ABSTRACT

ALNIZAMI, REEMA KHALID. Geometry Professional Development and Teacher Education: Research Synthesis. (Under the direction of professor Paola Sztajn).

This study examined professional development models for in-service teachers and teacher education courses for pre-service teachers that addressed the content of geometry, with attention to the integration of technology into such programs or courses. This study is designed as a research synthesis in which written work on professional development programs and teacher education courses on the subject of geometry were collected, screened, coded, and analyzed. Recommendations and suggestions from the literature review on the effectiveness of PD and on the use of technology with geometry instruction suggested the problem formulation which resulted in translating the research questions of this study into a set of variables defined in the codebook. Papers on geometry PD were collected and then coded using the codebook. The results obtained were of a descriptive nature. The results indicated that a scarce amount of details are provided about geometry PD in the coded papers. Results also indicated differences between papers that reported on in-service teacher professional development models and those that reported on teacher education settings for pre-service teachers.

Geometry Professional Development and Teacher Education Settings: Research Synthesis

by
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BIOGRAPHY

Reema Alnizami, raised in Jordan, is daughter of Fatima Alotoum and Khalid Bani Mustafa. She graduated in 1996 from Jordan University with a degree of Business Administration. She moved to USA in 1999 and became a US citizen in 2000. She came back to school in 2006 to earn an undergraduate degree in Mathematics Education from North Carolina State University in 2010. Presently, she is getting her maters degree in Mathematics Education at North Carolina State University.

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CHAPTER 1: INTRODUCTION

Rationale

Many researchers agree that enhancing mathematics teachers' knowledge is one of the most significant factors contributing to the advancement of student understanding of mathematics. In fact, Sowder (2007) stated, "A variety of recently published documents support the notion that the key to increasing students' mathematical knowledge and to closing the achievement gap is to put knowledgeable teachers in every classroom" (p. 158). Learning opportunities for teachers can occur throughout their career starting when they are pre-service teachers and continuing after they become in-service teachers; therefore in this study, I was interested in investigating published papers that focused both on professional development interventions for in-service teachers and on teacher education settings for pre-service teachers. Throughout this thesis, I will use the PD abbreviation to refer to both pre-service and in-service teacher education opportunities. When a distinction between pre-service and in-service is needed, I will explicitly note that. Different factors contribute to the effectiveness of various PD opportunities. Such factors, related to both the content and format of PD, are the objects of interest in this study.

This study examined the design of PD that addressed the content of geometry, with attention to the integration of technology into PD activities. The focus on geometry with particular attention to technology is due to the belief that "educators need to be knowledgeable about their content as well as the ways technology can transform the delivery of that content" (Spires, Weibe, Young, Hollebrands, & Lee, 2009, p. 15). Researchers, such as Powers & Blubaugh (2005), have confirmed the emergence of a growing focus on the use of technology in teacher education settings. Furthermore, the results of different studies on the use of technology in the learning and teaching of mathematics indicate “that the use of technology promotes students’ understandings of mathematics and recommend the incorporation of technology into the teaching and learning of mathematics” (Hollebrands, 2003, p. 70). The use of technology should be for the goal of enhancing students’ deep understanding of the mathematical concepts being taught.

The particular questions of interest to this study are:

1) To what extent are recommendations for effective professional development present in the design of PD that focus on geometry as reported in published papers?
2) Is there a trend in the types of goals for geometry PD as reported in published papers?
3) In what ways do PD that focus on geometry integrate technology either as a learning tool for teachers or as a tool to be used with students as reported in published papers?
4) When technology is reported to be integrated, what types of technology are used? In particular, what percentage of PD that focus on geometry and integrate technology use dynamic geometry software?
5) As reported in published papers, in what ways are recommendations from research about the use of dynamic geometry software in instruction incorporated in PD that focus on geometry and integrate dynamic geometry software?

The findings in this study provided a description of the field of geometry PD that is presented in published papers. Beyond the research questions of interest in this study, the results of the study indicated differences between papers reporting on professional development programs for in-service teachers and papers that reported on teacher education settings for pre-service teachers.

Overview of the Study and Thesis Organization

To address the research questions of this study, descriptive data on geometry PD were coded and analyzed. This study is designed as a research synthesis. Research synthesis is defined as a study that "focus[es] on empirical studies and seek[s] to summarize past research by drawing overall conclusions from many separate investigations that address related or identical hypotheses" (Cooper, 1998, p. 3). Cooper, Hedges, and Valentine (2009) distinguished between literature review and synthesizing research. Whereas synthesizing research is the process of extracting and combining data from different sources in a methodical and systematic manner, for them, a review of the literature is not necessarily systematic in selecting and working with research reports. For research synthesis, findings or other information found in reports, articles, book chapters, etc. are considered the data for the study. Further, when researchers conduct research synthesis, they examine all of the collected data that emerge from the literature search using their established search criteria, which is not necessarily true for literature reviews in which researchers construct their argument by selecting a subset of papers and studies to review.

In this study, written work on professional development programs and teacher education courses in the subject of geometry were collected, screened, and coded, and then results were analyzed. Different from the synthesis proposed by Cooper and his colleagues, this is a synthesis of the designs used in the PD reported in the papers—not of the research findings about such courses and programs. That is, in this study, I examined what researchers reported about the design of their PD instead of the usual synthesis analysis of what was reported for the research design and findings. Since the focus was not on research findings, whether the papers collected for this synthesis were reports of empirical studies or not was not one of the screening criteria for the selection of published papers to be coded.

In regard to the organization of this synthesis study, my research draws on Cooper's (1998) 5 stages of a research synthesis. His stages comprise of (a) formulating the problem, (b) collecting the data, (c) evaluating the collected data, (d) analyzing and interpreting the data, and (e) displaying the results. The chapters that follow are based on these steps and recommendations from Cooper (1998). Each of Cooper's stages were incorporated within the different chapters of this thesis in the following manner: problem formulation is the literature review chapter, Chapter 2, which includes a description of how the research questions were examined and concludes with the formulation of the codebook representing the variables of interest. The explanations of how I carried out the data collection, evaluation, and analysis are included in my methodology, presented in Chapter 3. The results from the data analysis and interpretation are discussed in Chapter 4, the findings. The discussion in chapter 5 includes the interpretation of the findings and the display of my final conclusions. The discussion chapter also includes the limitations and implications of my study.

CHAPTER 2: LITERATURE REVIEW

Problem Formulation

This literature review was compiled with the purpose of supporting the coding process used for inspecting collected papers on geometry PD. The goal of the review was to highlight important and established knowledge from the field that needed to be considered in the analysis of the studies selected for examination in this research synthesis. This review served as the basis for the design of the codebook used in the study.

Because of the nature of the research synthesis, and to separate the literature review from the synthesis process, this review was built based on previous, well-established reviews and seminal papers. Thus, in the literature review, I searched for important features that needed to be considered in the subsequent coding of papers selected through a search of databases like ERIC and PsychInfo.

The literature reviews from Clements, Sarama, Yelland, and Glass (2008) and Hollebrands, Laborde, and SträBer (2008) were the initial resources used for the review related to geometry. These two recent reviews focused on the use of technology in the learning and teaching of geometry. Therefore, the two reviews were chosen as main initial references to this study on the use of technology with geometry at the elementary, middle and secondary level. Subsequently, a subset of the relevant authors whose works were reviewed by Clements et al. and by Hollebrands et al. were consulted to complete the literature review part of this thesis. Selected authors for further review were those who examined the integration of dynamic geometry environments in the teaching and learning of geometry.

In addition to the articles related to geometry, other selected seminal papers on teacher education were also examined. For instance, Sowder (2007) was consulted on the goals for PD, which should address the needs of teachers, and Wei, Darling-Hammond, Andree, Richardson, and Orphanos' (2009) work was chosen as a main paper for putting together a list of criteria for effective PD. Similarly to the process used for the geometry review, papers mentioned in these two reviews of PD were subsequently reviewed.

The literature review part of this thesis has five main themes, focusing on (1) effective professional development and teacher education, (2) preparing teachers to work with technology, (3) dynamic geometry software (DGS), (4) geometric transformations and DGS, and (5) the use of DGS with proof activities. For the geometry part, this order of sections was used because each theme builds upon the previous ones. The reason for dedicating separate sections for geometric transformations and proof teaching with DGS is because of the vast amount of research that has investigated the use of DGS for the teaching and learning of these two topics. Many of these studies' findings suggested the academic advantages of the use of DGS with these two particular topics.

Effective professional development and teacher education.

> "Teaching excellence is the most powerful influence on achievement" (Harding, & Parsons, 2011, p. 53).

After conducting a review of research studies in teacher professional development conducted both in the USA and in other nations, Wei et al. (2009) concluded that the following criteria were found to advance the effectiveness of teacher professional development: “1. Professional development should be intensive, ongoing, and connected to practice” (p. 9). “2. Professional development should focus on student learning and address the teaching of specific curriculum content” (p. 10). “3. Professional development should align with school improvement priorities and goals” (p. 10). “4. Professional development should build strong working relationships among teachers” (p. 11). Thus, format, content, alignment and relations are features of professional development that have been established in a previous review of the literature as important.

