that thicknesses below 50 µm achieved this effect while imposing very limited parasitic damping on the system. A typical torsional membrane was 25 µm x 6.35 mm x 101.6 mm (1/1000” x 1/4” x 4”) in dimension, as shown in Figure 6.2. Current market value for the material ranges around $0.03 for each strip.

Figure 6.2: Long and Thin PVDF Strips
The long and thin PVDF strip permitted the system to be elastically deformable subject to external aerodynamic loading. Materials with higher modulus of elasticity would restrict the system’s response to the limited aerodynamic loading. Additionally, a higher modulus of elasticity would likely be accompanied by a significant reduction in elongation. PVDF offers the precise combination between stiffness and elasticity for the prescribed application. A torsional- membrane was cyclically loaded within a working prototype continuously up to 1 million cycles to evaluate its fatigue-life. No plastic deformation was detected.

The PVDF strips were cut to dimensions by use of a VersaLASER cutting machine, shown in Figure 6.3. The laser cutting machine was essential in achieving uniform cuts in the material. Prior to implementing the laser cutter, sharp blades and scissors were used to achieve the desired cuts in the PVDF sheet. It was observed that the non-uniform cuts produced notches which resulted in stress risers. Provided that the thin membrane is cyclically loaded, it was undesirable to have stress risers provided they could tear the membrane or induce localized plastic deformation, affecting the overall stiffness response.

Figure 6.3: VersaLASER Cutting Machine
6.2 Coil Stator

The coil stators employed in this investigation were both made in-house and procured from external sources, in particular musical instruments. The main advantage of using commercial coils rather than fabricating the devices in-house, as shown in Figure 6.4, was the packing density. As demonstrated in Chapter 4, for maximizing output power it was desirable to employ sufficient turns in order to achieve working voltages while minimizing overall coil resistance. In order to achieve this goal it was important to maximize the packing density of the coil with the selected wire gauge. In other words, loose packing represents an opportunity for increasing the resistance of the wire (minimizing internal losses).

![Home-Made 30 AWG Gauge Coil Consisting of 300 Turns](image)

Figure 6.4: Home-Made 30 AWG Gauge Coil Consisting of 300 Turns

For a limited coil volume it is always desirable to employ low resistance wire subject to voltage limitations. The coil requires a minimum number of turns such that the device
generated workable voltages. A typical electromagnetic transducer for musical applications is shown in Figure 6.5. Commercial pickups typically have DC resistances in the order of 1-10 kΩ, 1,000–10,000 turns and wire diameters between 63.5 µm (0.0025”) and 127 µm (0.005”). This investigation employed a variety of coils in examining which combination resulted in the best possible power-voltage combination. Very low voltages were undesirable since they require boosting to drive loads and add losses resulting from conversion inefficiencies.

![Figure 6.5: Single-Coil Electric Guitar Magnetic Pickup](image)

The procured electromagnetic coils varied in regards to their DC resistance and number of turns. An important design parameter was matching the geometry of the coils to the area of the magnets in maximizing flux. It was necessary to remove all magnetic components from the purchased coils such that only the conductors were left behind. Although the magnets would increase the flux density in the coils, resulting in higher voltages, leaving them within the transducers would also affect the dynamics of the system. This is because at operating proximity the coil magnets would interact with the device magnets offsetting the aeroelastic force balance.
As it was demonstrated in Chapter 4, power optimization in coil design is independent of wire gauge and number of turns for a fixed volume. The main advantage for using commercial pickups was the high packing density with the desired geometry. Round wire has a maximum packing density of 90.69% when wound in a hexagonal arrangement. Provided that 20.28% of wire volume is insulated (42 AWG), this shows that the maximum bare copper per volume achievable is approximately 72.3%. These results demonstrate the practical limitations associated with typical magnetic wire. Improvements may be achieved by implementing square or rectangular magnetic wire and further minimizing the insulation layer thickness.

The final coil that was employed in this investigation consisted of a 42 gauge wire with a nominal diameter of 71.1 µm (0.0028”). This included an insulating coating with nominal thickness of 7.62 µm (0.0003”) to prevent the coil from shorting. The nominal resistance for this wire gauge is 1652 Ohms/1000 ft. The coil consisted of 7,000 turns. Physical experimentation demonstrated that packing density was the most important factor in selecting the inductor coil, in particular at locations in close proximity with magnetic flux provided the magnetic field strength varies inversely proportional to the distance squared. The total coil impedance was 4,800 Ω as shown in Figure 6.6. No variation in impedance was observed below 100 Hz.
6.3 Thin Airfoil

The thin airfoil is a critical component provided it interfaces the system with the moving fluid. Additionally, the computer modeling (Chapter 5) revealed that the aeroelastic energy harvester has a moment of inertia threshold. As such, there’s a limited moment of inertia the system may drive while sustaining stable oscillations. In regards to the proposed solution, the moment of inertia was affected by two major components, mainly the 1) thin airfoil and 2) permanent magnets. The total moment of inertia in the system was extremely sensitive to the airfoil geometry provided that the center of mass is offset.

The functional requirements for the thin airfoil included sufficient structural rigidity and minimal density. These requirements are typically encountered in the aerospace industry for similar reasons. Candidate materials included a variety of plastic and composite materials. Some of the plastics that were considered included acrylonitrile butadiene styrene (ABS),
poly(methyl methacrylate) (acrylics), polystyrene (foam) and polyethylene terephthalate (PETG). Alternately, carbon fiber was also considered as a candidate material in satisfying the desired requirements as shown in Figure 6.7. The main difficulties associated with implementing carbon fiber included poor machinability and high density resulting from epoxy matrix.

The best material choice for the thin airfoil component consisted of the laser machined acrylic. Despite having moderate density (1.18 g/cm³), the material was chosen due to its ease of machinability by use of laser processes. Traditional machining of thin-shells structures, similar to that of a small airfoil, is very difficult provided the parts are hard to fixture. The VersaLASER cutting machine (Figure 6.3) eliminated fabrication problems by allowing full 3-axis machining capabilities of acrylic. This permitted thin-airfoils to be machined to specified thicknesses and geometries as shown in Figure 6.8. The standard acrylic sheets were reduced from 1.53 µm (0.06”) to 0.51 µm (0.02”) in minimizing the added moment.
Another advantage of employing both the laser cutter and acrylic was the ability to control the material stiffness. During the machining process it was determined that there was significant correlation between stiffness and cutting direction. This allowed the airfoil to be machined with increasing stiffness along the critical direction, which supported the aerodynamic loading. The total weight and mass moment of inertia of the extended airfoil was well tolerated by the torsional spring such that it did not deform statically. The extensions in the side of the airfoils were used when modifying the aspect ratios during the damping studies. When increasing the span and reducing the chord length of the airfoil, it was important to prevent the airfoil from interfering with the torsional membrane.

6.4 Permanent Magnets

This investigation considered several permanent magnets for the rotor. All magnets were procured from the same source (K&J Magnetics) and mostly consisted of Neodymium-based block magnets. As stated previously, the computation studies revealed that the system could tolerate significant increases in moment of inertia prior to hampering system dynamics.
The main advantage of using long slender magnets was that the moment of inertia could be minimized provided the mass is both balanced and close to the axis of rotation. The first iterations of the energy harvester consisted of two magnets with a total weight of 3.84 grams (1.92 g each). The final prototype consisted of two larger magnets, as shown in Figure 6.9, which resulted in a total weight of 15.36 grams (7.68g each). The 4x weight increase did not affect the targeted nonlinear dynamics.

