ABSTRACT

JUN, ZHANG. Development of Failure Criteria for Asphalt Concrete Mixtures under Fatigue Loading. (Under the direction of Dr. Murthy N. Guddati and Dr. Y. Richard Kim).

For asphalt concrete under fatigue loading, the damage process can be generally categorized into pre-localization, localization and post-localization, which spans the modeling approaches from continuum damage to fracture. In this thesis, the different mechanisms associated with these three regimes are reviewed and the proper transition points are also investigated through experimental observations, both qualitatively and quantitatively. It is concluded that the drop of phase angle correctly characterizes the occurrence of macro-fracture and the modeling of asphalt concrete under fatigue loading can be separated into two regimes, the first governed by viscoelastic continuum damage (VECD) model capturing material’s behavior up to the drop of phase angle, and the rest governed by a fracture model representing macro-crack propagation.

The criterion that adheres to the failure definition based on phase angle is also thoroughly studied to predict fatigue life for asphalt mixture. Approaches based on stiffness and dissipated energy are both explored and compared, leading to a new energy based failure criterion. The proposed failure criterion is shown to be consistent with VECD model and is able to predict the mixture’s fatigue life over a wide range of temperatures using one single characteristic relation.
Development of Failure Criteria for Asphalt Concrete Mixtures under Fatigue Loading

by

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DEDICATION

To my dear parents, family and friends.
BIOGRAPHY

Jun Zhang was born in Hefei, a middle city in China where he spent the first eighteen years of his life. In 2004, he obtained the opportunity to join University of Science and Technology of China and became a student majored in applied mechanics. He received his Bachelor of Science in 2008 and in the same year, he came to United States and started the graduate study in the research group of Dr. Y. Richard Kim and Dr. Murthy N. Guddati.
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Chapter 1 Introduction

Under cyclic loading below 25°C, the dominant damage mechanism in asphalt concrete is fatigue cracking. The development and propagation of fatigue cracks inside a pavement structure allow water to enter the system freely, which can severely accelerate the rate of pavement deterioration and significantly reduce the structure’s service life. Thus, it is crucial to understand the behavior of asphalt concrete under fatigue loading.

Figure 1.1 Schematic representation of three phases in a fatigue test.

Depending on the characteristic length of the fatigue crack and the corresponding governing mechanism, the damage process of fatigue cracking in asphalt concrete can be categorized into three phases: pre-localization, localization and post-localization. A schematic
representation of these three phases in a stiffness degradation diagram for a fatigue test is shown in Figure 1.1.

*Pre-localization* is regarded as the continuum damage state, which is manifested by the initiation and propagation of micro-cracks inside the material. *Localization* and *post-localization* refer to the interaction and coalescence of micro-cracks and the propagation of macro-cracks, respectively. In general, the definition of *post-localization* is similar to that of *fracture* because it also deals with definitive macro-cracks, whereas *localization* includes the entire transition region from distributed micro-cracks to localized macro-cracks. The exact cut-off points between these three consecutive phases usually are difficult to define. Over the past few decades, the pre-localization and post-localization behavior of asphalt concrete has been studied intensively using continuum damage models and fracture models. In comparison, the stage of localization is least understood because of its ambiguous definition and the complexity of its role. To bridge the modeling of asphalt concrete from continuum damage to fracture, it is important to understand the underlying mechanisms of these three stages and the relationships among them.

**1.1 Background**

*1.1.1 Pre-localization and continuum damage mechanics*

During pre-localization, micro-cracks spread throughout the structure, with the number and distribution of these micro-cracks dependent on both material heterogeneity and applied load. Because the micro-cracks considered at this stage are usually smaller or comparable to the
size of the representative volume element (RVE), the microstructure plays an important role in the propagation of these micro-cracks. It is very difficult to quantify precise information about these cracks, even using advanced digital imaging tools, such as the digital image correlation (DIC) method. Instead, the material behavior is studied in a more smeared way. Continuum damage theories ignore specific microscale behavior and attempt to characterize the material using macroscale observations, i.e., the net effect of microstructural changes on observable properties. An example is the viscoelastic continuum damage (VECD) model, which was developed for viscoelastic material such as asphalt concrete. The VECD model is based on the work potential theory proposed by Schapery (1990), which essentially lumps the entire micro-crack distribution into a single damage parameter, S, and quantifies the evolution of damage through the thermodynamics of irreversible process. Based on extensive research conducted thus far, the VECD model has been proven to be successful in characterizing the pre-localization behavior of asphalt concrete (Chehab et al. 2002).

1.1.2 Post-localization and fracture mechanics

Post-localization starts with the formation of macro-cracks and is investigated in terms of fracture mechanics, which approaches macro-crack behavior using definitive crack geometry. In general, fracture in quasi-brittle materials such as asphalt concrete develops in a process zone ahead of the crack tip. This fracture process zone (FPZ) is a strong nonlinear region with intense damage. Schapery (1974) developed the general theoretical framework to predict the time-dependent growth of cracks and flaws in linear viscoelastic and isotropic media with the failure zone ahead of the crack tip. Another type of effective model that can
describe the nonlinear fracture process at the outset of a pre-existing crack is the cohesive crack model (CCM) (Dugdale 1960, Barenblatt 1962). For cyclic loading, research generally has focused on characterizing the crack growth rate using external conditions and material properties. For example, Pairs’ law (Paris and Erdogan 1963) relates the crack growth rate to the stress intensity factor at the crack tip, as follows:

\[
\frac{da}{dN} = C(\Delta K)^n
\]  

(0.1)

where

- \(a\) = crack length,
- \(\Delta K\) = stress intensity factor, and
- \(C, n\) = material parameters.

1.1.3 Nonlocal formulation

Even though continuum damage mechanics and fracture mechanics describe different damage processes of asphalt concrete, they nonetheless share similarities between their intrinsic mechanisms, as listed in Table 1.1 (Lee et al. 2000), which appear to indicate that they can be combined easily. One hypothesized simple approach is to extend the continuum damage model for fracture directly, because fracture can be considered as the extreme case when damage goes into infinity and stiffness reduces to close to 0 at a material point.

However, research has found that the direct extension of the local continuum damage model to fracture may cause a problem called mesh sensitivity (Bazant 1976). Mesh sensitivity occurs when the constitutive relationship reaches the softening branch, where one stress point
may correspond to two strain values. Without a proper limiter, strain and damage will localize to a region of zero volume as the mesh refines and the total dissipated energy for fracture vanishes.

Table 1.1 Continuum Damage Mechanics versus Fracture Mechanics

<table>
<thead>
<tr>
<th>Types of approaches</th>
<th>Continuum damage mechanics</th>
<th>Fracture mechanics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crack/damage driving force</td>
<td>( f = -\frac{\partial W}{\partial S} ) Thermodynamic force</td>
<td>( G = -\frac{\partial W}{\partial a} ) Energy release rate</td>
</tr>
<tr>
<td>Crack/damage evolution law</td>
<td>( \frac{dS}{dt} = A(f)^\alpha )</td>
<td>( \frac{da}{dt} = A(G)^\alpha = A(f)^\alpha )</td>
</tr>
</tbody>
</table>

Note: \( a = \) crack length, \( W = \) strain energy, \( S = \) damage parameter, \( A, \alpha = \) material parameters.

A computationally efficient way to overcome mesh sensitivity is to adopt a nonlocal approach (Pijaudier-Cabot et al. 1987, Bazant et al. 1988). The key idea of nonlocal formulation is to assume that the stress or damage at a given point does not depend only on the strain at that point, but a weighted average of the points in its neighborhood. This neighborhood is scaled by an internal length parameter that is related to the size of the heterogeneities (Bazant and Pijaudier-Cabot 1989). The internal length defined in the nonlocal formulation prevents damage to localize into one arbitrary element and helps to characterize the size of the process zone. In addition, the spatial average method in nonlocal formulation also provides the framework for incorporating crack interaction and coalescence, which is a significant mechanism in the localization process. In summary, the nonlocal approach provides a method to simulate the damage evolution from the diffusive mode to the localized mode both numerically and theoretically.
1.2 General approach

From the above discussion, it appears that one plausible procedure to link the continuum damage to fracture in a seamless manner is to use the nonlocal VECD model. Employing this hypothesis, the work for this study initially focused on understanding micro-cracking formation, interaction and coalescence in order to develop the proper nonlocal formulation. However, during the investigation, it was realized that the nonlocal approach may not be required, or may not be the most efficient way to accomplish the task, for three reasons. First, a decision had since been made to focus exclusively on cyclic loading conditions. During the study, it is found that the mesh sensitivity, which is a critical issue for monotonic loading conditions, no longer appeared to exist under cyclic loading conditions. Second, experimental observations seemed to indicate that, for practical engineering purposes, the VECD model is able to represent the behavior up to localization or even during localization. Third, even if the nonlocal damage model is able to simulate the localized damage and fracture, it may still be better to switch to a fracture type of approach for post-localization for the sake of computational efficiency, because averaging all the nodes within the internal length to track the damage zone propagation usually requires expensive calculations.

Given these arguments, the research approach has been modified to focus on two regimes, namely the VECD regime representing pre-localization and localization, and the fracture regime representing the macro-crack propagation. Thus, the objectives of the research have been simplified and reduced to two major parts: (a) determination of the transition point between the VECD and fracture regimes, referred to as the *failure criterion* in this work, and
the development of a fracture model that can simulate cracking in damaged materials past the failure point.

The main contribution of the thesis is the solution of objective (a), and objective (b) is left for future work. The thesis also includes the work prior to simplification of the objectives, which was aimed at the micromechanical understanding of crack interactions and localization with the hope of developing a nonlocal model. This part of the thesis should provide some basis for further work on developing fracture models that include micro-crack interaction, thereby leading to the solution of objective (b), and perhaps even to a model for the localization regime that is more accurate than the local VECD model.

1.3 Thesis organization

The thesis is organized as follows: Chapter 2 provides an overview of the theoretical background for the VECD model and nonlocal approach. Chapter 3 briefly explains the experimental tests and the corresponding observations. Chapter 4 includes the exploratory investigations into the pre-localization and localization regions via micromechanical methods. Chapter 5, which is the main contribution of this thesis, presents the development of the failure criterion, which involves predicting the transition between the VECD and fracture regions. Chapter 6 is dedicated to some preliminary investigations into the post-localization region. Chapter 7 presents conclusions and recommendations for future research.
Chapter 2 Theoretical Background

2.1 Viscoelastic continuum damage (VECD) model

The VECD model is a mechanistic model that predicts the progression of material damage and resulting stiffness reduction under monotonic as well as cyclic loads. The theoretical framework of the VECD model consists of three major principles: 1) the elastic-viscoelastic correspondence principle that simplifies the viscoelastic problem into an elastic one, 2) continuum damage mechanics based on the work-potential theory for modeling the effects of micro-cracking on global constitutive behavior, and 3) the time-temperature superposition (t-TS) principle with growing damage to include the joint effects of time/rate and temperature. A detailed explanation of each principle is given below, after a brief review of the theory of linear viscoelasticity.

2.1.1 Linear viscoelastic theory

Linear viscoelastic (LVE) materials exhibit both time- and temperature-dependent behavior. The material’s response is dependent not only on the current load input, but also on all the past input history. The constitutive relationship for LVE materials generally is expressed in the form of a convolution integral:

$$\sigma = \int_0^t E(t-\tau) \frac{d\varepsilon}{d\tau} d\tau$$  \hspace{1cm} (1.1)

$$\varepsilon = \int_0^t D(t-\tau) \frac{d\sigma}{d\tau} d\tau$$  \hspace{1cm} (1.2)

where
\( E(t) \) = the relaxation modulus,

\( D(t) \) = the creep compliance, and

\( \tau \) = the integration variable.

