ABSTRACT

KONDYAN, SERGEY SERGO. Evaluating the Effects of FDI on Growth in the Context of One and Two-Sector Endogenous Growth Models. (Under the direction of John J. Seater.)

In my dissertation research I develop a theoretical framework that focuses on the pure effects of FDI on growth in the absence of technology transfers or other channels through which the spillover effects of FDI can operate. By isolating the effects of FDI on growth from the presence of the alternative channels of transmission I address the issue of what FDI by itself does for growth through its primary function of capital accumulation. I study growth effects of FDI in the context of an endogenous growth model with one and two sectors of production where growth is generated by perpetual accumulation of capital. As the results suggest, the growth effects of FDI depend on technological differences across countries. If countries are technologically the same then FDI does not generate any growth effects for either home or host country. If, however, countries are technologically different with the home country being technologically advanced in the FDI conducting sector then the home country will not experience any growth effects, whereas the host country will face negative growth effects due to reduction of the marginal product of capital resulting from the process of capital accumulation through FDI. The transitional behavior of both countries in the presence of FDI is different from the dynamics of closed economy, one and two-sector models with endogenous growth. The difference comes from the fact that the behavior of the variables in the host country depends on the behavior of the variables in the home country through the FDI channel which suggests that the transitional behavior of host and home countries should be analyzed together, through the multidimensional system of differential equations describing the transitional path of both countries. The
dimensionality of this dynamic system does not lead to analytical solutions, and I use model calibration techniques to study the transitional behavior of both countries.
Evaluating the Effects of FDI on Growth in the Context of One and Two-Sector Endogenous Growth Models

by
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DEDICATION

To my wife Karine

and

my daughters Anahit and Christine
BIOGRAPHY

Sergey Kondyan was born in Ninotsminda, Georgia. He graduated from Yerevan Polytechnic Institute in 1991 with Electric Engineer Diploma. He continued his education in the State Engineering University of Armenia in the area of Control System Engineering and received his Master of Engineering diploma in 1994. Continuing his education in the area of Control System Engineering in the same university in 1996 he completed the program and received Engineer-Researcher diploma.

While he was in the Graduate School Sergey Kondyan combined his education with part time administrative job at Yerevan University of Management and Information Technology (YUMIT), and starting from September 1996 he moved to the full time appointment in the position of the Deputy Dean of the Preliminary Department.

Sergey Kondyan started his teaching career at YUMIT in 1996, where he taught Principles of Economics course and supervised several diploma works, first in the Department of Economics and Finance, and later in the Department of Applied Economics and System Analysis at YUMIT.

In 2006 Sergey Kondyan received full Graduate Teaching Assistantship to pursue master studies in economics and continued his education in the Department of Economics at East Carolina University. He completed the program within one year and received his Master of Science in Economics degree in December 2006.

In 2007 he received full graduate teaching assistantship to pursue PhD degree in Economics at NCSU. During the years of education at NCSU Sergey Kondyan developed fields in Economic Growth and Industrial Organization. At NCSU he also worked as an instructor for Principles of Macroeconomics, and Introduction to Agricultural and Resource Economics courses.
In the fall 2011 Sergey Kondyan worked as an Adjunct Assistant Professor, in the Department of Economics at Elon University.

Sergey Kondyan is married and has two beautiful daughters.
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Chapter 1

A One-Sector Model with Foreign Direct Investment

1.1 Introduction

There has been a substantial increase in the flow of foreign direct investment (FDI) in recent years. For the United States alone direct investment abroad increased from $209,392 million in 1999 to $328,905 million in 2010, which translates to a 57% increase in US capital outflows over the indicated period.\(^1\)

Despite quite significant increases in FDI flows, there is little research on the different aspects of FDI, in particular the effects of FDI on growth. The objective of the current work is to study growth effects of FDI through the process of capital accumulation, which is a primary benefit that FDI brings to a host country. To the best of my knowledge this work is the first study of the growth effects of FDI through its primary function of capital accumulation.

\(^1\)Source: www.bea.gov.
In the existing literature the theoretical and policy arguments on the positive effects of FDI on growth are considered mainly through the positive spillover mechanism of FDI in the form of technological transfers, training of the labor force, and networks etc. On empirical grounds, however, there is no strong evidence that FDI promotes growth through spillover mechanisms.

Lipsey (2002) addresses different aspects of the potential effects of FDI in his review of the empirical literature on the subject. Among other things, he considers the empirical evidence on the productivity spillovers from FDI and argues that the evidence of productivity spillovers is mixed.

Micro empirical evidence based on the studies using firm-level data for different countries supports Lipsey’s claim. Blomstrom and Wolff (1994), for example, found that increased foreign presence contributed to an increase in productivity of Mexican firms relative to US productivity levels between 1965 and 1982. On the contrary, Aitken and Harrison (1999), using plant level data on Venezuela, argue that FDI has a negative effect on productivity of domestically-owned plants, even though it may increase productivity of the plants receiving FDI. Harrison and McMillan (2003), using firm data on the Ivory Coast, provide evidence that not only are domestic firms more credit constrained compared to foreign firms but also the excessive borrowing by foreign firms in the domestic market “exacerbates domestic firm credit constraints”, making the argument on positive productivity spillovers of FDI quite weak.

In terms of the macroeconomic literature, the studies do not find systematic evidence that FDI in itself raises growth. Given the relevance of the current work to the macroeconomic literature I would like to discuss macroeconomic results on the effects of FDI in more details.

Borensztein, De Gregorio and Lee (1998) argue that FDI can become an important
channel of technology transmission and therefore contribute to faster growth only in the host countries that have some minimum threshold stock of human capital. Their empirical approach is built on the theoretical framework of endogenous growth models with variety expansion, discussed among others by Barro-and Sala-i-Martin (2004). The stock of physical capital in the economy is an aggregate of the varieties of the capital goods. The endogenous growth in the model is therefore generated by the accumulation of the aggregate capital stock through the process of expansion of the varieties of capital goods. The varieties can be expanded by both domestic and foreign firms. New varieties require adaptation of technology and thus incur fixed setup costs, which negatively depend on the ratio of foreign firms to the total number of firms in the economy. The latter assumption captures the notion that foreign firms bring advanced technology to the host country. Borensztein, De Gregorio and Lee show that under their model specification FDI increases growth through reducing the costs of introducing new varieties and increasing the rate of introduction of new varieties. What is important for their empirical analysis is the positive association between the growth rate of the economy and the level of human capital that the economy is endowed with. The authors argue that the positive relationship between human capital and growth suggests that the effect of FDI on growth can be stronger in countries with higher endowments of human capital; the argument they tested empirically. Even though this conclusion directly follows from the specification of the production technology, nevertheless empirical analysis using panel data on FDI flows from industrialized to 69 developing countries, covering the periods from 1970 to 1979 and 1980 to 1989, provides robust evidence that the effect of FDI on growth depends on the level of human capital in the host country. This evidence combined with the other results of empirical analysis leads to a more general conclusion that the positive growth effects of FDI are only possible in the presence of “sufficient absorptive capability of the
advanced technologies” in the host countries.

Alfaro, Chanda, Kalemi-Ozcan and Sayek (2004, 2009) provide more evidence on the “absorptive capability” argument by exploring the role financial markets play in fostering the effects of FDI on growth. Their empirical study (2004) suggests that the growth effects of FDI are possible only in the presence of well-developed financial markets that lead to easier adaptation of advanced technologies brought in by FDI.

In their later study (2009) Alfaro, Chanda, Kalemi-Ozcan and Sayek develop a theoretical formalization of their empirical framework on the role of financial markets in promoting the positive effects of FDI on growth through backward linkages that generate positive spillovers for the whole economy.\(^2\) As they argue, the failure of researchers to find empirical evidence of positive technological spillovers of FDI can be linked to the idea that foreign firms want to use the ownership advantage that they have to “outperform local firms”. Their approach, therefore, is to model benefits from FDI as “occurring via linkages and not through technology spillovers”.

Similar to Borensztein, De Gregorio and Lee (1998), the theoretical structure developed by Alfaro, Chanda, Kalemi-Ozcan and Sayek is essentially an open-economy version of variety expansion model with endogenous growth.\(^3\) They use a small open economy setup characterized by two industry streams. “The downstream industry” produces the final consumption good by using two intermediate inputs that differ from each other based on their country of ownership. The production of these intermediate inputs in its turn constitutes ”the upstream industry” and requires a combination of skilled and unskilled labor and an array of differentiated inputs. The evolution of the varieties of those intermediate inputs generates the mechanism of endogenous growth in the model. The

\(^2\)Backward linkages are defined as: “contacts between domestic suppliers of intermediate inputs and their multinational clients in downstream sectors”.

\(^3\)see Barro and Sala-i-Martin, 2004.
link between foreign firms and the level of development of financial markets in the host country arises from the need of foreign firms to finance their upfront investment into developing the new varieties of the capital inputs. The costs of borrowing resources from the domestic financial markets reflect the degree of the development of financial markets in the host country. The results of the calibration exercise show that an increase in the share of FDI contributes more to growth in financially developed countries, supporting the claims that there is no evidence on the exogenous effects of FDI on growth and that the ability of FDI to affect growth depends on the “absorptive capability” of the host country, which in this particular study is captured by the level of financial development of the host country.

Earlier work by Carkovic and Levine (2002) also supports the conclusion that there is no robust evidence on the positive effects of FDI on growth. As they argue, “there is not reliable cross-country empirical evidence supporting the claim that FDI per se accelerates growth”.

Even though on empirical grounds the evidence for the exogenous effect of FDI on growth is weak if not absent at all, the theoretical framework explaining this phenomenon and addressing the issue of what FDI itself does to growth seems to be missing. My dissertation work fills in this gap in the literature by developing a theoretical framework focusing on the pure effects of FDI on growth through its primary function of capital accumulation. Researchers in fact do emphasize the importance of FDI as a source of capital accumulation but in the meantime they acknowledge the lack of a literature focusing primarily on the capital accumulation role of FDI.

In the abovementioned survey of the literature on FDI, Lipsey (2002) argues that in addressing the issues related to efficiency of foreign operations in the host country and the aspects related to the improvements in aggregate efficiency due to foreign operations,
researchers use measures of efficiency ranging from “value added per unit of labor input, the simplest, to value added per unit of labor and capital input and value of output per unit of labor, capital and intermediate product input”. As he continues “most authors seem to prefer the efficiency measures including capital input. The result is to ignore any host country benefit from the accumulation of physical capital, or from any advance in technology that consists of the adoption of more capital intensive methods of production or larger scale production”.

In the aforementioned work by Borensztein, De Gregorio and Lee (1998), the authors do test the effect of FDI on growth through the process of augmenting capital accumulation in the host country. In particular, they consider the effect of FDI on total investment, arguing that if the coefficient on FDI equals 1 then it suggests that FDI does not increase total investment in a host country, while a coefficient more than one implies that FDI actually leads to a “crowding-in” effect. As their results suggest, the data show that FDI stimulates domestic investment and that there is a complementarity between foreign and domestic investment, however this result is not robust across different model specifications used by the authors.

The lack of work focusing on the effects of FDI on growth through its primary function of capital accumulation can be explained, at least to some extent, by the lack of evidence on the growth effects of capital accumulation. Jones (1995) argues that there is no empirical evidence on the positive long-run growth effects of investment. Easterly and Levine (2001) in their review of the empirical literature on growth conclude that there is no evidence that factor accumulation contributes to faster growth of output per worker.

In their recent study, Bond, Leblebicioglu and Schiantarelli (2010) reassess the empirical relationship between investment in physical capital and long-run growth. In their study of 75 countries for the period of 1960 to 2000 they conclude that there is in fact
evidence on the positive relationship between investment, measured as a share of GDP and long-run growth. Their methodology is based on controlling for country specific fixed effects and focusing on time-varying control variables. The advantage of this methodology is that it controls for country-specific factors such as political and institutional infrastructure that could potentially influence the relationship between investment and growth. As they conclude, “our results suggest that investment is at least an informative proxy, and are consistent with investment being an important channel through which deeper causes influence growth outcomes. Growth theories that predict a positive relationship between investment and long-run growth rates should not be dismissed as being grossly inconsistent with the empirical evidence”.

Lack of a literature on FDI, its increasing role, and disagreements between theoretical and empirical literature serve as important motivational factors for current work. In this work I develop a theoretical framework that focuses on the pure effects of FDI on growth in the absence of technology transfers or any other channel through which spillover effects can operate. By isolating the effects of FDI on growth from the presence of alternative channels of transmission I address the issue of what FDI by itself does for growth through its primary function of capital accumulation.

My theoretical framework is based on one and two sector endogenous growth model structures discussed in Barro and Sala-i-Martin (2004). I have developed an open-economy extension of both one and two sector endogenous growth models to study the growth effects of FDI. The choice of model structures is motivated by the following characteristics. Both one and two sector endogenous growth models have tractable structures. Endogenous growth is generated by perpetual accumulation of capital, which is exactly the structure that I need to study the growth effects of FDI from capital accumulation. There are two types of capital, usually assumed to be physical and human capital. In the
one sector model both types of capital are assumed to be perfect substitutes produced within the single sector of an economy. This simplified structure serves as a benchmark for study of FDI in the context of endogenous growth with accumulation of capital, which is discussed in the first chapter of my dissertation.

In the second chapter I utilize the simplified two sector structure of the Uzawa-Lucas model (Uzawa (1965), Lucas (1988)). The model separates the production of both types of capital into two different sectors of the economy. This structure allows one to distinguish between domestic and foreign investment in physical capital and domestic production of human capital. As emphasized in the literature, sectoral decomposition of the production process is an important aspect in the study of the growth effects of FDI. In particular, Lejour and Rojas-Romagosa (2006) argue that most policy prescriptions aimed to increase flows of FDI are sector specific, for example the Services Directive by the European Commission, and the GATS proposals aimed to reduce trade barriers in services. Alfaro (2003) shows that the growth effects of FDI differ substantially across different sectors of the economy, being negative in the primary sector, positive in the manufacturing sector and ambiguous in the services sector.

In the third chapter I extend my analysis to incorporate FDI into the more general structure of a two sector model. The richer framework of this model allows me to introduce FDI not only in physical capital, but also in the other type of capital, which can be interpreted as human capital or any other type of capital that augments labor but is not embodied in labor.4

Another important characteristic of this class of endogenous growth models is that the closed economy versions of one and two-sector endogenous growth models generate very interesting patterns of transitional dynamics. Adopting the structure of these models

4For the detailed discussion see Yenokyan, Seater and Arabshahi (2011).
allows one to focus not only on the long-run but also on the transitional effects of FDI on growth.

Across all three models I assume that the production process is described by a perfectly competitive structure to avoid the complications arising from potential strategic games played by countries in an attempt to manipulate rates of return on capital. There is also no borrowing or lending via credit markets either within or across countries.

The results of the analysis suggest that technological differences across countries play an important role in determining the growth effects of FDI from capital accumulation. If countries have the same level of technology then there are no long-run growth effects of FDI. This result is consistent with the aforementioned empirical evidence on the absence of exogenous effects of FDI on growth.

If, however, countries are technologically different then the host country will experience negative growth effects from FDI.

The transitional behavior of both countries in the presence of FDI is different from the dynamics of the closed economy one or two sector models with endogenous growth. The difference comes from the fact that the behavior of the variables in the host country depends on the behavior of the variables in the home country through the FDI channel. This suggests that the transitional behavior of host and home countries should be analyzed together, through the multidimensional system of differential equations describing the transitional path of both countries. Even though the dimensionality of the dynamic system does not lead to an analytical solution, I use model calibration techniques to characterize behavior of the variables along the transitional path.
1.2 The Setup of the Model

Before I introduce FDI into the framework of a one sector endogenous growth model I will briefly summarize the setup of the model under autarky, discussed in Barro and Sala-i-Martin (2004, chapter 5).

1.2.1 The Model Under Autarky

In autarky the total output produced in the economy ($Y$) is used for consumption and domestic investment in two types of capital, K and H. Under the assumption of a Cobb-Douglas production process the resource constraint for an economy is written as:

$$Y = AK^\alpha H^{1-\alpha} = C + I_K + I_H$$  \hspace{1cm} (1.1)

where $I_K$ and $I_H$ are gross investments in the K and H type of capital respectively.

Assuming that both types of capital depreciate at the same rate $\delta$, gross investment is:

$$I_K = \dot{K} + \delta K$$  \hspace{1cm} (1.2)

and

$$I_H = \dot{H} + \delta H$$  \hspace{1cm} (1.3)

Household preferences are described by the following CRRA utility function:

$$U = \int_0^\infty U(C_t)e^{-\rho t}dt$$  \hspace{1cm} (1.4)

where

$$U(C_t) = \frac{C_t^{1-\theta} - 1}{1-\theta}$$
The optimization problem faced by a country can be written as the maximization of (1.4) subject to the resource constraint (1.1) and accumulation conditions (1.2) and (1.3), which leads to the following present value Hamiltonian.

\[ V = \frac{C_t^{1-\theta} - 1}{1 - \theta} e^{-\rho t} + \mu_1(I_K - \delta K) + \mu_2(I_H - \delta H) + \lambda(\alpha K^\alpha H^{1-\alpha} - C - I_K - I_H) \]

The growth rate of consumption is derived from the necessary conditions of this optimization problem:

\[ \dot{C} = \frac{1}{\theta} \left( A\alpha \left( \frac{K}{H} \right)^{\alpha-1} - \delta - \rho \right) \]

The no arbitrage condition requires equalization of the marginal products of K and H capital, which leads to the solution of K to H ratio as follows:

\[ A\alpha \left( \frac{K}{H} \right)^{\alpha-1} = A(1 - \alpha) \left( \frac{K}{H} \right)^\alpha \]

\[ \frac{K}{H} = \frac{\alpha}{1 - \alpha} \quad (1.5) \]

Constancy of \( \frac{K}{H} \) implies equalization of the growth rates of K and H. It is also shown that growth rates of C and Y are constant and equal to the growth rates of K and H. Substituting the expression for \( \frac{K}{H} \) into the solution for the growth rate of consumption will result in the solution for the common growth rate of this autarkic economy given by:

\[ \gamma_{Au} = \frac{1}{\theta} \left( A\alpha^\alpha (1 - \alpha)^{1-\alpha} - \delta - \rho \right) \quad (1.6) \]

The important argument in the discussion of the transitional dynamics of a one-sector endogenous growth model is the assumption of irreversible investment, which mathematically is written as: \( I_K \geq 0 \) and \( I_H \geq 0 \). Irreversibility of investment implies that the
country cannot dissinvest in either type of capital. Therefore, each time the country deviates from the value of the K to H ratio given in (1.5) one of the constraints of irreversible investment becomes binding and country sets that investment equal to 0.

The inequality constraints create the main foundation in the study of transitional dynamics of the autarkic version of the one sector model discussed above. In particular, as it follows from the assumption of irreversible investment, only one type of capital will be accumulated along the transition reducing the expression in (1.1) to either

\[ Y = A K^\alpha H^{1-\alpha} = C + I_K \]

or

\[ Y = A K^\alpha H^{1-\alpha} = C + I_H \]

depending on the deviation of the ratio of K to H capital from its steady state value.\(^5\)

### 1.2.2 The Open-Economy Setup of the Model

In this section I incorporate FDI into a macro framework using the structure of the one sector endogenous growth model with two types of capital discussed above. Here I assume that the two types of capital are physical and human capital and that the country conducts foreign investment only in physical capital.

To introduce FDI I need to distinguish between capital ownership and the capital used in the production process.

Note that in the production process a country can only use the capital located within its borders. Thus I can distinguish between total K type capital ownership and K capital

\(^5\)For the detailed discussion of the transitional dynamics of a closed economy, one-sector endogenous growth model see Barro and Sala-i-Martin (2004), chapter 5.
used in the production process as follows:

\[
\hat{K}_{it} = \bar{K}_{it} - \tilde{K}_{it}, \hat{K}_{jt} = \bar{K}_{jt} + \tilde{K}_{it}.
\] (1.7)

where \(\hat{K}_{it}\) denotes K capital produced in home country, \(\bar{K}_{it}\) denotes the total K capital ownership for country i, and \(\tilde{K}_{it}\) is capital created as an outcome of FDI. Similarly, \(\hat{K}_{jt}\) is the total K type capital used in the production in country j, and \(\bar{K}_{jt}\) is the total K type capital ownership in country j. Here I explicitly assume that index i denotes the home country that conducts FDI and index j denotes the host country - recipient of FDI.

The intuition behind the above expressions is as follows: if a country chooses to invest abroad, then its total capital ownership will consist of capital produced domestically and capital produced in the foreign country as an outcome of foreign direct investment. On the other hand if country receives FDI then its total capital ownership will consist of capital accumulated through the process of domestic investment only, while capital used in production will be equal to the capital stock created through both domestic and foreign investment.