![Figure 6.9: N42 Grade Neodymium Permanent Magnet](image)

50.8 mm x 6.35mm x 3.175 mm (2” x 1/4” x 1/8”)

The magnets employed in this investigation were all N42 grade, which dictates the maximum strength the material may be magnetized to. Higher grades (N50 and N52) were unavailable for the required geometrical specifications. With dimensions of 50.8 mm x 6.35mm x 3.175 mm (2” x 1/4” x 1/8”), the magnets encompassed a surface field of 3332 Gauss and a pull force of 45.99 N (10.34 lbs), which corresponds to the force required to remove the magnet from a steel plate. The finalized device employed two magnets in balancing the mass about the torsional membrane. The pull force between both magnets at a
distance of 635 µm (0.025”) (airfoil and membrane) is approximately 111.2 N (25 lbs). This allowed the magnets to stay attached at high accelerations.

6.5 Rigid Fixture

The rigid fixture was designed to serve various functions, mainly to 1) grip the torsional membrane, 2) provide structural rigidity to the system, 3) permit alignment capabilities, 4) allow tension adjustments and 5) rotational capabilities (angle of attack).

Some of the functional components of the system are shown in Figure 6.10, which shows the frontal view of the structure. From a structural standpoint, the fixture required sufficient rigidity to support the membrane’s initial tension, which was not more than 44.5 N (10 lbs). From a calibration’s standpoint, it required that the grippers be able to swivel in order to align the membrane subject to tension.

The structure was constructed principally out of 6061-T6 aluminum. The alignment system consisted mainly of T-slotted extrusions, which aided in guiding the grippers while
applying tension and locking the final position. The pivoting structure consisted of a tension rod and two support blocks as shown in Figure 6.11. The purpose of the pivoting system was to allow rotation of the main structure while conducting angle-of-attack experiments. This was accomplished by positioning the system at the desired angle of attack and tightening the tension rod. The support blocks were machined in order to support the moments produced by the structure during rotation. Overall, the structure encompassed the required stiffness required to promote the self-sustained aeroelastic oscillations.

![Figure 6.11: Pivoting Structural Blocks](image)

### 6.6 Coil Fixture

The coil fixture is the final component found on the proposed oscillator and prototype. Unlike the membrane grippers, which provide two-dimensional calibration capabilities, the coil fixture required full 3-dimensional calibration. This requirement is based on the fact that the coil must be positioned in order to maximize the capture of magnetic flux.
The optimal position which was determined experimentally consisted of positioning the magnets in such a way such that electromagnetic induction was maximized during the cycles. This was determined by examining the instantaneous velocity of the permanents magnets during one cycle. The computation models revealed that the maximum velocity is achieved when the system is close to its maximum amplitude.

The large magnets were therefore positioned in such a way that the high flux areas were cut by inductor coils at maximum velocity. This complex requirement was satisfied by both designing and rapid-prototyping a coil holder that included three-dimensional calibration capabilities. The component was design in PTC Pro/ENGINEER as shown in Figure 6.12. The main functionality of the system was holding the coil at a specified position with enough rigidity to counter the electric damping during energy conversion.

![Figure 6.12: Coil Fixture Design](image)

The mechanism which was design and fabricated for calibrating the coil fixture in three-dimensions is shown in Figure 6.13. The figure shows that the structure has a total of 6 butterfly nuts which may be adjusted individually to set the position of the coil fixture. The
calibration began by first setting the X-Y position by sliding the fixture along the T-slotted frame and iteratively setting the height. The Y position was selected such that the magnet’s faces matched the cross-sectional area of the coil at 90 degrees. The Z calibration was performed last provided it may be accomplished independently. The procedure focused on minimizing the air gap between the magnets and coil without causing rotational interference.

Figure 6.13: Coil Fixture Calibration
Chapter 7: Experimental Results

The core of this investigation consisted of evaluating both computational models and prototypes for optimizing the proposed energy harvester. Limitations associated with constructing prototypes for each of the design parameters motivated the development of a numerical model. Additionally, the objective of the numerical model was not only to evaluate the designs but also provide additional insight regarding the complex aeroelastic behavior that drives the system. The construction of the computational model was outlined in Chapter 5. The main challenges associated with constructing the numerical model consisted of significant meshing requirements and intricate fluid-structure-interaction (FSI) modeling. Conversely, design and fabrication of the physical prototypes was outlined in Chapter 6.

This chapter will elaborate on both results stemming from physical and computational experimentation. The computational model served as a powerful tool in evaluating the system. As stated previously, it is not possible to model the aeroelastic phenomenon by use of traditional mathematical modeling because the underlying nonlinear forces result from turbulent aerodynamics. So far, there are no modeling alternatives besides computational fluid dynamics (CFD) for examining this problem. This chapter will address a variety of experimental studies, mainly the 1) planform area, 2) coil position, 3) airfoil shape, 4) angle of attack and airspeed, 5) electromagnetic damping and 6) moment of inertia. The results of each study were then employed in developing design guidelines for optimizing the proposed energy harvester.
7.1 Airfoil Planform Study

This study examines the effect of employing different planforms for the proposed aeroelastic energy harvesting solution. It focuses on designing the span (width) and chord (length) of the airfoil. In essence, this study investigates the effect of employing airfoils with different aspect ratios and sizes. It is expected that, for a fixed system, increasing the planform area (via the span or chord) will result in higher aerodynamic forces and damping at the expense of frequency. Conversely, it is expected that decreasing the planform area in the airfoil will result in higher frequencies which in turn likely increase electromagnetic damping from the transducer. The main objective of this study is to determine the appropriate balance between the span and chord in the airfoil and its effects on system performance.

The methodology for conducting this study consisted on varying the chord length of the airfoil while retaining a constant planform area. By following this approach, increasing the chord length resulted in proportional reductions in span. This study was conducted by use of both computational models (Chapter 5) and experimental prototypes (Chapter 6) evaluated at a 10 degree angle of attack and 4 m/s airspeed. The physical outcome of varying the chord length (while retaining the planform area constant) on airfoil geometries is depicted in Figure 7.1. The figure shows four different geometries (all with a planform area of 12.90 cm² or 2 in²) with chord and span lengths as specified on Table 7.1. The airfoils where individually fixed to the permanent magnets so that it was possible to interchange them in the system while retaining structural parameters (spring stiffness). The strength of the permanent magnets was sufficient in holding the airfoils in place during experimentation.
Table 7.1: Constant Planform Airfoil Geometries

<table>
<thead>
<tr>
<th>Chord</th>
<th>Span</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3.175 cm (1.25”)</td>
</tr>
<tr>
<td>3</td>
<td>2.54 cm (1.00”)</td>
</tr>
<tr>
<td>2</td>
<td>1.91 cm (0.75”)</td>
</tr>
<tr>
<td>1</td>
<td>1.27 cm (0.50”)</td>
</tr>
</tbody>
</table>

The four airfoils in Figure 7.1 reveal the practical limitations associated with employing certain geometries. On both extremes of the spectrum the airfoil geometries are infeasible. Very small chord lengths require long spans which challenge the structural rigidity of the airfoil. On the other hand, very long chords result in prohibitive moments of inertia which challenge the structural stiffness of the system (spring). This forms the foundation for searching for solutions around permissible geometries. Practical experimental revealed that thin (under 152.4 mm or 0.06”) plastic airfoils with aspect ratios (chord/span) lower than (1) deflected under gravity due to insufficient structural rigidity. Alternately, airfoils with aspect ratios in the range of (4) resulted in static spring deflections due to gravity.