Although the recommendations for effective professional development presented in Wei et al. (2009) particularly targeted in-service teacher education, this study extends them to consider the design of pre-service teacher education courses as well. Ingvarson, Beavis, and Kleinhenz (2007) reported on the findings of a study that “investigated the characteristics of effective initial teacher education programs, as reported by” (p. 351) 1147 teachers who recently graduated from teacher education programs. Ingvarson and colleagues report that their findings about characteristics of effective pre-service teacher education courses were consistent with recommendations from the literature on characteristics of effective professional development programs for in-service teachers. Some of the characteristics found by Ingvarson and colleagues for effective teacher education courses included an emphasis on content knowledge for teachers, working in teams, and attention to student learning. Smith (2001) added that settings, in which activities were connected to practice such as when teachers worked in real classrooms or artifacts were taken from real classrooms, could be used with both in-service and pre-service teachers.

Part of the format of PD, duration, seems to be an important feature of PD, as stated by Weiss et al. (2009):

> An analysis of well designed experimental studies found that a set of programs which offered substantial contact hours of professional development (ranging from 30 to 100 hours in total) spread over six to 12 months showed a positive and significant effect on student achievement gains. (p. 9)

They also concluded that a total of about 80 professional development hours or more would have more potential to allow teachers to "put the given teaching strategies into practice" (Wei et al., p. 9) than a program with a less number of hours would do. The findings in a 7-year study of "the relationship between professional development and teachers' attitudes, preparedness, and classroom practice in mathematics" (Heck, Banilower, Weiss, & Rosenberg, 2008, p. 113) were consistent with the findings reported by Wei and colleagues. Heck and colleges also found that the longer the hours of professional development provided for teachers, the higher were the teachers' attitudes and perceptions of their ability to teach mathematics.

The intensity and continuity of the program can be determined by looking at the total time span of the program, as well as the total number of contact hours. However, the duration of the PD by itself does not determine its effectiveness. It is also important to attend to what teachers are doing during the PD. One important feature of PD that emerged from literature was whether tasks selected for PD were designed and implemented in ways that are guided by suggestions from research. For example, Garet, Porter, Desimone, Birman, and Yoon

(2001) concluded that PD that used tasks that actively engaged teachers in learning were more effective than PD settings in which teachers were told how to teach or lectured about the mathematical content. Beyond tasks, focusing PD on content is also important. The findings in the study by Heck et al. (2008) suggested that more "content-based focused professional development than most teachers currently experience would have beneficial effects" (p. 145). According to Garet et al. (2001) the content focus of a professional development intervention related to the degree of emphasis during the intervention on the participants' knowledge of particular content, such as focusing on the teaching and learning specific to the content of geometry in a math professional development.

A list of goals for mathematics professional development interventions was suggested in Sowder's (2007) review of the literature. Of those goals were: developing teachers' content knowledge, developing teachers' understanding of various instructional and pedagogical approaches used in mathematics classroom, developing teachers' understanding of how children learn particular topics in mathematics, developing alignment of mathematics instruction to a particular curriculum, developing teachers' understanding of school or district pacing guides for organizing mathematical instruction, and developing teachers' understanding and use of technology for teaching (Sowder, 2007).

In summary, in this study, I integrated recommendations from literature for in-service and pre-service teachers in the codebook that was used to code the collected written work on professional development and teacher education courses. I considered that effective math PD is intensive and continuous, focuses on mathematical content, connects the content addressed to classroom practice, actively engages teachers in the content of PD, and has student learning as one of its goals.

Because focusing on student learning is an important component of effective PD, in subsequent sections, after discussing teacher preparation to work with technology, I focus on what is known from the literature about student learning of geometry with technology. Such knowledge about student learning should be made available for teachers through PD.

Preparing teachers to work with technology.

Laborde, Kynigos, Hollebrands, and Strässer (2006) summarized the history of the integration of technology in the teaching of geometry. When technology was first used for the learning of geometry, it was used without consideration of particular learning objectives or targeting particular learner needs. The teacher's role as a facilitator was absent in this initial stage as well, leading Hollebrands et al. (2008) to note that "Twenty years ago the notion of microworld could have been interpreted as implying spontaneous learning just by interacting with the microworld" (p. 178). Over time, it was learned that spontaneous interaction with the technology by the user did not guarantee that learning of geometry occurred.

Educators and researchers determined that just promoting interaction between the students and the technology was not sufficient to achieve desired learning goals and started investigating ways in which the alliance of teachers' role, design of the geometric activity, and the use of technology affected students' learning of geometry (Laborde et al., 2006). Since research recommends integrating technology in the teaching of geometry and teachers play a fundamental role in shaping this integration, it is important for teachers to receive proper training on the incorporation of technology with geometry (Coffland & Strickland, 2004). Besides, according to Lavicza, Hohenwarter, Jones, Lu, and Dawes, (2010) most teachers were not willing to use technology in their classroom if they were not trained adequately on the use and integration of the particular technology.

Lee and Hollebrands (2008) stated “Whether technology will enhance or hinder students’ learning depends on teachers’ decisions when using technology tools, decisions that are often based on knowledge gained during a teacher preparation program” (p. 326). Therefore, PD should be carefully designed to inform teachers on how to use technology in the classroom. Lee and Hollebrands explained the importance of preparing teachers to effectively use technology in the teaching of mathematics by stating the following:

> By integrally developing teachers’ understanding of mathematics, pedagogy, and technology with a focus on student thinking, we will help teachers develop a more complete picture of what is needed when teaching mathematics with technology and, in turn, be prepared to make informed decisions about appropriate uses of technology. (p. 328)

PD in geometry should focus on preparing teachers to effectively use technology in mathematics classes. One assumption that guided this study is that it is important to consider recommendations from research in the design of PD that incorporate technology as a way to enhance teachers' knowledge on the effective use of technology with geometry.

Dynamic geometry software (DGS).

"Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena" (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010, p. 74). DGS is "a computer microworld with Euclidean geometry as the embedded infrastructure" (Lopez-Real & Leung, 2006 ,p. 665), such as Cabri, Geometer's Sketchpad (GSP), Cinderella, GeoGebra, Geometry Assistant, Euclid, DrGeo, Grace, Jeometry, GeoLOG, and GeoNET. Some are available for free, such as GeoGebra, while others, such as GSP, need to be purchased.

Lee and Hollebrands (2006) synopsized the following recommendations suggested by different researchers about designing technologies to be used in teaching mathematics:

> Technology should provide tools for students that allow them to test ideas and receive feedback, engage in playful mathematical exploration, and directly manipulate objects. Underwood et al. (2005) agree with these suggestions and also recommend that technological tools should support multiple solution strategies and approaches, employ multiple representations, and make links between representations obvious. In working with interactive diagrams (applets) that are embedded in digital textbooks, Yerushalmy (2005) suggests different functions of such diagrams — which typically include multiple representations of a concept — to narrate, complement or elaborate text found in the digital textbooks. (p. 254)

All the above mentioned characteristics are present in the design of dynamic geometry environments or dynamic geometry software (DGS), which indicates that DGS are important for supporting students' learning of geometry—and consequently are important for teachers.

Prior to the use of DGS in teaching geometry, representations of geometric figures were almost limited to static figures that were either found in textbooks or drawn by students or teachers. Now, images seen on computer screens are not static anymore; "In an age when we become surprised if something on our computer screen *doesn't* move, it might seem natural, even obvious, that the static figures from Euclidean geometry should give way to dynamic ones" (Scher, 2000, p. 42).

When a shape is created in DGS, students "can investigate a large number of examples" (Forsythe 2009, p. 12). Sanchez & Sacristan (2003) found that a construction of a geometric shape in DGS could be thought of as a construction of a "general case." On the other hand, constructions using pencil-and-paper created a geometric shape that was a "particular case," that is, one representation of the general case. Therefore, using DGS in instruction enhanced students' ability to generalize. Achieving this idea of generalization within DGS heavily depends on the design of the construction activities used with DGS. When appropriately used, "DGS-use widens the range of possible activities, provides an access route to deeper reflection and more refined exploration and heuristics than in paper and pencil geometry" (Straesser, 2001, p. 332).

Erbas and Yenmez (2011) concluded from the results of a study with sixth grade students who worked on inquiry-based activities using DGS "that dynamic geometry environments were not only effective but also necessary tools for independent inquiry and investigations in geometry" (p. 2472). In their study, students' retention of the learned concepts with the use of DGS was higher than those of the control group, who did not use technology with the tasks. Furthermore, "students in the experimental group showed great interest and motivation in learning geometry compared to those in the control" (p. 2471).

A study on the use of DGS with geometry suggested that "dynamic geometry was valued for the contribution it could make to guiding students to discover mathematical properties for themselves" (Ruthven, Hennessy, & Deaney, 2008, p. 305). Moreover, using DGS is almost like playing on the computer; therefore, "users sense their own role in shaping and crafting their understanding of mathematics. Ultimately, they come to see mathematics less as a collection of rules and procedures and more as an ongoing human endeavor" (Scher, 2000, p. 48). Many researchers have found potential for DGS in fostering students' understanding of analyzing and understanding relationships between geometric concepts. Aydin and Monaghan (2011) concluded that, "The use of GeoGebra (or any dynamic geometry software) enables, even forces, actions with mathematical relationships" (p. 9).

Beyond research with K-12 students about the importance of DGS, work conducted with teachers also suggested advantages for using DGS with geometry. For example, Baki, Kosa, and Guven (2011) explained the results of a study conducted with pre-service teachers who used Cabri 3D with geometric tasks. They explained that use of Cabri 3D enhanced the spatial visualization skills of the pre-service teachers; that is, it enhanced their "ability to mentally manipulate, rotate, twist, or invert a pictorially presented stimulus object" (McGee, 1979, p. 893).