If the LVE material is introduced by cyclic loading and reaches the steady state, the constitutive relationship can be written in a simple form using the complex modulus \((E^*)\) and phase angle \((\phi)\). For example, if the applied strain is

\[
\varepsilon = \varepsilon_0 \sin(\omega t),
\]

then the LVE stress response would be

\[
\sigma = \sigma_0 \sin(\omega t + \phi)
\]

where

\[
\varepsilon_0 \quad = \text{strain amplitude},
\]

\[
\sigma_0 \quad = \text{stress amplitude},
\]

\[
\omega \quad = \text{loading frequency}, \text{ and}
\]

\[
\phi \quad = \text{the time shift between the stress and strain signals}.
\]

The stress maintains the same shaped curve as that of the strain but with a shift in the time domain. The ratio between the harmonic forms of stress and strain is complex valued and is referred to as the \textit{complex modulus} \((E^*)\). Thus, the ratio between the stress amplitude \((\sigma_0)\) and the strain amplitude \((\varepsilon_0)\) would be \(|E^*|\), which characterizes the material’s stiffness, as shown in Equation (1.5). The phase angle \((\phi)\) characterizes the lag of time between the stress and strain.

\[
|E^*| = \frac{\sigma_0}{\varepsilon_0}
\]
2.1.2 Time-temperature superposition principle

The stiffness of viscoelastic material is dependent on both time (rate of loading) and temperature. Therefore, if the full range of stiffness is to be captured, then experimental tests need to be performed at multiple temperatures and loading frequencies, which is a process that is both time consuming and difficult to accomplish. However, it is found that the effects of temperature and loading frequency are not necessarily exclusive. For example, the same complex modulus can be obtained either at a lower temperature or higher loading frequency. The t-TS principle utilizes this fact and provides a method to combine the effect of time and temperature into a single joint parameter. The combined parameter is called reduced frequency, which is defined as:

\[ f_R = f \times a_T \]  \hspace{1cm} (1.6)

where

- \( f_R \) = reduced frequency,
- \( f \) = the frequency in Hz, and
- \( a_T \) = the shift factor.

\[ \log(a_T) = \alpha_1 T^2 + \alpha_2 T + \alpha_3 \]  \hspace{1cm} (1.7)

where

- \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) = the coefficients, and
- \( T \) = temperature.

The basic idea behind reduced frequency is that by horizontal shifting the experimental data at various temperatures, a single master curve can be formed to represent the material’s
behavior in its full range. The shifted amount for each temperature is determined by the shift factor, and the shifted frequency becomes the reduced frequency.

Figure 2.1 Measured $|E^*|$ at different temperatures and physical frequencies.

Figure 2.2 Shifted $|E^*|$ mastercurve at reduced frequency.
2.1.3 Elastic-viscoelastic correspondence principle

The correspondence principle states that viscoelastic problems can be solved by elastic formulations when physical strains are replaced by pseudo strains (see e.g., Schapery 1984). Pseudo strain is defined as:

\[
\varepsilon^R = \frac{1}{E_R} \int_0^t E(t-\tau) \frac{d\varepsilon}{d\tau} d\tau
\]  

(1.8)

where

- \( \varepsilon^R \) = pseudo strain,
- \( \varepsilon \) = physical strain,
- \( E_R \) = the reference modulus, typically taken as one, and
- \( E(t) \) = the relaxation modulus.

Compared to Equation (1.1), it is observed that the relationship between stress and pseudo strain is simply:

\[
\sigma = E_R \varepsilon^R
\]  

(1.9)

The above equation has the same form as Hooke’s law for linear elastic material. A correspondence is thus set between the stress-strain constitutive relationship of the elastic and viscoelastic materials. In reality, pseudo strain is simply the LVE stress response for the given strain input. If there is no damage, the stress should be equal to the pseudo strain. One important advantage of introducing pseudo strain is that it can remove the time effects due to viscoelasticity, which provides a clearer picture for quantifying the damage.
Figure 2.3 presents the behavior of a LVE material under monotonic tension testing. In the stress-strain plot, the deviation from the initial modulus starts at a very early stage, which seems to indicate that damage has occurred already. However, by using the correspondence principle in the stress-pseudo strain plot, it is easy to find that this nonlinearity is due only to a time effect related to the viscoelasticity. That is, no damage actually occurs up to the stress of 500 kPa.

![Stress-strain and Stress-pseudo strain plots](image)

Figure 2.3 Schematic explanation of correspondence principle in monotonic loading:
(a) stress-strain plot, and (b) stress-pseudo strain plot.

2.1.4 Viscoelastic continuum damage theory

As the loading continues, micro-cracks gradually initiate and develop within the microstructure, and the material becomes “damaged”. Continuum damage theory characterizes the damage through macroscopic observation while ignoring the detailed microscale behavior. In other words, continuum damage theory focuses only on the net effect
of the change in the material’s microstructure to its macroscale property. The two essential parameters that continuum damage theory tries to quantify and bridge are the effective stiffness and damage. Effective stiffness represents the material’s integrity and can be easily measured in experiments. Damage, on the other hand, is sometimes difficult to quantify and generally relies on semi-empirical models grounded in rigorous theories. One of the theories is the work potential theory developed by Schapery (1990) for elastic materials with growing damage, and it is based on the thermodynamics of irreversible process. In this theory, damage is defined as an internal state variable (ISV) that accounts for microstructural changes in the material. For a viscoelastic material, the correspondence principle is applied first to uncouple the time-dependency associated with viscoelasticity. After transforming the physical strain to pseudo strain, the viscoelastic problem can be modeled within the same formula as is used for elastic materials. The basic equations for Shapery’s damage theory are:

the pseudo strain energy density function,

\[ W^R = f(\varepsilon^R, S) \]  

(1.10)

the stress-pseudo strain relationship,

\[ \sigma = \frac{\partial W^R}{\partial \varepsilon^R} \]  

(1.11)

and the damage evolution law,

\[ \frac{dS}{dt} = (- \frac{\partial W^R}{\partial S})^a \]  

(1.12)

where

\[ W^R \] = the pseudo strain energy density,
\( \varepsilon^R \) = pseudo strain,

\( S \) = the damage parameter (internal state variable), and

\( \alpha \) = the damage evolution rate.

For uniaxial mode of loading, the pseudo strain energy density function can be written as:

\[
W^R = \frac{1}{2} (\varepsilon^R)^2 C
\]  

(1.13)

where \( C \) is called the effective stiffness and is the only variable that is a function of damage \( S \) in Equation (1.13). Hence, the damage evolution law in Equation (1.12) becomes:

\[
\frac{\partial S}{\partial t} = \left( -\frac{1}{2} (\varepsilon^R)^2 \frac{\partial C}{\partial S} \right)^\alpha
\]  

(1.14)

Following the approach proposed by Lee and Kim (1998) and applying the chain rule in Equation (1.15), the damage can be evaluated at each time step using Equation (1.16). Note that the effect of time-temperature superposition is taken account by replacing the physical time with reduced time.

\[
\frac{dC}{dS} = \frac{dC}{dt} \frac{dt}{dS}
\]  

(1.15)

\[
dS_i = \left( -\frac{1}{2} (\varepsilon^R)^2 \Delta C_i \right)^\frac{\alpha}{1+\alpha} \left( \Delta \xi \right)_i \frac{1}{1+\alpha}
\]  

(1.16)

where

\( \Delta \xi \) = reduced time interval.

### 2.1.5 Simplification of the VECD model formulation for cyclic loading

The rigorous VECD model requires the calculation of the pseudo strain (\( \varepsilon^R \)) and effective stiffness (\( C \)) at each time step in order to track precisely the damage evolution. This
requirement is a very expensive procedure for cyclic loading. To gain a good cyclic pulse definition, usually at least 50 data points per cycle are needed. In this case, an average test with 20,000 cycles to failure would require the analysis of 100,000 data points. Although this task is not absolutely impossible using advanced computers, it is nonetheless cumbersome and time consuming.

In response to this problem, Underwood (2010) developed the simplified VECD model for the application of cyclic loading. Under fatigue loading, damage usually does not propagate much during one cycle. Thus, effective stiffness is evaluated only at the end of each cycle instead of at each data point. According to this rigorous approach, the effective stiffness should be defined as the total pseudo strain based on the secant modulus ($C^*$) at the peak stress, which is shown in Figure 2.4. However, if the permanent pseudo strain does not accumulate significantly, the secant modulus can be approximated by the cyclic magnitude-based stiffness ($F$). The mathematic definition of both stiffness values and their relationship is given in Equations (1.17) through (1.19).
Figure 2.4 Schematic view of stress, pseudo strain and pseudo stiffness definitions.

\[
C^* = \frac{\sigma_{0,ta}}{\varepsilon_{m}^R} = \left(\frac{\sigma_{0,ta}}{\varepsilon_{0,ta}^R + \varepsilon_{s}^R}\right)^* I
\]

(1.17)

\[
F = \frac{\sigma_{0,ta}}{\varepsilon_{0,ta}^R} = \frac{\sigma_{0,pp}}{\varepsilon_{0,pp}^R} = \left(\frac{\sigma_{0,ta}}{\varepsilon_{m}^R - \varepsilon_{s}^R}\right)^* I
\]

(1.18)

\[
C^* = F \frac{\varepsilon_{m}^R - \varepsilon_{s}^R}{\varepsilon_{m}^R}
\]

(1.19)

where

\[
\varepsilon_{m}^R = \text{the absolute pseudo strain at peak},
\]

\[
\varepsilon_{0,ta}^R = \text{the pseudo strain tension amplitude},
\]

\[
\varepsilon_{0,pp}^R = \text{the peak-to-peak pseudo strain amplitude},
\]

\[
\varepsilon_{s}^R = \text{the permanent pseudo strain},
\]
If cyclic magnitude-based stiffness \((F)\) is utilized, the pseudo strain can be also calculated in a simplified form. Instead of using the convolution integral, the pseudo strain is calculated under the so-called *steady-state assumption*, and its peak-to-peak value is calculated as:

\[
\varepsilon_{0,pp}^R = \varepsilon_{0,pp} \ast |E^*|_{LVE}
\]

where

\[
\varepsilon_{0,pp} = \text{peak-to-peak strain}, \quad \text{and} \quad |E^*|_{LVE} = \text{the linear viscoelastic dynamic modulus defined at a specific reduced frequency.}
\]

### 2.2 Nonlocal damage theory

As indicated in the introduction, the initial idea is to attempt to utilize nonlocal damage theory to bridge the gap between VECD and fracture mechanics. While the approach has deviated from this, although unrelated to the rest of the thesis in the present form, for the sake of completeness of reporting, a summary of nonlocal damage theory is given in this section, along with the numerical example that lead to the conclusion that cyclic loading does not computationally necessitate nonlocal damage theory. This section contains discussion on (a) the issue of mesh sensitivity for local damage models, (b) a summary of nonlocal formulations that can fix the problem, and (c) an example, done after the fact, indicating that mesh sensitivity is not a problem when the load is cyclic and damage accumulation is gradual.
2.2.1 *Mesh sensitivity*

Standard local constitutive models tend to encounter numerical problems in simulating strain-softening behavior during fracture. This problem is referred to as *mesh sensitivity*, which means that the simulated results depend significantly on the element size and mesh geometry. To explain this phenomenon, a simple one-dimensional (1-D) example is given. Consider a straight bar with a constant cross area \( A \) and total length \( L \) (Figure 2.5 (a)), and the material’s constitutive law is assumed to be linear elastic up to the peak, followed by linear softening Figure 2.5 (b)).