Figure 7.1: Constant Planform Thin Plate Airfoils
The frequency results for the planform area study are displayed in Figure 7.2. The results show that for constant planform areas, the oscillator’s frequency is approximately related to the reciprocal of chord length as shown below:

\[(f \cdot c)_A \propto C\]  \hspace{1cm} (7.1)

where \(f\) is the observed frequency, \(c\) is the chord length, \(A\) is the planform area and \(C\) is an arbitrary constant. The accuracy of this result is best for the numerical model in contrast to the experimental prototypes. Figure 7.2 shows two inverse proportions which correspond to two different constants from Equation 7.1. It is important to emphasize that this results are for an unloaded system. That is, these systems are not converting any mechanical energy into the electrical domain by use of electromagnetic transduction. These results highlight the aeroelastic nature of the proposed energy harvester and provide a simple approach for tuning the frequency.

Figure 7.2: Frequency (Hz) vs. Chord Length (m) for Constant Planform Areas
The results in Figure 7.2 show that varying airfoil geometry while retaining planform areas provides a practical approach for tuning unloaded aeroelastic systems. The approach may be further extended to study the loaded system itself. This was accomplished by fixturing an inductor coil while driving a load. The dummy load consisted of a variable resistor matching the internal impedance of the coil. Both aeroelastic and electromechanical damping ratios were measured independently for the 12.90 cm² (2 in²) and additional 19.35 cm² (3 in²) planforms. The geometries for the additional 4 prototypes (19.35 cm² planforms) consisted of the following chord to span ratios: 1.75:1.714, 1.5:2.0, 1.25:2.5 and 1.0:3.0.

![Matching Point for 19.35 cm² (3 in²)](image)

Figure 7.3: Aeroelastic and Electromagnet Damping

This study proposed and demonstrated that varying chord lengths subject to constant planform areas is a practical approach for matching aeroelastic and electrical damping in
aeroelastic energy harvesting. To the author’s knowledge, there are no established alternatives for achieving this goal. The main challenge associated with finding a solution is that for each planform area airfoil there exists one unique solution as demonstrated in Figure 7.3. Any other alternatives, such as varying the span or chord independently, represent intractable approaches for matching the damping in the system. The results in Figure 7.3 also demonstrate that for a fixed system, reductions in planform areas push the airfoil design into the high aspect ratios (chord/span) regime and decrease total damping. Conversely, increasing the planform areas pushes the airfoil design into the lower aspect ratio (chord/span) regime at the expense of total system damping. This result presents a major guideline for optimizing the proposed aeroelastic energy harvester.

7.2 Coil Position Study

The coil positioning study intends to investigate the effect of positioning the magnets with respects to the proposed coil design. The design consists of two block magnets with opposite poles facing the torsional membrane such that the magnets themselves are used for fixturing. A simple analysis into the moment of inertia of common magnet geometries revealed that balanced magnets (square cross-sections) are advantageous over alternative configurations. This was achieved by using two rectangular magnets that form a square cross-section when coupled. This study aimed at examining how this configuration interacted with the proposed coil subject to the observed system dynamics. The employed magnet configuration is shown in Figure 7.4. The figure demonstrates that this setup results in 4 planes of interest, 2 maximum magnetic flux planes and 2 minimum magnetic flux planes.
The figure demonstrates that at rest, the proposed coil-magnet configuration results in minimum magnetic flux interaction. This simple observation was verified experimentally by use of iron cores. The cores were rotated around the device to gauge for relative magnetic interaction. For the proposed configuration, not only was the magnetic field displaced at the neutral position, but also the inductor coil cross sectional area. Provided that both the minimum and maximum magnetic field ($B$) and area perpendicular to the magnetic field ($A$) occur at 0 and 90 degrees, respectively, an expression for the magnetic flux may be approximated as follows:

$$\Phi_B = B \cdot A = B \cdot \sin(\alpha) \cdot A \cdot \sin(\alpha) \propto \sin^2(\alpha) \quad (7.2)$$
where $\alpha$ represent the displacement angle relative to the magnets resting position. In electromagnetic induction, the power generated by the proposed aeroelastic energy harvester is proportional to the square rate of change in the magnetic flux $(d\Phi_B/dt)^2$. As such, a proportionality expression for the rate of change in magnetic flux may be derived as follows:

$$d\Phi_B/dt \propto \cos(\alpha) \cdot \sin(\alpha) \cdot (d\alpha/dt)$$  \hspace{1cm} (7.3)$$

where $(d\alpha/dt)$ represents the instantaneous angular velocity of the system. This result may be then extended to show that for the proposed torsional electromagnetic energy harvester, which consists of an air core, power output is directly proportional to angular position and velocity, as shown below:

$$Power\ Output \propto \cos^2(\alpha) \cdot \sin^2(\alpha) \cdot (d\alpha/dt)^2$$  \hspace{1cm} (7.4)$$

This expression may be then utilized to analyze a phase space results obtained by use of the computational model. A typical phase space output from the model is shown in Figure 7.5, which encompassed the angular velocity of the airfoil with respect to its position for a system at a 10 degree angle of attack and 4 m/s airspeed. The phase space diagram shows that there is inherent variability associate with turbulent nature of the energy harvester. As observed, there is approximately a 20 degree maximum peak-to-peak variation for the simulated model. The figure demonstrates that maximum angular velocity occurs roughly at 50 degrees, which corresponds to approximately half the maximum amplitude. This result
highlights the complex periodic nature of the proposed energy harvester in contrast to simple sinusoidal waveforms with maximum velocity at 0 degrees.

Figure 7.5: Phase Space - Angular Velocity (rad/sec) vs. Position (Degrees) at 10 Degrees for 4 m/s Airspeed

The phase space results from the computational model may then be employed in evaluating the instantaneous power resulting from electromagnetic induction and how it relates to amplitude for the proposed configuration. The proportionality expression (Equation 7.2) for the rate of change in magnetic flux with respect to angular position is shown in
Figure 7.6. The figure demonstrates that maximum power is generated at approximately 45 degrees. The results demonstrate that low amplitude vibrations (which are marked by the system startup) result in roughly a third the rate of change in magnetic flux, as compared to the large amplitude vibrations.

Figure 7.6: Electromagnetic Induction vs. Position (Degrees) at 10 Degrees for 4 m/s Airspeed

These results demonstrate that the maximum rate of change in magnetic flux corresponds with the maximum angular velocity in the proposed energy harvesting system. The most important conclusion from this study is that high amplitude vibrations are required in maximizing the rate of change in magnetic flux. That is, for the proposed configuration, it
is imperative that the energy harvesting system be designed in order to achieve full 180
degree peak-to-peak amplitude to maximize power output. As it will be shown in the next
studies, there are several strategies that may be employed in increasing the amplitude of the
proposed energy harvesting system. It will be also demonstrated that increasing the airspeed
sufficiently enough to achieve the target amplitudes is the best strategy.

7.3 Airfoil Shape Study

The objective of this study is to examine the effects of employing different airfoil
shapes for the proposed aeroelastic oscillator. Initially, the system was examined by
employing standardized symmetrical airfoils, in particular the National Advisory Committee
for Aeronautics (NACA) 0012 airfoil. This airfoil was used primarily because of its large
body of experimental data regarding static conditions. This investigation later determined
that is was necessary to minimize the moment of inertia of the airfoil such that larger
magnets could be employed. To this end, laser machining processes were used to thin-out
acrylic plates to the desired thickness. From a practical standpoint, the best geometries that
may be employed are thin flat plates, mainly because of fabrication issues. Additionally, it
was observed that uneven machining of thin plastics easily resulted in warping due to
residual stresses in the material.