Because of the importance of DGS and of professional learning tasks for teachers, one idea of interest in this thesis concerned the types of geometric tasks recommended by literature to be used with DGS, such as those recommended by Hollebrands and Dove (2011). Construction of a dynamic figure in DGS provided the user with a chance to examine different properties and relationships resulting from the construction. Furthermore, construction under purposeful constraints can provide an opportunity to reason about the particular relationships resulting form those constraints. Also, generalization about geometric properties and relationships, which require a higher-level of reasoning, could be aided using DGS (Hollebrands & Dove, 2011).

In summary, the use of DGS environments along with "carefully designed tasks, sensitive teacher input, and a classroom environment" (Jones, 2000, p. 81) allows the users to dynamically explore many geometric shapes, helps them with discovery and generalization, enhances spatial visualization skills, and helps them understand relationships between different geometric concepts, all in a fun and investigative environment. Therefore, it is important to consider whether and how PD is preparing teachers to use DGS in geometry.

In the remaining sections, I focus on the use of DGS and summarize findings related to features of these environments that seem most promising to promote the learning of geometry. Specifically, I present findings related to three important uses of DGS: dragging, transformations, and proofs. Because of their importance for student learning of geometry, I consider these finding important for teacher PD.

Dragging in DGS.

DGS environments are described by Clements et al. (2008) as computer software that "allow students to alter original objects by moving components, such as vertices and edges, to different locations on the screen" (p. 110). These movements are called dragging, which is "the option to select a point and move it continuously in a plane" (Talman & Yerushamly, 2004, p. 95). This continuous movement is dynamically connected with other representations on the computer screen that are related to the dragged part. When an object on the screen is dragged or altered, any constructions, measurements, or transformations that were applied to the original object change accordingly and instantly. Marrades and Guttierez (2000) viewed dragging as a useful and distinctive function of DGS. Furthermore, three types of dragging behaviors were recognized by Arzarello, Olivero, Paola, and Robutti (2002), which could be experienced to formulate, investigate, and systematically test a conjecture—respectively. First, " wondering dragging" (p. 68) is random and allows users an opportunity to experience the problem. Second, "bound dragging" (p. 68) is experienced when users attempt to validate their conjecture. Third, systematic dragging is used to obtain empirical evidence that would prove or disprove a conjecture.

Dragging allowed students to promptly inspect different cases and attend to invariants in geometric shapes. The dragging capability could also reveal to the user dependency relationships between objects constructed first and objects constructed later (Hollebrands, 2007; Talmon & Yerushalmy, 2004). For example, the parent-child property special to DGS foreshadows conditional statements. If angle ABC is constructed in DGS and then an angle bisector is constructed, dragging A or C changes the magnitude of angle ABC, however congruency of the two parts of the angle remains invariant. The following conditional statement can express this dependency relationship: given angle ABC, the angle bisector splits ABC into two congruent parts. “If properties of figures are not conceived as dependent, a deductive reasoning has no meaning” (Laborde, 2000, p. 157); therefore, Jones (2000) recommended that construction activities in DGS should foster an awareness of the dependency between properties.

DGS also allows the user to construct a shape that represents a certain figure, and then drag parts of the shape to illustrate drawings of different cases. It is of importance here to note that “drawing refers to the material entity while figure refers to a theoretical object” (Laborde, 1993, p. 49). Exploring a figure in DGS by dragging while paying attention to different properties and relationships can help arrive at generalizations about geometric properties (Hollebrands & Dove, 2011).

Clements et al. (2008) and in Marrades and Guiterrez (2000) reported that requesting students to use DGS for creating figures that do not get damaged during dragging can help the students understand construction of figures. An example of such figures is a rectangle that preserves its properties as dragged. The goal of exploring using such constructions is to move students away from the limited concept of drawing and towards the general concept of figure, which also requires an understanding of definitions of the constructed figures.

Furthermore, dragging shapes that are constructed in DGS with certain constraints can help the user explore relationships and examine properties of the constructed shape that result from the constraints (Hollebrands & Dove, 2011).

The dragging function has such importance in DGS that Talman and Yerushalmy (2004) endorsed teaching geometry learners about the dragging function in DGS as an “explicit subject of discussion” (p. 116). In conclusion, dragging in DGS can be beneficial if used for the purposes of testing construction, observing invariance, exploring properties and relationships, and/or arriving at generalizations about geometric properties. And thus should be highlighted in PD.

Geometric transformations and DGS.

In the Common Core State Standards (2010), geometric transformations are one of the important topics to be taught in geometry. Students are expected to learn and understand geometric transformations as functions and to distinguish between the different transformations in terms of preservation of angles and distances. Also stated in the Mathematics Standards, “the concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation” (p. 74). Using DGS to explore reflections and rotations has greater potential in developing conceptual understanding for these transformations than using manipulatives (Clements et al. 2008).

Hollebrands (2004) concluded from working with tenth grade students who explored rotation, reflection, translation, and dilation activities while using Geometer’s Sketchpad software that “teachers should provide opportunities for students to attend to relationships among the center, pre-image, and corresponding image points" (Hollebrands, 2004, p. 211) while working on transformation tasks. The use of GSP to rotate a geometric object can foster awareness of relationships, as students need to choose the center of rotation and the object to be rotated, and then they need to decide the degree and direction of rotation. After the rotation is performed in GSP, the teacher can prompt students to pick any point on the pre-image and use the measure command to compute the distance from that point to the center. When the students then drags the pre-image point, they should notice that the distance is invariant. Furthermore, the students need to highlight the line of reflection and then select Mark Mirror from GSP's Transform menu, which can help them pay attention to the significance of the line of reflection (Hollebrands, 2004). Hollebrands (2003) also suggested that understanding transformations as functions is important for learning geometric transformations.

Glass (2001) worked with middle school students who interacted with pre-constructed images that had been transformed. Observations of students' reasoning while interacting with the pre-constructed objects provided evidence that the dynamic nature of the software promoted structural understanding of geometric transformation.

Hollebrands and Dove (2011) advocated that the use of DGS while inspecting transformations through equivalence tasks help students make connections between single transformations and compositions of transformations. In such tasks, students are asked to come up with a single transformation that is equivalent to multiple-composed transformations. In attempting to answer this question, students might use the measuring tools, add vectors, and/or reason geometrically about different transformations.

Literature-recommended DGS transformation activities include construction of transformations using DGS menu, exploring pre-constructed transformations, inspecting equivalency of transformations, and/or emphasis on transformations as functions. However, thinking of transformations as functions goes beyond the use of DGS to include reasoning about transformations in general.

Geometric proof and DGS.

"Proof is the essence of mathematics" (Laborde, 2000, p. 151). Establishing the validity of geometric ideas can be obtained using geometric proof. However, "students' difficulty with reasoning and justification in the context of proofs in geometry has been well documented" (Wares, 2007, p. 599). Therefore, researchers have investigated how to foster students' learning and understanding of proof.

Many researchers, such as de Villiers (2004), Oner (2008), and Sanchez and Sacristan (2003), argued that using carefully designed activities along with DGS fosters students' learning and understanding of geometric proof. However, the use of DGS environments in the geometry classes modified the design of the proving activity and, in turn, the incorporation of DGS in proofs affected the views on how proof should be taught. Sanchez & Sacristan (2003) suggested that students explore conjectures in DGS and then construct deductive proof to explain the empirical results obtained from DGS. Still, success in emphasizing the need for proof heavily relied on the ways the interactive software was used, rather than in the use of the software itself (Vincent, 2005).

Proof activities that can be used with DGS should be designed such that investigation in DGS does not replace the need for proof. Students might see the empirical evidence provided by DGS as an excuse for not needing to prove. Chazan (1993) explored high school students' "understandings of the similarities and differences between the measurement of examples and deductive proof" (p. 359). The findings indicated that the students in his study viewed empirical evidence as an alternative for proof. According to Marrades and Guiterrez (2000), when DGS is used, teachers need to use proving activities that foster the shift from empirical evidence to deductive reasoning.

Teachers should create activities and extension questions that encourage students' curiosity about the reasons why they thought their conjectures were true, which in turn could motivate the students to prove their conjectures (Christou, Mousoulides, Pittalis, & Pitta-Pantazi, 2004). Hadas, Hershkowitz, and Schwarz (2000), in addition to Hölzl (2001), suggested using activities purposefully designed such that students were guided to arrive at a conclusion which they discovered was in contradiction with their hypothesis after exploration in DGS. The purpose was to provoke the students' curiosity, thus creating the need for deductive proof. From using such activities with students, Hadas et al. concluded that the activities could "exemplify a design in which learning in a DG environment opens opportunities for feeling the need to prove, rather than considering proof as superfluous" (p. 148).

Wares (2007) suggested using DGS to explore conjectures that the students did not yet assert to be true. When the students know a conjecture is true, students will probably feel that providing a proof for that conjecture is "meaningless and purposeless" (p. 1). He suggested two options for creating proof activities: the first was to let the students come up with their own conjecture and then investigate it in DGS to come up with a proof or a counter example. In both cases, the conjectures were *unusual* in that they were not the types of conjectures usually found in textbooks. Because students were not sure of the truth-value of the conjecture, the function of proof in this case was discovery.

Transforming that idea of using unusual tasks to the teacher-preparation level, in her study analyzing interaction between pre-service teachers and students, Crespo (2000) recommended that the use of "unfamiliar mathematical tasks [with pre-service teachers], that is, tasks that challenged and extended pre-service teachers' own understanding of mathematics" helped pre-service teachers learn to accept different answers from their students instead of expecting one correct way of solving a mathematical problem.

