ABSTRACT

WIEST, JENNIFER HANNEN. Nonlinear Control Strategies for a Teleoperated Cardiac Ablation Catheter Actuated by Shape Memory Alloy Tendons: System Modeling, Controller Synthesis, and Experimental Validation. (Under the direction of Gregory D. Buckner).

Endocardial ablation is commonly used to treat atrial fibrillation, a cardiac arrhythmia that affects nearly 3 million Americans. These ablation procedures often use manually-actuated catheters with limited degrees of freedom, and are therefore tedious, time-consuming, and result in significant X-ray exposure to the patient and clinical staff. Teleoperated (i.e. robotic) catheters have the potential to enhance catheter maneuverability in the heart, reduce procedure times, and improve patient outcomes. Previous work has focused on the design and fabrication of shape memory alloy (SMA) actuated robotic catheters for this application. Widespread clinical adoption of robotic catheter technology, however, requires precise and robust catheter control through an intuitive user interface. Real-time control is complicated not only by the highly nonlinear, hysteretic nature of shape memory alloy actuators, but also by catheter compliance and multivariable system coupling. Closed-loop control requires accurate feedback of the catheter’s location and orientation within the beating heart, must reject hemodynamic and contact disturbances, and must accommodate reference trajectories that are unknown a priori. This dissertation details the development of a complete closed-loop control system for surgical evaluation of a teleoperated cardiac ablation catheter actuated by SMA tendons.

First, the development of an indirect intelligent sliding mode controller (IISMC) for a reduced-order system (a flexible beam actuated by single or dual offset SMA tendons) is presented. This system exhibits many of the control challenges of the robotic catheter (actuator hysteresis, model uncertainties, etc.), but is more tractable. The IISMC algorithm manipulates applied voltage, enabling temperature control in the SMA tendons, to produce bending in the flexible beam. Hysteresis compensation is achieved using a hysteretic recurrent neural network (HRNN), which maps the nonlinear, hysteretic relationships between SMA temperatures and bending angle. Incorporating this HRNN into a variable structure control architecture (sliding mode control) provides robustness to model uncertainties and parameter variations. Experimental results demonstrate precise tracking of
a variety of reference trajectories, with superior performance compared to an optimized PI controller. Robustness to parameter variations and disturbances is shown.

The robotic catheter is a direct extension of the flexible beam system: it consists of two bending segments, each actuated by four offset pull wires (which are attached externally to SMA tendons), allowing four degree-of-freedom motion. This four-fold increase in system inputs and outputs makes implementing the IISMC on the robotic catheter significantly more challenging. Also, in vivo position sensing requires advanced imaging technologies with limited precision, bandwidth, and accuracy. Because the catheter requires higher actuation forces and utilizes larger diameter SMA tendons, system bandwidth is significantly reduced. For these reasons, multiple model-based control strategies (including IISMC) were developed, investigated, and compared for the robotic catheter. Incorporating the best features of these initial control architectures, a hybrid control strategy was developed to compensate for actuator hysteresis, bandwidth limitations, and coupling between system inputs. Experimental results demonstrate robust and accurate tracking.

A real-time path optimization and control strategy is developed to take advantage of the two segment catheter’s redundancy. Catheter tip locations and orientations are optimized using parallel genetic algorithms to produce continuous ablation paths with near normal tissue contact through physician-specified points. Simulated and experimental results demonstrate efficient generation of ablation paths and optimal reference trajectories. Closed-loop control of the SMA-actuated catheter along optimized ablation paths is validated experimentally.
Nonlinear Control Strategies for a Teleoperated Cardiac Ablation Catheter Actuated by Shape Memory Alloy Tendons: System Modeling, Controller Synthesis, and Experimental Validation

by
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A dissertation submitted to the Graduate Faculty of North Carolina State University in partial fulfillment of the requirements for the degree of Doctor of Philosophy

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DEDICATION

To my mother, Debbie.
Jennifer Hannen Wiest was born and raised in Beavercreek, OH. The daughter of an electrical engineer and a computer scientist, she went to the University of Cincinnati in 2004 to pursue an engineering degree. Jennifer competed for UC as a member of the cross country and track and field teams, before graduating with her Bachelor of Science degree in Mechanical Engineering in 2009. After six quarters of mandatory co-op during her undergraduate career, she realized that graduate school was the next step for her. In the fall of 2009, Jennifer came to North Carolina State University to begin work toward her doctoral degree in Mechanical Engineering.
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Chapter 1

Introduction

1.1 Motivation

Atrial fibrillation (AF) is the most common cardiac arrhythmia, affecting over 2.7 million Americans, and is a leading cause of thrombus and stroke [1], [2]. AF occurs when the heart’s normal electrical impulses are disrupted by random electrical activity, Figure 1.1b, preventing organized contraction of the heart’s upper chambers, causing only partial blood evacuation to the lower chambers. Treatment typically requires isolating these abnormal electrical impulses by forming scars in the atrial tissue. Published success rates of open-heart procedures (i.e. Cox Maze [3]) exceed 90%, but the surgeries are highly invasive and complex, encompassing the risks and long recovery associated with open-heart surgery [4]. Alternatively, minimally-invasive catheter ablation can be used to create lesions by burning or freezing the atrial tissue, Figure 1.1c.

Current catheter ablation procedures typically use a single degree-of-freedom, manually actuated catheter, Figure 1.2. The physician’s ability to efficiently navigate this catheter is hindered by its limited degrees of freedom, and requires the physician to simultaneously manually advance, rotate, and bend the catheter. As a result, catheter ablation procedures are tedious, time-consuming, and result in significant X-ray exposure to the patient and medical team [5], [6]. Success requires highly skilled catheter manipulation in the beating heart, and continuous surface contact for transmural and continuous lesions [7], [8].
Figure 1.1: Heart experiencing: (a) normal sinus rhythm, (b) atrial fibrillation caused by abnormal electrical impulses, (c) catheter ablation to form lesions to block unwanted electrical activity, and (d) return to normal sinus rhythm.

Figure 1.2: Standard manually-actuated cardiac ablation catheter.
1.2 SMA-Actuated Catheter

Teleoperated (i.e. robotic) catheters have the potential to enhance catheter maneuverability within the heart, increase ranges of motion, and improve positioning accuracy and precision. Additionally, increased contact stability during ablation may be possible. Potential clinical benefits include reduced procedure times, less radiation exposure for the physician and patient, and higher success rates.

Previous work focused on the design, optimization, and fabrication of shape memory alloy (SMA) actuated robotic catheters for endocardial ablation [9], [10]. Initial prototypes consisted of a central flexible beam directly actuated by offset SMA tendons, Figure 1.3. Joule heating was used to cause the SMA tendons to contract, causing a bending moment. Four offset tendons enabled omnidirectional bending. The addition of a second bending segment (containing four additional tendons) increased the catheter’s maneuverability and control over tip location and orientation.

(a)          (b)

Figure 1.3: Initial SMA-actuated catheter with SMA tendons incorporated into bending segments: (a) single segment design with central beam actuated by four offset SMA tendons; (b) two segment prototype.
Initial prototypes confirmed the feasibility of SMA actuation and control; however they did not result in a clinically viable product. Practical ablation catheters include several additional components (e.g. thermocouples, ablation electrodes, sensing electrodes, etc.) and require complete isolation between electrical components and \textit{in vivo} hemodynamics. Embedding all required ablation components into a 2.7 mm diameter catheter, along with the SMA tendons and their electrical lines, presents a major fabrication challenge. Additionally, restricting SMA tendons to the length of the bending segments limits the range of motion.