![Diagram of a straight bar](image)

**Figure 2.5** Schematic plot of 1-D bar problem: (a) straight bar, and (b) constitutive relationship.

The bar is loaded in tension by an applied displacement \( u \) at one end. Before softening occurs, all the strain is distributed uniformly throughout the bar. When \( u_0 = L \varepsilon_0 \), the force transmitted to the bar reaches its maximum, \( F = A f_t \). After that, if the applied displacement continues to increase, softening will occur, and the stress in the bar will begin to decrease. Due to the requirement of static equilibrium, the stress in the cross-section along the entire
bar should be decreasing all the time. According to constitutive law, this can be achieved either with increasing strain (softening) or with decreasing strain (unloading). After the peak stress, there are always two strains corresponding to the same stress value (Figure 2.6 (a)), and any piecewise constant strain jumping between these two strains satisfies the equilibrium (Figure 2.6 (b)). Thus, the strain is not necessarily uniform along the bar. If the cumulative length of the softening region in the bar is denoted by $L_s$, and the unloading region by $L_u = L - L_s$, then any combination satisfies the boundary condition, $u = L_u \varepsilon_u + L_s \varepsilon_s$, which represents a valid solution. Therefore, the solution can take infinite paths after the peak stress.

![Figure 2.6 Possible stress and strain states after peak stress: (a) two strain values corresponding to one stress, and (b) piecewise constant strain profile.](image_url)

If the strength in a small region of the bar is slightly less than in the other part of the bar, then when the applied force reaches the reduced strength of the small region, the remaining portion will still be in the elastic region. As the displacement increases, the small region will begin to soften, but the other parts are only unloading. The total displacement is given by
integrating the small strain over the large unloading region and the large strain in the small softened region. However, since the size of the softening region is approximated by at least one finite element, the integrated displacement will depend significantly on the element size, leading to significant errors in the predicted softening curves for realistic mesh sizes. This process is illustrated in Figure 2.7. This phenomenon is referred to as mesh sensitivity.

![Figure 2.7 Mesh sensitivity: (a) strain profiles with number of elements, and (b) load-displacement curve for different number of elements (Jirásek 2007).](image)

2.2.2 Basic nonlocal formulations

Nonlocal formulation is a good strategy to avoid mesh sensitivity by incorporating a characteristic length to prevent the localization of the strain to an arbitrary, small volume. In physics, this characteristic length describes the minimum thickness of the localized region and is measured in experiments as approximately three times the maximum aggregate size (Bazant et al. 1989). The basic idea behind the nonlocal approach is to replace a certain variable by its nonlocal counterpart. The nonlocal counterpart is obtained from the weighted
average of the points in a spatial neighborhood of the point under consideration. If $f(x)$ is some “local” field in a domain $V$, the corresponding nonlocal field is defined as:

$$
f(x) = \int_V \alpha(x, \xi) f(\xi) d\xi
$$

(1.21)

where

$$
\alpha(x, \xi) = \text{the weight function.}
$$

The weight function defines the interaction between the source point, $\xi$, and the target point, $x$. The largest distance between $\xi$ and $x$ where $\alpha(x, \xi) \neq 0$ is called the interaction radius. Via nonlocal formulation, the previous localized strain in one element is forced to spread within a certain width, and the dissipated energy for fracture is kept at a finite value. Figure 2.8 compares the simulations of notched beams using both local and nonlocal approaches. From the results, it is clear that the nonlocal approach effectively eliminates the mesh sensitivity and provides the correct convergence of post-localization behavior.
2.2.3 Mesh sensitivity for cyclic loading

Currently, most mesh sensitivity studies are focused on monotonic loading only. During cyclic loading, damage is accumulated in a much more gradual manner and the applied boundary condition is also not increasing monotonically. Hence, it can be assumed that the damage evolution in cyclic loading is generally more stable and the mesh sensitivity in cyclic loading may not exhibit as severe as monotonic loading. To illustrate this, a numerical experiment is performed under cyclic loading. The simulated geometry is chosen to be a square plate with a circular hole in the center. The left edge of the plate is fixed and cyclic force is applied at the right edge with constant amplitude in the horizontal direction. Since there is only little damage propagation within each cycle and the total number of cycles to reach failure is relatively large, a Fourier-Finite Element (FFE) approach is utilized instead of traditional time stepping method for computational efficiency (Eslaminia et al. 2011). The constitutive relation of the material is assumed to be the VECD model. In FFE, damage is updated after every 100 cycles and the stiffness reduction at the end of each loading interval is written as an explicit integral of experienced pseudo strain during this loading interval. Three different meshes are used in the finite element simulation, which are all shown in Figure 2.10. Damage is updated at every 100 cycles and the value of averaged C is calculated by dividing the amplitude of the applied force to the calculated amplitude of displacement at right edge. Because of the stress concentration induced by the hole, damage is not uniformly distributed within the structure and strain localization will occur at the top and bottom of the hole. A shot of the damage contour is shown in Figure 2.9. The computational analysis is set to stop automatically when the minimum local element stiffness
reduces to 0.05, which is considered as the indication of local fracture. From the result in Figure 2.11, it is found that the simulated results are almost the same among these three meshes. The mesh sensitivity is not actually observed. Hence, it can be concluded that the mesh sensitivity becomes not significant for the case of cyclic loading since damage is updated in a gradual way. The local formulation is sufficient to simulate the material behavior up to failure for cyclic loading and the nonlocal approach may not be needed.

Figure 2.9 Damage contour of the plate with a central hole in simulation.
Figure 2.10 Three different meshes for the mesh sensitivity study of cyclic loading.

Figure 2.11 Simulated results for three different meshes under cyclic loading.
Chapter 3 Experimental Observations

This chapter briefly summarizes the experimental methods and observations for this research. The experiments involved in this study were performed by fellow Masters student Tian Hou (2009) and current doctoral student Mohammad Reza Sabouri at North Carolina State University.

3.1 Materials

The research for this study focuses on two types of mixtures. One is the S9.5C mix from the NCDOT HWY-2007-7 Mechanical-Empirical Pavement Design Guide (MEPDG) local calibration project, and the other is the VTe30LC mix from the New England reclaimed asphalt pavement (RAP) project. The reason for choosing these two mixtures is that a large number of tests have been done on these mixes, which help to provide a comprehensive picture. The properties of these two mixtures are summarized in Table 3.1.

<table>
<thead>
<tr>
<th>Mix Type</th>
<th>Binder</th>
<th>AC (%)</th>
<th>NMAS(mm)</th>
<th>RAP (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S9.5C</td>
<td>PG70-22</td>
<td>5.2</td>
<td>9.5</td>
<td>Non-RAP</td>
</tr>
<tr>
<td>VTe30LC</td>
<td>PG64-28</td>
<td>6.61</td>
<td>9.5</td>
<td>30</td>
</tr>
</tbody>
</table>

3.2 Test Methods

Direct tension tests were conducted on cylindrical specimens using an asphalt mixture performance tester (AMPT). To investigate fatigue, controlled crosshead (CX) cyclic testing
was chosen as the test mode. Details of the experimental procedure can be found in Hou (2009) and Underwood (2010). The advantage of CX cyclic tests is that they allow the specimen to run to complete failure while still effectively limiting the viscoplasticity to a certain extent. Furthermore, CX cyclic tests are easier to conduct on cylindrical specimens compared to traditional controlled strain cyclic tests. During CX cyclic tests, the machine actuator’s displacement is programmed to reach a constant peak value at each cycle. However, because of the change in compliance ratio between the specimen and the machine, the actual on-specimen strain increases in both magnitude and mean value with cycles. Generally, CX cyclic tests employ a mixed mode of loading, which is neither purely controlled stress nor purely controlled strain. After a few initial cycles, the mean stress within the specimen stabilizes at approximately zero, and the next loading mode is similar to tension-compression cyclic loading. Figure 3.1 shows the on-specimen stress and strain history for the initial five cycles.

Figure 3.1 Strain and stress profile for CX testing at initial cycles.
The dynamic modulus and phase angle are tracked throughout the loading history using Equation (1.22) and Equation (1.23), respectively. The evolutions of the mean stress and mean strain also are recorded.

\[ |E^*| = \frac{\sigma_i}{\varepsilon_i} \]  \hspace{1cm} (1.22)

where

\[ \sigma_i \] = stress amplitude at cycle \( i \), and \n\[ \varepsilon_i \] = strain amplitude at cycle \( i \).

\[ \varphi_i = \Delta t_i / 2\pi f \]  \hspace{1cm} (1.23)

where

\[ \Delta t_i \] = the time shift between the stress and strain signals, and \n\[ f \] = testing frequency.

### 3.3 Definition of failure point

The definition of fatigue failure for laboratory testing has always been a controversial issue, especially in the case of controlled strain mode of loading when usually no catastrophic failure or fracture happens. Traditional fatigue analysis determines failure to be the point at which the material’s modulus reduces to 50% of its initial value, and the number of cycles when modulus reduces to 50% of the initial modulus is denoted as \( N_{f50} \). This definition is always under debate due to its arbitrary assumption.
From Figure 3.2, it is obvious that the $N_{f50}$ for the VTe30LC mix is still far from the failure of the material. In this sense, the fatigue life, which is number of cycles to failure, defined by $N_{f50}$ is not consistent in the real damage state and cannot correctly characterize the actual service life of the structure.

Reese (1997) suggested a new approach to define fatigue failure using the upper limit of the phase angle. During cyclic loading, the measured phase angle usually exhibits a stable increase if damage occurs and is then followed by a sharp decrease. Fatigue life, or number of cycles to failure, $N_f$, is defined as the cycle when the sharp decrease occurs (Figure 3.3). This approach is believed to be more theoretically based, because it is tracked through the material’s viscoelastic property, and this reverse in direction must represent a transformation in the dominating mechanism inside the material. Qualitatively, it is also found that the damage state at the number of cycles to failure, $N_f$, is much more consistent compared to the
$N_{f50}$ between the same two mixtures. Hence, throughout this thesis, the failure point is defined based on the phase angle criterion.

Figure 3.3 Fatigue life defined by Reese’s (1997) approach.

Figure 3.4 Location of $N_f$ for two different asphalt mixtures.
3.4 Permanent pseudo strain

As mentioned earlier, the CX cyclic test employs a mixed mode of loading that accumulates significant mean strain during its history. However, the simplified VECD model discussed in Section 2.1.5 takes into account only the magnitude of the stress and strain. Thus, it is necessary to check the corresponding permanent pseudo strain in order to evaluate the relative error associated with it. Before any simplification is applied, the pseudo strain is calculated rigorously through time-stepping so that the permanent pseudo strain also is recorded throughout the history.

Instead of calculating the convolution integral in Equation (1.1), a state variable approach is utilized to reduce the computation time. The state variable approach transforms the process of convolution into a recursive form (Simo et al. 1998). The relaxation modulus of the material $E(t)$ is represented in the Prony series as:

$$E(t) = E_{\infty} + \sum_{i=1}^{M} E_i e^{-\frac{t}{\rho_i}}$$

where

$E_{\infty} =$ the relaxation modulus at $t = \infty$, and

$E_i =$ the Prony coefficients that correspond to the relaxation times, $\rho_i$.