The study consisted of computationally comparing the aeroelastic oscillations of a
NACA 0012 airfoil to that of a simple rectangular flat plate. The objective was to determine
if there were any significant differences between the two shapes. To keep the model
consistent, equal meshing and setup parameters were used for both models as depicted in
Figure 7.7. To increase sensitivity, the torsional spring constant was increased and the mass moment of inertia was reduced in order to achieve higher frequency vibrations.

Figure 7.7: Meshing of the Flat Plate

The response for both aeroelastic systems is shown in Figure 7.8. The diagram demonstrates that both signals are very similar. The RMS amplitude of the NACA 0012 was calculated to be 63.35 degrees while that of the flat plate was 64.15 degrees. The calculated frequencies for this system were 11.11 Hz for the NACA 0012 airfoil and 10.75 Hz for the rectangular plate. The slight variation is mainly attributed to a displaced moment of inertia in between the airfoil and flat plate. The geometry of the flat plate was calculated by retaining the chord of the airfoil while matching the thickness in order to retain the original volume.
The overall result of doing so was that the moment of inertia for the NACA 0012 airfoil was slightly higher provided this shape has more mass concentrated in its leading edge.

![Figure 7.8: Aeroelastic Response of NACA 0012 vs. Flat Plate](image)

The results for this study demonstrated that there is negligible difference between employing standardized symmetrical airfoil or flat plates. This result was also observed experimentally. Additionally, the most important factor in considering airfoil shapes is that they need to be thinned out the maximum amount while retaining structural rigidity in optimizing the proposed energy harvester. From a practical standpoint, standardized airfoil shapes are not desirable because they taper down towards the trailing edge which corresponds to the mounting location. The fabrication process which was described in
Chapter 6 successfully machined down a 1.52 mm (0.06”) acrylic plate to a thickness of 762 μm (0.03”). It was determined experimentally, that for this material, further reductions in thickness challenge the structural rigidity of the airfoil. In other words, further machining down of a 762 μm (0.03”) thin airfoil is not feasible.

**7.4 Angle of Attack and Airspeed Study**

This section investigates the effect of the proposed energy harvester with respect to the angle of attack of the airfoil and incoming airspeed. It is expected that high airspeeds would result in higher amplitude vibrations due to larger energy inputs into the system. Figure 7.9 demonstrates experimental results obtained for a prototype that consisted of a 3.81 cm x 2.54 cm x 0.85 cm (1.5” x 1” x 1/3”) airfoil and a 13.97 cm x 0.64 cm x 26 μm (5.5” x 0.25” x 1/990”) PVDF membrane. The prototype was loaded with two large slender magnets which resulted in a total weight of 15.36 grams (7.68g each). The applied tension on the membrane was 33.36 N (7.5 lbs). Figure 7.9 illustrates the measured amplitude with respect to airspeed (m/s) and angle of attack (degrees).
This figure demonstrates the overall relationship of a fixed-design aeroelastic oscillator with respect to airspeed and angle of attack. The experimental procedure for achieving these results consisted of subjecting the aeroelastic energy harvester to a preset airspeed and measuring both the amplitude and frequency of oscillations while varying the angle of attack. The procedure was repeated nine times for all of the different airspeeds which resulted in 81 experiments. The presented values correspond to averages due to light amplitude variation during each cycle. The results demonstrated that in general, average amplitudes increases with airspeed as expected. The accompanying frequency plot for all experimental conditions is shown in Figure 7.10.
In contrast to the amplitude results, the frequency diagram showed very small variation with respect to the angle of attack and airspeed. The result confirmed preliminary observations that the frequency of oscillation was primarily driven by structural parameters and planform area. In analyzing and combining results, it was necessary to compute the acceleration for each experiment. This is because previous analytical work has demonstrated that system acceleration is directly proportional to the power generated by an energy harvester. The experimental results for system acceleration (rad/sec^2) are shown in Figure 7.11.
The acceleration results exhibited important trends, classified as group 1 and 2, as shown in Figure 7.11. Group 1 corresponds to a set of low airspeed experiments and group 2 to that of high airspeeds. The diagram shows that performance between these groups is approximately inversely related. In regards to group 1, the performance of the energy harvester was found to be superior over the high angle of attack systems. This is because the aerodynamic forces driving the system are highly dependent on angle of attack. As such, low angle of attack systems are exposed to significant lower forces at their resting point. On the other hand, high angle of attack systems are subjected to large pulsating forces at their
equilibrium point. It is the disparity in aerodynamic forces that results in the observed
dynamics at low speeds.

This inverse result was observed for high airspeeds as demonstrated by group 2 in
Figure 7.3. These results indicate that high angle of attack systems are slightly superior over
their respective low angle of attack counterparts. The acceleration results also demonstrate
that at a critical airspeed, in this case 4.75 m/s, both systems perform equally. After crossing
this critical airspeed, low angle of attack systems showed limited increased performance.
This is because aerodynamic forces are proportional to the airspeed squared. As such, the
aerodynamic forces at low angles of attack are high enough to deform the torsional spring.
The larger amplitudes arise provided that the torsional work is higher. That is, the airfoil is
engaged by the flow throughout a longer stroke.

The results from the angle of attack study demonstrate that system performance on a
fixed-design is dependent on both the angle of attack and airspeed. For this particular design,
high angles of attack were advantageous over the higher airspeeds while low angles of attack
systems outperform at lower speeds, all else in the system unchanged (moment of inertia and
spring stiffness). Assuming that acceleration is application dependent, there are two major
design guidelines (for a fixed system) which may be deduced from this study, mainly that 1)
high angle of attack systems may be employed for controlling amplitude by increasing
airspeed while retaining frequency and 2) low angle of attack designs demonstrate better
performance over a decreasing airspeed spectrum leading to improvements in the systems
overall robustness.
7.5 Electromagnetic Damping Study

This study consisted on evaluating the aeroelastic energy harvester’s performance with respect to electromagnetic damping. Electromagnetic damping in the system corresponds to energy that is converted from the mechanical domain into the electrical domain. Once the energy is converted into the electrical domain, some is dissipated by the electromagnetic transducer while the rest is transferred into the load. The total external damping that a system may withstand is dependent on the underlying phenomena driving the system. In this case it is aeroelastic in nature. The main challenge of working with aeroelastic phenomena is that the system’s response is often unpredictable provided it is mainly driven by periodic vortex shedding.

The effect of varying external damping (power conversion) on the oscillator is shown in Figure 7.12. The results correspond to experiments conducted by use of the computational model. Increasing electromagnetic damping can result from three mains sources, mainly 1) more copper in the inductor coil, 2) equal magnets of higher grade and 3) tighter air gap between the magnets and coil. This study examines the aeroelastic effect of increasing energy generation while retaining all structural components constant.
The damping results (Figure 7.12) demonstrate that energy conversion affects the behavior of the system distinctly for different airspeeds at constant angles of attack. This solution highlights the system’s sensitivity to varying one parameter. It was expected that increasing damping would decrease the amplitude of vibrations. However, the study revealed that amplitude may be diminished in three different modes in response to damping, as shown in Figure 7.12, mainly in a controlled (1), sustained (2) and accelerated (3) mode. In controlled (1) mode, which corresponds to airspeeds of 4 m/s, the numerical model demonstrates that the amplitude decreases linearly with increasing damping. This result
would be typical for a simple inertial oscillator. The other modes, which correspond to the higher airspeeds, demonstrated increased sensitivity to high damping.