The usual accumulation conditions for \(\hat{K}_{it}\), \(\bar{K}_{it}\) and \(\hat{K}_{jt}\) are:

\[
\dot{\hat{K}}_{it} = I_{\hat{K}_{it}} - \delta \hat{K}_{it}, \dot{\bar{K}}_{it} = I_{\bar{K}_{it}} - \delta \bar{K}_{it}, \dot{\hat{K}}_{jt} = I_{\hat{K}_{jt}} - \delta \hat{K}_{jt}.
\] (1.8)

From (1.7) it follows that the accumulation of total capital stock owned by country i will be given as:

\[
\dot{\hat{K}}_{it} = \dot{\bar{K}}_{it} - \dot{\tilde{K}}_{it}
\]

Using this equality I can now derive the resource constraint for country i in the presence of FDI. The total output in country i will be used for consumption, domestic
investments in the K and H type capital and foreign investment in the K type capital. In addition, country i receives returns on the capital that it owns in the host country, which will be given by the term \( r_{jt} \hat{K}_{it} \).

The overall resource constraint for country i can therefore be written as:

\[
Y_{it} + r_{jt} \hat{K}_{it} = C_{it} + I_{K_{it}} + I_{H_{it}} + I_{\hat{K}_{it}} \tag{1.9}
\]

where \( r^\hat{k}_{jt} = \alpha_j A_j \hat{K}_{jt}^{\alpha_j - 1} H_{jt}^{1 - \alpha_j} \) is the rental rate on physical capital in the host country.

The optimization problem that country i faces can be summarized as maximization of (1.4) subject to the above resource constraint.

So, the present value Hamiltonian (PVH) and the necessary conditions for country i will be given by:

\[
J_{it} = \frac{C_{it}^{1 - \theta} - 1}{1 - \theta} e^{-\rho t} + \psi_{it}(I_{K_{it}} - \delta \hat{K}_{it}) + \mu_{it}(I_{H_{it}} - \delta H_{it}) \\
+ \phi_{it} \left[ A_i \hat{K}_{it}^{\alpha_i} H_{it}^{1 - \alpha_i} - C_{it} - \delta \hat{K}_{it} - I_{H_{it}} - I_{\hat{K}_{it}} + r_{jt} \hat{K}_{it} \right]
\]

\[
\frac{\partial J_{it}}{\partial C_{it}} = C_{it}^{1 - \theta} e^{-\rho t} - \phi_{it} = 0 \tag{1.10}
\]

\[
\frac{\partial J_{it}}{\partial I_{K_{it}}} = \psi_{it} - \phi_{it} = 0 \tag{1.11}
\]

\[
\frac{\partial J_{it}}{\partial I_{H_{it}}} = \mu_{it} - \phi_{it} = 0 \tag{1.12}
\]

\[
\frac{\partial J_{it}}{\partial I_{\hat{K}_{it}}} = -\phi_{it} \left[ A_i \alpha_i \hat{K}_{it}^{\alpha_i - 1} H_{it}^{1 - \alpha_i} - \delta \right] \tag{1.13}
\]

\[
\frac{\partial J_{it}}{\partial \hat{K}_{it}} = -\psi_{it} [-\delta] - \phi_{it} r_{jt} \tag{1.14}
\]
\[ \dot{\mu}_{it} = -\frac{\partial J_{it}}{\partial H_{it}} = -\mu_{it} \left[ -\delta - \phi_{it} \left[ A_i (1 - \alpha_i) \dot{K}_{it}^{\alpha_i} H_{it}^{-\alpha_i} \right] \right] \quad (1.15) \]

Initial and transversality conditions are omitted for simplicity.

To write the optimization problem for the host country it will be useful to simplify the resource constraint as follows. I will start with the following expression:

\[ Y_{jt} = C_{jt} + I_{\dot{K}_{jt}} + I_{H_{jt}} - I_{\dot{K}_{it}} + r_{jt} \tilde{K}_{it} \quad (1.16) \]

The above expression incorporates the flow of FDI from the home country given by \( I_{\dot{K}_{it}} \), which I can rewrite using the equation (1.8) as follows:

\[ Y_{jt} = C_{jt} + \dot{\hat{K}}_{jt} - \dot{\tilde{K}}_{jt} + \delta \hat{K}_{jt} - \delta \tilde{K}_{jt} + I_{H_{jt}} + r_{jt} \tilde{K}_{it} \quad (1.17) \]

Using the definitions in (1.7) I can simplify the above expression as:

\[ Y_{jt} = C_{jt} + \hat{K}_{jt} + \delta \tilde{K}_{jt} + I_{H_{jt}} + r_{jt} \tilde{K}_{it} \quad (1.18) \]

Finally, I can express the resource constraint in the host country as an accumulation condition for the domestically produced capital \( \tilde{K}_{jt} \).

\[ \dot{\tilde{K}}_{jt} = Y_{jt} - C_{jt} - I_{H_{jt}} - r_{jt} \tilde{K}_{it} - \delta \tilde{K}_{jt} \quad (1.19) \]

This final expression is used to write the present value Hamiltonian and the necessary conditions for country j:

\[ J_{jt} = \frac{C_{jt}^{1-\theta} - 1}{1 - \theta} e^{-\rho t} + \mu_{jt} (I_{H_{jt}} - \delta H_{jt}) \]
\[ + \phi_{jt} \left[ A_j \hat{r}_{j}^{\alpha_j} H_{jt}^{1-\alpha_j} - C_{jt} - \delta \bar{K}_{jt} - I_{H_{jt}} - r_{k_{jt}} \bar{K}_{it} \right] \]

\[ \frac{\partial J_{jt}}{\partial C_{jt}} = C_{jt} e^{-\rho t} - \phi_{jt} = 0 \quad (1.20) \]

\[ \frac{\partial J_{jt}}{\partial I_{H_{jt}}} = \mu_{jt} - \phi_{jt} = 0 \quad (1.21) \]

\[ \dot{\phi}_{jt} = -\frac{\partial J_{jt}}{\partial K_{jt}} = -\phi_{jt} \left[ \alpha_j A_j \hat{r}_{j}^{\alpha_j} H_{jt}^{1-\alpha_j} - \delta \right] \quad (1.22) \]

\[ \dot{\mu}_{jt} = -\frac{\partial J_{jt}}{\partial H_{jt}} = -\mu_{jt} \left[ -\delta \right] - \phi_{jt} A_j (1 - \alpha_j) \hat{r}_{j}^{\alpha_j} H_{jt}^{-\alpha_j} \quad (1.23) \]

Again, the initial and transversality conditions are omitted for simplicity.

### 1.3 The Balanced Growth Path Solution

From the equations (1.11), (1.12), and (1.21) it follows that there is a bang-bang control problem in investment. The necessary conditions for the choices of domestic and foreign investment do not depend on the investment itself.

Each of the corresponding costate variables represents the marginal value of an investment: \( \psi_i \) is a marginal value of foreign investment for country \( i \), \( \phi \) is a marginal value of the domestic investment in K type capital and \( \mu \) is a marginal value of the investment into H type capital for either country. The ratios of these costate variables represent internal relative values of one type of investment in terms of the other. The ratio of \( \frac{\psi_i}{\phi} \) is the relative price of foreign investment in terms of the domestic investment for country \( i \) and the ratio of \( \frac{\psi}{\phi} \) is the internal price of domestic investment in H capital in terms of the domestic investment in K capital in either country.

The balanced growth path (BGP) solution requires that all variables are either constant or grow at a constant rate. If marginal values of all investments do not equalize on
the BGP then constraints of irreversible investment can become binding for some types of investment, violating the requirement of balanced growth. Therefore, for all types of investment to be positive their marginal values must be equalized, otherwise a country will have an incentive to invest in one type of capital and set investment in another type of capital equal to 0 to satisfy inequality restrictions given by \( I_{K_{it}} \geq 0, I_{\tilde{K}_{it}} \geq 0, I_{H_{it}} \geq 0, I_{\tilde{K}_{jt}} \geq 0, \) and \( I_{H_{jt}} \geq 0). Equalization of the marginal values of all investments implies constancy of the internal prices as well; however marginal values are not necessarily equal along the transition. They change according to their laws of motion given by equations (1.13), (1.14), (1.15), (1.22) and (1.23).

I follow the approach discussed in Barro and Sala-i-Martin (2004) and first proceed with the BGP solution where all investments are positive.

If all types of investment are positive for country \( i \) then the necessary conditions for the optimal control problem will be given by (1.10) - (1.15). I can start with the solution for the growth rates of the costate variables.

From (1.12) and (1.15) I can write:

\[
\frac{\dot{\mu}_i}{\mu_i} = \delta - A_i (1 - \alpha_i) \hat{K}_i^{\alpha_i} H_i^{-\alpha_i} \tag{1.24}
\]

Then by combining (1.11) and (1.14) I will get:

\[
\frac{\dot{\psi}_i}{\psi_i} = \delta - r_j \tag{1.25}
\]

Finally from (1.13) it follows:

\[
\frac{\dot{\phi}_i}{\phi_i} = \delta - A_i \alpha_i \hat{K}_i^{-\alpha_i} H_i^{1-\alpha_i} \tag{1.26}
\]
Using the necessary condition for the choice of consumption and combining it with the above solution for the growth rate of costate variable \( \phi \) I obtain the growth rate of consumption in country \( i \) as a function of the ratio of \( \frac{\hat{K}}{H} \) as follows:

\[
\frac{\dot{C}_i}{C_i} = \frac{1}{\theta} \left( A_i \alpha_i \hat{K}_i^{\alpha_i-1} H_i^{1-\alpha_i} - \delta - \rho \right)
\] (1.27)

I have already argued that if all investments are strictly positive, then the no-arbitrage condition requires equalization of the marginal values of each investment, which also implies equalization of the growth rates of all costate variables. By setting them equal I can solve for the value of the ratio of \( \hat{K} \) to \( H \) as follows:

\[
A_i \alpha_i \hat{K}_i^{\alpha_i-1} H_i^{1-\alpha_i} - \delta = A_i (1 - \alpha_i) \hat{K}_i^{\alpha_i} H_i^{-\alpha_i} - \delta = r_j - \delta
\] (1.28)

\[
A_i \alpha_i \hat{K}_i^{\alpha_i-1} H_i^{1-\alpha_i} = A_i (1 - \alpha_i) \hat{K}_i^{\alpha_i} H_i^{-\alpha_i}
\]

\[
\left( \frac{\hat{K}_i}{H_i} \right)^* = \frac{\alpha_i}{1 - \alpha_i}
\] (1.29)

It follows from the above solution that this ratio is constant on the BGP and is similar to the solution of \( K \) to \( H \) ratio under autarky (see equation (1.5)). The important difference between these two ratios is the notion of \( K \) type capital represented in each of them. In the presence of FDI \( \hat{K} \) measures the total capital stock produced within the country, which is not equal to the capital stock owned by country \( i \). Under autarky the two concepts of \( K \) type capital stock (capital owned and capital used in production) coincide.

Having solved for the ratio, I can substitute it into the solution for the growth rate of consumption for country \( i \) and rewrite it as:
\[
\left(\frac{\dot{C}_i}{C_i}\right)^* = \frac{1}{\theta} \left[\alpha_i^{\alpha_i} A_i (1 - \alpha_i)^{1-\alpha_i} - \delta - \rho\right] \tag{1.30}
\]

where \( r_i^* = \alpha_i^{\alpha_i} A_i (1 - \alpha_i)^{1-\alpha_i} \) is the steady state rate of return on capital in country \( i \).

Following similar steps for country \( j \) I can solve for the growth rate of consumption in country \( j \).

First, using (1.21) and (1.23) I can derive the growth rate of the costate variable \( \mu_j \):

\[
\frac{\dot{\mu}_j}{\mu_j} = \delta - A_j (1 - \alpha_j) \hat{K}_j^{\alpha_j} H_j^{-\alpha_j} \tag{1.31}
\]

Then from (1.22) I can derive the growth rate of costate variable \( \phi_j \) equal to:

\[
\frac{\dot{\phi}_j}{\phi_j} = \delta - \alpha_j A_j \hat{K}_j^{\alpha_j-1} H_j^{1-\alpha_j} \tag{1.32}
\]

Combining the necessary condition for choice of consumption (1.20) with the above solution for the growth rate of \( \phi \) I can write the solution for the growth rate of consumption as:

\[
\frac{\dot{C}_j}{C_j} = \frac{1}{\theta} \left[\alpha_j A_j \hat{K}_j^{\alpha_j-1} H_j^{1-\alpha_j} - \delta - \rho\right] \tag{1.33}
\]

Using (1.31) and (1.32) I can solve for the ratio of \( K \) to \( H \) type capital in country \( j \) as follows:

\[
A_j \alpha_j \hat{K}_j^{\alpha_j-1} H_j^{1-\alpha_j} = A_j (1 - \alpha_j) \hat{K}_j^{\alpha_j} H_j^{-\alpha_j}
\]

\[
\left(\frac{\hat{K}_j}{H_j}\right)^* = \frac{\alpha_j}{1 - \alpha_j} \tag{1.34}
\]

Again, the ratio of \( K \) to \( H \) capital is constant and is similar to the solution under autarky (1.5) that I was referring to earlier. Similar to the case with country \( i \) the differ-
ence in the ratios is related to the concept of K capital. Since country j is a recipient of FDI, \( \hat{K} \) represents not only capital produced by country j but also the capital stock in country j accumulated through FDI.

Substituting (1.34) into (1.33) we get the growth rate of consumption in country j.

\[
\left( \frac{\dot{C}_j}{C_j} \right)^* = \frac{1}{\theta} \left[ \alpha_j A_j (1 - \alpha_j)^{1-\alpha_j} - \delta - \rho \right]
\]  

Similarly \( r_j^* = \alpha_j A_j (1 - \alpha_j)^{1-\alpha_j} \) is the steady state rate of return on capital in country j.

Comparison of the growth rates of consumption for either country in the presence of FDI given by (1.30) and (1.35) with the growth rate of consumption under autarky in (1.6) reveals that there are no growth effects of FDI. There are, however, level effects of FDI for both home and host countries. Country i has increased its total K type capital stock, by accumulating capital through both domestic and foreign investment. I have already argued that the ratio \( \hat{K}_i \) for country i is the same as under autarky, but it only reflects K type capital accumulated domestically.

For the host country the ratio \( \hat{K}_j \) also reflects K type capital produced within country, but in the presence of FDI it incorporates K type capital owned by the home country, which suggests that host country owns a smaller stock of capital in the presence of FDI compared to the scenario under autarky.

To finalize the BGP solution I show (see Appendix A) that on the BGP growth rates of the variables are equalized across countries, such that there exists a common growth rate defined as \( \gamma^* \).

Note that equalization of the growth rates across countries does not necessarily imply that the countries have identical technologies. Under this structure of the model tech-
nological differences across countries are captured by both total factor productivity and factor share parameters. From the equalization of the growth rates across countries it follows that \( r^*_i = r^*_j \) or alternatively:

\[
A_i \alpha_i^\alpha_i (1 - \alpha_i)^{1 - \alpha_i} = A_j \alpha_j^\alpha_j (1 - \alpha_j)^{1 - \alpha_j}
\]

which implies that not only technologically identical countries can grow at the same rate on the BGP in the presence of FDI. Countries can grow at the same rate but still differ in terms of TFP and factor share parameters.

The BGP solution of the model in the presence of FDI generates at least two interesting results: it follows that FDI does not generate any growth effects for both home and host countries, and that equalization of the growth rates across countries in the presence of FDI can be achieved even for technologically different countries.

The above expression on the relationship between technological parameters on the BGP in the presence of FDI across countries raises an interesting question on the behavior of the model if the technological parameters of both countries do not satisfy that requirement with equality such that one of the countries has a higher long-run rate of return on capital.

Note that as it follows from \( A_i \alpha_i^\alpha_i (1 - \alpha_i)^{1 - \alpha_i} \) and \( A_j \alpha_j^\alpha_j (1 - \alpha_j)^{1 - \alpha_j} \) the steady state rental rate of capital is determined by both TFP and the factor share parameters of each country. If one of the countries has a higher long-run rate of return on capital then the investment in that country is always more productive compared to another country. This, in turn, implies that in the absence of barriers to openness, a country with a lower rate of return on capital has an incentive to continuously shift its resources to the country where the rate of return on capital is higher. Under this scenario there will be no BGP
in the usual sense. A continuous shift of resources from the home to the host country will lead to a BGP solution only asymptotically when the home country’s production goes out of existence. So, overall there are two possible BGP solutions: a BGP solution with both countries producing and the home country investing in the host country and a BGP solution that exists asymptotically when the host country produces while the home country’s domestic production is going out of existence. Whichever solution will prevail depends on the relationship between technological parameters of both countries.

In the next two chapters of my dissertation I will further explore the importance of cross country technological differences by studying the effects of FDI in the context of richer models with two sectors of production.

1.4 Transitional Dynamics

In this section I will discuss the solution of the model under the assumption that constraints of nonnegative investments are binding. Again, I will follow the approach discussed in Barro & Sala-i-Martin (2004). In particular, in the context of the closed economy they consider scenarios when an economy starts with a ratio of capital stocks $\frac{K_0}{H_0}$ that is different from its BGP value. If the country has more of the K type capital than H type capital compared to the BGP value, then the country will set investment in K type capital equal to zero and accumulate only H type capital because the rate of return on H type capital is higher than rate of return on physical capital.

Similarly, if a country starts with a higher level of H type capital and a lower level of K type capital compared to the BGP values then as it follows from the definition on the rate of return from (1.9) the rate of return on K type capital is higher than the rate of return on H type capital so the country will set investment into H type capital equal to
zero and accumulate K type capital only. As Barro & Sala-i-Martin (2004) show in the context of a closed economy one sector model, deviation of the $K/H$ ratio from the BGP value leads to the violation of the irreversibility constraints such that either $I_K = 0$ or $I_H = 0$.

In analysis of the transitional behavior of the current model I will use the same approach to study the behavior of the economy when one of the countries deviates from the BGP value of the ratio of K to H types of capital.

Under the open economy framework with FDI the constraints of nonnegative investment can be written as: $I_{K_{it}} \geq 0$, $I_{H_{it}} \geq 0$, $I_{\tilde{K}_{it}} \geq 0$, $I_{\tilde{K}_{jt}} \geq 0$, and $I_{H_{jt}} \geq 0$.

Under the BGP solution with all types of investment being strictly positive I showed that the ratios of K type capital to H type capital in each country has the same value as the closed economy ratio of K to H capital. Also, the condition that must be satisfied on the BGP for the constraints of nonnegative investment not to be binding is the equalization of the marginal products of both types of capital not only within but also across the countries.

Now, suppose that the host country which is the recipient of the FDI from the home country deviates from the BGP ratio of K to H capital by having less K than H. A lower level of the total K type capital stock in the host country compared to the BGP value implies that the rate of return on K type capital is higher in the host country compared to the BGP value of the common rate of return. So, if $\frac{K_{jt}}{H_{jt}} < \frac{\alpha_j}{1-\alpha_j}$ then the host country has an incentive to set investment in H type capital equal to zero, such that $I_{H_{jt}} = 0$ and accumulate K type capital only.

As it is shown in appendix A the only possible scenario for the home country is to keep $\frac{\bar{K}_i}{\bar{H}_i} = \frac{\alpha_i}{1-\alpha_i}$ for this world economy to asymptotically converge to the steady state.

So, if in the host country $\frac{K_{jt}}{H_{jt}} < \frac{\alpha_j}{1-\alpha_j}$ then in the home country the following condition...
will hold \( \frac{\dot{K}_i}{H_i} = \frac{\alpha_i}{1-\alpha_i} \).

As the ratio of \( \frac{\dot{K}}{H} \) in country i is consistent with the BGP value, it follows that the rates of return on both types of capital are also consistent with the BGP values. However, in the host country the rate of return on K type capital is higher than the common rate of return in both countries on the BGP. Since in the home country the \( \frac{\dot{K}}{H} \) ratio is on its BGP value, the home country will be willing to keep its ratio at that level\(^6\) and use its resources to invest into K type capital in the host country, where the rate of return on K type capital is higher.

Under the assumption of common depreciation rates the home country can keep the ratio of \( \frac{\dot{K}}{H} \) on the BGP level by setting investment in both types of capital equal to zero. Therefore, the resource constraint for the home country can be written as:

\[
\dot{K}_{it} = A_i \dot{K}_{it}^\alpha \dot{H}_{it}^{1-\alpha} - C_{it} - \delta \dot{K}_{it} + r\dot{K}_{jt} \dot{K}_{it} \tag{1.36}
\]

while the resource constraint for the host country can be written as:

\[
\dot{K}_{jt} = Y_{jt} - C_{jt} - r\dot{K}_{jt} \dot{K}_{it} - \delta \dot{K}_{jt} \tag{1.37}
\]

I can write the present value Hamiltonian and the set of necessary conditions for each country as follows.