The pseudo strain can be calculated from:

$$\varepsilon^{B(n+1)} = \frac{1}{E_R} \left[ \eta_0^{n+1} + \sum_{i=1}^{M} \eta_i^{n+1} \right]$$

(1.25)
\[ \eta_{0}^{n+1} = E_{e} \left( \varepsilon_{0}^{n+1} - \varepsilon_{0}^{0} \right) \]  
(1.26)

\[ \eta_{i}^{n+1} = e^{-\Delta t/\rho_{i}} \eta_{i}^{n} + E_{e} e^{-\Delta t/2\rho_{i}} \left( \varepsilon_{0}^{n+1} - \varepsilon_{0}^{n} \right) \]  
(1.27)

where

\[ \eta_{i}^{n+1} = \text{the internal state variable for element } i \text{ at time step } n+1, \]

\[ \varepsilon_{0}^{n+1} = \text{physical strain at time step } n+1, \text{ and} \]

\[ \Delta t = t_{n+1} - t_{n}. \]

The internal state variable \( \eta_{i}^{n+1} (i = 1...M) \) records the history of the material up to the time \( t_{n+1} \). Figure 3.5 shows the stress versus pseudo strain calculated from the rigorous scheme. It is found that even though significant physical strain accumulates during the process, the actual permanent pseudo strain is kept at a low level, even to failure. The pseudo-hysteresis loop almost centers at zero with a growth in the hysteresis band. Hence, it is appropriate to simplify the calculation of pseudo strain using the steady state formulation shown by Equation (1.20), and the instantaneous secant pseudo stiffness \( (C_{s}^{*}) \) at each cycle can be approximated by the cyclic magnitude-based stiffness \( (F) \), which is:

\[ C_{s}^{*} \approx F = \frac{\sigma_{0,pp}}{\varepsilon_{0,pp} \ast I} = \frac{\sigma_{0,pp}}{E_{\ast} \ast \varepsilon_{0,pp} \ast I} \]  
(1.28)

In Equation (1.28), the secant pseudo stiffness is simply the dynamic modulus at cycle \( i \) normalized by the linear viscoelastic complex modulus at the corresponding reduced frequency.
Figure 3.5 Stress vs. pseudo strain at different cycles in CX testing.
Chapter 4 Micromechanical Damage Modeling

This chapter contains the summary of thoughts related to micromechanical understanding of damage. While not directly related to the main contribution of the thesis (Chapter 5), it is presented for the sake of completeness, and with the expectation that it may be useful for future research.

4.1 Introduction

As a parallel to the VECD model, the so-called micromechanical damage model also describes a material’s behavior during the distributed damage stage prior to localization. Unlike the VECD model that smears all the damage into a single parameter $S$, the micromechanical method quantifies damage using detailed micro-crack distributions. Hence, the damage may no longer be a scalar value but has a specific physical representation. The advantages of the micromechanical model are that it could provide a better understanding of the different stages of the damage process and also provides a platform for crack interaction and coalescence to be considered in terms of localization behavior. In addition, the micromechanical damage model can capture the effects of the microstructure to some degree, because in a predamaged material such as asphalt concrete, it is believed that the initiation and development of micro-cracks are related to the size and orientation distribution of the aggregate structures. For example, Krajcinovic (1986) proposed a micromechanical model that relates the material’s behavior to the volume fraction and size distribution of aggregate particles, which determines the initial distribution of the micro-cracks under monotonic loading.
In the micromechanical damage model, damage is usually modeled as the summation effect of the opening of active micro-cracks under external loading, which generates an additional strain component and makes the apparent stiffness value decrease. Kachnov (1992) provides a comprehensive review of efforts to determine the effective elastic properties of micro-cracked solids. Although most micromechanical theories have been developed for elastic materials, their conclusions can also be applied to viscoelastic materials under cyclic loading, whereby the pseudo stiffness is calculated using the same procedure as is used for elastic materials.

4.2 Micromechanical modeling of asphalt concrete

Asphalt concrete is a complex material with heterogeneities at various scales. Wittmann (1983) states that the mechanical components of concrete can be observed on at least three different scales: micro, meso, and macro (Table 4.1). Extending this terminology to asphalt concrete, the explored micromechanical model focuses on the material at the meso-scale. In the micromechanical damage model, asphalt concrete is considered to be an assemblage of micro-cracks embedded in a statistically homogeneous matrix. The matrix consists of asphalt binder and a distribution of aggregate particles, and the property of the matrix is dependent on the combination of these two constitutive components. The reason for this separation at meso-scale is that micro-cracks are the major damage source investigated in micromechanical modeling.
Table 4.1 Hierarchy of Structure Scales Defining the Mechanical Responses of Concrete

<table>
<thead>
<tr>
<th>Scale</th>
<th>Volume element</th>
<th>Defect</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Micro</td>
<td>Cement paste, xerogel, aggregate</td>
<td>Atomic voids,</td>
<td>Material science</td>
</tr>
<tr>
<td></td>
<td></td>
<td>crystal defects</td>
<td>models</td>
</tr>
<tr>
<td>Meso</td>
<td>Unit cell containing statistically valid samples of phases</td>
<td>Micro-cracks,</td>
<td>Micromechanical</td>
</tr>
<tr>
<td></td>
<td></td>
<td>large pores</td>
<td>models</td>
</tr>
<tr>
<td>Macro</td>
<td>Concrete specimen</td>
<td>Macro-cracks</td>
<td>Fracture mechanics</td>
</tr>
</tbody>
</table>

Even though the theories of damage mechanics and micromechanics together build the framework for relating damage stiffness values to any given distribution of microcracks, the investigation here that employs the micromechanical damage model adheres to the simplest case for a qualitative understanding only, because quantification of the microstructure and the distribution of microcracks are not the main focus of this thesis.

If the microcracks are generally small in size and the overall microcrack density stays at a low level, the reduced material stiffness can be written as an explicit function of microcrack density using the non-interactive dilute solution in micromechanics (Nemat-Nasser and Hori 1993). In the non-interactive dilute form, no interaction is considered between adjacent micro-cracks, and the effect of all the microcracks is implemented in the model simply as a superposition of each individual microcrack. The complete expression for the effective stiffness value as a function of micro-crack size distribution and orientation distribution is derived in Krajcinovic (1996).
In general, the distribution of micro cracks can be determined using two approaches. One approach is an advanced visual method, such as using an scanning electron microscope (SEM) to track the microcracks directly, and the other is a parameter study of the relationship between the distribution of microcracks and the distribution of aggregate particles. Due to the limitation of the current research scope, neither of these two approaches was adopted in this study. Instead, the orientation of the microcracks is assumed to be that of uniform alignment, which means that the microcracks are perpendicular to the loading axis. In reality, this assumption is reasonable for the case of tensile loading, because all the microcracks tend to propagate perpendicular to the maximum principle stress. The effective modulus of an elastic body that contains aligned penny-shaped cracks, as determined by Nemat-Nasser and Hori (1993), is written as:

\[
\frac{E}{E_0} = \left(1 + f \frac{16(1 - \nu^2)}{3}\right)^{-1} 
\]

(2.1)

\[
f = \sum_{\alpha=1}^{n} f_\alpha, \quad f_\alpha = M_\alpha a_\alpha^3, \quad M = \sum_{\alpha=1}^{n} M_\alpha
\]

(2.2)

where

\[
E \quad = \text{damaged modulus with microcracks},
\]

\[
E_0 \quad = \text{undamaged modulus without microcracks},
\]

\[
f_\alpha \quad = \text{density of microcracks with radius } a_\alpha,
\]

\[
M_\alpha \quad = \text{number of microcracks with radius } a_\alpha,
\]

\[
f \quad = \text{overall microcrack density, and}
\]

\[
M \quad = \text{total number of microcracks}.
\]
The overall microcrack density $f$ can also be written in a probability form:

$$ f = N \int_{a^-}^{a^+} p(a) \, da = N < a >^3 $$

(2.3)

where

- $a^+, a^-$ = maximum and minimum microcrack length at current time,
- $p(a)$ = probability of microcrack with length $a$, and
- $\langle a \rangle$ = equivalent average microcrack length.

In reality, the ratio between the damaged and undamaged modulus values as expressed in Equation (2.1) is the same value as for the pseudo stiffness during cyclic loading, so

$$ C = \frac{E}{E_0} $$

(2.4)

By comparing Equation (2.1) into Equation (2.4), it is found that the overall microcrack density $f$ is actually proportional to the additional compliance, which is calculated by $\frac{1}{C} - 1$.

The density of the microcracks has been linked with the material’s macroscopic properties. Hence, even without advanced visual tools, the status of the micro-crack evolution inside the material can still be evaluated through macroscopic observations.
4.3 Evolution of micro-crack density

Because the additional compliance can be regarded as a reflection of the overall micro-crack density, it is interesting to plot the additional compliance with the number of cycles. Figure 4.2 shows the typical evolution history of the additional compliance under CX cyclic loads. It is observed that the additional compliance generally does not start from zero in the heterogeneous material, as it does in asphalt concrete, which means that damage exists even before the load is applied. This pre-existent damage generally is comprised of the initial defects within the structure that occur during the casting process.
According to the history of the additional compliance, the development of micro-cracks can be categorized into three general stages, if the history of the additional compliance is fitted locally into a power functions as

\[ f = AN^\alpha \]  

(2.5)

where

\[ N \quad = \text{number of cycles, and} \]
\[ A, \alpha \quad = \text{fitted parameters.} \]

The three stages are the deceleration stage, linear stage and acceleration region, respectively, depending on the value of \( \alpha \).

(1) Deceleration stage (\( \alpha < 1 \))
During the deceleration stage, the additional compliance grows in a power law function, and the overall microcrack density increases at a reduced rate. The reduced rate in the micro-crack evolution generally can be explained by the toughening mechanism that is due to the heterogeneous microstructure. When the size of the crack is relatively small compared to the amount of heterogeneity, its propagation cannot be quantified by the traditional stress intensity factor or the J-integral, because the stress at the crack tip is within the area of local fluctuation. From experimental observations, cracks of this type generally are found to propagate with a reduced rate that is caused by the increasing crack resistance, such as crack deflection or aggregate shielding. Cracks at this scale are referred to as “small fatigue cracks” in Suresh (1998), which shows an abnormal propagation behavior compared to “normal fatigue cracks”.

(2) Stable linear stage ($\alpha = 1$)

As the micro-cracks grow in size, their behavior begins to transition to that of cracks of regular size. At the same time, the number of propagating micro-cracks also stabilizes when that number reaches a certain value. Hence, the overall crack density at this stage evolves at a constant rate.

(3) Acceleration stage ($\alpha > 1$)

When the evolution rate begins to accelerate, it is assumed that micro-crack interactions have begun to occur and that the micro-cracks begin to propagate rapidly. During this stage, the material’s behavior becomes unstable and can no longer be characterized by the dilute distribution. It is interesting to observe that the drop in phase angle happens much later in the acceleration stage. Hence, it may be hypothesized that the drop in phase angle
does not actually correspond to the start of localization but to the macro-fracture. The ending point of the stable linear stage is defined as the localization point, which signifies the occurrence of crack interaction and will be discussed in later chapters.
Chapter 5 Development of the Failure Criteria

5.1 Introduction

The failure criterion defines the applicable regions associated with continuum damage theory and indicates the occurrence of fracture. In engineering, the failure criterion is important because it characterizes a mixture’s fatigue life, which is an essential parameter in evaluating the mixture’s ability to resist fatigue damage. However, most continuum damage models cannot predict failure automatically, which means that certain external criteria must be applied. Through the research in this thesis, fatigue failure is defined as the cycle at which the phase angle drops. The failure criterion developed in this chapter also adheres to this definition.