An alternative design was developed to ease fabrication and increase the catheter’s range of motion. Collaboration with a commercial catheter manufacturer (Cirtec Medical Systems, Los Gatos, CA) enabled rapid development of more clinically viable catheter prototypes. These prototypes utilize offset pull wires, similar to those used in manually-steerable catheters, which run the length of the catheter and are crimped to SMA tendons in the catheter handle, Figure 1.4. The SMA tendons are contained in a tube posterior to the catheter handle; their length and diameter were optimized to meet bending requirements. The final prototype consists of eight SMA tendons, which are independently actuated to produce four degree-of-freedom bending in the flexible catheter body.
Widespread clinical adoption of this technology requires robust, real-time control of the catheter’s bending segments. A joystick in the catheter handle, Figure 1.4, allows the physician to interactively specify reference points and trajectories. However, real-time control is complicated by the highly nonlinear, hysteretic behavior of the thermally-activated SMA tendons (see Section 1.3), hemodynamic and contact disturbances, and limitations with sensing and visualization. Furthermore, the multivariable control problem presented here introduces additional challenges: axial compression, coupling between tendons, tendon slack, and redundancy in catheter tip positioning.
1.3 SMA Control

In recent decades, a variety of medical devices (including coronary stents, catheter guide wires, eyeglass frames, etc.) have utilized the super-elastic properties of SMAs [11]. Even more recently, the actuation and self-sensing capabilities of SMAs have expanded their application to micro-scale and macro-scale robotics [12]. SMA actuators exhibit relatively large specific energies (700-2500 J/kg [13]) and strain recoveries (up to 8% [13]), making them attractive options for limited space, lightweight applications. Because the power densities of SMA actuators are significantly higher than those of small electric motors, 90-100 W/kg v. 1.5-15 W/kg [14], they have the potential to revolutionize the design, actuation and control of medical robotic systems (including upper-extremity prostheses [15] and minimally-invasive surgical robots [16], [17]).

However, bandwidth limitations and control challenges have prevented the widespread adoption of SMA actuators in robotic systems. SMA actuation results from temperature-induced phase transformations, practically achieved through Joule heating and convective cooling, which are often slow, creating bandwidth limitations. A dynamic relationship exists between applied power and resulting SMA temperature, introducing significant phase lags into actuation, increasing the potential for control overshoot. Additionally, SMA behavior is highly nonlinear and hysteretic, no one-to-one relationship exists between input temperature and output stress and strain (Figure 1.5), further complicating control.
Figure 1.5: Constant stress SMA hysteresis example: (a) temperature-induced phase transformation from martensite $M^+$ to austenite $A$, causing the tendon to contract, and (b) hysteretic relationship between temperature and strain.

Proposed control methods to date often rely on some form of hysteresis compensation [18], [19], [20], [21] and/or account for slow heating and cooling [22], [23], [24]. Song et. al developed a hybrid controller for a single SMA tendon which combines a neural network SMA model for hysteresis compensation, linear and nonlinear feedback terms for robustness, and a first order feedforward term for dynamic path tracking [25]. Shameli et. al proposed a PID-P$^3$ controller which includes a term proportional to the tracking error cubed for reduced settling time and overshoot [26]. Teh et. al incorporated an anti-sack component and a rapid-heating mechanism for increased system bandwidth of antagonistic SMA actuators [27].

While some have incorporated physics-based SMA models into control architectures [21], [28], [29], many rely neural network, fuzzy, or neuro-fuzzy based approaches due to their computational efficiency, learning capabilities, and adaptability. Kumagai et. al developed a feedforward controller based on a dynamic model generated by a neuro-fuzzy system [30]. Lei et. al experimentally evaluated and compared three neural-fuzzy based algorithms on a planar control application utilizing four SMA actuators [31]. Asua et. al combined a neural network for hysteresis compensation with a PI controller for a micropositioning application [32]. Kha and Ahn developed and experimentally evaluated a self-tuning fuzzy PID controller for SMA actuators [33].
1.4 Research Objectives

The overall goal of this work is to develop and demonstrate a closed-loop control system for surgical evaluation of the SMA-actuated cardiac ablation catheter. The following research objectives address this goal:

1. Model and analyze the electrical, mechanical, and thermal behavior of the SMA-actuated catheter
2. Develop real-time control platform (hardware and software) to interface between the physician and catheter
3. Derive control strategies that enable robust, closed-loop control of the SMA-actuated catheter
4. Develop path optimization strategies that enable the physician to efficiently navigate the SMA-actuated catheter and ensure adequate tissue contact during ablation

1.5 Outline of Dissertation

This dissertation is organized as follows:

Chapter 2: Catheter Actuation: Design, Analysis, and Modeling

This chapter begins with a detailed description of the SMA-actuated catheter. Hardware integration and software development are discussed. Open-loop data is presented and analyzed, and a thorough discussion of control challenges is provided. Heat transfer is analyzed and models governing temperature dynamics are derived. Finally, an application of this temperature model to increase system bandwidth is discussed.
Chapter 3: Single Degree-of-Freedom Bending Actuator Control

Control of reduced-order, SMA-actuated bending systems, which exhibit many of the control challenges of the full catheter prototype, is discussed. This chapter describes the development of an indirect intelligent sliding mode controller (IISMC) for single input, single output and two input, single output SMA bending actuators. The controller incorporates a hysteretic recurrent neural network for hysteresis compensation and utilizes a control law based on SMA temperature dynamics. Experimental results are presented and IISMC performance is compared to a PI controller. Additionally, disturbance rejection properties of the controller are investigated.

Chapter 4: Single Segment Catheter Control

This chapter discusses controller synthesis for the distal bending segment of the SMA-actuated catheter, treating the proximal segment as passive. Several nonlinear control strategies (including IISMC) are developed, experimentally evaluated, and compared. All strategies (except for the baseline PID) incorporate a model for hysteresis compensation and use a control law based on SMA temperature dynamics. Incorporating the best features of each controller, a hybrid control approach is presented which combines a model-based feedforward term for hysteresis compensation, a PID feedback term for robustness, and a third term added to improve system response time. Experimental results are presented for a wide variety of reference trajectories to quantify performance.

Chapter 5: Dual Segment Catheter Path Optimization and Control

In this chapter, control is extended to two segments and the associated performance advantages and control challenges are discussed. A real-time path optimization strategy is developed where optimal reference angles are computed to produce continuous ablation paths with near normal tissue contact through physician-specified points. The hybrid control approach discussed in Chapter 4 is extended to account for force coupling between bending
segments. Additionally, a modified version of this controller is presented to account for heat transfer between adjacent tendons. Experimental optimization and tracking results are presented.

Chapter 6: Conclusions

This chapter provides concluding remarks and offers recommendations for future work. Three potential areas for improvement are discussed: system bandwidth, position sensing, and haptic feedback.
Chapter 2

Catheter Actuation: Design, Analysis, and Modeling

2.1 Introduction

Surgical evaluation of the SMA-actuated catheter requires development of a complete closed-loop control system, including hardware integration, software development, and user interface design. This chapter begins with a detailed description of catheter design and fabrication. Hardware integration for \textit{in vivo} position sensing is described and the kinematics required for feedback control are derived. Controller software development is detailed and user interface features are described.

Before closed-loop control algorithms can be derived, it is first necessary to model, analyze, and understand the open-loop catheter behavior. This chapter presents a series of experimental tests conducted on the SMA-actuated catheter and reduced-order test beds. Analysis of individual actuator dynamics and overall catheter behavior is conducted. Finite-element analysis is used to simulate heat transfer within the catheter. Experimental and simulated results are used to develop models governing catheter behavior.
2.2 Catheter Design and Fabrication

The SMA-actuated catheter, shown in Figure 2.1, consists of two bending segments: the distal segment containing the catheter tip, and the adjacent proximal segment. The tip of each segment contains a pull ring, where four pull wires are attached offset from the catheter’s neutral axis (Figure 2.1b). These pull wires run the length of the catheter body, and are mechanically crimped in the handle to SMA tendons, which can be independently actuated to produce desired catheter bending. The SMA tendons are housed inside a copper tube posterior to the catheter handle and rapid-prototyped spacers are used to guide the tendons during actuation, shown in Figure 2.2a. The distal SMAs are 0.381 mm diameter Flexinol actuator wire (Dynalloy, Inc. Tustin, CA) and the proximal SMAs are 0.508 mm. These relatively large diameter tendons are necessary to meet force requirements for 180° and 90° bending in the distal and proximal segments, respectively, but heat and cool slowly, decreasing system bandwidth.

Performance of initial catheter prototypes was bandwidth limited due to slow SMA cooling. Additionally, heat transfer between adjacent tendons was high, further impeding catheter performance. To improve system bandwidth and reduce thermal coupling, the catheter was redesigned to incorporate active cooling for the SMA tendons, significantly increasing cooling rates. Active cooling is achieved through a concentric tube heat exchanger, shown in Figure 2.2b, where an aluminum tube is placed over the copper tube containing the tendons and water at 0°C is run through the annulus.
Figure 2.1: SMA-actuated catheter showing (a) proximal and distal bending segments (b) internal view of catheter tip showing distal pull wires connection to pull ring, and (c) catheter handle with joystick for user input.