It is important to emphasize that the drops in amplitude observed in groups 2 and 3 were also accompanied by an increase in cycle frequency. This variation may be observed in Figure 7.13, which characterizes the acceleration for the different conditions. Although the highest sensitivity and acceleration decline was observed at higher speeds (6 m/s), it was also shown that at high damping levels the oscillating mode outperforms the rest. This result is not beneficial in regards to the proposed energy harvester provided that excessive damping only results in transducer inefficiencies as depicted by the generic models. This study is also particularly important because it shows that the proposed energy harvesting device cannot tolerate significant amounts of external damping. As such, iron cores may not be employed in increasing the magnetic flux through the coil.

The acceleration results show that higher modes of vibration are achievable (group 2 on Figure 7.13) by subjecting the system to both high damping and airspeeds. Additionally, before arriving to this mode of vibration, the results also show that the acceleration decreases drastically, as depicted by group 1 in Figure 7.13. It is hypothesized that subjecting the aeroelastic oscillator to high damping at high airspeeds leads to over-damping in the down-stroke which inhibits the targeted large amplitude vibrations. At this point, it is expected that different aerodynamic phenomena would dominate overall system dynamics. As stated throughout this investigation, there are many ways to achieve self-sustained aeroelastic oscillations. For the proposed solution, we are mainly concerned with dynamic stall.
The study revealed that aeroelastic response to external damping is both complex and sensitive to airspeed. The largest variation was observed at both high external damping and airspeeds, which resulted in amplitude reductions. This result is similar to that observed in the angle of attack study, where device performance is heavily dominated by airspeeds. In achieving large amplitude vibrations (required for design), lower airspeed design was shown to tolerate higher external damping, while higher airspeed systems favored low damping. In practice, the external damping on the system is easily calibrated by varying the load resistance or air gap between the coils and magnets. Increasing the load resistance (beyond matching impedance) results in decreasing damping on the system. This variation is accompanied by lower power generation and higher transducer efficiency.
This study demonstrates that the proposed energy harvester performs best with minimal damping. As shown in the Figure 7.13, maximum acceleration was observed when the external damping matched the parasitic damping which is consistent with the generic models. Ideally, energy harvester design would encompass variable electrical loading capabilities. If such an active system could ever be implemented, it would be able to maximize power output by dynamically varying the load subject to external (airspeed) conditions. A controller would vary the load while examining the power gradient. For the proposed energy harvester, this is relatively simple provided that the coil behaves like a low-pass filter. To achieve such a goal, the power consumption of the active controller would have to be self-sufficient in order to sustain a net positive output. Scaling up might be necessary in order to overcome the power requirements of a micro-controller.

7.6 Moment of Inertia Study

The moment of inertia study was primarily carried out in order to investigate the proposed energy harvester’s response to inertial loading in a fixed-design resulting from both the airfoil and permanent magnets. Preliminary results indicate that increasing the magnetic flux for an electromagnetic energy harvester is positively unbounded. That is, the higher the magnetic flux, the higher the power density. As such, the permanent magnets were identified as crucial components in maximizing power output. Within the same magnetic grade, higher volume magnets result in higher magnetic fluxes. This section primarily examined the effects of increasing the size of a magnet, which directly affects the total moment of inertia in the system, by maximizing power output.
The moment of inertia characterizes the system’s resistance to angular rotation. Increments in the moment of inertia can result from a variety of reasons, mainly by varying the 1) density distribution of the material, 2) geometry of the component and 3) axis of rotation. This investigation dealt mainly with variations in moments of inertia arising from increasing the size of the magnets. In contrast, the airfoil design did not require careful examination provided it’s an unconstrained minimization problem. The lower the moment of inertia resulting from the airfoil the better, because then larger magnets may be used to compensate.

The minimal moment of inertia for the airfoil was achieved by machining a low density polymer by using a laser cutter as depicted in Chapter 6. This process resulted in a total moment of inertia of 2.5E-7 kg m² for the airfoil. This result was particularly high because the axis of rotation is displaced by a half-chord distance in the airfoil. In contrast, maximizing power density by using larger magnets is unconstrained. At some point, for a fixed-design, the added moment of inertia inhibits the system from rotating at the required rate, thus hampering the nonlinear aeroelastic phenomena driving the system. This study aimed at examining the dynamic effects of increasing the moment of inertia in the system resulting from permanent magnets. The result for a 10 degree system at various airspeeds is shown in Figure 7.14.
Figure 7.14: Amplitude (Degrees) Response to Moment of Inertia (kg m²) at 10 Degrees

(Group 1 demonstrates target area for permanent magnets.)

The system’s amplitude response to increasing moment of inertia is shown in Figure 7.14. The behavior of a fixed system to increasing moment of inertia was examined by starting at a base level, which corresponds to the airfoil’s moment of inertia. The figure demonstrates that the system’s response to moment of inertia is different for the various airspeeds. It also shows that the higher the airspeed, the less added moment of inertia the system can tolerate prior to experiencing a drop in amplitude. As stated previously, the proposed energy harvester required high amplitudes in order for the magnets to cut the coils at the maximum speed and flux. Thus, an operational region, as depicted by group 1 on the
Figure 7.14, may be identified for the proposed design at a 10 degree angle of attack. The objective is to increase the size of the permanent magnets without disrupting the underlying system dynamics.

The largest permanent magnet configuration consisted of two 50.8 mm x 6.35mm x 3.175 mm (2” x 1/4” x 1/8”) grade N42 blocks. The total mass of each block was 7.68 grams. In the proposed design, the magnets are positioned in such a way that the opposite poles are in contact with the membrane. This allows the magnets to stay attached at high angular accelerations while the device is operational. The second advantage of using two magnets on a torsional membrane was the ability to align the rotational axis of the magnets with the center of mass. This can be accomplished by using rectangular magnets with a height of width/2 or a width of height/2, such that the two magnets construct a square cross-section when attached. For the described magnets, the total moment of inertia was of 1.05E-7 kg m². Hence, the total moment of inertia for the proposed prototype was 3.6E-7 kg m², significantly less than the critical value of 1E-6 kg m² observed in the numerical model for airspeeds of 6 m/s at a 10 degree angle of attack.

The acceleration results for the mass moment of inertia at 10 degrees followed the same trend that was observed for previous studies, as shown in Figure 7.15. That is, increasing the moment of inertia resulted in diminishing acceleration which is directly proportional to power density. The airfoil’s moment of inertia is also shown in the figure which corresponds to the fixed or initial moment of inertia in the system. Once the airfoil design is constraint, there is limited inertia that may be added into the system by the
permanent magnets without disrupting the overall dynamics and inhibiting the required large amplitudes.

Figure 7.15: Acceleration (rad/sec\(^2\)) vs. Moment of Inertia (kg m\(^2\)) at 10 Degrees

The moment of inertia study demonstrated that the aeroelastic energy harvesters can tolerate significant increases in moment of inertia prior to interfering with the activating phenomena. It is important to emphasize that the moment of inertia is proportional to the square of the height and width of the magnets. As such, maximizing the magnetic flux density by increasing the cross sectional area of the magnets has a cost. The effect of increasing the thickness of a magnet along the axis of interest is approximately linear with respect to the surface field. That is, doubling a magnets thickness will typically result in an
increase of approximately 1.5x-2x in surface field. Conversely, increasing the magnet’s dimensions along non-critical axes (e.g. length), does no result increases in the magnets surface field along the axis of interest (e.g. thickness).

