

ABSTRACT

CAYTON, CHARITY SUE-ADAMS. Teachers' Implementation of Pre-Constructed Dynamic Geometry Tasks in Technology-Intensive Algebra 1 Classrooms. (Under the direction of Dr. Karen F. Hollebrands).

Technology use and a focus on 21st century skills, coupled with recent adoption of Common Core State Standards for Mathematics, marks a new challenge for mathematics teachers. Communication, discourse, and tools for enhancing discourse (NCTM, 1991, 2000) play an integral role in successful implementation of technology and mathematics standards.

Dynamic geometry environments represent a tool that has the potential to enhance the discourse within mathematical discussions. Also, Stein, Engle, Smith and Hughes (2008) described five practices for orchestrating productive mathematical discussions intended to help teachers incorporate student thinking into classroom discussions. The 1:1 laptop learning environment provides a context that has the potential to examine the intersection of classroom discourse, pedagogical practices, and technology use.

This study examined the nature of mathematical discourse, extent to which teachers used the five practices, examined design and technology features of pre-constructed dynamic geometry tasks and how teachers use them, and explored the combined influence of discourse, the five practices, and technology use on level of cognitive demand when teachers implement pre-constructed dynamic geometry tasks. The study followed a qualitative, multicase observational research design, where a purposeful sample of three teachers represented individual cases. All participants taught Algebra 1 in two high schools within the same 1:1 computing, high needs school district during the study.

The conceptual framework combined the Mathematical Task Framework (Smith & Stein, 1998) and the five practices for orchestrating productive mathematical discussions into

a dynamic model characterizing task implementation and use of the five practices in 1:1 computing classrooms. The Task Analysis Guide (Smith & Stein, 1998), Sinclair's (2003) design principles for pre-constructed dynamic geometry sketches, and the IQA for Academic Rigor: Mathematical Rubric for Potential of the Task were used to evaluate the potential level of cognitive demand of pre-constructed dynamic geometry tasks. Participants were observed for three teaching sets (Simon, Tzur, Heinz, Kinzel, and Smith, 2000), and excerpts of the video that included use of pre-constructed dynamic geometry tasks served as episodes for analysis. Discourse was analyzed using a modified version of the Oregon Mathematics Leadership Institute Classroom Observation Protocol and Boaler and Brodie's (2004) question types. Drijvers, Doorman, Reed, & Gravemeijer (2010) teacher technology orchestration types were utilized to describe how teachers used pre-constructed dynamic geometry tasks during implementation. The five practices served as codes for interview, group planning, and classroom observation data to characterize the extent of teachers' use of the five practices. The triangulation of discourse, technology use, and use of five practices was compared to the IQA for Academic Rigor: Mathematical Rubric for Implementation of the Task to evaluate the implemented level of cognitive demand of the pre-constructed dynamic geometry tasks.

Findings indicated that teachers relied on teacher to whole class as the predominant mode of mathematical discourse. Questions and statements were the most common type of mathematical discourse, and questions were used slightly more often than statements. Higher level questions were posed more often than lower level questions, but teachers' incorporation of the five practices varied greatly. In terms of technology use, teachers most often orchestrated the pre-constructed dynamic tasks by discussing the screen or explaining the

screen. Teachers used sliders and hide/show buttons to focus student attention and facilitate development of mathematical concepts. Given that pre-constructed dynamic geometry tasks adhered to design principles, two themes emerged for level of cognitive demand; 1) when teachers employed higher level questions/statements in conjunction with a discuss the screen technology orchestration type the implemented level of cognitive demand remained high and 2) when teachers employed lower level questions/statements in conjunction with an explain the screen technology orchestration type the implemented level of cognitive demand decreased.

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Teachers' Implementation of Pre-Constructed Dynamic Geometry Tasks in
Technology-Intensive Algebra 1 Classrooms

by
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A dissertation submitted to the Graduate Faculty of
North Carolina State University
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

Mathematics Education

Raleigh, North Carolina

2012

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DEDICATION

First and foremost, I dedicate this work to my husband, Darren, for your unwavering love, support, and encouragement. Second, I dedicate this work to my children, Zach and Macayla. My vision for your learning has always been my inspiration for growth as a mother and a mathematics educator. Third, to Sybil Faye Cayton, better known as Big Mama, none of this would have been possible without your daily support in our home over the past four years. Finally, to my parents, Sue Faught and William Adams, both of you modeled and instilled in me the core values that govern my life. These values not only guided me toward my dreams, but also helped me bring them to fruition. I love you all!

BIOGRAPHY

Charity Sue-Adams Cayton was born in New Bern, North Carolina on September, 4, 1974. Charity is the daughter of Sue Faught and William Adams. She was raised in Craven County, North Carolina and graduated from West Craven High School in 1992. She received a North Carolina Teaching Fellows Scholarship to attend East Carolina University, where she earned her Bachelor of Science in Mathematics in May, 1996. At graduation, she was honored with the University Award, given to recognize outstanding graduating seniors. After graduation, Charity accepted a teaching position at D. H. Conley High School in Greenville, North Carolina.

On July 13, 1996, Charity married Darren Jay Cayton, and they welcomed their first child, Zachary Hunter, into the world in the spring of 1997. In the fall of 2001, Mrs. Cayton earned *National Board Certification in Adolescent and Young Adult Mathematics*, which she renewed in 2011. Mr. and Mrs. Cayton were blessed with their second child, Macayla Daryn, in the spring of 2002. As her teaching career progressed, Charity received the *Kathryn Hodgkin Award for Teaching Excellence* in 2004, and was a state finalist for the *Presidential Award for Excellence in Mathematics and Science Education* in 2005. While teaching full-time, she earned her Master of Arts in Mathematics Education from East Carolina University in May, 2007. Students' understanding of multiple representations of linear functions served as the focus of her thesis research. Mrs. Cayton left the classroom in the fall of 2008 to pursue a doctorate in mathematics education at North Carolina State University.

Charity worked as a graduate teaching assistant and supervised student teachers when she began her doctoral studies at North Carolina State University. She was also awarded a

Provost Fellowship to help with her first year of doctoral studies. As a graduate teaching assistant, she taught *Introduction to Mathematics Education* for three semesters, and served as an assistant in *Technology in Mathematics Education* for one semester. In January, 2010, Mrs. Cayton became a graduate research assistant with *Scaling Up STEM Learning with the VCL*, under the supervision of Dr. Karen Hollebrands.

Charity accepted a one year research associate position with *Scaling Up STEM Learning with the VCL* in August, 2012. In the future, she hopes to obtain a faculty position where she can work with prospective and practicing mathematics teachers during their undergraduate and graduate studies, respectively.

ACKNOWLEDGMENTS

My mantra throughout my adult life has been, “I can do all things through Christ who strengthens me” (Phillipians 4:13). This belief has never held truer than during the past four years. The path has been long and arduous, but He has been with me every step of the way. For that I am eternally grateful.

While God has been my spiritual rock, my husband, Darren, has been my physical rock. Thank you for your love and support throughout this process. You have often taken a back seat to my school work and the kids, but you have always been understanding and patient with me. You understand me like no one else, and you have believed in me even when I did not believe in myself. I love you and thank God for you every day.

Zach and Macayla, I cannot believe how much each of you have grown over the past four years. When I began this process, Zach was entering middle school, and Macayla had just finished kindergarten. Now, Zach is taller than me, has his driving permit, and his voice is deeper than his daddy’s. You have grown into a fine young man, and I thank you for the clinical interviews and teaching experiments you participated in during my course work. Love you, Wildman. Macayla you amaze me and keep me humble as I see the obstacles you overcome on a daily basis. You have transformed in to a vibrant, young lady with a love of learning and a passion for life. Love you, my little Pinkie Pie.

Mom and Joe, Daddy and Barbara, I could not have asked for more loving and caring parents. Mom, you are the perfect example of how far hard work, determination, and faith can take you. Dad, you blessed me with my curly locks, ‘math brain’, and book sense. Both of you overcame so much to provide me with opportunities that you never had. You instilled

in me the value of education and set me on this path, long before any of us knew exactly where it would lead. Joe and Barbara, thank you for loving me as if I were your own. I have never felt like a step-daughter, and I hope you have never felt like a step-parent. Also, to my wonderful mother-in-law, Sybil, you have been an angel on earth for my family. You have treated me like your own daughter and loved and cared for Zach and Macayla since they were born. I can only hope to be a portion of the mother-in-law and grandmother that you have been.

To my chair, Karen, you provided academic and emotional support, and welcomed me into your family. I have always been amazed at your patience and how you balance the many facets of your life. I am thankful for all you have done, and I am very excited that I will be able to mentor with you for another year as a research associate. Hollylynne, I remember meeting you and Karen for the first time at PME-NA in Toronto. I could not imagine back then the influence you would have on my life. Thank you for helping me transition from the high school classroom to the doctoral program and always having an open door with a box of tissues. Allison, thank you for sharing your insight and lessons learned with me, not to mention always giving wonderfully therapeutic hugs. Paola, thank you for helping me think deeply about professional development and teacher learning. Roger, I appreciate the fact that you joined my committee at a very critical time and helped me transition from a PhD student to a PhD candidate.

I also thank my former colleagues, especially Dr. Robin Angotti, Renea Baker, Lisa Beale, and Jennifer Girona. Robin, you have been my mentor and friend since the first day I began teaching. Thank you for always pushing me to pursue excellence. Renea and Lisa you

are exemplary mathematics educators and lifelong friends. I am blessed to have each of you in my life. Jenny, I thank you for helping me think and live outside of the box. Don't worry, John will be finished with his program sooner than you think.

As a high school teacher, I had the opportunity to teach approximately 1500 students. Each one of you contributed in some way to the educator I am today, so I thank each of you. For those that are no longer with us, thank you for helping me remember how precious life is and to cherish every moment with my family and loved ones.

Finally, I thank the wonderful teachers that participated in this study. Without you none of this would be possible. I am grateful to each of you for opening your classroom to me. Your students are fortunate to have mathematics teachers that are willing to be lifelong learners and bring innovation into the classroom.

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CHAPTER 1: INTRODUCTION

Background

The use of technology and a focus on 21st century skills has become an increasingly important facet of education for teachers and students. Evidence of this can be seen in the creation of standards pertaining to technology use by teachers and students, as well as criteria for mathematics teacher educators in the preparation of mathematics teachers. The International Society for Technology in Education (ISTE) produced the National Educational Technology Standards for Teachers (NETS-T) (2008) and the National Educational Technology Standards for Students (NETS-S) (2007) to provide guidelines for how teachers and students should be using technology in the classroom. While these standards relate to the use of technology broadly in education, the Association of Mathematics Teacher Educators (AMTE) offers recommendations about the use of technology in the mathematics classroom. AMTE aligns their position with the National Council of Teachers of Mathematics (NCTM) technology principle and directly addresses the preparation of mathematics teachers to teach with technology and outlines the types of experiences that should be included to accomplish this goal. While their suggestions are written in reference to teacher candidates, practicing teachers may or may not have encountered such experiences as components of their mathematics teacher education programs. Two ways that mathematics teacher educators may accomplish the goal of preparing practicing mathematics teachers to teach with technology are through graduate courses and professional development. The latter of these, professional development, represents the learning context for teachers in this study.

The recent adoption of the Common Core State Standards for Mathematics (CCSSM) (2010) in all but five states (as of August 2012) marks a new challenge for mathematics teachers. The NCTM process standards and the strands of mathematical proficiency from the National Research Council's report *Adding It Up* were used to build the Standards for Mathematical Practice specified in the CCSSM. Communication is one of five process standards from NCTM. This implies that teachers will need to communicate effectively with their students to successfully implement the CCSSM.

The *Professional Standards for Teaching* (NCTM, 1991) emphasized the role of discourse in the classroom by including the teacher's role in discourse, the student's role in discourse, and tools for enhancing discourse as three of six essential teaching standards. NCTM (2000) resonated the importance of discourse with a description of mathematics teaching and learning that engages students, facilitates students' ability to create and share conjectures, encourages evaluation of ideas presented (by self and others), makes connections among ideas, and reorganizes understanding.

Research has been conducted that describes specific pedagogical practices that support teachers' facilitation of mathematical discussions. Stein, Engle, Smith and Hughes (2008) focused on helping teachers incorporate student thinking into classroom discussions, and included five practices for orchestrating productive mathematical discussions. According to Smith and Stein (2011),

The five practices were designed to help teachers to use students' responses to advance the mathematical understanding of the class as a whole by providing teachers with some control over what is likely to happen in the discussion as well as more time

to make instructional decisions by shifting some of the decision making to the planning phase of the lesson (p. 7).

The five practices include *anticipating* student solution strategies to a task, *monitoring* students thinking as they work on a task, *selecting* student strategies to include in classroom discussions, *sequencing* the selected student strategies in a meaningful way, and *connecting* the strategies presented. Smith and Stein emphasize the setting of mathematical goal(s) and selecting appropriate mathematical tasks to help achieve said goal(s) as essential aspects of planning that must take place prior to implementing the five practices.

Technology, specifically dynamic geometry environments, represents a tool that has the potential to enhance the discourse contained in mathematical discussions. Technology may provide visual images of mathematical ideas, facilitate the organization and analysis of data, provide efficient and accurate computations, and support student investigation in all areas of mathematics by allowing students to focus on decision making, reflecting, reasoning, and problem solving. The 1:1 laptop learning environment provides a context that has the potential to examine the intersection of classroom discourse, pedagogical practices, and technology use. Within this environment, individual teachers and students are provided with access to laptop computers both during and after the school day. Discourse (e.g., Cobb, Wood, Yackely, Nicholls, Wheatley, Trigatti, et al., 1991; North Central Regional Educational Laboratory, 2000; Stein & Lane, 1996) and 1:1 computing (Argueta, Huff, Tingen, and Corn, 2011) have been linked to positive student learning outcomes; however, the intersection of discourse and 1:1 computing within mathematics classrooms has not been sufficiently studied. One area of particular importance is first year high school algebra.

Algebra 1 serves as a gateway for students to reach higher level mathematics. Research focused on Algebra 1 classrooms operating on a 1:1 learning model that includes dynamic geometry technology represent a learning environment that may add to body of knowledge regarding discourse and 1:1 computing.

Teachers face a daunting task to effectively incorporate technological standards (NETS-T and NETS-S) and mathematical standards (CCSSM) in a manner consistent with the vision of mathematics created by NCTM (1991, 2000). AMTE's (2006) propositions about the important role of technology in preparing future mathematics teachers provides insight into how to build specific types of experiences into mathematics teacher education programs that align with these standards. Yet depending upon where and when practicing teachers received their education, they may not have participated in educational experiences that align with AMTE's propositions. This is especially true for teachers employed in rural areas with limited access to universities.

Purpose of the Study

The overarching question that guided this study was, "*How do teachers implement pre-constructed dynamic tasks in technology intensive Algebra 1 classrooms?*" The purpose of this study is to 1) characterize the nature of mathematical discourse, 2) describe the extent to which teachers use the five practices for orchestrating productive mathematical discussions, 3) examine design and technology features of pre-constructed dynamic geometry tasks and how teachers use them, and 4) explore the combined influence of discourse, the five practices, and technology on level of cognitive demand when teachers implement pre-

constructed dynamic geometry tasks. As such, the following research question will be answered:

When involved in technology-intensive, research-based, professional development on

Algebra:

- a. What types of mathematical questions and statements are central to the mathematical discussions that occur among students and the teacher when implementing pre-constructed dynamic geometry tasks?
- b. To what extent do teachers utilize the five practices for orchestrating productive mathematical discussions to support their implementation of pre-constructed dynamic geometry tasks?
- c. What are the design and technological features of pre-constructed dynamic geometry tasks and how do teachers use them during implementation?
- d. What is the overall influence of a-c on the implemented level of cognitive demand of the pre-constructed dynamic geometry tasks?

To answer these research questions, a qualitative, observational multicase study was conducted in a small, rural, high needs school system. The school system had been participating in a 1:1 laptop initiative for approximately four years at the time of the study. The study involved three teachers within two different high schools, where each teacher represented an individual case. Data included teacher interviews, group planning sessions, video-taped classroom observations, pre-constructed dynamic geometry tasks, and field notes. Within case and cross case analysis were instrumental in responding the research questions.

Significance and Limitations of the Study

The significance of this study may be found in several areas. First, the context of rural, high needs schools represents an understudied demographic of teachers and students. Second, this study focuses on the use of pre-constructed dynamic geometry tasks in a novel environment, Algebra 1. To clarify, the pre-constructed tasks focus on content specific to standards based instruction for Algebra 1, and they were created using dynamic geometry software. Third, this study seeks to integrate three essential aspects of the 1:1 laptop mathematics learning environment (i.e., discourse, pedagogical practices that support mathematical discussions, and technology use) to examine level of cognitive demand when teachers implement pre-constructed dynamic geometry tasks. A limitation of the study is the restricted time frame for the study. It is well documented that changes in teachers' practices occurs over extended periods of time (Loucks-Horsley, Stiles, Mundry, Love, and Hewson, 2010). Because this study will take place over a single semester, longitudinal data and analysis will not be possible.

Subjectivity Statement

I left the classroom after teaching high school mathematics for 12 years. During my tenure as a teacher, I focused on conceptual learning using multiple representations and reform-oriented pedagogical practices. I also invited apprentices and student teachers into my classroom because I believe the best way to learn about teaching is through experience and reflection. At the onset of my graduate work, I served as both a teaching assistant for introductory mathematical methods courses and supervised student teachers. The methods course focused on the use of high level tasks and NCTM's *Principles and Standards for*

School Mathematics. At the time of the study, I had served one and one-half years as a graduate research assistant for the grant that provided the professional development for participants in this study. My experiences in the classroom and graduate school provided a wealth of knowledge to draw from throughout this study.

During group planning meetings and classroom observations, I was a participant observer. I facilitated interactions among myself and the teachers and provided support in the classroom throughout the semester. During pre and post observation interviews, I tried to focus on reflection to have teachers think about their emerging practice. Two teachers had participated in earlier phases of the professional development project, so we had an existing rapport. This familiarity helped to facilitate rapport building with the remaining participant.

CHAPTER 2: LITERATURE REVIEW

The purpose of this chapter is to outline research related to teachers' professional development in technology intensive environments. The first two sections of the literature review focus on research pertaining to the context for this study, 1:1 laptop teaching environments and mathematics professional development. Then literature relevant to the role of discourse in teaching and learning mathematics precedes a discussion of pedagogical practices to promote mathematical discussions and level of cognitive demand. The chapter concludes with research findings related to technology professional development, dynamic geometry environments, and tools for examining teachers' use of dynamic geometry environments.

Teaching and Learning in 1:1 Environments

The use of technology was cited by NCTM (2000) as one of six core principles of teaching and learning school mathematics. "The existence, versatility, and power of technology make it possible and necessary to reexamine what mathematics students should learn as well as how they can best learn it" (p. 24). A particular learning environment that personifies the vision of *Principles and Standards for School Mathematics* (NCTM, 2000), where every student access to technology to facilitate mathematics learning under the instruction of a skilled teacher, is the 1:1 laptop learning environment. Within this environment, individual teachers and students are provided with access to laptop computers both during and after the school day. Argueta, Huff, Tinggen, and Corn (2011) performed a meta-analysis using state executive reports from six major statewide 1:1 laptop initiatives throughout the United States, including Florida, Maine, Michigan, North Carolina,

Pennsylvania, and Texas, as well as Henrico County, Virginia. The evaluation studies from these states “found a positive relationship between laptop use and various aspects of the teaching and learning process” (p.11). Evaluators of 1:1 initiatives observed that 1:1 laptop initiatives had the potential to positively influence instructional practice of teachers and student outcomes. For example, a high level of implementation of laptop use was shown to increase student engagement, motivation for learning, and interest in school. Conversely, a low implementation of laptops led to student distraction, as well as indifference and/or negativity with respect to laptop use. With regard to student outcomes, student achievement scores were significantly higher when laptops were used at least once a week. Also, technology skills of students’ and teachers’ improved along with critical thinking, communication, collaboration, and self-directed learning skills when high implementation was present. The meta-analysis also documented changes in pedagogy and classroom practices. Teachers increased their use of technology to develop learning materials, higher order questioning increased, and their instructional practices began to shift away from more traditional approaches toward teaching with more reform-oriented approaches (e.g., student-centered instruction, experiential, hands-on learning activities, and project-based learning) (Argueta et al, 2011).

The evaluation report for North Carolina used a mixed-methods approach involving 1) student, teacher, and administrator surveys, 2) interviews with technology facilitators, and 3) document analysis with test scores, rubrics, and video. According to Corn (2009), North Carolina teachers echoed findings from the meta-analysis that student engagement was increased when using technology, but it could also be a distraction. Also, teachers began to

shift their role in the classroom toward becoming facilitators and coaches. The role of high quality professional development was also discussed in relation to successful 1:1 implementation. Within the meta-analysis, schools with an emphasis on professional development had higher implementation, and the structure of the professional development was critical. Built-in teacher training days and teacher accountability for implementing learned practices were characteristics of professional development in higher implementing schools. This contrasted with lower implementing schools that participated in short professional development sessions during or after school that emphasized proficiency with a particular product. The evaluation report for North Carolina concurred with the need for quality professional development and expressed teachers' need to collaborate and share successful lessons, specific to the 1:1 learning environment. Supporting professional development, having reasonable expectations for integration, modeling of technology use, providing resources and support, and communication were important recommendations for leadership that were included in the North Carolina evaluation report.

The meta-analysis and North Carolina's evaluation report was not specific to mathematics classrooms; however the description of teaching and learning in 1:1 environments is consistent with technology standards for teachers and students, the *Professional Standards for Teaching Mathematics* (NCTM, 1991), the *Professional Standards for School Mathematics* (NCTM, 2000), and CCSSM (2010). The next section describes research findings related to mathematics professional development and reporting practices for professional development studies

Mathematics Professional Development: Research and Reporting

Much has been learned in relation to effective professional development in mathematics education. Research supports that professional development is more effective when teachers have a vested interest (Elmore, 2002 and Guskey, 2003). Further, effective professional development needs to be on-going and coherent; focused on actively learned content combined with how students learn that content; connected to teaching practice, school, and system-wide goals; and promote collective participation among teachers as a community of learners (Darling-Hammond, Wei, Andree, Richardson & Orphanos, 2009; Desimone, 2009; Elmore, 2002; Garet, Porter, Desimone, Birman & Yoon, 2001; Guskey, 2003; Heck, and Banilower, Weiss & Rosenberg, 2008; & Kennedy, 1998). There also needs to be open communication with all stakeholders in a school system (Elmore, 2002).

Consistent with these findings, Polly and Hannafin (2010) synthesized empirically based studies and recommendations to create a learner-centered professional development (LCPD) framework that could be useful for providing professional development and conducting empirical research on teacher learning. Their framework was characterized by a focus on student learning, teacher-ownership, development of content and pedagogical knowledge, collaboration, ongoing support, and reflection. Various technologies were included in the framework that enhanced teachers' and researchers' ability to examine student learning outcomes and teachers' practice, reconcile differences in beliefs and practices, scaffold in-class implementation of technology, and document the impact of LCPD.

Reporting on mathematics professional development. Sztajn (2011) reviewed mathematics professional development studies published in the *Journal for Research in*

Mathematics Education and organized 21 features into five categories (based on Loucks-Horsley et al., 1998). These categories included, 1) knowledge and beliefs, 2) context, 3) goals, 4) critical issues, and 5) strategies. The first category pertains to researchers' knowledge and beliefs about student learning, teacher learning, and adult learning. The next section discusses research pertaining to adult and teacher learning. Various metaphors for adult learning will be described before focusing the discussion of adult learning around teacher learning. The methodology chapter will utilize the remaining four categories suggested by Sztajn (2011) to discuss the Algebra professional development context for this study.

Adult learning. Before embarking on a study involving teachers, attention must be given to how teachers learn. Since teachers are a subset of the adult learning community, literature related to adult learning will be discussed prior to focusing specifically on teacher learning. Daley (1999) studied novices and experts in the field of nursing to examine how they learn to change their practice as they gain more experience. Building upon Dreyfus and Dreyfus (1980, 1985), five stages were described to characterize career development, 1) novice, 2) advanced beginner, 3) competent, 4) proficient, and 5) expert. Novice professionals were found to govern their practice with rule-oriented behaviors, whereas advanced beginners begin to differentiate among situations. However, advanced beginners continue to have great difficulty distinguishing what is important and what is not important when making decisions. Between the three to five year mark, professionals transition into the competent stage, characterized by the ability to organize and plan activities, remain consciously aware of the plan, and feeling that they possess the ability to cope with

unpredictable situations. Once a professional moves into the proficient stage, they begin to have a holistic sense of the work they are doing. At the highest stage, experts possess an intuitive understanding of a given situation and focus on the main issues without wasting considerable time on extraneous aspects of a situation. Two skills that experts draw upon are similarity recognition across situations and realizing that not all tasks, observations and interventions are equally important. As a professional transitions through the five stages, their working paradigm changes from abstract principles to concrete past experiences. They begin to view situations as part of a connected whole and become an involved performer, rather than viewing situations as discrete and unconnected or being a detached observer, respectively. Daley (1999) noted that the predominant theme within the findings was that nurses learned to how to learn within the context of their practice. Novices relied on more formal means such as review of policy/procedures, attendance at continuing education programs, and reading journals. Experts relied more on informal means situated within their practice for gaining knowledge, such as consulting with peers and other health care professionals. Experts also referred to themselves in one of two ways, serving their clients or a resource to their peers, depending on their work context. Novices did not fully comprehend how they came to construct knowledge, whereas experts not only understood how they learned, but also how they created and used knowledge for themselves within the context of their practice. Contextual factors that novices noted as hindrances to their learning included insufficient time, insufficient in-service education opportunities, and low staffing. Experts cited systemic issues (e.g., politics, resources, and organizational structure) as obstacles to their learning.

Daley (2001) expanded upon the previous study by including twenty members from each of four professions (social workers, lawyers, adult educators, and nurses) to examine how learning becomes meaningful across various professions. Professionals reported that experiences, continuing professional education, and interactions with colleagues contributed to their growth and refinement of meaningful knowledge. Transfer of learning was not seen as an outcome of educational opportunities, rather transfer was viewed as an integral part of the meaning-making process. For example, new information gained through continued professional education was added to their professional knowledge via thinking about, acting upon, and identifying feelings related to the new information. Before information was to become meaningful, it needed to connect with other concepts and be used in practice. “In other words, incorporating new knowledge is a recursive, transforming process, rather than a simple, straightforward transfer of information from one context to another” (Daley, 2001, p. 50). Similarly, Webster-Wright (2009) argued for a shift in terminology and a move toward more situated, holistic research regarding the learning of professionals. Research from five broad categories of professionals (teaching, health, business, social sciences, and science) was utilized to describe an alternative to professional development, referred to as *authentic professional learning*. Authentic professional learning was defined as “the lived experience of continuing to learn as a professional” (p. 713). According to Webster-Wright (2009),

Here, practice is not a situation separate from the professional, but a social, dynamic, and integral part of being a professional working in the current context. Such a conceptualization is congruent with many qualitative research approaches involving a holistic sociocultural orientation (p. 725).

This is contrary to the assumptions of much professional development and research that maintains that knowledge is gained outside of professional practice, and then incorporated into professional practice and that learning can be mandated through attendance and engagement in professional development programs. Within authentic professional learning knowledge cannot be viewed as a commodity, rather knowledge is situated in practice and cannot be separated from the individuals experiences and social participation. Three mechanisms for authentic professional learning include learning through experience, learning from reflective action, and learning mediated by context. This view of knowledge and how professionals gain knowledge aligns with the researcher's view and is consistent with research pertaining specifically to teachers.

Teacher learning. Berliner (1992) utilized the five stages described by Dreyfus and Dreyfus' (1980, 1985) to describe teachers' transition along the teaching continuum and noted seven, empirically supported characteristics that expert teachers demonstrated. According to Berliner (1992), expert teachers 1) excel mainly in their own domain and particular contexts, 2) develop automaticity for repetitive operations, 3) are more sensitive to task demands and social situations, 4) are opportunistic in problem solving, 5) view representations of problems and situations as qualitatively different, 6) possess fast and accurate pattern recognition capabilities, and 7) perceive meaningful patterns in their domain of experience. However, teacher learning represents a messy problem space involving the continuum across teacher education, professional development, student outcomes, value based decisions, and intersection of multiple fields of research (i.e., learning theory, teaching, teacher knowledge, teacher induction, professional development, change, and school culture)

(Feiman-Nemser, 2008). Within this messy workspace Feiman-Nemser (2008) outlines a framework for teacher learning that includes learning to think, know, feel, and act like a teacher that takes into consideration the interconnectedness of content, process, and context of learning to teach. To *think* like a teacher, one must examine their beliefs, transition to pedagogical thinking, and develop meta-cognitive awareness. To *know* like a teacher represents a broad range of knowledge. Content knowledge, how to teach content to diverse learners, understanding student development and the influence of culture and language, curriculum, pedagogy, classroom organization, assessment, and the broad purposes of schooling, as well as how these purposes affect their work were listed as necessary components of knowing like a teacher. Depth and breadth of knowledge, as well as how teachers hold their knowledge organized around conceptual frameworks, were important aspects pertaining to teacher knowledge. Also knowledge *for* teaching (learned outside of practice) was contrasted with knowledge *of* teaching (learned in the context of practice) as integral components of teacher knowledge. To *feel* like a teacher involved deeply personal issues dealing with professional identity, self-efficacy, teachers' vision versus the reality of teaching, and dispositions. To *act* like a teacher, one must possess skills, strategies, and routines that inform decisions about what to do and when. "Ultimately teachers must learn to integrate ways of thinking, knowing, feeling, and acting into a principled and responsive teaching practice" (Feiman-Nemser, 2008, p. 699). This framework of teacher learning intersects with cognitive science and other mathematics education research. Lave and Wenger's (1991) concept of teacher learning and communities of practice and Putnam and

Borko's (2000) situative view of cognition serve as two powerful and connected metaphors for teacher learning. Lave (1996) states,

The argument developed by Etienne Wenger and myself (Lave & Wenger, 1991) is that learning is an aspect of changing participation in changing "communities of practice" everywhere. Wherever people engage for substantial periods of time, day by day, in doing things in which their ongoing activities are interdependent, learning is part of their changing participation in changing practices. This characterization fits schools as well as tailor shops. There are not distinguishable "modes" of learning, from this perspective, because however educational enterprises differ, learning is a facet of the communities of practice of which they are composed (p. 150).

Putnam and Borko (2000) echo this view of teacher learning and include specific reference to tools within their situative view of cognition. "Cognition is (a) situated in particular physical and social contexts; (b) social in nature; and (c) distributed across the individual, other persons, and tools" (Putnam and Borko, 2000, p. 4). Similar to the research on adult learning in general, teacher learning is not viewed as a passive process where knowledge is an imparted commodity, rather it is learned in practice and cannot be separated from experiences and social participation. In reference to practicing teachers, Putnam and Borko (2000) described situations for learning that encompassed staff developers working with teachers in their classrooms, teachers bringing issues and examples from their classrooms to group discussions, and summer workshops focused on teachers learning of subject matter; however, specific goals for teacher learning should guide the specific site.

For example, summer workshops appear to be particularly powerful settings for teachers to develop new relationships to subject matter and new insights about individual students' learning. Experiences situated in the teachers' own classrooms may be better suited to facilitating teachers' enactment of specific instructional practices. And, it may be that a combination of approaches, situated in a variety of contexts, holds the best promise for fostering powerful, multidimensional changes in teachers' thinking and practices (p. 7).

They further state that utilizing multiple contexts that incorporate summer workshops, emphasizing theoretical and research-based ideas, with on-going support during the year represents a promising model for assisting teachers in incorporating these ideas into their classrooms. One means of providing support and sharing knowledge may be accomplished by creating discourse communities where members with diverse backgrounds and types of knowledge come together to share and learn from one another's expertise. This is consistent with the third aspect of situative cognition that knowledge is distributed across individuals, persons and tools; however, one must be cognizant of what Richardson (1992) referred to as the *agenda-setting delimita* where the staff developer would like to see particular changes in teachers' practice, but also meaningfully involve teachers in deciding what changes need to be made. "This dilemma is analogous to one faced by the classroom teacher who wants to empower children to build upon their own thinking while simultaneously ensuring that they learn expected subject-matter content" (Putnam and Borko, 2000, p. 9). The aspect of tools represents the final element of teacher learning within the situative view of cognition. Two types of tools, performance and pedagogical, were described based on the work of Salomon

(1993). Performance tools “enhance or change how a task is accomplished” and pedagogical tools “focus primarily on changing user’s competencies” (Putnam and Borko, 2000, p. 10). Video cases represent one type of pedagogical tool. Video cases have been shown to positively impact teacher learning and growth and may serve as a powerful tool for reflection about teaching practice (Sherin & van Es, 2009). One powerful aspect of video cases is that teachers are able to see teaching practice in an environment similar to their own.

Dynamic geometry environments may serve as both a performance and pedagogical tool. Dynamic geometry environments may be used by teachers to create dynamically linked representations. This provides teachers with new ways to present mathematical content. However, teachers must consider how to use the dynamic geometry environment to facilitate student learning, which changes the teacher’s pedagogical competencies. A more detailed discussion of dynamic geometry environments as a tool will be included in the section on dynamic geometry environments and technology use. The next section examines the role of discourse in teaching and learning mathematics.

Similar to Daley (2001), teachers’ creation of new knowledge (e.g., technology, pedagogical practices) was recursive. The learning process involved continuing professional education, classroom experiences, and interactions with colleagues and opportunities were given for teachers to interact throughout the process. All of which were noted by Daley to support growth in meaningful knowledge. This study also aligned with Webster-Wright’s (2009) notion of authentic professional learning because 1) it was situated in teachers’ practice, 2) it took a holistic look at what was occurring in the classroom, and 3) it was a

lived experience for teachers as they learned to incorporate new technology and pedagogical practices.

Role of Discourse in Teaching and Learning Mathematics

The purpose of this portion of the literature review will 1) define mathematical classroom discourse, 2) discuss patterns of mathematical classroom discourse and analysis, 3) share findings related to discourse and student learning, and 4) discuss research on how to develop discursive mathematics classrooms.

Defining mathematical classroom discourse. This section serves three purposes. First, a discussion of *Discourse* and *Mathematical Discourse* (with an intentional capital *D*) will be presented to briefly describe the backdrop for *mathematical classroom discourse* (with an intentional lowercase *d*). Second, *discourse*, as it is defined in various studies, will be synthesized to formulate a definition for *mathematical classroom discourse*.

Discourse and mathematical discourse. Sfard (2000) reported that some research utilized *Discourse* (emphasis on *D*) to connote a broad view of communication that applies across contexts. Gee (as cited in Moschkovich, 2003) defines *Discourse* in the following way:

A Discourse is a socially accepted association among ways of using language, other symbolic expressions, and ‘artifacts’, of thinking, feeling, believing, valuing and acting that can be used to identify oneself as a member of a socially meaningful group or ‘social network’, or to signal (that one is playing) a socially meaningful role. (p. 326).

Thus, *Discourse* in the broader sense describes mechanisms, verbal or otherwise, utilized by individuals to situate themselves as vetted participants within a particular community. Each community is characterized by the discursive practices that members are expected to employ. Further, discursive practices are dynamic rather than static, and these practices often share commonalities with other communities (Sfard, 2000). According to Kieran (2001), *Discourse* occurs both publically and privately (i.e., among individuals in the community or with oneself).

Similarly, “*Mathematical Discourse* (emphasis added) includes not only ways of talking, acting, interacting, thinking, believing, reading, writing but also mathematical values, beliefs, and points of view” (Moschkovich, 2003, p. 326) that cannot be described by a unique set of homogeneous practices. Participation in *Mathematical Discourse* means an individual speaks and acts consistently with “the ways that mathematically competent people talk and act when talking about mathematics” (p. 326). Sfard (2001) described mediating tools and meta-discursive rules that regulate communication. Mediating tools shape the content, or object-level, aspects of discourse. Meta-discursive rules are the “molders, enablers, and navigators of the communicational activities” (p. 28). These elements control the content of exchange and the flow of the exchange, respectively. *Mathematical Discourse* may take place among members of a mathematics community or with oneself while doing mathematics. It is important to note that beliefs (Baxter, Woodward, Voorhies & Wong, 2002; Clement, 1997; Forman & Ansell, 2001; Nathan & Knuth, 2003) and sociomathematical norms (Clement, 1997; Voigt, 1995; Yackel, Cobb & Wood, 1991) undergird the various modes of *Mathematical Discourse*.

Mathematical classroom discourse. The term *discourse* varied in meaning throughout the mathematics education literature reviewed for this study. Definitions of *discourse* included 1) general definitions (including any type of communication), 2) verbal definitions (talk, utterances, discussion, and dialog), and 3) specific definitions that outlined

Table 1: Definitions of Discourse

Source	Definition
Bereiter (1994, p. 9)	progressive discourse-understandings are being generated that are new to the local participants and that the participants recognize as superior to their previous understandings.
Clement (1997, p. 3)	meaningful mathematical discussion-listening on both sides (teacher and students), all parties (teacher-students or students-students) involved, and questioning rather than assuming meaning
Cobb, Boufi, McClain & Whitenack (1997, p. 258)	reflective discourse-characterized by repeated shifts such that what the students and teacher do in action subsequently becomes an explicit object of discussion
Lo, Wheatley & Smith (1991, p. 1)	mathematical discussion-student interpretation of the mathematical task is the focus of the activity, student-to-student interaction encouraged, teacher's role is to facilitate student-to-student communication rather than guide or evaluate
Nathan & Knuth (2003, p. 204)	productive discourse-forms of social exchange which provide participants with an avenue to construct and build on correct conceptions through their interactions with other class members
Pirie & Schwartzberger (1988) as reported by Leonard (1999, p. 3)	purposeful talk on a mathematical subject in which there are genuine contributions and interactions

Note: Groves & Doig (2004) utilize Bereiter's (1994) definition, Jones & Tanner (2002) utilize Cobb et al.'s (1997) definition.

the nature of discourse that must take place. Table 1 presents a summary of six definitions that emerged during this review of literature.

In an effort to synthesize these definitions for the focus of this literature review, I propose the following definition, *mathematical classroom discourse* refers to verbal interactions that focus on mathematics and serve to clarify, challenge, expand, investigate, or justify content, solution strategies, or contributions from self and others. This definition resonates with the type of teaching and learning described by NCTM (1991, 2000) and encompasses all members involved in the learning context, consistent with all six views of discourse presented in Table 1. The verbs utilized give further detail to Leonard's (1999) description of purposeful talk and provide means to build new knowledge that is superior to previous knowledge, similar to Bereiter (1994) and Nathan and Knuth (2003). Further, successful participation in discursive settings requires listening by all parties (Clement, 1997) to allow for information provided to serve as the object of subsequent discussions (Cobb et al., 1997). Finally, the nature of *mathematical classroom discourse* described requires that the teacher serve as an arbitrator in the exchange of ideas (Lo et al., 1991), rather than the primary speaker or provider of knowledge.

Patterns of mathematical classroom discourse and analysis tools. Research supports that patterns involved in *mathematical classroom discourse* may influence student learning. One pattern, Initiation-Reply-Evaluation/Inquiry-Response-Evaluation (IRE), refers to an interaction where a student or teacher initiates a discussion, another individual replies to the initial query, and evaluative statements are offered in reference to the individual's response (Blanton, Berenson & Norwood, 2001; Rosales, Orrantia, Vicente, & Chamoso,

2008). A slight variation of this pattern, Inquiry-Response-Feedback (IRF) was introduced by Inagaki, Morita, & Hatano (1999) and differs from IRE because the statements offered by the teacher or student after the response are designed to illicit feedback from others to continue the discussion, rather than the evaluative statements that tend to end an IRE unit. Other studies chose to characterize the direction of *mathematical classroom discourse* between participants. Teacher-to-student and student-to-student interactions were often studied in an effort to document changes in discursive patterns over time in either whole group or small group settings (Baxter et al., 2002; Leikin & Zaslavsky, 1997; Webb, Nemer & Ing, 2006). One study (Leikin & Zaslavsky, 1997) included interactions with not only teacher and student, but also learning materials. Student-to-learning material, student-to-learning material-to-student, and student-to-learning material-to-teacher were three additional directions of interactions. Research by Heibert and colleagues (Hiebert, Gallimore, Garnier, Hollingsworth, Jacobs, Chui, et al., 2003; Hiebert & Stigler, 2000) reported that most classrooms in the United States follow an IRE pattern in reference to teacher and student interactions. Within the IRE discursive pattern, Franke, Kazemi, & Battey (2007) noted that teacher-to-student and student-to-student talk focused on processes and procedures to arrive at solutions, rather than student thinking and strategies when determining a solution. Chazen (2000) noted that mathematical discourse in high school mathematics classrooms was more teacher-centered than other subjects, but maintained that it was one of three themes related to the heart of teaching algebra.

Franke, et al. (2007) also describes a different pattern of discourse, multidimensional, that focuses on making sense of mathematical ideas. Teachers' questioning during

multidimensional discourse delves into students' mathematical thinking and sharing of their thinking. Boaler and Brodie (2004) examined types of questions posed by teachers (Table 2) to provide a finer grained analysis for what was occurring in secondary mathematics classrooms. The types of questions teachers pose shape the “nature and flow of classroom discussions and the cognitive opportunities offered to students” (p. 780). They found that

Table 2: Teacher Question Types (Adapted from Boaler and Brodie, 2004)

	Question Type	Description
1	Gathering information, leading students through a procedure	Requires immediate answer. Rehearses known facts/procedures. Enables students to state facts/procedures.
2	Inserting terminology	Once ideas are under discussion, enables correct mathematical language to be used to talk about them.
3	Exploring mathematical meanings and/or relationships	Points to underlying mathematical relationships and meanings. Makes links between mathematical ideas and representations.
4	Probing, getting students to explain their thinking	Asks students to articulate, elaborate, or clarify ideas.
5	Generating discussion	Solicits contributions from other members of the class.
6	Linking and applying	Points to relationships among mathematical ideas and mathematics and other areas of study/life.
7	Extending thinking	Extends the situation under discussion to other situation where similar ideas may be used.
8	Orienting and focusing	Helps students to focus on key elements or aspects of the situation in order to enable problem solving.
9	Establishing context	Talks about issues outside of mathematics in order to enable links to be made with mathematics.

most teachers employed information gathering questions, but Smith and Stein (2011), suggest that question types 3, 4, and 5 may be helpful to teachers in facilitating classroom discussions. The use of these types of questions aligns well with multidimensional discourse. In multidimensional discourse, processes and procedures may be included in the discussion; however, the goal is to utilize student ideas and questions to drive the mathematical discourse. This type of discourse requires students to employ a higher level of cognitive demand when participating in mathematical classroom discourse in either whole group or small group settings. Lampert & Cobb (2003) reported that both whole group and small group discussions provided the opportunity for students to make sense of the mathematical ideas being presented when participating in multidimensional discourse.