Figure 2.2: SMA tendons in tube posterior to catheter handle showing (a) proximal and distal tendons held offset by spacers, and (b) tendons enclosed in inner copper tube of a concentric tube heat exchanger used to actively cool SMAs.
2.3 Hardware and Software Integration

2.3.1 Hardware Setup

The SMA-actuated catheter was interfaced with hardware, shown in Figure 2.3, to create the complete closed-loop control system. Pulse-width modulation (PWM) is used to regulate electrical power in each SMA tendon based on commands sent from a custom C++ control application. The controller tracks reference bending \((\theta_p, \theta_d)\) and orientation \((\phi_p, \phi_d)\) angles for each segment (Figure 2.4), which can be either pre-programmed or provided real-time using a 2D joystick in the catheter handle. An EnSite NavX System (St. Jude Medical, St. Paul, MN) provides feedback of catheter position required for closed-loop control.

![Diagram](image)

**Figure 2.3:** Experimental setup for closed-loop catheter control showing PWM amplifier for powering SMA tendons and EnSite NavX System for real-time catheter position feedback.
2.3.2 Sensing and Feedback

The EnSite NavX System (Figure 2.5a), used clinically for cardiac ablation procedures, was modified to provide real-time measurement of catheter electrode positions \((x, y, z)\) relative to electrode patches strategically placed on the patient. For laboratory testing, the catheter is submerged in a saline-filled custom tank fabricated with EnSite-compatible patch electrodes (Figure 2.5c).
Figure 2.5: (a) EnSite NavX showing 3D rendering of porcine heart, (b) EnSite electrode patch placement for live pig trials, and (c) catheter submerged in saline-filled custom tank fabricated with EnSite-compatible patch electrodes.

Real-time \((x, y, z)\) data from electrode pairs, located at the distal segment tip and the proximal segment tip and base, is feedback to the controller software. The data is filtered (using a 3rd-order Butterworth filter) to reduce noise and then used to calculate the system’s generalized angles \((\theta, \phi, \theta_D, \phi_D)\). Multiple methods exist for converting between Cartesian and generalized coordinates. In Chapter 5, the complete, coupled, two-segment catheter kinematics are derived. Here, a local method is presented which is often more practical for endoatrial control, where only a portion of the distal segment enters the cardiac chamber.

The local approach uses tip electrode pairs to calculate bending segment orientation relative to its unactuated orientation, illustrated in Figure 2.6a for the distal bending segment (the derivation is identical for both segments). A transformation matrix converts global \(XYZ\) coordinates to local \(X_bY_bZ_b\) coordinates with origin at the unactuated segment tip, Figure 2.6. The following three rotation matrices describe the transformation from \(X_bY_bZ_b\) to \(XYZ\)
$$R_z(\alpha) = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix}$$

$$R_z(\psi) = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(2.1)

where $\alpha = \text{atan}\left(\frac{y_1 - y_2}{x_1 - x_2}\right)$, $\beta = \text{atan}\left(\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}\right) / z_1 - z_2$, and $\psi$ sets the direction for $\phi = 0$. Defining the total rotation matrix $R = R_z(\alpha)R_y(\beta)R_z(\psi)$ gives the transformation matrix

$$T = R \begin{bmatrix} x_{b0} \\ y_{b0} \\ z_{b0} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} c_\alpha c_\beta c_\psi - s_\alpha s_\psi & -c_\alpha c_\beta s_\psi - s_\alpha c_\psi & c_\alpha s_\beta & x_{b0} \\ s_\alpha c_\beta c_\psi + c_\alpha s_\psi & -s_\alpha c_\beta s_\psi + c_\alpha c_\psi & s_\alpha s_\beta & y_{b0} \\ -s_\beta c_\psi & s_\beta s_\psi & c_\beta & z_{b0} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(2.2)

where $(x_{b0}, y_{b0}, z_{b0})$ are global coordinates of the local coordinate system origin. The inverse transformation is used to compute the local coordinates

$$\begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = T^{-1}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

(2.3)

and the bending and orientation angles, illustrated in Figure 2.6b, are calculated.
\[ \theta = \text{atan} \left( \frac{\sqrt{(x_{1b} - x_{2b})^2 + (y_{1b} - y_{2b})^2}}{z_{1b} - z_{2b}} \right) \]  
(2.4)

\[ \phi = \text{atan} \left( \frac{y_{1b}}{x_{1b}} \right) \approx \text{atan} \left( \frac{y_{2b}}{x_{2b}} \right) \]  
(2.5)

Figure 2.6: (a) Catheter in heart showing unactuated reference orientation and actuated catheter, and (b) electrode coordinates specified in the local coordinate system for calculating bending and orientation angles.

Although the EnSite NavX was not developed to provide real-time measurements for catheter control algorithms, experiments were conducted to establish its potential for this purpose. Specifically, the system’s measurement accuracy, precision, bandwidth and linearity were quantified by placing a plastic cube (8.5 cm sides) in the saline-filled tank and acquiring steady-state position data (15 Hz sample rate) corresponding to the cube’s corners and midpoints. Nonlinear measurement errors (Figure 2.7a) and low-frequency positional variations (Figure 2.7b) were observed even after implementing a 3rd-order Butterworth filter (0.75 Hz cutoff frequency). Collectively, these measurement limitations resulted in 7.1±3.5...
mm of position uncertainty and ±0.8 mm imprecision for the 2.7 mm diameter catheter. Despite these limitations, the EnSite system is presently the best option available for endocardial sensing, and was therefore chosen for real-time control of the SMA-actuated robotic catheter.

![Figure 2.7: EnSite NavX accuracy and precision: (a) comparison between known cubic geometry and measured tip locations (shown as 300 measured data points at each cube corner and midpoint); (b) close-up view of one 300 data point measurement showing variation within a 1.6 mm diameter sphere.](image)

### 2.3.3 Software Development

A custom C++ control application, including a graphical user interface (GUI), Figure 2.8, was developed using Visual Studio C++ (Microsoft Corporation, Redmond, WA). Real-time control algorithms allow the catheter to track reference angle trajectories provided by either: interactive joystick commands, manual GUI inputs, pre-programmed functions, or saved measurements. The GUI includes options to select a specific control algorithm (see Chapter 4) and input controller gains. Additionally, the GUI displays real-time plots of catheter orientation, tracking performance, and control effort; progress bars indicate tendon power and estimated tendon temperature (see Section 2.5).
Additional features aid the physician in creating endocardial ablation lesions. A GUI button commands the catheter to be held in its current location, and another set of buttons allow catheter orientations to be saved and later revisited. A third feature automatically generates a continuous curve (or ablation line) through physician-recorded points. For single segment control, this is simply a cubic spline between recorded angles. The dual segment catheter has increased maneuverability and an optimal path can be computed, as described in Section 5.2.

Figure 2.8: Graphical user interface (GUI) for closed-loop catheter control
2.4 Open-Loop Behavior

Several open-loop experiments were conducted to illustrate the modeling and control challenges of the SMA-actuated catheter. Figure 2.9 shows the transient and steady-state responses to actuating a single distal segment SMA tendon. A four second delay is seen between applied voltage extrema and bending angle extrema, Figure 2.9a. Because SMA tendons are actuated by temperature-induced phase transformations, and temperature changes are not instantaneous, this delay occurs, increasing the potential for control overshoot.

The steady-state hysteresis curve, Figure 2.9b, highlights additional control challenges. The term hysteresis implies no one-to-one mapping between inputs and outputs exists. This phenomenon is most extreme near 5 V, where a single input could result in any bending angle 0-180°, depending on the input history. Additionally, the steep slopes seen in the heating and cooling curves indicate that overshoot of reference bending angles is probable. If overshoot occurs, a large voltage decrease is required to recover hysteresis, shown by the wide plataues in Figure 2.9b. Finally, tendon slack prevents catheter movement for steady-state inputs under 3V. This slack increases response time, but is necessary to achieve 180° bending without over-stressing the antagonistic tendon.

Considering both the transient and steady-state responses, additional information can be inferred. Assuming the steady-state hysteresis curve, Figure 2.9b, is accurate, a 7 V input should achieve 150° bending. However, considering again Figure 2.9a, a 7 V input takes approximately 30 seconds to reach steady-state. This response time must be decreased, which requires initially applying a higher voltage and then later scaling back to the predicted steady-state value.