Country i:

\[
J_{it} = C_{it}^{1-\theta} \left[ 1 - \frac{1}{1-\theta} e^{-\rho t} + \phi_{it} \left[ A_i \dot{K}_{it}^\alpha \dot{H}_{it}^{1-\alpha} - C_{it} - \delta \dot{K}_{it} + r\dot{K}_{jt} \dot{K}_{it} \right] \right] \]

\[
\frac{\partial J_{it}}{\partial C_{it}} = C_{it}^{1-\theta} e^{-\rho t} - \phi_{it} = 0 \tag{1.38}
\]

\(^6\)See Appendix A.
\[ \dot{\phi}_{it} = -\frac{\partial J_{it}}{\partial \tilde{K}_{it}} = -\phi_{it} \left[ r_{K_{jt}} - \delta \right] \] (1.39)

\[ \lim_{t \to \infty} \phi_{it} \tilde{K}_{it} = 0 \] (1.40)

From (1.39) we can write:

\[ \frac{\dot{\phi}_{it}}{\phi_{it}} = \delta - r_{K_{jt}} \]

Differentiation of (1.38) with respect to time combined with the above expression leads to the solution for the growth rate of consumption in country i:

\[ \frac{\dot{C}_{it}}{C_{it}} = \frac{1}{\theta} \left[ r_{K_{jt}} - \delta - \rho \right] \] (1.41)

For country j the present value Hamiltonian and the set of necessary conditions with the modified resource constraint will be:

\[ J_{jt} = \frac{C_{jt}^{1-\theta} - 1}{1 - \theta} e^{-\rho t} + \phi_{jt} \left[ A_{j} \hat{K}_{jt}^{\alpha_{j}} H_{jt}^{1-\alpha_{j}} - C_{jt} - r_{K_{jt}} \tilde{K}_{it} - \delta \tilde{K}_{jt} \right] \]

\[ \frac{\partial J_{jt}}{\partial C_{jt}} = C_{jt}^{-\theta} e^{-\rho t} - \phi_{jt} = 0 \] (1.42)

\[ \dot{\phi}_{jt} = -\frac{\partial J_{jt}}{\partial \tilde{K}_{jt}} = -\phi_{jt} \left[ \alpha_{j} A_{j} \hat{K}_{jt}^{\alpha_{j} - 1} H_{jt}^{1-\alpha_{j}} - \delta \right] \] (1.43)

Following similar steps as for the home country I can solve for the growth rate of costate variable \( \phi \) in country j, which will be given as:

\[ \frac{\dot{\phi}_{jt}}{\phi_{jt}} = \delta - \alpha_{j} A_{j} \hat{K}_{jt}^{\alpha_{j} - 1} H_{jt}^{1-\alpha_{j}} \]

Using the above expression I can solve for the growth rate of consumption in country
\[ \frac{\dot{\hat{K}}}{\hat{K}} = \frac{A_i \hat{K}^{\alpha_i} H^{1-\alpha_i}}{\hat{K}} - \frac{C_i}{\hat{K}} - \delta + r_j \] (1.45)

Also, from equation (1.17) I can write:

\[ \frac{\dot{\hat{K}}}{\hat{K}} = Y_j - C_j - \delta \left( 1 - \frac{\hat{K}}{K_j} \right) - r_j \frac{\hat{K}}{K_j} + \frac{\dot{\hat{K}}}{K_j} \] (1.46)

From the above expression it follows:

\[ \frac{\dot{\hat{K}}}{\hat{K}} = A_j \left( \frac{\hat{K}}{H_j} \right)^{\alpha_j-1} - \frac{C_j}{K_j} - \delta \left( 1 - \frac{\hat{K}}{H_j} \right) \]

\[ -\alpha_j A_j \left( \frac{\hat{K}}{H_j} \right)^{\alpha_j-1} \frac{\hat{K}}{K_j} + \frac{\dot{\hat{K}}}{K_j} + \frac{\hat{K}}{K_j} \] (1.47)

The dynamic behavior of each country can be described in terms of the behavior of \( K \) type capital, \( H \) type capital and consumption along the transition. However, as it follows from (1.36) the accumulation condition for \( \tilde{K} \) depends on \( \hat{K} \) and \( \hat{K} \). To reduce the number of variables affecting transitional behavior of this world economy I will proceed with some manipulations of (1.36).

So, from equation (1.36) I can write:

\[ \frac{\dot{\hat{K}}}{\hat{K}} = A_i \hat{K}^{\alpha_i} H^{1-\alpha_i} - \frac{C_i}{\hat{K}} - \delta + r_j \] (1.44)
Finally, substituting (1.45) into (1.47) I get:

\[
\frac{\dot{\hat{K}}_{jt}}{\hat{K}_{jt}} = A_j \left( \frac{\dot{\hat{K}}_{jt}}{H_{jt}} \right)^{\alpha_j - 1} - C_{jt} - \delta \left( 1 - \frac{\dot{\hat{K}}_{it}}{H_{jt} \hat{K}_{jt}} \right)
\]

\[
+ \frac{\dot{\hat{K}}_{it} H_{jt}}{H_{jt} \hat{K}_{jt}} \left[ A_i \left( \frac{\dot{\hat{K}}_{it}}{H_{it}} \right)^{\alpha_i} \frac{H_{it} H_{jt}}{H_{jt} \hat{K}_{it}} - C_{it} - \delta \right]
\]

Dynamic equations describing the behavior of the variables along the transition process in the home country are:

\[
\frac{\dot{\hat{K}}_{it}}{\hat{K}_{it}} = -\delta
\]

\[
\frac{\dot{\hat{H}}_{it}}{\hat{H}_{it}} = -\delta
\]

\[
\frac{\dot{\hat{C}}_{it}}{\hat{C}_{it}} = \frac{1}{\theta} \left[ \alpha_j A_j \left( \frac{\dot{\hat{K}}_{jt}}{H_{jt}} \right)^{\alpha_j - 1} - \delta - \rho \right]
\]

\[
\frac{\dot{\hat{K}}_{it}}{\hat{K}_{it}} = A_i \left( \frac{\dot{\hat{K}}_{it}}{H_{it}} \right)^{\alpha_i} \frac{H_{it} H_{jt}}{H_{jt} \hat{K}_{it}} - C_{it} + \alpha_j A_j \left( \frac{\dot{\hat{K}}_{jt}}{H_{jt}} \right)^{\alpha_j - 1} - \delta
\]

For the host country the dynamic equations are:

\[
\frac{\dot{\hat{H}}_{jt}}{\hat{H}_{jt}} = -\delta
\]

\[
\frac{\dot{\hat{C}}_{jt}}{\hat{C}_{jt}} = \frac{1}{\theta} \left[ \alpha_j A_j \left( \frac{\dot{\hat{K}}_{jt}}{H_{jt}} \right)^{\alpha_j - 1} - \delta - \rho \right]
\]

\[
\frac{\dot{\hat{K}}_{jt}}{\hat{K}_{jt}} = A_j \left( \frac{\dot{\hat{K}}_{jt}}{H_{jt}} \right)^{\alpha_j - 1} - C_{jt} - \delta \left( 1 - \frac{\dot{\hat{K}}_{it}}{H_{jt} \hat{K}_{jt}} \right)
\]

\[
+ \frac{\dot{\hat{K}}_{it} H_{jt}}{H_{jt} \hat{K}_{jt}} \left[ A_i \left( \frac{\dot{\hat{K}}_{it}}{H_{it}} \right)^{\alpha_i} \frac{H_{it} H_{jt}}{H_{jt} \hat{K}_{it}} - C_{it} - \delta \right]
\]
It follows from the above dynamic equations that dynamic behavior in one country depends on the variables in the other country. Since the home country does not invest domestically the path of its consumption will depend on the accumulation of capital in the host country. Therefore I cannot study the dynamic behavior of each country separately, instead I need to discuss the dynamic path of this world economy in terms of a general dynamic system of equations characterizing joint dynamic behavior of this world economy.

The dynamic behavior of this world economy, consisting of two countries, can be described in terms of the following ratios: \( \frac{K_{jt}}{H_{jt}}, \frac{K_{it}}{H_{jt}}, \frac{C_{it}}{K_{it}} \) and \( \frac{C_{jt}}{K_{jt}} \), which results in a four dimensional system of differential equations.\(^7\)

The dimensionality of the system does not lead to analytical solutions; however some properties of this dynamic system can be studied through calibration exercises, which will be discussed in detail in the third chapter of this dissertation.

1.5 Conclusion

In this chapter I studied the effects of FDI on growth in the context of a one-sector endogenous growth model. To the best of my knowledge this work is the first study of the growth effects of FDI through its primary function of capital accumulation. The choice of the model is determined by my objective to focus on the capital accumulation role of FDI. The growth mechanism in the current model is driven by the accumulation of two types of capital K and H, which, in this class of models, are usually assumed to be physical and human capital respectively.

\(^7\)Note that ratio \( \frac{H_i}{H_j} \) stays constant along the transition path. Both \( H_i \) and \( H_j \) depreciate at the same rate \( \delta \) which keeps the ratio constant on the transition.
One of the limiting features of the model is its one-sector structure, which explicitly assumes that physical and human capital are produced within the single sector of an economy and therefore are perfect substitutes for each other. While this structure is mathematically more tractable, it limits the interpretation of the results. In particular, results of the BGP solution suggest that technological differences across countries seem to matter, but the restricting structure of a model does not allow one to completely explore the role that technological differences play in determining the growth effects of FDI.

Nevertheless the model serves as a useful benchmark for study of the growth effects of FDI. It introduces at least three important results that I further explore in the remaining chapters of my dissertation: in the current model there are no effects of FDI on growth through the process of capital accumulation, technological differences across countries may potentially play some role for growth effects of FDI and transitional behavior of countries in the presence of FDI differs from the dynamic behavior under autarky, because behavior of the variables in one country becomes affected by the behavior of the variables in the other country through the FDI channel.
Chapter 2

Uzawa-Lucas Model with Foreign Direct Investment

2.1 Introduction

In this chapter I incorporate FDI into the Uzawa-Lucas model structure (Uzawa (1965), Lucas (1988)) of a two-sector endogenous growth model. The main advantage of the current setup compared to the one-sector structure is that in the current setting the production of K and H types of capital is separated into two different sectors of the economy. In the closed economy version of the model the output of the first sector is used for the investment into K type capital and consumption, while the output of the second sector is used for the accumulation of H type capital only. Besides being more realistic this structure eliminates perfect substitution between both types of capital.

A two-sector structure can also introduce a richer framework addressing technological differences across countries: countries can be different not only in TFP and factor intensity parameters of the sector producing K type capital but they can also have dif-
ferent technologies in the sector producing H type capital. However, the Uzawa-Lucas model structure does not allow one to completely explore cross-country technological differences because, as I show later in this chapter, equalization of the rates of return on capital across countries leads to the equalization of the technological parameters in the sector producing good H.

The restrictive assumption of the current model is that the underlying structure of the Uzawa-Lucas model is a special case of a general two sector endogenous growth model and assumes that physical capital is not used in the production of human capital. Even though this assumption limits the results of the model, it simplifies the model substantially and makes it a good starting point for the introduction of FDI into an endogenous growth model with the sectoral decomposition of the production process.

As in the previous chapter I will start with a brief summary of the autarkic version of the model before introducing FDI.

2.2 The Setup of the Model

The detailed discussion of the closed economy version of the Uzawa-Lucas model can be found in Barro and Sala-i-Martin (2004, Chapter 5). Here I will briefly highlight the main results of the solution of the model.

2.2.1 The Model Under Autarky

There are two sectors in the economy: a sector producing good Y and a sector producing good H. Good Y is produced using a Cobb-Douglas production function, with both K
and H type capital contributing to the production process in that sector as follows:

\[ Y = AK^\alpha (uH)^{1-\alpha} \]

Under autarky the output of this sector is used for investment in K type capital and consumption which leads to the following resource constraint in this sector:

\[ Y = C + \dot{K} + \delta K = AK^\alpha (uH)^{1-\alpha} \]

The production structure of the second sector is simpler and assumes that K is not used in the production of H type capital such that:

\[ \dot{H} + \delta H = B(1 - u)H \]

The country’s preferences are given by (1.4), and therefore the present value Hamiltonian for the country is written as:

\[ J = C^{1-\theta} + \theta e^{-\rho t} + \nu \left( AK^\alpha (uH)^{1-\alpha} - C - \delta K \right) + \mu (B(1 - u)H - \delta H) \]

where \( \nu \) and \( \mu \) are the shadow prices of K and H capital respectively.

From the necessary conditions of the optimal control problem it follows that the growth rate of consumption is:

\[ \frac{\dot{C}}{C} = \frac{1}{\theta} \left[ \alpha A \left( \frac{K}{uH} \right)^{\alpha-1} - \delta - \rho \right] \]
It also directly follows from the steady state solution of the model that:

\[
\frac{K}{uH} = \left( \frac{\alpha A}{B} \right)^{\frac{1}{1-\alpha}}
\]

which suggests that the ratio of K type capital to the share of H type capital used in production of Y is constant in the steady state.

The solution for the steady state rate of return is determined from the equalization of the growth rates of C, K, H and Y and is given by:

\[r = B\]

The above expression implies that the common growth rate of the country under autarky is:

\[\gamma^* = \frac{1}{\theta} (B - \delta - \rho)\]  \hspace{1cm} (2.1)

The transitional behavior of the model is characterized by the set of dynamic equations describing behavior of \(\omega = \frac{K}{H}\), \(\chi = \frac{C}{K}\) and u and, as the analysis suggest, exhibits monotonic convergence to the steady state.

### 2.2.2 The Open-Economy Setup of the Model

To introduce FDI into the framework of the Uzawa-Lucas model I will use the definitions of capital used in production and capital ownership presented in (1.7). I continue assuming that FDI is conducted only in K type capital, and that H type capital is human capital produced in the educational sector.\(^1\)

\(^1\)This assumption is very common in the literature on two-sector endogenous growth models. See Barro and Sala-i-Martin (2004).
As in the previous chapter I will differentiate between countries using subscripts i and j, with i denoting the home country - conductor of FDI - and j identifying the host country - recipient of FDI. Note that at this point I arbitrarily assign subscripts i and j: I will discuss factors determining the direction of the flow of FDI in the next chapter that provides more a complete framework for the study of FDI.

Following a similar technique as in the previous chapter I can modify the resource constraint for country i as follows:

\[ Y_{it} + r_{jt} \tilde{K}_{it} = C_{it} + I_{\hat{K}_{it}} + I_{\tilde{K}_{it}} \]

If there is no FDI in H type capital the accumulation condition for H type capital for country i will be unchanged compared to autarky and given by:

\[ \dot{H}_{it} + \delta H_{it} = B_i(1 - u_{it})H_{it} \]

Both countries have the same preferences and maximize lifetime utility described by (1.4), subject to resource constraints in the Y and H sectors.

The present value Hamiltonian and the set of necessary conditions for country i will be given as:

\[
J_{it} = \frac{C_{it}^{1-\theta}}{1-\theta} e^{-\rho t} + \psi_{it} (I_{\hat{K}_{it}} - \delta \tilde{K}_{it}) \\
+ \phi_{it} \left[ A_i \hat{K}_{it}^{\alpha_i} (u_{it} H_{it})^{1-\alpha_i} - C_{it} - \delta \hat{K}_{it} - I_{\hat{K}_{it}} + r_{jt} \tilde{K}_{it} \right] \\
+ \mu_{it} \left[ B_i (1 - u_{it}) H_{it} - \delta H_{it} \right]
\]

where \( r_{jt} = \alpha_j A_j \hat{K}_{jt}^{\alpha_j} (u_{jt} H_{jt})^{1-\alpha_j} \) is the rate of return on physical capital for the host country.
\[
\frac{\partial J_{it}}{\partial C_{it}} = C_{it}^{-\rho} e^{-\rho t} - \phi_{it} = 0 \quad (2.2)
\]
\[
\frac{\partial J_{it}}{\partial I_{\hat{K}_{it}}} = \psi_{it} - \phi_{it} = 0 \quad (2.3)
\]
\[
\dot{\phi}_{it} = -\frac{\partial J_{it}}{\partial \hat{K}_{it}} = -\phi_{it} \left[ \alpha_i A_i \hat{K}_{it}^{\alpha_{it} - 1} (u_{it} H_{it})^{1 - \alpha_i} - \delta \right] \quad (2.4)
\]
\[
\dot{\psi}_{it} = -\frac{\partial J_{it}}{\partial K_{it}} = -\psi_{it} \left[ -\delta \right] - \phi_{it} r_{jt} \quad (2.5)
\]
\[
\dot{\mu}_{it} = -\frac{\partial J_{it}}{\partial H_{it}} = -\phi_{it} \left[ (1 - \alpha_i) A_i \hat{K}_{it}^{\alpha_{it}} (u_{it} H_{it})^{-\alpha_i} u_{it} \right] - \mu_{it} [B_i (1 - u_{it}) - \delta] \quad (2.6)
\]
\[
\frac{\partial J_{it}}{\partial u_{it}} = \phi_{it} (1 - \alpha_i) A_i \hat{K}_{it}^{\alpha_{it}} (u_{it} H_{it})^{-\alpha_i} H_{it} - \mu_{it} B_i H_{it} = 0 \quad (2.7)
\]

Here as well, initial and transversality conditions are omitted for simplicity.

To proceed with the solution I define all the shadow prices of this optimization problem: \( \psi_i \) is the shadow price of foreign investment for country \( i \), \( \phi_i \) is the shadow price of domestic investment in K type capital and finally \( \mu \) is defined as the shadow price of domestic investment in H type capital.

Equation (2.3) demonstrates the trade-off that a country faces between the choice of domestic and foreign investment in K type capital. If (2.3) holds with equality then country \( i \) is indifferent between these two types of investment. Under the assumption that irreversibility restrictions are not binding such that \( I_{\hat{K}_i} > 0, I_{H_i} > 0, I_{\hat{K}_j} > 0, I_{H_j} > 0, \) and \( I_{\hat{K}_i} > 0 \), it means that the home country will be investing both domestically and abroad.

However, as it follows from the equations (2.4) and (2.5) the shadow prices of domestic and foreign investment evolve according to their laws of motion and are not necessarily equal at any point in time. As in the previous chapter I will proceed with the steady
state solution first, when all investments are positive, such that (2.3) holds with equality and then discuss transitional behavior of this economy when equality in (2.3) no longer holds.

From (2.2) I can write:

\[
\frac{\dot{C}_{it}}{C_{it}} = -\frac{1}{\theta} \left[ \frac{\dot{\phi}_{it}}{\phi_{it}} + \rho \right]
\]

I can derive the growth rate of the costate variable \(\phi\) from (2.4), which is equal to:

\[
\frac{\dot{\phi}_{it}}{\phi_{it}} = -\alpha_i A_i \dot{K}_{it}^{\alpha_i - 1} (u_{it} H_{it})^{1-\alpha_i} + \delta
\]  
(2.8)

Substituting the above expression into the growth rate of consumption I will get:

\[
\frac{\dot{C}_{it}}{C_{it}} = \frac{1}{\theta} \left[ \alpha_i A_i \dot{K}_{it}^{\alpha_i - 1} (u_{it} H_{it})^{1-\alpha_i} - \delta - \rho \right]
\]  
(2.9)

Next, from (2.7) I can write:

\[
\phi_{it} (1 - \alpha_i) A_i \dot{K}_{it}^{\alpha_i} (u_{it} H_{it})^{-\alpha_i} = \mu_{it} B_i
\]  
(2.10)

Combining the above expression with (2.6) I can derive the growth rate of the shadow price \(\mu\):

\[
\frac{\dot{\mu}_{it}}{\mu_{it}} = -B_i + \delta
\]  
(2.11)

So far I have derived the growth rate of consumption, and the accumulation conditions for costate variables \(\phi\) and \(\mu\) for country \(i\). Before I summarize the BGP solution of the current model, I will proceed with the description of the optimal control problem and the necessary conditions characterizing the solution of this optimization problem faced by the host country.
The modification of the resource constraint for the host country also happens only in
the sector producing good Y. The host country receives a flow of FDI in K type capital
from the home country, conducts its own domestic investment in K and H type capital
and pays a return on the capital stock owned by home country. So, I can write the
modified resource constraint in the sector producing good Y for country j as follows:

\[ Y_{jt} = C_{jt} + I_{Kjt} - I_{Kjt} + r_{jt}K_{jt} \]  

(2.12)

Following manipulations similar to the ones discussed in the previous chapter I can
write the present value Hamiltonian and the necessary conditions for country j under the
assumption that all investments are positive as follows:

\[
J_{jt} = \frac{C_{jt}^{1-\theta}}{1-\theta} e^{-\rho t} + \phi_{jt} \left[ A_j (\dot{K}_{jt})^{\alpha_j} (u_{jt}H_{jt})^{1-\alpha_j} - C_{jt} - r_{jt}K_{jt} - \delta \right] \\
+ \mu_{jt} [B_j (1 - u_{jt})H_{jt} - \delta H_{jt}] 
\]

(2.13)

\[
\frac{\partial J_{jt}}{\partial C_{jt}} = C_{jt}^{1-\theta} e^{-\rho t} - \phi_{jt} = 0
\]

(2.14)

\[
\dot{\phi}_{jt} = -\frac{\partial J_{jt}}{\partial \dot{K}_{jt}} = -\phi_{jt} \left[ \alpha_j A_j \dot{K}_{jt}^{\alpha_j} (u_{jt}H_{jt})^{1-\alpha_j} - \delta \right]
\]

(2.15)

\[
\dot{\mu}_{jt} = -\frac{\partial J_{jt}}{\partial H_{jt}} = -\phi_{jt} \left[ (1 - \alpha_j) A_j \dot{K}_{jt}^{\alpha_j} (u_{jt}H_{jt})^{1-\alpha_j} u_{jt} \right] - \mu_{jt} [B_j (1 - u_{jt}) - \delta]
\]

(2.16)