5.2 Stiffness-based criterion

Damage and effective stiffness are the two essential, interrelated parameters in continuum damage models. Hence, the development of a failure criterion starts with the simplest assumption that material fails at a critical damage state, or at an equivalent critical stiffness value. However, asphalt concrete is a highly heterogeneous and time- and temperature-dependent material; therefore, it is not easy to define a universal stiffness-based criterion for different asphalt mixtures across different temperatures. Hou (2009) made an attempt to develop a failure envelope through the experimental observations of twelve different mixtures at various temperatures. The pseudo stiffness value at failure, which is the pseudo stiffness at the point where the phase angle drops, is expressed as a function of reduced
frequency and the nominal maximum size of aggregate (NMSA). After optimization, these two analytical functions of the failure envelope are proposed for RAP and non-RAP mixtures separately. Even though this failure envelope is logically reasonable, it is still under scrutiny due to the high variability shown in the experimental data. Moreover, the sensitivity of the proposed failure envelope with regard to new types of mixtures also needs to be investigated.

Figure 5.1 shows the distribution of the failure stiffness $C_f$ as a function of reduced frequency for the S9.5C and VTe30LC. Because the loading frequency for all the tests is the same (10 Hz), reduced frequency can be interpreted as the parameter that characterizes the temperature effect only. Asphalt concrete generally behave in a brittle manner at low
temperatures (high reduced frequency) and become more ductile when the temperature is elevated. Hence, it is reasonable to assume that failure stiffness is an increasing function of reduced frequency, because the tolerance for damage is low for cases of brittle material. Even though this brittle-to-ductile transition can be seen roughly in Figure 5.1, large scatter still exists within each reduced frequency.

5.2.1 Development of localization envelope

In Chapter 4, it was concluded that the drop in phase angle is related more to macro-fractures than to the beginning of micro-crack interactions and coalescence. Because the process of crack coalescence in heterogeneous materials such as asphalt concrete is believed to be highly random, it is prudent to look at the ending point of stable damage accumulation, which is referred to as the localization point. The relative locations of the localization point and failure point are shown in Figure 5.2.
As mentioned before, the localization point is defined as the deviation point in the history of the material’s additional compliance $(\frac{1}{c} - 1)$, which may be related to overall micro-crack density. To minimize the variability in the process of searching for localization points, a program has been written that locates all the localization points automatically (Figure 5.3). The methodology is as follows.

1) Calculate the slope of additional compliance $(\frac{1}{c} - 1)$ versus the number of cycles $N$ within a fixed window size.
2) Initially, the slope decreases monotonically at a reduced rate. When the slope becomes stable and varies within 5%, the corresponding cycle is taken as the starting point of the linear region.

3) The ending point of the linear region is defined as the point at which the slope has increased 20% compared to the previous cycle.

4) A linear regression is performed for all points inside the linear region.

5) The localization point is the point on the additional compliance history that is 0.01 above the regression line. A summary of the determined points of localization for all tests is given in Table 5.1.

Figure 5.3 Methodology for defining the point of localization.
Table 5.1 Summary of Failure Stiffness and Localization Stiffness for All Tests

<table>
<thead>
<tr>
<th>Material</th>
<th>Specimen Name</th>
<th>Temperature (°C)</th>
<th>Cycle at Localization ( (N_f) )</th>
<th>Localization Stiffness ( (G_f) )</th>
<th>Cycle at Failure ( (N_f) )</th>
<th>Failure Stiffness ( (G_f) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>S9.5C</td>
<td>S9.5C-26</td>
<td>5.00</td>
<td>51600</td>
<td>0.70</td>
<td>69990</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>S9.5C-27</td>
<td>4.90</td>
<td>124850</td>
<td>0.66</td>
<td>140700</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>S9.5C-41</td>
<td>4.60</td>
<td>880</td>
<td>0.71</td>
<td>1420</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>S9.5C-13</td>
<td>19.05</td>
<td>38100</td>
<td>0.44</td>
<td>44600</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>S9.5C-14</td>
<td>19.00</td>
<td>257960</td>
<td>0.46</td>
<td>308000</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
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<td>0.23</td>
<td>1241</td>
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</table>
Figure 5.4 presents the localization stiffness values obtained from the methodology as a function of reduced frequency. Compared to failure stiffness, $C_f$, the stiffness at localization, $C_l$, is much more consistent, and the brittle-to-ductile transition trend is also more explicit. In general, localization occurs at a high level of pseudo stiffness as the reduced frequency increases. It is also interesting to observe that the localization stiffness is an approximate linear function of reduced frequency in semi-log space, and the slopes of the function for these two mixtures are similar to each other. This observation happens to be in accord with the hypothesis for the development of the failure envelope in the work by Hou (2009). Hence, an analytical function is proposed for the envelope of localization instead of for failure. Note that because calibration data are not available for reduced frequencies greater than 10 or less...
than 0.01, it is assumed that the localization pseudo stiffness neither increases or decreases beyond this range. The localization envelope within this range is a linear function in semi-log space, whereas the slope is a function of NMSA, and the intercept is affected by the inclusion of RAP in the mixture. The piecewise fitting function is given in Equation (3.1), and the corresponding coefficients are listed in Table 5.2. The fitted localization envelope also is presented in Figure 5.5.

\[
C_f = \begin{cases} 
    a \log (0.01) + b & f_R \leq 0.01 \\
    a \log (f_R) + b & 0.01 \leq f_R < 10 \\
    a \log (10) + b & f_R \geq 10
\end{cases}
\] (3.1)

where

\[ f_R \] = reduced frequency, and
\[ a \] and \[ b \] = coefficients.

### Table 5.2 Coefficients for Localization Envelope

<table>
<thead>
<tr>
<th>Mix</th>
<th>NMSA</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>S9.5C (Non-RAP)</td>
<td>9.5</td>
<td>0.111082</td>
<td>0.541475</td>
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<td>VTe30LC (RAP)</td>
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<td>0.111082</td>
<td>0.376219</td>
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</tbody>
</table>
5.2.2 Fatigue test prediction

Even though localization provides a consistent damage state and distribution envelope, the ultimate purpose of the failure criterion is still to predict the fatigue life in terms of failure. After localization, there may still be considerable time for the material to reach failure, which means that the structure can still provide an adequate and even significant service life. Test results indicate that the required time from localization to failure is generally proportional to the total fatigue life. Hence, the relationship between the cycle at localization and the cycle at failure can be described by a linear function, which is shown in Figure 5.6.
The general procedure for predicting fatigue life using the localization envelope is a two-step process. The first step is to obtain the number of cycles at localization \((N_l)\) from the VECD model, once the analytical function of the damage characteristic curve is calibrated for the mix, the amount of damage can be calculated for a known pseudo strain history by assuming an initial damage value, e.g., 0.1. The predicted damage for a prescribed pseudo strain history is then as follows:

\[
S_{i+1} = S_i + \left( \frac{1}{2} \left( \varepsilon_0^{R,t,a} \right)^2 \frac{dC}{dS} \right)^\alpha K_1 d\varepsilon
\]

where

\[\varepsilon_0^{R,t,a} = \text{pseudo strain tension amplitude, and}\]
\[K_1 = \text{adjustment factor for tensile loading time.}\]

Based on the calculated damage history, the pseudo stiffness history is also obtained at each cycle through the analytical relationship between damage and stiffness. The cycle at localization \((N_l)\) is determined as the cycle when the pseudo stiffness reaches the value on
the localization envelope for the corresponding reduced frequency. Then, the second step is to extrapolate the number of cycles at localization ($N_l$) to the number of cycles at failure ($N_f$). The extrapolation relation is assumed to be a linear function for a given mixture that is irrespective of temperatures. However, the coefficients of the linear function may vary with mixture type. In this analysis, the extrapolate coefficients are determined separately for mix S9.5C and VTe30LC, which are shown in Figure 5.6. Figure 5.7 and Figure 5.8 present the measured and predicted fatigue life following the above procedure for all the tests and the associated prediction error is listed in Table 5.3. It can be seen that the method of localization generally provides a reasonable prediction of fatigue performance at all temperatures. However, this reasonable prediction is relied on the accuracy of developed localization envelope and also the extrapolation relation for the mixture, which usually requires quite a number of calibration tests.

Figure 5.7 Controlled-crosshead cyclic test simulation results: (a) S9.5C mix and (b) VTe30LC mix.
Figure 5.8 Comparison of measured and predicted fatigue life in (a) arithmetic scale and (b) log scale.
Table 5.3 Summary of Fatigue Test Prediction Results Using Localization Envelope

<table>
<thead>
<tr>
<th>Material</th>
<th>Specimen Name</th>
<th>Temperature (°C)</th>
<th>Measured $N_f$</th>
<th>Predicted $N_f$</th>
<th>Prediction Error (%)</th>
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5.3 Dissipated energy approach

The so-called dissipated energy approach is another favored approach in characterizing fatigue damage for asphalt mixtures. Branco et al. (2008) state that the dissipated energy approach has the potential to reduce the high variability in results commonly observed from laboratory fatigue tests. The basic idea behind the dissipated energy approach is to quantify
the energy consumed during damage accumulation and relate it to the final fatigue life of asphalt mixtures.

One of the earliest attempts to characterize fatigue damage using the dissipated energy concept for asphalt mixtures was made by Van Dijk et al. (1972), who assumed that the total cumulative dissipated energy until failure for a given material is constant. However, this hypothesis was not supported by experimental observations, and the total cumulative dissipated energy was later fitted into a power function of fatigue life (Van Dijk and Visser 1977). The reason for this change was the realization that the energy associated with viscoelasticity had not been separated out properly. Ghuzlan and Carpenter (2000) developed the concept of *ratio in dissipated energy change* (RDEC) and argued that it is not the dissipated energy itself but the change in dissipated energy between consecutive cycles that represents damage. A plateau value (PV) in the RDEC was identified through the loading history, and failure was defined as the cycle when the RDEC exhibits a significant increase. Shen and Carpenter (2005) later proposed that the relationship between PV and fatigue life is unique and insensitive to the mode of loading, or even to different types of mixtures. The concept of PV also is said to be successful in correcting the abnormal behavior at low damage found in traditional fatigue analysis.

Research also has been conducted using *dissipated pseudo strain energy* (DPSE), in which the effects of viscoelasticity are first uncoupled via the correspondence principle. Kim et al. (2003) employed the concept of DPSE in the characterization of micro-crack growth. Masad et al. (2008) related the rate of change in DPSE to the J-integral in fracture mechanics and
assessed the fatigue cracking potential of different asphalt mixtures. However, one significant dilemma in the DPSE approach was the proper determination of the viscoelastic phase angle and the differentiation of behavior between nonlinear viscoelasticity and damage. Si et al. (2002) developed the nonlinear reference modulus to estimate the true viscoelastic phase angle through initial cycles. The premise for this hypothesis is that the phase angle and concomitant dissipated energy in the first few cycles are due mainly to viscoelasticity, which may result in an underestimation of damage at high strain amplitudes. Masad et al. (2008) demonstrated that the nonlinear phase angle could be obtained via experiments through a careful selection of strain or stress amplitude. However, this method could be verified only though control strain testing and still did not provide a definitive limit between nonlinearity and damage.

Although the above approaches have all shown positive results in some sense, none of them has been calibrated for direct tension cyclic testing of asphalt mixtures. In addition, most of the methods listed above were investigated at only one temperature, and the definitions for fatigue failure are not consistent. Thus, in this section, two typical dissipated energy approaches are applied to the asphalt mixtures under CX cyclic loading at different temperatures. For consistency, the fatigue failure in the analysis is uniformly defined in terms of phase angle.
5.3.1. Total dissipated pseudo strain energy

For a material under cyclic loading, if the loading branch and unloading branch in the stress-strain plot do not collapse, the enclosed area inside the loop is called \textit{dissipated strain energy}.