To summarize, the airfoil’s moment of inertia should be first minimized (for fixed geometry) by employing novel materials and/or material processing. The objective of this process is to later provide a larger moment of inertia allowance for the permanent magnets. Once the practical limit for the airfoil is established, the results from this study indicated that the moment of inertia should be increased, via the magnets, until the aeroelastic phenomenon is interrupted. This corresponds to the observed drop in amplitude. The moment of inertia in the permanent magnets should be increased primarily by enlarging the cross sectional areas, thus achieving higher magnetic fluxes.
Chapter 8: Design Process and Power Analysis

This chapter focuses on integrating the results and guidelines that were obtained by conducting multiple studies on the proposed aeroelastic energy harvesting solution (Chapter 7). The studies revealed that overall system performance is complex in nature and highly sensitive to design parameters. This investigation proposes both the first electromagnetic membrane-based torsional energy harvester and its complementary optimization methodology. The optimization methodology was established by considering the inverse design problem provided that there does not appear to be any other tractable approaches.

This chapter will begin by considering the independent design guidelines which have been established for optimizing the proposed solution. These guidelines serve as the starting point for optimizing aeroelastic energy harvesters and consider independent variables. Following that, the dependent design parameters will be addressed for arriving at a final solution. A final prototype is then constructed and evaluated against relevant benchmarks. An overview of this chapter is outlined in Figure 8.1

Figure 8.1: Chapter Overview
8.1 Independent Design Guidelines

While some design guidelines are dependent on other system variables, such as planform area, others are independent of system variables. It is important to cover these independent design guidelines first provided they affect the design process downstream. Energy harvesting systems must be constructed with a solid foundation obtained from the numerical analysis in order to attain good performance. A weak foundation results in the development of suboptimal solutions. The following guidelines cover all the independent design variables in the system.

1. Inductor coil density: The amount of copper in a coil volume dictates the efficiency at which energy may be converted. The maximum theoretical packing density for round wire is 90.69%. By deducting approximately 20% for insulation the maximum packing density is then reduced to 70%. Poor winding due to overlap and tension can result in further density reductions (30-50%) which can severely impair transducer efficiency. The design guideline is that the amount (mass) of copper should always be maximized irrespective of wire gauge and number of turns for a fixed volume. For coils with many turns, it is preferable to employ computer controlled winding machines.

2. Permanent magnets: The surface field and size of the magnet dictates the efficiency at which energy may be converted. Bigger magnets have longer fields which are advantageous in air-core applications. Higher surface field results in increased flux at equal distances. The design guideline is that magnetic grade should always be
maximized for fixed volume magnets. For example, an N52 grade block is always preferable over an N42 block of equal dimensions. Higher magnetic fluxes are desirable provided that same voltages may be attained while reducing the amount of copper in the coil (resistance).

3. Air gap: The distance between the permanent magnets and inductor coils dictates the efficiency at which energy may be converted. Larger distances result in magnetic flux losses for air-core systems. For a system with fixed electrical damping, the air gap may be reduced such that the same power may be converted with less copper, thereby increasing the efficiency of the system. An effective design guideline is that the distance between the inductor coil and magnets should always be minimized without resulting in mechanical interference.

4. Airfoil moment of inertia: The moment of inertia of the airfoil and magnets comprises the system’s total moment of inertia. For a total allowance, it is always desirable to distribute as much as possible to the permanent magnets. Additionally, the airfoil’s moment of inertia is surprisingly high in contrast to the magnets provided the center of mass is greatly displaced. The design guideline is that for a fixed planform airfoil design, mass should always be minimized while retaining structural integrity. This problem is commonly faced in aerospace engineering and is usually solved by employing novel materials and/or manufacturing processes.

5. Torsional spring: The magnetic flux study demonstrated that high amplitude vibrations are required in order to maximize the rate of change in magnetic flux. As such, the target (90 degree) amplitudes may be achieved by simply increasing the
airspeed until required. Provided that the amplitude is fixed, maximizing the frequency results in higher power output. The tradeoff is that larger spring stiffness results in both higher frequencies and higher electromagnetic/aeroelastic damping. Because the device generates more power, higher airs speeds are required. The proposed design guideline is that spring stiffness is arbitrary provided that it affects the operational airspeed of the energy harvester. As such, torsional spring stiffness is application dependent. The stiffness should always be high enough to prevent the spring from deflecting in static equilibrium due to the airfoil and magnets.

### 8.2 Dependent Design Guideline

The dependent guidelines for optimizing the proposed energy harvester are covered in this section. One of the major contributions of this investigation was determining that there’s a tractable approach for optimizing aeroelastic energy harvesting solutions. Thus far, there are no other alternative for achieving this goal. This is achieved by solving the inverse design problem. That is, the design process begins by maximizing the amount of copper in the inductor coil and ends with determining the operational airspeed of the device. In contrast, designing a device for a particular airspeed is a very challenging because it requires accurate modeling of turbulent flow. The proposed methodology for determining the dependent design parameters consists of an iterative procedure that was established by use of the results obtained from the planform area study (Chapter 7).

The proposed methodology aims at determining the optimal coupling between the aeroelastic and electromagnetic domains in the system. This achieved by matching the
aeroelastic and electromagnetic damping in the dynamic system by modifying airfoil geometry subject to constant planform areas. This approach has the best tractability provided that the oscillator’s frequency is inversely related to the airfoils chord length subject to constant planform areas, as demonstrated in Chapter 7. Iteratively modifying both the chord and span lengths may also be used to arrive at a solution. This approach is difficult to implement provided that the sensitivity is highly nonlinear. In contrast, the proposed approach provides a practical approach for achieving optimized solutions.

The methodology consists of starting with the target planform area (desired airfoil geometry). From this starting point, both the aeroelastic and electromagnetic damping may be determined by use of the logarithmic decrement method. This is accomplished by performing the analysis on both the unloaded and loaded system. For the loaded system, the electrical load is set so that it matches the internal impedance of the inductor coil. This setup results in maximum power transfer. Once the damping ratios are calculated for both conditions, a decision is made whether to 1) increase chord/decrease span or to 1) decrease chord/increase span. If the electromagnetic damping is higher than the aeroelastic, 1) is the correct direction of travel. Likewise, if the aeroelastic damping inhibits the system, 2) is correct action.

The coil position study also revealed that is imperative for the system to achieve high amplitude oscillations for generating the maximum rate of change in magnetic flux. Additionally, the airspeed study revealed that the amplitude of vibrations is directly proportional to airspeed. As such, it’s always possible to increase airspeed to attain the desired amplitude. Furthermore, the study also revealed that for higher angles of attack (45
degrees), it is possible to increase amplitude while controlling frequency. This forms the basis for establishing a simple procedure for determining the operational airspeed of the device.