The Oregon Mathematics Leadership Institute (OMLI) was a five year project funded by the National Science Foundation that focused on K-12 school mathematics. The project involved summer institutes and follow-up activities during the school year. All activities were based on the “belief that understanding and facilitating meaningful mathematics achievement requires a focus on the learner and an emphasis on all levels of student discourse around important concepts in mathematics” (Weaver and Dick, 2006). This project developed an extensive classroom observation protocol specifically designed to examine student discourse in mathematics classrooms. Three levels of student mathematical discourse were included. *Modes* of mathematics discourse included student-to-teacher, student-to-student, student-to-group or class, and individual reflection. *Types* of mathematics discourse were hierarchical; 1) answering, 2) making a statement or sharing, 3) explaining, 4) questioning, 5) challenging, 6) relating, 7) predicting or conjecturing, 8) justifying, and 9)

generalizing. The third involved *tools* for mathematical discourse; 1) verbal, 2) gesturing/acting, 3) written, 4) graphs, charts, sketches, 5) manipulative, 6) symbolization, 7) notation, 8) computers, calculators, and 9) other. This observation protocol only dealt with student discourse, but the various levels include 1) who is speaking, 2) the nature of the exchange, and 3) tools used. Each of these levels represent mediating tools described by Sfard (2001). Year one results of the project indicated that students in participating teachers' classrooms participated in classroom discourse slightly more often, and the cognitive demand was slightly higher; however, these results were attributed to selection of participating teachers, not the professional development project. No further results from subsequent years were found regarding student mathematical discourse. Hollebrands, Cayton, and Patterson (2011) utilized a revised version of the OMLI Classroom Observation Protocol to examine mathematical discourse in 1:1 computing, high school, geometry classrooms. The revised protocol included teacher discourse within the mode of discourse. This amendment allowed the researchers to characterize, 1) who was speaking, and in what direction, 2) the nature of the exchange, and 3) tools that were used. Shifts in modes of discourse were found for both teachers in the study when technology, specifically *The Geometer's Sketchpad*, was used. Teacher initiated discourse decreased, and student initiated discourse increased in one classroom. The second classroom demonstrated a shift from teacher-to-whole class toward teacher-to-student discourse. Results did not show marked changes in the type of discourse, with or without technology use. The two main types of discourse were statements and questions.

According to the OMLI Classroom Observation Protocol, statements and questions occur on the lower end of hierarchical types of discourse; however, a finer grained analysis of statement and question types may provide important insights into the mathematical discourse that occurred (e.g., Boaler and Brodie, 2004). The revised OMLI Observation Protocol, combined with Boaler and Brodie's question types may be a powerful tool for analyzing multidimensional discourse.

Discourse and student learning. The era of accountability for students and teachers mandates continual growth in student achievement. Hiebert and Wearne (1993) studied six second grade mathematics classrooms (2 experimental, 4 control) during an instructional unit focused on place value, multi-digit addition, and multi-digit subtraction. They linked improved student achievement to classroom discourse, as measured by amount of teacher/student talk and types of questions, and reform teaching practices. Although these findings are not broadly generalizable, they do support the notion that teaching and learning environments that encourage classroom discourse and reform practices have the potential to increase student understanding in empirically measureable ways.

Walkowiak (2010) provided examples from research that support the positive impact of multidimensional discourse on student learning (e.g., Cobb, Wood, Yackely, Nicholls, Wheatley, Trigatti, et al., 1991; North Central Regional Educational Laboratory, 2000; Stein & Lane, 1996). According to Walkowiak (2010),

The North Central Regional Educational Laboratory (NCREL) (2000) compared mathematics teachers at higher performing schools to those at lower performing schools. Conversations between teachers and students at lower performing schools

were similar to the IRE pattern. In contrast, the mathematics teachers at the higher performing schools facilitated multidimensional discourse with their students by asking for their mathematical ideas, pushing for the mathematical meaning behind the tasks, and valuing student thinking (p. 25).

Also, the research component of the OMLI project addressed the question: *Can student achievement in mathematics be significantly improved by increasing the quantity and quality of meaningful mathematical discourse in mathematics classrooms?* Findings after the third year of professional development were initially inconclusive;

However, once implementation fidelity traits were taken into account, a positive relationship between project participation and student achievement emerged. The degree to which schools implement the practices promoted by the OMLI project is a significant positive predictor of student performance above and beyond what can be explained by the socioeconomic factor as indicated by the percentage of students who qualify for the free and reduced lunch program. This relationship is particularly acute at secondary levels...(Weaver and Dick, 2009, p. 57).

These findings reinforce the importance of mathematics classroom discourse, but the dilemma of how to create and sustain discursive environments remains. This study examines not only discourse, but also practices to facilitate discourse. Findings may provide further insight into this dilemma.

Developing discursive classrooms. The teacher, the student, and known support structures and constructs for *mathematics classroom discourse* each play integral roles in the development of discursive mathematics classrooms.

Developing teachers' mathematical classroom discourse. The teacher initially decides what is fundamental to the learning environment and, through either direct or indirect means, transfuses these fundamentals into the classroom. *Mathematical classroom discourse* represents one technique available to teachers that desire to create a teaching and learning environment consistent with the vision of NCTM (1991, 2000). A study conducted by Webb et al. (2006) showed that students' *mathematical classroom discourse* mirrored teacher practices when working in small groups. This finding is not surprising since teachers serve as mediators of students' experiences in the classroom (Baxter et al., 2002; Blanton et al., 2001; Cobb et al., 1997; Weber, Maher, Powell, and Lee, 2008), and teacher's beliefs greatly impact students' experiences with discursive practices (Clement, 1997; Nathan & Knuth, 2003).

Leonard (1999) identified four specific teacher variables—pedagogical content knowledge, teacher questioning strategies, use of instructional strategies, and norms—that impacted the amount and quality of *mathematical classroom discourse*. Consequently, the skill set of a particular teacher has the potential to greatly influence the success of *mathematical classroom discourse*. Professional development has been utilized to facilitate teacher growth with respect to classroom discourse. Herbel-Eisenmann and Cirillo (2009) conducted a professional development study with eight secondary mathematics teachers. The project occurred over a course of three years and utilized literature on classroom discourse to “engage classroom teachers in cycles of action research that could improve secondary school mathematics teaching and learning” (p. 7). Participating teachers not only engaged in action-based research, but they also wrote about and published their experiences. Three key findings

emerged by looking across teachers' work. First, the literature on classroom discourse had an influence on how the teacher-researchers thought about talk in their classrooms, and what they did in their classrooms. Second, reflection allowed teacher-researchers time and opportunity to think about what they would like to change and helped them make these changes. Third, community was an integral aspect of the professional development project.

Developing students' mathematical classroom discourse. Students represent a second factor to consider in reference to the development of *mathematical classroom discourse*. Research compiled for this literature review suggests one of two ways to develop habits of *mathematical classroom discourse* in students, direct or indirect transfer. Direct transfer says that expectations for *mathematical classroom discourse* do not transfer on their own and suggests methods may need to be employed where teachers model, bring attention to, and reinforce expectations for *mathematical classroom discourse*, treating them as part of the learning goals for the course (Huang, Normandia, & Greer, 2005; Webb et al., 2006). Indirect transfer means that acquiring habits of *mathematical classroom discourse* occurs as a natural consequence of being a participant in a learning environment patterned around desired habits (Sfard, 2000; Weber et al., 2008). Inagaki et al. (1999) found that transfer varied along cultural lines when comparing American and Japanese mathematics classrooms exhibiting direct and indirect transfer, respectively. Yet Sfard (2000) recognized that "it is...difficult to decide about the ways to teach those advanced meta-rules we eventually deem as indispensable" (p. 184). Also, Baxter et al. (2002) and Webb (1991) pointed to students' mathematical and linguistic ability as variables that may affect *mathematical classroom discourse*. Students need both mathematical understanding of content and

appropriate verbal means to communicate their ideas effectively. Deficiencies in either area may diminish the students' ability to actively participate in the learning environment.

Support structures and constructs for mathematical classroom discourse. Research suggests that specific support structures are required to sustain high levels of *mathematical classroom discourse*. Support structures that overtly impact *mathematical classroom discourse* and can be documented through the actual dialog include scaffolding (Baxter et al., 2002; Jones & Tanner, 2002; Nathan & Knuth, 2003; and Rosales et al., 2008), revoicing (Baxter et al., 2002 and Forman & Ansell, 2001), questioning/probing (Groves & Doig, 2004; Hiebert & Wearne, 1993; Jones & Tanner, 2002; and Piccolo, Harbaugh, Carter, Capraro, & Capraro, 2008), and use of robust learning materials (Groves & Doig, 2004; Heibert & Wearne, 1993; Leonard, 1999; Jones & Tanner, 2002; Webb, 1991; Weber et al., 2008).

The work of Cobb and colleagues contributed several underlying constructs that undergird *mathematical classroom discourse*; however, these constructs are not necessarily observable in verbal exchanges. Collective reflection refers to the “joint or communal activity of making what was previously done in action an object of reflection” (Cobb et al., 1997, p. 258). One example noted in this study involved a student justifying why the class had exhausted all possible combinations of two numbers that add to a given sum by organizing them systematically into number pairs. Parallel and equivalent interpretations, taken as shared, and intersubjectivity were explicated by Cobb, Yackel, & Wood (1992). Parallel and equivalent interpretations occur when a student offers an idea and all parties assume that others share the same underlying meaning of the idea. When this assumption is

true and no observable evidence exists to contradict the assumption the interpretation is considered to be equivalent. If evidence is observed that contradicts this assumption the interpretation would be parallel. In either parallel or equivalent interpretation, when all parties assume their view is shared by others the meaning is said to be taken as shared. During this process members also experience intersubjectivity, which refers to the assumption that others interpret what we say as we intended, unless evidence is provided to the contrary.

Summary. This portion of the literature review 1) defined mathematical classroom discourse, 2) discussed patterns of mathematical classroom discourse and analysis, 3) shared findings related to discourse and student learning, and 4) discussed research on how to develop discursive mathematics classrooms. It is appropriate to end this section with a quote that encompasses the complex nature, power, and influence of *discourse*:

Discourse refers to the ways of representing, thinking, talking, agreeing, and disagreeing that teachers and students use to engage. ... The discourse embeds fundamental values about knowledge and authority. Its nature is reflected in what makes an answer right and what counts as legitimate mathematical activity, argument, and thinking. Teachers, through the ways they orchestrate discourse, convey messages about whose knowledge and ways of thinking and knowing are valued, who is considered able to contribute, and who has status in the group. (NCTM, 1991, p. 20).

Five Practices for Orchestrating Productive Mathematics Discussions & Level of Cognitive Demand

Sfard (2000) noted two limitations inherent to discourse by emphasizing the extreme difficulty in establishing “appropriate measures of discipline and rigor in school mathematical discourse” (p. 184), and deciding how to teach elements of *mathematical classroom discourse*. A combined analysis of *mathematical classroom discourse*, using the modified version of the OMLI Classroom Observation Protocol and Boaler and Brodie’s (2004) question types, may provide a model that characterizes and potentially serve as a measurement tool for the rigor of mathematical discourse. One possible way to address the latter limitation may be found through professional development designed to increase teachers’ awareness of 1) elements of classroom discourse (i.e., revised OMLI codes for modes and types, question types) and 2) the five practices for orchestrating productive mathematical discussions (Smith & Stein, 2011).

Specifically, the five practices include anticipating, monitoring, selecting, sequencing, and connecting student responses within the context of a mathematics lesson. *Anticipating* refers to teachers’ proactive consideration of students’ likely responses to challenging mathematical tasks. However, anticipation involves much more than identifying the correct answer to a task. The teacher must consider 1) how students will mathematically interpret a problem, 2) correct and incorrect approaches to the task, 3) how these interpretations and strategies “relate to the mathematical concepts, representations, procedures, and practices that the teacher would like his or her students to learn” (p. 8). The practice of *monitoring* student responses requires teachers to be cognizant of students’

mathematical thinking and solution strategies during the process of solving the mathematical task. It is of great importance during this time that the teacher utilizes questioning strategies to elucidate and allow for clarification of student thinking, ensure that all students are engaged in solving the task, and ensure students are attending to important details necessary for solving the task. After anticipating and monitoring student responses, the practice of *selecting* particular students to present their solutions during whole-class becomes the focus for the teacher. The selection process should be informed by the mathematical goal(s) of the lesson coupled with how the teacher perceives each solution method will help accomplish said goal(s). Once particular students have been selected to share their solutions, the teacher must decide upon the *sequencing* of the solution methods. This practice provides the teacher with a myriad of options. The teacher may choose to sequence solution strategies from more concrete to more abstract. Another option may be to sequence student strategies by the particular solution strategy that the majority of students utilized to solve the task, then move on to valid but less prevalent solution strategies. Further, sequencing could be based on students' use of particular representations. Regardless of the how a teacher chooses to sequence presentation of student strategies, the sequencing should be aligned with the mathematical goal(s) for the lesson. The final practice focuses on *connecting* student strategies. During this practice teachers connect salient features throughout the various solution methods and tie all solution methods in with the mathematical goal(s) for the lesson.

While not presented as specific practices for facilitating mathematical discussions, the setting of mathematical goal(s) and selecting appropriate mathematical tasks to help achieve said goal(s) represent essential aspects of planning that must take place prior to implementing

Lower-level demands	Higher-level demands
<p><u>Memorization</u></p> <ul style="list-style-type: none"> • Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas, or definitions to memory • Cannot be solved by using procedures, because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure • Are not ambiguous. Such tasks involve exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlies the facts, rules, formulas, or definitions being learned or reproduced 	<p><u>Procedures with connections</u></p> <ul style="list-style-type: none"> • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas • Suggest, explicitly or implicitly, pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts • Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representation helps develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding
<p><u>Procedures without connections</u></p> <ul style="list-style-type: none"> • Are algorithmic. Use of the procedure is either specifically called for or is evident from prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done or how to do it. • Have no connection to the concepts or meaning that underlies the procedure being used • Are focused on producing correct answers instead of on developing mathematical understanding • Require no explanations or explanations that focus solely on describing the procedure that was used 	<p><u>Doing mathematics</u></p> <ul style="list-style-type: none"> • Require complex and non-algorithmic thinking – a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example • Require students to explore and understand the nature of mathematical concepts, processes, or relationships • Demand self-monitoring or self-regulation of one's own cognitive processes • Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task • Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions • Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required

Figure 1: Task Analysis Guide (Smith and Stein, 1998)

the five practices (Smith and Stein, 2011). Some teachers working with Smith and Stein (2011) referred to the setting of mathematical goals as ‘practice 0’, situating it as the foundation of the remaining five practices. In the Task Analysis Guide (Figure 1), mathematical tasks are organized by either lower-level or higher-level of cognitive demand, where cognitive demand refers to the amount of reasoning and thinking students will need to employ to solve a particular task. Within these two levels, tasks are further delineated into four categories, 1) memorization, 2) procedures without connections, 3) procedures with connections, and 4) doing mathematics. Teachers may use the Task Analysis Guide to evaluate the level of mathematical tasks during their planning for a lesson; however, the initial categorization of mathematical tasks is not static. Tasks initially appear in curricular/instructional materials, but they are implemented in classrooms by teachers and students with the eventual outcome of student learning (Figure 2).

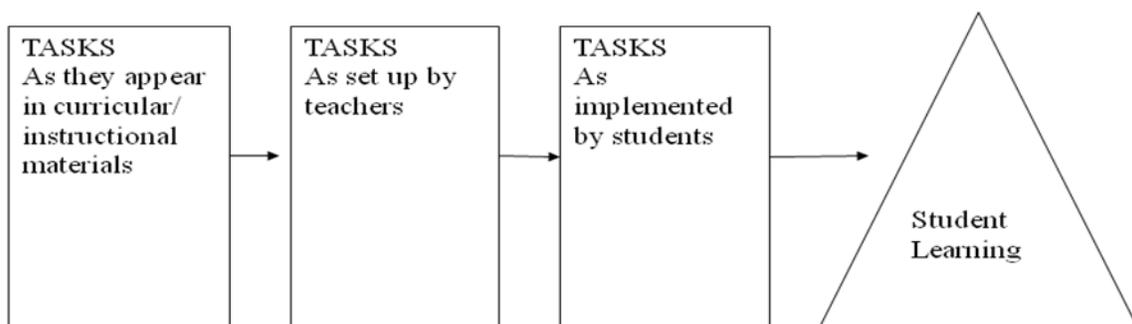


Figure 2: Mathematical Task Framework (Smith and Stein, 1998)

At each stage of the diagram the level of cognitive demand is subject to change based on the actions of the teacher and the students. For example, a task that appears in curricular

materials may be categorized at a lower-level of cognitive demand, but the teacher may alter the task or use questioning strategies during the implementation of the task that increase the level of cognitive demand for students. The reverse scenario is also possible. Similarly, students have the potential to increase or decrease the cognitive demand of a task during classroom implementation. The five practices include anticipating, monitoring, selecting, sequencing, and connecting student solution strategies during whole class discussions. Two essential practices that take place prior to implementing the five practices are setting mathematical goals and selecting appropriate mathematical tasks that require a high level of cognitive demand on the part of the student. By strategically aligning mathematical tasks with mathematical goals, teachers may be able to embed the discipline and rigor mentioned by Sfard (2000). Also, effective teacher modeling of the five practices would provide students with experiences being a participant in productive mathematical discussions that could possibly teach students how to conduct *mathematical classroom discourse* during small or large group discussions. A study by Stowbridge (2008) utilized the five practices to study teachers' use of formative assessment with students as they worked on mathematical problem solving tasks. This research was part of a larger Mathematics Problem Solving Model developed by the Northwest Regional Educational Laboratory that was intended to facilitate teachers' use of problem solving activities in middle school mathematics classrooms. However, the practice of sequencing, as it related to formative assessment, received the most attention. In reference to the five practices Stowbridge (2008) states:

As of yet, no research has been conducted examining how well teachers actually incorporate these practices into their work. Much research is needed to determine if

studying the five practices helps teachers to craft more mathematically productive sharing episodes in their own classrooms. It is still unknown if teachers actually do feel more in control of classroom discussions when they utilize the five practices (p. 66).

A study that examines the evolution of teachers' use of all of the five practices to orchestrate productive mathematical discussions would address the lack of research noted by Strowbridge.

Technology Professional Development, Dynamic Geometry Environments, and Tools for Examining Teachers' Use of Dynamic Geometry Software

Research on teachers and technology professional development. In a study by Bennison & Goos (2010), teachers identified learning *how to incorporate* various technologies in the classroom as their dominant need. Additionally, teacher confidence and positive beliefs regarding student learning with technology was higher for individuals that participated in professional development focused on using technology in teaching mathematics. Findings from Matzen & Edmunds (2007) showed that teachers increased their use of technology in constructivist ways after participation in technology focused professional development. Bennison and Goos (2010) utilized chi-square tests on frequency distributions (created from teachers' survey responses about computers and graphing calculators) and found that teachers with more experience, exclusively teaching mathematics in non-regional and non-rural areas of Australia, were more likely to have experienced technology professional development. Also, teacher confidence and positive beliefs regarding student learning with technology was higher for individuals that participated in

professional development focused on using technology in teaching mathematics. However, professional development and access to technology are necessary, but not sufficient conditions for implementing student-centered teaching practices.

Palak & Walls (2009) investigated links between teachers' beliefs and technology use in the classroom and how these relate to any shifts toward student-centered teaching practices. Teachers involved in the study came from technology rich schools in northern West Virginia that demonstrated a commitment to reform, professional development, and technology integration, and they were all current users of technology. The mixed methods analysis revealed three major results. First, teachers most often used technology in non-academic ways (preparation, management, and administrative purposes). Second, teachers using technology in a student-centered manner was rare, even when considering a technology-rich environment and documented student-centered beliefs. Finally, teachers' technology use supported existing teacher-centered practices. These findings are of particular importance when considering the potential for mathematical knowledge and practices when digital technologies are present. In their conclusions, Palak and Walls (2009) concluded that technology professional development should not only focus on integrating technology into classroom practice using student-centered pedagogy, but also attend to the context of teaching practice. Hence, well designed professional development continues to hold promise for facilitating teacher learning.

Dynamic Geometry Environments. Dynamic geometry environments (DGE's) represent a tool that encompasses aspects of both a performance and a pedagogical tool as described by Putnam & Borko (2000). DGE's serve as a performance tool because they

change how a task may be accomplished in a classroom; however, the dynamic nature of the tool may serve to alter student understanding of concepts (i.e., dynamically linked multiple representations of functions). Olive, Makar, Hoyos, Liew, Kosheleva, and Sträßer (2010) conducted a substantive review of literature to amend the didactic triangle (teacher, student, mathematical knowledge) into a didactic tetrahedron by adding technology. The tetrahedron metaphor was based on the claim that technology is transformative, rather than assimilated, which creates an additional didactic element. They defined *proceptual* knowledge as a merging of procedural and conceptual knowledge and argue that such knowledge and operational aspects, rather than notational aspects, of mathematics may be enhanced by the use of dynamic technologies in the classroom. For example,

At the secondary level dynamic geometry environments can (and should) completely transform the teaching and learning of mathematics. Dynamic geometry turns mathematics into a laboratory science rather than one dominated by computation and symbolic manipulation, as it has become in many of our secondary schools. As a laboratory science, mathematics becomes an investigation of interesting phenomena, and the role of the mathematics student becomes that of the scientist: observing, recording, manipulating, predicting, conjecturing and testing, and developing theory as explanations for the phenomena (Olive et al., 2010, p. 148).

Other studies have also focused on reform-oriented instructional practices in technology intensive environments that utilize dynamic geometry environments. Rutheven, Hennessy, & Deaney (2008) reported that teachers' instructional practice ranged from full integration of reform-oriented practices to partial use of reform-integrated practices to direct instruction

(demonstration and/or presentation) when using dynamic geometry environments. Teacher decisions regarding dynamic geometry to support guided discovery were mediated by 1) costs and benefits of students' use of software, 2) providing experience of a mathematical reference model for a topic, 3) promoting mathematically disciplined interaction with a generalized geometric system, 4) handling apparent mathematical anomalies of software operation, and 5) supporting learning through analysis of mathematical discrepancies. The authors cite pressure associated with standardized testing as one reason teachers continue to teach with traditional teaching practices. Becker, Ravitz, & Wong (1999) surveyed teachers across the United States and found that mathematics teachers viewed *The Geometer's Sketchpad* as the most valuable mathematics program for teaching; however, they most often incorporated traditional, skill-based software in their classes. Hanafin, Burruss, & Little (2001) examined teacher and student roles in a student-centered geometry class that utilized *The Geometer's Sketchpad*. Findings indicate that feelings associated with unpreparedness to facilitate and scaffold instruction was prohibitive to the teacher relinquishing control within their instructional practices. They also acknowledge that they provided no formal training beyond discussing strategies and describing expectations before asking the teacher to take on the role of facilitator. These findings support the findings of Argueta et al. (2011) that quality professional development and support are necessary components of implementing technology intensive instruction.

Tools for examining teachers' use of dynamic geometry environments. Sinclair (2003) reported five important design principles related to dynamic geometry sketches and learning materials for mathematics instruction. According to Sinclair,

In order to facilitate exploration a sketch must provide the means to address the questions and instructions. In general:

1. When a question aims to focus student attention, the sketch must provide the visual stimulus. It must *draw attention* through colour, motion, and markings.
2. When a statement prompts action, such as asking students to drag, observe or deduce, the sketch must contain the necessary provisions. It must *provide affordances* so that the student can take the required steps.
3. Questions that invite exploration are open-ended. In order to explore uncharted territory, the student requires a sketch that allows options. Thus, when a question invites exploration, the sketch must *provide alternate paths*.
4. A question can *surprise* – which may lead to further exploration; however, the teacher is not necessarily there to correct any misinterpretation. Thus, the sketch must *support experimentation* to unmask the confusion. It must be flexible enough to help students examine cases, yet constrained enough to prevent frustration.
5. Questions that check understanding are important parts of any learning situation. In the study tasks, the checking involved students looking together for the answer. Although peer-interactions were not discussed in this article, study results showed that the sketch aided this process by *providing a shared image* for students to consider and discuss.

An important underlying thread must be a focus on provisions that will help students learn how to use change to explore, and how to extend their visual interpretation

skills (p. 312).

These design principles may be taken into consideration by teachers when creating or evaluating pre-constructed dynamic geometry sketches, but these principles do not address what happens within the classroom during implementation of the task. Drijvers, Doorman, Reed, & Gravemeijer (2010) reported on six orchestration types that teachers demonstrated when using a java applet in their classrooms to teach the concept of function. The six orchestration types include *Technical-demo*, *Explain-the-screen*, *Link-screen-board*, *Discuss-*

Table 3: Teacher Technology Orchestration Types

Orchestration Type	Description
Technical-demo	Demonstration of tool techniques by the teacher.
Explain-the-screen	Whole class explanation by the teacher, guided by what happens on the computer screen
Link-screen-board	Teacher stresses the relationship between what happens in the technological environment and how this is represented in conventional mathematics of paper, book and blackboard.
Discuss-the-screen	Whole class discussion about what happens on the computer screen.
Spot-and-show	Student reasoning is brought to the fore through the identification of and deliberate use of interesting student work in the classroom discussion.
Sherpa-at-work	A Sherpa-student uses the technology to present his or her work, or to carry out actions the teacher requests.

Drijvers et al. (2010, p. 219-220)

the-screen, *Spot-and-show*, and *Sherpa-at-work*. The first three orchestration types are considered to be more teacher-centered orchestrations, and the last three orchestration types refer to more student-centered orchestrations. Table 3 describes each orchestration type in terms of exploitation mode within orchestration theory. These orchestration types provide insight into technological aspects of task implementation. The combination of design principles and orchestration types has the potential to provide a great description of dynamic geometry sketches before and during instruction.

Summary

This chapter served several purposes. First, research related to 1:1 laptop learning environments and mathematics professional development was shared. Then, attention was given to discourse, practices to support mathematics discussions and level of cognitive demand, and issues related to technology and dynamic geometry environments. The next chapter reiterates the purposes of this study, restates the research questions, discusses the context of the study, provides the overall framework, and details the methodology for data collection and analysis.

CHAPTER 3: METHODOLOGY

As stated in the introduction, the overarching question that guided this study was, “*How do teachers implement pre-constructed dynamic tasks in technology intensive Algebra 1 classrooms?*” The purpose of this study was to 1) characterize the nature of mathematical discourse, 2) describe the extent to which teachers use the five practices for orchestrating productive mathematical discussions, 3) examine design and technology features of pre-constructed dynamic geometry tasks and how teachers use them, and 4) explore the combined influence of discourse, the five practices, and technology on level of cognitive demand when teachers implement pre-constructed dynamic geometry tasks. As such, the following research question will be answered:

When involved in technology-intensive, research-based, professional development on Algebra:

- a. What types of mathematical questions and statements are central to the mathematical discussions that occur among students and the teacher when implementing pre-constructed dynamic geometry tasks?
- b. To what extent do teachers utilize the five practices for orchestrating productive mathematical discussions to support their implementation of pre-constructed dynamic geometry tasks?
- c. What are the design and technological features of pre-constructed dynamic geometry tasks and how do teachers use them during implementation?
- d. What is the overall influence of a-c on the implemented level of cognitive demand of pre-constructed dynamic geometry tasks?

In accordance with the work of Sztajn (2011), the researcher's knowledge and beliefs about adult and teacher learning were shared in the literature review. The discussion will now attend to the remaining categories for reporting on professional development studies. The context, program goals, critical issues, and strategies of the Algebra professional development will be discussed prior to discussing the research design, conceptual framework, data collection, and analysis.

Scaling Up STEM Learning with the Virtual Computing Lab

Conclusions from the body of literature on mathematics and technology professional development served as the foundation for the design of the Algebra 1 component of the Scaling Up STEM Learning with the VCL project (NSF ITEST Grant DRL-0929543). Dr. Karen Hollebrands served as a co-principal investigator for the project, and the researcher worked as a graduate research assistant with the project. The larger professional development project served as the context for this study. The remainder of this section includes a brief description of the overall project, program goals, critical issues, and strategies.

Context. Four public, rural school districts were recruited from existing 1:1 laptop initiatives in North Carolina. Two of these school districts had a history of not meeting state and national benchmarks for student achievement. Nine high schools within these districts were represented. In two cases only one high school from each district was represented; however, one of these cases was the only high school in the district, and the other was only one of two high schools in the district. The number of years of teaching experience ranged from two to greater than 30 years, as did their experience with *The Geometer's Sketchpad*.

Student demographics varied by district, but the overall population included 33% African American, 9.8% Latino, and 37.8% economically disadvantaged students.

Project goals. The project utilizes four interrelated interventions: mathematics software programs (*The Geometer's Sketchpad* and *Fathom*), teacher professional development, STEM role models, and cloud computing (VCL). The goal of these interventions is to increase student success in high school mathematics courses so that they are able to pursue STEM related college majors and careers.

Critical issues. The content focus of the professional development was the Algebra and Function objectives from CCSSM (2010). Efforts were made to include at least two participants from each school. For several schools, more than two teachers were present, but this was not possible for three of the participating high schools partly because of the small number of mathematics teachers at those schools. Professional development providers included Dr. Karen Hollebrands and the researcher. Teacher participation was voluntary, and participants were paid stipends, received in two installments, for attending the week-long summer institute and completing a follow-up online course. Communication with district level instructional staff and virtual computing lab staff occurred throughout the project. Feedback from teachers and project evaluators was utilized to shape professional development strategies, but project staff made all final decisions on what was included.

Strategies. A cohort of 22 Algebra teachers was recruited during the Spring of 2011. These teachers agreed to participate in a 2 year, multi-faceted professional development opportunity. Teachers participated in a week-long institute at the Friday Institute for Educational Innovation in July 2011. Each teacher was also provided with twelve hours of

online professional development throughout the 2011-2012 school year and was observed three times per semester if they were teaching Algebra 1 (Figure 3).

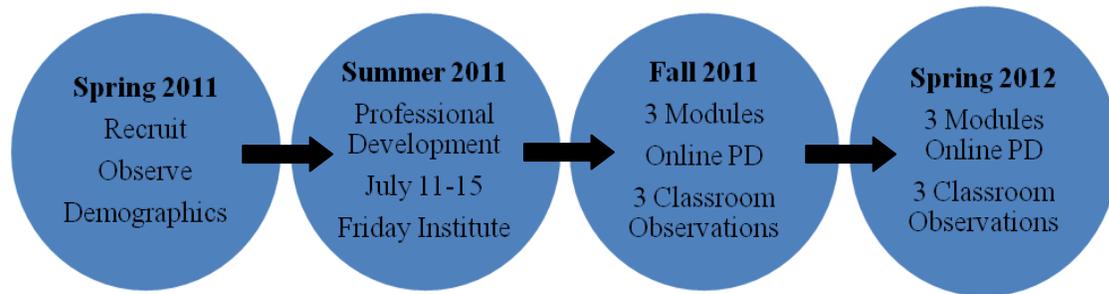


Figure 3: Scaling UP Stem Learning Algebra Professional Development Timeline

The first strategy was the summer institute. Project staff utilized the technological, pedagogical, and content knowledge (TPACK) (Koehler & Mishra, 2008) framework to plan, organize, and deliver material. The main technological tool incorporated during the summer institute was *The Geometer's Sketchpad* (Version 5.0). Pedagogical aspects of the institute focused on 1) identifying mathematical goals, 2) aligning these goals with high level dynamic geometry tasks, and 3) incorporating the practices of anticipating, monitoring, selecting, sequencing, and connecting student work (Smith & Stein, 2011). First year algebra content from the Common Core Standards for School Mathematics (2010) was the mathematical context for technological and pedagogical knowledge during the institute. Planning for the summer institute adhered to these technological, pedagogical, and mathematical tenets. Teachers were also provided with books to use with their students in their classes: *Teaching Algebra 1 with the Geometer's Sketchpad* and *Teaching Algebra 2*

with the Geometer's Sketchpad. Several activities from these books were used during the institute to model ways in which teachers might implement them with their own students. Instructors purposefully incorporated the five practices into their daily instruction with teachers. Further, instructors explicitly discussed and asked participating teachers to consider how the five practices and mathematical task framework applied to the instruction they were receiving.

Participating teachers read the first five chapters of *Five Practices for Orchestrating Productive Mathematics Discussions* (Smith & Stein, 2011). These chapters introduced, defined, and provided examples of use of the five practices in teachers' classrooms. These readings also explicated the Mathematical Task Framework and discussed the important relationship among mathematical goals, mathematical tasks, and the five practices. Teachers used the mathematical task framework as part of a sorting activity with 16 mathematical tasks on the first day of the summer institute, and they were required to use the mathematical task framework to guide their creation of the final project for the week. The final project for the week was to design, present, and share a pre-constructed dynamic geometry task that could be used with students in Algebra.

During the week, teachers also analyzed two video excerpts of classroom instruction using the five practices as a guide. Video cases serve as a powerful tool for reflection about teaching practice and have been shown to positively impact teacher learning and growth (Sherin & van Es, 2009). One powerful aspect of video cases is that teachers are able to see teaching practice in an environment similar to their own. However, due to the unique 1:1 laptop aspect of participating schools, video cases needed to be created that reflected the

classroom environment of participants, while also matching the focal points of the summer institute. The mathematical, pedagogical, and technological foci of the summer institute were linear functions, discourse, and *The Geometer's Sketchpad*, respectively. In order to achieve the best alignment possible with both teaching environment and professional development goals, the researcher collaborated with principal investigators and a teacher in one of the participating schools. The researcher designed, taught, and videotaped several lessons focused on systems of linear equations and inequalities, as well as families of functions using multiple representations. The resulting videos were used as part of the summer institute during July 2011 to allow teachers to analyze teaching practice utilizing the pedagogical practices described above.

A second strategy used during the professional development project was an online component moderated by the researcher. *Teaching Algebra with Geometer's Sketchpad* (Key Curriculum Press) was a Moodle-based learning environment consisting of six four-hour modules to be distributed throughout the school year. As part of each module teachers reviewed testimonial type videos featuring the creator of *The Geometer's Sketchpad* and teachers that have used the software with their students. The videos were followed by an instructional video demonstrating the particular features of *The Geometer's Sketchpad* that would be utilized during the module. Scaffolding was provided in the form of optional activities designed to assist novice users in their learning of the software. All activities could also be used with students since the activities are focused on algebraic content. The course also provided a discussion board where teachers may posted/replied to questions/comments posted by their peers; responded to questions posted by the moderator, and/or posted sketches

and/or teaching ideas. Each participant was required to make at least two postings during each module. Frequent monitoring of the discussion boards was utilized to promote a discursive environment and connected learning community. In addition to the discussion board, two products were required for each module. A sketch – designed for classroom use and directly connected to the algebraic content of the module – plus a journal reflection – based on a given prompt incorporating content, pedagogy, and technology – were required of each participant. Written constructive feedback was provided by the moderator on both products.

A final strategy included three classroom visits per teacher during each semester of the 2011-2012 school year, provided they were teaching Algebra 1. These observations served two purposes. First, the visits were used to assist teachers in any facet of classroom instruction sought by the participant. This consisted of general conversations about how the semester was progressing, locating resources or dynamic geometry tasks, and in one case helping co-plan and co-teach a lesson. Secondly, observations of classroom teaching provided continual feedback for participants as they incorporated *The Geometer's Sketchpad* and discourse into their Algebra I classes.

Summary. The Algebra 1 professional development described above was comprehensive in scope and adhered to recommendations found in research to design effective professional development. Research suggests that effective professional development is intensive, ongoing and connected to practice. It builds on existing school initiatives (like ubiquitous computing) and develops strong working relationships among teachers (Darling-Hammond, et al., 2009; Loucks-Horsley, et al., 1998). The design took into

consideration, and capitalized on, the unique teacher and student needs that arise in a 1:1 laptop learning environment. Particular attention was given to fostering a cohesive learning community within and among schools. The foci of the professional development addressed content, pedagogy, and technology coherently and continually via face-to-face and online interactions. Participation in this professional development provided teachers with a great opportunity to increase not only their learning, but also their students' learning.

Research Design

This study relied on qualitative methods to address the research questions. Bogdan and Biklen (2007) list five characteristics of qualitative research. Qualitative research is naturalistic, descriptive, concerned with process, inductive, and provides meaning about the context and/or individuals being studied. These characteristics allowed the researcher to delve deeply into the practice of teaching and learning in technology rich environments. A qualitative, observational multicase study served as the specific research design. In observational multicase studies, two or more participants are studied, and “the major data-gathering technique is participant observation (supplemented with formal and informal interview and review of documents) and the focus of the study is on a particular organization” (p. 60). Individual teachers represented a separate case. Utilizing individual teachers as a case gave credence to their specific teaching context and allowed for within case and cross case analysis. Individual cases were bounded by time (Fall 2011 semester) and location (Nottingham High School and Taggart High School). Multiple data sources allowed the researcher to provide an extensive description of teachers' use of mathematical discourse, the five practices for orchestrating mathematical discussions, technology use, and

implemented level of cognitive demand. These data sources included video-taped interviews, classroom observations, and planning sessions, as well as artifacts from classroom observations, planning sessions, and online professional development. Field notes were also kept for each interview, observation, and planning session to capture contextual aspects not observable on video.

Conceptual framework. The conceptual framework that guided this study stemmed from research conducted by Stein, et al. (2008). The mathematical task framework (Figure 2) and the five practices for orchestrating productive mathematical discussions were described earlier. The mathematical task framework detailed the dynamics of how tasks unfold during planning and implementation. The five practices represent tools that teachers may rely upon to facilitate productive discussion around tasks in their classroom. For the context of this study the aspect of technology was also considered (i.e., 1:1 computing learning environment and use of pre-constructed dynamic geometry tasks). The conceptual framework (Figure 4) represents the dynamic interplay among all of these and guided the researcher's choices regarding data collection and analysis.

The downward vertical direction of the conceptual framework mirrors the mathematical task framework. The top portion of the diagram represents tasks as they appear in their original form. Then tasks are set up by the teacher, and students begin to interact with the task. What happens in this phase connects to student learning. Within the diagram the five practices appear where they would occur. Anticipating is in the upper portion of the diagram, representing actions the teacher takes outside of the classroom to plan (e.g., group planning sessions). The remaining four practices occur within the classroom and facilitate

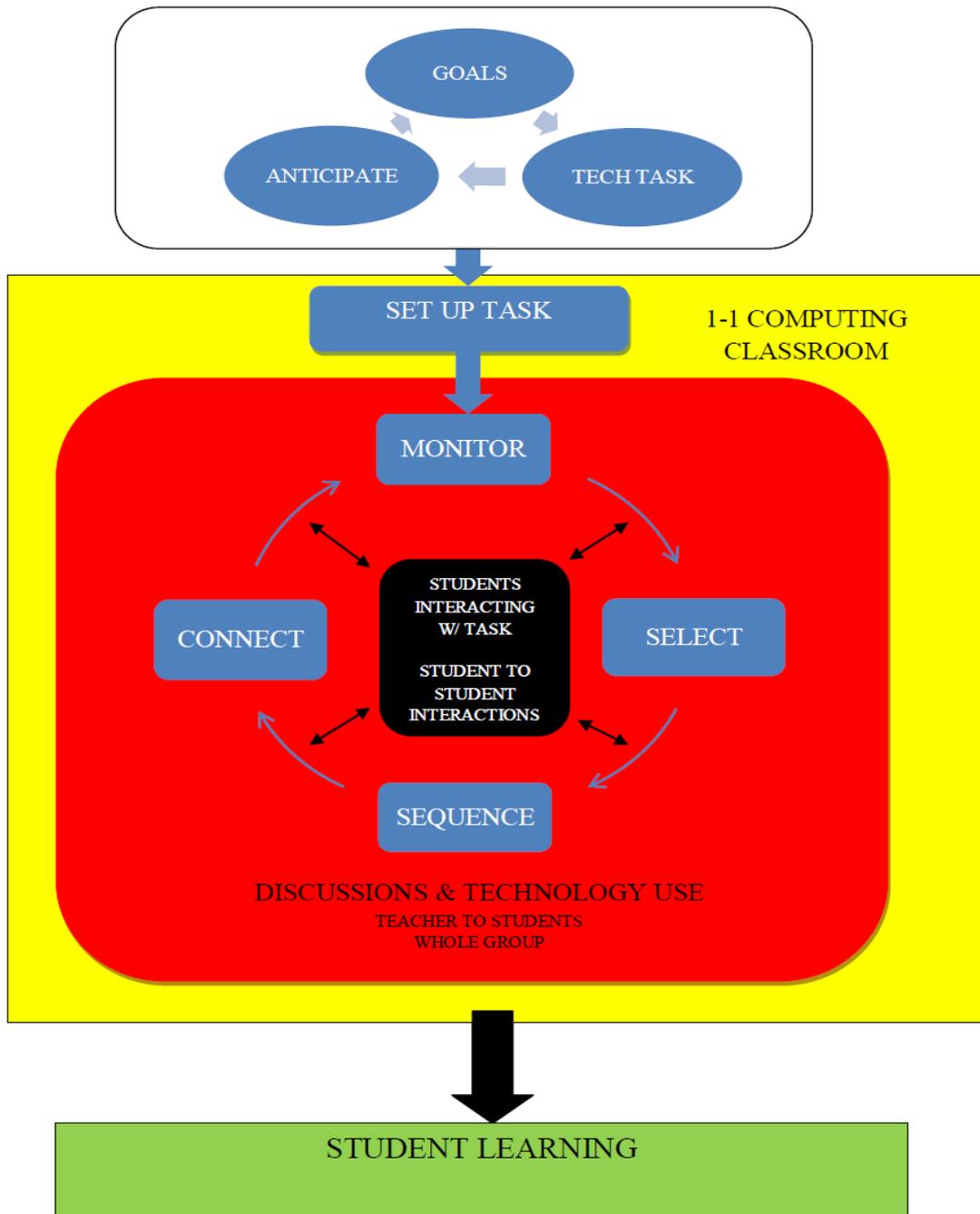


Figure 4: Dynamics of the Mathematical Task Framework & 5 Practices in 1-1 Computing Classrooms

discussions as students interact with the task. The practices are cyclic, and they are acted upon by what students are doing with the task. Because the tasks are pre-constructed dynamic geometry tasks, technology use is also a factor in the discussions.

Data collection and analysis also followed the diagram. In the planning portion of the conceptual framework, pre-constructed dynamic geometry tasks were collected from teachers and analyzed for potential level of cognitive demand and design principles. As tasks move into the classroom, the discussions that occurred during implementation of a particular task bounded an episode. Then episodes were analyzed using discourse codes, the five practices, and technology use. The consideration of each of these was used to determine the level of implemented cognitive demand.

Participant Selection

One school system from the larger professional development project was identified to be included in the study based on convenience. The decision of which school system to include in the study was based on the physical proximity of the school system to the researcher. The school system chosen was a small, rural school system in the eastern part of the state. The school system had participated in a 1-1 laptop learning initiative for approximately four years. At the time of the study, the school system was classified as one of the twelve lowest performing school systems in the state based on student performance on state mandated tests. Participants' teaching schedule needed to include Algebra 1 during the time of the study, and a purposeful sample of four teachers (across three different schools) was identified based on these criteria. All four teachers were asked to participate in the study; however, only three teachers chose to fully participate. Because this study was situated

within a larger scale up project, the teacher who could not fully participate was invited to attend planning sessions. The final sample included three teachers located at two high schools. Each school operated on a block schedule, meaning students enroll in 4 ninety-minute classes per semester.

Mrs. Lewis. Mrs. Lewis worked at a rural high school with 662 students, where 67% of students were African American, 26% were Caucasian, and 6% were Hispanic. Less than 1% identified as Asian or Native American. Student achievement on Algebra 1 state mandated testing ranked second out of four high schools in the school system for the 2008-2009, 2009-2010, 2010-2011 school years, with 63.2%, 65%, and 57.7% proficiency, respectively. Mrs. Lewis earned a Bachelor of Science in Mathematics Education and obtained her initial licensure in Secondary Mathematics. She was beginning her third year of teaching, all of which had taken place at the school described above. Her teaching experience included pre-Algebra, Algebra 1, Geometry, a third year Algebra course, and technical mathematics courses. She was currently teaching two sections of Geometry and one section of Algebra 1.

In addition to laptops, the classroom learning environment included several forms of technology. Mrs. Lewis' technological equipment included a document camera, LCD projector, and graphing calculators. In terms of software, she had access to student management software (i.e., view and manage student computers, send and receive files), *The Geometer's Sketchpad* (via Virtual Computing Lab), and office management programs (i.e., word processing, spreadsheets, power point). Web based tools that she had access to included Moodle for her class web pages, SAS Curriculum Pathways for lesson materials, and

ClassScape for state testing review. Mrs. Lewis reported facile use of all the technological resources that she had access to; however, she had not utilized *The Geometer's Sketchpad* in Algebra 1 prior to this study.