In addition to confirmation of mathematical facts, "Proof has many other important functions within mathematics, which in some situations are of far greater importance than that of mere verification" (de Villiers, 2004, p. 703). Christou et al. (2004), de Villiers, (2004), and Wares, (2007) agreed on a set of functions of mathematical proof. Those functions are validation, communication, "explanation, discovery, intellectual challenge and systemization" (Christou et al., 2004, p. 217). De Villiers (1999) explained that after students confirm with repeated investigations in DGS that a certain conjecture held, proof as a means for verification is most likely not persuasive for them. On the other hand, if asked to explain the reasoning behind their conclusions, constructing a proof might become necessary for the students as a method for explanation. Oner (2008) illustrated an example on proof for discovery where students were asked to prove or disprove a conjecture of which students did not know its truth-value. Proof for systemization refers to the idea that proofs involve organizing and putting together postulates, definitions, and theorems of Euclidean geometry.

When teachers view proof in light of the above listed functions, particularly the discovery and explanation functions, using DGS in the proving activity might foster students' understanding. However, in high school geometry, as Hölzl (2001) stated, "Often, it seems to me, DGS is used only in a *verifying* manner: that is, learners are just supposed to vary geometric configurations and confirm empirically more or less explicitly stated facts" (p. 65).

Researchers have investigated pre-service teachers' beliefs about incorporating DGS in proving activities. Jiang (2002) explained the results of a study in which two pre-service teachers formulated conjectures and then explored the conjecture in DGS to obtain insight for proof. The results indicated that both teachers improved in their Van Hiele level of thinking and their mathematical reasoning and proof aptitudes improved due to the use of DGS in exploring proof activities. Further, their beliefs about how proof should be taught changed. For instance, at the end of the study, regarding how students learn proof, one of the pre-service teachers stated, "knowing the vocabulary is not critical when trying to find a solution or an explanation/proof. What is important is the process of problem solving and conceptual understanding" (p. 728). However, when asked at the beginning of the study, the same teacher thought that it would be difficult to expect students to explore the geometric object in DGS before learning the name of the object they were exploring.

Michael de Villiers (2004) provided examples of DGS-based activities that could be used to introduce new concepts to student teachers and engage them in exploration and discovery. The activities were developed and revised by educators over a period of 25 years. One of the main goals of the activities was “developing understanding of varied meanings or roles of proof at the different levels (e.g. explanation, discovery, systematization, etc.)” (p. 710). The results obtained from using the activities with high school students and pre-service teachers indicated that using the activities contributed to the students’ understanding of geometric proof.

As indicated in this section, research signified that DGS had great potential in fostering students’ understanding of geometric proof. However, “it is the context in which the computer is a part of the teaching and learning arrangement that strongly influences the ways in which the need for proof does-or does not- arise” (Hölzl, 2001, p. 65). It is important to value proof as a tool for explaining empirical evidence, discovering new ideas, providing intellectual challenge, and organizing the geometric concepts. Recommendation from literature on the design of geometric proof activities used with DGS included exploring conjectures, including non-traditional conjectures in DGS and ending up with the construction of proof, using DGS to come up with conjectures and then prove or disprove them, and inquiry activities in DGS intentionally designed to confront users with contradictions or uncertainties.

Summary.

The research discussed in this review indicated that focusing on student learning is an important component of effective PD; therefore, knowledge about student learning should relate to what teachers learn in PD. Moreover, PD in mathematics should focus on preparing teachers to effectively use technology in the mathematics classes. One way to do so is by incorporating recommendations from research on the effective use of technology. Particularly, the use of DGS environments, along with carefully designed tasks, can enhance student understanding of geometric concepts, such as geometric transformations and proof. Many recent studies in the use of DGS with geometry found benefits from using DGS in teaching the concepts of geometric transformation and proof. Moreover, dragging in DGS is considered a valuable function of the technology.

Creating the Codebook

The research goal of interest for this study was to investigate the design (format and content) of professional development programs and teacher education courses in geometry as reported in published papers. The following five research questions were of interest in this study:

1. To what extent are the recommendations for effective professional development present in the design of professional development interventions and teacher education courses that focus on geometry as reported in published papers?
2. Is there a trend in the types of goals for geometry professional development interventions or geometry teacher education courses as reported in published

papers?

3. In what ways do professional development interventions or teacher education courses that focus on geometry integrate technology either as a learning tool for teachers or as a tool to be used with students as reported in published papers?
4. When technology is reported to be integrated, what types of technology are used? In particular, what percentage of the professional development interventions or teacher education courses that focus on geometry and integrate technology use DGS?
5. As reported in published papers, in what ways are recommendations from research about the use of DGS in instruction incorporated in professional development interventions or teacher education courses that focus on geometry and integrate DGS?

A coding book was created based on the literature review in an attempt to answer the research questions. (See Appendix A for a copy of the codebook, which includes description of each of the codebook items.) For each of the research questions, I used recommendations that emerged from the literature review to specify what should be coded in order to answer that particular question. Before starting to code all the collected manuscripts, a random sample of 20 papers was examined to determine the feasibility of coding the papers using the codebook. The first question concerned the features of effectiveness of PD. For this question, based on the review of the literature, I was interested in information related to the following aspects: span of the program, number of contact hours, active engagement of teachers in the PD, connections to practice, and what the goals of the PD were.

After examining this pilot sample of 20 papers, I determined that from the variables above, the active engagement factor would not be easily determined because most of the papers did not include sufficient details to determine what the teachers were doing throughout the PD hours. At that point, a decision was made to consider that active engagement occurred when there was an indication in the coded paper that participants of the PD were working on any kind of hands-on activity or discussion during the PD. Active engagement was not present in the PD if the author stated that the teachers were lectured during PD.

For the connections-to-practice variable, Smith's (2001) description of how a PD was considered to be connected to practice was adopted. If the author of the coded manuscript reported that teachers were involved in any activities in actual classrooms or worked with artifacts from actual classrooms, such as sample student work, videos, or curriculum materials as part of the PD, then connections to practice was considered an important part of the PD. On the other hand, when the author explicitly stated that the PD content was connected to practice, however none of the activities were connected to practice (that is, all examples explored were theoretical), then I considered that connections to practice to be only one among many activities.

The second research question addressed the goals of the programs, and the variables of interests for this question were whether the goals related to increasing teachers' content knowledge, pedagogical content knowledge, knowledge about student learning, alignment of curriculum materials, awareness of district or school pacing guides, and/or the use of technology. The third research question was less general than the first two in that it addressed the use of technology in geometry PD. Thus, for this question, the variable of interest was only the use of technology in the PD. The last question aimed at obtaining insights on the use of DGS in geometry PD. The list of recommendations on the use of DGS with geometry that emerged from the literature was included within the codebook in an attempt to answer this research question.

Most of the recommendations on the use of DGS resulting from the literature review were from studies aimed at integrating DGS in teaching geometry to K-12 students and were not necessarily intended for teacher education. However, I chose to combine those recommendations along with the ones that were intended for teacher education to develop the codebook. The underlying assumption for this choice was that the ways in which teachers teach their students certain mathematical concepts reflect the ways in which the teachers themselves learned those concepts. Further, I considered that the types of activities and settings in which teachers learn should be similar to those that teachers are expected to provide for their students (Borasi, 1999; Cobb, Wood, & Yackel, 1990; Langham, Sundberg, and Goodman, 2006; Steffe, 1990).

The codebook was organized around structure, goals, and content of PD. It is comprised of four categories: the first includes general information about the coded paper; the second is on the effectiveness of the PD and on the goals reported by the author; the third category, which is on the use of technology with geometry, was created to gain insight on the use of technology in the PD; the last category includes a list of recommendation by the literature on the use of DGS with geometry, which were inferred from the literature.

Recommendations and suggestions from the literature review on the effectiveness of PD and on the use of technology with geometry suggested the problem formulation that resulted in translating the research questions of this study into a set of variables defined in the codebook. In the next section, I explained how data were collected, evaluated, and analyzed.

CHAPTER 3: METHODOLOGY

In research synthesis, the goal of data collection is to search for relevant studies from varied databases using appropriate search terms; the goal of data evaluation is to determine which of the collected references or studies should be included or dropped from the list of references to be synthesized. Additionally, the goal of data analysis is to synopsize and combine research results (Cooper, 1998). In this study, Cooper's definitions of the processes for data collection, evaluation, and analysis were adapted as illustrated in this chapter and in the findings chapter.

Process for Data Collection

The data collected to be synthesized in this study is comprised of journal articles, reports, and books, all of which focused on geometry professional development programs and teacher education geometry settings. They were collected using Academic Search Premiere, Eric, PsychInfo, and Social Science Index databases. The search terms that were used within the above list of databases were as follows:

- The search terms in Eric, Academic Search Premiere, and PsychInfo databases were: ("Professional Development" OR "Professional Training" OR "Staff Development" OR "Teacher Education" OR "Faculty Development" OR "Teacher Workshops" OR "Institutes Training Programs") and ("Mathematics" OR "Mathematics Education" OR "Mathematics Instruction" OR "Mathematics Teachers")

  and "geometry" and 1992-2011 and English.