The single distal tendon, steady-state hysteresis curve presented here was recorded in a controlled setting (in air, with the catheter straight, and with all other tendons unactuated), and therefore the relationship is not valid globally. When multiple tendons are actuated simultaneously, heat transfer between tendons occurs (see Section 2.5), altering these steady-state voltage-bending angle relationships. Additionally, coupling between segments alters the voltage range required from each individual tendon to achieve the desired bending.
Lastly, the long flexible catheter will not remain straight when fed through the body, causing varying SMA tendon pre-strain while also increasing the potential for catheter twisting near the tip when actuated. This behavior is unpredictable (due to varying vascular anatomy) and significantly effects input-output hysteresis relationships.

![Figure 2.9: Open-loop results from actuation a single distal SMA tendon: (a) transient response comparing normalized voltage to normalized bending angle, and (b) steady-state hysteresis curve between applied voltage and bending angle.](image)

### 2.5 Thermal Analysis, Modeling, and Application

From Section 2.4, it is apparent that a thorough understanding, and accurate model, of SMA temperature dynamics could greatly improve system performance and actuation bandwidth. However, accurate SMA temperature measurement, particularly in embedded systems, is difficult. Thermocouples are commonly used for SMA temperature measurement, but are invasive, distorting local heat transfer properties, often resulting in inaccurate readings [34], [35]. Additionally, thermocouple attachment to a mobile, enclosed SMA tendon introduces costly fabrication challenges. Thermal imaging (using an infrared (IR) camera) is a non-invasive method occasionally used for SMA temperature measurement. However, IR
cameras have low accuracy [34], require extensive and precise setup for accurate results [36], [37], and, most relevant to this application, can only measure objects they can detect [37].

Because of the aforementioned embedded SMA temperature measurement challenges, alternative methods were developed for estimating temperature and developing governing models. This section details a series of experimental tests and finite element simulations conducted to understand and quantify heat transfer within the catheter. Using this information, both coupled and decoupled temperature models are derived for the system. Finally, an application of the temperature models to increase catheter bandwidth is discussed.

2.5.1 Experimental Testing

A series of experiments were conducted on reduced-order test beds to obtain valuable information regarding SMA temperature and its relationship to actuation force/displacement. First, a single spring-coupled SMA system, Figure 2.10, was actuated by a pre-programmed voltage profile. A load cell (MLP-10, Transducer Techniques, Temecula, CA), connected in series with the linear spring (k=8 N/cm), measured SMA force while and infrared camera (FLIR A32 with T197200 close-up lens, FLIR Systems, Wilsonville, OR) simultaneously measured SMA temperature.

![Figure 2.10: Single spring-coupled SMA experimental setup showing load cell used for force measurement and infrared camera used for temperature measurement]
Figure 2.11 shows the response to a series of varying-amplitude, varying-frequency step and ramp inputs. While there is a significant phase lag between applied voltage and temperature, extrema in temperature and force (or displacement) appear to occur simultaneously. Therefore, while the measured temperature shown here cannot be used to develop a complete temperature model for the embedded system (heat transfer to the surroundings is very different in that case), this experiment justifies using bending angle extrema information to develop the catheter temperature model.

A second experimental test bed was constructed to analyze heat transfer between the embedded SMA tendons. The setup is identical to the catheter posterior to the handle, with 8 SMA tendons inside the two concentric tubes, held in place with spacers. However, instead of crimping the SMA tendons to the catheter pull wires, they were crimped to linear springs, which were serially connected to load cells for SMA force measurement.

To investigate the effect of temperature coupling between tendons, a ramp input was applied to a single tendon while adjacent tendons were actuated at full power. Figure 2.12 confirms thermal coupling between tendons; actuation force (or displacement) for a single tendon increases when adjacent tendons are actuated.
2.5.2 Finite Element Simulations

Finite element analysis was used to simulate heat transfer within the concentric tubes. A representative cross-section of the heat exchanger and SMA tendons was modeled and ANSYS CFX was used to solve the heat transfer/fluid flow problem. While the actual system is 3D (fluid flow is perpendicular to the cross section modeled), simulation efficiency can be greatly improved by considering only the 2D cross section and assuming symmetry in the third dimension. To apply this symmetry condition, the coolant must be transformed from forced convection in an annulus to forced convection in crossflow over a cylinder.

The required cross-flow fluid velocity was calculated to ensure the heat transfer coefficients (and thus heat flux from the copper tube) are equivalent for the simulated and actual systems. First the actual system was considered: forced convection in an annulus. The pump for active cooling provides a volumetric flow rate of $V = 3.28 \frac{mL}{s}$. Using the properties of water at 0°C, the Reynolds number [38] is
\[ \text{Re}_{D,a} = \frac{\rho V D_h}{\mu A_c} \]  

(2.6)

where \( \rho \) and \( \mu \) are fluid density and viscosity, \( A_c \) is the cross-sectional area of the annulus, and \( D_h = D_o - D_i \) is the hydraulic diameter, calculated from the inner diameter of the outer aluminum tube \( D_o \) and the outer diameter of the inner copper tube \( D_i \). The calculated Reynolds number \( \text{Re}_{D,a} = 105.7 < 2300 \) indicates laminar flow. Therefore, considering the ratio of inner to outer tube diameter, the Nusselt number is found (\( Nu_{D,a} = 6.55 \) [39]) and the heat transfer coefficient is

\[ h_c = \frac{Nu_{D,a} k}{D_h} = 390.3 \frac{W}{m^2 K} \]  

(2.7)

where \( k \) is the thermal conductivity.

For the simulated system, forced convection in cross-flow over a cylinder, the Nusselt number [40] is

\[ Nu_{D,c} = 0.3 + \frac{0.62 \text{Re}_D^{1/2} \text{Pr}^{1/3}}{1 + (0.4 / \text{Pr})^{2/3}} \left[ 1 + \left( \frac{\text{Re}_D}{282,000} \right)^{5/8} \right]^{4/5} \]  

(2.8)

where \( \text{Pr} \) is the Prandtl number and the Reynolds number is

\[ \text{Re}_{D,c} = \frac{\rho V_\infty D_i}{\mu} \]  

(2.9)

where the cross-flow velocity \( V_\infty \) is to be determined. The heat transfer coefficient is then

\[ h_c = \frac{Nu_{D,c} k}{D_i} \]  

(2.10)
Equating the two heat transfer coefficients $h_a = h_c$, the required fluid velocity for cross-flow is calculated as $V_\infty = 0.0023 \frac{m}{s}$.

This velocity, along with the other boundary conditions, was imported into the CFX software and Joule heating was applied to the tendons to imitate the inputs shown in Figure 2.12. The results, shown in Figure 2.13, verify the temperature coupling assumed from the experimental force results. As shown in Figure 2.13a, tendons adjacent to fully actuated SMAs are significantly heated. However, it is also useful to note that non-adjacent tendons are only slightly heated.

![Figure 2.13: Simulated heat transfer in catheter cross-section showing: (a) Temperature distribution at t=60 sec while two adjacent tendons are active; (b) transient results comparing the effect of actuating adjacent tendons.](image)

2.5.3 Temperature Model

A combination of experimental and simulated results was used to derive dynamic temperature models for the catheter SMA tendons. Figure 2.13a showed that heat transfer between adjacent tendons is significant; however heat transfer between non-adjacent tendons is minimal. As a result, two separate temperature models were derived, one which assumes
tendons are decoupled, and one which accounts for coupling between tendons. The former, simpler model is used when only controlling a single segment of the catheter, as adjacent tendons are never actuated in that case. When both segments are actuated simultaneously, the coupled model is used.

### 2.5.3.1 Decoupled

Ignoring coupling between tendons, first-order temperature dynamics are assumed

$$\dot{T}(t) = -h(T(t) - T_\infty) + \gamma V(t)^2$$  \hspace{1cm} (2.11)

Recalling Figure 2.11, the heat loss coefficient $h$ is determined iteratively by equating transient extrema in bending angle to calculated extrema in temperature, Figure 2.14a. Once $h$ is known, the power coefficient $\gamma$ is calculated

$$\gamma = \frac{h(T_{ss} - T_\infty)}{(I_{ss} R)^2}$$  \hspace{1cm} (2.12)

where $R$ is tendon resistance and $T_{ss}$ is the steady-state temperature the SMA tendon reaches when applied with a constant current $I_{ss}$. This temperature is found using finite element simulations (see Section 2.5.2 and Figure 2.14b), giving $T_{ss} = 400K$ when $I_{ss} = 1.5A$.