Mrs. Lewis configured her classroom (Figure 5) to position herself at the front left corner of the room during instructional time. This decision was mitigated by placement of the overhead screen. This location allowed her to have a full view of students during instruction, but limited her view of their computer screens. In addition to her laptop, a document camera

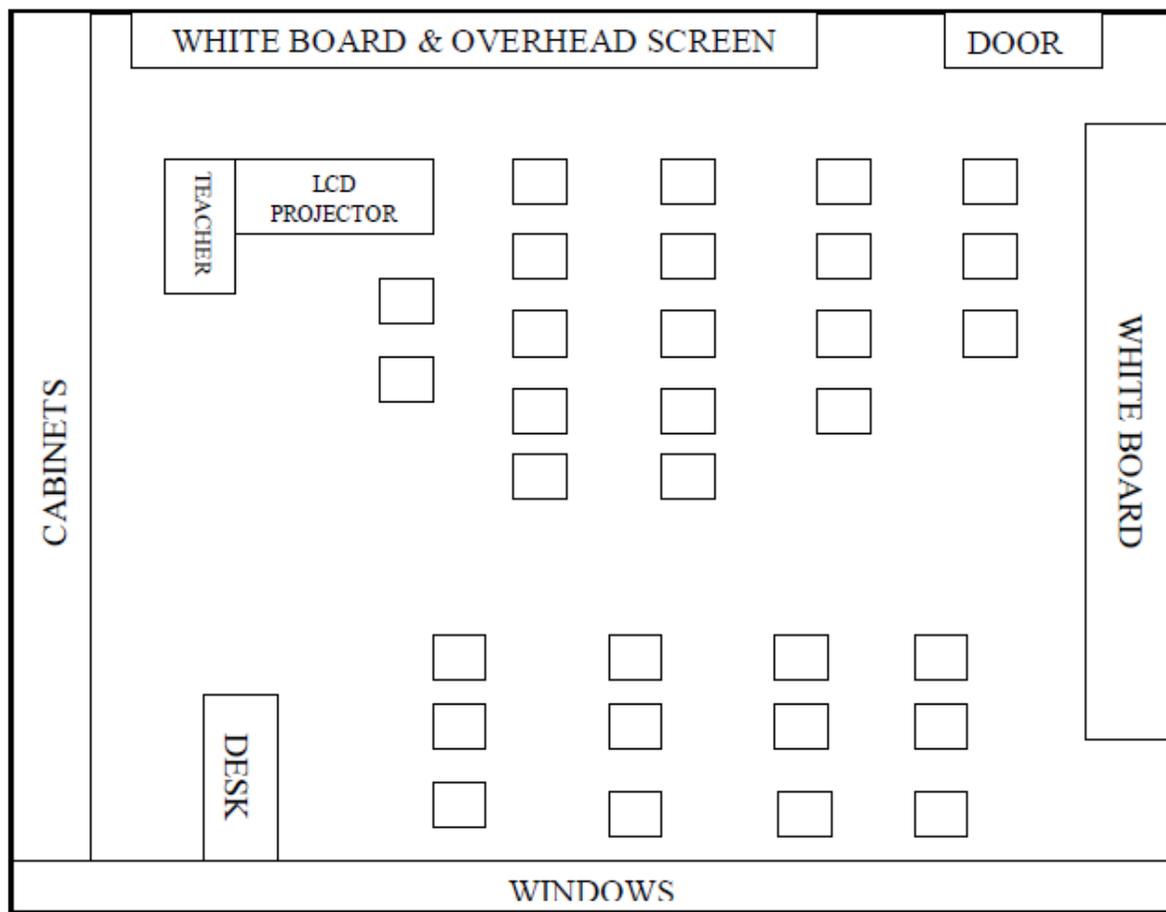


Figure 5: Classroom Configuration for Mrs. Lewis

was located on the raised desk Mrs. Lewis utilized during instruction. The LCD projector was situated on a cart adjacent to the teacher, which allowed her to connect and disconnect her computer easily throughout the day. All instructional activities were focused in this area of the room. The white board on the right hand wall was reserved for weekly agenda items and reminders of impending deadlines. She did have a separate teacher desk at the rear left corner of the room to use during her planning period.

Student desks were arranged in two sets of rows in the front and back of the room. Space between the sets of rows allowed Mrs. Lewis to easily navigate the room as students worked individually or in small groups. Space constraints and students' ability to see the board were noted by Mrs. Lewis as reasons for keeping the desks in rows, rather than small groups. Students were given paper copies of worksheets to accompany pre-constructed dynamic tasks, and each student was expected to complete their own work. The only pairing of students occurred in cases when an individual student did not have their laptop. However, Mrs. Lewis encouraged students to communicate with one another and work together during instruction.

Mrs. Lewis' participation in the larger professional development project was two-fold. She began her participation with the professional development project during the summer of 2010 as part of the Geometry cohort and chose to participate in the Algebra cohort beginning in the summer of 2011. This meant at the time of the study, she had completed three week-long summer institutes (2 Geometry, one Algebra) focused on incorporating *The Geometer's Sketchpad* into the classroom. Plus, one year of online professional development focused on the use of *The Geometer's Sketchpad* in teaching

Geometry. During the study, she completed one-half of an online professional development course focused on the use of *The Geometer's Sketchpad* in teaching Algebra. She had also used *The Geometer's Sketchpad* as part of course work for her undergraduate degree. During her initial interview, she reported being very comfortable with designing her own sketches, as well as finding appropriate pre-made sketches. Artifacts from the summer institute and online professional development courses corroborate her expressed comfort level.

Mrs. Patterson. Mrs. Patterson recently transferred to work at the same school as Mrs. Lewis. She previously taught at the same high school as the third participant, Mr. Phelps, for two years. Mrs. Patterson was entering her fifth year of teaching at the time of the study. She earned a bachelor's degree in Secondary Mathematics Education from a local university and held an initial teaching license. She taught second grade for two years at a private school and earned her credentials in early childhood before obtaining a high school position. Her teaching experience included Algebra 1, Geometry, and two levels of technical mathematics courses. She was currently teaching two sections of Algebra 1 and one section of a technical mathematics course.

Mrs. Patterson had access to and utilized the same technology tools in her classroom as Mrs. Lewis. She reported facile use of all the technological resources that she had access to; however, she had not utilized *The Geometer's Sketchpad* in Algebra 1 prior to this study. Mrs. Patterson's classroom arrangement (Figure 6) also positioned the teacher at the front of the room, but the overhead screen was mounted in the center of the white board at the front of the room. As such, she situated herself at the front center of the room during instructional time. Similar to Mrs. Lewis, this location allowed her to have a full view of students during

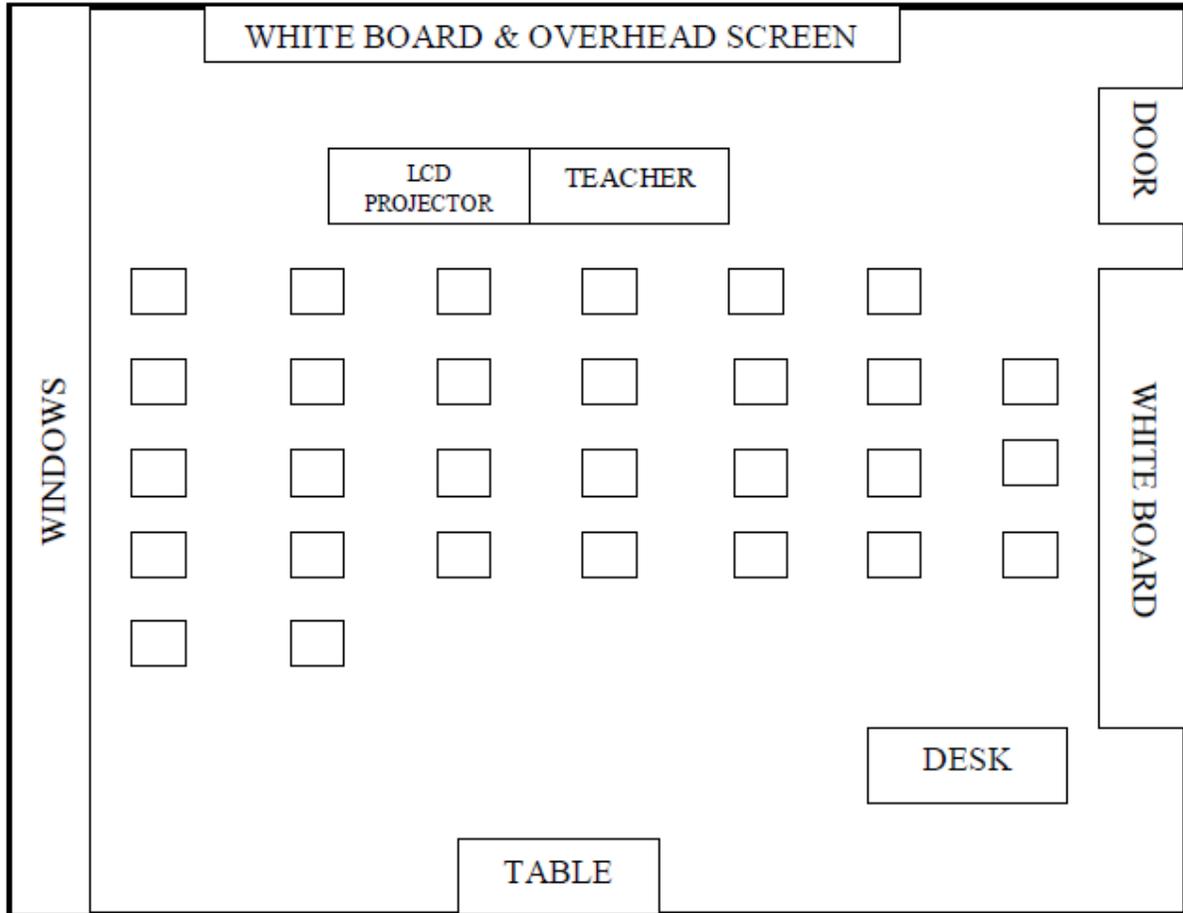


Figure 6: Classroom Configuration for Mrs. Patterson

instruction, but limited her view of their computer screens. In addition to her laptop, a document camera was located on the raised desk Mrs. Patterson utilized during instruction. The LCD projector was situated on a cart adjacent to the teacher, which allowed her to connect and disconnect her computer easily throughout the day. All instructional activities were focused in this area of the room. The white board on the right hand wall was reserved for weekly agenda items and reminders of impending deadlines. She did have a separate teacher desk at the rear right corner of the room to use during her planning period.

Student desks were arranged in rows, and space between the sets of rows allowed Mrs. Patterson to easily navigate the room as students worked individually or in small groups. Space constraints were noted by Mrs. Patterson as the main reason for keeping the desks in rows, rather than small groups, but she regularly instructed students to pair up (i.e., pull desks together) to complete their work using one laptop. Students were given paper copies of worksheets to accompany pre-constructed dynamic tasks. Each student was expected to complete their own work, but Mrs. Patterson encouraged students to communicate with one another and work together during instruction.

Mrs. Patterson's participation in the larger professional development project mirrored that of Mrs. Lewis. She began her participation with the professional development project during the summer of 2010 as part of the Geometry cohort and chose to participate in the Algebra cohort beginning in the summer of 2011. This meant at the time of the study, she had completed three week-long summer institutes (2 Geometry, one Algebra) focused on incorporating *The Geometer's Sketchpad* into the classroom, and one year of online professional development focused on the use of *The Geometer's Sketchpad* in teaching Geometry. During the study, she completed one-half of an online professional development course focused on the use of *The Geometer's Sketchpad* in teaching Algebra. She had also used *The Geometer's Sketchpad* as part of course work for her undergraduate degree. During her initial interview, she reported being very comfortable with designing her own sketches, as well as finding appropriate pre-made sketches. She attributed this to her undergraduate work and using the software in teaching Geometry and two levels of technical mathematics

courses. Artifacts from the summer institute and online professional development courses corroborate her expressed comfort level.

Mr. Phelps. Mr. Phelps worked at a rural high school with 378 students, where 87% of students were African American, 7% were Caucasian, and 6% were Hispanic. Student achievement on Algebra 1 state mandated testing ranked fourth out of four high schools in the school system for the 2008-2009, 2009-2010, 2010-2011 school years, with 23.2%, 29.6%, and 39.3% proficiency, respectively. Mr. Phelps became a mathematics teacher via lateral entry and held an initial teaching license in middle grades mathematics. He was beginning his fifth year of teaching at the time of this study. He taught three years at the middle school level and was beginning his second year at the high school level. His teaching experience included eight grade mathematics, introductory Algebra, and Algebra 1. He was currently teaching two sections of introductory mathematics and one section of Algebra 1.

Mr. Phelps access to technology included the same resources as other participants, but he also had an Interwrite pad, which was used to remotely control his laptop. He incorporated SAS Curriculum Pathways for lesson materials and ClassScape for state testing review; however, he did not utilize Moodle or have a class web page. He expressed that he was 60% comfortable with technology and expressed the desire to begin using Moodle to build a class website. His classroom (Figure 7) was the only example where student desks were arranged in groups. Mr. Phelps expressed a strong desire to create a discursive learning environment where students shared responsibility for learning outcomes. He stated that situating students in groups promoted discussion and working together, which was consistent with the learning environment he wanted to create. The horizontal row of desks in the back of the room was

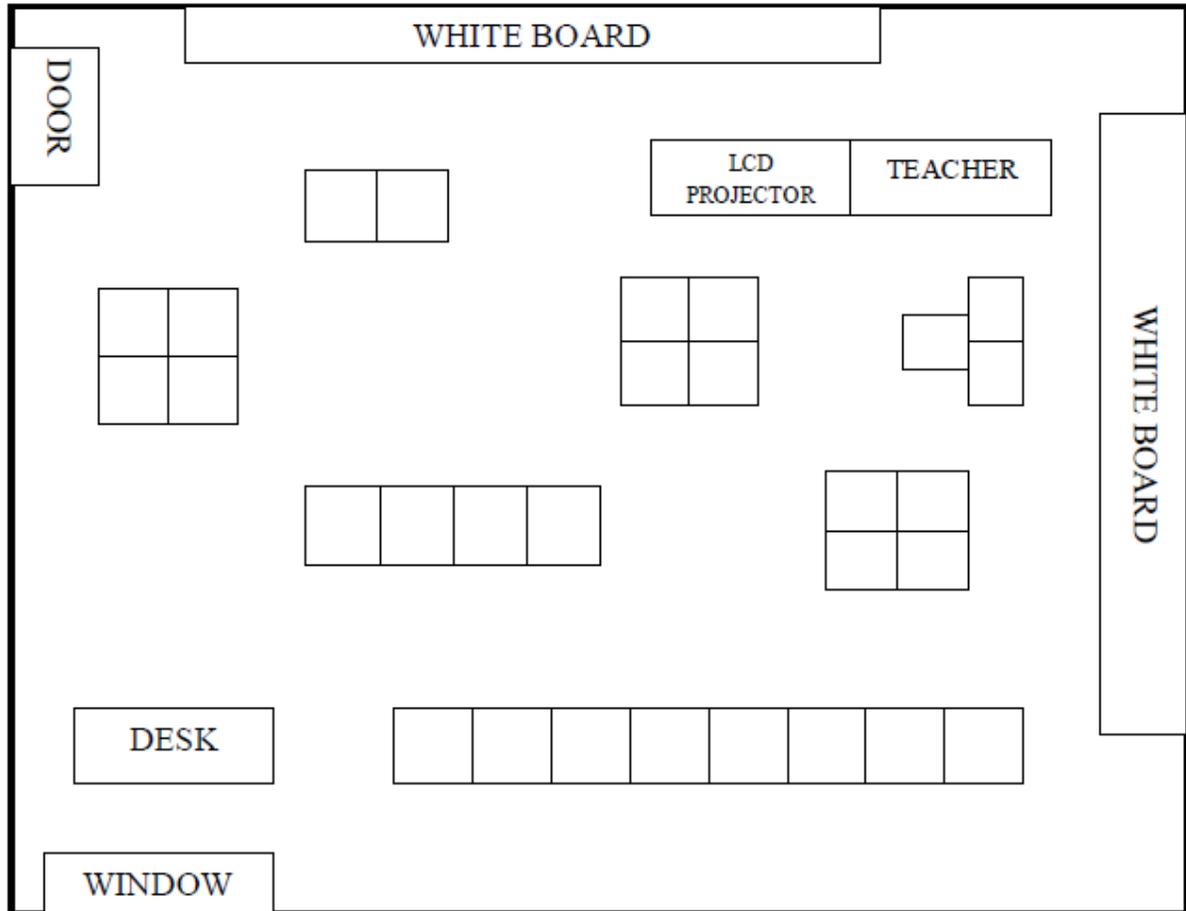


Figure 7: Classroom Configuration for Mr. Phelps

reserved for students that did not return research consent forms. This location prevented them from being captured on video during classroom observations. Mr. Phelps positioned the LCD projector on a table at the front right portion of the room and projected the image on the white board. He utilized his Interwrite pad to control his laptop, unless he was working with a pre-constructed dynamic task. At these times he manually controlled his laptop using the mouse pad. He also distributed paper copies of student worksheets for pre-constructed

dynamic tasks and expected individual students to complete their own work as part of a group.

Mr. Phelps chose to participate in the Algebra cohort beginning in the summer of 2011. This meant at the time of the study, he had completed one week-long summer institute focused on incorporating *The Geometer's Sketchpad* into Algebra classrooms. He was introduced to *The Geometer's Sketchpad* at the summer institute, and he completed one-half of an online professional development course focused on the use of *The Geometer's Sketchpad* in teaching Algebra during the course of the study. During his initial interview, he reported being 30% comfortable with using *The Geometer's Sketchpad*. Artifacts from the online professional development course corroborate his expressed comfort level.

Data Collection Methods

Data collection occurred from August 2011 through January 2012. Teacher interviews (initial/summative, pre/post-observation, and focus group), planning sessions, and classroom observations were documented using video and/or audio. Verbatim transcripts of classroom observations were created. Other artifacts included pre-constructed dynamic geometry tasks and field notes (descriptive and reflective) from interviews, planning sessions, and classroom observations. A complete overview of data collection, data analysis tools, and matching of data sources to research questions may be found in Appendix A. The triangulation of data from multiple data sources added robust evidence to support claims.

Teacher interview data. Initial & Summative interviews were audio recorded and followed a specified protocol (Appendix B). Initial interviews provided the researcher with specific background and context information, teachers' personal views/beliefs regarding how

students learn mathematics, incorporating the five practices, technology use, and the role of planning in their teaching practice. Information from initial interviews also assisted the researcher in designing initial group planning session. Summative interviews conducted at the conclusion of the Fall 2011 semester were utilized to document changes in teachers' context, personal views/beliefs regarding how students learn mathematics, incorporating the five practices, technology use, and the role of planning in their teaching practice.

Pre and post-observation interviews were also conducted to provide the researcher with information regarding teachers' use of the five practices before and after the implementation of a lesson. These interviews were audio-taped and followed the interview protocol in the revised OMLI Classroom Observation Protocol (Appendix C). Information from pre/post-observation interviews informed the content of planning sessions and facilitated teacher reflection on their practice. Finally, a focus group interview occurred after the culmination of data collection. The focus group was video-taped and consisted of seven questions (Appendix D). These questions were developed *a posteriori* based on themes that emerged throughout the semester. The focus group was useful for gaining multiple perspectives from participants in an environment where group participants stimulated talk and articulation of their individual views (Bogdin and Biklin, 2007).

Group planning session data. Three group planning sessions were conducted in September, October, and November 2011. These group planning sessions were documented using video and were designed to assist teachers in using the five practices and task analysis guide during their instruction. The form and content of these sessions was informed by initial interviews and teaching sets to best meet the needs of participants at the time of each session.

Data from the group planning sessions included video and pre-constructed dynamic geometry sketches. The use of video allowed the researcher to examine teacher and researcher interactions and documented teachers' thoughts about the five practices and technology features of tasks. Group planning sessions also helped facilitate the building of a community among teachers within the school system and documented teacher change during the semester. The pre-constructed dynamic geometry sketches provided the researcher means to analyze the design and technology features of task and potential level of cognitive demand.

Classroom observation data. Classroom observation data for teachers was collected three times during the semester. In order to capture a more accurate picture of teachers' practice three teaching sets were conducted. According to Simon, Tzur, Heinz, Kinzel, and Smith (2000):

A teaching set consisted of two classroom observations and three interviews: a pre-lesson interview with the teacher about the first lesson to be observed, an observation of the first mathematics lesson, a second interview in which the teacher was asked about the first lesson and about plans for the second lesson, an observation of the second lesson, and an interview about the second lesson (p. 583).

Each teaching set took place after group planning sessions and included materials from the sessions. Each observation was video-taped and pre/post-observation interviews were audio recorded. Video from the observations served to document teachers' use of pre-constructed dynamic tasks, incorporation of the five practices, and implemented level of cognitive demand. The focus of this study was to examine student to teacher and teacher to student discourse, rather than student to student discourse. As such, a microphone was placed on each

teacher during classroom observations. This allowed the researcher to capture teacher instruction and individual/group student to teacher interactions at a very high quality. One consequence was that student to student interactions were not audible on the video, but the inability to capture student to student interactions does not imply that students were not talking to one another. This was not viewed as a major dilemma due to the nature of typical classroom interactions for participants in this study. Each teacher utilized whole group instruction/discussions predominantly during observations. If students worked individually or in groups, the teacher often walked around to monitor student work and provide support when needed. The teacher microphone was very helpful in capturing these interactions, which were the focal point of this study.

Pre-constructed dynamic geometry tasks were collected to document any changes teachers made to the original task. Video from the observation was transcribed verbatim to allow for accurate discourse analysis.

Other artifacts. Descriptive and reflective field notes were kept for initial/summative interviews, group planning meetings, classroom observations, and the focus group. These documents provided the researcher with information about what occurred during a particular event and provided means for the researcher to consider how these events fit into the research process. Reflective field notes were very useful to guide next steps during the research process. Descriptive field notes helped the researcher recall specifics about events that were not captured by video and would otherwise be forgotten.

Data Analysis Methods

Teacher interview data analysis. Initial interviews were analyzed to document teachers' baseline views/beliefs and practices regarding learning mathematics, instructional practices, technology, and planning. Open coding (Strauss & Corbin, 1990) within these categories was utilized to accomplish this task. Keywords from teachers' statements were noted about learning mathematics, general instructional practices, technology, and planning. For example, if a teacher included the word 'facilitator' in their description of the teachers' role in the classroom, then that word was documented in the category on instructional practices. Teachers' statements regarding the five practices were coded using the name of the specific practice mentioned. The results were synthesized to create an initial snapshot of individual teacher. Furthermore, results were used to inform planning for the first group meeting. Summative interviews were analyzed using the same process. Results from summative interviews were compared to initial interviews to note changes in teachers' views/beliefs and practices over the course of the semester. Focus groups were coded in the same manner, and the data was used to identify broad themes related to support, teacher decisions regarding student access to pre-constructed dynamic tasks, and the five practices.

Pre and post-interviews were coded using the five practices as broad categories, and similar to initial/summative interviews, keywords from teachers' statements were recorded to capture what teachers' actually did within each category. For example, if a teacher said they were going to sequence student responses from least complex to more complex, this would have been coded as sequencing: least to most complex. Pre and post-observation data was compared to see how teachers' intentions matched what occurred during the lesson. Data

from these interviews was also compared across teaching sets to document changes in teachers' stated intentions and use of the five practices over the course of the semester.

Group planning session data analysis. Group planning sessions were coded using the five practices (as described for pre/post observation interviews) and open coding procedures. Open coding was especially useful for capturing teachers' opinions regarding students' abilities, obstacles to instruction, and decision making about technology use. All pre-constructed dynamic geometry tasks were coded for design principles (Sinclair, 2003) and potential level of cognitive (Task Analysis Guide, Figure 1; IQA for Academic Rigor: Mathematical Rubric for Potential of Task, Appendix F) to ascertain a base line potential level of cognitive demand (e.g., tasks as they appear in curricular materials). Teachers then had the option to utilize only specific tasks or modify the tasks further.

Classroom observation data analysis. Analysis of classroom observation data included several stages. First, pre-constructed dynamic geometry tasks were coded for design principles and potential level of cognitive demand. This was necessary in cases where teachers amended the original tasks from group planning sessions. Second, episodes were created by identifying portions of the video and transcript where the teacher and/or students were working on a particular pre-constructed dynamic geometry task. Hence, an episode was bounded by the portion of the lesson that was dedicated to a specific pre-constructed dynamic geometry task. This included teachers' set up of the task, time spent working on the task (small group and/or individual student), and whole class discussion of the task. Fourteen episodes were identified for Mrs. Lewis, 12 episodes were identified for Mrs. Patterson, and 11 episodes were identified for Mr. Phelps. All participants dedicated the majority of class

time during observations to the use of pre-constructed dynamic geometry tasks. Thus, the identified episodes accounted for the majority of instructional time. Each of these episodes was analyzed using the verbatim transcripts in conjunction with video. The revised OMLI Classroom Observation Protocol (Appendix C) was used to document the *mode*, *type*, and *tools* for each episode. Within *types* of discourse, questions and statements were coded using Boaler and Brodie's (2004) questions types. Boaler and Brodie included "utterances that had both the form and function of questions" (p. 776), which justified the decision to code statements using the same types. For example, 'tell me what you are thinking' would be coded as a statement: probing, versus 'what was your thinking process' would be coded as question: probing. In Boaler and Brodie's research there would have been no distinction between statements versus questions. Four types of statements emerged that did not fit into the question types. Teachers used statements to revoice student responses, encourage/praise, correct, and direct students to perform mathematical actions with the technology. These codes were added to types of statements. Responses to questions and statements also represented a large proportion of discourse *types*. The OMLI provided codes to capture the nature of these responses (answer, explanation, challenge, relating, predicting, justifying, and generalizing). These responses are considered to be hierarchical. A response coded as an *answer* indicates a low level response (i.e., "four"); regardless of question type, whereas an *explanation* would include not just a response, but also how an individual formed their response ("I added two plus two to get four"). A response coded as a *challenge* calls another individual's thinking into question in a constructive manner. *Relating* refers to connecting one's response with previously learned content. *Predicting* indicates a response that makes a

prediction or conjecture about what will occur based on previous observations. *Justification* refers to a response that moves beyond explanation and includes why certain mathematical conclusions were drawn. *Generalizing* represents the highest level of response because it moves beyond the present situation to include the general case (i.e., “all lines with a positive slope will increase from left to right on the coordinate plane”). As such the OMLI inherently delineated the quality of responses within the learning environment, so no further break down was required. Within *tools* for discourse, specific teacher actions with the technology were noted. For example, the teacher ‘dragged the slider toward the right, beginning at zero’ would be coded as an action/gesture within *tools*. Hence, a completed code would document *mode*, *type*, and *tool*. Such as, TW (*mode*)|Q:Exp (*type*)|V,C,G,A (*tools*) would indicate that the teacher addressed the whole class with a question intended to promote exploration. The question was verbalized, referenced the computer and pre-constructed task, and an action/gesture was performed. Also, the specific action would have been noted, which allowed the researcher to capture teachers’ use of dynamic multiple representations within a task. The multi-level coding scheme provided by the revised OMLI captured the various facets of *mathematical classroom discourse*.

In addition to discourse codes, the verbatim transcripts were used in conjunction with classroom video to code teachers’ use of the five practices. The researcher indicated on transcripts when the video demonstrated teachers’ implementation of monitoring, selecting, sequencing, and connecting.

Episodes were also coded using Drijvers et al. (2010) teacher technology orchestration types. Throughout an episode it was noted if a teacher was 1) providing a

technical demonstration of a feature within a pre-constructed task, 2) explaining to students what was occurring on the screen, 3) linking what was occurring on the screen with more conventional mathematics, 4) discussing with students what was occurring on the screen, 5) spotting and showing student work, or 6) allowing a Sherpa to demonstrate their work. All codes were entered in to an Excel worksheet to calculate percentages in reference to discourse codes. A detailed example of the coding scheme is included in Appendix I.

Other artifact data analysis. Field notes were not formally coded. They were reviewed often during the research and provided contextual information that assisted the researcher in defining cases. For example, descriptive field notes for classroom observations provided information about each teacher's classroom configuration, student demographics, flow of a lesson, how many students had laptops on a given day, and any issues that were occurring at the time.

Within case analysis. Each teacher represented a case. Within each case, findings from the analysis of each episode were compiled to draw conclusions regarding implemented level of cognitive demand. The Task Analysis Guide (Figure 3), the IQA Lesson Checklist (Appendix G), and the IQA for Academic Rigor: Mathematical Rubric for Implementation of Task (Appendix H) were used in to assign each episode a number 0-4. A threshold of 2.5 was utilized as the cut off for an implemented high level of cognitive demand. This is consistent with other research conducted with IQA materials (Boston & Wolf, 2006, and Matsumura et al., 2006). Episodes were then grouped by teaching set. The researcher looked across teaching sets chronologically to identify patterns and document change.

Cross case analysis. For cross case analysis, findings related to use of the five practices (RQ 1b) and technology (RQ 1c) were compared across teachers, rather than across teaching sets, to identify trends. The researcher grouped the findings in two ways to address research questions 1a and 1d. For RQ 1a, codes from the revised OMLI were compiled by teaching set. This allowed the researcher to look at changes in discourse, collectively, over the course of the semester. For RQ 1d, the researcher only considered common pre-constructed tasks across teachers. This controlled for variables related to design, potential level of cognitive demand, and potential orchestration types. The teachers taught six common tasks. Findings from the data analysis were grouped within tasks and examined for trends in *mode* and *type* of discourse, including question and statement types.

CHAPTER 4: CASE ANALYSIS

Mrs. Lewis

Teaching Set 1

This section describes the planning/collaboration process between the researcher and participants for the first teaching set. Mathematical goals, task descriptions, potential level of cognitive demand, and technological design principles of the tasks are discussed prior to reporting results for participants' selection and use of tasks in their classroom.

In preparation for the first teaching set, the researcher met with the three participating teachers as a group to review information from the summer institute (e.g., mathematical task framework, five practices, project website, and VCL access instructions), identify mathematical goals and find/design dynamic geometry tasks for teaching the selected mathematical goals. All participants were planning to introduce linear functions, so they identified objectives from state standards pertaining to slope, rate of change, graphing linear functions, and writing equations of linear functions. Activities from the summer institute, the online professional development course, and the *Exploring Algebra 1 with The Geometer's Sketchpad* (Key Curriculum Press, 2012) book were considered for inclusion in lessons. One activity from *Exploring Algebra 1 with The Geometer's Sketchpad*¹, which provided directions for students to create their own sketch, and two pre-constructed dynamic geometry sketches (*Slope Presentation Sketch* and *Slope-Intercept Sketch*) were selected from the materials described above to introduce students to slope and slope-intercept form of a line,

¹ For the purposes of this study only pre-constructed sketches were analyzed.

respectively. The *Slope Presentation Sketch* was chosen for two reasons. First, teachers created the sketch as the final project for the first module of the online course they were taking as part of the larger professional development project. Second, the sketch addressed the learning objectives for slope identified by the teachers. The *Slope-Intercept* was included because the teachers recalled the use of the first task of the sketch from the summer institute and stated this task would help teach the identified learning objectives. Both sketches were also evaluated by the teachers in reference to level of cognitive demand and opportunities to incorporate five practices within the lesson.

The *Slope Presentation Sketch* was created by teachers as part of the online professional development course. (Appendix E). The project description required teacher to create a sketch that included a free moving line, a visual representation of slope (slope triangle), and a calculation of the slope that utilized the formula $\frac{y_2 - y_1}{x_2 - x_1}$. The use of color and hide/show buttons to emphasize mathematical relationships in the sketch was encouraged. Based on the project requirements, the potential level of cognitive demand of the *Slope Presentation Sketch* was categorized (by the researcher) as a 3. In the absence of a corresponding student worksheet, the task has the potential to engage students in complex thinking and creating meanings for mathematical concepts, procedures, and/or relationships; however, there is not an explicit call for student reasoning and understanding. Also, students' identification of patterns to form generalizations and to use mathematical evidence to support conjectures will likely be mitigated by the teacher's use of questioning during implementation of the task. Further, the use of multiple representations and connections among the representations included in the sketch depend upon each participant's level of

comfort with *The Geometer's Sketchpad* and use of questioning during implementation, respectively. Features of the sketch that were included in the rubric adhered to the following design principles (Sinclair, 2003): *draw attention* (e.g., movement, color), *provide affordances* (movable line), *provided alternate paths* (multiple ways to create positive, negative, zero, undefined slope), *support exploration*, and *providing a shared image* for students.

Planning for day two of the teaching set built upon the mathematical concepts from day one to foster students' understanding of graphing and writing equations of lines in slope-intercept form. The task selected to accomplish the mathematical goals was part of a pre-constructed sketch created by the researcher for the summer institute. The task included two sliders that may be dragged to alter the slope (red slider) and y-intercept (blue slider) of a line (purple). Similar to the *Slope Presentation Sketch*, the potential level of cognitive demand for the *Slope-Intercept Task* (Figure 8) was evaluated at a 3 due to the absence of an accompanying student document to gauge the content of the discussion that may result during implementation. The task also adhered to the Sinclair's (2003) design principles by *drawing attention* (color), *providing affordances* (sliders), *providing alternate paths*, *supporting exploration*, and *providing a shared image*. Together the two tasks were intended to be used to develop students' understanding of slope, graphing lines in slope-intercept form, and writing equations of lines in slope-intercept form.

It should be noted that during the task selection process participants ranked both tasks at a high level of cognitive demand based on their experience with the mathematical task framework from the summer institute; however, they had the option to use the pre-

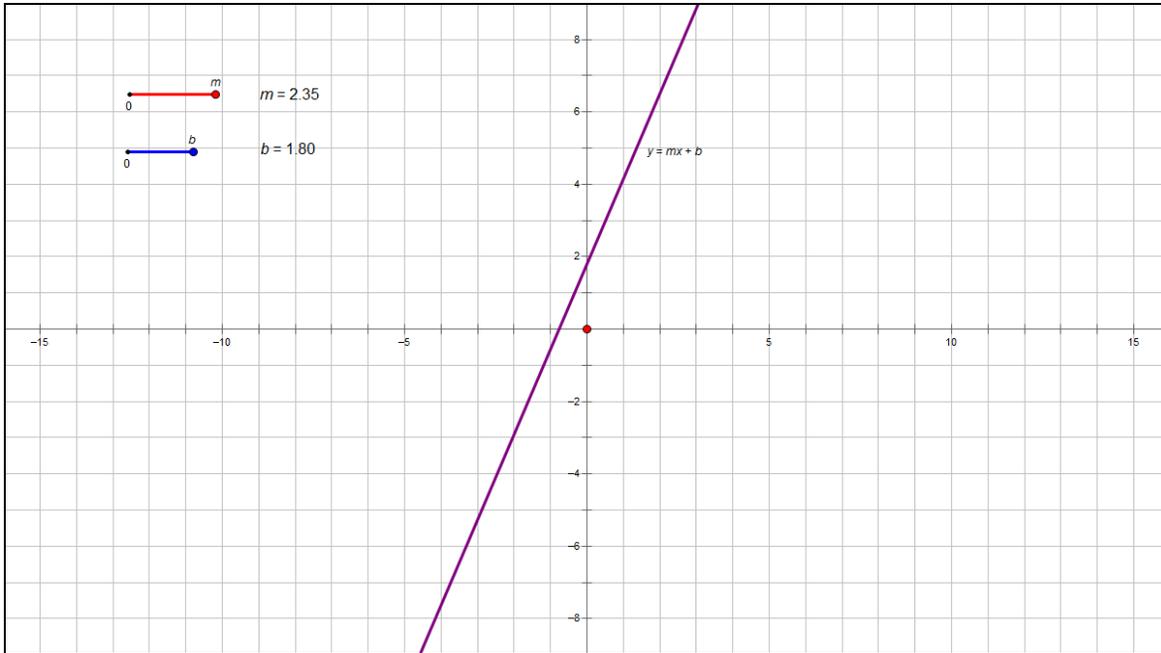


Figure 8: Slope-Intercept Task

constructed sketches from the group meeting as they appeared, alter them, and/or supplement the sketches. The following section describes Mrs. Lewis' final selection of pre-constructed dynamic geometry tasks, reports the potential cognitive demand and design principles of those tasks, details changes in the cognitive demand, characterizes the mathematical discussion that occurred, and notes teachers' use of the five practices and technology use to support mathematical discussions during implementation of the tasks.

Prior to implementation. Mrs. Lewis opted to utilize her *Slope Presentation Sketch* (Figure 9) and the *Slope-Intercept Task* (Figure 8) during the second day of the first teaching set. Both sketches were displayed by the teacher and opened on individual student laptops during the lesson. The *Slope Presentation Sketch* created by Mrs. Lewis was evaluated at a level 3 for potential of cognitive demand due to the absence of a student document that

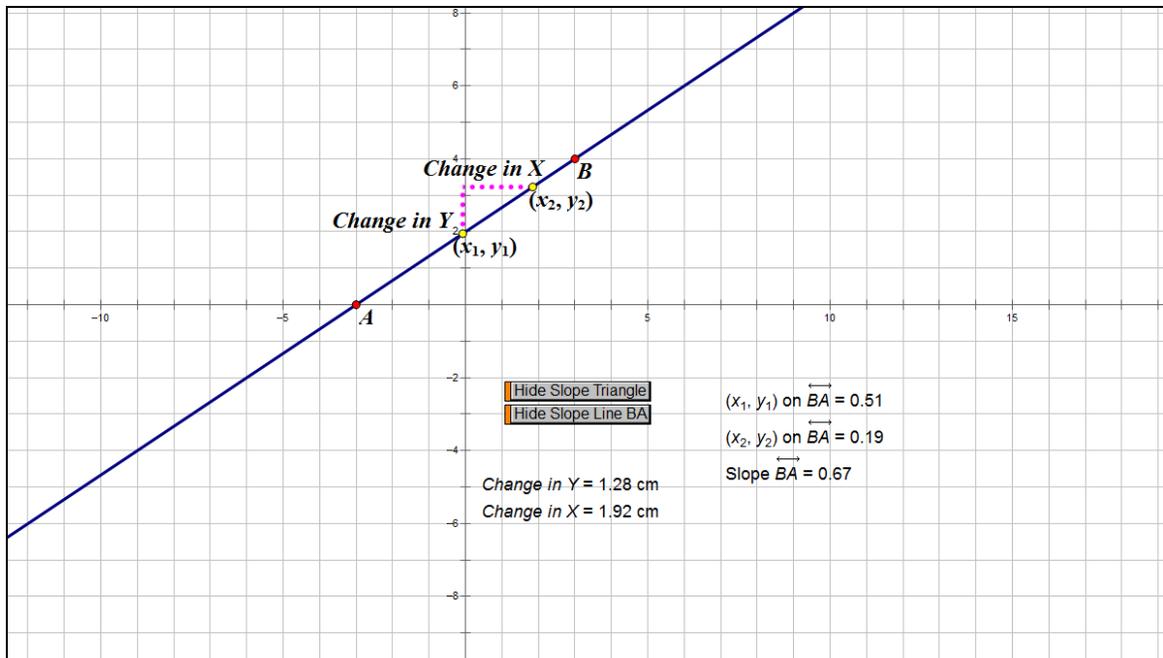


Figure 9: Mrs. Lewis' Slope Presentation Sketch

explicitly asked for student thinking. Her final sketch did not include the slope formula (as required by the online course), but her use of the slope triangle employed hide/show buttons that displayed dotted pink segments with labels “change in y” and “change in x”. The sketch (1) used color and motion to draw student attention, (2) possessed affordances to examine the slope of a line, (3) provided alternate paths for demonstrating various slopes, (4) support exploration, and (5) provided a shared image of slope for the class.

Mrs. Lewis did not alter the *Slope-Intercept Task* at all, so the potential level of cognitive remained a 3, and incorporated the design principles described previously.

Implementation of tasks. This section characterizes the mathematical discussions that occurred during implementation of the tasks described above. Evidence regarding Mrs. Lewis' incorporation of the five practices and technology use to support these discussions are

included before drawing conclusions for implemented level of cognitive demand for the tasks.

Mathematical discussions. Mathematical discussions were coded on three levels using a revised version of the Oregon Mathematics Leadership Institute Observation Protocol (OMLI). First, the *mode* of mathematical discourse was coded. Second, the *type* of mathematical discourse was coded. Third, the *tools* utilized during mathematical discourse were coded. In addition to these three levels of coding included in the OMLI, the *type* of mathematical discourse was further examined using categories for teacher question types (Boaler & Brodie, 2004) and open coding. The *tools* for mathematical discourse were also examined for technological actions and mathematical content during implementation. This level of analysis allowed the researcher to capture teachers' use of dynamic multiple representations in the pre-constructed task.

During the first teaching set, there were 225 documented interactions when pre-constructed dynamic tasks were utilized. Table 4 summarizes the *mode* and *type* of these interactions. Results indicate that *teacher-to-whole class* was the predominant *mode* during the mathematical discussions, and that *questions* and *statements* accounted for the majority of the *types* of interactions. Questions and statements were further broken down into categories to ascertain the role each played in the mathematical discussion (Table 5). Only one statement was provided by a student, so the percentages in the table accurately capture Mrs. Lewis' use of questions and statements during the first teaching set. Questions that focused on *exploring mathematical meanings and/or relationships* occurred most frequently. Mrs. Lewis also utilized questions to *gather information* and *probe student thinking*, respectively.

Table 4: Mrs. Lewis' Modes and Types of Mathematical Discourse for Teaching Set 1

Mode			Type		
Student to Teacher	54	24.00%	Answer	50 (49 from students)	22.22%
Student to Student	0	0.00%	Statement	58 (1 from student)	25.78%
Student to Group	0	0.00%	Explanation	19	8.44%
Student to Whole Class	2	0.89%	Question	78	34.67%
Student In Reflection	0	0.00%	Challenge	1	0.44%
Teacher to Group	0	0.00%	Relate	0	0.00%
Teacher to Whole Class	138	61.33%	Predict	3 (all from students)	1.33%
Teacher to Student	31	13.78%	Justify	1	0.44%
Teacher In Reflection	0	0.00%	Generalize	1	0.44%
Total	225	100.00%	Technology	14 (1 from student)	6.22%
			Total	225	100.00%

The remainder of questions *generated discussion, introduced terminology, and provided context*. In terms of statements, Mrs. Lewis most often displayed the discourse move of *revoicing* described by Chapin, O'Conner, and Anderson (2003), where she would repeat a student's response to the class. Her statements also served to *orient/focus* student attention, *encourage* students, prompt students to *explore* mathematical meanings and relationships, provide *information, direct* students' actions, *generate discussion, correct* student responses,

Table 5: Mrs. Lewis' Questions and Statements for Teaching Set 1

Questions			Statements		
Info	25	32.05%	Info	4	6.90%
Terminology	2	2.56%	Terminology	0	0.00%
Explore	29	37.18%	Explore	5	8.62%
Probing	18	23.08%	Probing	0	0.00%
Generating	3	3.85%	Generating	2	3.45%
Link/Apply	0	0.00%	Link/Apply	0	0.00%
Extend	0	0.00%	Extend	0	0.00%
Orient/Focus	0	0.00%	Orient/Focus	12	20.69%
Context	1	1.28%	Context	1	1.72%
Total	78	100.00%	Revoice	23	39.66%
			Encourage	6	10.34%
			Correction	2	3.45%
			Directive	3	5.17%
			Total	58	100.00%

and provide *context*. The following exemplar occurred during the second day of the first teaching set as students were exploring the *Slope-Intercept Task* (Figure 5). Mrs. Lewis demonstrated how to manipulate the sliders for m and b , then provided students with the following instructions:

T: Now...what I want you to do on your papers, I want you to move your m . I want you to tell me what m is doing and tell me what you think m is. I want you to manipulate your b . Tell me what b is doing on your paper and tell me what you think

b represents. Okay? So take about a minute to move it around and write down what you think m and b are.