- Social Sciences Index: (“Professional Development” OR “Professional Training” OR “Staff Development” OR "Teacher Education" OR “Faculty Development” OR “Teacher Workshops” OR “Institutes Training Programs”) in Topic and (“Mathematics” OR “Mathematics Education” OR “Mathematics Instruction” OR “Mathematics Teachers”) in Topic and “geometry” in topic

This search was restricted to papers published between 1992 and 2011 inclusively for the purposes of including the most recent works, and at the same time keeping the study manageable and the work well-timed. The choice not to include research published before 1992 was because “the first Handbook for Research on Mathematics Teaching and Learning (Grouws, 1992) contained no chapter devoted to teacher education” (p. 211, Sztajn, 2011). Thus, as noted by Sztajn, I considered that there was limited amount of published research on mathematics PD programs and teacher education courses prior to 1992.

A total of 248 references returned from this search. The breakdown in terms of types of publication was as follows: 168 journal articles, 26 reports, and 54 whole books.

The next stage of the data collection was the pre-screening. During the pre-screening, articles are selected for possible further analysis. The abstracts of the 248 collected works were inspected based on the following criteria:

1. The written work was not an edited book or a collection of written work by multiple authors. Such resources were eliminated because in a collection of work it could not be warranted that all chapters were related to this study.

2. The written work described a professional development for in-service mathematics teachers, or a workshop, course, or any form of setting designed for pre-service mathematics teacher education.
3. The strand of mathematics that was taught to the teachers was geometry, or geometry was one of the strands taught.

When the abstract of the pre-screened paper provided evidence that all of the above criteria were present in the paper, that paper was included in the list of papers to be screened. Otherwise, the paper was deemed irrelevant to this study and was dropped from the list. When any of the above criteria could not be determined through the abstract, the article was kept to make sure that no false assumption was made. A total of 92 written works passed the prescreening stage, including 77 journal articles, 11 reports, and 4 books.

To complete the data collection process, the collection process of the manuscripts of the 92 started through online downloads; if not found through online resources, the hard copies available at North Carolina State University libraries were borrowed or scanned. The manuscript of one of the papers that passed the prescreening stage could not be found and was eliminated. Also, two of the full texts were not in English and no English translation copy was found; therefore, their manuscript was not screened. At that point, a total of 89 manuscripts constituted the project's data set.

Process for Data Evaluation

After the full texts of the papers that passed the pre-screening process were collected, they were screened to determine relevancy to the study. This was necessary because some abstracts were either very short or the description provided in the abstract was not sufficient to determine relevancy. The following questions were used in the screening stage to determine relevancy of the full texts to the study:

1. Did the written work describe a professional development for in-service mathematics teachers, or a workshop, course, or any form of setting designed for pre-service mathematics teacher education?
2. Was the strand of mathematics taught to the teachers geometry? Or was geometry one of many strands being taught?

Similar to the pre-screening process, once an answer to any of the screening questions was found to be *no*, the paper was considered irrelevant. Also, for a manuscript that discussed a program on mathematics without any indication that geometry was one of the strands addressed, the answer to the 2nd screening question was *no.* This decision was made assuming that if the author did not include any remark about geometry, then probably little focus, if any, was on the content of geometry during the contact hours with the program participants. Of the 89 papers that were screened, 64 papers (57 journal articles, 5 reports, and 2 book sections) passed the screening stage and were subsequently coded using the codebook found in appendix A. Also, as part of the data evaluation process, the list of research articles used in writing this study's literature review was compared to the list of 64 papers that passed the pre-screening. It was found that two of the papers that were collected and passed the pre-screening were also cited in the literature review.

Process for Data Analysis

The codebook was created based on what was concluded from the literature review discussed in chapter 2. Each of the 4 categories of the codebook includes a number of items that allowed for multiple-choice answers. For some of the items, an option of *not reported* was included in the answers since it was anticipated that authors would not report on every aspect of the PD. However, not reporting information in a paper did not necessarily mean that the variable being coded for did not apply to the PD. After the codebook was finalized, it was entered into Qualtrics as an online survey. Qualtrics is an online-research software that allows the user to do online data collection and analysis. Qualtrics was chosen as the data analysis software because of its analysis capability and ease of use. Each of the papers that passed the screening stage was considered a subject that took the survey.

After all the data were coded, Qualtrics software provided descriptive statistical analysis of the data and the results were exported into Excel and word documents. Also, Excel was used for some of the analysis. Correlation analyses were not conducted because the data were mostly categorical. Additionally, the Qualtrics version used did not facilitate pairing the results from different codebook items, at least not directly, which made any study of correlations complex. The tables that do relate two variables from the codebook were not created directly within Qualtrics. Because the data obtained by coding the papers were a mix of categorical and numerical, a descriptive analysis was prepared including frequency counts and percentages for the various categories. These findings are discussed in the next section.

CHAPTER 4: FINDINGS

Results from the Data Analysis and Interpretation

The results of coding the papers are summarized in this chapter. Data analysis took place within a descriptive framework. The data shown here is a summarization of the coding results obtained from the 64 papers in my data set. The results are organized in terms of their relevance to each of the five research questions. From looking at the data, there seemed to be trends in the coding results obtained within each of the two groups of papers focused on in-service teachers and those focused on pre-service teachers. From now on in this paper, I will use the phrase in-service papers to refer to the collection of coded papers that were on professional development interventions for in-service teachers and pre-service papers to refer to papers that were on education settings for pre-service teachers.

In terms of the general information obtained, 45% of the coded papers were papers about in-service teacher education; 50% were papers about pre-service teachers; and only 5% (3 papers) were on teacher education settings for both in-service and pre-service teachers. When coding for the grade-band of the content of the PD, 66% of the papers did not report on this variable. A total of 22 papers (34%) reported the grade-band of the content of the PD. Table 1 provides a summary of these percentages of papers that mentioned offering geometry PD about content topics associated to a particular grade level or grade band. Some of the PD included content topics that were associated with more than one grade band as shown in Table 1.

Table 1

Grade-level of the Content of PD, Number of Papers, and Percentage of Papers

Grade level	Number of papers reporting grade level	% of 22 papers
K-5	4	18%
6-8	3	14%
9-12	8	36%
K-8	3	14%
6-12	2	9%
K-12	1	5%
5	1	5%

Looking at the frequencies presented in the Table 1, there were about twice as many PD that addressed grades 9-12 than any other category. However, when looking at all PD that addressed each grade band, 11 PD addressed grades 9-12; 9 PD addressed 6-8, and 8 PD addressed K-5. These frequencies for each grade-band are relatively comparable with some dominance for 9-12 grade band. This is correspondent with my expectations since geometry is taught in grades K through 12. On the other hand, it is noteworthy that some PD reported addressing a content focus for K-8, 6-12, and K-12. Addressing such a wide range of content topics to address all these grade levels might imply that very little focus was on deep mathematical thinking of the concepts addressed, especially that many of the PD had short duration and/or less-than-recommended contact hours.

Question 1: To what extent are the recommendations for effective professional development present in the design of professional development interventions and teacher education courses that focus on geometry as reported in published papers?

The variables of interest for this question are the following: the time span of the intervention, duration (measured by the total contact hours), the level of teacher engagement in activities during the PD, and connections of activities to teaching practice. Authors reported on all of these variables in only 8% of the coded papers: 5 papers. Table 2 provides a summary of the coding for the effective-PD variables reported in those 5 papers.

Table 2

Effective PD Variables for the 5 Papers that Reported on the Variables

Contact hours	Time span	Active engagement	Connections to practice
100	10 days	Participants Actively engaged	Connections were important part of activities
65	2 weeks	Participants Actively engaged	Connections were important part of activities
60	3 years	Participants Actively engaged	Connections were important part of activities
34	11 weeks	Participants Actively engaged	Connections were important part of activities
26	4 months	Participants Actively engaged	Connections were important part of activities

The papers that reported on all four variables of effective PD seem similar in that they all reported long contact hours, teachers active engagement with the PD activities, and activities connected to practice. However, the time span of the PD varied widely among the five papers. All of the 5 papers were about work with in-service teachers.

Of the 28 papers that reported the time span of the PD, 43% reported a span of 1 year or more, 7% reported a PD time span of 6 months, and the remaining 50% reported time spans of 4 months or less. In terms of hours, only 25% (16 papers) reported the total contact hours with teachers. Of those, 13% had 80 or more contact hours, 25% had contact hours of 40-80, and 62% had less than 40 contact hours. More than half of the PD for which the number of contact hours was reported had less than the minimum amount of time considered to be necessary for effective PD.

According to literature, associating adequate contact PD hours over a relatively long period of time has shown to be fruitful in terms of student achievement. In light of that, the total contact hours and the time span of PD were paired for the 11 papers that reported on each of the two variables as illustrated in Figure 1. Only one of the 11 papers was a pre-service paper, and the remaining 10 were all in-service papers.