Since country j conducts only domestic investment in K and H type capital, there are
only two costate variables defined for this problem: \( \phi_{jt} \) - the shadow price of good Y, and
\( \mu_{jt} \) - the shadow price of good H.
From (2.14) I can write:

\[
\frac{\dot{\phi}_{jt}}{\phi_{jt}} = -\alpha_j A_j K_j^{\alpha_j - 1} (u_{jt} H_{jt})^{1-\alpha_j} + \delta \tag{2.17}
\]

Using (2.13) I can derive growth rate of consumption as:

\[
\frac{\dot{C}_{jt}}{C_{jt}} = -\frac{1}{\theta} \left[ \frac{\dot{\phi}_{jt}}{\phi_{jt}} + \rho \right]
\]

Substituting the expression for the \( \frac{\dot{\phi}_{jt}}{\phi_{jt}} \) from (2.17) into the growth rate of consumption I get:

\[
\frac{\dot{C}_{jt}}{C_{jt}} = \frac{1}{\theta} \left[ \alpha_j A_j K_j^{\alpha_j - 1} (u_{jt} H_{jt})^{1-\alpha_j} - \delta - \rho \right] \tag{2.18}
\]

Similar to the solution for country \( i \) from (2.16) I can write:

\[
\phi_{jt} (1 - \alpha_j) A_j K_j^{\alpha_j} (u_{jt} H_{jt})^{-\alpha_j} = \mu_{jt} B_j \tag{2.19}
\]

Finally, taking into account the last expression and (2.15) I have:

\[
\frac{\dot{\mu}_{jt}}{\mu_{jt}} = \delta - B_j \tag{2.20}
\]

So, for the host country as well I have derived the expressions for the growth rates of costate variables and the growth rate of consumption, which I will use to summarize the BGP solution of the model in the presence of FDI.
2.3 Balanced Growth Path Solution

For country $i$ the domestic relative price of good $H$ in terms of good $Y$ is defined by the ratio of the costate variables $\phi_i$ and $\mu_i$ as $\frac{\mu_i}{\phi_i} = P_i$. Equalization of the rates of return on $K$ and $H$ type capital on the BGP requires that the relative price stays constant on the BGP as well. Constancy of the relative price in its turn implies that:

$$\frac{\dot{\mu}_i}{\mu_i} = \frac{\dot{\phi}_i}{\phi_i}$$

Using the expressions for the growth rates of the costate variables $\phi_i$ and $\mu_i$ derived in (2.8) and (2.11) respectively, I can write:

$$A_i \alpha_i \hat{K}_i^{\alpha_i-1} (u_i H_i)^{1-\alpha_i} = B_i$$

(2.21)

Then it follows that:

$$\left( \frac{\hat{K}_i}{u_i H_i} \right)^{\alpha_i-1} = \frac{B_i}{\alpha_i A_i}$$

In the meantime using the above definition of the relative price and the equality given by (2.10) I can write:

$$\left( \frac{\hat{K}_i}{u_i H_i} \right)^{\alpha_i} = P_i \frac{B_i}{A_i (1-\alpha_i)}$$

Dividing the above expressions by one another leads to the solution for the ratio of $K$ capital to the fraction of $H$ capital used in the $Y$ sector as a function of the relative price.

$$\left( \frac{\hat{K}_i}{u_i H_i} \right)^* = P_i \frac{\alpha_i}{1 - \alpha_i}$$

(2.22)

Having established the fact that $P_i$ is constant on the BGP, it follows from (2.22)
that \( \frac{K_i}{u_i H_i} \) is also constant on the BGP.

Using the earlier derived expression for the growth rate of consumption in (2.9) and the solution for the ratio \( \frac{K_i}{u_i H_i} \) given by (2.22), the growth rate of consumption for home country can be written as:

\[
\left( \frac{\dot{C}_i}{C_i} \right)^* = 1 \theta \left[ \alpha_i \alpha_i P_i^{\alpha_i - 1} A_i (1 - \alpha_i)^{1 - \alpha_i} - \delta - \rho \right] \tag{2.23}
\]

where \( \alpha_i \alpha_i P_i^{\alpha_i - 1} A_i (1 - \alpha_i)^{1 - \alpha_i} \) is the rate of return on physical capital in the home country.

On the other hand using (2.21) the growth rate of consumption can be written as:

\[
\left( \frac{\dot{C}_i}{C_i} \right)^* = 1 \theta \left[ B_i - \delta - \rho \right] \tag{2.24}
\]

Combining both expressions for the growth rate of consumption, it follows:

1. Long-run marginal product of capital in country \( i \) is equal to the total factor productivity of the sector producing good H.

2. The BGP solution for the domestic relative price of good H in terms of good Y can be obtained from the equalization of the above two expressions for the growth rate of consumption, such that:

\[
P_i = \left( \frac{A_i \alpha_i (1 - \alpha_i)^{1 - \alpha_i}}{B_i} \right)^{\frac{1}{1 - \alpha_i}} \tag{2.25}
\]

Going through similar steps as for the home country, I can summarize the BGP solution for the host country.

Here as well I define \( P_j = \frac{\mu_j}{\phi_j} \) as the domestic relative price of good H in terms of
good Y in the host country, which is constant on the BGP.

Using equations (2.17), and (2.20) for the host country I can write:

\[ A_j \alpha_j \dot{K}_j^{\alpha_j - 1} (u_j H_j)^{1-\alpha_j} = B_j \]  

(2.26)

Again, using (2.19) and (2.26) I can solve for \( \frac{\dot{K}_j}{u_j H_j} \) in the steady state as a function of \( P_j \) as follows:

\[ \left( \frac{\dot{K}_j}{u_j H_j} \right)^* = P_j \frac{\alpha_j}{1 - \alpha_j} \]  

(2.27)

Consequently I can rewrite the expression for the growth rate of consumption in the host country as:

\[ \left( \frac{\dot{C}_j}{C_j} \right) = \frac{1}{\theta} \left[ \alpha_j \frac{P_j^{\alpha_j - 1} A_j (1 - \alpha_j)^{1-\alpha_j} - \delta - \rho}{B_j} \right] \]  

(2.28)

where \( \alpha_j \frac{P_j^{\alpha_j - 1} A_j (1 - \alpha_j)^{1-\alpha_j}}{P_0} \) is the rate of return on physical capital in the host country. and

\[ \left( \frac{\dot{C}_j}{C_j} \right)^* = \frac{1}{\theta} \left[ B_j - \delta - \rho \right] \]  

(2.29)

Finally, from the equalization of the above two expressions for the growth rate of consumption, I can solve for the domestic relative price of good H in terms of good Y in country j as follows:

\[ P_j = \left( \frac{A_j \alpha_j (1 - \alpha_j)^{1-\alpha_j}}{B_j} \right)^{\frac{1}{1-\alpha_j}} \]  

(2.30)

As the above derivations suggest, the BGP solution of the open economy model with FDI is consistent with the solution of the autarkic Uzawa-Lucas model in that the TFP parameters of the sector producing good H, \( B_i \) and \( B_j \), represent the long run common rate of return in country i and j respectively. In the presence of FDI, however, the assumption of non-binding irreversibility constraints on the BGP requires equalization.
of the rates of return on both types of capital not only within a country but also across countries. In the context of the current model, equalization of the cross-country rates of return implies equalization of the TFP parameters in the sectors producing H-type capital in both countries. Therefore, I can conclude that for FDI to be present on the BGP, with one country investing in the other country, countries should have the same value of the TFP parameter in the sector producing good H. Since cross country technological differences in the second sector are captured only by TFP parameters, from the above result it immediately follows that for FDI to be present on the BGP countries should have the same level of technology in the sector producing good H.

Note that this result is a direct outcome of the restrictive technological structure of this model for the sector producing good H.

In particular, in the more general structure of the two-sector endogenous growth model discussed by Barro and Sala-i-Martin (2004, Chapter 5) technological differences across both sectors of the economy are reflected not only in TFP parameters but also in factor intensity parameters, which means that the equalization of the rates of return across countries does not necessarily require equalization of the levels of technology.\(^2\)

Note also that cross country equalization of the TFP parameters in the sector producing good H is a necessary condition for the existence of the BGP in the presence of FDI. If country \(j\) has higher TFP parameter in H sector then country \(i\) will have incentive to continuously shift its resources from domestic investment to foreign investment which can result in an unbalanced growth solution for the model.\(^3\)

To finalize the BGP solution of the current model I will just note that from the equalization of \(B_i\) and \(B_j\) immediately follows the equalization of the growth rates of

\(^2\)I will discuss this issue in more details in the third chapter of my dissertation.

\(^3\)Note that this result holds because under the structure of the current model the TFP parameter of H sector is equal to the long-run rate of return on capital in each country.
consumption across countries. Also, as in the solution for the one-sector model, it can be shown (see Appendix A) that growth rates of all variables are equalized across countries on the BGP. The common growth rate for the countries can be defined as:

$$\gamma^* = \frac{1}{\theta} (B - \delta - \rho)$$

where $B = B_i = B_j$

Immediate comparison with (2.1) reveals that as in one sector model, here as well FDI does not generate any long-run growth effects through the process of capital accumulation.

2.4 Transitional Dynamics

To study the transitional behavior of the Uzawa-Lucas model in the presence of FDI I will use the approach discussed by Barro and Sala-i-Martin (2004) that I have also used in the discussion of the transitional dynamics of the one-sector model with FDI.

The approach is based on the assumption that constraints of nonnegative investment become binding such that countries have incentives to set some of their investments equal to zero.

Here I assume that the host country deviates from the BGP ratio of K to H type capital by having less K than H such that: $\frac{\dot{K}_{jt}}{u_jH_{jt}} < P_j \frac{\alpha_j}{1-\alpha_j}$, where $P_j$ is defined as in (2.30). This scenario of the deviation of the K to H ratio in the host country is more relevant for the current model, because I still assume that FDI is conducted only in the sector producing good Y and the reduction of H type capital in the host country will not lead to the increased flow of FDI in the H sector.
Reduction in the K capital stock in the host country implies that the rate of return on K-type capital is higher in the host country compared to the BGP value of the common rate of return and that the host country has an incentive to set investment into H-type capital equal to zero and accumulate K type capital only.

Home country also has an incentive to allocate its resources towards foreign investment into K type capital, which is more profitable compared to domestic investment.

I have already shown (see Appendix A) that for the home country not to deviate from the BGP and to keep its ratio of K to H capital at its BGP value it needs to set domestic investments in K and H type capital equal to zero and use its resources to invest in K type capital in the host country, where the rate of return on K type capital is higher.

So, if in the host country I have \( \frac{\hat{K}_{jt}}{u_{jt}H_{jt}} < P_j \frac{\alpha_j}{1-\alpha_j}, I_{K_j} > 0, \text{ and } I_{H_j} = 0. \) In the home country I have \( \frac{\hat{K}_i}{u_iH_i} = P_i \frac{\alpha_i}{1-\alpha_i}, \) where \( P_i \) is defined as in (2.25), \( I_{K_i} = 0, I_{H_i} = 0 \) and \( I_{K_i} > 0. \)

Using the above conditions for home and host country I can derive the resource constraints for the countries as follows.

The accumulation condition for country i become:

\[
\dot{\hat{K}}_it = Y_{it} - C_{it} + r_{jt}\hat{K}_{it} - \delta \hat{K}_{it}
\]

Along the transition the optimization problem faced by country i can be summarized as maximization of lifetime utility defined in (1.4) subject to the above resource constraint.

Similarly, country j will be maximizing (1.4) subject to the following accumulation condition:

\[
\dot{\hat{K}}_{jt} = Y_{jt} - C_{jt} - r_{jt}\hat{K}_{it} - \delta \hat{K}_{jt}
\]
Then the present value Hamiltonian for each country can be written as:

\[ J_{it} = \frac{C_{it}^{1-\theta} - 1}{1 - \theta} e^{-\rho t} + \phi_{it} \left[ A_i \hat{K}_i^{\alpha_i} H_i^{1-\alpha_i} - C_{it} - \delta \hat{K}_i + r K_i \hat{K}_i \right] \]

and

\[ J_{jt} = \frac{C_{jt}^{1-\theta} - 1}{1 - \theta} e^{-\rho t} + \phi_{jt} \left[ A_j \hat{K}_j^{\alpha_j} H_j^{1-\alpha_j} - C_{jt} - r \hat{K}_j - \delta \bar{K}_j \right] \]

As one can see the above expressions are identical to the expressions for the present value Hamiltonian in the one-sector model with FDI which implies that the dynamic equations describing behavior of both countries along the transition are going to be identical to those discussed in chapter 1.

2.5 Conclusion

In this chapter I studied the effects of FDI on growth in the context of a model with two sectors of production, where the sectors are usually assumed to be the final output sector and the educational sector. The model follows the Uzawa-Lucas production structure, in which the educational sector does not use physical capital in the production process.

The model discussed in this chapter is an intermediate step between the simple structure of a one-sector model with FDI and a richer framework of the general two-sector model considered in the third chapter of this dissertation.

I continue assuming that FDI is conducted only in the sector producing good Y.

Similar to the one-sector model discussed in chapter one, here as well I show that FDI does not generate long-run growth effects, a result consistent with the empirical evidence on FDI. The results also suggest that for the BGP solution to exist in the presence of FDI countries should have the same level of technology in the sector producing good
H. This latter result is a direct outcome of the restrictive structure of the underlying Uzawa-Lucas production process, with K type capital not being used in the educational sector.

The model can also generate a solution with unbalanced growth if the host country has higher TFP parameter in the sector producing good H. Under the structure of the current model the TFP parameter of the H sector is consistent with the long-run rate of return on capital. In the absence of any economic or political barriers a higher long-run rate of return in the host country will imply that the home country has an incentive to continuously shift its resources towards foreign investment leading to unbalanced growth for the world economy.

Finally, transitional behavior of the Uzawa-Lucas model with FDI can be characterized by the same set of dynamic equations as in the one-sector model with FDI.

I will leave more detailed discussion of the transitional dynamics to the third chapter.
Chapter 3

The Model with Two Sectors of Production and FDI

3.1 Introduction

In this chapter I study the effects of FDI in the context of a two-sector endogenous growth model. The model generalizes the structure of the models discussed in the earlier chapters of my dissertation. It has two sectors of production similar to the Uzawa-Lucas structure discussed in chapter 2, however, both K and H type capital are essential factors of production in both sectors.

The model provides a richer framework for the comparison of technological differences across countries, which are going to play a role in determining the direction of the flow of FDI across countries. Unlike in the previous two chapters where I arbitrarily defined one of the countries as the conductor of FDI and the other country as the recipient of FDI, here I will derive the conditions determining which country will originate FDI and which country will receive it.
This model also generalizes my study of the growth effects of FDI through the process of capital accumulation by introducing FDI not only in K type capital, but also in H type capital.¹

I have argued in the earlier chapters that the transitional behavior in the presence of FDI should be analyzed for both countries together, which increases the dimensionality of the dynamic system describing behavior of countries in transition. I have conducted calibration exercises to evaluate the transitional behavior of this world economy, which I include in the discussion of this model.

The outline of this chapter will be similar to the previous chapters. First, I will briefly summarize closed economy version of the model discussed in Barro and Sala-i-Martin (2004, Chapter 5). Then I will proceed with the solution of the model in the presence of FDI both on the BGP and transition.

3.2 The Setup of the Model

As I have already mentioned my model is an open-economy extension of the two-sector endogenous growth model introduced by Rebelo (1991) and discussed by Barro and Sala-i-Martin (2004). Before introducing FDI I would like to briefly summarize the main structure and the results of the autarkic version of the model.

¹Yenokyan, Seater, Arabshahi (2011) consider a similar model structure with two countries trading factors of production. They treat H type capital as a factor of production that augments labor but is not embodied in the labor to avoid limitations of the trade in human capital (For details see Yenokyan, Seater, Arabshahi (2011)). Unlike their model, treating H type capital as human capital is not restrictive in the context of the current model; however I will also discuss more general treatment of H type capital later in this chapter.
3.2.1 The Model Under Autarky

The model is described by two sectors of production, producing good Y and good H. Both sectors use Cobb-Douglas constant returns to scale production technology, with both factors of production playing essential roles in the production process of each sector. The production functions take the following form:

\[ Y_t = A(v_t K_t)^\alpha(u_t H_t)^{1-\alpha} \]

\[ \dot{H}_t + \delta H_t = B((1 - v_t)K_t)^\eta((1 - u_t)H_t)^{1-\eta} \]

where A and B are total factor productivity (TFP) parameters, \( \alpha \) and \( \eta \) are factor share parameters and \( v \) and \( u \) are the fractions of K and H type of capital respectively used in the production of good Y. The technological differences between sectors are reflected in both TFP and factor share parameters.

The output of the sector producing good Y is used for the investment in K type capital and consumption, while the output of the other sector is used only for the investment in H type capital, leading to the following accumulation conditions for each type of capital:

\[ \dot{K}_t + \delta K_t = A(v_t K_t)^\alpha(u_t H_t)^{1-\alpha} - C_t \quad (3.1) \]

\[ \dot{H}_t + \delta H_t = B((1 - v_t)K_t)^\eta((1 - u_t)H_t)^{1-\eta} \quad (3.2) \]

Note that even though the current structure separates two types of capital into two different sectors and has a richer framework to capture technological differences across countries, the more complete structure of the production process would have three sectors of production instead of two. In particular, Bond and Trask (1997) study the effects of
trade on growth in the context of a small open economy model with endogenous growth and three sectors of production: one sector producing consumption good C, one sector producing good K and one sector producing H. They assume that H is a non-tradeable good, and focus on trade in two goods C and K, where C is a pure consumption good and K is a capital good.

Even though having three sectors of production introduces a more detailed structure on the production side, the authors show that under some specialization patterns the behavior of a small economy in the presence of trade becomes identical to the behavior of a general two-sector closed economy model with endogenous growth, where the authors are referring to the earlier study by Bond, Wang and Yip (1996).

On the other hand, being more complete, the three-sector production structure leads to a higher number of state variables in the model, represented by the stocks of K and H type capital in each sector. To avoid complications in the analysis of the transitional behavior of the model resulting from a higher number of state variables, and given the fact that the main results of the model are the same under both production structures, I used the more simplified framework of a two sector endogenous growth model discussed by Barro and Sala-i-Martin (2004, Chapter 5) to generalize my study of the growth effects of FDI through capital accumulation.

Households’ preferences in the model are described by the same CRRA utility function I introduced in Chapter 1, such that the optimization problem faced by the households is defined as the maximization of the overall utility given by (1.4) subject to the above two accumulation conditions (3.1) and (3.2).

What is relevant for the further discussion is the growth rate of the country and the solution of the ratio of the two types of capital in each sector under autarky on the BGP,
which are presented below.

The autarkic growth rate of the country is:

$$\gamma = \frac{1}{\theta} \left[ A^{\frac{\eta}{1-\alpha+\eta}} B^{\frac{1-\alpha}{1-\alpha+\eta}} \alpha^{\frac{\alpha \eta}{1-\alpha+\eta}} (1 - \alpha)^{\frac{(1-\alpha) \eta}{1-\alpha+\eta}} (1 - \eta)^{\frac{(1-\alpha)(1-\eta)}{1-\alpha+\eta}} - \delta - \rho \right]$$  \hspace{1cm} (3.3)

The ratios of K to H type capital are:

$$\frac{vK}{uH} = P \frac{\alpha}{1 - \alpha}$$

$$\frac{(1-v)K}{(1-u)H} = P \frac{\eta}{1 - \eta}$$

where P is defined as the relative price of good H in terms of good Y under autarky and on the BGP it is equal to:

$$P = \left( \frac{A^{\alpha}(1 - \alpha)^{1-\alpha}}{B^{\eta}(1 - \eta)^{1-\eta}} \right)^{\frac{1}{1-\alpha+\eta}}$$  \hspace{1cm} (3.4)

Before introducing FDI into the above described framework I would like to discuss the issue related to the nature of H type capital used in the production process, because in this chapter I will also consider FDI in H type capital.

As I have argued in the introduction to this chapter, in the presence of FDI the interpretation of H type capital is still linked to the concept of human capital; however doing so restricts the application of the model to the case of FDI in physical and human capital only. Mulligan and Sala-i-Martin (1993) argue that H type capital can be treated as “human capital, embodied and disembodied knowledge, public capital, quality of products, number of varieties and financial capital”. These interpretations can potentially increase the scope of application of the current model.
Here I will follow the discussion by Yenokyan, Seater, and Arabshahi (2011) that generalizes the concept of H type capital and treats it as any type of capital that augments labor but is not embodied in labor.

It directly follows from the production functions that the main conceptual difference between these two types of capital is related to the fact that H type capital augments unskilled labor without being embodied in the labor, whereas K type capital does not augment labor.

Usually, the capital that augments labor is treated as human capital embodied in the labor, however Peretto (2007) develops a framework in which labor used in the production of a final good gets augmented by the improved quality of the intermediate goods. In particular, the production function for the final good that Peretto imposes is given by:

\[
F = \int_{0}^{N} R_{i}^{\theta}(Z_{i}^{\alpha} Z^{1-\alpha} L_{i})^{1-\theta} di
\]

where \( R \) is the quantity of intermediate goods, \( Z_{i} \) is the quality of the intermediate good \( R \) and \( Z \) is the average quality of the varieties of intermediate good \( R \).