Mrs. Lewis' *statements* prompt students to *explore* as she addressed the class and directed them to make a conjecture regarding m and b in reference to the line included in the task. She then walked around to *monitor* her students as they manipulated the sliders. After a few minutes elapsed, Mrs. Lewis pulled the class back together to discuss their findings.

T: Now, most of you should have already...have played around with it enough that you have some type of idea. All right, so what do you think our m represents? All right, Jake?

Jake: Run. (1)

T: All right. So Jake says that it represents run. Okay? Let's see. So he says the m ...if I can get a marker that wants to work for me. (*Writes $m = \text{run}$ on the board.*) The m represents run is what he's saying. Okay? So looking at m , if I pull mine, is it just moving to the left or right? 'Cause isn't that what run is? (*Teacher moving slider for m during last two statements.*) (2)

Jake: Yeah

T: It is?

Ss: No!

T: Run...Okay? So run moves left to right. Correct?

Ss: Yeah.

T: Is it just moving left to right? (*Continues to move slider for m .*) (3)

Ss: No.

T: I don't see moving just left and right at all. I see it...What is changing?

S1: The motion.

T: All right. So look at it at this point. Okay? And then I'm gonna move it up. What changed from that...the one I just showed you to the one I...now? (*Manipulates slider for m in a positive direction to illustrate two lines where the second line has a larger positive slope.*) (4)

S2: The line.

T: Here's the line. I'm gonna change my m . I'm gonna increase my m . What is increasing along with m ? (*Manipulates slider for m again to show a reference line, then a line with a larger positive slope.*) (5)

Kalyn: The slope.

T: Who said that?

S: Kalyn.

T: Kalyn. The slope. My slope. My steepness. Isn't my steepness getting a little bit higher as we move the m ? (6)

S: Yes.

T: So what does m represent? It doesn't just represent run. It represents? (7)

S: Rise.

T: The rise over the run, which is? What is my rise over my run? What is the rise over run called? What's the name? What's the key word for it? (8)

S: Slope

Mrs. Lewis began the discussion by asking a *question* to *generate discussion* and *prompt further participation* from the class regarding m , which is followed by Jake's *prediction/conjecture* that m represents "run" (1). She then used her *statements*, coupled with a *notation*, to *revoice* Jake's response before using *questions* and technology to *explore mathematical relationships*, *orient/focus* student attention, and *challenge* his response (2 & 3). During the next two interactions (4 & 5), Mrs. Lewis used a series of *statements* and *questions*, along with strategic technology use, to *orient/focus* students' attention on important features of the problem situation, while *probing* their thinking. When Kalyn provided a correct response, Mrs. Lewis used her *statements* to *revoice* the student's response, as well as *relate* the association of steepness to slope that was learned earlier in the *Slope Presentation Sketch* (6). She then attempted to *connect* the two students' responses (7), prior to using *questions* to have students use correct *terminology* (8). *Wait time* was also utilized as a discourse move by the teacher during this discussion, and throughout the first teaching set.

Five practices. Mrs. Lewis actions during this exemplar show that she employed *monitoring*, *selecting*, *sequencing*, and *connecting* of student responses. She did not *anticipate* the incorrect response given by Jake ($m = \text{run}$) in either the planning session or in her pre-observation interview; however, she chose to use it as a way to begin a class discussion about the influence of m on the graph of the line that resulted in a correct response.

Technology use. This exemplar also demonstrates Mrs. Lewis' use of *discuss the screen* and is very representative of the way Mrs. Lewis utilized technology throughout the

first teaching set. She demonstrated five of six orchestration types during the first teaching set (Drijvers et al., 2010). For example, *tech demo* was utilized when she manipulated the sliders in the *Slope-Intercept Task* before instructing students to explore the influence of m and b on the graph of the line. She *explained the screen* when introducing the *Slope Presentation Sketch* by pointing out the line, the two points on the line, and their coordinates before stating, “So if I have those two points, I can find out the slope of my line”. She *linked the screen and board* when she wrote formal mathematical notations for slope on the white board. The predominant orchestration type was *discuss the screen* because she consistently made very strategic use of the technology to orient/focus student attention on important features or changes in the sketch.

Finally, she used *Sherpa-at-work* by having two students demonstrate their solution strategies during the *Slope Presentation Sketch* for examples of a line with negative and zero slope, respectively. The students had been given time to explore the pre-constructed task and find examples for a line with negative slope, and Mrs. Lewis had been walking around to monitor students’ work. She then selected a young lady to present her example for the class using the teachers’ laptop. The following exemplar captures Mrs. Lewis use of *Sherpa-at-work*:

T: Come on, Kendra, you can do it. Come show me what line you found. (*Student hesitates.*) Come on. All you got to do is move it for me. Show me the line you found.

You can use my computer. Drag that B and show me what line you found.

S: (*Student comes to the front of the classroom and inadvertently drags the entire line, rather than point B to create an example of a line with negative slope.*)

T: Oops. You dragged the line. Select B. Highlight B and drag it. Show me what you found.

S: (*Student drags the point B to create an example of a line that has a negative slope.*)

T: So she found a negative slope. So what direction is my slope going?

Mrs. Lewis then used the opportunity to discuss the direction of the line when the slope is positive and negative, based on earlier manipulation of the task and the student's example of a line with negative slope. After this discussion, Mrs. Lewis challenged students to create an example of a line with zero slope. She then selected a male student to demonstrate how he manipulated the line to show a line with zero slope. His demonstration was used to surmise that a line with zero slope will be horizontal before exploring the slope of vertical lines. The only orchestration type not observed was *spot-and-show*.

Implemented level of cognitive demand. When implementing the *Slope Presentation Sketch*, Mrs. Lewis was able to maintain the level of cognitive demand of the task at a level 3. She used the features of the sketch (hide/show features for slope triangle and slope measurement), to facilitate students' understanding of the direction of a line with a particular slope (i.e., positive, negative, zero, and undefined); however, the slope formula was not included in the sketch, nor did the teacher write the formula on the board as part of the discussion. The absence of this representation diminished students' opportunity to make connections among multiple representations for slope (i.e., rise, vertical change, $y_2 - y_1$). After the *Slope Presentation Sketch*, the class completed computational work for finding slope given a graph or the coordinates of two points.

The *Slope-Intercept Task* followed the computational work described above. As described in the exemplar, Mrs. Lewis' use of questioning while facilitating students' use of the task and during whole class discussions increased the implementation level of cognitive demand from a 3 to a 4.

Teaching Set 2

This section describes the planning/collaboration process between the researcher and participants for the second teaching set. Mathematical goals, task descriptions, potential level of cognitive demand, and technological design principles of the tasks are discussed prior to reporting results for Mrs. Lewis' selection and implementation of tasks in her classroom.

Preparation for the second teaching set was modified based on feedback from participating teachers after the planning and implementation of the first teaching set. Participating teachers expressed concerns regarding time to prepare pre-constructed sketches and accompanying student materials for the second teaching set, which focused on quadratic functions. The researcher responded to these concerns by reviewing state objectives, summer institute materials, *Exploring Algebra 1 with The Geometer's Sketchpad*, and the online professional development course for potential activities prior to the group meeting. This process resulted in a multi-page pre-constructed sketch that relied heavily on sliders to explore concepts related to quadratic functions (i.e., influence of coefficients in vertex and standard form, axis of symmetry, vertex, maximum, minimum, roots/zeros) and application problems involving maximum, minimum, and zeros. Each page of the complete sketch represented a separate task to address individual mathematical goals. The sketch was an adaptation of a project submission (during the summer institute by a non-participating

teacher in the larger professional development project) and the final project for the second module of the online course. Specific attention was given to Sinclair's (2003) design principles and level of cognitive demand in the creation of the sketch.

This pre-constructed sketch was presented at the group planning meeting for participating teachers to explore and make suggestions for alterations to the sketch. The meeting included two out of three participating teachers, the researcher, and one non-participating teacher. This non-participating teacher was invited to all planning meetings based on his involvement in the larger professional development project and the fact he taught Algebra 1 in the same school district; however, this was the only planning session he attended. Once teachers had an opportunity to explore the pre-constructed sketch, four changes were made. First, all measurements in the sketch were updated to show accuracy to the tenths place, rather than the hundredths place. Second, two separate tabs using vertex form of a quadratic function were added to focus students' attention specifically on (a) changes in the quadratic coefficient, a , and resulting changes in the graph of the function and (b) changes in h and k and the resulting changes in the graph of the function. The original sketch had one tab with three separate sliders for exploring a , h , and k , which was kept in the final version to be used as a tool for summarizing students' findings in class. Third, the coordinate grid was hidden in all tabs to make questions for students embedded in the sketch easier to read. Finally, questions for students embedded in the sketch were copied to a word document to serve as an accompanying student worksheet. Each pre-constructed dynamic tasks for the two day teaching set was evaluated at a high potential level of cognitive demand and adhered to Sinclair's (2003) design principles. Teachers had the option to use all or

selected parts of the sketch and amend the student worksheet for their instruction during the second teaching set. The next section describes Mrs. Lewis’ use of the lesson materials.

Prior to implementation. Mrs. Lewis elected to use four tasks from the pre-constructed activities on the first day of the teaching set (Figures 10-13) and two tasks on the second day (Figures 14 &15). The student worksheet including the original questions from the selected tasks in the pre-constructed sketch and omitted the questions related to tasks not planned for use. Mrs. Lewis included extra problems between *min/max* (Figure 12) and *roots/zeros* (Figure 13) tasks, as well as after *roots/zeros* task, to provide an opportunity to teach students how to use their graphing calculator to find a minimum, maximum, and zeros. Mrs. Lewis’ practice questions were written as quadratic expressions following both the *min/max* and *roots/zeros* tasks. This was interesting because the notation of the practice

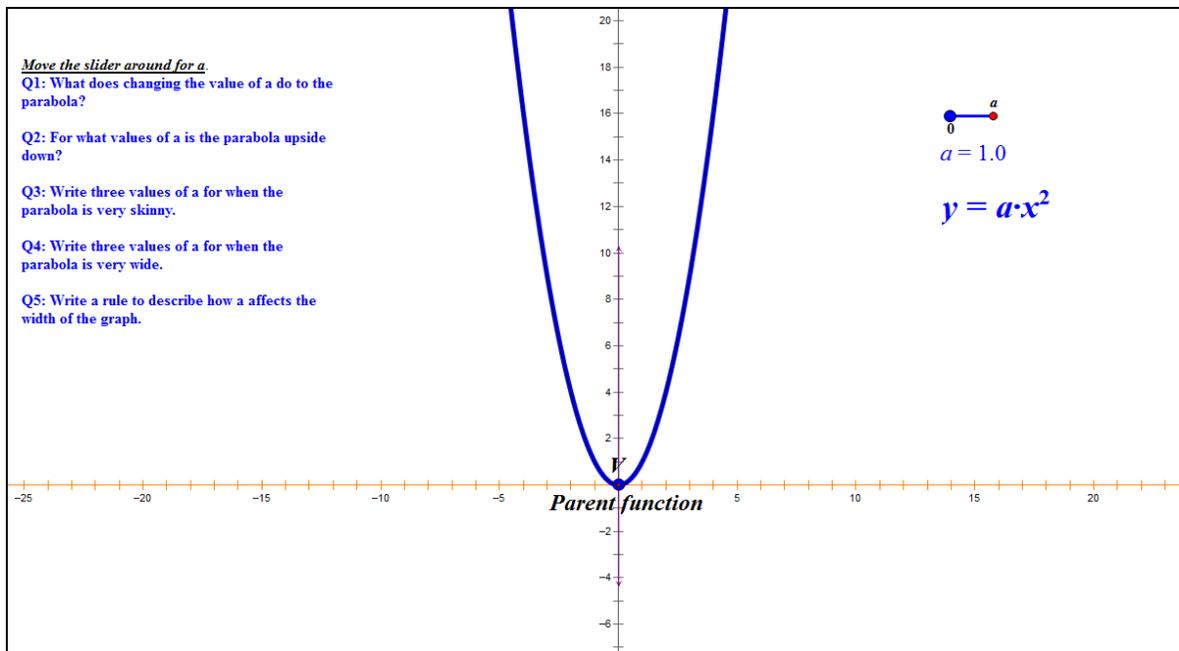


Figure 10: Quadratic Coefficient Exploration – Graphing $y = ax^2$

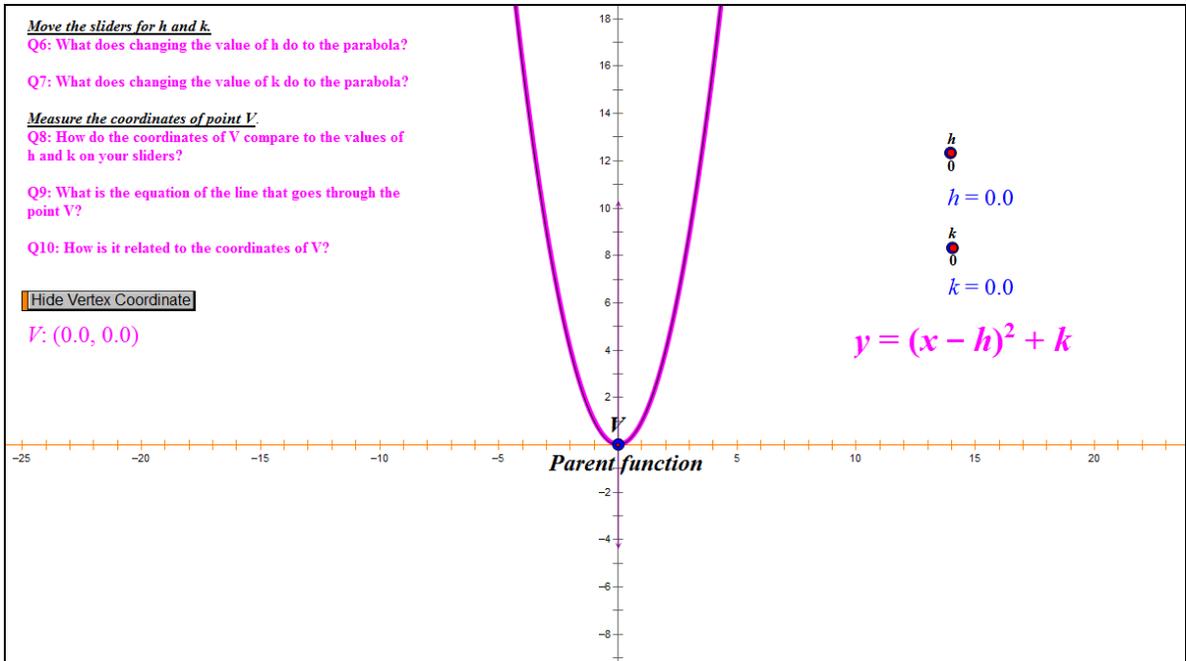


Figure 11: Vertical/Horizontal Transformation, Axis of Symmetry-Graphing $y = (x - h)^2 + k$

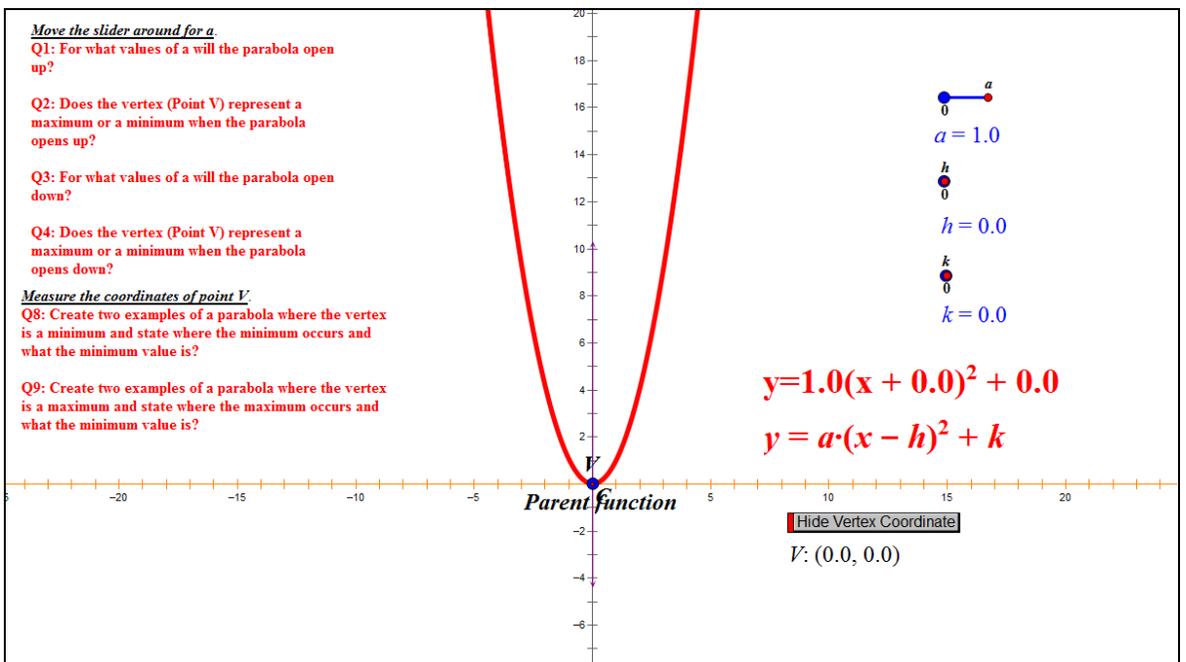


Figure 12: Minimum & Maximum Exploration

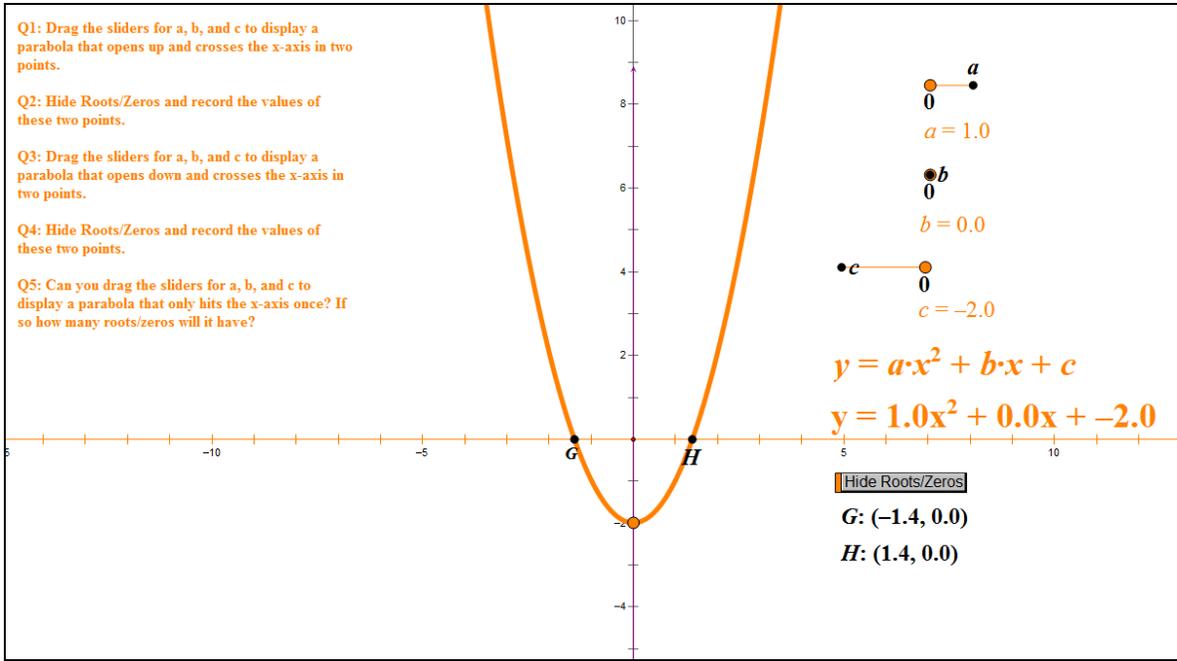


Figure 13: Roots/Zeros Exploration

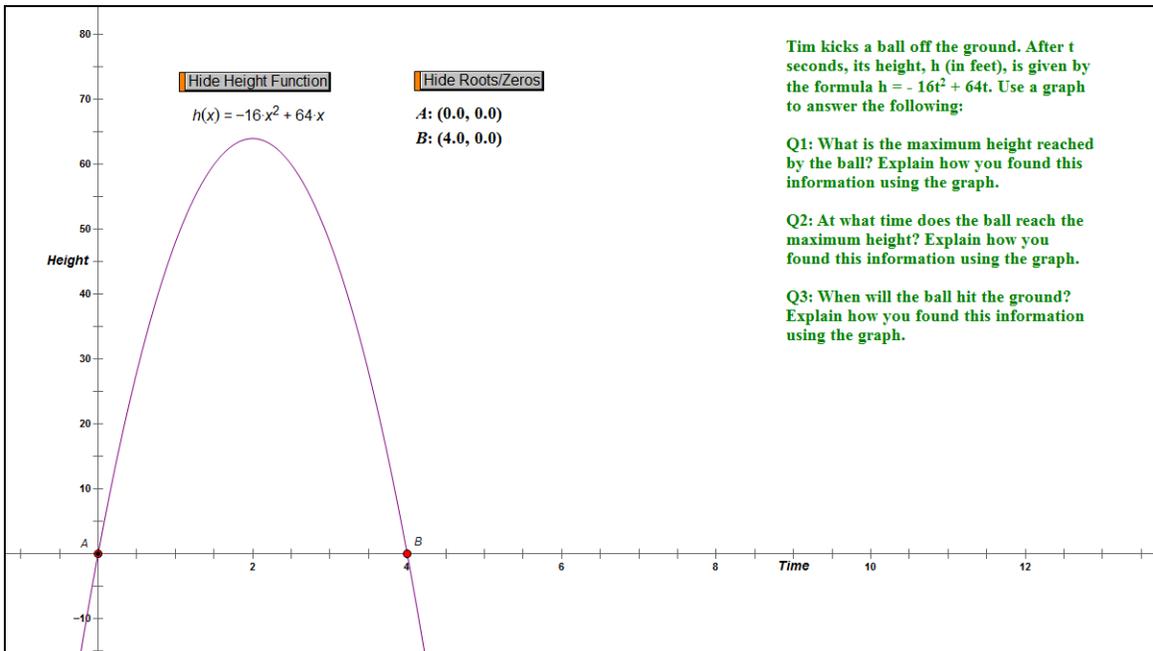


Figure 14: Quadratic Applications – Projectile Motion

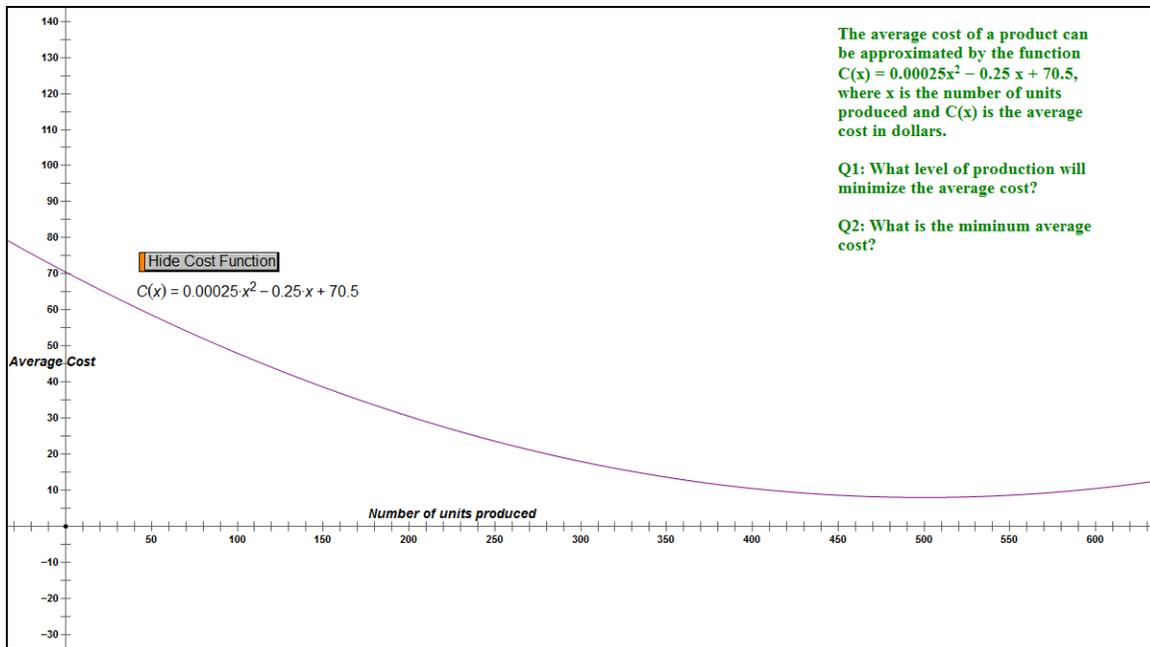


Figure 15: Quadratic Applications – Minimize Cost

questions was not mathematically correct. Writing the practice problems as quadratic expressions would be appropriate if the students were asked to factor the expressions. When finding a minimum/maximum or roots/zeros the practice problems should have been written as quadratic functions and equations, respectively. The student handout for the second day included additional problems for reviewing how to use the graphing calculator prior to the questions from the pre-constructed sketch for *Projectile Motion* (Figure 14) and *Minimize Cost* (Figure 15). Additional multiple choice and short answer practice questions were included for students to complete after the use of the pre-constructed tasks.

Mrs. Lewis did not modify any of the questions that were included with the six pre-constructed tasks she selected from the original sketch to use with her students. As such, the high potential level of cognitive demand for each pre-constructed task was retained in her

adaptation of the original activities. The tasks used on day one of the teaching set included (1) use of color to *draw attention* to changes within and among tasks and (2) sliders to *provide affordances, provide alternate paths, and support exploration*. Coupled with the student worksheet, these elements create a *shared image* for students. The two pre-constructed tasks used during the second day were more focused on students applying their understanding of minimum, maximum, and roots/zeros to projectile motion and minimizing cost. In the applications tasks, hide/show buttons were used to (1) *draw attention* to the graph, equation, and roots/zeros, (2) *provide affordances* for students to explore the problem situation, and (3) *support exploration* as student interacted with the sketch. The axes were named appropriately for the problem situation, so students were *provided alternate paths* (Hide/Show for roots/zeros or the graph) to arrive at solutions. All of these elements combined to create a *shared vision* for the class. In retrospect, one aspect missing from both application sketches was a point at the maximum and minimum of the graphs, respectively. This should have been included and linked to a hide/show button to *provide alternate paths* (i.e., hide/show values, estimate from graph, or use graphing calculator) for finding and interpreting the coordinates of the maximum and minimum point on each graph.

Implementation of tasks. This section characterizes the mathematical discussions that occurred during implementation of the tasks described above. Evidence regarding Mrs. Lewis' incorporation of the 5 practices and technology use to support these discussions are included before drawing conclusions for implemented level of cognitive demand for the tasks.

A critical change occurred in the Algebra 1 classes between the first and second teaching set that impacted Mrs. Lewis' use of *The Geometer's Sketchpad* with her students. School administrators and the math department decided to re-arrange all Algebra 1 classes into homogenous groups based on individual student performance on recent benchmark tests. After the change took place, Mrs. Lewis only retained five of her original students and 13 new students were added. Because only these five students had access to the Virtual Computing Lab to utilize *The Geometer's Sketchpad*, Mrs. Lewis opted to display all pre-constructed tasks on her computer to facilitate whole class discussions.

Mathematical discussions. During the second teaching set, there were 595 documented interactions when pre-constructed dynamic tasks were utilized. Table 6 summarizes the *mode* and *type* of these interactions. Similar to the first teaching set, results indicate that *teacher-to-whole class* was the predominant *mode* during the mathematical discussions, and that *questions* and *statements* accounted for the majority of the *types* of interactions. Questions and statements were further broken down into categories to ascertain the role each played in the mathematical discussion (Table 7). Only 11 statements and six questions were initiated by students; however, these were limited to informational questions and statements. Hence, Table 7 accurately represents Mrs. Lewis' use of higher level questions and statements and includes student use of informational questions and statements. Questions designed to promote *exploration* of mathematical meanings and/or relationships and *information* gathering questions occurred with equal frequency, followed by questions to *probe* student thinking, *orient/focus* student attention, *link/apply* mathematical ideas, provide

Table 6: Mrs. Lewis' Modes and Types of Mathematical Discourse for Teaching Set 2

Mode			Type		
Student to Teacher	165	27.73%	Answer	145 (144 from students)	24.37%
Student to Student	0	0.00%	Statement	169 (11 from students)	28.40%
Student to Group	0	0.00%	Explanation	52 (1 from student)	8.74%
Student to Whole Class	0	0.00%	Question	196 (6 from students)	32.94%
Student In Reflection	0	0.00%	Challenge	1	0.17%
Teacher to Group	0	0.00%	Relate	7	1.18%
Teacher to Whole Class	324	54.45%	Predict	2 (all from students)	0.34%
Teacher to Student	106	17.82%	Justify	4 (1 from student)	0.67%
Teacher In Reflection	0	0.00%	Generalize	14	2.35%
Total	595	100.00%	Technology	5	0.84%
			Total	595	100.00%

context, and *extend* thinking beyond the present situation. Again, Mrs. Lewis' statements most often displayed the discourse move of *revoicing*, followed by statements intended to *orient/focus* student attention; however, the number of information providing statements more than tripled compared to the first teaching number of information providing statements more than tripled compared to the first teaching set. Interestingly, there was an increase in student to teacher informational statements from the first teaching set (e.g., one for teaching

Table 7: Mrs. Lewis' Questions and Statements for Teaching Set 2

Questions			Statements		
Info	71	36.22%	Info	37	21.89%
Terminology	3	1.53%	Terminology	1	0.59%
Explore	71	36.22%	Explore	3	1.78%
Probing	19	9.69%	Probing	2	1.18%
Generating	4	2.04%	Generating	1	0.59%
Link/Apply	8	4.08%	Link/Apply	0	0.00%
Extend	5	2.55%	Extend	0	0.00%
Orient/Focus	9	4.59%	Orient/Focus	40	23.67%
Context	6	3.06%	Context	3	1.78%
Total	196	100.00%	Revoice	67	39.64%
			Encourage	9	5.33%
			Correction	5	2.96%
			Directive	1	0.59%
			Total	169	100.00%

set one compared to 11 for teaching set two) that accounted for nearly one-third of the informational statements. The fact that students did not have access to the pre-constructed tasks combined with the increased number of tasks may have contributed to the increase of informational statements from both the students and the teacher. During the first teaching set students may not have verbalized their thoughts aloud because they were exploring the technology on their own prior to the whole class discussion. Working on tasks simultaneously with the teacher, may have contributed to students verbalizing their thoughts. Mrs. Lewis' statements also served to *encourage* students after participating, *correct* student

responses, *explore* mathematical relationships, provide *context*, and *probe* student thinking while providing *terminology*, *generating discussion*, and *directing* student actions.

The following exemplar occurred during instruction related to $y = (x - h)^2 + k$ task and demonstrates the predominant pattern of interactions during the first day of the teaching set. Overall, this task was designed to have students explore and generalize the following mathematical relationships: how changing h and k transform the graph of the parabola, connection between the coordinates of the vertex and the values of h and k , the location and equation of the line of symmetry, and how the equation relates to the coordinates of the vertex. This particular exemplar pertains to the discussion regarding how changing the value of h transforms the graph of the parabola.

T: Now remember our vertex form was $a(x - h)^2 + k$. So now we're gonna look and see what h and k do. All right? So the first question for h and k , it says move the sliders for h and k . Well here's my parent function right here. (*Mrs. Lewis changed the graph of the parent function to bold using display menu in GSP*). Does everybody remember my parent function? The...where we start from; the original, right? Okay? Where it all began. All right, so we have our parent function. Now I'm gonna move h . What happens when I move h ? What happens to my parabola? (*Mrs. Lewis manipulates slider for h*). (1)

S: One side is negative, the other side is positive. (*Other students responding too, but other responses not distinguishable*).

T: So it moves from the left to the right. All right, so let's talk about it. If...okay.

Now obviously if I move...my h is on the right, it's positive. So does my graph stay

on the right too? (*Manipulating slider for h left and right, remaining in positive values*). (2)

S: No.

S: Yes.

T: It doesn't?

S: Yes.

T: Does it stay on the right side of the x ...the y -axis? (*Manipulating slider for h left and right, remaining in positive values*). (3)

S: No.

S: Yeah.

T: Yeah it does. As long as...look. Here are my values of h right here. (*Mrs. Lewis went to the board and pointed to the slider for h*). Watch the values and see what happens. Okay? As I decrease the value of h , does it stay on the right-hand side? As long as it's positive? (*Manipulating slider for h left and right, remaining in positive values*)

Ss: Yeah. (4)

T: All right. So let's go below zero and go to negative. What happens to the graph now? (*Mrs. Lewis moved the slider for h to negative values*).

Ss: Negative value.

Ss: Goes to the left. (5)

T: Goes to the left and the negative value. Okay? So what value do you think the x relates to? Or excuse me, the h relates to? (6)

S: Positive. (*Other students responding too, but other responses not distinguishable*).

T: So you're telling me when it's on the left...

S: It's negative.

T:...it goes negative and when the h values are positive it goes to the right? Do we agree on that? (7)

Ss: Yes.

T: That is what I asked you. So when h is positive, the graph goes to the right. When h is negative, the graph goes to the...

Ss: Left.

T: Left. (7)

Mrs. Lewis began the discussion by reminding students of the notation for vertex form and quickly proceeded to tell them that the class would be exploring h and k . Before manipulating the slider for h , she *oriented/focused* student attention on the graph of the parent function to provide a reference for comparison. She *informed* the class she was going to move h , and then asked a question designed to *explore* the problem situation and *generate* discussion (1). After students responded, she stated that the parabola moved left and right. Mrs. Lewis then began to manipulate the pre-constructed sketch and use questions/statements to *orient/focus* student attention and gather *information* on how the value of h influences the position of the graph of the parabola in relation to the y -axis (2, 3, 4 & 5). Once sufficient information had been gathered, she posed a higher level question (6) to *generate* discussion, *probe* student thinking, and *prompt further participation* in reference to h ; however, Mrs. Lewis reverted back to *information* gathering questions to finish the discussion (7).

Five practices. The lack of student access to the technology may have inhibited Mrs. Lewis' ability to implement the five practices during the second teaching set. Only the practice of anticipating was noted during a pre-observation interview. None of the remaining four practices were observed on the first day of the teaching set because students were working on the task simultaneously with the teacher. This prohibited Mrs. Lewis from monitoring student solution strategies, which in turn eliminated the opportunity to employ the remaining practices (selecting, sequencing, connecting). On the second day of the teaching set, monitoring, selecting, sequencing, and connecting were observed when Mrs. Lewis introduced the *projectile motion* task (Figure 14). After reading the problem situation to students, she instructed students to draw a sketch of the situation on their paper. She walked around to *monitor* student work and *probe* students' thinking regarding their sketches. She then *selected* two students' work to show to the class using the document camera. The more accurate and detailed sketch was presented last in the *sequence*. She then *connected* their sketches to the pre-constructed task by superimposing the features of their work onto the white board where the pre-constructed task was displayed.

Technology use. As in the first teaching set, Mrs. Lewis used the attributes of the pre-constructed tasks strategically. The exemplar detailed how she changed the parent function display to bold. This helped *draw attention* to important features within the sketch and *orient/focus* students on important aspects of the problem situation. On day one, her manipulation of the sliders and use of the hide/show buttons within each task was critical to the development of mathematical concepts related to quadratic functions because this was the only access students had to the pre-constructed dynamic tasks. Her manipulation of these

technological features, paired with her use of questions/statements, led to very productive mathematical discussions throughout the tasks. Her main orchestration type was to *discuss the screen*, but she also utilized *explain the screen*. The previous discourse exemplar provides an example of Mrs. Lewis use of *discuss the screen*. During interactions student thinking, as indicated by answers provided, guided the progress of the conversation. Thus, the level of cognitive demand to reach conclusions remained on the students. The following exemplar captures one of the few occasions where Mrs. Lewis demonstrated the use of *explain the screen*. This exemplar occurred toward the end of the *Projectile Motion* task.

T: So do we understand that the x -axis is the time and the y -axis is the height? So it is asking us at what time does it reach a maximum height. So it's asking us to correlate time down here. *(Draws a dashed vertical line from vertex of parabola to the x -axis)*. Right? At what time...I don't know if you can see it very well. What time, how many seconds, 'cause it's in seconds, did it take? We started at zero seconds and as the ball moved it went to one second and then it reached a maximum height at two seconds.

So we have two seconds. Two seconds that it took to reach a maximum height.

Student thinking did not factor into the process of finding out how long it took for the projectile to reach a maximum height. Mrs. Lewis utilized the graph of the parabola to illustrate how various parts of the graph contributed to the solution; however, she provided all information. This shifted the cognitive demand to Mrs. Lewis, rather than her students.

An examination of the *action* codes, within *tools* for discourse, showed that Mrs. Lewis did not take full advantage of making connections among multiple representations within the tasks. She assisted students in making very explicit connections among sliders,

their labels (i.e., a , h , and k), and their influence on the graph of a parabola. She also connected coordinates of the vertex to the values of h and k for the sliders, but she did not make explicit connections to how the coordinates of the vertex were related to values for h and k in the dynamic or symbolic representation of the function.

On day two Mrs. Lewis' use of the technology was limited to the use of the hide/show buttons built in to the quadratic application tasks. There were two hide/show buttons associated with the *projectile motion* task. The hide/show button for the graph was used first, and Mrs. Lewis utilized the display of the graph to create context and estimate values for the maximum height of the projectile, the time the projectile reached the maximum height, and when the projectile hit the ground. The last of these was then compared to the values from the hide/show button associated with the zeros of the function. In these instances, Mrs. Lewis made explicit connections among the graphical and numeric representations, but no connection was made to the symbolic representation. For example, the maximum height of the projectile, and when it reached the maximum height, were emphasized by drawing in horizontal and vertical segments to the y -axis and x -axis, respectively, before writing the answer as a coordinate. In the case of the projectile hitting the ground the graph was utilized to find the coordinates and compared to the values displayed with the hide/show button. However, the relationship between the time the projectile reached the maximum height and the value of the actual height was never discussed in reference to input and output for the function that modeled the path of the projectile. Similarly, the time representing when the projectile hit the ground was never discussed as an input that made the function value zero, which coincided with the point lying

on the x -axis. This point was not brought up in the *roots/zeros* task either. The symbolic representation was only used after discussions involving the pre-constructed tasks. The function was used as an input for $y =$ in the graphing calculator, and the calculate menu was used to compute the maximum value and zeros in *Projectile Motion* and the minimum value in *Minimize Cost*.

Implemented level of cognitive demand. During the first day of instruction, Mrs. Lewis utilized the tasks $y = ax^2$ (Figure 10), $y = (x - h)^2 + k$ (Figure 11), *min/max* (Figure 12), and *roots/zeros* (Figure 13) to introduce her students to quadratic functions. She was able to maintain a high level of cognitive demand for each pre-constructed task during implementation by pushing her students to observe changes happening in the sketch and then discussing these observations as a class. She rephrased questions and manipulated the sketch when necessary to focus student responses and connect their answers to the learning goals embedded into each pre-constructed task. Further, she relied more often on *discuss-the-sketch* and *link-screen-board*, and less on *explain-the-screen* for her orchestration types, while continually pushing her students to draw conclusions based on what they were seeing take place on the computer screen.

Mrs. Lewis continued to maintain a high level of cognitive demand for each task on the second day of instruction. She began by asking students to make a sketch of the projectile motion based on the prompt contained in the worksheet: *Tim kicks a ball off the ground. After t seconds, its height, h (in feet), is given by the formula $h = -16t^2 + 64t$.* She then selected two examples of student work to display with the document camera and connected these examples to the graph in the pre-constructed task *Projectile Motion* (Figure 14) by

drawing elements of the students' sketches on the white board where the task was displayed. The following exemplar illustrates how Mrs. Lewis utilized two students' pencil and paper sketches to begin a mathematical discussion of the situation.

T: (*Student sketch of path of ball displayed using document camera.*) Walter is showing me that the guy is kicking the ball in the air and it goes up and then it gradually comes back down. Okay? So let's look at this one. This one is pretty good too. Okay? We are kicking the ball and it's coming back up. So what is...if I were to connect all of...Okay? If I connected, okay, he's showing me each instance the ball was at, right? So the ball was going up and he's showing me each instance in which the ball comes back down. If I were to connect...can I draw on your paper?

S: Yes.

T: If I were to connect the balls and the motion of the balls, what do they make?

S: A parabola.

T: A parabola. So does everybody see if I connected the balls that Tim kicked that it would make a parabola? (*Students agree.*) Now does this parabola open upward or downward?

S: Downward.

T: Okay. Does it have a maximum or minimum?

S: Maximum. It has a maximum point in which Tim kicked it in the air, right? All right, so let me get your papers back and now we are gonna use this to talk about it (*referring to the Projectile Motion prompt*).

Mrs. Lewis utilized student thinking, as demonstrated by their pencil and paper sketch, to provide contextual information for the task before connecting the images in the sketches to more formal mathematical representations (e.g., parabola, opening upward, maximum). This effort to *link-screen-and-board* also represented the only use of *spot-and-show* by any teacher in any teaching set. Mrs. Lewis continued to *discuss-the-screen* when implementing the final pre-constructed task, *Minimize Cost* (Figure 15), and only switched to *explain-the-screen* as a last resort in either task (e.g., when students could not respond appropriately after several attempts to rephrase questions and/or re-focus student attention to pertinent elements of the task).

Teaching Set 3

The mathematical content for the third, and final, teaching set was exponential functions. The researcher, Mrs. Lewis, and Mr. Phelps met; however, Mrs. Patterson was sick, so she was sent all materials and notes from the meeting via email. Similar to the second teaching set, pre-constructed tasks and student materials adhered to state standards and were compiled/created by the researcher prior to the group planning session for the third teaching set. The activities consisted of two tasks that Mr. Phelps had co-created and presented at the summer institute (Figures 16 & 17) and three word problems involving interest, depreciation, and population growth (Figures 18-20). The word problems were released items from previous state Algebra 1 tests, and student handouts included prompts for each task.