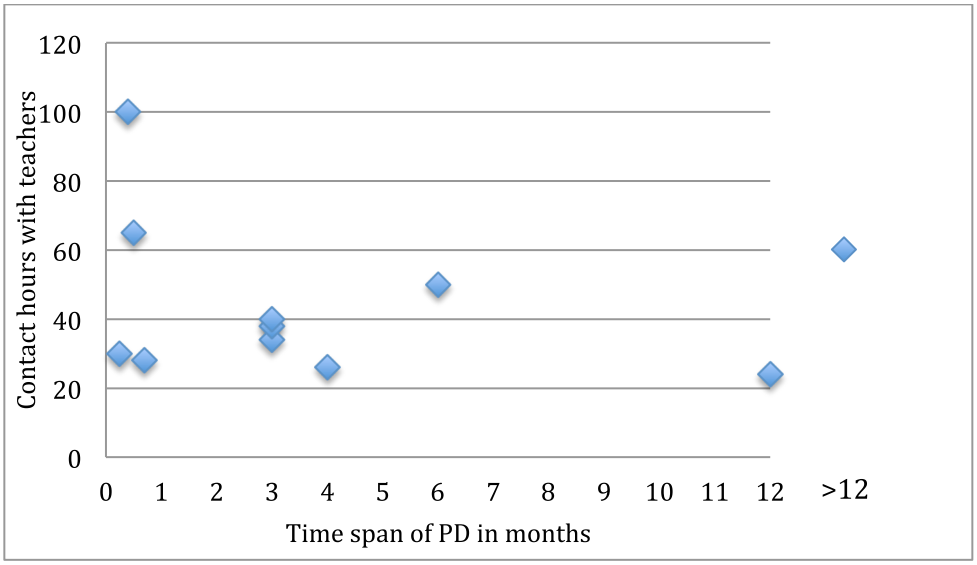

Figure 1. Time span of PD and total contact hours with the participants.

Figure 1 shows that very few of the coded papers reported on both of the time-related variables that are deemed important and were within the parameters recommended by literature– having contact hours between 30 and 100 and time spans from 6 to 12 months. The two variables of time span and contact hours of PD were as recommended by the literature in only 18% of the papers that reported on both variables, which is relatively small if linked to the literature-documented value of longer duration and wider time span of PD. Most of the PD that reported on both variables did include a substantial amount of hours, but these hours were often packed in less than a semester of work.

A total of 52 of the coded papers reported on the type of engagement of the PD participants. Fifty-one of those papers reported that teachers were actively engaged in working on activities during the PD hours. In terms of connections of activities to practice, only in 17 papers (27 % of the entire 64 coded papers) were there indications that connections to practice were important parts of the PD activities. Most of the 17 papers were in-service papers, which is expected given that it is probably more feasible for in-service teachers than for pre-service teachers to use activities that are connected to practice because in-service teachers have their own experiences in classrooms, which they can relate to PD activities.

Overall, in relation to effective PD, most papers did not provide information on all variables considered important for the effectiveness of the PD. Type of engagement was the variable most often reported, with papers indicating that teachers were actively engaged during the PD (which in my coding means they were not only lectured). Span was reported in about half of the paper, and it was about evenly split between PD that lasted more or less than 6 months. Papers that referred to in-service teachers were more likely to discuss connections to practice.

Question 2: Is there a trend in the types of goals for geometry professional development interventions or geometry teacher education courses as reported in published papers?

I initially conjectured that most, if not all, of the papers would provide indications of what the goals of PD were. However, the results of coding showed that about a quarter of the papers included no information on the goals of the PD. Of the 47 papers that reported the goals of the PD, subject matter and pedagogical content knowledge were the most frequent goals – subject matter knowledge was reported as a goal in 68% of the 47 papers and pedagogical content knowledge was reported in 66% of them. The frequency of reporting of each goal is summarized in Table 3, with some papers reporting on more than one goal.

Table 3

Goals of PD and Number of Papers in which the Goal was Reported

Goal of PD	Number of papers
Subject matter knowledge or mathematics content	32
Pedagogical content knowledge in mathematics	31
Student learning	11
Curricular Materials	3
District or School Pacing Guides	0
Technology	14
Other	0

Although the literature suggested the importance of addressing curriculum materials and district guides as goals for PD, there are few PDs in geometry that addressed these goals; this could be related to issues concerning either the participants of the PD or the PD providers. Perhaps teachers do not express interest in taking such PD, or designers did not value such goals as much as they valued the other goals listed in Table 3.

Interestingly, there were differences across in-service papers and pre-service papers in terms of the types of goals reported. While subject matter knowledge was reported as a goal for PD in most of the pre-service papers, the majority of the in-service papers reported pedagogical knowledge as the goal of the program. This trend indicates that perhaps the field is focusing on offering prospective teachers opportunities to learn more geometry, considering that they will need this knowledge to teach. Then later, there might be a perception that practicing teachers know the content and need to learn about how to better teach it, or how to know geometry for teaching.

Question 3: In what ways do professional development interventions or teacher education courses that focus on geometry integrate technology either as a learning tool for teachers or as a tool to be used with students as reported in published papers?

The results showed that about half of the papers about geometry PD did not report any incorporation of technology in the PD. The breakout of the percentages of papers (out of total of 64 paper coded) for each of the ways technology was incorporated, if any, are represented in Figure 2. Thirty-five papers reported that technology was used during the PD and in most of these papers, technology was incorporated as tool to support teacher learning in the PD.

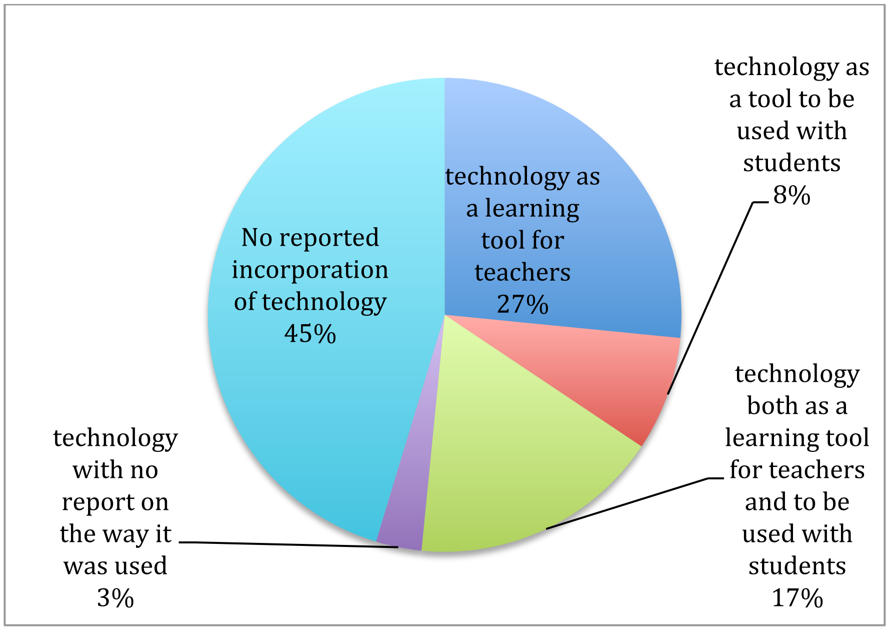

*Figure 2***.** Incorporation of Technology and Percentages of Papers that Reported on the Way Technology was Incorporated.

Having about half of the papers reporting using technology as a learning tool for teachers implies that PD designers find value for the use of technology in teachers' learning of geometry. Most of those that reported using technology only as a learning tool for teachers were papers about pre-service education. This is not surprising if related the findings displayed bellow in table 6, that shows that pre-service courses had a stronger emphasis on mathematical content knowledge than in-service PD did. Thus in these courses, technology seems to be an important tool for teachers to learn geometry content.

Question 4: When technology is reported to be integrated, what types of technology are used? In particular, what percentage of professional development interventions or teacher education courses that focus on geometry and integrate technology use DGS?

A majority of 89% of the geometry PD that used technology used DGS, indicating the way in which DGS is becoming a fundamental part of this type of PD. The remaining papers reported using other types of technologies, such as photo editing software. In the papers that reported using DGS, participants' prior experience with the technology was mostly not reported; only 7 out of the 31 papers that reported using DGS provided information on the teachers' experiences with the technology used in the PD. Four of those papers reported that most or all participants had no prior experience with DGS, which is surprising given the growing spread of the use of DGS in mathematics education.

Question 5: As reported in published papers, in what ways are recommendations from research about the use of DGS in instruction incorporated in professional development interventions or teacher education courses that focus on geometry and integrate DGS?

Twenty-two of the papers that reported incorporating DGS in PD activities discussed attention the dragging function of DGS. In the 22 papers, dragging was mentioned as being used for the purpose of exploring relationships and properties in 75% of the papers; for the purpose of observing invariants in 58%; for the purpose of arriving at generalization in 33%; and for the purpose of testing invariants in 17%.

On the geometric topic addressed with DGS, a total of 28 papers that reported using DGS included information on the geometric topic addressed in PD. Geometric proof was the most frequently reported topic in the geometry PD that used DGS, and the second most frequently reported topic was geometric transformations. These results are not surprising given the importance of proof and transformations found in the literature. Table 4 provides a summary on the geometric topics addressed in papers that reported using DGS. Some of the other topics that were reported as being addressed with DGS were modeling an isomorphism, spatial visualization, tessellations, solid geometry, area, and construction of 2-D geometric shapes.

Table 4

Geometric Topic Addressed in PD and the Associated Number of Papers, and Percentages

Geometric topic addressed	Number of papers	percent of the (31) papers reported using DGS
Geometric proof addressed with DGS.	12[b]	39%
Geometric transformations addressed with DGS.	8	26%
Other[a]	14	45%

[a]Those papers either did not report the geometric topic addressed with DGS or the topic addressed with DGS was other than proof or transformations.

[b]Three of those papers also reported using DGS with the topic of transformations with the participants.

Most of the PD that used DGS with the topic of geometric transformations (63% of them) reported using activities that asked teachers to transform shapes using DGS menu, which was suggested by Hollebrands (2004) to have potential in fostering an awareness of geometric relationships. Additionally, 13% reported using activities that involved exploring pre-constructed transformations. Likewise, 13% reported using activities that involved emphasis on transformations as functions. In 25% of the 8 papers that reported using DGS with the topic of transformation, the nature of the geometric transformation activities could not be determined based on the information provided in the papers.