It follows from the above expression that technological progress aimed at the improvement of the quality of intermediate good \( R \), will augment labor in the final good production sector without being embodied in the labor.

In a recent study, Ji and Seater (2010) used the framework developed by Peretto to study the effects of trade on growth. They draw a distinction between the amount of intermediate good, its quality and the R&D ability embodied in that good. The authors show that the growth effects of trade are driven by the embodied R&D ability, which in turn depends on R&D productivity. The rich framework of the model by Ji and Seater

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\(^{2}\)See Barro and Sala-i-Martin (2004).
generates a wide variety of results on the potential welfare and growth effects of trade. In particular, the authors argue that the welfare and growth effects of trade can be positive or negative in either the short-run or the long-run depending on the initial quality of the imported intermediate good and its embodied R&D ability. For example, if a country imports a low-quality good it can face an initial reduction in welfare, however if the imported good has high R&D ability then the import of such a good can generate positive long-run growth effects.

As the results of the model suggest, not only does trade affect growth but growth has its effect on trade through the change in specialization pattern as an outcome of the technological progress improving the quality of the intermediate goods.

The structure developed by Ji and Seater that distinguishes between the quantity of intermediate good and the progress embodied in it leads to a potential extension to their work of assessing both welfare and growth effects of the flow of intermediate goods through FDI rather than the trade channel, which can be interesting because, as I have argued earlier, in the macro literature the growth effects of FDI are linked to the productivity spillovers resulting at least to some extent from better technologies introduced in the host countries by multinational enterprises (MNE).

Another important point here is that Ji and Seater’s framework shows the endogeneity in the relationship between trade and growth, something that is a relevant issue in the literature on FDI and growth as well.

Having said that, I would argue that to focus on the pure effects of FDI on growth I need a simpler structure that can still sustain the conceptual differences arising between two types of capital used in production.

Yenokyan, Seater and Arabshahi (2011) in their study of the effects of trade on growth show that under some simplifying assumptions the more complicated production struc-
ture introduced by Peretto and developed by Ji and Seater can be reduced to the simpler structure of a two sector endogenous growth model, with H type capital taking the role of the average quality of intermediate goods (Z from the above framework) and augmenting labor without being embodied in it. Though much simpler, the model developed by Yenokyan, Seater and Arabshahi, is a useful approximation to the richer structure of Ji and Seater’s model, and therefore can serve as a benchmark case of introducing and studying the pure effects of FDI on growth, exactly the objective that I have in mind. Incorporating FDI into the richer framework of Ji and Seater’s model with the explicit modeling of the technological progress through R&D process can become an interesting extension of the current work and is left for future research.

3.2.2 The Open-Economy Setup of the Model

Once a country decides to invest abroad it faces decisions about how much to consume, how much to invest domestically and how much to invest abroad. This additional decision on foreign investment modifies the accumulation conditions in (3.1) and (3.2).

The new accumulation conditions for a country planning to invest abroad in both types of capital will be given by:

\[
\dot{K}_t = A(v_t \dot{K}_t)\alpha (u_t \dot{H}_t)^{1-\alpha} - C_t - \delta \dot{K}_t - I_{\dot{K}_t} + r \dot{K}_t \dot{K}_t
\]  

(3.5)

\[
\dot{H}_t = B \left[ (1 - v_t) \dot{K}_t \right] ^\eta \left[ (1 - u_t) \dot{H}_t \right] ^{1-\eta} - \delta \dot{H}_t - I_{\dot{H}_t} + r \dot{H}_t \dot{H}_t
\]  

(3.6)

Note that to derive these accumulation conditions I have extended the definitions of capital ownership and capital used in the production process introduced in chapter 1. See equation (1.7). Here I simply add the distinction between H type capital ownership and
H type capital used in the production process similar to what I did in chapter 1 and chapter 2 for K type capital such that:

\[
\hat{H}_{it} = H_{it} - \bar{H}_{it}, \quad \hat{H}_{jt} = H_{jt} + \bar{H}_{jt},
\]

(3.7)

where \(\hat{H}_{it}\) denotes H capital produced in the home country, \(\bar{H}_{it}\) denotes the total H capital ownership for country i, and \(\bar{H}_{it}\) is capital created as an outcome of FDI. Similarly, \(\hat{H}_{jt}\) is the total H type capital used in production in country j, and \(\bar{H}_{jt}\) is the total H type capital ownership in country j.

In particular, the investing country will be producing using \(\hat{K}\) and \(\hat{H}\), the capital stock produced domestically, and will be receiving a return on capital created through the process of foreign investment \(\bar{K}\) and \(\bar{H}\). It is important to emphasize here that at this point I only assume that the country, after being in autarky, is planning to open and engage in FDI and I am just stating the modified optimization problem faced by that country. This new optimization problem is now defined as maximization of (1.4) subject to (3.5) and (3.6). Whether or not the country will engage in FDI and whether or not the country will decide to invest in both types of capital will follow from the nature of the necessary conditions of this optimization problem.

The present value Hamiltonian under the assumption that \(I_{\hat{K}_i} > 0, I_{\hat{H}_i} > 0, I_{\hat{K}_j} > 0, I_{\hat{H}_j} > 0, I_{\bar{K}_i} > 0, I_{\bar{H}_j} > 0,\) and \(I_{\bar{K}_i} > 0\) will be given:

\[
J_t = \frac{C_t^{1-\theta} - \frac{1}{1-\theta} e^{-\rho t} + \psi_t(I_{\hat{K}_i} - \bar{\delta} \bar{K}_t) + \lambda_t(I_{\hat{H}_i} - \bar{\delta} \bar{H}_t)}{1 - \bar{\theta}} + \phi_t \left[ A(v_t \hat{K}_i)^{\alpha} (u_t \hat{H}_t)^{1-\alpha} - C_t - \bar{\delta} \bar{K}_t - I_{\bar{K}_i} + r_{\bar{K}_j} \bar{K}_t \right] \\
+ \mu_t \left\{ B \left[ (1 - v_t) \hat{K}_i \right]^{(1-\alpha)} \left[ (1 - u_t) \hat{H}_t \right]^{1-\bar{\eta}} - \delta \hat{H}_t - I_{\hat{H}_i} + r_{\hat{H}_j} \bar{H}_t \right\}
\]
where
\[ r_{K_{jt}} = \alpha_j A_j (v_{jt} \hat{K}_{jt})^{\alpha_j-1} (u_{jt} \hat{H}_{jt})^{1-\alpha_j} \] (3.8)
and
\[ r_{\hat{H}_{jt}} = (1 - \eta_j) B_j \left[ (1 - v_{jt}) \hat{K}_{jt} \right]^{\eta_j} \left[ (1 - u_{jt}) \hat{H}_{jt} \right]^{-\eta_j} \] (3.9)
are rates of return on \( K \) and \( H \) type capital respectively in the potential host country; \( \phi \) and \( \psi \) are the marginal values of domestic and foreign investment in \( K \) type capital; \( \mu \) and \( \lambda \) are the marginal values of domestic and foreign investment in \( H \) type capital.

The necessary conditions of this dynamic optimization problem will be written as:

\[ \frac{\partial J_t}{\partial C_t} = C_t^{-\theta} e^{-\rho t} - \phi_t = 0 \] (3.10)
\[ \frac{\partial J_t}{\partial I_{\hat{K}_t}} = \psi_t - \phi_t = 0 \] (3.11)
\[ \frac{\partial J_t}{\partial I_{\hat{H}_t}} = \lambda_t - \mu_t = 0 \] (3.12)

\[ \dot{\phi}_t = -\frac{\partial J_t}{\partial \hat{K}_t} = -\phi_t \left[ \alpha A v_t (v_t \hat{K}_t)^{\alpha-1} (u_t \hat{H}_t)^{1-\alpha} - \delta \right] - \mu_t B \eta (1 - v_t) \left[ (1 - v_t) \hat{K}_t \right]^{\eta-1} \left[ (1 - u_t) \hat{H}_t \right]^{-\eta} \] (3.13)

\[ \dot{\psi}_t = -\frac{\partial J_t}{\partial \hat{K}_t} = -\psi_t [-\delta] - \phi_t r_{\hat{K}_{jt}} \] (3.14)

\[ \dot{\lambda}_t = -\frac{\partial J_t}{\partial \hat{H}_t} = -\lambda_t [-\delta] - \mu_t r_{\hat{H}_{jt}} \] (3.15)

\[ \dot{\mu}_t = -\frac{\partial J_t}{\partial \hat{H}_t} = -\phi_t \left[ (1 - \alpha) A (v_t \hat{K}_t)^{\alpha} (u_t \hat{H}_t)^{1-\alpha} a_t \right] - \mu_t \left\{ B (1 - \eta) (1 - u_t) \left[ (1 - v_t) \hat{K}_t \right]^{\eta} \left[ (1 - u_t) \hat{H}_t \right]^{-\eta} - \delta \right\} \] (3.16)
\[
\frac{\partial J_t}{\partial v_t} = \phi_t A (v_t K_t)^{\alpha-1} (u_t H_t)^{1-\alpha} \\
- \mu_t B \eta_t K_t \left[ (1 - v_t) K_t \right]^{\eta-1} \left[ (1 - u_t) H_t \right]^{1-\eta} = 0 \tag{3.17}
\]

\[
\frac{\partial J_t}{\partial u_t} = \phi_t (1 - \alpha) A (v_t K_t)^{\alpha} (u_t H_t)^{-\alpha} H_t \\
- \mu_t B (1 - \eta_t) H_t \left[ (1 - v_t) K_t \right]^{\eta} \left[ (1 - u_t) H_t \right]^{-\eta} = 0 \tag{3.18}
\]

Focus on the necessary conditions (3.11) and (3.12). Those are the necessary conditions resulting from the choice of foreign investment in K and H capital respectively. It follows from the equations that the choice of investment does not depend on the investment itself, which is known in the literature as a bang-bang control problem\(^3\) and is an outcome of the linearity of the Hamiltonian in investment.\(^4\) The equations (3.11) and (3.12) suggest that the marginal value of foreign investment will be positive if the marginal value of the additional unit of capital created abroad is higher than the marginal value of the additional unit of capital created at home, implying \(\psi > \phi\) and \(\lambda > \mu\). Similarly, the marginal value of foreign investment will be negative if the marginal value of each additional unit of capital created abroad is less than the marginal value of the additional unit of capital created at home. This result suggests that if \(\psi > \phi\) the country has an incentive to invest abroad in K type capital and if \(\lambda > \mu\) then the country has an incentive to invest abroad in H type capital. The opposite will be true if \(\psi < \phi\) or \(\lambda < \mu\), in which case it will be in the interests of the other country to become a foreign investor.\(^5\) Finally, there is a possibility that \(\psi = \phi\) and \(\lambda = \mu\), in which case marginal

\(^3\)Similar results are obtained as a part of the solution in the one-sector model with FDI and the Uzawa-Lucas model with FDI.

\(^4\)Note that it is possible to eliminate the bang-bang control problem by introducing, for example, quadratic adjustment costs of investment, however I want to keep the structure of the model as simple as possible to focus on the pure effects of FDI.

\(^5\)Here I will restrict my attention to the scenario where one of the countries can be identified as an investor and the other country as the recipient of the foreign investment.
values of domestic and foreign investment are the same, and the country is indifferent between staying under autarky and investing abroad. If we consider the scenario such that both countries are initially under autarky and then they decide to invest abroad, then faced with the equality conditions in (3.11) and (3.12) they will remain under autarky. However, starting with the inequality in (3.11) or in (3.12) and having initiated foreign investment, the country under consideration will eventually be faced with the equalization of the marginal values of domestic and foreign investment in the long-run. Given that the country is indifferent between staying under autarky and investing abroad under the equalization in (3.11) and (3.12) implies that having initiated foreign investment the country will continue its investment abroad even when (3.11) and (3.12) hold with equality.

Note that equalization of the values of the costate variables follows from the necessary conditions of the optimization problem. Suppose $\psi > \phi$. Combining (3.13) and (3.17) it follows that the growth rate of the costate variable $\phi$ will be written as:

$$\frac{\dot{\phi}_t}{\phi_t} = \delta - r_{K_t}$$

where $r_{K_t} = \alpha A(v\hat{K}_t)^{\alpha-1}(u\hat{H}_t)^{1-\alpha}$ is the rate of return on physical capital.

Similarly, it follows from (3.14) that the growth rate of the costate variable $\psi$ can be written as:

$$\frac{\dot{\psi}_t}{\psi_t} = \delta - \frac{\phi}{\psi} r_{K_{jt}}$$

It can be shown also that the growth rate of the costate variable $\psi_j$ can be expressed in terms of the growth rate of $\phi_j$,\(^6\) which is the shadow price of K type capital in the

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\(^6\)See next section for the detailed discussion.
host country. The growth rate of $\phi$ can be written as:

$$\frac{\dot{\phi}_t}{\phi_t} = \delta - r_{K_t}$$

It clearly follows that the growth rates of the shadow values of domestic and foreign investment are negative if the marginal products of capital are higher than $\delta$. If it’s more efficient to do intertemporal transfer of resources in the foreign country because of the higher value of $r_{K_j}$ compared to $r_{K_i}$, then $\psi$ will be declining faster than $\phi$ leading to the equalization of their values.

Note also that equalization of the values of the costate variables implies the equalization of the rates of return on capital across countries. I have already argued that it is possible to sustain foreign investment even under the equalization of (3.11) and (3.12) which is consistent with the equalization of the cross country rates of return. I will show in the next section that the solution with FDI under the equalization of the rates of return takes the form of the balanced growth path where all variables grow at the same constant rate.

In the meantime, based on the above discussion, I can establish that country $i$ will be defined as a home country and a foreign investor in K type capital if:

$$r_{K_i} < r_{K_j} \quad (3.19)$$

the opposite will be true if the above inequality is reversed.

Following the same logic implies that country $i$ will become a foreign investor in H type capital if:

$$r_{H_i} < r_{H_j} \quad (3.20)$$
I will return to the above conditions establishing the directions of the flow of investment once I have discussed the solution for the BGP in the presence of FDI. It will allow me to express conditions in (3.19) and (3.20) in terms of the underlying parameters of the model and discuss the importance of the technological differences across countries in establishing the directions for the flow of FDI from home to host country.

As I have already mentioned I will proceed with the solution where both domestic and foreign investments are positive and the rates of return are equalized across countries. Then I will discuss possible scenarios when the equalization of the rates of return no longer holds and the distinction between home and host countries becomes more important.

### 3.3 The Balanced Growth Path Solution

Having established that index i denotes the home country and index j denotes the host country, I can rewrite the present value Hamiltonian and the necessary conditions for each country separately.

For the home country the equations will be similar to the equations (3.10) – (3.18), with the only difference that having made the decision to invest abroad, the country under consideration is now defined as a home country and should be described using index i.

So, for the home country the present value Hamiltonian and the necessary conditions are:

\[
J_{it} = \frac{C_{it}^{1-\theta} - 1}{1 - \theta} e^{-\rho t} + \psi_{it}(I\hat{K}_{it} - \delta\hat{K}_{it}) + \lambda_{it}(I\hat{H}_{it} - \delta\hat{H}_{it})
\]

\[
+ \phi_{it} \left[ A_i(v_{it}\hat{K}_{it})^{\alpha_i}(u_{it}\hat{H}_{it})^{1-\alpha_i} - C_{it} - \delta\hat{K}_{it} - I\hat{K}_{it} + r_{K_{it}}\hat{K}_{it} \right]
\]

\[
+ \mu_{it} \left\{ B_i \left[ (1 - v_{it})\hat{K}_{it} \right]^{\eta_i} \left[ (1 - u_{it})\hat{H}_{it} \right]^{1-\eta_i} - \delta\hat{H}_{it} - I\hat{H}_{it} + r_{H_{it}}\hat{H}_{it} \right\}
\]
\[ \frac{\partial J_{it}}{\partial C_{it}} = C_{it}^{-\theta} e^{-\rho t} - \phi_{it} = 0 \] (3.21)

\[ \frac{\partial J_{it}}{\partial I_{K_{it}}} = \psi_{it} - \phi_{it} = 0 \] (3.22)

\[ \frac{\partial J_{it}}{\partial I_{H_{it}}} = \lambda_{it} - \mu_{it} = 0 \] (3.23)

\[ \dot{\phi}_{it} = -\frac{\partial J_{it}}{\partial K_{it}} = -\phi_{it} \left[ \alpha_i A_i (v_{it} K_{it})^{\alpha_i - 1} (u_{it} \hat{H}_{it})^{1-\alpha_i} \right] \]

\[ -\mu_{it} B_i \eta_{it} (1 - v_{it}) \left[ (1 - v_{it}) \hat{K}_{it} \right]^{n_{it} - 1} \left[ (1 - u_{it}) \hat{H}_{it} \right]^{1-\eta_{it}} = 0 \] (3.24)

\[ \dot{\psi}_{it} = -\frac{\partial J_{it}}{\partial K_{it}} = -\psi_{it} [-\delta] - \phi_{it} r_{K_{jt}} \] (3.25)

\[ \dot{\lambda}_{it} = -\frac{\partial J_{it}}{\partial H_{it}} = -\lambda_{it} [-\delta] - \mu_{it} r_{H_{jt}} \] (3.26)

\[ \dot{\mu}_{it} = -\frac{\partial J_{it}}{\partial H_{it}} = -\phi_{it} \left[ (1 - \alpha_i) A_i (v_{it} K_{it})^{\alpha_i} (u_{it} \hat{H}_{it})^{-\alpha_i} u_{it} \right] \]

\[ -\mu_{it} \left\{ B_i (1 - \eta_{it}) (1 - u_{it}) \left[ (1 - v_{it}) \hat{K}_{it} \right]^{n_{it}} \left[ (1 - u_{it}) \hat{H}_{it} \right]^{-\eta_{it}} - \delta \right\} = 0 \] (3.27)

\[ \frac{\partial J_{it}}{\partial v_{it}} = \phi_{it} \alpha_i A_i \hat{K}_{it} (v_{it} \hat{K}_{it})^{\alpha_i - 1} (u_{it} \hat{H}_{it})^{1-\alpha_i} \]

\[ -\mu_{it} B_i \eta_{it} \hat{K}_{it} \left[ (1 - v_{it}) \hat{K}_{it} \right]^{n_{it} - 1} \left[ (1 - u_{it}) \hat{H}_{it} \right]^{1-\eta_{it}} = 0 \] (3.28)

\[ \frac{\partial J_{it}}{\partial u_{it}} = \phi_{it} (1 - \alpha_i) A_i (v_{it} \hat{K}_{it})^{\alpha_i} (u_{it} \hat{H}_{it})^{-\alpha_i} \hat{H}_{it} \]

\[ -\mu_{it} B_i (1 - \eta_{it}) \hat{H}_{it} \left[ (1 - v_{it}) \hat{K}_{it} \right]^{n_{it}} \left[ (1 - u_{it}) \hat{H}_{it} \right]^{-\eta_{it}} = 0 \] (3.29)

Note that the present value Hamiltonian and the first order conditions are written for the general case where all investments are positive. Within this framework I can identify two cases: when the home country invests only in K type capital abroad and when the home country invests only in H type capital abroad. In the first case if the country invests
only in K type capital I will get $I_{\hat{H}_it} = 0$, $\hat{H} = \bar{H}$ and $\bar{H} = 0$.

Similarly, when the home country invests only in H type capital, I will get $I_{\hat{K}_it} = 0$, $\hat{K} = \bar{K}$ and $\bar{K} = 0$.

The equalization of the values of costate variables in equations (3.22) and (3.23) leads to the simplification of the above necessary conditions of the optimal control problem.

Taking the time derivative of (3.21) leads to the following equation for the growth rate of consumption:

$$\frac{\dot{C}_i}{C_i} = -\frac{1}{\theta} \left[ \frac{\dot{\phi}_i}{\phi_i} + \rho \right]$$

(3.30)

From (3.24) the growth rate of the costate variable $\phi$ will be written as:

$$\frac{\dot{\phi}_i}{\phi_i} = \delta - \alpha_i A_i \left[ \frac{v_i \hat{K}_i}{u_i \hat{H}_i} \right]^{\alpha_i - 1} v_i - (1 - v_i) \frac{\mu_i}{\phi_i} \eta_i \frac{(1 - v_i) \hat{K}_i}{(1 - u_i) \hat{H}_i}^{\eta_i - 1}$$

(3.31)

From the equation (3.28) it follows that:

$$\alpha_i A_i \left[ \frac{v_i \hat{K}_i}{u_i \hat{H}_i} \right]^{\alpha_i - 1} = \frac{\mu_i}{\phi_i} \frac{(1 - v_i) \hat{K}_i}{(1 - u_i) \hat{H}_i}^{\eta_i - 1}$$

(3.32)

Using the above relationship to simplify (3.31) I will obtain the growth rate of the costate variable $\phi$ as follows:

$$\frac{\dot{\phi}_i}{\phi_i} = \delta - \alpha_i A_i \left[ \frac{v_i \hat{K}_i}{u_i \hat{H}_i} \right]^{\alpha_i - 1}$$

(3.33)

where

$$\alpha_i A_i \left[ \frac{v_i \hat{K}_i}{u_i \hat{H}_i} \right]^{\alpha_i - 1} = r_{K_it}$$

(3.34)

Similarly using equations (3.27) and (3.29) I can solve for the growth rate of the
costate variable $\mu$, which can be written as:

$$\frac{\dot{\mu}_i}{\mu_i} = \delta - \frac{\phi_i}{\mu} (1 - \alpha_i) A_i \left[ \frac{v_i K_i}{u_i H_i} \right]^{\alpha_i}$$  \hspace{1cm} (3.35)

Note that $\frac{\phi_i}{\mu_i}$ represents the ratio of the costate variables, where $\phi$ is the shadow value of domestic K capital and $\mu$ is the shadow value of domestic H type capital. Therefore this ratio of the costate variables can be defined as the inverse of the internal price of H type capital in terms of K type capital or alternatively the inverse of the internal price of good H in terms of good Y in country i. Therefore I can rewrite $\frac{\dot{\mu}_i}{\mu_i}$ as:

$$\frac{\dot{\mu}_i}{\mu_i} = \delta - \frac{(1 - \alpha_i) A_i \left[ \frac{v_i K_i}{u_i H_i} \right]^{\alpha_i}}{P_i}$$  \hspace{1cm} (3.36)

The equalization of the marginal values followed from the equations (3.22) and (3.23) implies that $P_i$ should be constant.