The *MP3 Download Task* (Figure 16) used color to *draw attention, provided affordances* for exploring through the use of hide/show buttons, and *provided a shared image*

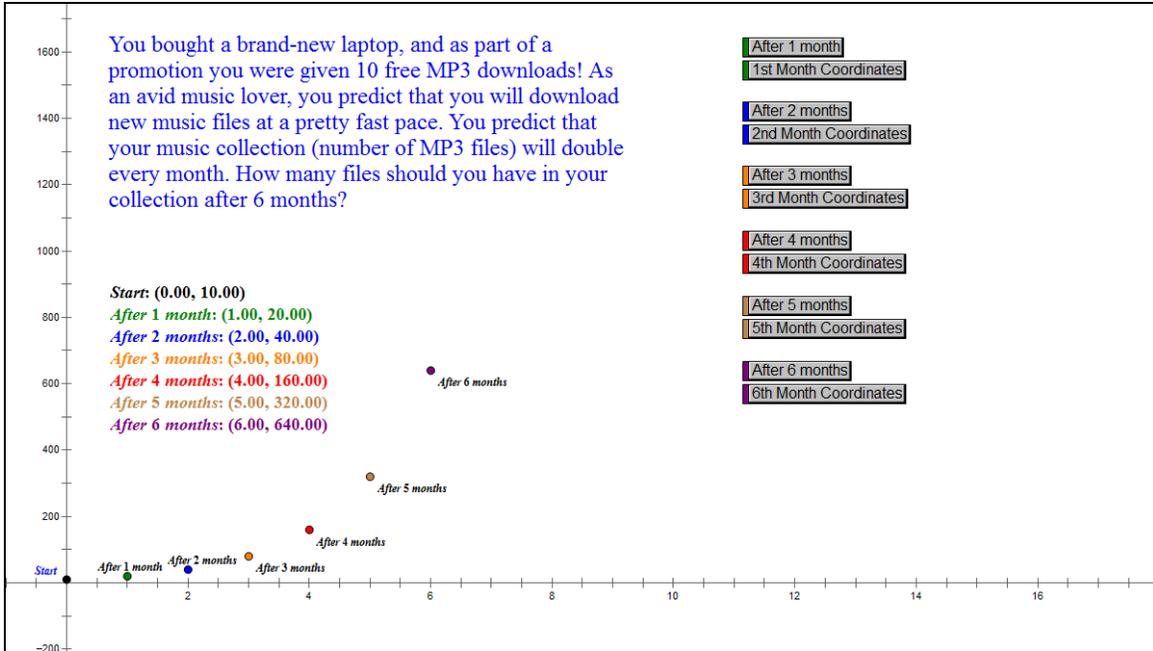


Figure 16: MP3 Download Task

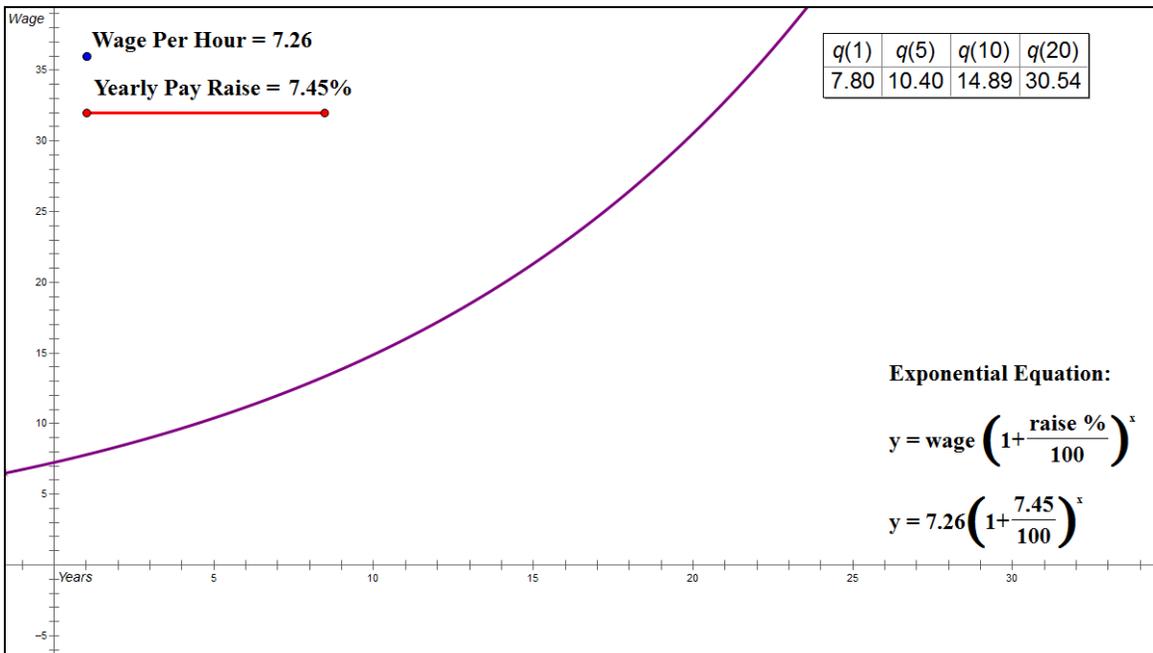


Figure 17: Pay Raise Task

for the class. The task was evaluated at a high potential level of cognitive demand by the teachers and the researcher because it involved procedures (doubling) with connections (i.e., pattern in points and context). The *Pay Raise Task* (Figure 17) adhered to several research based design principles for pre-constructed dynamic sketches. The use of color (e.g., red and blue sliders make purple graph) *drew attention*, and the presence of sliders, a table, and exponential equations *provided affordances* for exploration. These elements allowed for students to use *alternate paths* and *supported student exploration* to arrive at conclusion. Collectively, these attributes *provided a shared image* for the class. This task was evaluated at a high potential level of cognitive demand because students were asked to solve a problem they had never seen before and were explicitly asked for the reasoning that led to their conclusion about which type of pay raise would be the better choice (hourly wage or percent of salary) over time.

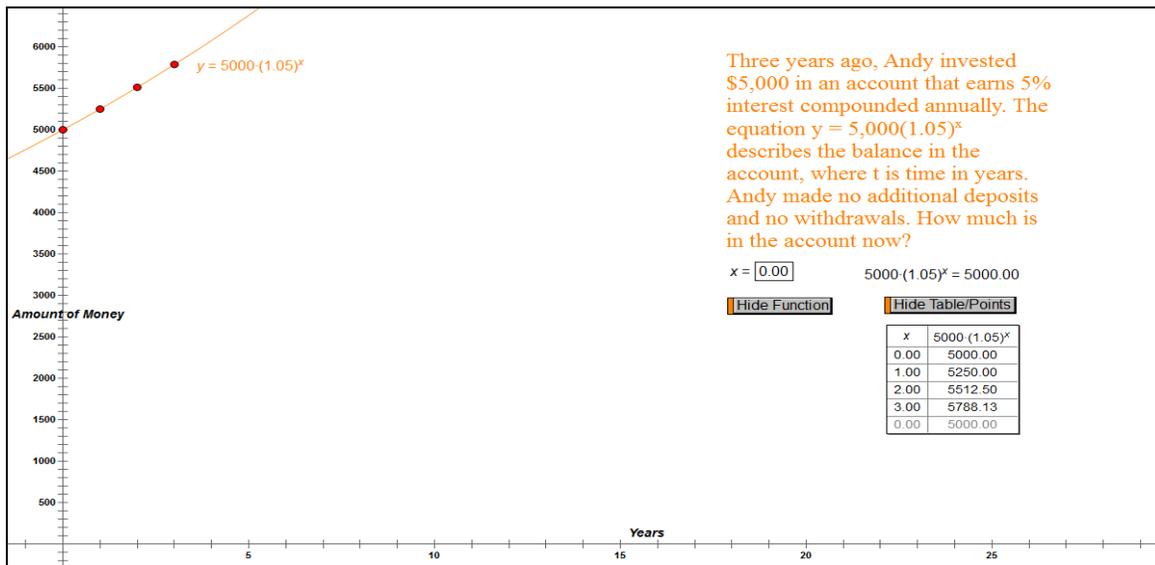


Figure 18: Exponential Applications – Interest

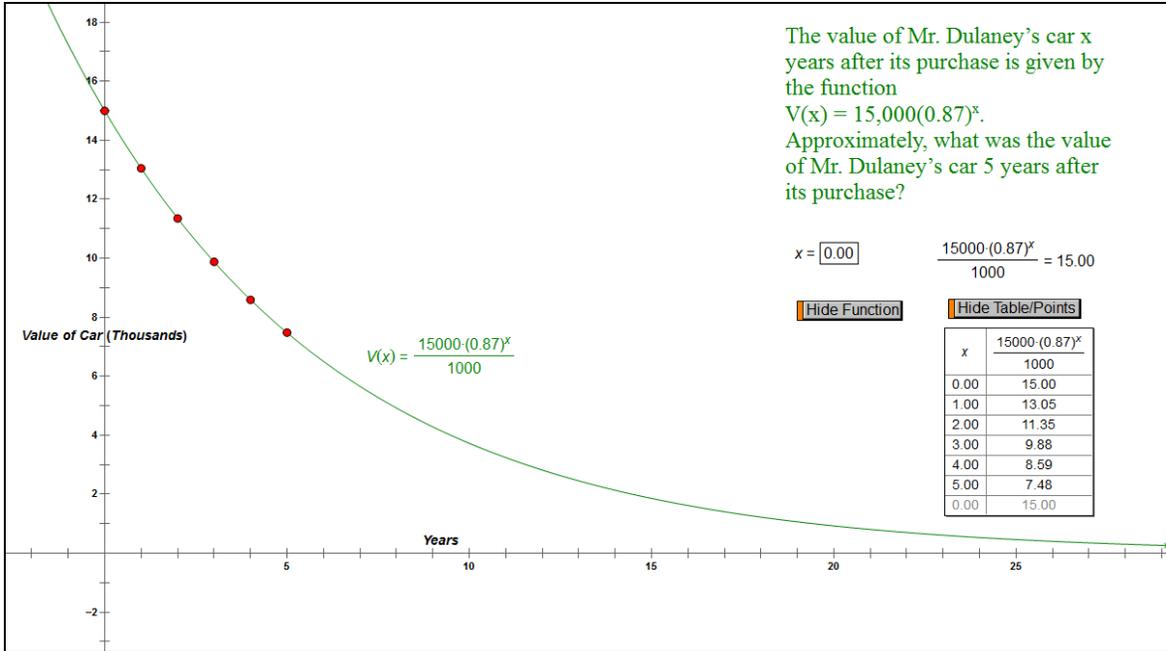


Figure 19: Exponential Applications – Depreciation

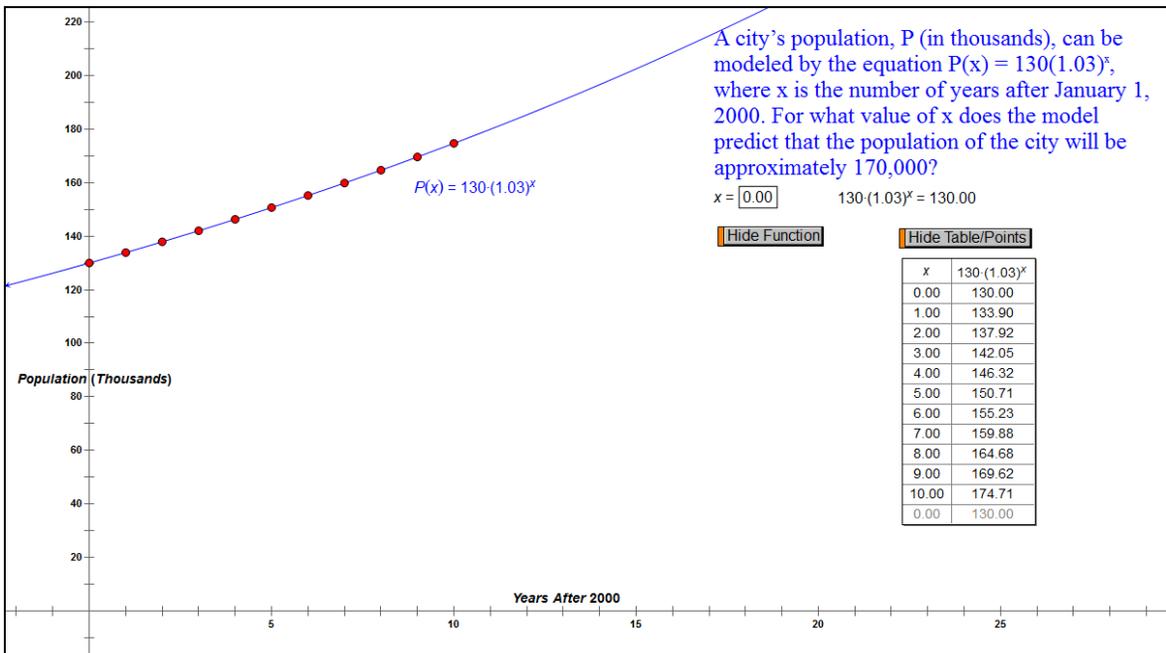


Figure 20: Exponential Applications – Population

The researcher created three tasks (Figures 18-20) containing questions from released Algebra 1 state test items that were deemed to have a high potential level of cognitive demand (procedures with connections) by the researcher and participants. Each task was designed to *draw attention* by changing color between tasks. The inclusion of the parameter, equation, table, and hide/show buttons were intended to *provide affordances, alternate paths, and support student exploration*, while *providing a shared image* for the class. All three participating teachers chose to use the pre-constructed tasks and accompanying student materials in the format presented at the group meeting.

Mathematical discussions. During the third teaching set, there were 652 documented interactions when pre-constructed dynamic tasks were utilized. Table 8 summarizes the *mode* and *type* of these interactions. Results indicate that *teacher-to-whole class* was the predominant *mode* during the mathematical discussions, and that *questions* and *statements* accounted for the majority of the *types* of interactions, followed by *answers* and *explanations*. Students contributed only 11 statements and two questions, which were low level, but they contributed most of the answers provided. Questions and statements were further broken down into categories to ascertain the role each played in the mathematical discussion (Table 9). Student contributions to statements and questions were limited to informational, so the information included in the table is very representative of Mrs. Lewis' use of higher level questions and statements. *Information* gathering questions occurred most frequently, followed by the use of questions that *explored* mathematical meanings or relationships. Mrs. Lewis also used questions to *probe* student thinking, *link/apply* reasoning,

Table 8: Mrs. Lewis Mode and Type of Mathematical Discourse for Teaching Set 3

Mode			Type		
Student to Teacher	188	28.83%	Answer	163 (162 from students)	25.04%
Student to Student	0	0.00%	Statement	190 (11 from students)	29.19%
Student to Group	0	0.00%	Explanation	77 (6 from students)	11.83%
Student to Whole Class	0	0.00%	Question	208 (2 from students)	31.95%
Student In Reflection	0	0.00%	Challenge	1	0.15%
Teacher to Group	0	0.00%	Relate	3 (1 from student)	0.46%
Teacher to Whole Class	354	54.29%	Predict	4 (all from students)	0.61%
Teacher to Student	109	16.72%	Justify	3 (1 from student)	0.46%
Teacher In Reflection	1*	0.15%	Generalize	2	0.31%
Total	652	100.00%	Technology	0	0.00%
	*Does not get coded for Type		Total	651	100.00%

provide *context*, *generate discussion*, *orient/focus* student attention, and *extend* thinking beyond the present situation. In terms of statements, Mrs. Lewis most often displayed the discourse move of *revoicing*, along with providing *information* and *orienting/focusing* students' attention. Additionally, her statements served to provide *context* for the problem

Table 9: Mrs. Lewis Question and Statement Types for Teaching Set 3

Questions			Statements		
Info	98	47.12%	Info	59	31.05%
Terminology	0	0.00%	Terminology	0	0.00%
Explore	58	27.88%	Explore	0	0.00%
Probing	17	8.17%	Probing	0	0.00%
Generating	7	3.37%	Generating	2	1.05%
Link/Apply	11	5.29%	Link/Apply	0	0.00%
Extend	2	0.96%	Extend	1	0.53%
Orient/Focus	4	1.92%	Orient/Focus	36	18.95%
Context	11	5.29%	Context	12	6.32%
Total	208	100.00%	Revoice	66	34.74%
			Encourage	6	3.16%
			Correction	5	2.63%
			Directive	3	1.58%
			Total	190	100.00%

situation, *encourage* students after participating, *correct* student thinking, *direct* student actions, *generate discussion*, and *extend* thinking beyond the present situation.

The following exemplar occurred during the second day of the teaching set after the researcher and Mrs. Lewis discussed the implementation of the *MP3 Download Sketch*. When implementing the *MP3* task, Mrs. Lewis emphasized that the growth pattern was “not a constant growth” and that “it doesn’t increase the same amount every month”. When asked about these statements during the post-observation interview, she indicated that these statements were intended to have students think about linear vs. exponential growth, but she

was not convinced that her students had made the connection between the two concepts. As a result, Mrs. Lewis decided to revisit the *MP3* task the next day to explore this comparison further and ensure students made the desired mathematical connections. She opened the discussion by asking students how many MP3 files they had initially and provided the following introduction:

T: Ten. Okay? That's how many free he got with the laptop. And then after ten, we predicted that you will double your music every month. Okay, I'm gonna take...we're gonna compare how this situation relates back to our constant, linear equation.

Similar to day one of the teaching set, Mrs. Lewis had the students tell her the values for each month and wrote them as coordinate pairs on the board. She also plotted and connected the points so that the graph of the line was superimposed on the *MP3 Download sketch* that was displayed on the whiteboard.

T: Now obviously mine looks kind of crooked 'cause I'm not perfect, right? What kind of function do I have here? What kind of graph do I have here? (1)

S: Linear.

T: It's linear. It's a line. Right? So if I continued on (*draws line beyond plotted points*), this line's gonna keep growing at a rate...and now is that line gonna...is it gonna be the same rate every month? (2)

Ss: No.

T: It's not?

S: Yes.

T: Okay. Explain...if you think it's not, then explain to me why. (3)

S: Because...

T: Huh? Did I...when I say is it the same rate, did I add the same amount of downloads every month? (4)

S: Yes.

T: Yes. How many did I add every month? (5)

S: 25.

T: Okay? So for every month that I moved, I went up 25 downloads, right? So there's my first, there's my second. (*Drawing slope steps between points on the line.*) Okay? It's like the steps. You remember when we were doing linear equations...we talked about it? It's that constant change every month. It changed 25 downloads, right? (6)

S: Yes.

T: Okay, so that slope...that rate of change is the same. So how does that rate of change, uh, how is that different from the exponential model that we did yesterday? Could somebody tell me just in your own words, how is the exponential model different yesterday? When we did this MP3 download, what was different about it than this linear line? (7)

Ss: It was ten...It was twenty...

T: Okay, so I hear it's ten every...Okay (*students still answering*). You said what?

S: It doubled every month.

T: It doubled every month. Okay? Now we say it doubles every month, doesn't the doubling...isn't that determined based off of the one previously, how many we had?

(8)

S: Yes.

T: So if I start off with 10, me doubling's only gonna go to 20. But if I start off with 30, my doubling's gonna go to 60. So let's go back and look at this. (*Uses hide/show buttons to display exponential growth in sketch*). Let's go Month 1, 2, 3, 4, 5, and 6. And I'm gonna show the coordinates as well. Now...the difference for this one...this one starts off here and it goes up and it shoots up that way. Right? So, this one is not linear. It has a curve. Am I...everybody understand that? (9)

At this point, Mrs. Lewis still did not appear convinced that her students understood the difference in the rate of change, so she had them compute the successive differences between the exponential model (double each time; +10, +20, +40...) and the linear model (+ 25 each time) for downloads.

T: So for this one, do you see that every month we...at the same rate went up. Right? We had the same rate: the same rate of change. Okay? Then, when we did the exponential model, did we go up the same amount every month? (10)

Ss: No.

She concluded by saying,

So instead of increasing by the same amount each time, by each x , right? X was the month. We increase...we doubled it, which caused us to have this curve instead of a constant, linear line. (*Pointing to each on the board*). (11)

Mrs. Lewis opens with two *information* gathering questions (1 & 2) to ascertain if students' recognize the model of the new situation as linear and their understanding about the rate of change. When the students verbalized different opinions, she focused on the incorrect

response and *challenges* the validity of their response (3). An alternate view of this interaction might have been to code the teacher's response as a *statement* that *probes* student thinking; however, the use of codes for type of mathematical discourse (i.e., challenge) supersede sub-codes for statements. However, when students are unable to explain their reasoning, she rephrases her challenge into *information* gathering questions (4 & 5). Mrs. Lewis then provided further *explanation* and referenced the graphical representation, including successive slope triangles, to finish the discussion of constant rate of change. The next portion of the exemplar (7) demonstrates, Mrs. Lewis' use of higher level questioning to *generate discussion* about the differences between the linear situation and the exponential model from the previous day, and she *revoiced* incorrect, then correct responses before again *explaining* how doubling effects the model compared to adding the same amount each month (8-11). As in the second teaching set, Mrs. Lewis displayed the pattern of posing higher level questions before rephrasing these questions into more information gathering questions to break the problem situation down for students.

5 practices. The lack of student access to the technology may have inhibited Mrs. Lewis' ability to implement the five practices during the third teaching set. During her pre-observation interviews, she anticipated potential mathematical responses and difficulties for her students. Because students worked on tasks simultaneously with the teacher, Mrs. Lewis was unable to physically monitor student solution strategies, which in turn eliminated the opportunity to employ the remaining practices (selecting, sequencing, connecting).

Technology use. Mrs. Lewis use of technology varied depending on the type of question that was being presented. During the first two tasks, she was developing the concept

of exponential functions with her students for the first time and used the hide/show buttons and sliders within each task, respectively, in alignment with this purpose. While presenting both the *MP3 Download Task* (Figure 16) and the *Pay Raise Task* (Figure 17), she utilized the features imbedded in each sketch to *draw attention*, *support exploration*, and *provide a shared image* for the class as she introduced them to the concept of exponential functions. For example, hide/show buttons in the *MP3 Download Sketch* were used to *discuss the screen* on day one of the teaching set to support/confirm student responses about the number of downloads per month and to introduce students to both the numeric and graphical pattern of exponential data (see discourse exemplar). These same attributes were utilized on day two of the teaching set to revisit linear vs. exponential functions (see exemplar above). She also made a critical move with the sliders embedded into the *Pay Raise Task*. When she was exploring the option of an hourly wage increase, she moved the slider for *Yearly Pay Raise* to zero, which made the dynamic line in the sketch appear horizontal. Then as the slider for *Wage per Hour* was manipulated Mrs. Lewis and the class discussed that the trend was for the line to move up the vertical axis, but the long term growth was stagnant until the boss gave another hourly increase. The following exemplar details the class discussion:

T: Okay. Now...what I want you to see, okay, here is my hourly wage. Okay. I'm gonna...I'm gonna decrease my pay per year and get it as close to zero as possible. (*Moves slider that represents a percentage increase per year toward zero, which straightens graph to a horizontal line.*) Now...so, with my hourly wage, ok, for my hourly wage let's say that you start out at \$9.00 an hour. Okay? As close as I can get. As the years go on, okay, so at five years am I still making \$9.00 per hour?

S: Yes.

T: You ain't even looking. Turn around. At...at five years, am I still making \$9.00 an hour? (*Points to graph with marker at five years.*)

S: Yes.

T: Okay. What about ten years? Am I still making...

S: Yes

T: ...so do I have any significant increase?

S: No.

T: So let's look. Let's say that they give me a raise and they tell me they're gonna increase my wage to \$11.00 per hour. Am I still gonna be making \$11.00 per hour no matter what time?

Ss: Yeah.

While this was a critical move, Mrs. Lewis relied solely on the dynamic graphical representation by asking student about the value at five and ten years. The sketch also contained a dynamic table that displayed the salary for a given situation at one, five, ten, and twenty years. This representation, nor the dynamic equation, was utilized when discussing hourly wage increase. A few minutes later, when exploring the *Yearly Pay Raise* option, she moved the *Wage per Hour* to the initial position, which displayed a horizontal line that represented minimum wage (\$7.25). The following exemplar details the discussion:

T: So now let's talk about salary. (*Referring to slider for percentage increase.*) So, as I move...with salary, what do you notice about the function?

S: It's raising.

T: All right. So is it raising at a significant amount?

S: Yes.

T: So you're telling me...so now that you've seen this increase, would you rather have a pay increase in your hourly wage or would you rather have an increase in your salary every year?

Ss: Salary.

T: I would rather have an increase in my percentage of my salary every year. So watch. So...yes? So my hour...if I had an hourly wage, I would still be making the same hourly wage no matter, um, what it...how many years I've been working 'til they gave me another pay increase, right? But if you chose to have an increase every year by percentage, what happens? (*Moving slider for percentage increase.*)

S: Increases a lot.

T: I mean, it...it's doing a big increase, right?

S: No.

T: So. You're telling me from five to ten years I'd still be making the same amount?

Ss: No response.

T: (*Moves slider for hourly increase to demonstrate change in graph for a \$2.00 raise.*) I'm let's say when I gave you, uh, two dollars extra, I...did it increase very much?

Ss: No.

T: No. So let's if I increase my pay raise...okay? (*Moves slider for percentage of increase.*) If my percentage every year gets higher what happens?

S: Goes up.

T: Look at five years. At five years here is my amount and then look at ten years. Is that a big difference? (*Uses pointer tool and graph to look at difference from 5 to 10 years.*)

S: Yes.

T: I mean, it's an increase, right? It's not a huge difference. But look at five years and twenty years. Is five years and twenty years a big difference?

Ss: Yes.

She again emphasized the graphical representation to compare/contrast the two options, without referring to the dynamic table or equation. In each case, hourly raise and percentage increase, Mrs. Lewis made a strategic move that assisted students in comparing growth to consider the better of the two salary options. However, the only reference to the dynamic equation involved Mrs. Lewis *orienting/focusing* students' attention to the placement of the percentage to discuss why the percentage was divided by 100. Her use of attributes changed dramatically when implementing the last three tasks.

The last three tasks were created from released items from the Algebra 1 state exam and focused on applications of exponential functions (i.e., interest appreciation, car depreciation, and population growth). A parameter linked to a dynamic expression, a hide/show button for the graph with function label, and a hide/show button for a table of values with points plotted on the graph was included in the sketch. The intent was for students to explore the situations using the parameter or the graph, make conjectures, and utilize the table/points to confirm their results. The necessary tools were included to explore

the situation and make connections among word, numeric, graphical, and symbolic representations. Mrs. Lewis used questioning to assist students in making connections between the word problem and the values in the symbolic representation (e.g., initial value, percent of increase), but this only required the prompt, not any use of the tools in the sketch. At no point did she utilize the parameter included in the sketch to explore the problem situation; rather she utilized the table in the graphing calculator. She only revisited the sketches after the solution was found to *explain the screen* by looking at the exponential form of the function, the shape of the graph displayed by the hide/show button, and the table of values with corresponding points. The following exemplar occurred after students had solved the *Depreciation Task* utilizing the graphing calculator:

T: Okay. I hear you. So my exponential...so does everybody see...now let's look at the graph of my exponential. (*Uses hide/show button to show equation, table, and graph in the pre-constructed task.*) Before, if I had the exponential, it would have looked something like this, right? (*Draws in increasing, concave up exponential graph on the board.*) It would have grown. But with decay, it's doing a complete flip flop and we're gonna start at the higher amount and then we're gonna go down until it plateaus off and slowly increases...decreases. Okay does everybody see the difference?

S: Yes, ma'am.

Similar to the *Pay Raise* task, focused solely on the graphical representation contained in the task. The difference in this situation was that Mrs. Lewis did not rely on any of the attributes contained in the pre-constructed tasks to explore or solve the problems. Nor did she solicit

student thinking to compare the present exponential decay situation to the previous exponential growth task. The use of *explain the screen* once again shifted the cognitive demand toward Mrs. Lewis, rather than remaining on her students.

The pattern that emerged during these three application tasks was slightly different than implementation of the quadratic application tasks from day two of the second teaching. During the quadratic application tasks, she relied heavily on the graphs and hide/show features contained in the tasks to have students think about 1) why the model applied to the given context (i.e., open down for max, open up for min), 2) appropriate scale for the context, and 3) approximate solutions before moving to the graphing calculator confirm student conjectures. She did not, however, focus on any connections of the problem situation and the coefficients and/or constants within the function. This pattern appeared to reverse in the third teaching set because Mrs. Lewis stressed the connections from the problem context and the coefficients and constants in the function before going to the graphing calculator to solve. She then returned to the tools of the sketch to summarize findings, without ever using the parameter.

Implemented level of cognitive demand. The implemented level of cognitive demand was for the first four tasks was evaluated at a high level of cognitive demand, but the final task did not remain at a high level of cognitive demand.. Mrs. Lewis demonstrated the use of *explain the screen*, *link screen and board*, and *discuss the screen* during implementation. More predominant use of *explain the screen*, in isolation or in conjunction with *link the screen and board*, contributed to the overall implementation being more teacher centered, especially on the last three tasks. Mrs. Lewis' use of questions and statements were

also a factor. She utilized higher level questions to begin facilitating whole class discussion. However, when rephrasing her questions in response to student input, she often lessened the cognitive demand by posing information gathering questions to guide students through mathematical objectives for each task. As stated before, this pattern may have been mitigated by lack of student access to pre-constructed tasks due to the class roster alterations.

Discussion of Mrs. Lewis

Mrs. Lewis was beginning her third year of teaching at the time of this study. All of her teaching experience occurred at the school where she was currently teaching. She entered the teaching profession by earning a four year degree in Secondary Mathematics Education from a local university and obtaining initial licensure to teach secondary mathematics. Her undergraduate studies employed the use of *The Geometer's Sketchpad*, she had participated in local professional development using *The Geometer's Sketchpad*, and she was participating in the larger professional development project focused on using *The Geometer's Sketchpad* in Algebra 1. As such, she reported being very comfortable with using *The Geometer's Sketchpad* personally, but she was less comfortable using the program with students. She demonstrated facile use of all technology in her classroom (e.g., document camera, LCD projector, graphing calculators, class website) and expressed that she could not imagine teaching without technology in general, or laptops. She stated that the teacher's role in the classroom was to serve as a facilitator to present material in different ways that students understand, show different methods, and support students during the learning process. In reference to the five practices for orchestrating productive mathematical discussions, she did not reference them at all in her initial interview, remaining focused on

the mathematical task framework. During her summative interview, she said that she did not attend to them specifically in her planning (i.e., this is how I plan to monitor, select, sequence, etc.), but she did keep them in mind. However, she indicated that her planning included thinking about the level of cognitive demand of tasks because tasks that were high level of cognitive demand promoted critical thinking in students. She also described her instructional planning in terms of thinking about what would be covered about a week in advance, then revisiting and “tweaking” individual lessons the night before.

Findings related to mathematical discourse, five practices, technology use, and level of cognitive demand corroborate much of the information Mrs. Lewis’ self-reported during her initial and summative interview. Consistent with her view of the teacher as facilitator, she relied on questioning most often across the three teaching sets. She utilized a variety of question types during each teaching set, and asked higher level questions (i.e., explore, probe, orient/focus, link/apply, generate) more often than information gathering questions during the first two teaching sets. For the final teaching set her use of higher level questions was approximately the same as her information gathering questions. Her use of the five practices was not consistent during observations. Pre and post-observation interviews indicated that she did anticipate students’ mathematical and technological approaches and difficulties throughout the semester, and the first teaching set included evidence of monitoring, selecting, sequencing, and connecting during implementation of the *Slope-Intercept* task (see exemplar for teaching set one). However, these four practices were not observed during the final two teaching sets. In terms of tasks, she created or adapted pre-constructed dynamic tasks in a manner that retained a high potential level of cognitive

demand, and she was able to maintain a high level of cognitive demand during implementation (using 2.5 as the threshold for high level of cognitive demand) in all but one task. Overall, she utilized pre-constructed dynamic tasks strategically to develop students' understanding in alignment with the mathematical goals for each lesson, but she deferred to the graphing calculator to find solutions for application based, state exam type questions, especially during the final teaching set.

Mrs. Patterson

Teaching Set 1

The mathematical goals for the first teaching set focused on slope and linear functions. Two pre-constructed dynamic geometry sketches were selected to introduce students to slope and slope-intercept form of a line. Please see the section titled "Teaching Set 1" for Mrs. Lewis for a more detailed description of the planning process, task selection, technology design principles, and potential level of cognitive demand.

Prior to implementation. Mrs. Patterson chose to implement her *Slope Presentation Sketch* (Figure 21) and the *Slope-Intercept Task* (Figure) on the first and second day of the first teaching set, respectively. Both sketches were displayed using the teacher's computer, and students accessed the files using *The Geometer's Sketchpad* via the virtual computing lab. Mrs. Patterson's *Slope Presentation Sketch* met all the requirements of the project rubric (Appendix E), so her sketch was evaluated at a 3 for potential level of cognitive demand. Her sketch also included each of Sinclair's (2003) design principles. On the second day of the teaching set, Mrs. Patterson included an electronic student document to accompany the *Slope-Intercept Task*. During the pre-observation interview, Mrs. Patterson expressed that her

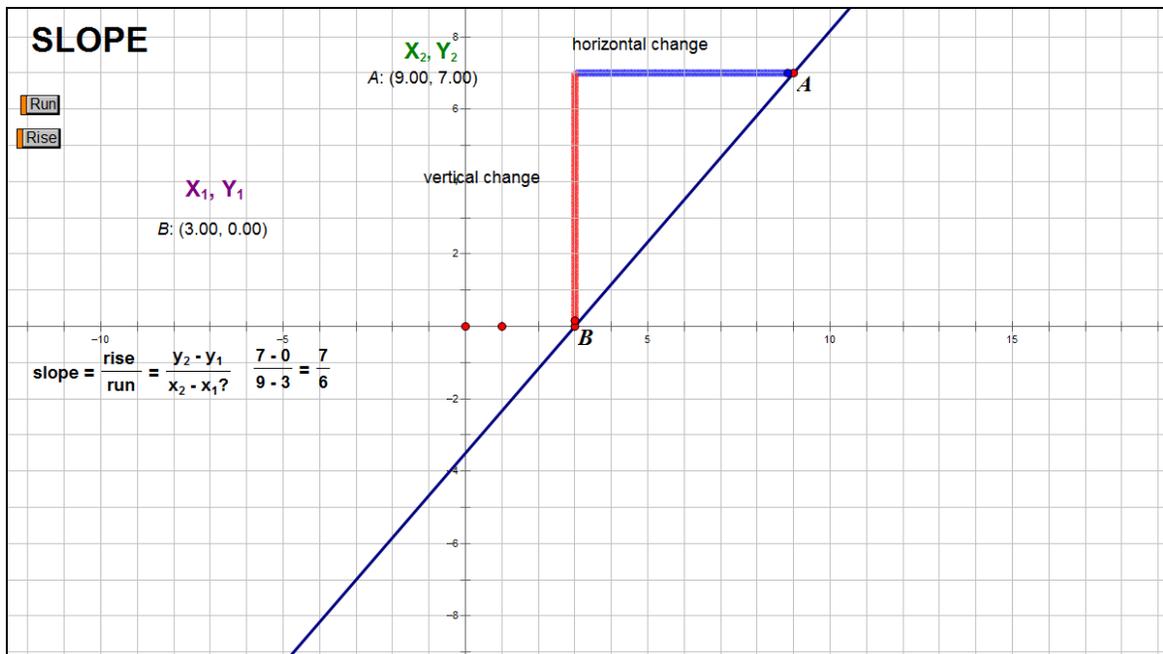


Figure 21: Mrs. Patterson’s Slope Presentation Sketch

intent was to have students complete the activity independently, and she would walk around and assist as necessary. The document contained questions to guide students’ exploration of slope-intercept form and required students to express their thinking. First, students were asked to manipulate the slider for m to make a positive, negative, and zero slope, then describe the appearance of the line. Second, students were instructed to do the same process with the slider for b . Finally, students were given five linear functions to create using the *Slope-Intercept Task* and re-create using paper and pencil on graph paper. Consequently, the potential level of cognitive demand was ranked at a 4 because student reasoning was explicitly requested and they were expected to transfer knowledge gained in the technology environment to graphing linear functions. Further, the alignment of the student document with the sketch adhered to appropriate design principles for promoting student thinking. For

example, the student document instructed students to manipulate the slider for m to positive, negative, and zero values and describe the line in their own words for each case. A similar question was asked regarding the slider for b . These questions combined with the pre-constructed task *supported exploration, provided alternate paths*, and took advantage of the *affordances provided* in the task (i.e., sliders).

Table 10: Mrs. Patterson’s Modes and Types of Mathematical Discourse for Teaching Set 1

Mode			Type		
Student to Teacher	58	14.25%	Answer	48 (42 from students)	11.79%
Student to Student	0	0.00%	Statement	157 (4 from students)	38.57%
Student to Group	0	0.00%	Explanation	63	15.48%
Student to Whole Class	0	0.00%	Question	138 (12 from students)	33.91%
Student In Reflection	0	0.00%	Challenge	0	0.00%
Teacher to Group	0	0.00%	Relate	0	0.00%
Teacher to Whole Class	243	59.71%	Predict	0	0.00%
Teacher to Student	106	26.04%	Justify	0	0.00%
Teacher In Reflection	0	0.00%	Generalize	0	0.00%
Total	407	100.00%	Technology	1	0.25%
			Total	407	100.00%

Mathematical discussions. During the first teaching set, there were 407 documented interactions when pre-constructed dynamic tasks were utilized. Table 10 summarizes the *mode* and *type* of these interactions. Results indicate that *teacher to whole group* was the predominant *mode* of discourse and *statements* and *questions* were the main *types* of discourse, respectively. Statements and questions were further broken down into categories to ascertain the role each played in the mathematical discussion (Table 11). Students contributed four statements and 12 questions that were all informational. Hence, Table 11

Table 11: Mrs. Patterson’s Questions and Statements for Teaching Set 1

Questions			Statements		
Info	102	73.91%	Info	43	27.39%
Terminology	3	2.17%	Terminology	1	0.64%
Explore	29	21.01%	Explore	0	0.00%
Probing	3	2.17%	Probe	0	0.00%
Generating	1	0.72%	Generate	0	0.00%
Link/Apply	0	0.00%	Link/Apply	0	0.00%
Extend	0	0.00%	Extend	0	0.00%
Orient/Focus	0	0.00%	Orient/Focus	50	31.85%
Context	0	0.00%	Context	0	0.00%
Total	138	100.00%	Revoice	37	23.57%
			Encourage	7	4.46%
			Correct	11	7.01%
			Directive	8	5.10%
			Total	157	100.00%

characterizes teacher and student use of informational statements and questions, as well as Mrs. Patterson's use of higher level statements and questions. The majority of *statements* utilized during the first teaching set 1) oriented/focused student attention on important aspects of the problem solving situations, 2) provided information, and 3) revoiced student responses. *Questioning* during the first teaching set focused mainly on gathering *information*; however, questioning was also utilized to encourage *exploration*, provide *terminology*, *probe* student thinking, and *generate* discussion. The following exemplar includes the entire whole class discussion of the *Slope Presentation Sketch* (Figure 18) on day one of the first teaching set. Prior to working with the sketch, the class had completed two examples that involved calculating slope using the slope formula in a contextual situation (e.g., finding the cost of renting a computer daily) and when given two points. Slope, rate of change, vertical change/horizontal change, rise/run, and the slope formula were written on the board by Mrs. Patterson prior. When Mrs. Patterson opened the sketch, she asked students questions to conclude that rise and run are associated with vertical and horizontal, respectively. She then moved to features built in to the sketch:

T: ... So this is what it's gonna look like when we have our rise. (*Uses hide/show button for rise to show vector that traces and labels vertical change.*) In this case it's going down. And then our run. (*Uses hide/show button for run to show vector that traces and labels horizontal change.*) And we have our horizontal change. So we have our vertical change and our horizontal change. Now this is made like a slope triangle. (*Points to screen at slope triangle.*) (1)

T: Now on a regular graph you can actually count these units. (*Points to screen.*)

Okay. You can count ‘em down and you can count ‘em across that way. (*Points to screen.*) But we have a formula there (*Points to formula.*), so rise is y_2 minus y_1 and run is x_2 minus x_1 . Now with the point we have A is $(0, -3)$ and then we have $(-4, 8)$. (*Used pointer tool to point at each point.*) When we calculate a slope, what do we get? (*Highlights slope calculation in sketch.*)

S: Negative 2.75 (2)

T: Negative 2.75. So what does our line look like when we have a negative slope?

Looks like it’s doing what?

T: Look at the line itself. (*Hides rise and run*) What does the line look like?

T: It’s called downhill. (*Runs finger down the line.*) Just like it’s going downhill if it has a negative slope. (3)

T: All right, what if we change it? (*Drags point A, but keeps slope negative.*) So notice as I change it or move the points, what happens to my slope?

S: It goes uphill.

T: My slope is actually changing too, isn’t it? So what if my line was a horizontal line? (*Drags point B to make horizontal line.*) What is my slope? (4)

S: Inaudible

T: Is it? Look at it. What is it? It’s right there. (*Highlights slope calculation.*)

S: Zero.

T: So if we have a horizontal line, what's the slope, Terrence? What is the slope if we have a horizontal line? Read it. It's right here, honey. It's slope. Right here. (*Points to slope calculation.*)

Terrence: Zero. (5)

T: Okay. What if I change it this way? (*Moves point B to make a line with positive slope.*) What is my slope then? Is it positive or negative?

S: Positive

T: It's...positive. And we call this?

S: Uphill. (6)

T: Uphill. So when our slope is positive it looks like this, like it's going uphill. (*Dragging finger along line.*) When it's negative, it's downhill. And when it is horizontal, what is our slope?

S: Zero. (7)

T: Zero. So what about if it's a vertical line? (*Moves point B to make a vertical line.*) What happens? What is that little thing right here? (*Points to $-\infty$, displayed as slope calculation.*) That's called undefined. Okay, because we have zero in our denominator. (8)

The first two elements of the exemplar (1 & 2) demonstrated Mrs. Patterson's use of two types of discourse, *statements* and *explanation*, before asking an *informational question*. She then *revoiced* a student's response before asking a *question* intended to *generate* discussion. When students did not respond, Mrs. Patterson used a *statement* to *orient/focus* student attention on important aspects of the problem situation and a *question* to *explore*

mathematical meaning. This did not elicit any student response, so she *answered* her own question and *explained* downhill is associated with negative slope (3). Mrs. Patterson continued to *prompt for student participation* by asking students to notice what was changing as she altered the line (4), but she did not acknowledge or correct the one student response given. Instead, she chose to focus on the changing value of the slope calculation before moving to the case of a horizontal line. Again, she received an incorrect answer, but she utilized *statements* to *orient/focus* two students' attention to attributes shown in the sketch and *questions* them to *explore* the link between a horizontal line and a slope of zero (5). Mrs. Patterson then used *information gathering questions* to discuss the graph of a line with positive slope (6) before *explaining* and using *informational questions* to summarize the relationship between slope and direction of the graph of the line (7). The last interaction demonstrated another attempt by Mrs. Patterson to *question* her students to *generate* discussion and *explore* the mathematical relationship between a vertical line and an undefined slope, but she quickly *stated* the answer to her own question by introducing the *terminology* undefined. She concluded by *explaining* that the slope is undefined because the denominator is zero (8).

Five practices. Based on classroom observations and interviews, Mrs. Patterson employed the practices of *anticipating* and *monitoring* during the first teaching set. Her responses during pre and post-observation interviews indicated that she correctly *anticipated* student strategies and difficulties, mathematically and technologically, before implementing the pre-constructed dynamic tasks. Also, during the *Slope-Intercept Sketch* she walked around the room to *monitor* student work to see if students were relying upon the *y-intercept*

and *slope* (as rise over run) to create paper and pencil graphs of the lines created by manipulating the sliders in the pre-constructed tasks. She also provided assistance to students as they worked. However, neither of the two episodes involving pre-constructed dynamic tasks demonstrated the use of *selecting*, *sequencing*, or *connecting* student responses.

Technology use. During the implementation of the *Slope Presentation Sketch*, Mrs. Patterson utilized *discuss-the-screen* and *explain-the-screen* orchestration modes to exploit the design principles of the dynamic sketch as part of a whole class discussion. The discourse exemplar above illustrated how Mrs. Patterson tried to *discuss the screen* by encouraging students to consider what was changing about the line by dragging the line to alter the slope (i.e., positive, negative, zero, undefined slope), but lack of student responses resulted in Mrs. Patterson *explaining* what was happening on the screen and making statements relating the value of the slope and the direction of the line. Also, she did not help students make explicit connections among representations of slope in the sketch (slope triangle and slope formula). She did state, “rise is y_2 minus y_1 and run is x_2 minus x_1 ” and referenced the fact that one could count the rise and run, but she never took the opportunity to link the various representations during the discussion.

When using the *Slope-Intercept Task*, Mrs. Patterson employed *tech-demo*, *discuss-the-screen*, *explain-the-screen*, and *link-screen-and-board* as orchestration types. Students had prior knowledge that m represented slope and b represented the y-intercept, so Mrs. Patterson began the activity by demonstrating how to manipulate the sliders. She then utilized *discuss-the-screen* to review students’ understanding about the value of m and the direction of the graph of the line. She attempted to *discuss-the-screen* with students to review

the influence of b on the graph of the line, but reverted to *explaining the screen* due to lack of quality student responses.

T: The next thing you are going to do is change your y -intercept. And I want you to notice what happens when I change this. Right now it is at 1. So if I change it to 0 what happens? (*Moves slider for y -intercept from 1 to 0.*)

S: The slope changes.

T: Well, the slope of the line doesn't change, does it?

S: No.

T: But what's happening to the line as I move it? (*Moves slider for y -intercept to the right then back toward zero.*) When I change B ; B became zero and at about zero what happened?