One of the issues of interest in this study was to determine if PD that addressed the topic of geometric proof used proof activities that highlight functions of proof beyond validation or confirmation as recommended by the literature. However, 5 of the 12 papers that reported the use of DGS to address geometric proof did not include any indication about the function of proof in the PD. The most frequently reported function of proof was the explanation function. Table 5 provides a summary of the reported functions and the number of papers in which the functions were reported. The table shows that the majority of PD that addressed proof with DGS addressed functions of proof beyond validation or confirmation.

Table 5

Reported Functions of Proof and Number of Papers

Reported function of proof	Number of papers
Explanation, discovery, and systemization	1
Explanation and systemization	2
Explanation and discovery	1
Discovery and systemization	1
Explanation	1
Validation or confirmation	1

In terms of the characteristics of proof activities that used DGS, 8 of the 12 papers reported the presence of one or more of the characteristics of DGS-based proof activities listed in the codebook as part of the design of PD activities. All of the 8 papers reported the use of activities that started with exploration in DGS and ended up with construction of proof. Two of those 8 papers also reported that DGS was used to come up with conjectures and then prove or disprove the conjectures. None of the remaining characteristics such as, asking participants to explore non-traditional conjectures using DGS or using DGS with proof activities that confront participants with contradictions or uncertainties, were mentioned in the papers.

Generally, in terms of the use of DGS in PD activities, geometric proof and transformations were the most frequently reported topics to be addressed in PD with DGS. In terms of proof, the explanation function was the most frequently reported function of proof, and only half of the characteristics of DGS-based proof activities listed in the codebook were mentioned in the papers. Attention to dragging was found in less than half of the papers that reported using DGS in PD.

Further results.

Although not related to the research questions initially posed for this study, insights were also gained on the results obtained across the two groups of papers combined in this research: papers about in-service professional development interventions and papers on pre-service settings. The coded papers were split into two groups: the in-service only papers (a total of 29 papers) and the pre-service only papers (a total of 32 papers). The percentages of coded papers that included evidence about the different coding variables were computed for each of the two groups of papers. A summary of the percentages is found in Table 6.

Table 6

Summary of Percentage of Papers that Reported on the Different Variables for the Two Groups of In-service and Pre-service Papers

Variable	% of in-service papers reported on the variable	%of pre-service papers reported on the variable
Grade-Band for content of PD	52% (of 29 papers)	19% (of 32 papers)
Span of time of PD	69% (of 29)	19% (of 32)
Total contact hours	48% (of 29)	3% (of 32)
Active engagement	79% (of 29)	84% (of 32)
Connections to practice	62% (of 29)	16% (of 32)
Reporting of goals	86% (of 29)	65% (of 32)
Incorporation of technology	62% (of 29)	50% (of 32)
Type of technology[a]	100% (of 18)	100% (of 16)
Teachers' experience with DGS[b]	25% (of 16)	14% (of 14)
Report on the dragging function[b]	21% (of 16)	43% (of 14)
Geometric topic addressed[b]	81% (of 16)	100% (of 14)
Type of transformation activities[c]	60% (of 5)	100% (of 2)
Function of proof[c]	50% (of 4)	62% (of 8)
Characteristics of proof activities[c]	50% (of 4)	75% (of 8)

Note. Papers on settings for both in-service and pre-service teachers were not included in this table.

[a]This variable was coded for only the papers that reported using technology, which were 18 in-service papers and 16 pre-service papers.

[a]This was coded only for papers that reported using DGS, which were 16 in-service papers and 14 pre-service papers.

[c]This was coded for papers that reported geometric transformation as the topic addressed with DGS, which were 5 in-service papers and 2 pre-service papers.
[c]This was coded for papers that reported proof as the topic addressed with DGS, which were 4 in-service papers and 8 pre-service papers.

Table 6 shows that the main differences between the two groups seem to be about time span, contact hours, and connections to practice, which is expected given that the 3 variables came from literature-recommendations for in-service programs. The upper part of Table 6 comprises of variables related to PD format, whereas the lower part contains variables related to PD content. The percentages in the upper part of the table are generally higher for in-service papers than for pre-service papers. The opposite is true for the lower part. Overall, while the in-service papers seemed to provide more details on the format of the PD, pre-service papers provided more details on the content addressed during PD than in-service papers did.

Very few of the papers reported conducting teacher-education settings in which participants comprised of both in-service and pre-service mathematics teachers. This indicates that designers and providers of geometry PD do not often get in-service and pre-service teachers together in the same education setting. Perhaps, it is less feasible to do so than to separate the two groups. Two of the papers that reported on PD that had both pre-service and in-service teachers indicated the goals of PD and only one incorporated DGS, in which the content addressed with DGS was geometric transformations. Two of those 3 papers had a time-span of one year. On the other hand, one reported the contact hours, which were only 6 hours. The results obtained from those papers were not very informative; they represented the variation present in the larger set of the coded papers. Also, since the sample is too small, no generalization about PD for both in-service and pre-service teachers can be made based on the obtained results from those 3 papers.

I was also interested in learning if, with time, the descriptions or designs of geometry PD changed, so I sorted the coded papers based on the publication date. However, there seemed to be a wide variation that did not seem to be correlated to the publication dates of the papers. The findings presented in this section are discussed in the next chapter, which is about what has been learned so far from this study, where we could go next, and the significance of the study.

CHAPTER 5: DISCUSSION

This study focused on a description of the field of PD for K-12 mathematics teachers in the content of geometry as reported in published papers retrieved through Academic Search Premier, Eric, Social Science Index, and PsychInfo databases from 1992 to 2011. Generally, there was a wide variation among descriptions of geometry PD in published reports. One of the most prevalent differences observed based on the results of this study was across the two groups of in-service papers and pre-service papers. Although the initial goals of this synthesis were focused on describing existing designs of geometry PD, the results revealed that many of the PD features that researchers discuss and perhaps hope to investigate when reading about an implemented PD were not mentioned in many of the coded papers.

The data obtained from coding substantiated that most papers on PD did not report on the elements that the literature suggested contributed to the effectiveness of PD. A very small percentage of the coded papers reported on all of the effective-PD variables used in the codebook; those were all in-service papers. Perhaps the fact that these were all in-service papers was because the literature, from which the recommendations on effective PD were taken from, was mostly focused on in-service teacher education. Although there were indications that what was important for in-service teachers was also important for pre-service teachers, PD designer and/or providers did not seem to agree that the same features should be reported and perhaps not even taken into consideration in the designs of the two types of teacher education experiences (in-service education and pre-service education). The contacthour and time-span variables seemed to be related to in-service PD more than they did to pre-service PD, perhaps because most of the work done with pre-service teachers is done within a college course setting, which usually has preset span and duration.

Since the literature suggested value for the use of technology with geometry in education, one of the goals of this study was to investigate the degree to which technology was used in geometry PD. Despite the calls for the importance of technology in the teaching and learning of geometry, I found that the use of technology is not yet a pervasive practice in PD. On the other hand, DGS was the type of technology reported to be incorporated in the majority of the geometry PD that were reported to incorporate technology. This indicated the awareness of PD designers of the potentials of using DGS in the learning and teaching of geometry. Still, it was surprising that teachers' prior experience with DGS was rarely reported in the papers that reported using DGS during PD, and when reported, most teachers had no prior experience with the technology.

The results obtained in this study on the geometric topics addressed in PD with DGS matched what was conjectured based on the literature. The literature review in this study indicated a prominence for the use of DGS with the topics of geometric proof and geometric transformations. Correspondingly, it was found from analyzing the coded data that geometric proof and geometric transformations were the most frequently reported geometric topics to be addressed in PD with DGS.

A set of characteristics about designs of geometric transformation and proof activities used with DGS were compiled in this study based on recommendations from literature. Very few of the compiled characteristics were present in the inspected PD reports. However, that did not necessarily imply that the PD, which did not use such activities or did not report on the nature of the activities, did not consider recommendations from research. Recommendations from research on the use of DGS with the topics of geometric proof and geometric transformations in PD might have been considered in the designs of activities, but in ways that have not been captured during the literature review done in this study. In terms of the function of proof, the results from analyzing the papers on geometry PD were promising in that the majority of PD, which addressed proof with DGS, did indeed address functions of proof beyond validation or confirmation. This indicates that the developers of those PD appreciate the value of understanding and acknowledging the different functions of geometric proof, particularly, when DGS was used.

A noteworthy observation on differences between the two groups of in-service and pre-service papers was that, while the in-service papers seemed to provide more details on the format of the PD, pre-service papers provided more details on the math content addressed during PD than in-service papers did. This could be related to differences in the audience for each group of papers or to other features, such as differences in nature of studies associated with each group.

Furthermore, it was observed that each of the coded papers reported on only one PD. This implied the limited amount of comparative studies in the documented field of geometry PD.

Limitations

The sample of coded papers in this study does not represent the entire field of geometry PD. It is biased in that it represents only those geometry PD that were reported in the papers published in the particular journals that are part of the databases that were used to conduct the search for this study: Academic Search Premiere, Eric, PsychInfo, and Social Science Index databases. I acknowledge that there are vast amounts of PD that were implemented with teachers, but no papers were written about them. Also, I recognize that there are other PD for which information was reported, but these papers were published in venues that were not captured through the databases used for data collection in this study. Also, I acknowledge that the search terms used in the collection process might have not captured all the relevant documented papers: however, I accepted that fact to maintain the study within achievable and manageable limits.