For a linear viscoelastic material under sinusoidal loading, the dissipated strain energy for a given cycle in the steady state can be approximated by the following elliptical formulation (Figure 5.9):

\[ W_i = \sigma_i \varepsilon_i \sin(\phi_i) \]  

(3.3)

where

- \( W_i \) = dissipated strain energy at cycle \( i \),
- \( \sigma_i \) = stress amplitude at cycle \( i \),
- \( \varepsilon_i \) = strain amplitude at cycle \( i \), and
- \( \phi_i \) = the phase angle between the stress and strain signals at cycle \( i \).
Figure 5.9 Dissipated strain energy for a given cycle in the steady state. However, dissipated strain energy does not correspond to damage exclusively; it also includes the energy that is associated with viscoelastic damping. The correspondence principle provides a method to eliminate the effect of viscoelasticity by replacing the physical strain with an equivalent pseudo strain. After the transformation, the newly formed hysteresis in the stress-pseudo strain space is called *dissipated pseudo strain energy*, or DPSE. For the same sinusoidal loading in the steady state, the physical strain ($\varepsilon$) and corresponding pseudo strain ($\varepsilon^R$) are:

$$\varepsilon = \varepsilon_0 \sin(\omega t)$$  \hspace{1cm} (3.4)

$$\varepsilon^R = E^* \varepsilon_0 \sin(\omega t + \varphi)$$  \hspace{1cm} (3.5)

The dissipated pseudo strain $W_i^R$ at cycle $i$ can be calculated as:

$$W_i^R = \pi \sigma_i \varepsilon_i^R \sin(\phi_i - \varphi)$$  \hspace{1cm} (3.6)
where

\[ E^* \] = the undamaged complex modulus at a given reduced frequency,

\[ \varphi_i \] = the apparent phase angle measured from testing at cycle \( i \), and

\[ \varphi \] = the true phase angle that is related to viscoelasticity.

In general, the hysteresis loop for the DPSE becomes narrower in shape compared to the dissipated strain energy because a viscoelastic portion is subtracted. If there is no damage, the experimentally measured apparent phase angle \( (\varphi_i) \) will be approximately equal to the true phase angle \( (\varphi) \) calculated from the linear viscoelastic solution. Hence, the pseudo hysteresis loop converges into a line, and the DPSE reduces to zero. Figure 5.10 shows the dissipated strain energy and DPSE for the initial cycles under low strain amplitude cyclic loading. From the figure, it is obvious that when no damage occurs, the pseudo loop vanishes, and the hysteresis loop in the stress-strain space is mainly due to viscoelasticity.

Figure 5.10 Dissipated strain energy and dissipated pseudo strain energy for low strain amplitude.
For an applied load that exceeds the material’s endurance limit, the measured apparent phase angle usually exhibits a stable increase as the cycle continues (Figure 3.3). If linear viscoelasticity is assumed, in which the true phase angle ($\varphi$) is kept constant under a fixed reduced frequency, the pseudo hysteresis will gradually increase because ($\varphi_i - \varphi$) increases with cycles. The pseudo hysteresis transforms from a line into an elliptical shape as it increases in size. Figure 5.11 shows that the stress-pseudo strain hysteresis varies with cycles under CX cyclic loading. The DPSE per cycle, which is the enclosed area of the loop, increases with cycles, and the incline of the loop also lowers due to a reduction in stiffness value.

Figure 5.11 Pseudo hysteresis under CX cyclic loading.
If the DPSE calculated by Equation (3.6) contributes only to damage at each cycle, the total DPSE up to failure, which is the summation of the DPSE at all cycles, would characterize the total energy required for fracture during cyclic loading. The total DPSE is calculated as:

$$TDPE = \sum_{i=1}^{N_f} W_i^R$$  \hspace{1cm} (3.7)

In the calculation of dissipated pseudo strain energy at cycle $i$ ($W_i^R$), the true phase angle, $\varphi$, is considered for two cases. In the first case, linear viscoelasticity is assumed, and the true phase angle is obtained directly from the Prony’s coefficients and set to be a constant in all the tests at the same temperature. In the second case, the true phase angle is evaluated from the initial cycles of each test separately. Experiments with polymers and asphalt binders have shown that the phase angle varies with applied strain even without damage (Knauss and Emri 1987, Masad et al. 2008). Hence, the value of the true phase angle is determined by collapsing the initial DPSE to zero and is set to be equal to the measured apparent phase angle, $\varphi_i$, for the initial cycles. The motivation for the second case is to compensate for the effects of nonlinear viscoelasticity to some extent.
Figure 5.12 Total dissipated pseudo strain energy for the VTe30LC mix at 13°C.

Figure 5.12 shows the total DPSE for four tests of the VTe30LC mix with different CX strain values at 13°C. It is found that no matter which method is used to calculate the true phase angle, the total DPSE significantly depends on the total fatigue life, which is inversely proportional to the applied CX strain. This finding means that something other than damage is inside the pseudo hysteresis that is cumulative by the number of cycles. Hence, the DPSE calculated through Equation (3.6) is not an accurate interpretation of damage, and the total DPSE cannot be used to characterize the fatigue failure in asphalt mixtures.
5.3.2 Rate of change in dissipated strain energy

Ghuzlan and Carpenter (2000) investigated damage by examining the change in dissipated strain energy between consecutive cycles. It is assumed that the energy due to viscoelastic damping within the dissipated strain energy at each cycle does not change much along the loading history. Hence, the change in dissipated energy between cycles represents damage, and the RDEC characterizes the percentage of energy that is consumed due to damage compared to the total dissipated energy at each cycle. The formulation for the RDEC is shown in Equation (3.8).

\[
RDEC = \frac{W_{n+1} - W_n}{W_n}
\]

(3.8)

where

\[RDEC = \text{the ratio of dissipated strain energy change, and}\]

\[W_{n+1}, W_n = \text{dissipated strain energy at cycle } n + 1 \text{ and } n.\]

Throughout the loading history, the relationship between the RDEC and the number of cycles can be described in terms of three regions: the initial reduced region, the stable region and the final failure region. The initial reduced region can be explained by the transition and stabilization of the material. During CX cyclic tests, the mean stress and mean strain are still varying significantly during this stage. The second, stable region corresponds to the steady state of damage accumulation and fatigue crack growth. The RDEC value at this stage is referred to as plateau value (PV). Failure is defined as the cycle when the RDEC starts to jump, that is, when a large contribution of energy to damage moves in an unstable manner. The fatigue life associated with the definition of the RDEC is written as \(N_{f,t}\), which is to be
distinguished by the failure defined by the phase angle $N_f$, which is number of cycles at the drop of phase angle. The results presented in Figure 5.13 show that the concept of RDEC also works for CX cyclic tests, and they also show the distinction of the three stages.

Before investigating further, it is worthwhile to evaluate the relationship between the fatigue life as determined by the RDEC and the fatigue life as determined by the phase angle. Figure 5.14 presents the number of cycles to failure that is determined using both approaches. It is interesting to observe that the actual results from these two approaches are quite similar, which means that the definition of fatigue failure by phase angle is also consistent with the dissipated energy explanation.

![Figure 5.13 History of RDEC during CX cyclic loading.](image-url)
The RDEC approach demonstrates a strong correlation between the PV and the number of cycles to fatigue failure ($N_{f_t}$). This finding makes sense because the PV characterizes the rate of energy input for damage, and the fatigue life $N_{f_t}$ reflects the total damage that the material can tolerate. The most recent relationship was developed by Shen and Carpenter (2003). It reveals a statistically unique relationship between the PV and number of cycles at 50% reduced initial stiffness ($N_{f_{50}}$), and this relationship is independent of the mode of loading, temperature and even mix type. However, as explained below, after detailed analysis, it was found that this unique relationship does not derive from the characteristic of the
material but from some pseudo effects of the fitting process. In addition, the fatigue life \( N_{f50} \) in the relationship is not an appropriate characterization of fatigue failure.

The fitting procedure comes with the calculation of the PV. In reality, because of test noise and variability, it is usually difficult to identify an absolute plateau region along the history of the RDEC. Hence, Shen and Carpenter (2005) proposed an analytical method instead. In the first step, the history of dissipated energy versus the number of cycles is fitted into a power function:

\[
DE_a = An^f
\]  

(3.9)

where

\( DE_a \) = dissipated strain energy at cycle \( a \),

\( n \) = number of cycles, and

\( A, f \) = fitting parameters.

Then, the RDEC can be simply calculated using the following equation:

\[
RDEC_a = \frac{DE_a - DE_b}{DE_a \cdot (b - a)} = \frac{1 - (1 + \frac{b-a}{a})^f}{b-a}
\]

(3.10)

where

\( RDEC_a \) = the ratio of dissipated energy change at cycle \( a \) compared to cycle \( b \).

If the RDEC is evaluated every 100 cycles, i.e., \( b - a = 100 \), the expression for the RDEC at cycle \( a \) becomes:

\[
RDEC_a = \frac{1 - (1 + \frac{100}{a})^f}{100}
\]

(3.11)
Then, it is assumed that the number of cycles at 50% reduced initial stiffness \((N_{f50})\) is located at stable stage II, which is generally observed through testing. So, \(PV\) is approximately equal to the value of the RDEC at \(N_{f50}\); thus:

\[
P V = \frac{1 - (1 + \frac{100}{N_{f50}})^f}{100}
\]

Given the expression in Equation (3.12), the \(PV\) can be calculated for all the tests once the Figure 5.15 shows the results of the \(PV\) and \(N_{f50}\) obtained using this method for the CX cyclic tests. It seems that the relationship between the \(PV\) and \(N_{f50}\) for the direct tension tests is also consistent with the proposed universal relationship. However, after careful examination of the procedure, it is found that this statistically unique relationship is not caused by the real nature of the material, but only by a pseudo effect of the fitting process.

For example, if \(N_{f50} \gg 100\), then \((1 + \frac{100}{N_{f50}})^f \approx 1 + f \cdot \frac{100}{N_{f50}}\), so \(PV \approx -\frac{1}{N_{f50}}\). The relationship between the \(PV\) and \(N_{f50}\) is incorporated inherently in the calculation of the \(PV\). So, this statistically unique relationship does not actually reflect the real characteristic of the material.

In order to exclude the effects of the fitting process, the \(PV\) is re-evaluated directly from the test results. The \(PV\) is calculated by averaging the values of the RDEC in the plateau region (Stage II in Figure 5.13), and the relationship between the \(PV\) and \(N_{f50}\) is re-plotted in Figure 5.16. From the figure, it is seen that a correlation still exists between the \(PV\) and \(N_{f50}\) for
each mixture and each temperature. However, these relationships do not necessarily coincide uniquely and may depend on both the mixture type and temperature.

Figure 5.15 Relationship between PV and \( N_{f50} \) for direct tension tests.