Since $P_i$ is defined as the ratio of $\frac{\mu_i}{\phi_i}$, its growth rate will be equal to the difference in the growth rates of these costate variables. Equalization of the growth rates of these costate variables under the condition that $P_i$ is constant will lead to the solution for the ratio of $\frac{v_i K_i}{u_i H_i}$ in country i:

$$\delta - \alpha_i A_i \left[ \frac{v_i K_i}{u_i H_i} \right]^{\alpha_i-1} = \delta - \frac{(1 - \alpha_i) A_i \left[ \frac{v_i K_i}{u_i H_i} \right]^{\alpha_i}}{P_i}$$

$$\frac{v_i K_i}{u_i H_i} = P_i \frac{\alpha_i}{1 - \alpha_i}$$  \hspace{1cm} (3.37)

Finally, acknowledging the fact that the internal price of good H in terms of good Y is consistent with the autarkic price level in the economy and using (3.33) and (3.37)
allows one to write the growth rate of consumption in country $i$ as follows:

\[
\frac{\dot{C}_i}{C_i} = \frac{1}{\theta} \left\{ \alpha_i A_i \left[ P_i \frac{\alpha_i}{1 - \alpha_i} \right]^{\alpha_i - 1} \right. - \delta - \rho \bigg\} \tag{3.38}
\]

or alternatively

\[
\frac{\dot{C}_i}{C_i} = \frac{1}{\theta} \left[ A_i \alpha_i^{\alpha_i} (1 - \alpha_i)^{1-\alpha_i} P_i^{\alpha_i - 1} - \delta - \rho \right] \tag{3.39}
\]

where the value of $P_i$ is determined by equation (3.4) and can be written as:

\[
P_i = \left( \frac{A_i \alpha_i^{\alpha_i} (1 - \alpha_i)^{1-\alpha_i}}{B_i \eta_i^{\eta_i} (1 - \eta_i)^{1-\eta_i}} \right)^{\frac{1}{1-\alpha_i + \eta_i}} \tag{3.40}
\]

For the host country I will start by writing down the production functions for goods $Y$ and $H$ in the presence of FDI:

\[
Y_{jt} = A_j (v_{jt} \hat{K}_{jt})^{\alpha_j} (u_{jt} \hat{H}_{jt})^{1-\alpha_j} \tag{3.41}
\]

\[
\hat{H}_{jt} + \delta \hat{H}_{jt} = B_j \left[ (1 - v_{jt}) \hat{K}_{jt} \right]^{\eta_j} \left[ (1 - u_{jt}) \hat{H}_{jt} \right]^{1-\eta_j} \tag{3.42}
\]

Country $j$ spends its output of sector one on domestic consumption, domestic investment in physical capital and on the payment of returns on foreign investment received from the home country. Therefore the resource constraint in sector one will be given by:

\[
Y_{jt} = C_{jt} + I_{K_{jt}} - I_{K_{it}} + r_{K_{jt}} \hat{K}_{jt} \tag{3.43}
\]

I can simplify equation (3.43) using the notations introduced in (1.7) and (1.8), so that I can write:
\[ Y_{jt} = C_{jt} + \dot{K}_j - \dot{K}_i + \delta \dot{K}_j - \delta \dot{K}_i + r_{\dot{K}_{jt}} \dot{K}_{it} \]  
(3.44)

\[ Y_{jt} = C_{jt} + \dot{K}_j + \delta \dot{K}_j + r_{\dot{K}_{jt}} \dot{K}_{it} \]  
(3.45)

\[ \dot{K}_j = Y_{jt} - C_{jt} - r_{\dot{K}_{jt}} \dot{K}_{it} - \delta \dot{K}_j \]  
(3.46)

where equation (3.46) is expressed as an accumulation condition for \( \bar{K} \) capital, defined as the K type capital stock owned by the host country.

Similarly, in the second sector the resource constraint in country \( j \) will be:

\[ \dot{H}_{jt} = B_j \left[ (1 - v_{jt}) \dot{K}_{jt} \right]^{\eta_j} \left[ (1 - u_{jt}) \dot{H}_{jt} \right]^{1-\eta_j} - r_{\dot{H}_{jt}} \dot{H}_{it} - \delta \dot{H}_{jt} \]  
(3.47)

Given the resource constraints (3.46) and (3.47) I can write the present value Hamiltonian and the necessary conditions for country \( j \) as follows:

\[ J_{jt} = C_{jt}^{\frac{1-\theta}{1-\theta} - 1} e^{-\rho t} + \phi_{jt} \left[ A_j (v_{jt} \dot{K}_{jt})^{\alpha_j} (u_{jt} \dot{H}_{jt})^{1-\alpha_j} - C_{jt} - \delta \dot{K}_{jt} - r_{\dot{K}_{jt}} \dot{K}_{it} \right] \]

\[ + \mu_{jt} \left\{ B_j \left[ (1 - v_{jt}) \dot{K}_{jt} \right]^{\eta_j} \left[ (1 - u_{jt}) \dot{H}_{jt} \right]^{1-\eta_j} - r_{\dot{H}_{jt}} \dot{H}_{it} - \delta \dot{H}_{jt} \right\} \]

\[ \frac{\partial J_{jt}}{\partial C_{jt}} = C_{jt}^{\frac{1-\theta}{1-\theta} - 1} e^{-\rho t} - \phi_{jt} = 0 \]  
(3.48)

\[ \dot{\phi}_{jt} = - \frac{\partial J_{jt}}{\partial \dot{K}_{jt}} = - \phi_{jt} \left[ \alpha_j A_j (v_{jt} \dot{K}_{jt})^{\alpha_j-1} (u_{jt} \dot{H}_{jt})^{1-\alpha_j} v_{jt} - \delta \right] \]

\[ - \mu_{jt} B_j \eta_j (1 - v_{jt}) \left[ (1 - v_{jt}) \dot{K}_{jt} \right]^{\eta_j-1} \left[ (1 - u_{jt}) \dot{H}_{jt} \right]^{1-\eta_j} \]  
(3.49)

\[ \dot{\mu}_{jt} = - \frac{\partial J_{jt}}{\partial \dot{H}_{jt}} = - \phi_{jt} \left[ (1 - \alpha_j) A_j (v_{jt} \dot{K}_{jt})^{\alpha_j} (u_{jt} \dot{H}_{jt})^{-\alpha_j} u_{jt} \right] \]
$$-\mu_{jt} \left\{ B_j (1 - \eta_j) (1 - u_{jt}) \left[ (1 - v_{jt}) \hat{K}_{jt} \right]^{\eta_j} \left[ (1 - u_{jt}) \hat{H}_{jt} \right]^{-\eta_j} - \delta \right\} \tag{3.50}$$

$$\frac{\partial J_{jt}}{\partial v_{jt}} = \phi_{jt} \left[ \alpha_j A_j \hat{K}_{jt} (v_{jt} \hat{K}_{jt})^{\alpha_j - 1} (u_{jt} \hat{H}_{jt})^{1 - \alpha_j} \right]$$

$$-\mu_{jt} B_j n_{jt} \hat{K}_{jt} \left[ (1 - v_{jt}) \hat{K}_{jt} \right]^{\eta_j - 1} \left[ (1 - u_{jt}) \hat{H}_{jt} \right]^{1 - \eta_j} = 0 \tag{3.51}$$

$$\frac{\partial J_{jt}}{\partial u_{jt}} = \phi_{jt} \left[ (1 - \alpha_j) A_j (v_{jt} \hat{K}_{jt})^{\alpha_j} (u_{jt} \hat{H}_{jt})^{-\alpha_j} \hat{H}_{jt} \right]$$

$$-\mu_{jt} B_j (1 - \eta_{jt}) \hat{H}_{jt} \left[ (1 - v_{jt}) \hat{K}_{jt} \right]^{\eta_j} \left[ (1 - u_{jt}) \hat{H}_{jt} \right]^{-\eta_j} = 0 \tag{3.52}$$

Following similar steps for country j I can express the growth rate of consumption in country j as:

$$\frac{\dot{C}_j}{C_j} = -\frac{1}{\theta} \left[ \frac{\dot{\phi}_j}{\phi_j} + \rho \right] \tag{3.53}$$

Using equations (3.49) and (3.51) after some manipulations I can write the growth rate of the costate variable \( \phi_j \) as:

$$\frac{\dot{\phi}_j}{\phi_j} = \delta - \alpha_j A_j \left[ \frac{v_j \hat{K}_j}{u_j \hat{H}_j} \right]^{\alpha_j - 1} \tag{3.54}$$

Note that using the definition in (3.8) I can rewrite the growth rate of \( \phi_j \) as:

$$\frac{\dot{\phi}_j}{\phi_j} = \delta - r \hat{K}_j \tag{3.55}$$

Let us for a moment focus on equation (3.25). It is straightforward to see that under the assumtion that \( \psi_i = \phi_i \) (3.25) reduces to:

$$\frac{\dot{\psi}_i}{\psi_i} = \delta - r \hat{K}_j$$
Comparison with (3.55) reveals that both equations are equivalent, which means that I can express the growth rate of consumption in country \( j \) as:

\[
\frac{\dot{C}_j}{C_j} = -\frac{1}{\theta} \left[ \frac{\dot{\psi}_i}{\psi_i} + \rho \right]
\]  

(3.56)

The equivalence between \( \phi_j \) and \( \psi_i \) is intuitively straightforward. In the absence of any asymmetric information, the marginal value of foreign investment in the home country should be the same as the marginal value of domestic investment in the host country.

Moreover, using equation (3.22) and (3.34) I can rewrite the growth rate of consumption in country \( j \) as:

\[
\frac{\dot{C}_j}{C_j} = \frac{1}{\theta} \left[ r_{ki} - \delta - \rho \right]
\]  

(3.57)

Finally, substituting (3.37) in the expression for \( r_{ki} \), I will write the growth rate of consumption in country \( j \) as:

\[
\frac{\dot{C}_j}{C_j} = \frac{1}{\theta} \left[ A_i \alpha_i (1 - \alpha_i)^{1 - \alpha_i} P_i^{\alpha_i - 1} - \delta - \rho \right]
\]  

(3.58)

where the value of \( P_i \) is determined again by equation (3.40)

Similarly, using equations (3.50) and (3.52) I can write that:

\[
\frac{\dot{\mu}_j}{\mu_j} = \frac{\phi_j}{\mu_j} A_j (1 - \alpha_j) \left( \frac{v_j K_j}{u_j H_j} \right)^{\alpha_j}
\]

Recognizing the fact that on the BGP the marginal values of domestic and foreign investment in \( H \) type capital should be equalized as well and using equalities in (3.22)
and (3.23) allows one to rewrite the above expression as:

\[
\frac{\dot{\mu}_j}{\mu_j} = \frac{\phi_i}{\mu_i} A_j (1 - \alpha_j) \left( \frac{v_j \dot{K}_j}{u_j \dot{H}_j} \right)^{\alpha_j} \tag{3.59}
\]

Finally from the equalization of the growth rates of \(\phi_j\) and \(\mu_j\) from equations (3.54) and (3.59) I get:

\[
\frac{v_j \dot{K}_j}{u_j \dot{H}_j} = P_i \frac{\alpha_j}{(1 - \alpha_j)} \tag{3.60}
\]

where again \(P_i\) is determined by (3.40).

Since \(v_i\) and \(u_i\) as well as \(v_j\) and \(u_j\) are constant on the BGP, then we can write: \(\dot{K}_i = \dot{H}_i\) and \(\dot{K}_j = \dot{H}_j\). Also, from the equalization of the rates of return and growth rates of consumption across countries follows the equalization of the growth rates of \(Y_i\), \(\dot{K}_i\), \(\dot{H}_i\), \(Y_j\), \(\dot{K}_j\), \(\dot{H}_j\), such that I can write the common growth rate of this world economy as:

\[
\gamma = \frac{1}{\theta} (\Omega - \delta - \rho) \tag{3.61}
\]

where

\[
\Omega = A_i^{\frac{\eta_i}{1 - \alpha_i + \eta_i}} B_i^{\frac{1 - \alpha_i}{1 - \alpha_i + \eta_i}} \alpha_i^{\frac{\alpha_i \eta_i}{1 - \alpha_i + \eta_i}} (1 - \alpha_i)^{\frac{(1 - \alpha_i) \eta_i}{1 - \alpha_i + \eta_i}} \eta_i^{\frac{(1 - \alpha_i) \eta_i}{1 - \alpha_i + \eta_i}} (1 - \eta_i)^{\frac{(1 - \alpha_i) (1 - \eta_i)}{1 - \alpha_i + \eta_i}}
\]

and (3.61) is derived by substituting (3.40) into (3.39).

Therefore the solution under the equalization of the marginal values of domestic and foreign investment takes the form of the BGP where variables are either constant or grow at a constant rate.

Comparison of the common long-run growth rate in the presence of FDI in (3.61) with the growth rate under autarky in equation (3.3) reveals that both are identical for
country \( i \). So, there are no long-run growth effects for the home country.

For the host country the growth effects may or may not be present depending on the technological differences across countries. One possibility is that both countries have the same level of technology captured by the TFP parameters and factor share parameters such that: \( A_i = A_j, B_i = B_j, \alpha_i = \alpha_j, \) and \( \eta_i = \eta_j \). This would imply that both countries have the same growth under autarky and they have the same long-run growth rate in the presence of FDI. In this case FDI may have only transitional effects on growth. As I argued above if countries have identical rates of return under autarky then neither country has an incentive to initiate FDI and countries will remain under autarky. The only reason for one of the countries to start investing in the other country under the scenario of equal technology is that the host country deviates from its autarkic equilibrium by having more of one type capital compared to another (for example by having more \( H \) than \( K \) due to a natural disaster for instance). This situation will lead to a temporarily higher rate of return on \( K \) type capital in the host country, which can serve as an incentive for the home country to initiate investment in \( K \) type capital abroad. Eventually, however, the host country will restore its stock of \( K \) type capital through both domestic and foreign investment and return to its long-run equilibrium level of the ratio of \( K \) to \( H \) capital and to its long-run growth rate.

More interesting, however, is the scenario where countries do differ in terms of their technology. In this case the autarkic growth rate of the host country will be different from the autarkic growth rate of the home country, which implies that there are in fact growth effects for the host country because, as I have showed above, the common growth rate in the presence of FDI will be equal to the autarkic growth rate of the home country.

If the host country had a higher rate of return under autarky compared to the home country then it implies that the growth rate of consumption was higher under autarky
than in the presence of FDI for the host country. The equalization of the growth rates of all the variables under autarkic equilibrium and the existence of the common growth rate with FDI implies that the host country will experience a reduction in its long-run growth rate once open to FDI.

The long-run growth effects of FDI discussed above are consistent with the empirical evidence on the effects of FDI. As I have discussed earlier in the paper, the empirical evidence does not support the existence of a systematic effect of FDI on growth. As my results suggest, the countries with the same level of technology will not experience any long-run growth effects from FDI.

On the other hand Alfaro (2003) in her study of the sectorial effects of FDI shows that growth effects of FDI tend to be negative for the primary sector. Her theoretical arguments explaining the negative effects of FDI in the primary sector are linked to the lower ability of the primary sector to benefit from the positive spillovers associated with FDI. In my model I have eliminated the spillover mechanism by construction to focus on the pure effects of FDI; however, the negative effect of FDI on growth in the host country can still be present if countries are technologically different.

So, my next step is to explore the nature of the technological differences across countries.

For countries to be technologically different and have the BGP solution in the presence of FDI, two conditions should be satisfied:

- In the autarky the marginal product of capital should be higher in the host country compared to the home country for FDI to flow from home country to host:

\[ r_i < r_j \]
On the BGP in the presence of FDI the marginal products of capital should be equalized across countries:

\[ r_i = r_j \]

I will first focus on the rates of return on K type capital, explicitly assuming that the host country receives FDI only in the sector producing good Y.

The equalization of the marginal products of K type capital can be written as:

\[ \alpha_i A_i \left( \frac{v_i K_i}{u_i H_i} \right)^{\alpha_i - 1} = \alpha_j A_j \left( \frac{v_j K_j}{u_j H_j} \right)^{\alpha_j - 1} \]

In the presence of FDI we have:

\[ \frac{v_i K_i}{u_i H_i} = P_i \frac{\alpha_i}{1 - \alpha_i} \]

and

\[ \frac{v_j K_j}{u_j H_j} = P_j \frac{\alpha_j}{1 - \alpha_j} \]

Substituting these expressions into the expressions for the rates of return on capital gives:

\[ A_i \alpha_i^{\alpha_i} (1 - \alpha_i)^{1 - \alpha_i} P_i^{\alpha_i - 1} = A_j \alpha_j^{\alpha_j} (1 - \alpha_j)^{1 - \alpha_j} P_i^{\alpha_j - 1} \]

where \( P_i \) is given by equation (3.40)

From the above expression it follows:

\[ \frac{A_i \alpha_i^{\alpha_i} (1 - \alpha_i)^{1 - \alpha_i}}{A_j \alpha_j^{\alpha_j} (1 - \alpha_j)^{1 - \alpha_j}} = P_i^{\alpha_j - \alpha_i} \tag{3.62} \]

From the condition that in autarky the marginal product of capital in the host country
j should be higher than the marginal product of capital in the home country it follows that:

$$\alpha_i A_i \left( \frac{v_i K_i}{u_i H_i} \right)^{\alpha_i - 1} < \alpha_j A_j \left( \frac{v_j K_j}{u_j H_j} \right)^{\alpha_j - 1}$$

In autarky we have:

$$\frac{v_i K_i}{u_i H_i} = P_i \frac{\alpha_i}{1 - \alpha_i}$$

and

$$\frac{v_j K_j}{u_j H_j} = P_j \frac{\alpha_j}{1 - \alpha_j}$$

Substituting these expressions into the expressions for the rates of return on capital under autarky I get:

$$A_i \alpha_i (1 - \alpha_i)^{1 - \alpha_i} P_i^{\alpha_i - 1} < A_j \alpha_j (1 - \alpha_j)^{1 - \alpha_j} P_j^{\alpha_j - 1}$$

Then I can rewrite it as:

$$\frac{A_i \alpha_i (1 - \alpha_i)^{1 - \alpha_i}}{A_j \alpha_j (1 - \alpha_j)^{1 - \alpha_j}} P_i^{\alpha_i - 1} < P_j^{\alpha_j - 1}$$

Using (3.62) I can rewrite the above expression as:

$$P_i^{\alpha_j - \alpha_i} P_i^{\alpha_i - 1} < P_j^{\alpha_j - 1}$$

Simplifying these expressions leads to:

$$P_i^{\alpha_j - 1} < P_j^{\alpha_j - 1}$$
Given that $0 < \alpha_j < 1$, it follows that

$$P_i > P_j$$

In terms of the parameter values this means that:

$$\left( \frac{A_i \alpha_i^{\alpha_i}(1 - \alpha_i)^{1-\alpha_i}}{B_i \eta_i^{\eta_i}(1 - \eta_i)^{1-\eta_i}} \right)^{\frac{1}{1-\alpha_i+\eta_i}} > \left( \frac{A_j \alpha_j^{\alpha_j}(1 - \alpha_j)^{1-\alpha_j}}{B_j \eta_j^{\eta_j}(1 - \eta_j)^{1-\eta_j}} \right)^{\frac{1}{1-\alpha_j+\eta_j}}$$

Note that under the simplifying assumption that the factor share parameters equalize across sectors, the above inequality reduces to:

$$\frac{A_i}{B_i} > \frac{A_j}{B_j}$$

This simplification suggests that if FDI is considered between countries with different levels of technology only in the FDI conducting sector, then the home country should have a higher level of technology than the host country, at least in terms of its TFP parameter.

A similar conclusion will be reached if I explicitly assume that the host country receives FDI only in the sector producing good H and that technological differences across countries apply only to that sector.