The list of recommendations on the use of DGS that was compiled in this study was based on the literature reviewed in this thesis, which was a small proportion of all the literature that was written on the use of DGS with geometry. Although that list was compiled based on work done by experts in the field of K-12 geometry instruction, that list could not have been an inclusive list of all documented recommendations.

Generally, many characteristics of PD designs that I considered of interest were not reported in many of the coded papers. I have two hypotheses regarding this; perhaps the characteristics were present in the PD, but the authors did not report on them. However, it is also possible that these characteristics might have not been considered in the design of the

PD and therefore the author did not mention them. Results obtained in this study on the description of PD are limited by the fact that relatively high percentages of papers did not report on some of the variables being measured, such as total contact hours and teachers' prior experience with the technology. Therefore, in the analysis, I focused only on the papers that reported on the variable being inspected. For example, when I found no mention of a certain variable in a coded paper, the coding result for that particular variable was *not reported* for the particular inspected PD. Moreover, although I did not draw results in terms of the particular variable on that portion of the papers, the variable of interest might have not been present in the design of the PD and, hence, the author did not report it. My choice to state that the decision could not be determined about the coded variable in such cases was for credibility reasons.

Another limitation of this study is that even when the coded paper included a report on a variable, the coder's interpretation of the meaning of the text might have been different from the authors' intended meaning. Additionally, due to different writing styles between the authors of the different coded papers, consistency in coding certain variables might have been hindered across the coded papers. However, this limitation was minimized as much as possible through careful attention to explicit indications from the coded papers about the coded variables.

Although it is important to acknowledge these limitations, there is still much to be learned from this research synthesis as illustrated the previous section. This research synthesis adds to research in that it provides a description and evaluation of the type of data provided by a set of published journal articles, reports, and book chapters on geometry professional development models and teacher education settings.

Implications

Since the literature indicated that many teachers used computers in the geometry class for the purposes of drill and practice, which is not the most efficient way of using the technology (Clements et al., 2008), and since the results of this study indicate that many geometry PD were not reported to have trained teachers in the use of technology with geometry, PD developers should focus on informing mathematics teachers on more efficient ways of using technology with geometry to take advantage of the technological tools, including DGS, and to support teachers' learning of what research indicates about how they can take advantage of technology to improve students' understanding of geometry.

Further examination is needed to obtain more insight on the incorporation of research recommendations in the designs of geometry PD. I am interested in conducting similar research syntheses that describes PD designs in other mathematical topics, and then compare results across different mathematical topics to investigate any differences or similarities. It would also be interesting to conduct a study in which factors that might have been correlated to the patterns obtained in this study are examined.

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APPENDIX

Appendix A – The Codebook.

Category 1: General information

Item 1: Reference in APA format:____________________________

Item 2: In-service or pre-service

Explanation: Indicate whether the model was designed for pre-service teachers, in-service teachers, or both.

Coding decision: Choose one

1) Professional development program for in-service teachers
2) Teacher education course for pre-service teachers
3) Program with participation of both in-service and pre-service teachers.

Item 3: Grade-band for the content of PD

Explanation: Indicate the grade band of the content addressed in the PD.

Coding decision: Choose all that apply

1) K; 2) 1^{st}; 3) 4^{th}; 4) 5^{th}; 5) 6^{th}; 6) 7^{th}; 7) 8^{th}; 8) 9^{th}; 9) 10^{th}; 10) 11^{th}; 11) 12^{th}.

Category 2: Effective teacher PD and goals of the PD

Item 4: Total span of time

Explanation: Indicate the total amount of time that passed from the beginning to the end of the PD, even if the participants did not meet all the time during the span of the program.

Coding Decision: Enter length of time from beginning of PD or course to the end of it. Include appropriate unit (days, weeks, months, or years)

1) Enter length____________________

2) Not reported

Item 5: Exact contact hours

Explanation: Total number of hours spent on PD activities. Coding decision: If days were reported, consider a day as 6 hours for professional development and 3 hours of teacher education courses.

Enter number of hours_______________

Item 6: Active engagement

Explanation: Indicate whether the PD was designed to actively engage teachers in activities.

Coding decision: Choose one

1) Teachers were actively engaged in working on activities for most of the program time

2) Teachers were not actively engaged in working on activities during the program

3) Not reported

Item 7: Connections between activities and the practice of teaching

Explanation: Indicate whether there was an intentional attempt to connect PD activities to the practice of teaching or to use the practice-based activities or artifacts in the PD. This connection to practice can be specifically stated in the PD or it can be noted by the choice of activities that are about teaching or related to the practice of teaching.

Coding decision: Choose one

1) Connection to practice was an important part of activities
2) Connection to practice was one among many parts of the activities
3) There were no activities connected to practice
4) Connections to practice were not reported

Item 8: Reporting of goals

Explanation: If the author(s) explicitly state the goals of the program, choose “yes”; otherwise choose “no”.

Coding decision: Choose one

1) Yes
2) No

Item 9: Goals of PD or course

Explanation: the author(s) might use words such as goals, aims, or a synonym of either to state the goals of the PD. In the case of imperial studies, we are not interested in the goals of the research. We are interested in the goals of the work being done with the teachers.

Coding decision: Choose all that apply

1) Subject matter knowledge or mathematics content (Sowder Goal 2): Develop teachers' in-depth understanding of mathematics or specific concepts within mathematics.
2) Pedagogical content knowledge in mathematics (Sowder Goal 4): Develop teachers' understanding of various instructional and pedagogical approaches used in mathematics classroom.
3) Student learning (Sowder Goal 3): Develop teachers' understanding of how children learn particular topics in mathematics.
4) Curricular Materials: Develop alignment of mathematics instruction to a particular, specific curriculum, usually offered when a new textbook or supplemental instructional material is adopted by the school or the district.
5) District or School Pacing Guides: Develop teachers' understanding of school or district pacing guides for organizing mathematical content over the year.
6) Technology: Develop teacher's understanding and use of technology for teaching mathematics.
7) Other ____________________

Category 3: Technology and geometry

Item 10: Incorporation of technology

Explanation: Indicate if technology was incorporated in the PD activities as a learning tool for teachers, as a tool that can be used with students, or as both.

Coding decision: Choose all that apply

1) Technology was incorporated as a learning tool for teachers
2) Technology was incorporated as a tool to be used with students
3) Technology was incorporated both as a learning tool for teachers and as a tool to be used with students
4) Technology is incorporated with no report on the way it was incorporated
5) No reported incorporation of technology

Item 11: Type of technology

Explanation: If technology was included, indicate the type of technology included.

Coding decision: Choose all that apply

1) Dynamic geometry software (DGS)
2) Other: ____________________
3) Not reported

Item 12: Teachers' experience with the technology:

Explanation: if DGS is used in the PD, determine whether the teachers have prior experience on the use of the particular technology or not.

Coding decision: Choose one

1) Most or all participants had some prior experience on the use of the particular technology.
2) Most or all participants had no experience on the use of the technology
3) Not reported

Category 4: The use of dynamic geometry software

Item 13: The dragging function in DGS.

Explanation: If DGS was used, determine the purpose of using the dragging capability either as explicitly stated by the author or as concluded from the context of the activities.

Coding decision: Choose all that apply:

1) Dragging for the purpose of exploring relationships and properties
2) Dragging and observing invariants
3) Dragging to test construction.
4) Dragging to arrive at generalization
5) No attention to the dragging function in activities
6) Cannot be determined from context

Item 14: Geometric topic addressed with DGS

Explanation: State the geometric topic addressed and whether DGS was used with that topic.

Coding decision: Choose all that apply

1) Geometric proof was one of the topics addressed along with DGS as a learning/teaching tool.
2) Geometric transformations were one of the topics addressed along with DGS as a learning/teaching tool.
3) Other: __________________
4) Topic was not reported

Item 15: Geometric transformation

Explanation: Determine the types of transformation activities used with DGS as explicitly stated by the author or as concluded from the context of the activity.

Coding decision: mark all that applies

1) Activity asked to transform shapes using DGS menu
2) Activity involved exploration of pre-constructed transformations
3) Activity involved examining or coming up with equivalencies of transformations
4) Activity involved emphasis on transformations as functions including focus on domain and variables.
5) Cannot be determined from context.

Item 16: Function(s) of proof

Explanation: Determine the function of the proof activity as explicitly stated by the author or as concluded from the context of the activity.

Coding decision: Mark all that apply

1) Proof for validation and confirmation
2) Proof for the function of explanation
3) Proof for discovery
4) Proof for communication
5) Proof for systemization
6) Cannot be determined from context

Item 17: Characteristics of the proof activity as recommended by research

Explanation: If topic addressed with DGS is proof, state the characteristics of proof activity(s) discussed or worked on in PD or course. The author might state that the design of the activity has any of the following Characteristics; or if the activity was included in the paper, the answer can be deduced by inspecting the activity.

Coding decision: Chose all that apply.

1) Proving activity started with exploration and ended up with construction of proof
2) Participants were provided with non-traditional conjectures to explore using DGS and then prove them; nontraditional conjectures are conjectures that are not commonly seen in textbooks.
3) Participants asked to use the technology to come up with their own conjectures and then prove or disprove them.
4) The use of inquiry activities in DGS, intentionally designed to confront participants with contradictions or uncertainties.
5) Other: author(s) explicitly state the use of DGS based on recommendations from research________________________
6) Not reported