8.3 Design Process and Power Results

The completed design process is outlined in Figure 8.2. As stated previously, this investigation determined that the only approach for optimizing the proposed energy harvester consisted on evaluating the inverse design process. This is because the aeroelastic response of the system to design parameters lacks a closed-form expression. As such, the proposed design methodology is iterative and begins by considering certain independent design guidelines. The design process then ends by establishing the operational speed of the device.
The power density and efficiency results for three different designs iterations are shown in Figure 8.3 and 8.4, respectively. The average power output was calculated by measuring the quadratic mean voltage across each resistance point. As such, the power output results presented in this section corresponds to the real power delivered to the electrical load. The computed power output was then normalized by the sweep area of each
airfoil to determine the power density. For a 0 degree angle of attack, the sweep area corresponded to double the planform area. This is because the airfoil oscillates 90 degrees up and down sweeping twice the area of the airfoil. For 45 degree angles of attack, the sweep area is $8^{1/2}$ times the airfoil planform area. This is because the airfoil rotates back more than 90 degree relative to the incoming air. The power density results were then further normalized by the energy input to arrive at the efficiency results shown in Figure 8.4.

The three systems were subject to varying electrical loads for maximizing power output. The lower efficiency system corresponds to a previous design iteration which
employed low-grade magnets and an arbitrary planform area. The second highest efficiency system corresponds to a model that employed higher-grade magnets and an arbitrary airfoil design. The final iteration (best efficiency) was achieved by following the design methodology derived (Figure 8.2) in this investigation. The model achieved the desired 90 degree amplitude at a 5 m/s airspeed. The maximum power output on the load (6-8 kΩ) was 1.46 mW and was harvested over an area of 0.0032 m² (5 in²).

Figure 8.4: Efficiencies for Three Design Iterations
The optimal design, shown in Figure 8.4, consisted of a chord length of 0.0304 m (1.2 in) and a span of 0.0635 m (2.5 in). This design was determined by matching the electromagnetic and aeroelastic damping in the system subject to the highest density inductor coil and highest-grade permanent magnets. The operation airspeed was then determined to be 5 m/s. The efficiency results (Figure 8.4) demonstrate that for a high density inductor coil, magnetic flux and airfoil optimization results in significant performance improvements. Although the 0.6% efficiency is very low, this is a significant result for harvesting kinetic energy from atmospheric air at 5 m/s provided that small-scale conventional systems have problems operating in these wind speeds.

In comparing the efficiency results to additional small-scale systems, two small-scale rotary devices, shown in Figure 8.6, were examined at 5 m/s. The first device consisted of a
sweep area of 0.0045 m² (3 in. diameter rotor) while the second was 0.0026 m² (2 in. diameter rotor). The finalized aeroelastic energy harvester consisted of a total sweep area of 0.0038 m² (6 in²). As such, the sweep area of the proposed energy harvester was between both rotary devices. In comparing devices, it is important to take into consideration scale and equal energy input. The smaller fan was analyzed first. Surprisingly, the system did not spin while subject to 5 m/s airspeed. The torque generated by the blades was not sufficient to overcome the mechanical friction in the shaft. Even after a manual start-up and while driving no electrical load, the system failed to remaining spinning and locked-in.

Figure 8.6: Rotary Devices with Similar Sweep Areas

The second (larger) device also failed to start when subject to a 5 m/s airspeed. In contrast to the smaller system, this device did spin after a manual start-up. The efficiency results for this device are shown in Figure 8.7. A maximum efficiency of 0.36% was observed at 300 Ω, which corresponds to the internal impedance of the brushless DC motor.
If is important to note that the observed efficiency was very low for the rotary system at the examined speed. This is because rotary devices experience sharp declines in efficiency at low speeds. At higher speeds, the efficiency in rotary systems increases drastically and outperforms any other existing techniques.

![Figure 8.7: Rotary Device with Similar Sweep Area at 5 m/s](image)

### 8.4 Cost of Power

This section aims at examining the practicality and applicability of the proposed energy harvesting solution. As depicted in the previous sections, the proposed solution demonstrated potential applicability in harvesting low speed airflow. This is mainly because the device is frictionless and does not have a succinct airspeed threshold. Additionally,
electromagnetic damping is usually negligible when the device is positioned at its resting state due to the arrangement of the magnets and coil. This allows the system to startup from zero velocity in practically “unloaded” state.

The maximum power and power density for the propose energy harvesting solution were 1.46 mW and 377 mW/m², respectively, for a 5 m/s airspeed. The average cost of the device was then approximated by summing its individual components, 1) 2 N42 magnets at $2.30 each, 2) PVDF membrane at $0.03, 3) acrylic airfoil for $0.05 and 4) induction coil for $6.99. This list did not include the cost for the structural frame provided its design and materials were chosen arbitrarily. As such, a minimal value for the device would be in the order of $11.50. From this result, a lower bound on the cost of power may be then approximated to 7.85 $/mW.

In contrast, mass production of the standard personal computer 76.2 mm (3”) DC brushless fans has decreased their respective market value to approximately $5-6. As such, the cost of power for that particular device may be approximated to 4 $/mW, which is significantly less than the proposed system. The main advantage of rotary devices is that they do not require expensive rare-earth permanent magnets. This is because the devices employ iron-cores in focusing the magnetic flux. It is important to re-emphasize that cost of power values were calculated at 5 m/s airspeed and decreasing or increasing the operational speed of the device would vary these results. For rotary devices, the cost of power decreases with airspeed until a minimum is reach. In contrast, the cost of power increases asymptotically near below the cut-off airspeeds.
8.5 Scaling Effect

The last section of this chapter describes some of the results obtained by investigating the scaling effect on the proposed energy harvesting solution. This was accomplished by employing an inductor coil with half the copper volume as the finalized iteration. The result of doing so was that the system’s electromagnetic damping was reduced. As such, it was then possible to match the damping by use of a smaller planform area airfoil, as illustrated in Figure 8.8. The employed airfoil consisted of a planform area of 0.0013 m² (2 in.²). The thin airfoil was fabricated by using the same laser cutting technique as previous iterations and resulted in a 762 µm (0.03 in) thickness. The required high amplitudes oscillations were observed at a slightly higher airspeed of 5.5 m/s.

Figure 8.8: Scaled Down System with Planform Area of 0.0013 m² (2 in.²)
The maximum power output for the device was 1.27 mW at a 5.5 m/s airspeed. Although the average output power for this device is lower than what was observed in the previous iterations, it was collected over a smaller area. As such, it is necessary to normalize the result by the sweep area of the device. For a total sweep area of 0.0021 m$^2$ (1.6 in.$^2$), this result corresponded to a power density of 604 mW/ m$^2$. This time, the power density is higher than the previous iterations. This result, like the average output power, may also be misleading since it does not take into account the energy input into the system. As such, the power density must be normalized by the energy input into the system to arrive at an efficiency result. By doing so, we arrive at a total efficiency of 0.58% for this design iteration.

This result demonstrates that no efficiency improvements were observed when scaling the system (planform area) by a factor of 0.75. This result is in line with what has been observed in commercial wind turbines since their inception. It is important to note that although the efficiency is independent of scale, the cost of power is not. The cost of power for all wind generators varies approximately linearly with scale. That is, doubling the critical dimensions in the system will result in twice its capital cost. In contrast, the power output varies proportionally to the square of the critical dimension, provided that is dominated by the sweep area. Hence, doubling the critical dimensions results in 4 times the power output. For that reason, commercial wind turbines are built as large as possible subject to practical limitations.
Chapter 9: Conclusions

This investigation presents a novel membrane-based torsional aeroelastic energy harvesting solution for harvesting low-speed airflow. The approach consists of extracting kinetic energy from atmospheric air by use of nonlinear aeroelastic phenomena and converting it into the electrical domain by employing electromagnetic transduction. The work was motivated by the desire to improve efficiency in harvesting kinetic energy from low-speed airflow. To overcome standing limitations in existing approaches, this investigation set forward to examine a novel vibrations-based torsional solution. The proposed solution was modeled, optimized and prototypes tested. This chapter will address the major accomplishments for this investigation and its main contributions. Potential future work will also be discussed.