In particular, under autarky it will imply:

$$B_i \eta_i^{\eta_i}(1 - \eta_i)^{1-\eta_i} P_i^{\eta_i} < B_j \eta_j^{\eta_j}(1 - \eta_j)^{1-\eta_j} P_j^{\eta_j}$$
On the other hand in the presence of FDI on the BGP the following should hold:

\[ B_i \eta_i (1 - \eta_i)^{1-\eta_i} P_i^{\eta_i} = B_j \eta_j (1 - \eta_j)^{1-\eta_j} P_j^{\eta_j} \]

Following the same steps as in the previous case I will get that for countries to be technologically different and to generate FDI in H type capital the following condition should hold:

\[
\left( \frac{A_i \alpha_i (1 - \alpha_i)^{1-\alpha_i}}{B_i \eta_i (1 - \eta_i)^{1-\eta_i}} \right)^{\frac{1}{1-\alpha_i+\eta_i}} < \left( \frac{A_j \alpha_j (1 - \alpha_j)^{1-\alpha_j}}{B_j \eta_j (1 - \eta_j)^{1-\eta_j}} \right)^{\frac{1}{1-\alpha_j+\eta_j}}
\]

Under the simplifying assumption that factor share parameters are equal across sectors the above condition reduces to:

\[
\frac{A_i}{B_i} < \frac{A_j}{B_j}
\]

which, under the assumption that technological differences only apply to H sector, in turn implies that \( B_i > B_j \) and that the home country has a higher level of technology compared to the host country in the FDI conducting sector.

The above discussion suggests that, under the condition that countries are technologically different, FDI can be conducted only in one of the sectors such that: if

\[
\left( \frac{A_i \alpha_i (1 - \alpha_i)^{1-\alpha_i}}{B_i \eta_i (1 - \eta_i)^{1-\eta_i}} \right)^{\frac{1}{1-\alpha_i+\eta_i}} > \left( \frac{A_j \alpha_j (1 - \alpha_j)^{1-\alpha_j}}{B_j \eta_j (1 - \eta_j)^{1-\eta_j}} \right)^{\frac{1}{1-\alpha_j+\eta_j}}
\]

then the home country invests only in K type capital, since it has a higher level of technology in the sector producing K type capital.
If, however,

\[
\left( \frac{A_i^{\alpha_i}(1 - \alpha_i)^{1-\alpha_i}}{B_i^{\eta_i}(1 - \eta_i)^{1-\eta_i}} \right)^{\frac{1}{1-\alpha_i+\eta_i}} < \left( \frac{A_j^{\alpha_j}(1 - \alpha_j)^{1-\alpha_j}}{B_j^{\eta_j}(1 - \eta_j)^{1-\eta_j}} \right)^{\frac{1}{1-\alpha_j+\eta_j}}
\]

then the home country will conduct FDI only in H type capital, because it will have a higher level of technology in the H sector.

So, even though the host country receives a flow of investment from the home country that has superior technology in the sector conducting FDI, the host country still experiences a decline in its growth rate, because of the reduction in the marginal product of capital. The growth effects of the FDI, therefore, substantially differ from the growth effects of trade in factors of production discussed by Yenokyan, Seater and Arabshahi (2011). In their model countries trade goods (K and H) that are factors of production. Essentially, trade takes the form of the exchange in the flow of investment across countries, with each country specializing in the production of one of the goods (Y or H) on the BGP according to their pattern of comparative advantage. As Yenokyan, Seater and Arabshahi show, if the BGP exists then trade increases the growth rates of both trading partners because countries are able to substitute their own production by more efficient production of their trading partner. Under the setup of the current model with one-way flow of investment through FDI, the home country does not experience any growth effects, while effects for the host country can be either negative if country has a lower level of technology (as captured by the TFP parameter) in the FDI receiving sector compared to the home country; or the host country does not experience any long-run growth effects due to FDI if it has the same level of technology as the home country.
3.4 Transitional Dynamics

The central idea behind the analysis of the transitional behavior of both countries is that the deviation of the ratio of both types of capital from its BGP value violates the irreversibility constraints and therefore changes the pattern of production and the behavior of both countries along the transition.\(^7\) There are two possible cases to consider based on the directions of the deviation of the ratio of both types of capital; a country can deviate from the BGP value of its ratio of K to H by having less K than H or having less H than K. Note that the two cases are not symmetric, because the output of the H sector is used only for investment in H type capital, whereas the output of the Y sector is used for the investment in K capital and consumption. As the results suggest, the type of the good the country is investing in matters in determining the pattern of production and therefore transitional behavior of both countries. For simplicity I will focus on the case when deviations of the ratio of K to H capital occurs only in the host country.

Suppose the host country, which is the recipient of the FDI from the home country, deviates from the BGP ratio of K to H capital by having less K than H. The lower level of the total K type capital in the foreign country compared to the BGP value implies that the rate of return on K type capital is higher in the foreign country compared to the BGP value of the common rate of return across both countries. So, if \(\frac{\nu_j^i \dot{K}_j}{\hat{u}_j H_j} < P_i \frac{\alpha_j}{(1-\alpha_j)}\) then the foreign country has an incentive to set investment in H type capital equal to zero, such that \(I_{Hjt} = 0\) and accumulate K type capital only.

In transition, under the condition that the rate of return on K type capital is higher in the host country, the home country also shuts down its domestic investment in K type capital, such that \(I_{Kjt} = 0\), which implies: \(\frac{\dot{K}_j}{K_{jt}} = -\delta\).

\(^7\)I have already discussed the idea behind the irreversibility constraints in the first chapter of my dissertation. For more details see Barro and Sala-i-Martin (2004).
Note, however, that under the assumption that there are no deviations in the K to H ratio from its BGP value in the home country, shutting down domestic investment in K-type capital only will lead to the deviation of the home country from the BGP. Depreciation of K type capital and accumulation of H type capital in the home country will lead to unbalanced growth in the home country. To keep the home country on the BGP and to focus on the transitional behavior in the host country, I will assume that the home country shuts down investment in both types of capital and only invests abroad in K type capital.\(^8\) Under the assumption that the depreciation rates are the same for both types of capital, shutting down investment in both types of capital would imply that the ratio of K to H capital and the relative price of good H in terms of good Y will stay constant in the home country.

Under the above assumptions the new budget constraint for the home country will take the following form:

\[
\dot{\bar{K}}_{it} = A_{ii} \bar{K}_{it}^{\alpha_i} \bar{H}_{it}^{1-\alpha_i} - C_{it} - \delta \bar{K}_{it} + r_{Kjt} \bar{K}_{it} \tag{3.63}
\]

Then the present value Hamiltonian and the set of the necessary conditions for the home country takes the following form:

\[
J_{it} = \frac{C_{it}^{1-\theta}}{1-\theta} e^{-\rho t} + \phi_{it} \left[ A_{ii} \bar{K}_{it}^{\alpha_i} \bar{H}_{it}^{1-\alpha_i} - C_{it} - \delta \bar{K}_{it} + r_{Kjt} \bar{K}_{it} \right]
\]

\[
\frac{\partial J_{it}}{\partial C_{it}} = \frac{C_{it}^{1-\theta}}{1-\theta} e^{-\rho t} - \phi_{it} = 0 \tag{3.64}
\]

\[
\phi_{it} = -\frac{\partial J_{it}}{\partial \bar{K}_{it}} = -\phi_{it} \left[ r_{Kjt} - \delta \right] \tag{3.65}
\]

\(^8\)See Appendix A for the details.
\[
\lim_{t \to \infty} \phi_{it} \bar{K}_{it} = 0 \quad (3.66)
\]

From (3.65) I can write:
\[
\frac{\dot{\phi}_{it}}{\phi_{it}} = \delta - r_{Kjt}
\]

Substituting the growth rate of costate variable \( \phi \) into the growth rate of consumption I get:
\[
\frac{\dot{C}_{it}}{C_{it}} = \frac{1}{\theta} \left[ r_{Kjt} - \delta - \rho \right] \quad (3.67)
\]

For the host country the condition that it has more H capital than K leads to the following resource constraint:
\[
\dot{\bar{K}}_{jt} = Y_{jt} - C_{jt} - r_{Kjt} \bar{K}_{it} - \delta \bar{K}_{jt} \quad (3.68)
\]

Note that under the scenario that the host country has more H capital than K compared to its BGP value, each country operates only one sector of production, such that the setup of the model essentially becomes similar to the setup of a one-sector endogenous growth model.

Therefore, I can write the present value Hamiltonian and the set of the necessary conditions for the host country as:
\[
J_{jt} = \frac{C_{jt}^{1-\theta}}{1 - \theta} e^{-\rho t} + \phi_{jt} \left[ A_j \dot{K}_j^{\alpha_j} H_{jt}^{1-\alpha_j} - C_{jt} - r_{Kjt} \bar{K}_{it} - \delta \bar{K}_{jt} \right]
\]
\[
\frac{\partial J_{jt}}{\partial C_{jt}} = C_{jt}^{-\theta} e^{-\rho t} - \phi_{jt} = 0 \quad (3.69)
\]
\[
\dot{\phi}_{jt} = -\frac{\partial J_{jt}}{\partial \bar{K}_{jt}} = -\phi_{jt} \left[ \alpha_j A_j \dot{K}_j^{\alpha_j - 1} H_{jt}^{1-\alpha_j} - \delta \right] \quad (3.70)
\]

Similar to the solution for the home country, the growth rate of costate variable \( \phi \) in
country j will be given as:

$$\frac{\dot{\phi}_{jt}}{\phi_{jt}} = \delta - \alpha_j A_j \hat{K}_{jt}^{\alpha_j - 1} H_{jt}^{1 - \alpha_j}$$

Substituting the above expression into the growth rate of consumption I will get:

$$\frac{\dot{C}_{jt}}{C_{jt}} = \frac{1}{\theta} \left[ \alpha_j A_j \hat{K}_{jt}^{\alpha_j - 1} H_{jt}^{1 - \alpha_j} - \delta - \rho \right]$$

(3.71)

Further solution provides dynamic equations for both home and host country as follows.

**Home country:**

$$\frac{\dot{K}_{it}}{K_{it}} = -\delta$$

$$\frac{\dot{H}_{it}}{H_{it}} = -\delta$$

$$\frac{\dot{C}_{it}}{C_{it}} = \frac{1}{\theta} \left[ \alpha_j A_j \left( \frac{\hat{K}_{jt}}{H_{jt}} \right)^{\alpha_j - 1} - \delta - \rho \right]$$

$$\frac{\dot{K}_{it}}{K_{it}} = A_i \left( \frac{\hat{K}_{it}}{H_{it}} \right)^{\alpha_i} H_{it} \frac{\hat{K}_{jt}}{H_{jt}} - \frac{\dot{C}_{it}}{C_{it}} + \alpha_j A_j \left( \frac{\hat{K}_{jt}}{H_{jt}} \right)^{\alpha_j - 1} - \delta$$

**Host country:**

$$\frac{\dot{H}_{jt}}{H_{jt}} = -\delta$$

$$\frac{\dot{C}_{jt}}{C_{jt}} = \frac{1}{\theta} \left[ \alpha_j A_j \left( \frac{\hat{K}_{jt}}{H_{jt}} \right)^{\alpha_j - 1} - \delta - \rho \right]$$

$$\frac{\dot{K}_{jt}}{K_{jt}} = A_j \left( \frac{\hat{K}_{jt}}{H_{jt}} \right)^{\alpha_j - 1} - \frac{\dot{C}_{jt}}{C_{jt}} - \delta \left( 1 - \frac{\hat{K}_{it} H_{jt}}{\hat{K}_{jt} K_{jt}} \right)$$
The interdependence of the dynamic behavior of both countries follows directly from
the above equations and requires that the transitional paths of both countries are ana-
lyzed together.

The dynamic behavior of this world economy, consisting of two countries, can be
described in terms of the following ratios: \( \frac{\hat{K}_{it}}{H_{jt}}, \frac{\hat{K}_{jt}}{H_{jt}}, \frac{c_{it}}{K_{it}} \) and \( \frac{c_{jt}}{K_{jt}} \).

To describe the solution along the transition I will use the following simplifying no-
tation:

\[
\hat{k}_i = \frac{\hat{K}_{it}}{H_{jt}}, \hat{k}_j = \frac{\hat{K}_{jt}}{H_{jt}}, \tilde{k}_i = \frac{\tilde{K}_{it}}{H_{jt}}, h_i = \frac{H_{it}}{H_{jt}}; \ c_{it} = \frac{c_{it}}{K_{it}}; \text{ and } c_{jt} = \frac{c_{jt}}{K_{jt}}.
\]

To derive the dynamic system describing the transitional behavior of this world economy
I need to derive the growth rates of the above ratios.

For the growth rate of \( \frac{k_{jt}}{k_j} \) and \( \frac{k_{jt}}{k_i} \) I can write:

\[
\frac{\dot{k}_{jt}}{k_{jt}} = A_j \left( \frac{\hat{K}_{jt}}{H_{jt}} \right)^{\alpha_j - 1} - \frac{c_{jt}}{K_{jt}} - \delta \left( 1 - \frac{\hat{K}_{jt}}{H_{jt}} \right)
\]

\[
+ \frac{\hat{K}_{it}}{H_{jt}} \frac{H_{jt}}{K_{jt}} \left[ A_i \left( \frac{\hat{K}_{it}}{H_{it}} \right)^{\alpha_i} \frac{H_{it}}{H_{jt}} \frac{H_{jt}}{K_{it}} - \frac{C_{it}}{K_{it}} - \delta \right] + \delta
\]

or alternatively

\[
\frac{\dot{k}_{jt}}{k_{jt}} = A_j \frac{k_{jt}^{\alpha_j - 1}}{k_{jt}} - c_{jt} + A_i \frac{k_{jt}}{k_{jt}} h_i \frac{1}{k_{jt}} k_{jt} - \frac{\tilde{k}_{jt}}{k_{jt}} c_{it}
\]  

(3.72)

and

\[
\frac{\dot{k}_{it}}{k_{it}} = A_i \left( \frac{\hat{K}_{it}}{H_{it}} \right)^{\alpha_i} \frac{H_{it}}{H_{jt}} \frac{H_{jt}}{K_{it}} - \frac{C_{it}}{K_{it}} + \alpha_j A_j \left( \frac{\hat{K}_{jt}}{H_{jt}} \right)^{\alpha_j - 1}
\]
or alternatively
\[
\frac{\dot{k}_{it}}{k_{it}} = A_i \hat{k}_{i}^{\alpha_i} h_i \frac{1}{k_{it}} - c_{it} + \alpha_j A_j \hat{k}_{jt}^{\alpha_j-1}
\] (3.73)

Similarly, for the growth rates of \(c_{it}\) and \(c_{jt}\) I can write:
\[
\frac{\dot{c}_{it}}{c_{it}} = \frac{1}{\theta} \left[ \alpha_j A_j \left( \frac{\hat{K}_{jt}}{H_{jt}} \right)^{\alpha_j-1} - \delta - \rho \right] - \left[ A_i \left( \frac{\hat{K}_{it}}{H_{it}} \right)^{\alpha_i} \frac{H_{it}}{H_{jt}} \frac{H_{jt}}{K_{it}} - \frac{C_{it}}{K_{it}} + \alpha_j A_j \left( \frac{\hat{K}_{jt}}{H_{jt}} \right)^{\alpha_j-1} - \delta \right]
\]
or
\[
\frac{\dot{c}_{it}}{c_{it}} = \left( \frac{1 - \theta}{\theta} \right) \alpha_j A_j \hat{k}_{jt}^{\alpha_j-1} + \delta \left( \frac{\theta - 1}{\theta} \right) - \frac{\rho}{\theta} A_i \hat{k}_{i}^{\alpha_i} h_i \frac{1}{k_{it}} + c_{it}
\] (3.74)

And similarly
\[
\frac{\dot{c}_{jt}}{c_{jt}} = \frac{1}{\theta} \left[ \alpha_j A_j \left( \frac{\hat{K}_{jt}}{H_{jt}} \right)^{\alpha_j-1} - \delta - \rho \right] - A_j \left( \frac{\hat{K}_{jt}}{H_{jt}} \right)^{\alpha_j-1} + \frac{C_{jt}}{K_{jt}} + \delta \left( 1 - \frac{\hat{K}_{it} H_{jt}}{H_{jt} K_{jt}} \right)
\]
\[- \frac{\hat{K}_{it} H_{jt}}{H_{jt} K_{jt}} \left[ A_i \left( \frac{\hat{K}_{it}}{H_{it}} \right)^{\alpha_i} \frac{H_{it}}{H_{jt}} \frac{H_{jt}}{K_{it}} - \frac{C_{it}}{K_{it}} - \delta \right]
\]
or
\[
\frac{\dot{c}_{jt}}{c_{jt}} = \frac{1}{\theta} \left[ \alpha_j A_j \hat{k}_{jt}^{\alpha_j-1} - \delta - \rho \right] - \left\{ A_j \hat{k}_{jt}^{\alpha_j-1} - c_{jt} + \frac{\hat{k}_{it}}{k_{jt}} \left[ A_i \hat{k}_{i}^{\alpha_i} h_i \frac{1}{k_{it}} - c_{it} \right] - \delta \right\}
\]

Finally, I can rewrite the last expression as:
\[
\frac{\dot{c}_{jt}}{c_{jt}} = \left( \frac{\alpha_j - \theta}{\theta} \right) \alpha_j A_j \hat{k}_{jt}^{\alpha_j-1} + \delta \left( \frac{\theta - 1}{\theta} \right) - \frac{\rho}{\theta} + c_{jt} - A_i \hat{k}_{i}^{\alpha_i} h_i \frac{1}{k_{jt}} + \frac{\hat{k}_{it}}{k_{jt}} c_{it}
\] (3.75)
I can write the equations (3.72), (3.73), (3.74), and (3.75) in terms of logged variables as follows:

\[
\frac{d \ln \hat{k}_{jt}}{dt} = A_j e^{(\alpha_j - 1) \ln \hat{k}_{jt}} - e^{\ln c_{jt}} + A_i \hat{k}_i e^{-\ln \hat{k}_{jt}} - e^{\ln \hat{k}_{it} - \ln k_{jt} + \ln c_{it}} \tag{3.76}
\]

\[
\frac{d \ln \tilde{k}_{it}}{dt} = A_i \hat{k}_i e^{-\ln \tilde{k}_{it}} - e^{\ln c_{it}} + \alpha_j A_j e^{(\alpha_j - 1) \ln \hat{k}_{jt}} \tag{3.77}
\]

\[
\frac{d \ln c_{it}}{dt} = \left(\frac{1 - \theta}{\theta}\right) \alpha_j A_j e^{(\alpha_j - 1) \ln \hat{k}_{jt}} + \delta \left(\frac{\theta - 1}{\theta}\right) - \frac{\rho}{\theta} + e^{\ln c_{it}} - A_i \hat{k}_i e^{-\ln \tilde{k}_{jt}} \tag{3.78}
\]

\[
\frac{d \ln c_{jt}}{dt} = \left(\frac{\alpha_j - \theta}{\theta}\right) A_j e^{(\alpha_j - 1) \ln \hat{k}_{jt}} + \delta \left(\frac{\theta - 1}{\theta}\right) - \frac{\rho}{\theta} + e^{\ln c_{jt}} - A_i \hat{k}_i e^{-\ln \tilde{k}_{jt}} + e^{\ln \hat{k}_{it} - \ln k_{jt} + \ln c_{it}} \tag{3.79}
\]

The dynamic system with differential equations (3.76), (3.77), (3.78), and (3.79), admits the following form of log-linear approximation around the steady-state:

\[
\begin{pmatrix}
\frac{d \ln \hat{k}_{jt}}{dt} \\
\frac{d \ln \tilde{k}_{it}}{dt} \\
\frac{d \ln c_{it}}{dt} \\
\frac{d \ln c_{jt}}{dt}
\end{pmatrix}
= 
\begin{pmatrix}
m_{11} & -m_{12} & -m_{13} & -m_{14} \\
m_{21} & -m_{22} & -m_{23} & 0 \\
m_{31} & m_{32} & m_{33} & 0 \\
m_{41} & m_{42} & m_{43} & m_{44}
\end{pmatrix}
\begin{pmatrix}
\ln \hat{k}_{jt} - \ln \hat{k}_{jt}^{ss} \\
\ln \tilde{k}_{it} - \ln \tilde{k}_{it}^{ss} \\
\ln c_{it} - \ln c_{it}^{ss} \\
\ln c_{jt} - \ln c_{jt}^{ss}
\end{pmatrix}
\]

The dynamic system can be written as:

\[
\dot{z}_t \cong M_z t
\]
where $M$ is the coefficient matrix from equation above:

$$M = \begin{pmatrix}
m_{11} & -m_{12} & -m_{13} & -m_{14} \\
m_{21} & -m_{22} & -m_{23} & 0 \\
m_{31} & m_{32} & m_{33} & 0 \\
m_{41} & m_{42} & m_{43} & m_{44}
\end{pmatrix}$$

and $z$ is the deviation vector from steady-state:

$$z_t = \begin{pmatrix}
\ln \hat{k}_{jt} - \ln \hat{k}_{jss} \\
\ln \hat{k}_{it} - \ln \hat{k}_{iss} \\
\ln c_{it} - \ln c_{iss} \\
\ln c_{jt} - \ln c_{jss}
\end{pmatrix}$$

The solution of this system can be written as:

$$z_t \cong e^{Mt}z_0 \quad (3.80)$$

The determinant of the matrix $M$ is negative, implying that the system admits saddle point trajectory taking to steady-state.