S: No response.

T: Watch your line. So let's change it to 5. (*Moves slider for y -intercept to 5.*) What happens? What happened to the line? Kayla what happened to the line?

Kayla: It got longer.

T: It got what?

Kayla: Longer.

T: It's not necessarily getting longer. Look at what happens to the line.

S: It moves.

T: The line is moving. Where is it moving?

S: Up.

T: All right. Let's try a negative. I'm gonna move this over so you can see when I move this negative. (*Moves the y-intercept slider to the right of the screen to allow more room to drag.*) So we got 5. I'm gonna make b ...notice b still over here. (*Movers slider for y-intercept to -3.*) So b is negative now. So when I go negative, I'm gonna go to negative 3. What happened? Are you noticing what's changing?

Mrs. Patterson showed several more examples and questioned students about where the line crossed the y -axis for each example. She would also point out the value of b for each example. Students never verbalized that the value of b corresponded to where the line crosses the y -axis, so Mrs. Patterson finally *explained*, "Whatever b is, is where your line is gonna cross the y -axis". Once the activity transitioned into individual student work (i.e., students were asked to create graphs for five lines with sliders and transfer to graph paper), she relied on *explain-the-screen* when assisting students as she monitored their work. Mrs. Patterson utilized *link-screen-and board* by coupling the dynamic pre-constructed task with a more traditional mathematical task of graphing lines using paper and pencil.

T: For number 1, I need to change my sliders on my program to, what did I say for number 1? What is the equation? (*Writes $y = 4x + 2$ on the board.*) So what is the slope? Which one represents slope?

S: m .

T: Remember it is $y = mx + b$, so m represents slope. So I'm gonna change m to be 4. (*Moves slider for m to 4.*) So I have slope being 4. What is my y -intercept in that equation?

S: 2.

T: So my *y-intercept* is 2. So I'm gonna change b to 2. (*Moves slider for b to 2.*) Now we are gonna graph that on your paper. What are you gonna do? How are you gonna do that? (*Does not wait for a student response.*)

T: So when you're graphing these equations you need to start with your *y-intercept*. So if your *y-intercept* is 2, I want you to plot a point at 2. (*Students request graph paper. Teacher passes out graph paper and restart explanation.*)

T: An we're graphing a line $y = 4x + 2$. We already know our *y-intercept* is 2. So at 2 we are gonna plot a point. Now, what is my slope?

S: Fraction.

T: So what is understood to be a fraction?

S: 4.

T: If my slope is 4, if m is equal to 4, how would I write that as a fraction?

S: 4 over 1

T: 4 over 1. So now which is my rise?

S: 4.

T: The top number is my rise. So my rise is 4 and my run is 1. So if I am gonna graph, I have to start back at this point and go up how many?

Ss: 4.

T: 4. One, two, three, four, over one and plot a point. Now, those are the two points I need to make that line. (*Students request rulers. Teacher passes out rulers and finishes explanation.*)

T: So you're gonna do the same thing with the next three that you have on the board. Once you graph it there (*referring to Slope-Intercept Task*) you are gonna be seeing where it crosses the *y-axis* and what you need to do, what your line should look like when you graph it.

In this exemplar, Mrs. Patterson used the *Slope-Intercept Task* to have students create a graph of the line using sliders before graphing the line with pencil and paper. The pre-constructed task served as a check for students to link what they saw on their computer screen to a more traditional task of graphing linear equations.

Implemented level of cognitive demand. Mrs. Patterson relied on *statements* and *explanations*, as well as *information gathering questions*, to *explain-the-screen* during her use of the *Slope Presentation Sketch*. Her sketch adhered to the project rubric and research based design principles, and she utilized the affordances built into the sketch during implementation. Unfortunately, she omitted representational connections within the sketch. Collectively, these actions diminished the potential high level of cognitive demand to a low level of cognitive demand during implementation.

For the *Slope-Intercept Task*, the implemented level of cognitive demand decreased but remained high. Mrs. Patterson's use of *questions to discuss-the-screen* with students during the introduction of the activity maintained the potential high level of cognitive demand, but she provided *statements to explain-the-screen* to students during the graphing portion of the task. This occurred during individual student work and whole class discussions for the graphing portion of the task, which encompassed the majority of the time for the task. Mrs. Patterson stated in her pre-observation interview that she wanted students to make the

connection of how to graph lines in slope-intercept form using pencil and paper from the sliders sketch, but her *explanations* and *statements* reduced the cognitive demand for the students, such that the full potential of the task was not met.

Teaching Set 2

The mathematical goals for the second teaching set focused on quadratic functions. Six pre-constructed dynamic geometry sketches were selected to introduce students to quadratic functions and their applications. Please see the section titled “Teaching Set 2” for Mrs. Lewis for a more detailed description of the planning process, task selection, technology design principles, and potential level of cognitive demand.

Prior to implementation. During pre-observation interviews, Mrs. Patterson expressed her intentions to utilize the same pre-constructed tasks that Mrs. Lewis implemented on day one and two of the second teaching set (Figures 10-15). Her student worksheet included the original questions from the selected tasks in the pre-constructed sketch and omitted the questions related to tasks not planned for use. Similar to Mrs. Lewis, Mrs Patterson included extra problems between *min/max* (Figure 12) and *roots/zeros* (Figure 13) tasks, as well as after *roots/zeros* task, to provide an opportunity to teach students how to use their graphing calculator to find a minimum, maximum, and zeros; however, Mrs. Patterson altered the coefficients and format of the questions on her student handout. Recall that Mrs. Lewis’ practice questions were inappropriately written as quadratic expressions. Mrs. Patterson adhered to correct mathematical notation by expressing her practice questions as quadratic functions (*min/max*) and quadratic equations (*roots/zeros*). The tasks and student handout for day two of the teaching set were identical for both teachers. The student handout

for the second day included additional problems for reviewing how to use the graphing calculator prior to the questions from the pre-constructed sketch for *Projectile Motion* (Figure 14) and *Minimize Cost* (Figure 15). Additional multiple choice and short answer practice questions were included for students to complete after the use of the pre-constructed tasks.

Like Mrs. Lewis, Mrs. Patterson retained all the original questions with the six pre-constructed tasks she selected to use with her students. As such the high potential level of cognitive demand was retained in her adaptation of the original activities. The tasks used on day one of the teaching set included (1) use of color to *draw attention* to changes within and among tasks and (2) sliders to *provide affordances, provide alternate paths, and support exploration*. Coupled with the student worksheet, these elements create a *shared image* for students. The two pre-constructed tasks used during the second day were more focused on students applying their understanding of minimum, maximum, and roots/zeros to projectile motion and minimizing cost. In the applications tasks, hide/show buttons were used to (1) *draw attention* to the graph, equation, and roots/zeros, (2) *provide affordances* for students to explore the problem situation, and (3) *support exploration* as student interacted with the sketch. The axes were named appropriately for the problem situation, so students were *provided alternate paths* (Hide/Show for roots/zeros or the graph) to arrive at solutions. All of these elements combined to create a *shared vision* for the class. Recall that, one aspect missing from both application sketches was a point at the maximum and minimum of the graphs, respectively. This should have been included and linked to a hide/show button to *provide alternate paths* (i.e., hide/show values, estimate from graph, or use graphing

calculator) for finding and interpreting the coordinates of the maximum and minimum point on each graph.

Implementation of tasks. Although Mrs. Patterson taught at the same school as Mrs. Lewis, her class roster for the second teaching set was not altered by the rearrangement of students into homogeneous groups because there was not another section of Algebra 1 being taught during the period she was being observed. During implementation she only utilized five of the six intended tasks. On day one, the whole class discussion for $y=(x - h)^2 + k$ encompassed several of the questions included with the *min/max* task, so Mrs. Patterson proceeded to the practice questions mentioned above. She then returned to the use of *The Geometer's Sketchpad* with the *roots/zeros* task. She did implement both *Projectile Motion* and *Minimum Cost* tasks on day two of the teaching set, as planned.

Mathematical discussions. During the second teaching set, there were 588 documented interactions when pre-constructed dynamic tasks were utilized. Table 12 summarizes the *mode* and *type* of these interactions. Results indicate that *teacher to whole class*, *teacher to student*, and *student to teacher* were the modes of discourse during the second teaching set, and *questions*, *statements*, and *answers* accounted for the majority of the type of discourse that occurred. *Explanations* were also utilized as a type of discourse, but the percentage was cut by more than half when compared to the first teaching set for Mrs. Patterson. It is also noteworthy that the percentage of *questions* increased, while the percentage of *statements* decreased. This resulted in a reversal of *questioning* and *statements* as predominant types of discourse when compared to the first teaching set. The use of questions and statements was further broken down into categories to ascertain the role each

Table 12: Mrs. Paterson’s Mode and Type of Mathematical Discourse for Teaching Set 2

Mode			Type		
Student to Teacher	152	25.85%	Answer	137 (125 from students)	23.30%
Student to Student	0	0.00%	Statement	159 (7 from students)	27.04%
Student to Group	0	0.00%	Explanation	50 (1 from student)	8.50%
Student to Whole Class	0	0.00%	Question	237 (18 from students)	40.31%
Student In Reflection	0	0.00%	Challenge	1	0.17%
Teacher to Group	0	0.00%	Relate	2	0.34%
Teacher to Whole Class	270	45.92%	Predict	1 (from a student)	0.17%
Teacher to Student	166	28.23%	Justify	0	0.00%
Teacher In Reflection	0	0.00%	Generalize	0	0.00%
Total	588	100.00%	Technology	1	0.17%
			Total	588	100.00%

played in the mathematical discussion (Table 13). Students contributed seven statements and 18 questions, which were all informational. Table 13 reflects teacher and student use of low level questions and statements, along with Mrs. Patterson’s use of higher level questions and statements.

Table 13: Mrs. Patterson’s Questions and Statements for Teaching Set 2

Questions			Statements		
Info	83	35.02%	Info	38	23.90%
Terminology	8	3.38%	Terminology	3	1.89%
Explore	75	31.65%	Explore	1	0.63%
Probing	36	15.19%	Probe	9	5.66%
Generating	9	3.80%	Generate	0	0.00%
Link/Apply	3	1.27%	Link/Apply	0	0.00%
Extend	6	2.53%	Extend	0	0.00%
Orient/Focus	17	7.17%	Orient/Focus	40	25.16%
Context	0	0.00%	Context	0	0.00%
Total	237	100.00%	Revoice	49	30.82%
			Encourage	6	3.77%
			Correct	7	4.40%
			Directive	6	3.77%
			Total	159	100.00%

When *questioning* students, Mrs. Patterson relied most heavily on *information gathering*, but she also utilized higher level questions intended to *explore* mathematical relationships and *probe* student thinking. Questions that *oriented/focused* student attention, *generated discussion*, introduced *terminology*, *extended* current thinking, and *linked/applied* reasoning were present, but to a lesser extent. Her use of *statements* was used most often to *revoice* student thinking, followed by *orienting/focusing* and *informational* statements. She also demonstrated the use of statements to *probe*, *correct*, *encourage*, and *direct* her students. Statements that introduced *terminology* and promoted *exploration* were also observed.

The following exemplar occurred during instruction related to *Graphing $y = (x - h)^2 + k$* and demonstrates Mrs. Patterson's predominant pattern of interactions during the first day of the teaching set. Recall, this task was designed to have students explore and generalize the following mathematical relationships: how changing h and k transform the graph of the parabola, connection between the coordinates of the vertex and the values of h and k , the location and equation of the line of symmetry, and how the equation relates to the coordinates of the vertex. This particular exemplar pertains to the discussion regarding how changing the value of h and k transform the graph of the parabola.

T: It says, what does changing the value of h do to the parabola? And mine are right on top of one another, but I'm gonna move it. (*Moves dragging point for slider that was superimposed on the zero point, then manipulating the slider for h .*) Oh! What happened? (*Continues to manipulate slider for h .*)

S: It's moving.

T: It's moving. So if I go to the right, what happens? (*Moving the slider for h to the right.*) (1)

S: It moves.

S: It goes up.

T: It goes to the...? (*Moving the slider for h to the right.*) (2)

S: Right.

T: Right. So if I move it to the left? (*Moving the slider for h to the left.*) What happens if I go past zero? (3)

S: It gonna go to the left

S: Negative.

T: Then it's on the negative x -axis. Okay. (4)

S: That's what I said.

S: So that's right?

T: Yes ma'am. That's called answering the question. Who said they don't get it? (5)

S: What do we write?

T: So what did you...what happened? What does changing the h value of the parabola do?

S: Moved it.

T: It moved what? Either left or right. And so how...when did it move left?

S: When h was negative.

T: When h was negative. When did it move right?

S: When h was positive.

T: When h was positive. Okay. (6)

T: All right. So what's the next question? What does changing the value of k do to the parabola? Well let's try that one. Oh, the whole thing is moving. (*Accidentally moved the entire slider because the dragging point was situated over the zero point.*) Let's see if I can get it off. (*Re-adjusts slider so dragging point is not on zero point.*) Ah.

So when I moved k , what happened? (*Manipulating slider for k .*) (7)

Ss: It moves up. It moves down.

T: It goes up and down. So if...when k is positive, it's where? (*Moving slider for k on the right hand side of zero.*)

S: Up. (8)

T: Up. How would you describe up though? Up above what? (*Moving slider for k on the right hand side of zero.*)

S: Origin

S: Positive. (9)

T: Up above...it's positive, but how is it positive? It's up above what axis? (*Moving slider for k on the right hand side of zero.*)

S: X. (10)

T: So if I go past zero, what do you think will happen? (*Moving slider for k toward the left and going to the left hand side of zero.*)

Ss: Negative.

Ss: It's gonna go down.

T: It's gonna go below the x -axis. (11)

At this point, Mrs. Patterson continued to work with the sketch to have students examine the coordinate of the vertex compared to the values of h and k .

During the first interaction (1), Mrs. Patterson began with a question designed to promote *exploration*, provided an *informational* statement, and then posed a question to *generate discussion* based on what students were seeing happen in the sketch. Based on students' responses, she posed an *informational* question (2). When she received the correct answer, she *revoiced* the student's response and posed two questions that promoted *exploration* of mathematical relationships (3). Mrs. Patterson then *revoiced* the combination of two students' responses (4). She then *encouraged* students for their participation and

responded to a students' question (5). When addressing the confused student, she employed a series of *informational* questions and statements to *revoice* student responses to help them clarify their understanding (6). Similar to the first interaction, Mrs. Patterson began the discussion for k with two questions to promote *exploration* and *generate* discussion (7) before *revoicing* student responses and posing a question to *orient/focus* students' attention to the location of the parabola (8). She then posed a series of three questions to *probe* student thinking (9 & 10) about the location of the parabola, and finished with an *informational* question (10). Mrs. Patterson concluded the discussion of h and k with a combined statement/question that *oriented/focused* student attention to *generate* discussion, followed by a combined *revoicing* of two students' responses (11).

Five practices. Similar to the first teaching set, Mrs. Patterson employed the practices of *anticipating* and *monitoring* during the second teaching set. Her responses during pre and post observation interviews indicated that she correctly *anticipated* student strategies and difficulties, mathematically and technologically, before implementing the pre-constructed dynamic tasks. Also, during all tasks she walked around the room to actively *monitor* student work and provide assistance to students as they worked. However, none of the episodes involving pre-constructed dynamic tasks demonstrated the use of *selecting*, *sequencing*, or *connecting* student responses.

Technology use. Mrs. Patterson's class roster was not affected greatly by the reorganization of students into homogenous groups, so she opted to have her class access pre-constructed tasks on the first day of instruction. She allowed adequate time for students to interact with the pre-constructed tasks prior to initiating whole class discussions (*discuss-*

the-screen), and her use of the attributes built into the tasks was strategically aligned with the mathematical goals. Lack of student responses sometimes led to Mrs. Patterson *explaining-the-screen*; however, during explanations she continued to re-focus students' attention to mathematical connections among representations in the sketch. In the following exemplar, Mrs. Patterson was trying to facilitate a discussion regarding the relationship between the coordinates of the vertex point, represented as (x, y) , and the maximum/minimum of a parabola, represented as (h, k) .

T: It says: How does the coordinates of V compare to the values of h and k on your sliders? So how are you gonna find that? You see where V is? Does everybody see V on their graph? (*Points to vertex.*)

S: Yes.

T: All right. So now we need to measure the coordinates of V . So what I want you to do is click on V so that it is highlighted, then go up to *Measure* and go to *Coordinates*. And what does that tell you? It says what?

S: Inaudible.

T: Well mine is a little different than yours. But yours says what?

S: Inaudible.

T: So you V is right on what?

S: Origin.

T: Origin. So what are the coordinates for the origin?

S: *Zero, zero.*

T: *Zero, zero.* So what do you notice? Look at h and k . (*Student asks a question, so teacher goes to student to help with a technology question. Then returns to her question about h and k .)*

T: What I want you to notice is when you look at h and k , what do you notice about the coordinates of h and k ? What do you notice about the coordinates?

S: Same.

T: They are the same. So what does that tell you? Which one is x ?

S: Inaudible.

T: And so that is h . Which one is y ?

S: 2.1.

T: So we don't...now what else can I say about that point? If I look at the parabola, what did...what would I say about the point? This point right here. (*Points at vertex, but students do not respond.*)

T: Well what did the parabola do? (*Traces graph of the upward opening parabola in the pre-constructed task.*)

S: It came down.

T: It came down. At this point what happened? (*Pointing at vertex.*)

S: It curves up.

T: It goes back up. Now, remember the *axis of symmetry* we talked about? What is it going through? It's going through that point. (*Pointing at vertex.*) What would I call that?

S: Vertex.

T: Okay. So what is a vertex? So these coordinates are the same as? (*Pointing to values for h and k .*) The h and k are the same as the coordinates of the vertex.

When students failed to respond to her initial discussion questions, she did not provide them with the answer, rather she refocused their attention to the values of x and h , then y and k . At this point students were able to confirm the values for each pair were the same. Mrs. Patterson then pushed students to name the vertex before *explaining* that the coordinates of h and k are the same as the vertex. She then uses the task to help students realize that the maximum or minimum of a parabola occurs at the vertex.

On day two of the second teaching set, Mrs. Patterson most often chose to *explain-the-screen* during the *Projectile Motion* and *Minimize Cost* tasks before instructing students to solve using the graphing calculator. The following exemplar took place during the *Projectile Motion* task:

T: (*Shows graph in pre-constructed task using hide/show button and read prompt.*)

Tim kicks a ball off the ground after t seconds his height is given by the formula.

Now, here is the formula $-16t^2 + 64t$. What do you think t represents?

S: x

S: Time.

T: Time. It says time in seconds. So you would use x in the calculator, right. Well the whole point is he kicked the ball and the ball went up and then it came back down. So the first point, what is the maximum? Now why did I ask the maximum height?

S: To figure out how high the ball is...

T: We're talking about the highest point of that parabola, so reached by the ball.

Explain how you find this information in the graph. So how do you think you are going to do that? (*No student response.*)

T: You're talking about the maximum height. So when we do the maximum, what are we finding?

S: The vertex.

T: The vertex. The maximum height. So looking...let me pull this over. (*Adjusts screen to better view graph of parabola.*) Which axis represents height and which axis represents time? (*Does not wait for student responses.*) So I have height here (*points to y-axis*) and time here (*points to x-axis*). So if I did this and found the vertex, it says what is a maximum height reached by the ball. Which one of those variables do you think will represent the height?

S: h

T: But when we put it in the calculator, h is gonna be our what?

S: y

T: Notice it is gonna be our y . Notice, where is the height? The y -axis, and time is on your x -axis. So I want you to go ahead and find the vertex. (*Students use graphing calculator to solve.*)

In essence, the tasks served to provide students with a *shared image* of the problem situation.

As the exemplar illustrates, the prompt contained in each task was read by the class before showing the graphs for each problem situation. Brief discussion occurred to reinforce why

they were calculating a maximum or a minimum, but the direction quickly shifted to finding solutions using the graphing calculator or symbolic calculations to find the vertices.

Implemented level of cognitive demand. During implementation, Mrs. Patterson was able to maintain a high level of cognitive demand for tasks implemented on day one of the second teaching set. Her reliance on *questioning* and *discuss-the-screen* with the $y = ax^2$, $y = (x - h)^2 + k$, and *roots/zeros* tasks not only kept students engaged, but also placed more of the cognitive demand on the students. When she did revert to *explain-the-screen*, she continued to draw student attention to multiple representations presented in the tasks. She also limited her use of *tech-demo* (teacher centered) to one occurrence by demonstrating how to manipulate a slider in the first task.

Implementation for each tasks during the second day of instruction failed to maintain a high level of cognitive demand. Mrs. Patterson continued to try to *discuss-the-screen*, but she ran into similar issues with students responding to her questions. The difference on day two of the teaching set was that Mrs. Patterson was more focused on getting students to arrive at a correct answer when utilizing *explain-the-screen* with the *Projectile Motion* and *Minimize Cost* tasks. She also utilized *link-screen-board* to have students compute the values for the coordinates of the vertex/minimum/maximum using either the graphing calculator or the formula $x = -b/2a$ (combined with substitution in the original function); however, she provided few connections back to the graph of the problem situation or explanation of where the formula originated.

Teaching Set 3

The mathematical goals for the third teaching set focused on exponential functions. Five pre-constructed dynamic geometry sketches were selected to introduce students to exponential functions and their applications. Please see the section titled “Teaching Set 3” for Mrs. Lewis for a more detailed description of the planning process, task selection, technology design principles, and potential level of cognitive demand.

Mathematical discussions. During the third teaching set, there were 386 documented interactions when Mrs. Patterson utilized pre-constructed dynamic tasks were utilized. Table 14 summarizes the *mode* and *type* of these interactions. *Teacher to whole class*, *student to teacher*, and *teacher to student* represented the most prevalent *modes* of mathematical discourse. *Questions* and *statements* were represented almost equally, followed closely by *answers*, with respect to the *type* of mathematical discourse. *Explanations* decreased again when compared with teaching set two. Questions and statements were further broken down into categories to ascertain the role each played in the mathematical discussion (Table 15). Six informational statements and six informational questions were contributed by students. Table 15 is reflective of teacher and student use of informational questions and statements, and Mrs. Patterson’s use of higher level questions and statements. Mrs. Patterson continued to utilize *information gathering* questions most often with her students. Questions intended to *explore* mathematical meaning and relationships and *probe*

Table 14: Mrs. Patterson Mode and Type of Mathematical Discourse for Teaching Set 3

Mode			Type		
Student to Teacher	117	30.31%	Answer	105 (102 from students)	27.20%
Student to Student	0	0.00%	Statement	121 (6 from students)	31.35%
Student to Group	0	0.00%	Explanation	15 (1 from student)	3.89%
Student to Whole Class	1	0.26%	Question	135 (6 from students)	34.97%
Student In Reflection	0	0.00%	Challenge	0	0.00%
Teacher to Group	0	0.00%	Relate	7 (1 from student)	1.81%
Teacher to Whole Class	171	44.30%	Predict	2 (from students)	0.52%
Teacher to Student	97	25.13%	Justify	0	0.00%
Teacher In Reflection	0	0.00%	Generalize	0	0.00%
Total	386	100.00%	Technology	1	0.26%
			Total	386	100.00%

student thinking were also posed frequently. The use of questions to *orient/focus* student attention, *generate* discussion, provide *context*, and *link/apply* or *extend* thinking were present sporadically. Mrs. Patterson's statements served to *revoice* student responses approximately twice as often as providing *information* and *orienting/focusing* student attention. The balance of statements throughout the teaching set were posed to *correct* and *probe* student thinking, *encourage* students after participation, and provide *context*. Mrs.

Table 15: Mrs. Patterson Questions and Statements for Teaching Set 3

Questions			Statements		
Info	62	45.93%	Info	26	21.49%
Terminology	0	0.00%	Terminology	0	0.00%
Explore	35	25.93%	Explore	0	0.00%
Probing	20	14.81%	Probe	4	3.31%
Generating	5	3.70%	Generate	0	0.00%
Link/Apply	2	1.48%	Link/Apply	0	0.00%
Extend	2	1.48%	Extend	0	0.00%
Orient/Focus	6	4.44%	Orient/Focus	24	19.83%
Context	3	2.22%	Context	2	1.65%
Total	135	100.00%	Revoice	55	45.45%
			Encourage	4	3.31%
			Correct	5	4.13%
			Directive	1	0.83%
			Total	121	100.00%

Patterson only *directed* students to perform a mathematical action once. The following exemplar was taken from a whole class discussion during implementation of the *Interest* task (Figure 18). Mrs. Patterson had introduced the task and questioned students about 1) the initial deposit into the account, 2) what the value 1.05 represented in terms of percentage of growth, and 3) how to calculate the value of the account after three years, with the parameter. The exemplar begins as she showed her students graphical, symbolic, and numeric representations and pushed them to find the value of the account after 10 years:

T: All right. So now we're gonna show...I'm gonna show you the function. (*Does not show the graph of the function with hide/show button, despite comment.*) I'm gonna show you the table values. (*Shows the table of values with hide/show button.*) So at three years, how much?

Ss: \$5788.13. (1)

T: All right. Now I'm gonna ask you a question. See how it gave you the table and it stopped at 3? (*Pointing to table of values.*) How many...how much money would be in the account after 10 years?

S: At least, uh, ten...

T: I want you to figure it out. Look at it. You had this before. You played with it. (2)

The next exemplar took place after Mrs. Patterson circulated to help a few students with the question about how much money would be in the account after 10 years. She then pulled the class back together to achieve a consensus for the answer. Before moving into the next task, she pushed her students to think about patterns in the graphs of the first three tasks and make a prediction about the graph for the next task:

T: So when you put in 10...(*Types 10 in for the parameter value, which also shows in the table.*)

T: \$8,144.47.

Ss: Yeah. (3)

T: Okay. All right. Any questions on that? So now I'm gonna move over and show you the graph of it. (*Shows graph of function with hide/show button and moves sketch to display a better view of the graph.*)

S: So that's the answer?

T: That's the answer for 10. All right. Now somebody...Matt, you're really...you've been really paying attention. Tell me something about the graph. Explain something to me you've noticed about the graph. The last few problems we've done, what has been happening? (4)

S: *No Response.*

T: All right. For the first problem, we had...he was collecting CD's and he got more CD's. He kept collecting. So what happened to his collection?

S: It grew.

T: It increased. So it went up. So when we did interest, it increased. When we did the pay raise, it increased. So what do you notice about the graph? (5)

S: It's going up.

T: It's going up, so we see an increase. Now, what do you think the graph would look like if we had a decrease?

S: Go down. It's gonna go down.

T: So how would it be going down? Here it is coming in and it's going up. (*Motions along graph for interest in sketch.*) How would you describe going down?

S: Come from the top and down.

T: Ah, it would come from the top and go down. (*Motions with hands along coordinate plane in sketch.*) Okay. So now, let's go to depreciation. (6)

Prior to the first interaction (1), Mrs. Patterson changed the parameter value in the sketch to three. She stated that she was going to show students the table, but for an unknown reason

she changed her mind and chose to *orient/focus* student attention on the table to confirm their answer to the prompt. She continued to *orient/focus* student attention on the limitations of the current table of values and asked the students to *link/apply* their current understanding of interest to how much money would be in the account at the end of 10 years (2). She concluded the interaction by *encouraging* students. Once Mrs. Patterson pulled the class back together, she utilized the parameter to confirm the class consensus (3). Her next interaction demonstrated the use of *statements* to *orient/focus* student attention on the graph of the situation and *generate* discussion, followed by a question to *generate* discussion (4). When the student failed to respond, Mrs. Patterson *related* the current problem solving context previous tasks and employed questions to *link/apply* current reasoning (5). Once the student responded, she *prompts students for further participation* using questions to *probe* student thinking and statements to *orient/focus* student attention. This resulted in a student accurately *predicting* the shape of a decreasing exponential function, which was used as a segway into the *Depreciation* task (6).

Five practices. Similar to the first and second teaching set, Mrs. Patterson employed the practices of *anticipating* and *monitoring* during the third teaching set. Her responses during pre and post observation interviews indicated that she correctly *anticipated* student strategies and difficulties, mathematically and technologically, before implementing the pre-constructed dynamic tasks. Also, during all tasks she walked around the room to actively *monitor* student work and provide assistance to students as they worked. However, none of the episodes involving pre-constructed dynamic tasks demonstrated the use of *selecting*, *sequencing*, or *connecting* student responses.

Technology use. When implementing the *MP3* task and the *Pay Raise* task, Mrs. Patterson utilized the attributes of the tasks (i.e., hide/show button and sliders) to *explain-the-screen*. The following exemplar comes from the *MP3* task:

T: (*Teacher reads prompt. Teacher and students have verbalized the number of downloads per month and displayed points and coordinates with hide/show buttons.*)

Notice this graph. (*Traces points with pointer tool.*) So it starts out small. So it's starting to grow. So how many do you think we will have in 7 months?

S: Same amount.

T: Why would we have the same amount, Robert? What did we say in the problem?

That it's doing what?

Robert: Increasing.

T: It's increasing, but how much? It's doubling. So by how many should it have after 7 months?

Robert: 1280.

T: 1280 because it is doubling. But now what is happening to the graph? It started small and then it's getting big really, really fast. (*Moves to Pay Raise task.*)

As the exemplar demonstrates, she did emphasize the general pattern of exponential growth, but made no connections to students' prior knowledge of linear growth. During the *Pay Raise* task, she demonstrated the use of *link-screen-and-board* with the exponential function to explain why the percentage was written as a ratio out of 100. As the discourse exemplar illustrated, Mrs. Patterson's orchestration type switched to *discuss-the-screen* during the *Interest* task. She utilized the parameter, table of values, and graph of the function to discuss

and solve each problem. She did not maintain this pattern with the *Depreciation* and *Population* tasks. In fact the only element of the task she utilized during the *Population* task was the prompt to have students identify information that would assist them in using the graphing calculator to solve.

Implemented level of cognitive demand. The implemented level of cognitive demand for the first three tasks during the third teaching set was evaluated at a high level of cognitive demand, but the final two tasks were implemented at a low level of cognitive demand. Mrs. Patterson's reliance on *statements to explain-the-screen* during the first two tasks decreased the potential level of cognitive demand, but the overall implementation was at a high level of cognitive demand. Her transition to *discuss-the-screen* and use of *questioning* during the *Interest* task maintained the high level of cognitive demand for the task; however, the level of cognitive demand began to wane during the the sketch for depreciation. For the final task (population growth) was only used to glean information from the prompt and discuss the context of the situation, which caused the level of cognitive demand to ebb once again.

Discussion of Mrs. Patterson

Mrs. Patterson was entering her fifth year of teaching at the time of the study. She earned a bachelor's degree from a local university in Secondary Mathematics Education and held an initial teaching license. She taught second grade for two years at a private school and earned her credentials in early childhood before obtaining a high school position. She was entering her third year teaching in the school district, but her first two years were spent at the high school where Mr. Phelps currently taught. She transferred to the same school as Mrs.

Lewis in late summer. She described the teacher's role in the classroom as a combination of authority, facilitator, and lecturer during initial and summative interviews. She indicated that sequencing and connecting were the most important of the five practices because sequencing allowed students to see multiple ways to approach a problem and "it comes down to connecting" to make sure all students understand. Referring to the five practices, she "just wants to be good at it, and do it every time." She stated that it was often necessary to introduce concepts with low level tasks and help students grow toward high level tasks. She demonstrated facile use of all technology in her classroom (i.e., laptops, LCD projector, document camera, class website) and "loved *Geometer's Sketchpad*". She learned how to use *The Geometer's Sketchpad* during her undergraduate studies, had participated in local professional development focused on *The Geometer's Sketchpad*, and joined the larger professional development project to increased her skill level with *The Geometer's Sketchpad*. She was the only teacher to mention the desire to use a student management program to potentially monitor and display student solution strategies; however, issues stemming from the re-imaging of teacher and student computers over the summer, made the program more difficult for teachers to use than in past years. In previous years, teachers could create a class once per semester using their roster, and students could sign-in to that class on a daily basis. The teacher could then use the student management system to send and receive files to/from students, monitor and lock student computers if necessary, and display student computer screens. After the re-imaging process, the program did not always load on student computers or recognize student computers when students tried to sign-in. One consequence of these

issues was that Mrs. Patterson relied on the class website, rather than use the student management system.

Findings based on Mrs. Patterson's implementation of pre-constructed dynamic tasks demonstrated her struggle to maintain a high level of cognitive demand during the three teaching sets. Discourse analysis showed fluctuations in her use of statements versus questions as her predominant type of discourse with students. When she relied more heavily on questioning, in lieu of statements, she attained higher levels of cognitive demand for specific tasks. She also utilized higher level questions more often than information gathering questions during these same tasks. Her use of pre-constructed dynamic tasks was aligned with the mathematical goals for each lesson, but on several occasions she used tasks to teach a skill (e.g., *Slope-Intercept* task and graphing lines) or abandoned the task to have students solve using either the graphing calculator or symbolic manipulations. This finding was consistent with Mrs. Patterson's statements regarding the use of low level questions initially and working up to high level tasks. Documented use of the five practices was limited to anticipating and monitoring, despite her comments about the importance of sequencing and connecting.

Mr. Phelps

Teaching Set 1

The mathematical goals for the first teaching set focused on slope and linear functions. Two pre-constructed dynamic geometry sketches were selected to introduce students to slope and slope-intercept form of a line. Please see the section titled "Teaching

Set 1” for Mrs. Lewis for a more detailed description of the planning process, task selection, technology design principles, and potential level of cognitive demand.

Prior to implementation. Mr. Phelps did not utilize the *Slope-Intercept Task* (Figure 8) during the first teaching set; however, he did utilize his *Slope Presentation Sketch* (Figure 22) during the first day of the teaching set. He chose to use the sketch on his computer to display to students, rather than each student opening the sketch on their computers.

Mr. Phelps did not follow the rubric for the *Slope Presentation Sketch* assignment during month one of the online professional development, and his novice skills with the technology were evident in the elements that were included in his sketch. He did use color to *draw attention* to what appeared to be a slope triangle for the movable line; however, he

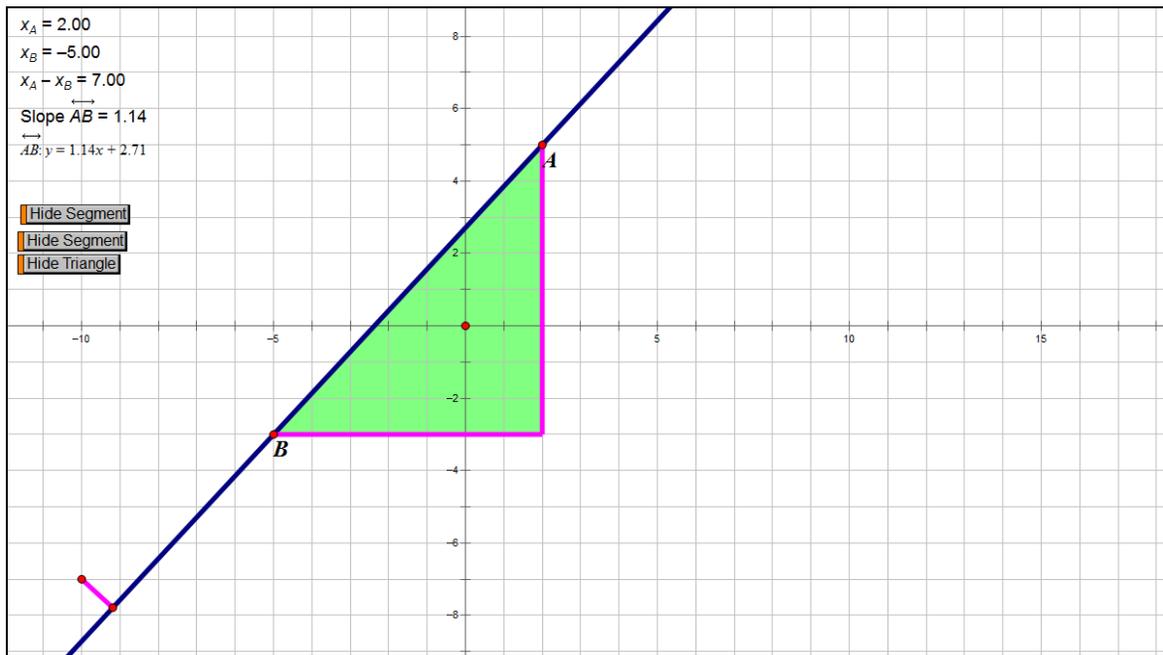


Figure 22: Mr. Phelps Slope Presentation Sketch

did not include labels for the segments of the slope triangle and when either of the two points used to construct the line were dragged the slope triangle was no longer a right triangle. He also neglected to utilize the slope formula in the sketch. The combination of these errors prevented the sketch from *providing affordances, providing alternate paths, and supporting exploration* for students. He did include the measurement of the slope of the line and the equation of the line, which had the potential to help students make connections to the value of the slope of the line and the direction of the graph of the line. Mr. Phelps' sketch provided a *shared image* for his class. Unfortunately, the shared image was a very fragmented and incomplete view of the slope of a line. Hence, the sketch did not receive a potential high level of cognitive demand.

Mathematical discussions. During the first teaching set, there were 55 documented interactions when Mr. Phelps utilized his *Slope Presentation Sketch*. Table 16 summarizes the *mode* and *type* of these interactions. Results indicate that *teacher to whole class* was the predominant *mode* of discourse, with minimal *student to teacher* and *teacher to student* interactions. *Statements* comprised the largest percentage of the *type* of discourse, followed by *questions*, and *explanations*. Statements and questions were further broken down into categories to ascertain the role each played in the mathematical discussion (Table 17). No statements and only one informational question was contributed by a student, so Table 17 demonstrates that Mr. Phelps relied more heavily on *statements* than *questions* for the *type* of discourse. Most of his statements were used to *orient/focus* student attention on important aspects of the situation. He also utilized statements to provide *information, revoice* student

Table 16: Mr. Phelps' Mode and Types of Mathematical Discourse for Teaching Set 1

Mode			Type		
Student to Teacher	7	12.73%	Answer	6 (5 from students)	10.91%
Student to Student	0	0.00%	Statement	24	43.64%
Student to Group	0	0.00%	Explanation	8 (1 from student)	14.55%
Student to Whole Class	0	0.00%	Question	14 (1 from student)	25.45%
Student In Reflection	0	0.00%	Challenge	0	0.00%
Teacher to Group	0	0.00%	Relate	0	0.00%
Teacher to Whole Class	46	83.64%	Predict	0	0.00%
Teacher to Student	2	3.64%	Justify	0	0.00%
Teacher In Reflection	0	0.00%	Generalize	0	0.00%
Total	55	100.00%	Technology	3	5.45%
			Total	55	100.00%

responses, provide *context*, and promote *exploration* of mathematical relationships. In terms of *questioning*, he posed questions intended to promote *exploration* of mathematical relationships, followed by *probing* student thinking and *information* gathering questions. There was one use of a question to *orient/focus* student attention to important aspects of the situation.

Table 17: Mr. Phelps' Questions and Statements for Teaching Set 1

Questions			Statements		
Info	2	14.29%	Info	4	16.67%
Terminology	0	0.00%	Terminology	0	0.00%
Explore	9	64.29%	Explore	1	4.17%
Probing	2	14.29%	Probe	0	0.00%
Generating	0	0.00%	Generate	0	0.00%
Link/Apply	0	0.00%	Link/Apply	0	0.00%
Extend	0	0.00%	Extend	0	0.00%
Orient/Focus	1	7.14%	Orient/Focus	14	58.33%
Context	0	0.00%	Context	2	8.33%
Total	14	100.00%	Revoice	3	12.50%
			Encourage	0	0.00%
			Correct	0	0.00%
			Directive	0	0.00%
			Total	24	100.00%

The following exemplar was taken from the whole class discussion of Mr. Phelps' *Slope Presentation Sketch*. Prior to the exemplar, Mr. Phelps had animated the point attached to the line and discussed how the point going up the line was similar to a car on a roller coaster. He then discussed x_A , x_B , and $x_A - x_B$, as the "rate of change between x_A and x_B ". He then posed the following question:

T: So everybody here knows if that's a positive...can you tell me if there's a positive or negative slope and how you know? Say it...say if it's a positive or a negative. (*A line with positive slope displayed in sketch.*) (1)

S: Positive.

T: Postive. How do you...just by looking at it, how do you know it's a positive slope?

(2)

S: Going up.

T: It's going uphill, right? What about...what about the coordinates? Does that make a difference? (3)

S: Yes.

T: The coordinates are the points. The points that's on the line, does that make a difference? (No response from students.) (4)

T: All right. So let's show...let's show what I call the rate of change. (Used hide/show buttons to display horizontal segment, then vertical segment, without labels.) My triangle looks like this. From here to here, that's all you do when you're counting rate of slope. (Points to segments.) You're calculating the rate of change from this and this. (Motions horizontal, then vertical.) And there's my slope triangle. (Used hide/show button to show interior of triangle.) (5)

Mr. Phelps opened the discussion with a question intended to *explore* the link between the graph of an increasing line and the knowledge that the line has a positive slope, but he quickly rephrases the question to gather *information* (1). Once a student responded correctly, he *revoiced* their response and posed a question to *probe* student thinking (2). A student responded by *explaining* that the line is “going up”, which Mr. Phelps *revoiced* before asking a question to *explore* the relationship between the coordinates of the points on the line and the slope (3). A student provided a response to the question, and Mr. Phelps used a *statement*

to *orient/focus* student attention on coordinates as points on the line before restating his earlier question (4). When students failed to respond, he abandoned the question and utilized hide/show buttons to display the “rate of change” and slope triangle (5). This represented the corpus of the whole class discussion of slope.

Five practices. Mr. Phelps anticipated that students would be able to identify the graphs of lines with positive and negative slope during implementation of his *Slope Presentation Sketch*, but he made no mention of horizontal or vertical lines during his pre-observation interview. No uses of the practices of *monitoring*, *selecting*, or *sequencing* were noted, nor did the whole class discussion demonstrate any use of *connecting* student responses.

Technology use. As the exemplar demonstrates, Mr. Phelps employed *explain-the-screen* more often than *discuss-the-screen*. He begins by asking students questions regarding what they see displayed in the sketch, which is consistent with a *discuss the screen* orchestration. He then reverts to *explaining the screen* when referring to the coordinates and the slope triangle. Also, Mr. Phelps *Slope Presentation Sketch* was missing several elements, based on the rubric for the project. He included measurements for the *x-coordinates* of two points on the line and their difference, which he referred to as the “rate of change between x_A and x_B ”. Similar measurements for the *y-coordinates* were omitted. The sketch was also missing a calculation of slope and labels for the vertical and horizontal change. Further, the slope triangle “broke” when you dragged either point on the line. Mr. Phelps expressed awareness of this during the post-observation interview. He asked if there was a way to construct a slope triangle that moved with the line and stated he did not move the line in his

sketch because his triangle only worked for the case he displayed. This explained why he never dragged the points to display a graph that had a negative, zero, or undefined slope. The use of the sketch concluded with Mr. Phelps hiding the slope triangle and referencing the equation of the line and the measurement of the slope of the line. He moved the entire line, so students' attention was drawn to the changing y-intercept of the line, instead of the slope of the line. Recall, the summer institute was Mr. Phelps' introduction to *The Geometer's Sketchpad*, and this was the first project for the online course. His inexperience with *The Geometer's Sketchpad* influenced the design and orchestration of his *Slope Presentation Sketch* and resulted in a fragmented and incomplete discussion of the concept of slope of a line.

Implemented level of cognitive demand. Mr. Phelps use of *statements* and *explain the screen* as predominant discourse and technology orchestration types contributed to a decrease in level of cognitive demand during implementation. Additionally, his deviations from the project rubric and absence of appropriate design principles quickly became apparent during this part of the lesson and contributed to a low level of cognitive demand.