![Figure 2.14](image_url)

**Figure 2.14:** Decoupled temperature model parameter calculation: (a) single tendon transient data comparing normalized measured bending angle to normalized calculated temperature, and (b) ANSYS CFX response to actuating a single tendon with 1.5 A.
2.5.3.2 Coupled

Next coupling between tendons is considered and the first order temperature model (2.11) is modified to include heat addition to the SMA from the surroundings

\[
\dot{T}_i(t) = -h(T_i(t) - T_\infty) + \gamma V_i(t) + \kappa(T_{i-1}(t) - T_\infty + T_{i+1}(t) - T_\infty)
\]  

(2.13)

where \( T_{i-1}(t) \) and \( T_{i+1}(t) \) are the two adjacent SMA tendon temperatures. The model coefficients \( h \), \( \gamma \), and \( \kappa \) are optimized (using MATLAB’s \textit{fmincon}) to fit simulated results. Because the proximal SMA tendons have a larger diameter than the distal tendons, separate coefficients must be found. Additionally, it was determined that a better fit could be achieved by using separate coefficients for heating and cooling. Therefore, 10 constants were optimized: \( h_{\text{P,heat}} \), \( h_{\text{P,cool}} \), \( h_{\text{D,heat}} \), \( h_{\text{D,cool}} \), \( \gamma_{\text{P,heat}} \), \( \gamma_{\text{P,cool}} \), \( \gamma_{\text{D,heat}} \), \( \gamma_{\text{D,cool}} \), \( \kappa_{\text{P}} \), and \( \kappa_{\text{D}} \).

Figure 2.15b-e compares simulated results to model-calculated temperatures for four SMA tendons: Tendon 1 (distal), Tendon 2 (proximal), Tendon 5 (distal), and Tendon 6 (proximal), with their locations shown in Figure 2.15a. These results were obtained using three input sets: (i) actuating Tendons 1 and 6 with a ramp input, (ii) actuating Tendons 1 and 6 with a ramp input, and Tendons 2 and 5 full power, and (iii) actuating Tendons 1 and 6 with a ramp input, and Tendons 2, 8, 5, and 7 full power. The results show close correlation between simulated results and model predictions, validating the derived model. The model successfully incorporates coupling between adjacent tendons, with increases in temperature seen as more tendons are actuated.
Figure 2.15: Comparison between simulated and calculated temperature using the coupled temperature model: (a) an example ANSYS CFX solution showing numbering scheme given to the tendons, (b)-(e) simulated and calculated temperature for Tendons 1, 2, 5, and 6 for three sets of inputs: (i) actuating Tendons 1 and 6 with a ramp input, (ii) actuating Tendons 1 and 6 with a ramp input, and Tendons 2 and 5 full power, and (iii) actuating Tendons 1 and 6 with a ramp input, and Tendons 2, 8, 5, and 7 full power.
2.5.4 Temperature Dependent Voltage Limits

SMA response times decrease as applied voltage increases. However, continuous high voltage will over-heat the SMA tendons causing material failure. Therefore, typical applications limit the power supply voltage to a safe continuous limit. Because this application requires large diameter SMA tendons, these fixed voltage limits impede heating.

Alternatively, dynamic voltage limits are implemented which permit high voltage initially for fast response, then reduce voltage output when SMA temperature exceeds a safe limit. The temperature model, derived in Section 2.5.3, estimates SMA temperature online, which is used to limit the PWM signal sent to the microcontroller

\[
P_{PWM_{MAX}} = -6.5 \, \text{atan} \left( 25 \left( \frac{T - T_e}{T_{lim} - T_{\infty}} - 1 \right) \right) + 90.0
\]  

(2.14)

where \( T_{lim} \) is the SMA temperature limit to prevent over-heating. The numerical coefficients given in (2.14), and a limit of \( T_{lim} = 350K \), give the distribution shown in Figure 2.16, where the power supply is set to 1.25 times the continuous voltage limit, decreasing response time by approximately 50%. Altering these coefficients changes peak voltage and the transition rate.

![Figure 2.16: PWM limit for a SMA tendon as a function of estimated SMA temperature](image-url)
2.6 Conclusions

This chapter detailed the design and fabrication of the SMA-actuated catheter, along with the experimental setup for closed-loop control, including hardware integration and software development. Analysis of open-loop catheter behavior highlighted several control challenges, which motivated the need for advanced model-based control algorithms. Due to the nature of shape memory alloys, specifically their temperature-induced phase transformations, a thorough understanding of catheter heat transfer proved essential to developing governing models. Experimental tests and finite element simulations enabled derivation of coupled and decoupled SMA-temperature models. Implementing temperature-dependent voltage limits, which utilize these models for online SMA temperature estimation, decreases catheter response time by approximately 50%. The next three chapters describe temperature model integration into closed-loop control algorithms.
Chapter 3

Single Degree-of-Freedom Bending Actuator Control

3.1 Introduction

The SMA-actuated catheter, described in Chapter 2, is an eight input, four output multivariable system. Developing, implementing, and testing model-based controllers for this entire system is complex, time-consuming, and costly. To reduce implementation complexity, a more tractable system was constructed to test potential controllers. The reduced-order system consists of a flexible beam actuated by one or two offset SMA tendons. This system captures many of the control challenges of the catheter (i.e. actuator hysteresis, tendon slack, model uncertainties), but reduces implementation complexity (i.e. sensing, kinematics, thermal modeling). Additionally, while the controller developed in this chapter was designed with the catheter in mind, the potential applications of this novel control system extend far beyond the catheter and the simplified system model (to surgical instruments, prosthetic limbs, pipe inspection tools, cleaning equipment, etc.).

The reduced-order system uses either a single SMA tendon or an antagonistic pair of SMA tendons for actuation. Real-time control of SMA actuated devices is complicated by the highly non-linear, hysteretic relationship between electrical input power and output stress and strain. While numerous SMA models have been developed, most are computationally intensive and are difficult to utilize for real-time control. Heuristic control algorithms (like
PI or PID), occasionally augmented with a form of hysteresis compensation, are frequently utilized for SMA actuated devices. More advanced control strategies make use of nonlinear, hysteretic, or neural network SMA models [41]. In [42], the Preisach model is used for hysteresis compensation and incorporated into a PID controller. In [43] and [44], the hysteretic recurrent neural network (HRNN) is used for hysteresis compensation combined with an auto-tuned PID controller. Song et. al [45] and Tai and Ahn [46] use neural networks combined with a sliding mode controller to regulate the displacement of a single SMA tendon.

Further complicating system performance and control, SMA actuation results from temperature-induced phase transformations, which creates bandwidth limitations: while temperature increases (via Joule heating) can be achieved relatively quickly, temperature decreases (via conduction or free convection to the surroundings) are often slow. To overcome this, antagonistic actuator configurations can be utilized, enabling bi-directional actuation to reduce the time delays associated with passive cooling.

Research over the past decade has focused on the design, modeling, and prototyping of systems actuated by antagonistic SMA components [47], [48], [49], [50]; however precise control is still a challenge. In addition to the inherent nonlinearities and hysteresis of SMA, antagonistic configurations can introduce new control challenges (e.g. tendon slack [51]) and the need for multivariable control strategies. Some control algorithms for antagonistic SMA actuation rely on force feedback for position control [51], [21], [52], which may be impractical for specific applications. Others are based on heuristic controllers (PID or fuzzy logic) [53]; few model-based schemes have been developed [54].

This chapter introduces an indirect intelligent sliding mode controller (IISMC) for an SMA-actuated plant: a flexible beam deflected by one or more offset SMA tendons. The sliding mode control (SMC) law manipulates SMA voltage to track reference SMA temperatures corresponding to desired bending angles. Single input, single output and multivariable implementations are presented.
A hysteretic recurrent neural network (HRNN) is used to map the nonlinear, hysteretic relationship between SMA temperature and bending angle. Like the well-known Preisach model [55], the HRNN consists of a weighted sum of operators, in this case conjoined sigmoid activation functions. However, the HRNN uses simple recurrence within each neuron to capture the directional dependence of the model. The distinct advantages of the HRNN over other hysteresis models have been established in the literature [56] - [57]. Veeramani et al. demonstrated the HRNN’s superior generalization capabilities to non-training data compared to a traditional radial basis network [56]. Lien et al. compared the HRNN to a rate-dependent Prandtl-Ishlinskii model for hysteretic piezoelectric actuators, with results indicating 50% less error for the HRNN [57].