I can write (3.80) in spectral decomposition form as follows:

$$z_t \cong e^{Mt}z_0 = \Gamma e^{\Lambda t} \Gamma^{-1}z_0$$
or alternatively:

\[
\begin{pmatrix}
\ln \hat{k}_{jt} - \ln \hat{k}_{j}^{ss} \\
\ln \hat{k}_{it} - \ln \hat{k}_{i}^{ss} \\
\ln c_{it} - \ln c_{i}^{ss} \\
\ln c_{jt} - \ln c_{j}^{ss}
\end{pmatrix}
\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & e^{\mu_{1}t} & 0 & 0 \\
0 & 0 & e^{\mu_{3}t} & 0 \\
0 & 0 & 0 & e^{\mu_{4}t}
\end{pmatrix}
\Gamma^{-1}
\begin{pmatrix}
\ln \hat{k}_{0} - \ln \hat{k}_{j}^{ss} \\
\ln \hat{k}_{0} - \ln \hat{k}_{i}^{ss} \\
\ln c_{0} - \ln c_{i}^{ss} \\
\ln c_{0} - \ln c_{j}^{ss}
\end{pmatrix}
\]

(3.81)

where \( \Gamma \) and \( \Gamma^{-1} \) are right- and left-eigenvectors of transition matrix \( M \) respectively, and \( \mu \)-s are the eigenvalues of transition matrix \( M \).

The high dimensionality of this system does not lead to an analytical solution but I can study the transitional behavior of this world economy using model calibration techniques.

The parameter values that I use in the calibration exercise are: \( \alpha_{j} = 0.34; \alpha_{i} = 0.28; \eta_{i} = 0.27; \eta_{j} = 0.3; \theta = 2; \rho = 0.01; \delta = 0.0125; A_{i} = 0.8; A_{j} = 0.76; B_{i} = 0.2; B_{j} = 0.3. \)

The value of \( \rho \) corresponds to a 4 percent annual interest rate, the depreciation rate is assumed to be 5 percent. The values of the parameter \( \alpha \) in each country are chosen to be around 0.3, which is a usual value assumed for the share of physical capital in the Cobb-Douglas production function. The values of the parameter \( \eta \) are chosen to be less than \( \alpha \) to correspond to the assumption that sector producing good \( Y \) is more intensive in \( K \) type capital compared to the sector producing good \( H \).

The remaining parameter values are chosen to satisfy two restrictions that should hold: the marginal product of capital in the host country should be higher under autarky, and the marginal products of capital in each country should equalize on the BGP in the presence of FDI.

The results of this calibration exercise suggest that the above matrix generates real
eigenvalues, implying monotonic convergence of the variables to the steady-state.\(^9\) The graphs in (Figure 3.1 and Figure 3.2) depict the realizations of each value of the vector \(z\) in each period \(t\). As it follows from the graphs the deviations of the variables from their steady-state values converge monotonically to the steady-state. Moreover, the convergence pattern is consistent with the one discussed in Barro and Sala-i-Martin in the context of Uzawa-Lucas model (2004, Chapter 5). In particular the authors argue that if the average product of capital is higher than on the BGP (the scenario that we observe in the host country in my model) then the ratio of \(C\) to \(K\) is declining monotonically to the steady state while \(K\) to \(H\) monotonically rises to it. As it follows from the Figures 1 and 2 below, the pattern of the transition is in line with the pattern demonstrated in Barro and Sala-i-Martin’s work.

Now I will turn to the case when the host country deviates from its BGP value of \(K\) to \(H\) ratio by having more \(K\) type capital than \(H\). The important difference from the previous case is that both home and host country cannot set the investment in \(K\) capital equal to 0, because the output of the sector producing good \(Y\) is used for consumption. To sustain their consumption each country needs to accumulate \(K\) type capital in addition to \(H\) type capital. Therefore under this case of the dynamics each country will operate both sectors of production and the setup of the model will be similar to the setup of a two sector endogenous growth model. Note also that if instead of FDI, countries were trading with each other then it would have been possible for a country to invest only in \(H\) type capital as long as it could import the flow of investment in \(K\) type capital from its trading partner.\(^{10}\)

Overall, I conclude that the direction of the deviation of the ratio of \(K\) to \(H\) capital in

\(^9\)for more discussion see Barro and Sala-i-Martin, 2004 page 587.

\(^{10}\)for the discussion of the similar scenario of dynamics in the presence of trade see Yenokyan, Seater, Arabshahi (2010).
the host country determines the production structure in each country along the transition with countries operating either under a one or two sector production structure.

3.5 Conclusion

There was a significant increase in the flow of FDI worldwide over the last twenty years. Despite the increased importance of FDI there is, however, no common agreement in the literature on the growth effects of FDI. This dissertation contributes to the literature in the area of FDI by focusing on its pure effects on growth through its primary function of capital accumulation.

In this chapter I have utilized the structure of a two sector endogenous growth model introduced by Rebelo (1991) and discussed by Barro and Sala-i-Martin (2004). Given the richer structure of the current model compared to the models discussed in the earlier chapters, I have shown that there are no long-run growth effects of FDI for countries that are technologically the same. If countries are technologically different, then the home country does not experience any long-run growth effects whereas the host country experiences a negative growth effect because of the reduction in the marginal product of capital that occurs in the host country as it accumulates capital through FDI.

I have considered two possible scenarios of the transitional dynamics that depend on the direction of the deviations of the ratio of two types of capital in the host country from its BGP value. If the host country deviates from the BGP by lacking K type capital then the production structure in each country is characterized by the operation of only one sector producing good Y used for the investment in K type capital. In this case the transitional behavior of countries becomes complicated since the variables in one country depend on the variables in the other country. As a result the behavior of the
world economy is described by a four dimensional system of differential equations that does not lead to an analytical solution. So, I used model calibration techniques to study the behavior of this world economy.

Finally, under the scenario that the host country deviates from the BGP by having more K than H type capital both countries need to operate both sectors of production to produce not only H type capital but also the output of sector Y that is used not only for the accumulation of K but also for consumption. This leads to a general conclusion that the direction of the deviations of K to H ratio from the BGP value in the host county determines the pattern of the production process in both countries along the transition path.
Figure 3.1: Deviation of \( \hat{k}_{jt} \), and \( \tilde{k}_{it} \) from steady-state
Figure 3.2: Deviation of $c_{it}$, and $c_{jt}$ from steady-state
Figure 3.3: The transition path of $\hat{k}_{jt}$, and $\tilde{k}_{it}$
Figure 3.4: The transition path of $c_{it}$, and $c_{jt}$
Figure 3.5: The transition path of $r_{\hat{K}_{jt}}$, and $\frac{c_{jt}^i}{c_{jt}}$
Chapter 4

Concluding Remarks

There has been a significant increase in FDI flows worldwide over the last twenty years. Despite the increased importance of FDI there is, however, no common agreement in the literature on the growth effects of FDI. On theoretical grounds the benefits of FDI for the host country are linked to the positive spillovers resulting from FDI especially in the form of technology transfers, however on empirical grounds there is no strong evidence on the systematic relationship between FDI and growth. The increasing importance of FDI and disagreements between theoretical arguments and empirical evidence present in the current literature served as important motivational factors for my dissertation research.

My dissertation research focuses on the study of the dynamic effects of FDI on growth. I have used the structure of one- and two-sector endogenous growth models (Uzawa-Lucas model and a general two-sector endogenous growth model) as discussed in Barro and Sala-i-Martin (2004). In this class of models, endogenous growth is generated by perpetual accumulation of two types of capital, K and H. This property allows me to focus on the growth effects of FDI arising from its primary function of capital accumulation. To the best of my knowledge this is the only study that focuses on the effects of FDI through
capital accumulation by isolating this channel from the traditional spillover mechanisms discussed in the existing literature. By eliminating the role of positive spillovers resulting from FDI, I address what FDI itself does for growth through its primary function of capital accumulation.

In the first chapter of my dissertation I study the dynamic effects of FDI on growth using the framework of a one-sector endogenous growth model with physical and human capital, as discussed in Barro and Sala-i-Martin (2004, Chapter 5). There are two important results that I want to emphasize here. First, I show that the technological differences across countries, as captured by total factor productivity and factor share parameters, play an important role in determining the growth effects of the flow of FDI. Second, the transitional behavior of the variables in one country depends on the evolution of the variables in the other country. This means that one should study the dynamic behavior of both home and host countries together with a single system of differential equations characterizing the transitional path of the world economy. The number of control and state variables of the problem leads to a four dimensional system of differential equations describing behavior under transition. Even though the system does not have an analytical solution, model calibration techniques can be used to characterize the transitional dynamics of the model.

In the second chapter of my dissertation I further explore the relationship between FDI and growth. I focus on the effects of FDI under the assumption that the production process in each country takes place in two different sectors, with one of those sectors producing output used for consumption and investment into physical capital and the other sector producing human capital. The production structure of the model under autarky is based on the Uzawa-Lucas version of a two-sector endogenous growth model with physical and human capital. For simplicity I assume that FDI occurs only in the
sector producing physical capital. Given the nature of the production structure and the requirement that the marginal products of capital be equalized across countries in the absence of any barriers to openness, it follows that any growth effects of FDI are transitory in nature and overall there are no long-run growth effects of FDI, under the assumption that countries have the same technology in the sector producing H type capital. The absence of the growth effects of FDI is consistent with the empirical evidence on the issue.

In the third chapter of my dissertation I use a generalized structure of a two-sector endogenous growth model to extend the initial analytical framework in several directions. First, I allowed for both countries to be technologically different not only in terms of their total factor productivity parameters of each sector but also in terms of factor share parameters. I also discuss the scenario in which the home country can invest in either the K or H type of capital. Under the framework of this generalized model it still holds that the growth effects of FDI depend on technological differences across countries. If countries are technologically the same then FDI does not generate any growth effects for either home or host country. If, however, countries are technologically different, with the home country being technologically advanced in the FDI conducting sector, then the home country will not experience any growth effects, whereas the host country will face negative growth effects from FDI.

The transitional behavior of both countries in the presence of FDI is different from the dynamics of the closed economy two-sector models with endogenous growth. The interdependence of the variables across countries through the FDI channel again leads to a multidimensional system of differential equations describing the transitional path of both countries. The results of the calibration exercise show monotonic convergence of the variables of both countries to the steady state.
There are several directions of extension of my current research. First, I can extend the generalized version of the model by analyzing the effects of government policy, in particular the effects of a tax on capital income. There are two important restrictions imposed by the solution of the model in the presence of FDI under the assumption that countries are technologically different from each other: the marginal product of capital in the host country should be higher than the marginal product in the home country under autarky, and marginal products of capital should be equalized along the BGP in the presence of FDI. The introduction of public policy may change the direction of flow of FDI by affecting the inequality restrictions. Another possible extension is to introduce adjustment costs for foreign investment. The plausible underlying assumption is that the capital stock created as an outcome of foreign investment may require some additional costs to be adjusted to the production environment of the host country. Introduction of the adjustment costs of FDI will also have mathematical implications for the solution of the problem, since it will eliminate linearity of the resource constraint and therefore linearity of the present value Hamiltonian in foreign investment.

I am also interested in studying the effects of FDI on growth in the context of a richer framework that will explicitly focus on the process of Research and Development aimed at improving the quality of intermediate inputs in the home country and in the host country by means of FDI.
REFERENCES


Appendix A

A.1 Equalization of the Growth Rates on the BGP in the presence of FDI

From (1.29) it follows that \( \frac{\dot{K}_i}{K_i} = \frac{\dot{H}_i}{H_i} \).

Similarly, from (1.34) it follows that \( \frac{\dot{K}_j}{K_j} = \frac{\dot{H}_j}{H_j} \).

Using the expression for the production function introduced in (1.1) and expression for the growth rate of consumption given in (1.27) I will get:

\[
\frac{\dot{C}_i}{C_i} = \frac{1}{\theta} \left( \alpha_i \frac{Y_i}{K_i} - \delta - \rho \right)
\]

Given that the growth rate of consumption should be constant on BGP it follows that \( \frac{\dot{K}_i}{K_i} = \frac{\dot{Y}_i}{Y_i} \).

Therefore for the home country I have:

\[
\frac{\dot{Y}_i}{Y_i} = \frac{\dot{K}_i}{K_i} = \frac{\dot{H}_i}{H_i}
\]

Similarly, for the host country combining the expression for the production function with the solution for the growth rate of consumption for this country in (1.33) I can
write:

\[
\frac{\dot{C}_j}{C_j} = \frac{1}{\theta} \left( \alpha_j \frac{Y_j}{K_j} - \delta - \rho \right)
\]

which again implies that \( \frac{\dot{K}_j}{K_j} = \frac{\dot{Y}_j}{Y_j} \).

So, for the host country I have the same result that:

\[
\frac{\dot{Y}_j}{Y_j} = \frac{\dot{K}_j}{K_j} = \frac{\dot{H}_j}{H_j}
\]

Next, using the relationship between different definitions of K type capital \( \hat{K}_i = \bar{K}_i - \tilde{K}_i \) I can write:

\[
\frac{\hat{K}_i}{H_i} = \frac{\bar{K}_i - \tilde{K}_i}{H_i - \tilde{K}_i} = \frac{\hat{K}_i}{H_i}
\]

I have already shown that \( \frac{\hat{K}_i}{H_i} \) is constant on BGP, which implies that \( \frac{\bar{K}_i}{H_i} \) and \( \frac{\tilde{K}_i}{H_i} \) must be constant too. So, I can conclude that:

\[
\frac{\dot{K}_i}{K_i} = \frac{\dot{H}_i}{H_i} = \frac{\dot{K}_i}{K_i} = \frac{\dot{K}_i}{K_i} \quad (A.1)
\]

Following similar reasoning for the host country I can write:

\[
\frac{\hat{K}_j}{H_j} = \frac{\bar{K}_j - \tilde{K}_i}{H_j} + \frac{\hat{K}_i}{H_j}
\]

On BGP \( \frac{\hat{K}_j}{H_j} \) is constant, implying that \( \frac{\bar{K}_j}{H_j} \) and \( \frac{\tilde{K}_i}{H_j} \) must be constant as well. So, I can conclude that:

\[
\frac{\dot{K}_j}{K_j} = \frac{\dot{H}_j}{H_j} = \frac{\dot{K}_i}{K_i} \quad (A.2)
\]
The last thing left to show in this section is how all these growth rates are related to the consumption growth rate.

No arbitrage condition requires equalization of the rates of return on capital across countries, which under the assumption of common depreciation rate and common rate of time preference implies:

\[
\frac{\dot{C}_i}{C_i} = \frac{\dot{C}_j}{C_j}
\]

Also, as it follows from (1.19):

\[
\frac{\dot{K}_j}{K_j} = \alpha_j \left( \frac{Y_j}{K_j} - \frac{C_j}{K_j} - r_j \frac{\dot{K}_i}{K_j} \right) - \delta
\]

The above expression suggests that \( \frac{C_j}{K_j} \) is constant on BGP and \( \frac{\dot{C}_i}{C_i} = \frac{\dot{K}_j}{K_j} \), from which immediately follows that growth rates of all variables are equalized across countries on BGP.

A.2 Behavior of the Home Country along the Transition

As I already mentioned in section (1.4) if the host country which is the recipient of the FDI deviates from the BGP ratio of K to H capital by having less K than H type capital then lower level of the total K type capital stock in the host country compared to the BGP value implies that the rate of return on K type capital is higher in the host country compared to the BGP value of the common rate of return. So, if \( \frac{\dot{K}_j}{H_j} < \frac{\alpha_j}{1-\alpha_j} \) then host country has an incentive to set an investment in H type capital equal to zero, such that
\( I_{H_{it}} = 0 \) and accumulate K type capital only.

Now consider the behavior of the home country which is the conductor of FDI. Obviously, when the rate of return on K type capital in the host country is higher than the common rate of return in both countries on the BGP there is an incentive for home country to invest abroad. Home country will set the domestic investment of K type capital equal to zero and invest in K type capital in the host country. BGP solution, however, requires that the home country also set the H type investment equal to zero.

To see this assume that the home country continues investing in H type capital instead of setting it to zero. The total derivatives of the marginal products of each type capital are given below.

\[
\dot{r}_{K_{it}} = A_i \alpha_i \hat{K}_{it}^{\alpha_i - 1} H_{it}^{1 - \alpha_i}
\]

\[
\frac{\partial r_{K_{it}}}{\partial t} = \alpha_i (\alpha_i - 1) A_i \hat{K}_{it}^{\alpha_i - 2} H_{it}^{1 - \alpha_i} \frac{\partial \hat{K}_{it}}{\partial t} + \alpha_i (1 - \alpha_i) A_i \hat{K}_{it}^{\alpha_i - 1} H_{it}^{-\alpha_i} \frac{\partial H_{it}}{\partial t}
\]

\[
\dot{\hat{K}}_{it} = \alpha_i (\alpha_i - 1) A_i \left( \frac{\hat{K}_{it}}{H_{it}} \right)^{\alpha_i - 1} \frac{\hat{K}_{it}}{K_{it}} + \alpha_i (1 - \alpha_i) A_i \left( \frac{\hat{K}_{it}}{H_{it}} \right)^{\alpha_i} \frac{\hat{H}_{it}}{K_{it}}
\]

\[
\frac{\dot{r}_{K_{it}}}{r_{K_{it}}} = \frac{\alpha_i (\alpha_i - 1) A_i \left( \frac{\hat{K}_{it}}{H_{it}} \right)^{\alpha_i - 1} \frac{\hat{K}_{it}}{K_{it}} + \alpha_i (1 - \alpha_i) A_i \left( \frac{\hat{K}_{it}}{H_{it}} \right)^{\alpha_i} \frac{\hat{H}_{it}}{K_{it}}}{\alpha_i A_i \left( \frac{\hat{K}_{it}}{H_{it}} \right)^{\alpha_i - 1} + \alpha_i A_i \left( \frac{\hat{K}_{it}}{H_{it}} \right)^{\alpha_i - 1}}
\]

\[
\frac{\dot{r}_{K_{it}}}{r_{K_{it}}} = (\alpha_i - 1) \frac{\dot{K}_{it}}{K_{it}} - (\alpha_i - 1) \frac{\dot{H}_{it}}{H_{it} K_{it}}
\]

\[
\frac{\dot{r}_{K_{it}}}{r_{K_{it}}} = (\alpha_i - 1) \left[ \frac{\dot{K}_{it}}{K_{it}} - \frac{\dot{H}_{it}}{H_{it}} \right] > 0
\]
Similarly,

\[ r_{Ht} = A_i (1 - \alpha_i) \hat{K}_{it}^{\alpha_i} H_{it}^{-\alpha_i} \]

\[
\frac{\partial r_{Ht}}{\partial t} = \alpha_i (1 - \alpha_i) A_i \hat{K}_{it}^{\alpha_i - 1} H_{it}^{-\alpha_i} \frac{\partial \hat{K}_{it}}{\partial t} + (-\alpha_i) (1 - \alpha_i) A_i \hat{K}_{it}^{\alpha_i} H_{it}^{-\alpha_i - 1} \frac{\partial H_{it}}{\partial t}
\]

\[
\dot{r}_{Ht} = \alpha_i (1 - \alpha_i) A_i \left( \frac{\hat{K}_{it}}{H_{it}} \right)^{\alpha_i} \frac{\dot{\hat{K}}_{it}}{\hat{K}_{it}} - \alpha_i (1 - \alpha_i) A_i \left( \frac{\hat{K}_{it}}{H_{it}} \right)^{\alpha_i} \frac{\dot{H}_{it}}{H_{it}}
\]

\[
\dot{r}_{Ht} = \frac{\alpha_i (1 - \alpha_i) A_i \left( \frac{\hat{K}_{it}}{H_{it}} \right)^{\alpha_i} \frac{\dot{\hat{K}}_{it}}{\hat{K}_{it}}}{(1 - \alpha_i) A_i \left( \frac{\hat{K}_{it}}{H_{it}} \right)^{\alpha_i}} - \frac{\alpha_i (1 - \alpha_i) A_i \left( \frac{\hat{K}_{it}}{H_{it}} \right)^{\alpha_i} \frac{\dot{H}_{it}}{H_{it}}}{(1 - \alpha_i) A_i \left( \frac{\hat{K}_{it}}{H_{it}} \right)^{\alpha_i}}
\]

\[
\frac{\dot{r}_{Ht}}{r_{Ht}} = \alpha_i \frac{\dot{\hat{K}}_{it}}{\hat{K}_{it}} - \alpha_i \frac{\dot{H}_{it}}{H_{it}}
\]

\[
\frac{\dot{r}_{Ht}}{r_{Ht}} = \alpha_i \left[ \frac{\dot{\hat{K}}_{it}}{\hat{K}_{it}} - \frac{\dot{H}_{it}}{H_{it}} \right] < 0
\]

So, if we assume that the home country continues investing in H type capital it will deviate from the balance growth path. The figure (A.1) illustrates the paths of the marginal products of both types of capital in both countries.
Figure A.1: Evolution of the marginal products of K and H type capital