Teaching Set 2

The mathematical goals for the second teaching set focused on quadratic functions. Six pre-constructed dynamic geometry tasks were selected to introduce students to quadratic functions and their applications. Please see the section titled "Teaching Set 2" for Mrs. Lewis for a more detailed description of the planning process, task selection, technology design principles, and potential level of cognitive demand.

Prior to implementation. Mr. Phelps did not choose to make any alterations to any materials for the second teaching set. He planned to go through the pre-constructed tasks in the order presented for both day one and day two and disseminate the student handout in original form. As described in the earlier, each task for the two day teaching set were evaluated at a high potential level of cognitive demand and adhered to Sinclair's (2003) design principles.

Implementation of tasks. For unknown reasons, Mr. Phelps decided to utilize the $y = a(x - h)^2 + k$ pre-constructed task to answer all questions related to the first three tasks contained in the two days of pre-constructed tasks, and he chose to display and manipulate the pre-constructed task on his computer only. While he did utilize the *min/max*, *Projectile Motion*, and *Minimize Cost* tasks, he did not implement the *roots/zeros* task.

Mathematical discussions. During the second teaching set, there were 439 documented interactions when pre-constructed dynamic tasks were utilized. Table 18 summarizes the *mode* and *type* of these interactions. Results indicate that *teacher to whole class* was the predominant *mode* of discourse, followed by almost equal *teacher to student* and *student to teacher* interactions. There were also three instances where a *student addressed the whole class*. Similar to the first teaching set, *statements* were employed more often than *questions*, with regard to the *type* of mathematical discourse. Statements and questions were further broken down into categories to ascertain the role each played in the mathematical discussion (Table 19). Students contributed 14 statements and 30 questions during discussions that were all lower level. Hence, Table 19 demonstrates both teacher and student use of lower level questions and statements, in addition to Mr. Phelps' use of higher

Table 18: Mr. Phelps' Mode and Type of Mathematical Discourse for Teaching Set 2

Mode			Type		
Student to Teacher	126	28.70%	Answer	95 (77 from students)	21.69%
Student to Student	0	0.00%	Statement	173 (14 from students)	39.50%
Student to Group	0	0.00%	Explanation	21 (1 from student)	4.79%
Student to Whole Class	3	0.68%	Question	121 (30 from students)	27.63%
Student In Reflection	0	0.00%	Challenge	0	0.00%
Teacher to Group	0	0.00%	Relate	5	1.14%
Teacher to Whole Class	182	41.46%	Predict	1 (from student)	0.23%
Teacher to Student	127	28.93%	Justify	2 (both from students)	0.46%
Teacher In Reflection	1*	0.23%	Generalize	11 (2 from students)	2.51%
Total	439	100.00%	Technology	9 (2 from students)	2.05%
	*TIR does not get coded for type		Total	438	100.00%

level questions and statements. Statements were utilized to *orient/focus* student attention and *revoice* student responses in almost equal measure. Mr. Phelps also used statements to *encourage* and *direct* students' participation during mathematical discussions. Statements designed to *generate* discussion, *explore* mathematical relationships, and provide *information*

Table 19: Mr. Phelps' Questions and Statements for Teaching Set 2

Questions			Statements		
Info	56	46.28%	Info	10	5.78%
Terminology	5	4.13%	Terminology	3	1.73%
Explore	32	26.45%	Explore	10	5.78%
Probing	17	14.05%	Probe	1	0.58%
Generating	8	6.61%	Generate	11	6.36%
Link/Apply	1	0.83%	Link/Apply	0	0.00%
Extend	0	0.00%	Extend	0	0.00%
Orient/Focus	2	1.65%	Orient/Focus	43	24.86%
Context	0	0.00%	Context	0	0.00%
Total	121	100.00%	Revoice	42	24.28%
			Encourage	26	15.03%
			Correct	7	4.05%
			Directive	20	11.56%
			Total	173	100.00%

were also noted. There were occasions when statements served to *correct* student responses, introduce *terminology*, and *probe* student thinking. Questioning during the second teaching set most often sought to gather *information*, but *exploring* mathematical relationships, *probing* student thinking, and *generating* discussion, collectively, accounted for a slightly larger percentage of question types than *informational* questions. Questions designed to introduced *terminology*, *orient/focus* student attention, or required students to *link/apply* their current understanding were also observed. The following exemplar occurred on the first day of the teaching set when students were exploring the influence of the quadratic coefficient on

the graph of the parabola. Mr. Phelps displayed the sketch for the whole class and had instructed students to “write down what’s happening as I move a to the right and as I move a to the left.” He manipulated the slider for a to the right and left several times before walking around the room to monitor the responses students had written on their paper. The excerpt begins as Mr. Phelps pulled the class together to discuss their findings.

T: All right. Okay. I’m gonna start with Jake. Shh. All of them had good answers. I just went down this row ‘cause I wanted to see what they were thinking. Jake what did you write for what happens when I change the values of a ?

Jake: It increases and decreases the parabola. (1)

T: Okay, parabola. He said it increases and decreases. All right. Juakim, what...

Juakim: Gets closer to the y -axis and value decreases and increases.

T: Okay. He said it increases, decreases. Does it get close to the y value?

Juakim: y -axis (2)

T: Okay. Y -axis. I’m sorry. All right. Silas will be my last response. Shh. What did you say Silas?

Silas: The line moves but the vertex never moves. (3)

T: He said the line moves but the vertex never moves. When he’s saying the line, is he talking about...are you talking about the actual parabola?

Silas: Yes. (4)

T: What's happening...now I open this up for people, just based on their response.

What's happening, Josh, as I move the value of a ? (*Moving slider for a to the right, greater than zero.*)

S: It gets greater than zero...my bad Josh. (5)

T: Look. I'm still increasing. What's happening to this parabola? (*Moving slider for a to the right, greater than zero.*)

Josh: Gets thinner then bigger.

T: It gets what?

S: Thinner.

T: Thinner?

S: Yeah. (6)

T: Okay. Let's use narrow. It's getting more narrow. Okay. So as I continue to go back close to 0, it widens up. (*Moving slider left, toward zero*) And when I get to my negative number, what happened? (*Moved slider for a into negative values.*)

S: It flips.

T: It flips over...(7)

As the discussion continued, Mr. Phelps relied on questioning students to have them classify the vertex as a maximum or minimum based on the direction the parabola opened and to generalize that when a is negative or positive the parabola has a maximum or minimum, respectively. This episode greatly contrasts Mr. Phelps use of the *Slope Presentation Sketch* during the first teaching set. He opened the discussion by *selecting* a student to respond to his question about what was happening as he moved a right and left. He also *encouraged*

students for their participation, and made his *monitoring* actions known to the students before the student provided an *explanation* for what he observed (1). Mr. Phelps *revoiced* the student's response and *generated discussion* by selecting another student to respond. Once the student provided his *explanation* of what he observed, the response was *revoiced* and *probed* for clarification. In this instance the student *corrected* the teacher's use of "y value" to "y-axis", which the student originally stated (2). Not only did Mr. Phelps *revoice* the student's response, he also apologized for his misunderstanding. He then selected a third student to respond and *generate discussion* (3). Again *revoicing* was used before *probing* Silas' thinking to clarify what was meant by "the line" (4). To further *generate discussion*, Mr. Phelps allowed anyone in the class to volunteer a response. Although he selected Josh to respond, a student interjected her explanation; however, she quickly apologized for responding out of turn (5). Once the teacher refocused on Josh, the student explained what he observed, and Mr. Phelps used several questions to *probe* Josh's thinking and clarify his response (6). In the final exchange, the *terminology* of "narrow" was introduced. Then, Mr. Phelps *explained* how the parabola widened as the slider for a approached zero. One last *exploration* question was posed for students to link the effect of a negative value for a with the appearance of the graph of the parabola, and a correct student response was *revoiced* to conclude the exemplar (7).

Five practices. During his pre-observation interview, Mr. Phelps *anticipated* probable student responses and difficulties for pre-constructed tasks. As the exemplar demonstrated he employed *monitoring* of student thinking and *selected* several student responses to highlight during the discussion. When asked about *sequencing* during the post-observation interview,

Mr. Phelps indicated that he selected student responses based on increased levels of accuracy and detail. This was supported by the use of Josh's response last. However, the *connecting* of student responses was not observed. It is interesting to note that Mr. Phelps chose to make students aware of his use of monitoring of student strategies and encouraged them for their responses to his question.

Technology use. The discourse exemplar provided a nice example of how Mr. Phelps relied on *discuss-the-screen* when implementing pre-constructed dynamic tasks throughout the second teaching set. Mr. Phelps also used technology in a manner that was strategically aligned with several mathematical goals. During implementation of $y = a(x - h)^2 + k$ and *min/max* tasks, the focus remained on developing students' understanding of the influence of a on the graph of the parabola, what values of a produced a minimum or maximum, and that the minimum and maximum occur at the vertex of the parabola. However, the influence of h and k on the graph, nor their connection to the vertex of the parabola was discussed. This was due to Mr. Phelps' limited understanding of vertex form of the parabola. He approached the researcher during the observation to ask about h and k and what he was supposed to help students understand regarding their connection to the graph of the parabola. After a brief discussion, he decided to move on to the *min/max*, *Projectile Motion*, and *Minimize Cost* tasks. During the last two tasks he introduced students to the formula $x = -b/2a$ (combined with substitution in the original function), similar to Mrs. Patterson. He did, however, connect the formula to students' prior knowledge of the quadratic formula, unlike Mrs. Patterson. Also, Mr. Phelps selected a student (Josh) to provide his graphical explanation for when the projectile hit the ground.

T: All right, now question 3. This I've seen this question several times [on exams].

When will the ball hit the ground?

Josh: Oh, I know.

T: And I want you to be able to tell me how to explain it. When will the ball hit the ground?

Josh: 4 seconds.

T: How do you know that?

Josh: 'Cause on the x -axis there were...there would be a zero.

T: Okay. Let me see what you are talking about. Let me see what you are referring to.

(Goes over to see students computer screen.) Ah. Okay. Right.

Josh: Right.

T: Okay. Explain again, Josh.

S: Because you threw the ball at 0 seconds.

T: Okay. Let me get my...let me get my graph up and then you can explain from that.

Okay? Come on up, since you want to show off. There's my graph, Josh. You can use mine. *(Student walks up to the display of the task on the whiteboard.)*

Josh: The ball starts at zero, goes in the air to 64 ft. But if it goes up, it comes back down, and it took 4 seconds to hit the ground. *(Tracing path of ball on graph as he talks and points out important parts.)*

T: Absolutely. Awesome. Awesome. Everybody understand what he just did?

This exemplar demonstrates Mr. Phelps' use of *Sherpa at work*; however, a general method for how to find roots/zeros was not discussed. The *Minimize Cost* task was posed to students,

and they were instructed to use similar methods (formula and substitution) to solve. The pre-constructed task was only utilized to display the prompt and graph.

Implemented level of cognitive demand. During the first day of the teaching set, Mr. Phelps utilized questioning more than statements to *discuss the screen*, and he did not initiate any discussions until after students had been given time to observe changes as he manipulated the task and arrive at their own conclusions. He monitored student work during this time and selected specific students to share their conclusions with the class to facilitate the whole class discussion. Mr. Phelps often pushed his students for further explanation during the discussion and re-voiced student responses to summarize mathematical concepts from the discussion. These actions resulted in Mr. Phelps maintaining a high level of cognitive demand during the $y = a(x - h)^2 + k$ task, but he did not have time to include any other tasks the first day.

The second day of instruction began with both students and Mr. Phelps accessing the pre-constructed task $y = a(x - h)^2 + k$. On the previous day, students had correctly concluded that positive values for a resulted in the parabola opening up and negative values for a resulted in the parabola opening down; however, they had not formalized their conclusions related to the values of a that either vertically stretched or compressed the graph of the parabola. The following exemplar illustrates how Mr. Phelps utilized *explain-the-screen* to help students see the relationship between the absolute value of a and the vertical stretch or compression of the graph of the parabola.

T: So looking at...looking at what we've got here, when we start with the positive values of a , look at what we got. You...you noted yesterday that as a increases, then

the, uh, parabola got more narrow, right? (*Moving the slider for a to positive values greater than 1.*) But we did not conclude yesterday when I go to the negative side that it still continues to get more narrow but we still...we got negative numbers now. (*Moves the slider for a to negative values less than -1.*) so what I want you to understand now is the negative...is the absolute value of a that determines how narrow the parabola is, okay?

The class then moved on to the *Min/Max* task. Students were given adequate time to explore and answer the questions within the task as Mr. Phelps monitored student work. During this time he asked questions to *discuss the screen* and push students to think about what was happening in the task that would help them draw conclusions, rather than directing them to conclusions. He employed *orient/focus* statements or questions to *discuss-the-screen* with students as he monitored student work during the task. Thus, he was able to maintain a high level of cognitive demand for the students.

The lesson concluded with the use of the *Projectile Motion* and *Minimize Cost* tasks. The level of cognitive demand remained high during the first application task because Mr. Phelps *discussed-the-screen* and utilized procedures with connections to the quadratic formula and the pre-constructed task. The specific procedure involved finding the x-coordinate of the vertex using the formula $x = -b/2a$, and then using substitution in the original function to find the y-coordinate of the vertex. While this was an example of *link-screen-board* similar to Mrs. Patterson, Mr. Phelps specifically related the prescribed formula to previous work with the quadratic formula. This contrast helped maintain a high level of cognitive demand. He also demonstrated the use of *Sherpa-at-work* by selecting a student to

go to the board and explain how he used the graph of the projectiles motion to locate the point where it hit the ground. The final application task was begun, but was not completed in the time remaining. Students were told to use previous methods to complete the task. Thus, the level of cognitive demand was not high. Overall, Mr. Phelps was able to achieve an overall high level of cognitive demand for the teaching set.

Teaching Set 3

The mathematical goals for the third teaching set focused on exponential functions. Five pre-constructed dynamic geometry tasks were selected to introduce students to exponential functions and their applications. Please see the section titled “Teaching Set 3” for Mrs. Lewis for a more detailed description of the planning process, task selection, technology design principles, and potential level of cognitive demand.

Mathematical discussions. During the third teaching set, there were 318 documented interactions when pre-constructed dynamic tasks were utilized. Table 20 summarizes the *mode* and *type* of these interactions. Results indicate that *teacher to whole class* was the most prominent *mode* of discourse, followed by *student to teacher* and *teacher to student* in almost equal measure. Similar to the first and second teaching set, *statements* were utilized most during discussions. *Questions*, *answers*, and *explanations* represented the majority of the remainder of *types* of discourse. There were several instances of *relating*, two *challenges* and *predictions/conjectures*, and one observed *justification* and *technology* interaction.

Statements and questions were further broken down into categories to ascertain the role each played in the mathematical discussions (Table 21). Students contributed 12 statements and nine questions that were lower level. Hence, Table 21 reflects students’ and teachers’ use of

Table 20: Mr. Phelps Mode and Type of Mathematical Discourse for Teaching Set 3

Mode			Type		
Student to Teacher	76	23.90%	Answer	55 (49 from students)	17.30%
Student to Student	0	0.00%	Statement	124 (12 from students)	38.99%
Student to Group	0	0.00%	Explanation	41 (3 from students)	12.89%
Student to Whole Class	0	0.00%	Question	86 (9 from students)	27.04%
Student In Reflection	0	0.00%	Challenge	2	0.63%
Teacher to Group	0	0.00%	Relate	6	1.89%
Teacher to Whole Class	167	52.52%	Predict	2 (both from students)	0.63%
Teacher to Student	75	23.58%	Justify	1 (from a student)	0.31%
Teacher In Reflection	0	0.00%	Generalize	0	0.00%
Total	318	100.00%	Technology	1	0.31%
			Total	318	100.00%

lower level questions and statements, and Mr. Phelps' use of higher level questions and statements.

In reference to *statements*, *revoicing* and *orienting/focusing* statements occurred most, with equal frequency. *Information*, *context*, and *encouraging* statements comprised the majority of remaining statements. *Probing*, *exploring*, *generating*, and *directive* statements were also noted. One occurrence of *correcting* and *terminology* statements were observed. In

Table 21: Mr. Phelps Questions and Statements for Teaching Set 3

Questions			Statements		
Info	29	33.72%	Info	20	16.13%
Terminology	2	2.33%	Terminology	1	0.81%
Explore	23	26.74%	Explore	5	4.03%
Probing	17	19.77%	Probe	7	5.65%
Generating	10	11.63%	Generate	4	3.23%
Link/Apply	0	0.00%	Link/Apply	0	0.00%
Extend	0	0.00%	Extend	0	0.00%
Orient/Focus	0	0.00%	Orient/Focus	30	24.19%
Context	5	5.81%	Context	13	10.48%
Total	86	100.00%	Revoice	30	24.19%
			Encourage	9	7.26%
			Correct	1	0.81%
			Directive	4	3.23%
			Total	124	100.00%

terms of *questioning*, Mr. Phelps utilized *information* gathering questions most often, but the combined usage of *exploring*, *probing*, and *discussion generating* questions greatly exceeded the use of lower level questions. *Context* and *terminology* questions also occurred.

The following exemplar took place during implementation of the *Pay Raise* task. Before opening the task, Mr. Phelps discussed the prompt with students and provided context to help student understand the problem situation. He then provided expectations for the students to discuss the problem with one another and gave students approximately 10 minutes to consider both pay options, decide which option they would choose, and provide a

written explanation for their choice. During this time, he walked around to monitor students' responses to the prompt, provide clarification, and ask questions to probe their thinking. He pulled the class back together to discuss a waitress and a sales person as examples of professions that get paid based on a percentage (i.e., tips and commission), then selected a student that chose hourly wage increase for their pay raise.

T: I want you, uh, Latasha...

Latasha: I ain't got no explanation.

T: No explanation. I want you to tell me what you want your wage to change to. It's \$8.25 right now. What do you want it to be and you know, he said he's gonna offer you a wage. Just give me a reasonable number now. I don't want no thousand. You ain't...that ain't happening in the real world.

Latasha: At least \$10.00. (1)

T: \$10.00 an hour. Okay. Watch this. No, she said \$10.00 an hour. That's a reasonable number. Boss comes to her, says, "Hey, I'm gonna give you a raise. I'm gonna raise you a \$1.25...that's...\$1.75. That's \$10.00." So here's...looks what...this is your line, this is your curve. Look what this would look like.

S: No, no. It's gonna be \$10.50. (2)

T: $\$1.75 + \8.25 is \$10.00. Okay. So I'm going up. Look, look, look, look, look. I'm going up to \$10.00. (*Moving slider for wage per hour from \$8.25 to \$10.00.*) Okay. I'm about right...I'm close. I...I'll stay there. I'll stay. I'll give you \$10.02. I'll give you that. (*Slider adjusts to \$9.99 as teacher moves hand off mouse pad.*) You started

at \$8.25. Now you're \$9.99. Okay. Listen. Look at what...how her...how her number changed with a graph of wages change. (3)

T: How many say a percent? Latwan, what would...that's 4% now. I'm gonna give...I'm paying you 4% of your wages. Like a commission. So whatever you make I'm gonna give you 4% of that and that's what you're gonna take home. What would you want your percent increase to be?

Latwan: 20%.

T: 20%! Boy, what you think...(laughing). Okay. I'll...a realistic number. Huh? You at 4 now. What you...

S: 10

T: Let's say he was a top Kirby sales person for that month and let's say we raise him to 8%. Let's double. Let's double. Let's look. Let's look. Look at what happens to this line based on the percent. (*Moves slider for percent increase from 4% to 8%.*) (4)

S: That's what I put.

T: Oh. Let me start back. Let me pull this back to zero. (*Moves slider for wage per hour back toward initial value; however doesn't realize both sliders selected, so both move to the left.*)...So let's go as close to straight as...yeah...as I can. All right. So you're at wage \$6.00. She went from \$6.00 to \$8.25 to \$10.00. So she got a decent...decent raise. (*Goes to move slider for percentage increase, realizes both sliders selected so clicks in white space to deselect everything. Re-selects percentage increase slider.*) (5)

T: I kept you at \$10.02 this time, Latasha. I put you back. Uh, he said his percent increase to 8%. Watch what happens. (*Drags slider for percentage increase from 4% to 8%.*)

S: What I tell you!

T: 8%. As close as I can get. 7.99%. Do you see the drastic difference that it would look like if I change my percent increase versus my wage increase? (6)

This exemplar is very representative of how Mr. Phelps orchestrated the pre-constructed tasks during the third teaching set. He chose to display all tasks for the class, instead of students opening files on their computer, and he provided time for students to consider each prompt before initiating a whole class discussion. During the exemplar, Mr. Phelps *generated* discussion by selecting two students with different opinions regarding the type of pay raise each would prefer (1 & 4). The first student he selected stated that she did not have an explanation. Rather than accept that response from the student, he pushes her for further participation. He then *revoiced* her response and provided *encouragement* for her participation before adding *context* to the situation (2). When a student tried to correct Mr. Phelps arithmetic, he *explained* how he arrived at an hourly wage of \$10.00 and *oriented/focused* student attention on the movement of the slider, as well as the changes in the graph (3). He then *generated* further discussion by selecting a student that chose a percentage of salary increase. Once the student was persuaded to use a realistic value for his percent increase, Mr. Phelps moved the slider to illustrate the change from 4% to 8% (4). As seen in (5), he intended to adjust the slider for wage per hour to zero and make the graph as

flat as possible. This was very similar to Mrs. Lewis' implementation of the same task. However, both sliders were selected and moved simultaneously as he dragged the slider for wage per hour. Mr. Phelps utilized this opportunity to reinforce the changes in the graph when the hourly wage was adjusted, then he realized that both sliders were selected and fixed the technological issue. He finished the exemplar by moving the slider for percent increase to contrast the two situations (6).

Five practices. During his pre-observation interview, Mr. Phelps *anticipated* student strategies and misconceptions that occurred during the teaching set. For example, in the *Pay Raise* task, he stated that students would most likely pick the hourly increase because of their familiarity with being paid by the hour at work. Throughout implementation, he followed a similar pattern demonstrated in the second teaching set, where he asked students to consider the problem situation, make conjectures or draw conclusions, and provide their reasoning. He walked around to *monitor* students' choice and reasoning, and then he *selected* and *sequenced* student responses. During the third teaching set, the technology mishap of selecting both sliders may have inadvertently led to a better *connection* between the two student strategies. When he re-demonstrated the change in the graph for the hourly wage increase, he reinforced the slower growth demonstrated in the graph just prior to adjusting the slider for percent increase, which displayed a greater increase in the graph. He was able to compare/contrast the two solution strategies, unlike the second teaching set.

Technology use. Mr. Phelps utilized *discuss the screen*, *explain the screen*, and *tech demo* during the third teaching set. The discourse exemplar from the *Pay Raise* task is representative of the way Mr. Phelps orchestrated technology by *discussing the screen* with

students; however, there were times he relied on *explaining the screen*. The following example occurred during the *Interest* task:

T: (*Mr. Phelps had read the prompt and students had identified the situation as an exponential function.*) Let's show the function. (*Used hide/show button to display graph and equation, but his window was not large enough for the graph to be seen.*)

Or, we're showing the function. (*Decided to abandon graph and use hide/show button for table of values.*) Let's show the table. Look at this table of values. So we started out with 0. We were at 500...\$5000.00. First year it went...it grew from \$5000.00 to \$5250.00. The second year, of course it grew again. You see the growth? The third year it grew again and then from there we just left it. (*Referring to table of values only containing three years of data.*) It's growing exponentially.

Mr. Phelps chose to explain that the growth pattern in the table was exponential, rather than solicit student thinking regarding the problem situation.

Mr. Phelps' use of technology during the third teaching set was initially well aligned with the mathematical goals, and he utilized hide/show buttons (*MP3 Downloads & Interest*), sliders (*Pay Raise*), and parameters (*Interest*) to stimulate student understanding.

Unfortunately, his use of the pre-constructed tasks to develop student understanding tapered off during the final two tasks (*Depreciation* and *Population Growth*). He did not discuss or explain solution strategies or answers to either of these tasks. He did show the graph of the function and the table of values with plotted points; however, the pre-constructed tasks were quickly abandoned and students were instructed to work on homework.

Implemented level of cognitive demand. The implemented level of cognitive demand for the first three tasks achieved a high level of cognitive demand. Mr. Phelps' use of questioning and statements, coupled with his use of technology maintained a high level of cognitive demand during the *MP3 Downloads*, *Pay Raise*, and *Interest* task; however, this trend was not maintained during the final two tasks. A decrease in level of cognitive demand corresponded with decreased use of *discuss the screen* and questioning, in lieu of increased use of *explain the screen* and statements, in isolation or with *link screen board*.

Discussion of Mr. Phelps

Mr. Phelps was beginning his fifth year of teaching at the time of this study. He became a mathematics teacher via lateral entry and held an initial teaching license in middle grades mathematics. He taught three years at the middle school level and was beginning his second year at the high school level. He taught at a smaller high school within the same school district as Mrs. Lewis and Mrs. Patterson. In his initial and summative interviews, Mr. Phelps expressed his view of the teacher as a facilitator, motivator, and innovator. He wanted to create a classroom where mathematical discussions represented the majority of class time. In his words, "I can teach all day, but that does not ensure students learned", but for students, "if you can talk about it, you can teach someone else." Mr. Phelps reported the biggest change in his pedagogical practices over the course pertained to increasing rigor by taking what he learned in the summer institute and applying that to his classroom.

"The challenge was to push me first, so I could push them. And I feel like that was done. From the first initial test that we got at the summer institute. That was like...ooh, what is this? I know what that felt like. I did that to my students this

semester, and I gave them something that I know that they probably couldn't answer...but it wasn't about whether they could answer, it was about...getting you to feel how this feels to not know anything...then at the end of the semester, giving you the same thing and saying 'oh wow' look how far you've come with this information you did not know in the beginning to where you are now. I used that experience to create some despair, and I was ok with that. I wanted you to be frustrated a little bit, but I'd rather you do it now the first part of the semester, first day of the semester. I don't want you looking like this the last part of the semester. And that was kind of my strategy”.

The theme of rigor continued as he talked about how the mathematical task framework factored in to his teaching during the semester.

“The higher level of expectation you have for them the more they can achieve. Um, but if you are always asking questions on a lower level...and that's all they see. So when you get to the higher level of the test, they don't know what to do with that. So thinking about that at the forefront of the semester and doing it...giving like I said the rigor...giving that high level stuff, even if they don't know how to answer it. At least you see it. At least now we are gonna start talking about it and discussing it, so that by the end of the semester I don't have to cram with you on the review...because you've been doing it all semester.”

He was also excited about incorporating the five practices in his classroom. At the beginning of the semester, he stated that he felt comfortable anticipating student solution strategies, and

he wanted to work on monitoring. By the end of the semester, he reported using the practices daily.

“Because without the five practices...you can lose so much during instruction. You know...anticipating, that’s the planning part...um, you know beginning with the end in mind, that’s anticipating...I gotta know what to expect. And so with that, I have to anticipate what my students are doing. And then going from that, getting to connecting, there’s a bridge. There’s about three or four bridges to cross to finally get to the connecting piece. Um, the sequencing and the monitoring, all those were big...big little pieces in making the classroom work. My cognizance of that now, my memory, not thinking about, but actually doing them is very, very natural now.”

He viewed anticipating and monitoring as responsibilities for the teacher, but sequencing and connecting were shared between the teacher and the student. He summarized the contribution of sequencing and connecting to student learning by saying,

“Well, the learning part for me, I always take from the listening part. And so I know they listen to each other. So let’s talk about...sequencing, that’s the one I want. So if I ask a student a question and that student gives me the basic general answer that I expect students to give me. And I’m thinking about sequencing to get to the idea...students are listening to that first student. So they won’t, hopefully, repeat the same answer. They will have this ‘ok, I have to think about this a different way because if he ask me this question’, then he don’t want to say the same thing she said. I have to say something different, and that requires some thinking process.”

He further stated the importance of the sequencing process being “done openly, where students can hear” and then connecting the responses that were shared. In the future, he wanted to “get the students more involved with the five practices.” For example, he mentioned giving the students a task and asking,

“What do you think Sarah would say about this question? How do you think this student would respond? How do you think other students in the past responded to this question? So now they’re anticipating. And then the sequencing, ok how could we arrange this? Getting them to think about this. How could we arrange this in an order that would work for you? I’m gonna give you five scenarios, and I’m gonna let you sit at a table and arrange them from lower level to higher level.”

At the beginning of the semester, he reported being 60% comfortable with technology in general and 20-30% comfortable with *The Geometer’s Sketchpad*. He did not have a class website, but his classroom contained the same technologies as Mrs. Lewis and Mrs. Patterson. In his summative interview, he expressed that technology was essential and now comfortably embedded in his practice. “It’s just part of the culture of the classroom. And I think the difference in the Algebra 1 classes, that I have seen, the engagement is different as well.” He indicated that he was more comfortable with the use of *The Geometer’s Sketchpad*, especially the topics taught with *The Geometer’s Sketchpad* during the semester, and he felt he could create sketches for other topics he wanted to teach. In reference to using *The Geometer’s Sketchpad* with students he said,

“I think the things that we did with it...that, I know stuck. I know stuck with them because when they saw that test question, they were able to connect that back, back to

that graph we did in *Geo Sketchpad*. So that was a big plus because without that they wouldn't have connected it back to anything. So now I got a visual of it. I got a graphic presentation of it. I've got a visual of what it looked like on the graph. Now even if I don't know everything about it, I know enough to make a educated assumption and get started."

Mr. Phelps was the only teacher in the study that utilized an Interwrite Pad during instruction. He personally utilized it during every classroom observation and reported allowing students to use it to share information with the class on other occasions.

Findings from the three teaching sets show that Mr. Phelps demonstrated the most visible use of the five practices, and he acted as a facilitator and motivator as part of his teaching role. As the exemplars demonstrated, the students shared the mathematical authority with Mr. Phelps in the classroom. The whole class discussions displayed a back and forth sharing of ideas between Mr. Phelps and his students, which helped create the discursive classroom environment he desired. The mathematical discourse was characterized by statements more than questions; however, the intended purpose of a majority of these was similar to high level questions (i.e., orient/focus). Mr. Phelps utilized information, exploring, probing, and generating questions most often when he posed questions to his students. His implementation of pre-constructed tasks varied greatly across teaching sets (1, 3.25, and 2.4, respectively). The low level of cognitive demand for the first teaching set may have been mitigated by his inexperience with *The Geometer's Sketchpad* because the *Slope Presentation Sketch* that he created did not meet the requirements of the rubric, and contained features that "broke" when the moveable line was dragged. He was able to

maintain a higher level of cognitive demand during the last two teaching sets, which utilized pre-constructed dynamic tasks that were created by the researcher or co-created by Mr. Phelps at the summer institute. There was also evidence that Mr. Phelps' mathematical content knowledge may have limited his implementation of pre-constructed dynamic tasks in his classroom (e.g., h and k were skipped in teaching set two because he was not familiar with vertex form of quadratic functions).

CHAPTER 5: CROSS CASE ANALYSIS AND FINDINGS

The previous chapter shared findings for individual teachers, with regard to discourse, pedagogy, technology use, and level of cognitive demand. The purpose of this chapter is to answer the following research question:

When involved in technology-intensive, research-based, professional development on Algebra:

- a. What types of mathematical questions and statements are central to the mathematical discussions that occur among students and the teacher when implementing pre-constructed dynamic geometry tasks?
- b. To what extent do teachers utilize the five practices for orchestrating productive mathematical discussions to support their implementation of pre-constructed dynamic geometry tasks?
- c. What are the design and technological features of pre-constructed dynamic geometry tasks and how do teachers use them during implementation?
- d. What is the overall influence of a-c on the implemented level of cognitive demand of pre-constructed dynamic geometry tasks?

Two conceptual frameworks guided the data collection and analysis for this study. The mathematical tasks framework and the five practices for orchestrating productive mathematical discussions were adapted into a dynamic model for implementation of technological tasks in 1:1 computing classrooms (Figure 3). The interaction of mathematical discussions, five practices, and technology use were used to characterize the implemented level of cognitive demand when teachers utilized pre-constructed dynamic geometry tasks.

The remainder of this chapter relied on cross case analysis to characterize the questions and statements within mathematical discussions, use of the five practices and technology, and implemented level of cognitive demand.

Findings Related to Research Question 1a.

This section details overarching trends in teachers' use of questions and statements when implementing pre-constructed dynamic geometry tasks throughout the semester. Each teaching set is discussed separately before summarizing results across teaching sets.

Teaching set one. For teaching set one, all teachers utilized *teacher to whole class* most often as the *mode* of mathematical discourse. *Statements* and *questions* were used almost equally, but *statements* occurred more often. The majority of teachers' *statements* served to 1) *orient/focus* student attention, 2) *revoice* student responses, 3) provide *information*, 4) *encourage* students after participating, and 5) *correct* student responses, respectively. *Questions* posed during implementation were intended to 1) provide *information*, 2) *explore* mathematical relationships and/or relationships, 3) *probe* student thinking, 4) introduce *terminology*, and 5) *generate discussion*. The combined use of higher level questions (i.e., explore, probe, generate) did not exceed the use of lower level questions.

Teaching set two. For teaching set two, all teachers utilized *teacher to whole class* most often as the *mode* of mathematical discourse. The use of *questions* exceeded the use of *statements*. Similar to the first teaching set, *information* gathering questions, *exploration* questions, and *probing* questions represented the top three question types. These were followed by questions that *oriented/focused* student attention on important aspects of the problem situation and questions used to *generate discussion*. During the second teaching set,

the combination of higher level questions exceeded the occurrence of lower level questions. Teachers' use of *statements* most often *revoiced* student responses, *oriented/focused* student attention, provided *information*, *encouraged* students after participating, and *directed* students to take a mathematical action.

Teaching set three. For teaching set three, all teachers utilized *teacher to whole class* most often as the *mode* of mathematical discourse. Similar to the first teaching set, *statements* and *questions* occurred almost equally, but *statements* were slightly more prevalent. *Statements* were used to *revoice* student responses, provide *information*, *orient/focus* student attention, provide *context*, and *encourage* students after participation. *Questions* most often asked for *information*, asked students to *explore* mathematical meanings and/or relationships, *probed* student thinking, *generated discussion*, or provided *context*. Like the second teaching set, the use of higher level questions exceeded the use of lower level questions.

Summary. Throughout the teaching sets, all teachers relied on *teacher to whole class* as the predominant *mode* of mathematical discourse. *Student to teacher* and *teacher to student* were the next most frequent *modes* of mathematical discourse, respectively. *Questions* and *statements* occurred almost equally; however, *questions* were used slightly more often than *statements*. Questions were used most often to gather *information*, *explore* mathematical relationships and/or meanings, *probe* student thinking, *generate discussion*, and *orient/focus* student attention on important aspects of the problem solving situation. Overall, higher level questions were posed more often than lower level questions. Teachers utilized statements to *revoice* student responses, *orient/focus* student attention, provide

information, encourage students, and *direct* students to take mathematical action, respectively.

The combined use of questions and statements positively influenced the learning environment in several ways. First, students were frequently told to orient/focus their attention on important aspects of the mathematical situation, including how the mathematics was represented and dynamically changed in the pre-constructed task. Second, students were asked to think deeply about the mathematics under discussion, as evidenced by more prevalent use of higher level questioning. Third, teachers' use of revoicing and encouragement provided students with positive reinforcement during mathematical discussions. Teachers also utilized the discourse moves of wait time and prompting students for further participation while facilitating mathematical discussions. Overall, the mathematical discussions provided opportunities for, and supported, student participation and learning. In essence, teachers and students were participating in *mathematical classroom discourse*.

Findings Related to Question 1b.

Teachers' use of the five practices for orchestrating productive mathematical discussions varied greatly throughout the semester. Only one teacher demonstrated progressively visible use of the five practices over the course of the semester. Mr. Phelps only anticipated students' mathematical and technological strategies and difficulties for the first teaching set. He anticipated, monitored, selected, and sequenced students strategies during the second teaching set, but he did not connect the student strategies. He successfully employed all five practices in the final teaching set. Mrs. Patterson consistently anticipated

and monitored student solution strategies throughout the semester, but the practices of selecting, sequencing, and connecting student responses were not observed. Mrs. Lewis used all five practices during the first teaching set; however, anticipation was the only practice observed for the second and third teaching set. This may have been mitigated by students' lack of access to pre-constructed dynamic tasks, but student access to pre-constructed tasks does not ensure or prohibit the use of the five practices. For example, Mrs. Patterson's students had access to pre-constructed tasks on the first day of the second teaching set, while Mr. Phelps' students did not. Mrs. Patterson employed anticipating and monitoring. Yet, Mr. Phelps incorporated each of the practices, except connecting. These inconsistencies point to factors beyond student access to technological tasks. One explanation may be related to teachers' opportunities to reflect upon their use of the five practices. During her summative interview, Mrs. Patterson expressed her difficulty in thinking about her use of selecting, sequencing, and connecting following a lesson. She stated that the researchers' comments, based on field notes, helped her reflect on her use of the five practices during post-observation interviews. All three teachers suggested that viewing either themselves or others teaching, similar to experiences during the summer institute, would be beneficial in improving their use of the five practices. As part of their participation in the study teachers were offered individual support from the researcher outside of the group planning meetings. Mr. Phelps asked for successive levels of support throughout the semester. First, the researcher modeled the use of a pre-constructed dynamic geometry task and five practices with Mr. Phelps students. He and the researcher reflected on the planning and implementation. Next, he requested that we co-plan and co-teach a lesson involving pre-

constructed dynamic geometry tasks and five practices. He and the researcher reflected upon the experienced following implementation. This series of events occurred between the first and second teaching set, which coincided with marked improvement of his use of the five practices. Mr. Phelps expressed how critical these experiences were for him to gain comfort and confidence over the course of the semester during the final focus group. Mrs. Lewis, nor Mrs. Patterson, requested this level of individual support during the semester. They did acknowledge the value of these experiences during their summative interviews and focus group. Both teachers said that given the opportunity again they would take advantage of individual supports, similar to Mr. Phelps.

Findings Related to Research Question 1c.

Mrs. Lewis and Mrs. Patterson were experienced users of *The Geometer's Sketchpad*, whereas Mr. Phelps was introduced to *The Geometer's Sketchpad* during the summer before the study took place. The choice to provide teachers with pre-constructed dynamic geometry tasks for the second and third teaching set allowed the researcher to examine teachers' use of tasks, without dependence on teachers' individual ability to create tasks. This decision 'leveled the playing field' in terms of technology. The researcher created tasks in alignment with teachers' mathematical goals and attended to research based design principles to promote student learning. The focus then shifted to what teachers were doing with the pre-constructed tasks during implementation. *Orchestration Types* was one lens that was utilized to examine teachers' use of pre-constructed dynamic geometry tasks. Teachers employed *discuss the screen* and *explain the screen* most often during implementation. Teachers' use of *Link screen and board* facilitated connections between representations in the pre-constructed

tasks and more traditional mathematical representations. Surprisingly, *tech demo* did not play a prominent role in teachers' use of technology. The use of similar technological features across teaching sets may have decreased the need for this orchestration type. *Spot and show* and *Sherpa at work* were also used sparingly. Difficulties with student management software may have hindered teachers' ability to employ *spot and show*.

Discourse analysis included teachers' use of *tools* for mathematical discourse, which allowed the researcher to document teachers' individual use of technological features in pre-constructed dynamic geometry tasks. All teachers utilized *hide/show buttons* and *sliders*, in conjunction with *orient/focus* statements and *exploration* questions, to facilitate development of student's understanding of important mathematics. Examples include, the use of *hide/show buttons* to 1) emphasize vertical and horizontal change (*Slope Presentation Sketch*, Teaching Set One), 2) explore patterns and trends in data (*MP3 Downloads*, Teaching Set Three), 3) highlight mathematical relationships (*Roots/Zeros Task*, Teaching Set Two), and 4) present multiple representations (i.e., *Slope Presentation Sketch*, Teaching Set One; *Quadratic Application Tasks*, Teaching Set Two; *Exponential Application Tasks*, Teaching Set Three). Teachers used *dragging* with *sliders* to focus students' attention on dynamic changes in graphical and numeric representations to promote 1) understanding of quadratic functions (i.e., $y=ax^2$, $y=(x-h)^2+k$, *Min/Max*, and *Roots/Zeros*, Teaching Set Two) and 2) compare exponential versus linear growth (*Pay Raise Task*, Teaching Set Three). Students drew conclusions based on the instantaneous changes they were seeing in graphs of quadratic and exponential functions, compared to the location and values of the sliders. Specific to quadratic functions, students based conclusions for the coordinates of the vertex,

minimum/maximum, and roots/zeros on invariance across dynamic representations. In general, the use of pre-constructed dynamic tasks presented multiple representations of function concepts in a manner that could not be achieved otherwise. The dynamic nature of linked representations would not be possible with more traditional tools. For example, students were able to explore dynamic symbolic, graphical, and tabular representations in the *Pay Raise Task* (Teaching Set Three). Teachers focused on the dynamic function and the slider for *Yearly Pay Raise* to bring attention to where the rate of increase occurred in exponential functions, as well as why it was written as a ratio. Students were able to compare the two pay raise options by examining changes in the graph, function, and/or table to draw conclusions. The use of researched based design principles for pre-constructed dynamic geometry tasks was critical to drawing student attention, providing student with affordances to support experimentation/exploration, providing alternate paths, and creating a shared image for the class.

Findings Related to Research Question 1d

Discussion of level of cognitive demand was based on teachers' implementation of common pre-constructed dynamic geometry tasks. Six tasks were utilized by each participating teacher, 1) *Quadratic Coefficient Exploration, Graphing $y = ax^2$* , 2) *Quadratic Applications, Projectile Motion*, 3) *MP3 Downloads*, 4) *Pay Raise*, 5) *Exponential Applications, Interest Appreciation*, and 6) *Exponential Applications, Depreciation*. Cross case analysis considered the use of statements and questions, five practices, and technology in reference to implemented level of cognitive demand to identify trends in the data. Table 22 summarizes the findings for the cross case analysis.

Table 22: Summary of Questions/Statements, Five Practices, and Implemented Level of Cognitive Demand for Common Pre-constructed Dynamic Tasks

Task	Teacher	% HLQ *	% LLQ *	% HLS *	% LLS *	5 Prac **	Tech ***	ILCD
$y = ax^2$	Lewis	82.36	17.65	23.08	23.08	A	DS, ES, LSB	4
	Patterson	83.34	16.67	82.36	17.65	A, M	DS, ES	3
	Phelps	57.15	42.85	39.09	4.55	A, M, Sel, Seq	DS	4
Projectile Motion	Lewis	63.89	36.11	24.32	32.43	A, M, Sel, Seq, C	DS, ES, LSB, SS	3
	Patterson	56.24	43.76	36.37	18.18	A, M	DS, ES	3
	Phelps	50	50	44.44	33.33	A, M, Sel, Seq	DS, ES, SW	4
MP3 Downloads	Lewis	62.5	37.5	18.42	36.84	A	DS, ES, LSB	4
	Patterson	25	75	26.67	33.33	A, M	ES	2
	Phelps	78.57	21.43	46.67	13.33	A, M, Sel, Seq	DS, ES	3
Pay Increase	Lewis	85.29	14.71	33.33	33.33	A	DS, ES	3
	Patterson	48.28	51.72	28.57	28.57	A, M	ES	2
	Phelps	65.52	34.48	58.62	18.96	A, M, Sel, Seq, C	DS, ES, LSB	3
Interest	Lewis	47.22	52.78	25	25	A	DS, ES, LSB	3
	Patterson	58.14	41.86	27.66	14.89	A, M	DS, LSB	3
	Phelps	51.61	48.39	32.35	17.65	A, M, Sel, Seq	DS, ES, LSB	3
Depreciation	Lewis	65	35	46.88	28.13	A	DS, ES, LSB	3
	Patterson	61.9	38.1	0	23.53	A, M	DS, ES	3
	Phelps	72.73	27.27	38.46	7.69	A, M, Sel, Seq	ES, DS	2

Table 22 (continued)

Note: %HLQ – Percentage of High Level Questions, %LLQ – Percentage of Low Level Questions; %HLS – Percentage of High Level Statements; %LLS – Percentage of Low Level Statements; ILCD – Implemented Level of Cognitive Demand

* Percentage of High and Low Level questions should sum to 100%; Percentage of High and Low Level Statements may not sum to 100% because some types of statements are neutral (i.e., revoicing)

** A – Anticipate, M – Monitor, Sel – Select, Seq – Sequence, C - Connect

*** DS – Discuss the screen, ES – Explain the screen, LSB – Link screen and board, SS – Spot and Show, SW – Sherpa at work

First, percentage of questions and statements was compared, within each task and across teachers. Higher use of questioning during implementation did not always coincide with higher implemented level of cognitive demand, nor did higher use of statements imply lower implemented level of cognitive demand. As such, questions and statements were analyzed in reference to types to look for trends. The overarching trend was that when teachers utilized higher level questions and statements more often than lower level questions and statements the implemented level of cognitive demand was high. Two instances (out of 18) deviated from this trend. First, Mrs. Lewis attained a high level of cognitive demand for the *Quadratic Applications, Interest Task* when her use of questions and statements was equal and her use of higher level questions and statements did not exceed her use of lower level questions and statements. Second, Mr. Phelps did not attain a high level of cognitive demand when implementing the *Quadratic Applications, Depreciation Task*, despite employing higher level questions and statements more often than lower level questions and statements. Both teachers utilized *hide/show buttons* for the table to confirm students' answers to each prompt, so the discrepancy was not likely due to technology use. Neither

teacher utilized *selecting*, *sequencing*, nor *connecting*, so the five practices fail to provide insight into the discrepancy. One explanation may lie in each teacher's mathematical development of the problem situation. Mrs. Lewis and her students used 126 (36 statements, 36 questions) interactions to explore the mathematical situation before using the graphing calculator to find the solution. Mr. Phelps' discussion of the task involved 40 interactions (13 statements and 11 questions) and was more focused on finding a solution.