Although hysteresis compensation in SMA actuators has been explored previously [42], [45], [46], the approach presented here is novel because the HRNN maps temperature (instead of voltage or current) to bending angle, ensuring rate independence and precise mapping when reference trajectories are unknown a priori. Additionally, a sliding mode control law based on temperature dynamics ensures that the time delay between applied voltage and resulting SMA temperature is accounted for. The HRNN+SMC combination effectively addresses hysteresis and system nonlinearities, exhibits robustness to model uncertainties and parameter variations, and is computationally efficient, enabling real-time implementation.

### 3.2 Single Tendon IISMC

#### 3.2.1 Single Tendon Bending Actuator

The plant for the single input, single output (SISO) controller consists of a flexible beam actuated by a single SMA tendon, Figure 3.1. The SMA tendon is offset from the neutral axis of the beam by a fixed distance $a$. As the tendon contracts, the force generated creates a moment about the beam, causing it to bend to an angle $\theta$. A complete description of the system modeling is presented in [58].
3.2.2 SISO Controller

3.2.2.1 Controller Overview

The IISMC consists of a sliding mode control law that manipulates electrical input voltage \( V(t) \) to track a reference temperature \( T_r(t) \), Figure 3.2. This reference temperature is provided by an inverse hysteretic recurrent neural network (HRNN), while SMA temperature \( T_s(t) \) is observed based on measured bending angle \( \theta_m(t) \).
3.2.2.2 Sliding Mode Control Law

Because the flexible beam system is inherently nonlinear and uncertain, a variable structure control strategy is employed. An augmented integral sliding surface is defined

\[ s(t) = \lambda \int_0^t \left( T(\tau) - T_r(\tau) \right) d\tau + \left( T(t) - T_r(t) \right) \]  

(3.1)

where \( T(\cdot) \) and \( T_r(\cdot) \) are the actual and reference SMA temperatures, respectively, and the integral gain \( \lambda \) is chosen for desired tracking performance. First-order SMA temperature dynamics are assumed

\[ \dot{T}(t) = f\left(T(t)\right) + gu(t) \]  

(3.2)

where \( f\left(T(t)\right) \) and \( g \) are estimated by models \( \hat{f}\left(T(t)\right) \) and \( \hat{g} \) (as detailed in Section 3.2.2.3). This results in the equivalent control law

\[ u_{eq}(t) = \frac{\dot{T}_r(t) - \hat{f}\left(T(t)\right) - \lambda \left(T(t) - T_r(t)\right)}{\hat{g}}. \]  

(3.3)
Uncertainty exists between the modeled temperature dynamics \( \hat{f}(T(t)) \) and \( \hat{g} \) and the actual temperature dynamics \( f(T(t)) \) and \( g \). Additive uncertainty in \( \hat{f}(T(t)) \) is assumed to be bounded by \( F(T) \):

\[
|\hat{f}(T(t)) - f(T(t))| \leq F(T) \tag{3.4}
\]

Because \( g \) acts to scale the input \( u(t) \), multiplicative uncertainty is assumed using another known bound \( G \):

\[
G^{-1} \leq \frac{\hat{g}}{g} \leq G. \tag{3.5}
\]

For robustness, a switching term is added to the equivalent control law (3.3)

\[
u_{sw}(t) = -k \operatorname{sgn}(s(t)) \tag{3.6}
\]

where \( k \) is chosen to guarantee stability in the presence of these model uncertainties by ensuring that the derivative of the Lyapunov function

\[
V(s) = \frac{1}{2}s^2 \tag{3.7}
\]

is negative definite [59], resulting in the switching gain constraint

\[
k \geq \hat{g}^{-1}G(F(T) + \eta) + (G-1)|u_{eq}(t)| \tag{3.8}
\]

where \( \eta > 0 \) determines attractiveness to the sliding surface.

The switching term (3.6) can result in system chattering. To reduce chatter, the \textit{sign} function in the switching term can be replaced by the \textit{saturation} function

\[
u_{sw} = -ks\operatorname{sat}\left(\frac{s(t)}{\varphi}\right) \tag{3.9}
\]
where \( sat(\cdot) \) is defined by

\[
sat(y) = \begin{cases} 
  y, & \text{if } |y| \leq 1 \\
  \text{sgn}(y), & \text{if } |y| > 1
\end{cases}
\]

and the boundary layer thickness \( \varphi \) can be adjusted to address the tradeoff between tracking error and chatter.

The complete SMC law combines the equivalent control term (3.3) and the modified switching term (3.9):

\[
\dot{u}(t) = \frac{\dot{T}_r(t) - \dot{\hat{f}}(T(t)) - \lambda_p(T(t) - T_r(t))}{\hat{g}}
- k_{sat} \left[ \frac{\lambda_i \int_0^t (T(t) - T_r(\tau))d\tau + (T(t) - T_r(t))}{\varphi} \right] 
\]

where the integral gain \( \lambda \) in the sliding surface (3.1) is replaced with dual gains \( \lambda_p \) and \( \lambda_i \) that allow for the independent emphasis of proportional and integral control, respectively. The controller parameters \( k, \varphi, \lambda_p, \) and \( \lambda_i \) can be chosen to satisfy the tradeoff between response time and overshoot; for this system an iterative tuning process resulted in gains \( k = 24, \varphi = 40, \lambda_p = 0, \) and \( \lambda_i = 3. \)

### 3.2.2.3 SMA Temperature Model

The first-order SMA temperature model (3.2) can be derived using an energy balance

\[
m c \dot{T}(t) = -h_c A_c (T(t) - T_{\alpha}) + j(t) - \dot{x}_c(t) h_c m - \dot{x}_c(t) h_m
\]

The first term of the right hand side of (3.11) describes convective heat loss to the environment: \( h_c \) is the convection heat transfer coefficient between the SMA tendon and its
environment (temperature $T_\infty$), and $A_s$ is the cross-sectional area of the tendon. The second term describes heat addition through Joule heating $j(t)$, and the last two terms describe temperature change associated with SMA phase transformation, where $x_+(t)$ and $x_-(t)$ represent the martensite plus and martensite minus phase fractions, respectively, and $h_+$ and $h_-$ are the corresponding latent heats of phase transformation. Heintze and Seelecke determined the magnitudes of the latent heat terms to be small in comparison to the convective cooling and Joule heating [60]. Neglecting the latent heat terms:

$$
\dot{T}(t) = -\frac{h_+ A}{mc} (T(t) - T_\infty) + \frac{1}{mc} j(t).
$$

(3.12)

Because the convective and Joule heating coefficients vary significantly with environmental conditions, and because voltage $V(t)$ is generally more convenient to control than power $j(t) = \frac{V(t)^2}{R}$, the coefficients in (3.12) are lumped into parameters $h = \frac{h_+ A}{mc}$ and $\gamma = \frac{1}{mcR}$, where $R$ is the SMA’s electrical resistance. The temperature model can thus be expressed

$$
\dot{T}(t) = -h(T(t) - T_\infty) + \gamma V(t)^2
$$

(3.13)

where the heat loss coefficient $h$ and the power coefficient $\gamma$ are easier to determine experimentally (see Section 3.7). In standard form, the temperature dynamics become

$$
\dot{T}(t) = \dot{f}(T(t)) + \dot{g}u(t)
$$

(3.14)

where $\dot{f}(T(t)) = -h(T(t) - T_\infty)$, $\dot{g} = \gamma$, and $u(t) = V(t)^2$. 

40
3.2.2.4 SMA Temperature Observer

Because real-time SMA temperature measurement is impractical for the flexible beam system, a Luenberger observer is used [61]. The observer dynamics, which are based on the difference between the measured and observed bending angle, are

\[
\dot{T}_o(t) = - h (T_o(t) - T_\infty) + \gamma V(t)^2 + L (\theta_m(t) - \theta_o(t))
\]  
(3.15)

where \(T_o(t)\) is the observed SMA temperature, \(\theta_m(t)\) is the measured bending angle, and \(\theta_o(t)\) is the observed bending angle. The observer gain \(L\) is initialized to 2000 to ensure the observed temperature tracks the actual (unmeasured) SMA temperature during actuation.

3.2.2.5 Hysteretic Recurrent Neural Network

The sliding mode control law (3.10) requires a reference temperature associated with the reference bending angle. Additionally, the observer (3.15) requires bending angle as a function of observed tendon temperature. The correlation between tendon temperature and bending angle is highly nonlinear and hysteretic, lacking a one-to-one relation between the two. A two-phase hysteretic recurrent neural network (HRNN) is used to map this relationship. For a complete description of the HRNN and its application to SMA modeling, see [56].