Figure 5.16 Relationship between PV and \( N_{f50} \) without fitting for the two mixtures: (a) S9.5C and (b) VTe30LC.
In the plots for the PV and fatigue life, such as shown in Figure 5.16, the relationship curve can be shifted horizontally if the representation of the number of cycles to failure in the x-axis changes. In Shen and Carpenter (2003), the number of cycles to failure was seen as $N_{f50}$. The motivation behind using this definition is that, based on the experiments conducted on fatigue beam tests by Chuzlan (2006), $N_{f50}$ and $N_{ft}$ are found to be related through a linear function. Hence, the actual fatigue life $N_{ft}$ is replaced by $N_{f50}$ for the purpose of reducing the testing time. However, this replacement does not appear to work for the direct tension tests considered in this work. Figure 5.17 shows the relationship between the $N_{f50}$ and $N_{ft}$ for the two different asphalt mixtures under CX cyclic loading. Even though a linear relationship still exists between the $N_{f50}$ and $N_{ft}$ for all cases, the exact relationship coefficients may differ for the different mixtures and different temperatures. For example, for mix S9.5C, $N_{f50}$ and $N_{ft}$ are approximately equal to each other at all temperatures. But for mix VTe30LC, the ratio of $N_{f50}$ to $N_{ft}$ decreases as the temperature elevates. Thus, a simple extrapolation function between the $N_{f50}$ and $N_{ft}$ does not exist, and the use of $N_{f50}$ as the number of cycles to failure causes inconsistent deviations from the real fracture and failure of the material. In order to relate the PV to the actual failure of the material, the fatigue life on the x-axis should remain as the $N_{ft}$, and the corresponding relationship between the PV and $N_{ft}$ is regenerated, as shown in Figure 5.18. It is interesting to observe that, once the number of cycles to failure is defined consistently with respect to actual real-life failure, the relationships between the PV and $N_{ft}$ at different temperatures tend to collapse, which is a clue that it may be possible to develop a characteristic relationship between the rate of energy
dissipation and fatigue life. If this is the case, it will be much more efficient to characterize fatigue failure than the stiffness-based criterion, which usually requires multiple tests at different reduced frequencies to develop the failure envelope or localization envelope.

Figure 5.17 Relationship between $N_f$ and $N_{f50}$ for the two mixtures: (a) S9.5C and (b) VTe30LC.

Figure 5.18 Relationship between PV and $N_f$ without fitting for the two mixtures: (a) S9.5C and (b) VTe30LC.
5.4 Proposed failure criterion

Even though the RDEC approach tends to develop a characteristic relationship that links the ratio of energy dissipation and the fatigue life, and the results are not too objectionable, a significant obstacle remains that prevents this approach from being incorporated into the VECD model for further application. Up to now, all the dissipated energy approaches have been investigated primarily through experiments only. The histories of stress, strain and phase angle were all available for calculation. However, the current version of the VECD model focuses on the quantification of damage and effective stiffness, and the change of time dependency in terms of phase angle is not captured. Hence, the VECD model cannot predict the variation in phase angle, so the dissipated strain energy and the DPSE presented in Equations (3.3) and (3.6) cannot be obtained. Therefore, none of the mentioned dissipated energy approaches could be implemented with only the output from the VECD model.

However, this problem does not suggest that the VECD model should be modified or the change in phase angle must be included in the model right away. First, the change in phase angle in asphalt mixtures usually is observed to be much less compared to that found in asphalt binders. Second, it is the degradation of the stiffness that is of more interest than the change in time dependency for engineering purposes in cyclic loading. Finally, the exact mechanism for the variation in phase angle, whether it is nonlinear viscoelasticity, fatigue cracking or plasticity, remains unclear. Given these considerations, the VECD model is being maintained in its current form for modeling asphalt concrete’s behavior up to localization. However, a new energy measure that represents the rate of damage dissipation has been
developed. The newly proposed dissipated energy measure is compatible with the VECD model and focuses on the dissipated energy associated with stiffness reduction only.

5.4.1 General concept

Instead of looking at the hysteresis area inside the loop, which depends on the variation in phase angle, energy is evaluated in a cumulative sense. During cyclic loading, the maximum amount of stored pseudo strain energy within each cycle appears at the point of peak stress. This point also corresponds to the point of maximum pseudo strain. According to Figure 2.4, the maximum stored pseudo strain energy at cycle $i$ can be calculated as:

$$ (W^R_{max})_i = \frac{1}{2} (\sigma^R_{max})_i (\varepsilon^R_{max})_i = \frac{1}{2} (\sigma^R_{0,t,a})_i (\varepsilon^R_{0,t,a})_i. $$

(3.13)

In the rigorous form of the VECD model, the $\sigma^R_{0,t,a}$ and $\varepsilon^R_{0,t,a}$ can be linked through the magnitude-based cyclic pseudo stiffness, $F$

$$ \sigma^R_{0,t,a} = F R \varepsilon_{0,t,a}. $$

(3.14)

Hence, the maximum stored pseudo strain energy at cycle $i$ can be re-written as:

$$ (W^R_{max})_i = \frac{1}{2} (R F)_i (\varepsilon^R_{0,t,a})_i^2. $$

(3.15)

The maximum amount of stored pseudo strain energy reflects the material’s current ability to store energy. As the damage accumulates, the material loses stored energy for the same magnitude of applied maximum pseudo strain because of the reduction in pseudo stiffness, $F$. The newly proposed dissipated energy approach compares the current available maximum pseudo strain energy to the corresponding undamaged state at each cycle and assumes that the difference between them represents a cumulative loss of energy due to damage.
propagation. This dissipated energy is called \textit{total released pseudo strain energy} and is denoted as $W_C^R$ in the following analysis.

Because the total released pseudo strain energy relates only to the cumulative released pseudo energy that is due to stiffness reduction rather than to the pseudo hysteresis loop at each cycle, the elliptical hysteresis loop can be simplified into a line with the slope of the current pseudo stiffness. The total released pseudo strain energy is calculated by evaluating the enclosed triangular area between the current damage line and the virginal line without damage for the same maximum pseudo strain. The schematic representation for the $W_C^R$ is shown in Figure 5.19, and the formulation is given in Equation (3.16).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5_19.png}
\caption{Schematic representation of total released pseudo strain energy $W_C^R$ in the stress-pseudo strain space.}
\end{figure}
\[(W_c^R)_i = \frac{1}{2} (\varepsilon_{0,ta}^R)_i^2 (1 - F_i) \]  

(3.16)

where

\[(W_c^R)_i = \text{total released pseudo strain energy at cycle } i, \]

\[(\varepsilon_{0,ta}^R)_i = \text{pseudo strain amplitude at cycle } i, \text{ and} \]

\[F_i = \text{pseudo stiffness at cycle } i. \]

From Equation (3.16), it can be seen that the total released pseudo strain energy is determined by two factors: one is the pseudo strain amplitude, \(\varepsilon_{0,ta}^R\), and the other is the material’s pseudo stiffness, \(F\). In this way, the same amount of total released pseudo strain energy can be obtained at either a high applied pseudo strain or with a relatively large reduction in stiffness. Therefore, the total released pseudo strain energy \(W_c^R\) is a comprehensive energy measure, because it includes the information from both the external loading and the material itself.

Because the \(W_c^R\) represents the released pseudo strain energy in a cumulative sense, the derivative of the \(W_c^R\) with respect to time (cycle number) would represent the rate of released pseudo strain energy, which is released pseudo strain energy per cycle. The history of the \(W_c^R\) calculated from the experimental data for the CX cyclic tests is shown in Figure 5.20. The pseudo stiffness \(F_i\) in Equation (3.16) is approximated by the formulation of the simplified VECD (S-VECD) model shown in Equation (1.28).
For almost all the CX cyclic tests, the trend of the $W_c^R$ can be categorized into three regions: the initial region, the stable region and the final failure region. Consistent with the observations of additional compliance discussed in Section 5.2.1 and the RDEC in Section 5.3.2, the initial region and the final failure region can contribute to the reorientation of the material and unstable damage propagation, respectively. Interest is focused on the second, stable region in which the rate of releasing pseudo strain energy is almost a constant.

If the released pseudo strain energy is all dissipated for damage, the constant value of the released pseudo energy rate characterizes a steady rate of damage accumulation. For
continuous cyclic loading, this steady rate also reflects the material’s ability to resist fatigue damage. Hence, it would be interesting to see the relationship between this stable rate of damage accumulation and the final fatigue life. The stable rate of pseudo strain energy release is referred as $G^R$ for the remainder of this thesis.

In order to reduce the variability during the calculation process, the same methodology described in Section 5.2.1 is applied again to identify the stable linear region automatically. The results for the methodology are shown in Figure 5.21, and $G^R$ is calculated by a linear regression of the identified region.

Figure 5.21 Result of methodology in identifying stable region.
The relationship between \( G^R \) and the fatigue life \( N_f \) for all the tests are presented in Figure 5.22. For both mixtures, \( G^R \) and the fatigue life \( N_f \) are strongly correlated, and this correlation is not affected much by temperature. This finding proves the previous hypothesis that a characteristic relationship exists between the rate of damage accumulation and the fatigue life of a material. In general, the characteristic relationship can be regarded as a fundamental property of the material and is consistent over a wide range of temperatures. This concept is, in reality, also quite similar to the concept of the damage characteristic proposed in the VECD model, which states that a unique relationship exists between damage and stiffness for all loading conditions if viscoelastic damage is the dominant damage mechanism.

The characteristic relationships derived for mix S9.5C and mix VTe30LC are shown in Figure 5.23. Overall, the characteristic relationship reflects the material’s ability to resist fatigue failure. For example, for the same stable rate of pseudo-strain energy release, if one material has a relatively longer fatigue life, this material can tolerate more damage at failure than a material with a relatively short fatigue life. However, the characteristic relationship only quantifies the fatigue life via the rate of energy dissipation. If the evaluation of fatigue performance starts from the applied strain or stress, the bridge between the applied strain/stress and rate of energy dissipation has to be built first, which is not necessarily independent of temperature.
Figure 5.22 Relationship between stable rate of pseudo strain energy release $G^R$ and fatigue life $N_f$.

Figure 5.23 Characteristic relationship for the two mixtures: (a) S9.5C and (b) VTe30LC.
5.4.2 Fatigue test prediction

Once the characteristic relationship is obtained, it can be utilized with the VECD model to predict the fatigue life. As mentioned in Section 5.4.1, the advantage of using the characteristic relationship as the failure criterion is that it could significantly reduce the number of tests required for the development of the failure envelope. For the stiffness-based criterion, to develop the failure envelope or localization envelope for each mixture, fatigue tests must be conducted at multiple reduced frequencies, and within each reduced frequency, usually more than one test is necessary to reduce the variability. Hence, the total number of required tests may be quite considerable. However, to develop the characteristic relationship between the stable rate of pseudo strain energy release and the fatigue life, calibration tests are required at only one temperature (reduced frequency), because the relationship has been proven to be statistically consistent among the different temperatures. So, in practice, the characteristic relationship can be derived from four tests at a single intermediate temperature where viscoelastic damage is dominant.

Given the characteristic relationship between \( G^R \) and the fatigue life, together with the damage characteristic in the VECD model, the material’s fatigue life can be predicted for various loading conditions. To predict the fatigue life, the general format is as follows.

1. For a given pseudo strain history, predict the evolution of pseudo stiffness (\( F \)) using the VECD model.
(2) Input the history of the pseudo stiffness and maximum pseudo strain at each cycle into Equation \((3.16)\) and calculate the history of the total released pseudo strain energy, \(W_C^R\).

(3) Identify the stable linear region during the history of the \(W_C^R\) based on the proposed methodology and evaluate the stable rate of pseudo strain energy release \(G^R\).

(4) Determine the corresponding fatigue life \(N_f\) for the characteristic relationship for the given \(G^R\).

In order to evaluate the ability and accuracy of the proposed failure criterion, fatigue test prediction is conducted first on the available two mixtures. The analytical function of the damage characteristic curve and the corresponding coefficients for the S9.5C and VTe30LC mixes are listed in Table 5.4.

<table>
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<th>Test temperature (°C)</th>
<th>(a)</th>
<th>(b)</th>
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<tr>
<td>13</td>
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<table>
<thead>
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<th>Test temperature (°C)</th>
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<th>(C_{12})</th>
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<td>19</td>
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Table 5.4 Analytical Functions and Coefficients for Damage Characteristic Curve

Given the damage characteristic curves, the histories of the pseudo stiffness, \(F\), and total released pseudo strain energy, \(W_C^R\), can be predicted for all tests. The \(G^R\) determined from the predicted \(W_C^R\) is compared with the experimentally measured value shown in Figure 5.24. It is found that the predicted \(G^R\) are generally close to the measured results except at very
large load levels. Hence, the linearity of the characteristic relationship is ensured, even for
the predicted stable rate of pseudo strain energy release, and the applicability of the
proposed failure criterion is confirmed.