The variability in teachers' use of the five practices, described earlier, did not appear to influence implemented level of cognitive demand. A high level of cognitive demand was maintained in 15 out of 18 tasks. Mrs. Patterson represented two instances (*MP3 Downloads* and *Pay Increase*) of low cognitive demand, and Mr. Phelps represented the third (*Quadratic Applications, Depreciation*). In each of these cases teachers employed anticipation, but none of the remaining practices. However, Mrs. Lewis demonstrated the most limited use of the five practices during common tasks, and she maintained a high level of cognitive demand for all tasks. As such, no discernible trends were observed in teachers' increased or decreased use of the five practices and implemented level of cognitive demand.

Teachers' use of statements and questions to *orient/focus* student attention and *explore* mathematical meanings and/or relationships paralleled teachers' use of technology. This theme occurred across orchestration types (*discuss the screen* or *explain the screen*) and students' access to pre-constructed tasks. Below are excerpts to illustrate this trend.

Teaching Set Two

Mrs. Lewis: All right, everybody look up here. I'm gonna move my c again. Do you see that it's not crossing the x -axis? And then as I move it...it crosses twice. (*Moving*

slider for c in Quadratic Exploration, Roots/Zeros. Points to where graph crosses the x-axis.)...Now, it says show the roots and the zeros. So they've been shown right here. (Used hide/show button to show roots/zeros. Points to the displayed points.)...So what can...if I...I haven't actually told you what a root or a zero is yet, but based off what I've asked you to do, what do you think a root and a zero is?

Mrs. Patterson: So if I take a and I grab a , what happens to my graph?*(Moving slider for a in Quadratic Exploration, Graphing $y=ax^2$ right and left.)...But notice the value of a when it goes down. (Moving slider into negative values.) What is the value of a when it goes down? So if I go positive, which way is it turning? (Moving slider from negative values to positive values.)*

Mr. Phelps: Okay, so as I continue to go back close to zero, it widens up. And when I get to my negative number, what happened? *(Moving slider for a in Quadratic Exploration, Graphing $y=ax^2$ from right to left toward zero then negative values.)*

Teaching Set Three

Mrs. Lewis: So I'm gonna show you the function. *(Uses hide/show button in Exponential Applications, Interest to show the graph of the function.)* Now, if I looked at this function, this function looks very much like a linear equation. Right? *(Only a small portion of the graph displayed in the upper left corner of coordinate plane.)* All right, but what do you think is gonna happen as I move it up? *(Drags coordinate plane down to see upper portion of graph of the function.)* Is it...is it still increasing, do you think, at a constant rate, at a linear rate?

Mrs. Patterson: Well notice the coordinates. Notice the points as we plot these points.

What's happening? (*Using hide/show buttons for points and coordinates in MP3 Download task.*)

Mr. Phelps: Let me pull this back to zero. So you'll see everybody is flat. (*Pulled sliders in Pay Raise Task to initial values.*)...So you are at wage \$6.00. She went from \$6.00 to \$8.25 to \$10.00. (*Moved slider for Hourly Wage in accordance with statement.*)...Uh, he said he wanted his percent to increase to 8%. Watch what happens. (*Moved slider for Yearly Pay Increase from 4% to 8%.*)...Do you see the drastic difference that it would look like if I change my percent of increase versus my wage increase?

The alignment of statements and questions with technology use also adhered to stated mathematical goals. This combination provided students with opportunities to learn significant mathematics related to linear, quadratic, and exponential functions.

Discussion

Analysis pertaining to mathematical discourse, use of the five practices, and technology use provided insight into the dynamics of three technology intensive Algebra 1 classrooms; however, the depth of analysis contained in this study would not be practical for teachers or administrators to employ in a school setting. Evaluation tools that have been vetted at scale include the IQA Academic Rigor: Mathematics Rubric for Potential of the Task, IQA Lesson Checklist, and the IQA Academic Rigor: Mathematics Rubric for Implementation of the Task (Adapted from Boston & Wolf, 2006, and Matsumura et al., 2006) (Appendices F-H). The mathematical task framework and level of cognitive demand

served as the theoretical foundations for the development and evolution of these evaluation tools, which align well with the theoretical underpinnings for this study. Unfortunately, they do not specifically attend to unique elements of technological tasks implemented in technology rich classrooms. One option would be to adapt these tools to include details relevant to technology, similar to changes made to the OMLI to capture teacher to student discourse and technology use. The Mathematics Rubric for Potential of the Task could be adapted to include Sinclair's (2003) design principles for pre-constructed tasks to evaluate the potential of technology features of the task. Results from this study indicated that design principles did not always coincide with high level of cognitive demand during implementation, but these principles were present in all tasks where teachers maintained a high level of cognitive demand. Further, in the one instance where a pre-constructed task did not include Sinclair's design principles (i.e., Mr. Phelps' *Slope Presentation Sketch*) a high level of implemented cognitive demand was not achieved. While not conclusive evidence for connections between research design principles for pre-constructed dynamic tasks and level of cognitive demand, further inquiry may be warranted.

Also, orchestration types (Drijvers et al., 2010) could be added to the Lesson Checklist. Discuss the screen, spot and show, and Sherpa at work represent three student centered orchestration types that would "provided opportunities for students to engage with the high-level demands of the task" (Boston & Wolf, 2006, and Matsumura et al., 2006), which compliment actions that maintain or increase the level of cognitive demand. Conversely, tech demo, explain the screen, and link screen and board were noted to be three teacher centered orchestration types that serve to remove or reduce the high level of cognitive

demand for the task. Finally, comments related to teachers' use of the technological features within pre-constructed dynamic tasks could be added to the Mathematics Rubric for Implementation of the Task. Again, the purpose of this study was not to evaluate or amend existing classroom evaluation tools, but initial findings based on these three cases may warrant further investigation in this area.

Summary

Mathematical discourse, five practices for orchestrating productive mathematical discussion, and technology use influenced the implemented level of cognitive demand in several ways. The surface level type of mathematical discourse (questions or statements) did not demonstrate clear trends regarding level of cognitive demand; however, second level analysis into the type of questions and statements showed connections between more prevalent use of higher level questions and statements with high implemented level of cognitive demand. Teachers' use of the five practices varied greatly across participants and teaching sets, so no discernible trends were found pertaining to level of cognitive demand. With regard to technology use, teachers' utilized student centered and teacher centered orchestration types, and research design principles for pre-constructed dynamic tasks supported teachers' implementation. Individual teacher actions with technology features positively influenced student's opportunity to learn when aligned with higher level questions, statements, and mathematical goals. These findings may provide opportunity for future study into the adaptation of existing evaluation tools for technology intensive learning environments.

CHAPTER 6: CONCLUSIONS, IMPLICATIONS, AND LIMITATIONS

The overarching question, “*How do teachers implement pre-constructed dynamic tasks in technology intensive Algebra 1 classrooms?*” has been discussed using within case analysis and cross case analysis. Mathematical discourse, teachers’ use of the five practices for orchestrating productive mathematical discussions, and teachers’ use of technology were utilized to create a rich description of what was occurring in the classroom and how these aspects influenced the level of cognitive demand. The introduction for this study described several issues for both teachers and students pertaining to the teaching and learning of mathematics with technology. Content, technology, and process standards set the bar for what teaching and learning in technology intensive environments should look like; however, practicing teachers need support to achieve the vision of teaching and learning described by these standards. Professional development continues to represent a tool that may be used to support teachers. The teaching and learning context for participating teachers provided further impetus for this study. The combination of 1:1 computing, in rural school districts that have a high population of economically disadvantaged and minority students remains an at-risk and understudied population. The remainder of this chapter draws conclusions based on findings from the study and relates them to issues described in the introduction and mathematics education research. Implications for decisions made during the research process, future research suggestions, and limitations of the study are also shared.

Conclusions

Discourse. Teachers utilized teacher to whole group as the predominant mode of instruction, and questions and statements were the predominant types of discourse. The

second level analysis of question and statement types indicated that teachers most often utilized higher level questions, including types 3, 4, and 5 (i.e., exploring, probing, and generating discussion) that were noted by Smith and Stien (2011). Smith and Stein suggested these types of question may be useful in facilitating classroom discourse. These types of questions did prove useful for teachers as they facilitated discourse around pre-constructed dynamic geometry tasks. Teachers' used a variety of questions. Informational questions focused student attention on processes and procedures; however, the combined use of higher level questions delved into students' mathematical thinking and sharing of ideas. This pattern of discourse mirrored multidimensional discourse (Franke, et al., 2007), where the pattern of discourse focuses on making sense of mathematical ideas. This supports the argument that higher level questions, including types 3, 4, and 5, align well with multidimensional discourse. Within case and cross case analysis for level of cognitive demand indicated that when this type of discourse was present the level of cognitive demand was higher, consistent with findings from Franke, et al., 2007.

Leonard (1999) identified pedagogical content knowledge, questioning strategies, use of instructional strategies, and norms as teacher variables that influence the amount and quality of mathematical classroom discourse. Findings from within case analysis reinforced these findings in reference to questioning and instructional strategies. The quality of questioning in relation to discourse was discussed above. In terms of teachers' instructional strategies, teachers' technology orchestration types Drijvers, et al. (2010) influenced the quality and amount of discourse. When teachers' utilized discuss the screen to orchestrate technology use the quality and amount of discourse increased. The converse was found when

teachers employed explain the screen. The only measure of pedagogical content knowledge for this study was the five practices, and the results were inconclusive. Teachers demonstrated use of high quality/multidimensional discourse with and without explicit use of the five practices. In the literature the five practices have been presented as pedagogical strategies that support teachers' orchestration of productive mathematical discussions, not as necessary or sufficient practices for productive discourse. Mr. Phelps, however, represented one case where increased use of the five practices corresponded with higher levels of discourse and cognitive demand.

Teachers demonstrated the use of support structures for discourse that have been documented in mathematics literature. Scaffolding (Baxter et al., 2002; Jones & Tanner, 2002; Nathan & Knuth, 2003; and Rosales et al., 2008), revoicing (Baxter et al., 2002 and Forman & Ansell, 2001), questioning/probing (Groves & Doig, 2004; Hiebert & Wearne, 1993; Jones & Tanner, 2002; and Piccolo, Harbaugh, Carter, Capraro, & Capraro, 2008), and use of robust learning materials (Groves & Doig, 2004; Heibert & Wearne, 1993; Leonard, 1999; Jones & Tanner, 2002; Webb, 1991; Weber et al., 2008) were used by all teachers. Learning materials (i.e., pre-constructed dynamic geometry tasks) were robust, as measured by potential level of cognitive demand. Teachers employed scaffolding in their modifications and organization of tasks to be utilized during a lesson, and the remaining support structures (revoicing and questioning/probing) captured in the discourse analysis.

In terms of transfer of habits of mathematics classroom discourse, Mrs. Lewis and Mrs. Patterson appeared to rely more on indirect transfer, but Mr. Phelps was more explicit about his expectations for students as discourse participants. This was more aligned with

direct transfer (Huang, Normandia, & Greer, 2005; Webb et al., 2006). Mrs. Lewis and Mrs. Patterson deviated from the findings by Inagaki et al. (1999), where direct and indirect transfer was split across cultural lines. American teachers exhibited direct transfer, whereas Japanese teachers relied on indirect transfer.

Sfard (2000) noted the extreme difficulty in establishing “appropriate measures of discipline and rigor in school mathematical discourse” (p. 184). The researcher suggested using a combined analysis of discourse using a modified version of the OMLI and Boaler and Brodie’s (2004) question types to address this limitation. Analysis performed with these tools for this study added discipline and rigor. The use of three levels of coding (mode, type, and tool) captured multiple facets of classroom discourse. The researcher was able to document directionality of interactions, nature of interactions, and tools utilized during interactions. Breaking down questions and statements using question types added a fourth level that allowed the researcher to discern the pattern of discourse (i.e., IRE/F, multidimensional). A second limitation was deciding how to teach elements of discourse. Exposing teachers to OMLI codes for mode and type, combined with question types, and the five practices has the potential to address this limitation. Professional development could serve as a means to deliver this instruction to teachers. For example, strategies within the professional development could involve participants 1) reading the *The 5 Practices for Orchestrating Productive Mathematics Discussions*, 2) learning the definitions for mode and type of discourse from the OMLI, and 3) analyzing video cases in reference to the five practices and coding them with the OMLI.

Five practices. Stowbridge (2008) pointed out the lack of research on how well teachers incorporate the five practices in their work. Stowbridge specifically mentioned research on 1) teachers' knowledge of the five practices and if that knowledge helped them facilitate productive discussions and 2) if they feel more in control when using the five practices. This study contributes to the former in several ways. First, Mrs. Lewis exhibited the least awareness of the five practices at the beginning of the study. In her initial interview, she failed to name one of the practices on her own and only referenced them after the researcher had reviewed them. Her use of the five practices was limited throughout the study. Second, Mrs. Patterson showed awareness of all five practices and recognized the significance of connecting, but this awareness and recognition did not transfer into her classroom instruction. Finally, Mr. Phelps articulated the most awareness of the five practices and transferred these explicitly into his classroom. In some cases pointing out to students which practice he was employing. In the case of Mr. Phelps, he also felt more capable of regulating what was occurring in the classroom when using the five practices.

Technology use. All teachers in this study reported that technology was essential to their teaching practice, and they could not imagine teaching in non-1:1 learning environments. Their use of pre-constructed geometry tasks mirrored their beliefs. Teachers orchestrated technology by discussing the screen (student-centered) more often than explaining the screen (teacher-centered), and their use of tech demo (teacher-centered) was very limited. These findings contrast with Palak & Walls (2009). Teachers' use of pre-constructed dynamic geometry tasks included "observing, recording, manipulating, predicting, conjecturing and testing, and developing theory as explanations for the

phenomena” (Olive et al., 2010, p. 148). The use of pre-constructed dynamic geometry tasks was transformative and provided students with opportunities to develop proceptual knowledge.

Design principles factored into teachers’ use of pre-constructed dynamic geometry tasks. During the first teaching set, teachers created a *Slope Presentation Sketch* (Appendix E) to use with students when introducing slope. The presence of design principles varied, as did the potential level of cognitive demand. For example, Mr. Phelps’ sketch utilized color to draw attention and provided a shared image for the class; however, the sketch did not include features to support exploration, provide affordances, or provide alternate paths. The potential level of cognitive demand was low. This limited opportunities for using the sketch in a transformative way and decreased students’ opportunity to develop proceptual knowledge. All other tasks adhered to design principles. Teachers’ took advantage of design principles with varying degrees of success during implementation; however, any limitations were not due to sketch design. The presence of design principles appeared to be a necessary, condition for high potential level of cognitive demand, but not sufficient condition for successful implementation.

Level of cognitive demand. All of the conclusions from above led to two predominant themes for level of cognitive demand. Given that pre-constructed dynamic geometry tasks adhered to design principles:

1. When teachers employed higher level questions/statements in conjunction with discuss the screen for their orchestration type, the implemented level of cognitive demand increased.

2. When teachers employed lower level questions/statements in conjunction with explain the screen for their technology orchestration type the level of cognitive demand decreased.

These results are consistent with findings from (Sherman, 2011). This study examined level of cognitive demand in relation to technology use as an amplifier vs. reorganizer (Pea, 1985) in secondary classrooms. Results showed that the decline or maintenance of high level cognitive demand depended more on teachers' classroom practice than the role of technology during implementation. How prepared students were to engage in high level thinking tasks in general, how teachers anticipated students needs while using technology to engage with the task, and how teachers responded to student questions and difficulties were noted as three critical areas of classroom practice. The findings from this dissertation study speak to the latter two classroom practices noted by Sherman.

All participants' in this study effectively anticipated students' mathematical and technological needs, prior to task implementation. Evidence of this was documented in both group planning sessions and during pre & post-observation interviews. The use of anticipation helped teachers handle mathematical and technological difficulties with minimal interruptions to the learning environment. Also, teachers' use of questions and statements in conjunction with technology orchestration types speaks to effective and non-effective responses to student inquiries and difficulties. The use of discuss the screen, with high level questions and statements, illustrates a student-centered response to questions and difficulties that places the cognitive demand on the students and brings student thinking to the

foreground. Conversely, the use of explain the screen, with lower level questions and statements, places the cognitive demand on the teacher and student thinking is diminished.

Implications

Results from this study provide implications for classroom mathematics teachers, teacher educators, providers of professional development, and future researchers. For classroom mathematics teachers, technology orchestration types and high level question/statement types represent very efficient and accessible ways to prepare for and reflect upon classroom instruction. The technology orchestration types are not specific to a particular technological tool, so they may be applied to teachers' use of all technological tools within the learning environment. High level question/statement types are not specific to a particular mathematics course or topic, so they could also be applied in all levels of mathematics classes. For example, when planning for a lesson a teacher could focus on how to use technological tools in a manner consistent with student-centered orchestration types. Combining this with the incorporation of high level questions and statements would place a high level of cognitive demand on the students and make student thinking the driving force for facilitating classroom interactions and discussions.

Teacher educators could also utilize the conceptual framework and results from this study in methods courses for pre-service teachers. One could envision an apprenticeship methods course that incorporated the CCSSM and the *Five Practices for Orchestrating Productive Mathematics Discussions*, along with supplemental readings focused on technology design and orchestration (i.e., Sinclair, 2003 and Drijvers, et al, 2010). Such a course has the potential to assist pre-service teachers in 1) identifying mathematical content,

process and/or technological goals, 2) selecting/designing high level tasks aligned with identified goals, and 3) using the five practices in conjunction with technology orchestration types and high level questions/statements to implement the task with students. A critical part of the course would be reflecting upon implementation of the task to consider factors that contributed to the decline or maintenance of level of cognitive demand.

As mentioned earlier, providers of professional development could also use the conceptual framework and coding schemes from this study to assist classroom mathematics teachers. For example, strategies within the professional development could involve participants 1) reading the *Five Practices for Orchestrating Productive Mathematics Discussions*, 2) learning the definitions for mode and type of discourse from the OMLI, 3) learning technology design and orchestration strategies, and 4) analyzing video cases in reference to the five practices, question/statement types, technology design and usage, and coding them with the revised OMLI. Such a deep analysis of video cases may be used to assist classroom teachers in reflecting upon their own classroom interactions and discussions. These learning activities encompass the implications noted for pre-service teachers, while delving deeper into ways to evaluate ones' teaching practice using research tools. Teachers that participate in this type of professional development could be used as teacher leaders with a teachers training teachers model of professional development. This model may serve as an effective scale up initiative by allowing more teachers and schools to participate.

For future researchers, several methodological decisions were made that shaped this research and provide implications future inquiry. First, pre-post observation interviews were selected as the tool to capture teachers' reflections on their use of the five practices. Teachers

expressed that it was difficult to pinpoint their use of selecting, sequencing, and connecting the five practices using this medium. This difficulty may have been mitigated by the fact that post-observation interviews often took place during planning time or after school because teachers had class immediately after an observation. The researchers' field notes assisted in their reflections; however, future researchers that wish to study teachers' use of the five practices may want to consider stimulated recall interviews. This would allow teachers to revisit short episodes selected by the researcher to reflect upon.

This study chose to focus mainly on teacher to student and student to teacher interactions. Hence, the decision was made to have teachers wear a microphone. Also, the camera used for observations did not have a supplemental external microphone. As such, student contributions to classroom discourse were not always audible. In whole group discussions capturing student discourse depended on how loud a student was speaking and their proximity to camera location. In small groups or individual work, capturing student discourse was also mitigated by the location of the teacher. Student to student interactions and discussions took place during classroom observations, but these were not the focus of this study. When conducting discourse analysis, quality audio is essential. Future researchers interested in discourse analysis need to consider the focus group for the study and make appropriate decisions for recording equipment and microphone placement.

A third methodological decision pertained to what type of dynamic geometry tasks to analyze for the study. In addition to pre-constructed dynamic geometry tasks, teachers implemented activities where students used *The Geometer's Sketchpad* to create sketches. The decision to analyze only pre-constructed dynamic geometry sketches was made to

control for variables related to students' knowledge of and ability to use *The Geometer's Sketchpad*. Pre-constructed dynamic geometry tasks are more accessible for students, and the use of pre-constructed dynamic sketches assured that teachers and students shared a common image (Sinclair, 2003) for small or large group discussions. This decision narrowed the focus and allowed the researcher to delve deeply into a specific type of use with dynamic geometry software. However, this decision prevented the examination of trends related to student-constructed sketches using *The Geometer's Sketchpad*.

After the first teaching set, the decision was also made not to have teachers create their own pre-constructed tasks. Two issues mitigated this decision. First, variance in teachers' products for the *Slope Presentation Sketch* influenced what occurred during implementation (e.g., opportunities to explore mathematical relationships and facilitate meaningful discussions). Second, teachers expressed concern about time constraints that were prohibitive to creating their own tasks for the second and third teaching sets. The researcher selected or created all pre-constructed dynamic geometry tasks for the second and third teaching sets. The tasks were presented at group planning meetings and edited per teachers' suggestions. This decision supported teachers in their daily practice; however, the decision prevented the researcher from examining individual teacher's use of *The Geometer's Sketchpad* as a tool on a more general scope.

Limitations

One limitation of this study was time. This study took place over the course of one semester and does not speak to teacher change over a significant period of time. The focus of this study was on the teacher, so conclusions were limited to students' opportunities to learn.

Also, measures related to teachers' content knowledge and how they may have influenced implementation of pre-constructed dynamic geometry tasks were not considered.

Directions for Future Research

One avenue for future research may include following these teachers over the course of the professional development project to examine longitudinal changes. Another research opportunity may involve the creation and vetting of new/revised observation tools to evaluate level of cognitive demand. For example, the IQA instruments for Mathematical Rigor – potential, lesson checklist, and implemented – (Appendices F-H) do not currently include elements pertaining to technology, but they could be modified to include technology and tested in technology-intensive mathematics classrooms. This may allow for the creation of a tool that could be used at scale for teachers and administrators. One last area that provides opportunities for future research involves professional development and the limitations to discourse noted by Sfard (2000). Designers of professional development could involve participants in strategies that bring attention to the five practices, definitions for modes and types of discourse, and video analysis, and then investigate teachers' use of the five practices and resulting discourse over the course of the professional development project.

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APPENDICES

Appendix A

Research Question	Data Used	Data Analysis
When involved in technology-intensive, research-based, professional development on Algebra,		
a. What types of mathematical questions and statements are central to the mathematical discussions that occur among students and the teacher when implementing pre-constructed dynamic geometry tasks?	<ul style="list-style-type: none"> • Video and Transcripts from Teaching Sets 	<ul style="list-style-type: none"> • Revised OMLI Classroom Observation Protocol-Mode & Type • Brodie & Boaler's (2004) Question Types • Open Coding
b. To what extent do teachers utilize the five practices for orchestrating productive mathematical discussions to support their implementation of pre-constructed dynamic geometry tasks?	<ul style="list-style-type: none"> • Video and Transcripts from Teaching Sets • Initial/Summative interviews • Planning sessions • Field notes 	<ul style="list-style-type: none"> • Five Practices
c. What are the design and technological features of pre-constructed dynamic geometry tasks and how do teachers use them during implementation?	<ul style="list-style-type: none"> • Pre-constructed tasks from planning sessions • Pre-constructed tasks used during teaching sets • Video and Transcripts from Teaching Sets • Field notes 	<ul style="list-style-type: none"> • Sinclair (2003) Design Principles • Drijvers et al. (2010) Orchestration Types • Revised OMLI Classroom Observation Protocol-Tools
d. What is the overall influence of a-c on the implemented level of cognitive demand of pre-constructed dynamic geometry tasks?	<ul style="list-style-type: none"> • Pre-constructed tasks used during teaching sets • Video and Transcripts from Teaching Sets • Field notes 	<ul style="list-style-type: none"> • Triangulation of Analysis for a-c • Task Analysis Guide

Appendix B

Initial/Summative Teacher Interview Protocol

Background and Context Information:

1. How long have you been teaching at (name of school)?
2. Were you teaching before? Where? How long? What subjects have you taught?
3. What degrees, certifications, and/or licenses do you currently have?

Learning Mathematics:

1. Please describe your learning experiences with mathematics as a student.
2. Did any of your experiences involve technology? If so, what technology and how was it utilized?
3. In the experiences you described, what aspects of the learning experiences supported/hindered your learning?
4. What are your feelings towards mathematics now?
5. Can you identify students in your class(es) who you think possess similar experiences/feelings? How do you interact with them?
6. How do you think students' best learn mathematics?

Instructional Practices/Five Practices:

1. What do you feel is the teacher's role (your role) in the classroom?
2. During the Algebra Summer Institute, you were introduced to 5 practices for orchestrating productive mathematics discussions, do these practices factor in to the teacher's role (your role) in the classroom? If so, how? If not, why not?
3. You were also introduced to the Mathematical Task Framework, does the framework factor into your role as a classroom teacher? If so, how? If not, why not?
4. How do you think the 5 practices and the Mathematical Task Framework relate to how students learn?

Technology:

1. What specific technological tools do you have access to in your classroom?
2. Please describe your experiences teaching with technology?
3. What are your feelings toward teaching with technology?
4. How comfortable are you with teaching with technology in general and with The Geometer's Sketchpad?
5. Have you taught with Geometer's Sketchpad before? Describe your experience teaching with Geometer's Sketchpad.

6. Is there anything about teaching with technology, laptops, and Geometer's Sketchpad that does not fit with your visions of teaching and learning mathematics?
7. Do you feel there are any difference between how students learn mathematics in 1:1 classrooms utilizing Geometer's Sketchpad and student without access to these tools? If so, what are they?

Planning/Coaching:

1. Describe how you plan for teaching with the 5 practices and Geometer's Sketchpad?
 - a. Do you find yourself planning more (or less) than you have in the past?
 - b. What types of things do you think about when you're planning?
 - c. How do you decide when to incorporate Geometer's Sketchpad activities?
 - d. Do you collaborate with other teachers when planning? If so, can you describe how that came about, and what your collaboration involves?
2. How are students responding to the five practices and Geometer's Sketchpad?
 - a. Do you think there's an adjustment period? If so, please describe why you think it occurs, and how long it typically lasts?
 - b. What do students have to get used to?
3. How often do students talk about mathematics in your class(es)?
 - a. Do you try to get students to talk more about mathematics? How? Can you describe that process?
 - b. What types of norms did you establish at the beginning of the school year?
 - c. How did the students respond?
4. How comfortable are you teaching with the five practices and Geometer's Sketchpad?
 - a. What do you think is necessary to increase your comfort level?
 - b. Have you encountered any hindrances? If so, please describe them.
 - c. What types of support do you think would increase your comfort level and alleviate hindrances (e.g., model teaching, co-planning, co-teaching, outside support sessions, collaboration with other Algebra 1 teachers, etc.)?

Appendix C

Classroom Observation Summary Form

Lesson Context

Observer: _____ Date: _____

Teacher: _____ School: _____

Grade(s): _____ Course: _____

Unit/Topic: _____

Learning Objective: _____

Instructional Materials: _____

Math Class Began: _____

Math Class Ended: _____

Students: _____

Percent Minority: _____ %

Relationship to previous and future lessons:

Other comments regarding the lesson context:

Pre-observation Interview Questions

1. What are your mathematical goals for the lesson today?
2. What tasks are you utilizing to accomplish the mathematical goals and how will you be using them to achieve the mathematical goals?
3. Are there specific mathematical strategies/difficulties you anticipate students will use/experience during the lesson?
4. Are there specific technological strategies/difficulties you anticipate students will use/experience during the lesson?
5. How do you plan to monitor students' mathematical and technological progress during the lesson?
6. Is there anything in particular that I should know about the students in this class?

NOTE: Get specific instructional materials reference or a copy of the lesson plans.

Teacher Observation Notes

Teacher: _____ **Date:** _____

School: _____ **Class Period:** _____

Lesson Content: _____

Notes	Questions & Comments

Postobservation Interview Questions

1. How well were the stated mathematical goals reached during the lesson? Explain.
2. Did the tasks help accomplish the mathematical goals as intended?
3. Did the students exhibit the mathematical strategies/difficulties that you anticipated?
 - a. Were there specific mathematical strategies/difficulties that you did not anticipate students would use/experience during the lesson? If so how did you handle these?
4. Did the students exhibit the technological strategies/difficulties that you anticipated?
 - a. Were there specific technological strategies/difficulties that you did not anticipate students would use/experience during the lesson? If so how did you handle these?
5. What strategies did you use to monitor students' mathematical and technological progress during the lesson?
6. Which student strategies did you select to highlight during the lesson?
 - a. Why did you select those particular strategies and/or students?
7. How did you sequence student strategies during the lesson?
 - a. Why did you choose to sequence them in that manner?
8. How did you connect student strategies during the lesson?
9. What challenges did you confront in encouraging students to engage in the mathematical discussions?
 - a. What would help you in addressing these challenges?
10. What do you think the students learned from this lesson, and what they still need to learn?
11. What do you plan to do in the next lesson with your students?

Codes for Mathematical Discourse

Modes of Mathematical Discourse

Code	Definition	Explanation
ST	Student to Teacher	Student primarily addresses the teacher, even if entire class hears
SS	Student to Student	
SG	Student to Group	
SW	Student to Whole Class	
SIR	Student's Individual Reflection	
TG	Teacher to Group	
TW	Teacher to Whole Class	
TS	Teacher to Student	
TIR	Teacher's Individual Reflection	Teacher may ponder out loud at the board
N	None	There is no discourse (test or quiz, for example)

Types of Mathematical Discourse

Code	Definition	Explanation
A	Answering	Answer from student or from teacher
S	Making a statement or sharing	Individual does not share how or why
E	Explaining	A mathematical idea may be explained, but there is no justification
Q	Questioning	A student asks a question to clarify own understanding
C	Challenging	Student or Teacher makes a statement that challenges validity of an idea
R	Relating	Student indicates he/she has made a connection to prior knowledge
P	Predicting or Conjecturing	Based on student's own understanding
J	Justifying	Student provides justification for the validity of an idea
G	Generalizing	Shifts from specific case to general understanding
T	Technological	Focus is on the technology and its operation

Tools for Mathematical Discourse

Code	Definition	Explanation
V	Verbal	
A	Acting/Gesturing	Moves body to communicate mathematical idea
W	Written	A narrative is written to explain mathematical concept
G	Graphs, Charts, Sketches	Depicts idea or procedure
M	Manipulative	Physical object used to model the idea
S	Symbolization	Informal and nonmathematical
N	Notation	Formal math notation
C	Computers	Technology is used to communicate an idea
CA	Calculator	Calculator is used to communicate an idea
O	OTHER	

Classroom Observation Discourse Form

Evidence of Mathematical Discourse

Page: ____

Teacher: _____ Date: _____

Episode Type			Start/End Times	Observed	Episode Description	Discourse Codes			Tally
Large Group	Pairs/Small Group	Individual				Mode	Type	Tools	

Appendix D

Focus Group Protocol

1. Describe the types of supports you have received this semester.
 - a. Which ones were the most helpful? Why?
 - b. Which ones were the least helpful? Why?
2. How did you decide if *GSP* tasks would be ‘student-driven’ versus ‘teacher-driven’?
 - a. How much of a factor was the VCL?
3. Group planning meetings
 - a. In what ways have the group planning meetings been helpful?
 - b. Were there things that you would add that we did not do this semester?
 - c. As we continue, what could be used in place of the group meetings to assist you?
4. In addition to the group planning meetings, we worked together on an individual basis throughout the semester.
 - a. What was the most helpful? Why?
 - b. What would you alter? Why?
5. 5 Practices
 - a. Which practices were the easiest to incorporate during the semester? Why?
 - b. Which practices were the most difficult to incorporate during the semester? Why?
 - c. Which practices were the most helpful in planning and teaching? Why?
 - d. What help/supports do you need to as you continue to incorporate the 5 practices?
6. Do you feel there are differences in using the 5 practices in middle school versus high school?
 - a. If so, what are they?
 - b. If not, why not?
 - c. Are there differences in using them for a lesson versus for an individual task?
7. Do you feel there are differences in using the 5 practices in 1:1 classrooms versus traditional classrooms? Are there modifications based on the 1:1 environment?

Appendix E

Slope Presentation Project from Module 1 of Online Professional Development

Create a presentation sketch that illustrates slope. Your sketch should include a line that moves freely; a visual representation of slope, such as a slope triangle; and a calculation of the slope of the line using the slope formula. You may choose to use Hide/Show buttons and color as a way to highlight relationships in your sketch.

You can use this week's JavaSketch for inspiration, but you don't need to duplicate it. Create a sketch that you can use to introduce or review slope with your students. If you are having trouble constructing a slope triangle, refer to the hint below.

*Please include your last name and the week when you save your file (e.g.,
SmithZ_Project1.gsp).*

To submit your project, select your file using "Browse" or "Choose file" below, then click on "Upload this file." If you are using a Mac, do not use Internet Explorer to upload your project (use Safari or Firefox)

Hint: The x and y axes in Sketchpad are both lines. You can construct a line parallel or perpendicular to either axis. In this week's tutorial video, Janet Bowers demonstrates how to construct parallel and perpendicular lines in the segment "The Project".

[Insert your main body text here. Format depends on your style guide.]

Appendix F

IQA Academic Rigor: Mathematics Rubric for Potential of the Task (Adapted from Boston & Wolf, 2006, and Matsumura et al., 2006)

4	<p>The task has the potential to engage students in exploring and understanding the nature of mathematical concepts, procedures, and/or relationships, such as:</p> <ul style="list-style-type: none"> • Doing mathematics: using complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). <p>OR</p> <ul style="list-style-type: none"> • Applying procedures with connections: applying a broad general procedure that remains closely connected to mathematical concepts. <p>The task must explicitly prompt for evidence of students’ reasoning and understanding.</p> <p>For example, the task <i>may</i> require students to—</p> <ul style="list-style-type: none"> • solve a genuine, challenging problem for which students’ reasoning is evident in their work on the task; • develop an explanation for why formulae or procedures work; • identify patterns and form generalizations based on these patterns; • make conjectures and support conclusions with mathematical evidence • make explicit connections among representations, strategies, or mathematical concepts and procedures; and • follow a prescribed procedure in order to explain/illustrate a mathematical concept, process, or relationship
3	<p>The task has the potential to engage students in complex thinking or in creating meaning for mathematical concepts, procedures, and/or relationships. However, the task does not warrant a “4” because—</p> <ul style="list-style-type: none"> • it does not explicitly prompt for evidence of students’ reasoning and understanding; • students may be asked to engage in doing mathematics or procedures with

	<p>connections, but the underlying mathematics in the task is not appropriate for the specific group of students (i.e., too easy or too hard to promote engagement with high-level cognitive demands);</p> <ul style="list-style-type: none"> • students may need to identify patterns but are not pressed for generalizations; • students may be asked to use multiple strategies or representations, but the task does not explicitly prompt students to develop connections between them; and students may be asked to make conjectures but are not asked to provide mathematical evidence or explanations to support conclusions.
2	<p>The potential of the task is limited to engaging students in using a procedure that is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. There is little ambiguity about what needs to be done and how to do it. The task does not require students to make connections to the concepts or meaning underlying the procedure being used. The focus of the task appears to be on producing correct answers rather than developing mathematical understanding (e.g., applying a specific problem solving strategy, practicing a computational algorithm).</p> <p>OR</p> <p>The task does not require student to engage in cognitively challenging work; the task is easy to solve.</p>
1	<p>The potential of the task is limited to engaging students in memorizing or reproducing facts, rules, formulae, or definitions. The task does not require students to make connections to the concepts or meaning that underlie the facts, rules, formulae, or definitions being memorized or reproduced.</p>
0	<p>The task requires no mathematical activity.</p>

Appendix G

IQA Lesson Checklist

(Adapted from Boston & Wolf, 2006, and Matsumura et al., 2006)

Check each box that applies:

A ↑	The lesson provided opportunities for students to engage with the high-level demands of the task:	B ↓	During the lesson, the high-level demands of the task were removed or reduced:
<input type="checkbox"/>	Students engaged in the task in a way that addressed the teacher's goals for high-level thinking and reasoning.	<input type="checkbox"/>	The task expectations were not clear enough to promote students' engagement with the high-level demands of the task.
<input type="checkbox"/>	Students communicated mathematically with peers.	<input type="checkbox"/>	The task was not complex enough to sustain student engagement in high-level thinking.
<input type="checkbox"/>	Students had appropriate prior knowledge to engage with the task.	<input type="checkbox"/>	The task was too complex to sustain student engagement in high-level thinking (i.e., students did not have the prior knowledge necessary to engage with the task at a high level).
<input type="checkbox"/>	Teacher supported students to engage with the high-level demands of the task while maintain the challenge of the task.	<input type="checkbox"/>	Classroom management problems interfered with students' opportunities to engage in high-level thinking.
<input type="checkbox"/>	Students had opportunities to serve as the mathematical authority in the classroom.	<input type="checkbox"/>	Teacher provided a set procedure for solving the task.
<input type="checkbox"/>	Teacher provided sufficient time to grapple with the demanding aspects of the task and for expanding thinking and reasoning.	<input type="checkbox"/>	The focus shifted to procedural aspects of the task or on correctness of the answer rather than on meaning and understanding.
<input type="checkbox"/>	Teacher held students accountable for high-level products and processes.	<input type="checkbox"/>	Feedback, modeling, or examples were too directive or did not leave any complex thinking for the student.
<input type="checkbox"/>	Teacher provided consistent presses for explanation and meaning.	<input type="checkbox"/>	Students were not pressed or held accountable for high-level products and processes or explanations and meanings.
<input type="checkbox"/>	Teacher provided students with sufficient modeling of high-level performance on the task.	<input type="checkbox"/>	Students were not given enough time to deeply engage with the task or to complete the task to the extent that was expected.

<input type="checkbox"/>	Teacher provided encouragement for students to make conceptual connections.	<input type="checkbox"/>	Students did not have access to resources necessary to engage with the task at a high level.
<input type="checkbox"/>	Students had access to resources that supported their engagement with the task.	<input type="checkbox"/>	Other:
<input type="checkbox"/>	Other:		

Appendix H

IQA Academic Rigor: Mathematics Rubric for Implementation of the Task (Adapted from Boston & Wolf, 2006, and Matsumura et al., 2006)

4	<p>Students engaged in exploring and understanding the nature of mathematical concepts, procedures, and/or relationships, such as:</p> <ul style="list-style-type: none"> • Doing mathematics: using complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). <p>OR</p> <ul style="list-style-type: none"> • Applying procedures with connections: applying a broad general procedure that remains closely connected to mathematical concepts. <p>There is explicit evidence of students’ reasoning and understanding. For example, students may have—</p> <ul style="list-style-type: none"> • solved a genuine, challenging problem for which students’ reasoning is evident in their work on the task; • developed an explanation for why formulae or procedures work; • identified patterns and formed generalizations based on these patterns; • made conjectures and supported conclusions with mathematical evidence; • made explicit connections among representations, strategies, or mathematical concepts and procedures; • followed a prescribed procedure in order to explain/illustrate a mathematical concept, process, or relationship.
3	<p>Students engaged in complex thinking or in creating meaning for mathematical concepts, procedures, and/or relationships. However, the implementation does not warrant a “4” because—</p> <ul style="list-style-type: none"> • there is no explicit evidence of students’ reasoning and understanding; • students engaged in doing mathematics or procedures with connections, but the underlying mathematics in the task was not appropriate for the specific group of students (i.e., too easy or too hard to sustain engagement with high level cognitive demands); • students identified patterns but did not make generalizations; • students used multiple strategies or representations, but connections between different strategies/representations were not explicitly evident; • students’ made conjectures but did not provide mathematical evidence or explanations to support conclusions.
2	<p>Students engaged in using a procedure that was either specifically called for or its use was evident based on prior instruction, experience, or placement of the task. There was little ambiguity about what needed to be done and how to do it. Students did not make connections to the concepts or meaning underlying the procedure being used. Focus of the implementation appears to be on producing correct answers rather than developing mathematical understanding (e.g., applying a specific problem-solving strategy, practicing a computational algorithm).</p> <p>OR</p>

	Students did not engage in cognitively challenging work; the task was easy to solve.
1	Students engaged in memorizing or reproducing facts, rules, formulae, or definitions. Students did not make connections to the concepts or meanings that underlie the facts, rules, formulae, or definitions being memorized or reproduced.
0	Students did not engage in mathematical activity.

Appendix I

Example of Detailed Coding for Discourse

Episode	Mode	Type	Tool	Question Type	Statement Type	Explanation Type	Challenge	Generalization	Action	Notation
Slope Sketch	TW	E	V, C, G			Explain Screen				
D2 (25:00 - 39:45)	TW	Q	V, C, G	Info						
p. 8	TW	E	V, C, G, A						pointing to two generic ordered pairs written on board and corresponding points on line	
	TW	Q	V	Info						
	TW	Q	V	Info						
	TW	S	V		Directive					
	TW	Q	V	Explore						
	TW	S	V		Revoice					
	TW	Q	V	Explore						
	TW	Q	V	Explore						
	ST	A								
	TW	S	V		Revoice					

					ce					
	TW	Q	V	Explore						
	TW	S	V		Encourage					
	TW	Q	V	Explore						
	TW	T	V, C, G, A			Tech Demo			Used Hide/Show button to show numeric value for slope of the line; dragging point on the line to alter the slope	
	TW	S	V, C, G, A		Orient /Focus					
	TW	T	V, C, G, A			Tech Demo				
	TW	Q	V, C, G, A	Explore						
	TW	S	V, C, G, A		Revoice					
	TW	Q	V, C, G, A	Explore						
	TW	Q	V, C, G, A	Explore						
	TW	S	V, C, G, A		Orient /Focus					
	TW	Q	V, C, G, A	Explore						

	ST	A	V, C, G						
	TS	Q	V, C, G	Info					
p. 9	ST	A	V, C, G						
	TS	S	V, C, G		Revoic ce				
	TS	Q	V, C, G	Explo re					
	TW	E	V, G, N			Tells class slope is steepness			"slope: steepne ss"
	TW	S	V		Conte xt	hill vs. mtn			
	TW	E	V			chg y over chg x			
	TW	E	V, C, G, A			rise over run		H/S slope triangle; change in y & change in x	
	TW	E	V, C, G, N			rise up/dn, run left/rt			"rise/ru n"