The HRNN, illustrated in Figure 3.3, consists of interconnected neurons with conjoined activation functions. At time step \(q\), the output of each activation function is

\[
f_i(\hat{T}(q)) = \frac{1 - f_i(\hat{T}(q-1))}{1 + \exp \left( \left( T_{F,i} - \hat{T}(q) \right) \chi_i \right)} + \frac{f_i(\hat{T}(q-1))}{1 + \exp \left( \left( T_{R,i} - \hat{T}(q) \right) \chi_i \right)}
\]  
(3.16)

where \(\hat{T}(q)\) refers to SMA tendon temperature (either measured, observed, or reference) at the \(q\)th time step. The activation functions range between 0 and 1, where 0 refers to an inactive neuron and 1 refers to an active neuron. The parameter \(\chi_i\) controls how quickly the
output switches between the two values, with a high value \( \chi_i >> 1 \) leading to step changes.

A previously inactive neuron becomes active at the forward transition temperature \( T_{F,i} \). A previously active neuron becomes inactive at the reverse transition temperature \( T_{R,i} \).

The network output

\[
\hat{\theta}(q) = \sum_{i=1}^{N} w_i^2 f_i(\hat{T}(q))
\]  

(3.17)

is a weighted combination of these activation functions. Squaring the network weights \( w_i \) ensures a positive contribution from each neuron, and establishes a statistical distribution of SMA crystals based on their transformation temperatures:

\[
\sum_{i=1}^{N} w_i^2 = 1
\]  

(3.18)

Figure 3.3: Two-phase HRNN architecture relating SMA tendon temperature to beam bending angle.
The HRNN is trained to optimize network weights using open-loop data from the physical plant. The Levenberg-Marquardt algorithm [62] is used to solve the nonlinear optimization problem governed by the cost function

\[ P = \frac{1}{2} \sum_{q=1}^{Q} e(q)^2 \]  

(3.19)

where

\[ e(q) = \theta_m(q) - \hat{\theta}(q) \]  

(3.20)

represents the error between an experimentally measured bending angle \( \theta_m(q) \) and the HRNN-predicted angle \( \hat{\theta}(q) \). The optimization algorithm was implemented using MATLAB’s \textit{lsqnonlin} command (The MathWorks, Inc., Natick, MA) using a Jacobian defined by

\[ J_{q,i} = \frac{\partial e(q)}{\partial w_i} = -2w_i f_i. \]  

(3.21)

This HRNN maps observed temperature to observed bending angle; to determine reference temperature from reference bending angle requires inversion of the network. Because the analytical inverse of this network is difficult/impossible to evaluate, a bisection algorithm is used to approximate it. The bisection algorithm uses current system state information (activation states of the HRNN) and halves the SMA temperature range iteratively until a specified tolerance is achieved. The complete block diagram of the indirect intelligent sliding mode controller (IISMC) is shown in Figure 3.4.
3.2.2.6 Temperature Model Parameter Determination

The heat loss coefficient $h$ of the temperature model (3.13) can be determined experimentally from the system’s transient response, Figure 3.5a. A constant voltage is applied to the flexible beam system and then switched off at time $t_i$. The temperature dynamics are described by

$$\dot{T}(t) = -h(T(t) - T_w)$$

which is solved analytically

$$T(t) = (T_i - T_w)e^{-h(t-t_i)}$$

where $T_i$ is the temperature at time $t_i$. The heat loss coefficient is then determined by:
where \( T \) is the SMA temperature at some time \( t > t_i \). Because extrema in bending angle and temperature occur simultaneously, it is reasonable to assume that the bending angle decay rate is approximately equal to the temperature decay rate. Therefore, the heat loss coefficient can be approximated from

\[
h \approx -\frac{1}{t-t_i} \ln \left( \frac{\theta - \theta_f}{\theta_i - \theta_f} \right)
\]

(3.25)

where \( \theta_i \) and \( \theta \) are the measured bending angles at times \( t_i \) and \( t > t_i \), respectively and \( \theta_f \) is the final steady-state bending angle determined from Figure 3.5a.

Once the heat loss coefficient is determined, the power coefficient is calculated from the steady-state portion of a plant step response using

\[
\gamma = \frac{h(T_{ss} - T_x)}{V_{step}^2}
\]

(3.26)

where \( V_{step} \) is the voltage step input that results in steady-state SMA temperature \( T_{ss} \). The bending angle associated with a steady-state SMA temperature can be estimated from stress-strain data obtained at a specific SMA temperature \( T_{ss} \), Figure 3.5b. The functional relationship between stress \( \sigma \) and strain \( \varepsilon \) in the flexible beam system is calculated from

\[
\sigma = \frac{EI}{a^2A_c} (\varepsilon_p - \varepsilon)
\]

(3.27)

where \( EI \) is the bending stiffness of the beam, \( a \) is the tendon offset from the neutral axis, \( A_c \) is the cross-sectional area of the SMA tendon, and \( \varepsilon_p \) is the SMA pre-strain [58]. This relationship is superimposed on the constant temperature stress-strain hysteresis curve,
Figure 3.5b. The stress where this load line intersects the unloading curve is determined. From this stress the bending angle is calculated

$\theta_{ss} = \frac{aA L}{EI} \sigma_{ss}$

where $L$ is the length of the active SMA tendon [58]. A ramp voltage input of 24.5 mV/sec is applied to the plant until this bending angle is reached; this relatively slow input rate eliminates overshoot in the bending angle response. Finally, the known temperature and voltage are used in (3.26) to calculate $\gamma$.
3.2.3 SISO Methods

3.2.3.1 Simulation Methods

IISMC performance was simulated using the homogenized energy model (HEM) of SMAs; see [63] for a complete description. SMA tendon stress is determined by coupling the beam bending stress-strain relation (3.27) with the SMA stress-strain relation

\[ \sigma(t) = \frac{\varepsilon(t) - \varepsilon_T (x_+(t) - x_-(t))}{1 - x_+(t) - x_-(t)} + \frac{x_+(t) + x_-(t)}{E_A E_M} \]  

(3.29)

where \( x_+(t) \) and \( x_-(t) \) represent the fractions of the SMA tendon in the martensite plus (tension induced) and martensite minus (compression induced) phases, respectively. These phase fractions depend on SMA tendon temperature, see [63] for a complete discussion of their calculation. Once stress in the tendon is known, the bending angle is calculated from

\[ \theta(t) = \frac{aAL}{EI} \sigma(t). \]  

(3.30)

For simulations, the IISMC and plant model were implemented using MATLAB’s Simulink (The MathWorks, Inc., Natick, MA). Model parameters were optimized to fit open-loop experimental data obtained from the plant. HRNN training and testing data were generated by simulating plant responses to the inputs shown in Figure 3.6. This relatively slow, piecewise linear alternating voltage results in hysteretic temperature-bending angle data that captures the ascending and descending transition curves of the SMA. The HRNN was then trained by optimizing neuron weights, as outlined in (3.19)-(3.21). Finally, the neuron weights and transition temperatures were imported into the IISMC algorithm.
3.2.3.2 Experimental Methods

Experimental validation of the developed controller was conducted on the flexible beam system shown in Figure 3.7b. The system consists of a central flexible beam (0.5 mm diameter super-elastic Nitinol) with equally-spaced collets attached to hold the SMA tendon (0.127 mm diameter Flexinol, Dynalloy, Inc. Tustin, CA) at a fixed offset from the neutral axis. The tendon was encased in PTFE tubing to minimize friction and heating in the collets. For real-time implementation, the IISMC was programmed in Visual Studio C++ (Microsoft Corporation, Redmond, WA) and set to run at 50 Hz. During operation, commands are sent through serial communication to a microcontroller which regulates the voltage sent from the power supply (Agilent E3620A, Agilent Technologies, Santa Clara, CA) to the SMA tendons using a custom pulse width modulation (PWM) amplifier. Assuming constant-curvature deflection [58], the bending angle is calculated from 3D position sensors (trakSTAR 3D Magnetic Tracking System, Ascension Technology Corporation, Burlington, VT) which measure the location of the base and tip of the flexible beam. A complete schematic of the experimental setup is shown in Figure 3.7a.
Bending angle data was acquired for HRNN training using the same plant inputs of Figure 3.6. SMA tendon temperature was calculated using (3.13). The HRNN was then trained to optimize the network weights as outlined in (3.19)-(3.21); weights were then imported into the control code.