Figure 5.25 shows the characteristic relationship developed from the model prediction at
13°C and 19°C for mixes S9.5C and VTe30LC, respectively. The final results for the
predicted fatigue life are presented in Figure 5.26. It can be seen that the proposed failure
criterion also provides a reasonable prediction across the different temperatures. The largest
prediction error appears at 5°C for mix S9.5C. This is because the damage characteristic
curves between C and S at this temperature is not collapsing well with the representative
curve that is used for prediction at 19°C. The detailed prediction errors for all tests are
summarized in Table 5.5.

![Predicted Stable rate $G^R$ vs Measured Stable rate $G^R$](image1)

![Predicted Stable rate $G^R$ vs Measured Stable rate $G^R$](image2)

Figure 5.24 Comparison of stable rate of pseudo strain energy release $G^R$ between measured
and predicted results: (a) arithmetic scale and (b) log scale.
Figure 5.25 Characteristic relationship derived from predicted stable rate of pseudo strain energy release.

\[ y = 268721x^{-0.615} \quad R^2 = 0.9913 \]
\[ y = 903013x^{-0.726} \quad R^2 = 0.998 \]

Figure 5.26 Comparison of measured and predicted fatigue life in (a) arithmetic scale and (b) log scale.
<table>
<thead>
<tr>
<th>Material</th>
<th>Specimen Name</th>
<th>Temperature (°C)</th>
<th>Measured $N_f$</th>
<th>Predicted $N_f$</th>
<th>Prediction Error (%)</th>
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Chapter 6 Investigation of Post-localization Behavior

6.1 Nonlinear Viscoelastic Parameter

Throughout this thesis, the failure point of asphalt concrete under cyclic loading has been defined as the cycle at which the phase angle drops. Even though this definition is considered to be qualitatively logical and also appears to be consistently utilized in the dynamic modulus diagram (Figure 3.3), it is nonetheless worthwhile to investigate the underlying mechanisms that lead to failure and determine the major differences between a material’s behavior before and after the failure point, and then find the corresponding modeling approach for each behavioral pattern. Thus, a comparison is made here of the measured on-specimen stress and strain before and after the failure point. For simplicity, any points on the curves after the failure point are referred to as post-localization.

Figure 6.1 shows that the major feature of post-localization behavior is that the stress and strain signals begin to exhibit irregularity and that a relatively large deviation emerges that differs from the original single frequency function. However, this kind of irregularity is almost negligible even up to a few cycles before the drop in the phase angle. Hence, it may be hypothesized that the drop in phase angle is associated with the occurrence of a large amount of irregularity in the stress and strain signals.
As is known, the traditional method used to calculate the phase angle is to fit the stress and strain signals into a sinusoidal function first and then evaluate the phase shift between them. For example, if the measured stress and strain signals are fitted separately as

\[ \sigma(t) = A_\sigma \sin(\omega t + \varphi_\sigma) \]  
\[ \varepsilon(t) = A_\varepsilon \sin(\omega t + \varphi_\varepsilon) \]

then the phase angle is calculated by

\[ \varphi = \varphi_\sigma - \varphi_\varepsilon \]

However, if the measured stress and strain deviates significantly from a sinusoidal function, then the calculated phase angle derived from the traditional method becomes inaccurate because the fitted sinusoidal function is not a good representation of the actual experimental data.
To better quantify the extent of the deviation of the measured signals compared to the one frequency harmonic function, a parameter referred to as the *nonlinear viscoelastic parameter* (NVP) is introduced. According to linear viscoelasticity, if the applied load is a single frequency harmonic function, the material’s response should also be the same frequency harmonic function. Hence, any response other than this frequency can be attributed to nonlinear viscoelasticity. Take the stress signal as an example. If the stress can be expressed as a complete expansion,

\[ \sigma(t) = \frac{\sigma_0}{2} + \sigma_1 \sin(\omega t + \varphi_1) + \sigma_2 \sin(2\omega t + \varphi_2) + \sigma_3 \sin(3\omega t + \varphi_3) + \ldots \]  

(3.20)

then the NVP is defined as:

\[ NVP = \frac{\sigma_2 + \sigma_3 + \cdots + \sigma_n}{\sigma_1} \]  

(3.21)

If no significant deviation or irregularity exists in the signals, then the measured stress nearly coincides with the single frequency harmonic function (Figure 6.1 (a)). Hence, the only non-vanishing coefficients in Equation (3.20) are \( \sigma_0, \sigma_1, \varphi_1 \), and the NVP value is very close to 0. However, as the extent of the deviation and irregularity increases, the higher order terms with higher frequencies gradually emerge. Figure 6.2 presents the stress signals during post-localization. It is obvious that once the irregularity occurs, a single sinusoidal function is not sufficient to capture the material’s behavior, and the measured experimental data are better fitted using higher order terms.

It should be noted that the term *nonlinear viscoelasticity* as it is used here is not specific only to theories related to viscoelasticity. Instead, it is a more general concept that includes all the
nonlinear mechanisms that cannot be explained by linear viscoelasticity, such as plasticity or the fracture process. Based on the experimental data, it is found that the NVP value is generally higher at low temperatures, which is when the plasticity is supposed to be minimal. Hence, it is inferred that the NVP values calculated here are associated mainly with fracture, which is macro-cracking. Furthermore, the high NVP values at low temperatures are caused by the brittle nature of fracture, while at high temperature, the material behavior more ductile so that the signals associated with fracture are also smoother. Hence, the value of NVP at high temperature is also smaller.

Figure 6.2 Stress at post-localization and the fitting results with multiple frequencies.
Figure 6.3 shows the variation history of the phase angle and NVP for a single test during cyclic loading. It is found that the drop in phase angle occurs almost at the same time significant nonlinearity occurs. This observation also is seen consistently in other tests (Figure 6.4). Thus, it can be concluded that the drop in phase angle is highly correlated with the jump in nonlinearity. If this nonlinearity is assumed to be mainly attributable to fracture, then this phenomenon serves as further proof that the drop in phase angle actually characterizes the transition from micro-fracture to macro-fracture and is the proper indicator of the failure point. The NVP history also illustrates that, for the material’s behavior up to the failure point, the stress and strain signals are accurate enough to be considered as single frequency harmonic functions. Hence, the formulations for the VECD model are valid and can be used to model the material until the failure point.
6.2 Behavior during post-localization

It has been demonstrated that the signals measured after the drop in phase angle can no longer be fitted into a single frequency function, and the traditional method used to calculate the phase angle also is no longer accurate. Figure 6.5 indicates that, due to the irregularities in the stress shape, a consistent phase angle is no longer properly defined. The time shift between the stress and strain signals may vary with the location during a single cycle. The calculated phase angles at the different locations are presented in Figure 6.6. It is observed that, after the drop in phase angle, a consistent, defined phase angle no longer exists, and the
phase angle calculated using the traditional method is only an average of the time shifts within the cycle.

Figure 6.5 Evaluation of phase shift at different localizations.
Figure 6.6 Phase angle history calculated at different localizations.

Even though the measured stress and strain signals can no longer be used as input for the continuum damage model after the failure point, they can still be helpful in understanding the material’s behavior during post-localization. Figure 6.7 shows the stress and strain signals during post-localization. The responses prior to the failure point also are given in the same graphs for comparison. During post-localization, the maximum tensile stress gradually diminishes, and the symmetry in the stress magnitude with respect to tension and compression is violated. That is, the material loses the ability to carry a tensile load due to the propagation of macro-cracks. At the same time, the period of the tensile portion of the stress also seems to increase compared to the compressive portion. This phenomenon is
assumed to be caused by the plasticity or other nonlinearity inside the process zone at the crack tip that retards the closing of the macro-cracks. The strain signals do not exhibit much deviation compared to the previous sinusoidal function, unless the strain curves become blunt at the minimum point of the cycle, which is the maximum value at compression. This occurrence can also be attributed to the plasticity associated with macro-crack closure. From these observations, it is concluded that the measured stress and strain during post-localization are governed by the behavior of macro-cracks under cyclic loading.

![Figure 6.7 Comparison of stress and strain signals before and after the failure point.](image)

**6.3 Behavior outside the localization region**

It should be noted that during the actual fatigue testing of asphalt concrete, two types of failure patterns, middle failure and end failure, tend to occur, depending on the location of the finally formed macro-cracks. A schematic plot of these two patterns is given in Figure 6.8. In general, the mid-failure tests are considered to be good tests, because the macro-cracks form within the strain gauge, and the linear variable differential transducers (LVDTs)
capture the damage evolution throughout the entire loading history. The analysis used in this thesis is based on mid-failure tests only. For end-failure tests, macro-cracks localize beyond the experimental measurement range. Hence, the damage evolution is not recorded for the localization and post-localization regions. However, end-failure tests can be used to study the material’s behavior outside the localization region.

![Figure 6.8 Failure locations of CX cyclic tests: (a) middle failure and (b) end failure.](image)

As previously mentioned, after localization, the overall stress inside the specimens begins to reduce due to the propagation of macro-cracks. Hence, the material outside the localization region experiences unloading and healing. Figure 6.9 presents the history of the calculated phase angle and effective stiffness values for the end-failure tests. It is seen that both the stiffness and the phase angle recover to the virgin state after localization occurs. At the end of the test, when the stiffness of the region outside localization has returned to approximately $C = 0.95$, the measured strain can be compared to the predicted pseudo stress-based undamaged properties that are derived from linear viscoelastic theory; the results are
shown in Figure 6.10. It is found that the predicted responses are quite similar to the measured data, which means that the recovered material’s properties are close to the linear viscoelastic solution. Hence, it is concluded that, after localization occurs, the material’s behavior outside the localization region can be simulated via viscoelastic theory together with a healing model.

Figure 6.9 History of stiffness and phase angle for end-failure tests.
Figure 6.10 Comparison of measured strain and predicted strain obtained from LVE properties.
Chapter 7 Conclusion and Research Recommendations

In this thesis, behavior of asphalt concrete under fatigue loading is investigated through pre-localization, localization and post-localization (macro-cracking) stages, with specific emphasis the transition points between these stages. The failure point defined using phase angle is proven to correctly characterize the initiation of macro-fracture. After the failure point, the smooth shapes of the stress and strain signals are interrupted and the material’s behavior is mainly dominated by the propagation of macrocracks.

Development of failure criteria in predicting fatigue life is also intensively studied. It is found that by looking at the point of localization instead of failure, a much more consistent envelope of pseudo stiffness can be built as a function of reduced frequency. However, this type of approach generally requires verification test at multiple temperatures and is considered to be not efficiency enough. A new criterion that based on the concept of dissipated energy is proposed. The results show that the proposed failure criterion is consistent with VECD model and is able to predict the fatigue life for asphalt mixtures under multiple temperatures and applied strain levels based on a unified characteristic relation.

The proposed model on failure criterion needs to be further verified in the future. The model in predicting fatigue life is not just limited to CX cyclic loading. It would be interesting to apply it to simulate purely controlled strain cyclic tests and other modes of loading. Moreover, the extension of the proposed criterion for more complex loading conditions would also be a valuable research topic. The resulting failure model, combined with VECD model, could result in a powerful engineering tool to predict the degradation and failure of asphalt concrete under cyclic loading.
References