9.1 Summary

This investigation consisted of modeling, optimizing, fabricating and evaluating a novel energy harvesting solution. The proposed design achieved self-sustained aeroelastic oscillations by use thin torsional membranes and light rigid airfoils. The system was designed such that high-amplitude vibrations were achieved with respected to low-speed airflow. The vibrations were then harvested by use of low-damping electromagnetic transducers. The main challenge associated with optimizing the proposed solution involved the modeling of aeroelastic phenomena, which is driven by nonlinear aerodynamics. To overcome this challenge, this investigation employed a combination of both computational and
experimental results for developing a set of guidelines which were then used to fabricate and test energy harvesting systems.

A full 3-dimensional computer model was constructed in ANSYS to verify and examine the underlying phenomena driving the system. The model was constructed to scale and encompassed all components present in the physical prototype. This included the nonlinear torsional spring, external electromagnetic damping and moment of inertia. Some of the challenges associated with modeling the complex aeroelastic system consisted of 1) coupling fluid and solid domains, 2) maintaining mesh integrity during large deformations and 3) non-convergence resulting from poor time-step resolution and mesh size. In order to successfully run the model, both the time-step resolution and mesh size were decreased iteratively until the model converged. The overall result of attaining a full working model was that the computational requirements increased drastically.

The computational model was then used to investigate the aeroelastic response of a fixed-design system subject to variations in design parameters. Overall, the computational model was sufficient in developing guidelines for optimizing the proposed energy harvesting solution. The major achievements of the model were determining that 1) frequency is inversely related to chord length at constant planform areas, 2) the system can tolerate significant increases in moment of inertia and that 3) electromagnetic damping is best minimized and matched to aeroelastic damping. This information was then useful in developing a design methodology for designing the airfoil. The established design methodology consisted of varying the chord length in the airfoil subject to constant planform while examining both aeroelastic and electromagnetic damping. The electromagnetic
damping was set by imposing a resistive load on the transducer at equal impedance, a condition which results in maximum power transfer.

The established design methodology begins by defining independent components and ended by determining the operational airspeed of the device. The results demonstrated that, for three design iterations, implementing the design methodology resulted in improved results. The efficiency of the proposed system (0.6%) marginally outperformed that of the benchmarked rotary devices (0.36%) at low speeds. Although the efficiency of the proposed system is approximately 50% higher, the overall efficiency is still very low. The challenge still remains to develop small-scale system which can achieve cost-effective efficiencies. It is important to note that efficiency of rotary device is highly dependent on airspeed and the evaluated airspeed (5 m/s) is near the cut-off levels for that device. For higher airspeeds, rotary systems remain the best approach for extracting kinetic energy from moving fluids. The main advantage of rotary systems is that they employ iron-cores in maximizing magnetic flux. In contrast, aeroelastic energy harvesting systems require low damping arrangements (air-cores) to achieved self-sustained oscillations.

**9.2 Research Contributions**

This section aims to outline the major research contributions delivered by this investigation. The major contributions are mainly within the areas of energy harvesting, aeroelastic modeling and optimization, membrane-based torsional springs and harvesting low speed airflow.
9.2.1 Torsional Electromagnetic Aeroelastic Energy Harvesting

The primary contribution of this research was the development of the first torsional electromagnetic aeroelastic energy harvester. To achieve this goal, this investigation employed membrane-based torsional springs for 1) minimizing aeroelastic damping and 2) supporting large permanent magnets. The transducer was designed such that minimal damping occurred at the equilibrium point. These design considerations allowed the system to “start-up” at lower airspeeds and resonate towards target amplitudes. This novel system was developed for harvesting kinetic energy from low speed airflow.

9.2.2 Aeroelastic Modeling

This investigation successfully employed CFX-ANSYS in constructing a fluid-structure-interaction (FSI) model for examining the aeroelastic phenomena. The scale model was 3-dimensional and encompassed all of the components found in the real prototype. A major contribution of the model was confirming that the observed self-sustained aeroelastic oscillations result from periodic vortex shedding. To achieve this outcome, specialized meshing techniques were developed to support the large mesh deformations. An overall contribution of the model is that it may be employed in studying additional aeroelastic problems, within or outside the field of energy harvesting.

9.2.3 Design Methodology

In delivering a novel solution, this investigation outlined an original design methodology which may be employed for maximizing power output in aeroelastic energy
harvesters. The major contribution within the design methodology was the discovery that both electromagnetic and aeroelastic damping may be systematically matched by varying the aspect ratio of the airfoil while holding the planform area constant. As such, for structurally-fixed systems, total damping is dominated by the airfoil’s aspect ratio and area. Additionally, several critical guidelines were identified for designing the permanent magnets, coils and airfoil shape.

9.2.4 Component Extensions

The proposed energy harvesting solution encompassed several components. Additional contributions may be identified within the specific components. In this regards, the major contribution was the realization that large permanent magnets may be balanced and suspended in thin torsional membranes for achieving low damping torsional vibrations. This concept may easily be extended in the development of angular inertial generators. Another important contribution was the realization that variable electromagnetic damping may achieved by strategically positioning the permanent magnets with respect to the inductor coil.

9.2.5 Broader Contributions

The broader contributions of this investigation regard the development of new class of energy harvesting solutions. This corresponds to the use of torsional membranes and electromagnetic transducers for harvesting kinetic energy from moving fluids. The objective is to improve efficiency such that energy may be economically harvested from low speed airflow. Such systems would have significant impact in the areas of energy and
communications provided that the power source is readily available. This investigation contributed to this overall goal by providing novel ideas and optimization approaches which may be incorporated with additional methods to arrive at an economically-feasible solution.

9.3 Future Work

Future work from this investigation could focus on a variety of different areas. That is, the results from this investigation may be extended within aeroelastic energy harvesting and beyond. Within aeroelastic energy harvesting, future work may focus on employing advanced materials for constructing thin rigid airfoils. As demonstrated in this investigation, decreasing the mass moment of inertia provides significant opportunity for increasing the magnets sizes and reducing transduction losses. Additionally, further improvements may also be observed by employing specialized high-grade magnets, such as N50 or N52, for vibration-based energy harvesting applications.

Future work may also focus on making additional modifications to the arrangement between the permanent magnets and coils. Within this investigation, the employed strategy successfully minimized initial damping at the expense of delivering uniform damping throughout the oscillation cycles. As such, further research is required to determine whether evenly distributing the rate of change in magnetic flux is beneficial. Additionally, future work may focus of active damping strategies, such as employing solid state relays for systematically loading/unloading the system with optimal timing. For example, the system could be fully loaded only during the down stroke. As such, the airfoil would rotate quickly
during the upstroke to high torsional displacements and then be loaded as it retreats to
equilibrium.

Apart from aeroelasticity, results from some of the components may be employed in
future investigations. For example, the torsional membrane configuration may also be
employed for inertial generators. This configuration has the advantage that large permanent
magnets may be fixed and balanced in thin torsional membranes for achieving low-damping
torsional vibrations. This is significantly different from linear generators where the mass is
supported by the spring. In contrast, large mass moments of inertia may be supported without
deflecting the torsional spring. Furthermore, if the center of mass is not balanced the
generator may be employed in converting linear motion to torsional vibrations.
References