3.2.4 SISO Results

3.2.4.1 Temperature Model Parameters

A transient system step response, Figure 3.8a resulted in a heat loss coefficient $h = 0.637 s^{-1}$, calculated from (3.25). Constant temperature stress-strain data, originally published in [63] and reproduced here in Figure 3.8b, was obtained via tensile testing using the same SMA wire used for plant fabrication. The bending model parameters, Table 3.1, were measured during plant fabrication and resulted in the load line shown in Figure 3.8b. Using this graph, the steady-state stress $\sigma_{ss}$ and corresponding steady-state bending angle $\theta_{ss}$ were determined. From these values, the associated input voltage was determined and the power loss coefficient $\gamma = 2.47 \frac{K}{\sqrt{\gamma}}$ was computed from (3.26).
Figure 3.8: (a) Transient plant step response used to calculate heat loss coefficient and (b) SMA tendon stress-strain data at 80 C taken from [Crews] with superimposed load line for the flexible beam system model.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$EI$</td>
<td>Beam bending stiffness</td>
<td>$1.30 \times 10^{-4}$</td>
<td>Nm$^2$</td>
</tr>
<tr>
<td>$r$</td>
<td>SMA tendon radius</td>
<td>0.0635</td>
<td>mm</td>
</tr>
<tr>
<td>$a$</td>
<td>Tendon offset from neutral axis</td>
<td>1.0</td>
<td>mm</td>
</tr>
<tr>
<td>$L$</td>
<td>Active beam length</td>
<td>88.0</td>
<td>mm</td>
</tr>
<tr>
<td>$\varepsilon_p$</td>
<td>SMA tendon pre-strain</td>
<td>4.0</td>
<td>%</td>
</tr>
</tbody>
</table>

3.2.4.2 Simulation Results

A HRNN with 1561 neurons was initialized with 151 normalized forward transition temperatures $T_{F,i}$ ranging from 0 to 1 at equal intervals of 0.00667. Each forward transition temperature was paired with 21 reverse transformation temperatures $T_{R,j}$ ranging from $T_{F,i} - 0.0$ to $T_{F,i} - 1.0$ at equal intervals of 0.05, where only pairs with positive reverse transformation temperatures were implemented as neurons.
The simulated training data (2660 input-output samples) and testing data (2900 input-output samples) were normalized, and the HRNN was trained to optimize network weights. Figure 3.9 shows the simulated training and testing data along with the optimally trained HRNN prediction. The optimal solution resulted in a training cost of $1.39 \times 10^{-4}$ and a testing cost of $1.36 \times 10^{-4}$, reduced from initial values of 0.234 and 0.233, respectively, over 65 epochs. The constraint equation on network weights (3.18) was confirmed.

![Figure 3.9: Simulated HRNN predicted bending angle as a function of temperature compared to (a) training data and (b) testing data obtained from the flexible beam system model.](image)

The optimized HRNN weights and transition temperatures were implemented into the IISMC and simulations were conducted to demonstrate performance without temperature model uncertainties. Figure 3.10 shows precise tracking of a sinusoidal bending angle reference trajectory. Figure 3.10c illustrates how effectively the observed temperature tracks the reference temperature. Additionally, the observed temperature tracks the actual SMA temperature, validating the HRNN’s ability to generalize the system’s hysteretic characteristics.
3.2.4.3 Experimental Results

The HRNN training process to optimize network weights was repeated using experimental training data (2660 input-output samples) and testing data (2900 input-output samples). The optimal solution resulted in a training cost of $8.79 \times 10^{-5}$ and a testing cost of $2.71 \times 10^{-4}$, reduced from initial values of 0.212 and 0.200, respectively, over 27 epochs. The constraint equation (3.18) was satisfied to within 0.02. The resulting HRNN prediction, along with the experimentally generated training and testing data, are shown in Figure 3.11. The maximum error between experimental data (not used for HRNN training) and the inverse HRNN prediction was determined to be 11.3K; this error bound was utilized to establish the additive
and multiplicative uncertainly bounds \( F(T) \) and \( G \) in (4) and (5), respectively) used to ensure stability robustness of the sliding mode control algorithm.

![Figure 3.11: Experimental HRNN predicted bending angle as a function of temperature compared to (a) training data and (b) testing data obtained from the flexible beam system.](image)

Controller performance was evaluated using a 0.05 Hz sinusoidal reference, with results shown in Figure 3.12. The observed temperature tracked the reference temperature, Figure 3.12c, resulting in precise regulation of the reference trajectory, Figure 3.12a. The control input, Figure 3.12b, is composed of the equivalent control term \( u_{eq} \), Figure 3.12d, which accounted for 72% of the total input, and the switching term \( u_{sw} \), which ensured attractiveness to the sliding surface and overcame model uncertainties.
Figure 3.12: Experimental IISMC tracking results for a 0.05 Hz sinusoidal reference trajectory showing (a) bending angle, (b) controller generated plant input, (c) reference and observed SMA tendon temperature, and (d) contributions of the switching and equivalent control terms.

For comparison, the gains of a proportional+integral (PI) controller were optimized using a modified version of the Ziegler-Nichols method [64]. The gains were then manually tuned for optimal performance, resulting in a proportional gain of $k_p = 4.5$ and an integral gain of $k_i = 10$. Figure 3.13 and Figure 3.14 compare IISMC and PI controller performance for sinusoidal and ramp bending angle reference trajectories of various frequencies. These figures demonstrate superior tracking performance for the IISMC over the PI controller. This is evident at the peaks of the reference trajectories, where the PI controller corrected based on a small error in bending angle, while the IISMC compensated for hysteresis and corrected
based on the larger error in temperature. Computing the RMS tracking error over one period, Table 3.2, the IISMC resulted in 33.2 ± 5.3 % less error than the PI controller for the six reference trajectories tested.

While differences in tracking error between the IISMC and the PI controller are relatively small, a primary advantage of the IISMC is evident at directional changes, where the system must overcome actuator hysteresis. In tracking situations where rapid directional changes are common, the IISMC has a clear performance benefit. Additionally, larger diameter SMA tendons often required for higher force applications have longer heating and cooling times. In those cases it is expected that the performance differences would be even more apparent, as the IISMC regulates temperature and can thus account for large delays between input voltage and resulting actuation.

![Figure 3.13: Experimental IISMC and PI controller tracking performance against sinusoidal references of 0.05, 0.067, and 0.1 Hz frequencies.](image-url)
Table 3.2: Performance measures comparing IISMC and PI controller for sinusoidal and ramp reference trajectories of various frequencies.

<table>
<thead>
<tr>
<th>Reference Trajectory</th>
<th>RMS Tracking Error (deg)</th>
<th>RMS Control (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IISMC</td>
<td>PI</td>
</tr>
<tr>
<td>Sinusoidal, 0.05 Hz</td>
<td>0.37</td>
<td>0.54</td>
</tr>
<tr>
<td>Sinusoidal, 0.067 Hz</td>
<td>0.55</td>
<td>0.82</td>
</tr>
<tr>
<td>Sinusoidal, 0.1 Hz</td>
<td>0.99</td>
<td>1.30</td>
</tr>
<tr>
<td>Ramp, 0.05 Hz</td>
<td>0.43</td>
<td>0.70</td>
</tr>
<tr>
<td>Ramp, 0.067 Hz</td>
<td>0.61</td>
<td>0.99</td>
</tr>
<tr>
<td>Ramp, 0.1 Hz</td>
<td>0.99</td>
<td>1.49</td>
</tr>
</tbody>
</table>

3.3 Antagonistic IISMC

3.3.1 Antagonistic Bending Actuator

The system for the two input, single output (TISO) controller consists of a flexible beam actuated by an antagonistic pair of SMA tendons, Figure 3.15, where collets are again used to ensure that both SMA tendons remain a fixed offset $a$ from the beam’s neutral axis. The following nomenclature is used to delineate between the two tendons: $\text{SMA}_+$ refers to the
tendon which exerts a positive bending moment (resulting in a positive bending angle \( \theta > 0 \)) when actuated, while SMA. refers to the tendon which creates negative bending moments when actuated.

![Figure 3.15: Antagonistic SMA-actuated flexible beam system: (a) photograph showing output bending angle \( \theta \) of the deflected beam; (b) solid model showing collets used to maintain fixed SMA tendon offsets \( a \).](image)

### 3.3.2 TISO Controller

#### 3.3.2.1 Controller Overview

The antagonistic IISMC consists of two sliding mode control laws that manipulate input voltages \( V_+ (t) \) and \( V_- (t) \) to track SMA reference temperatures \( T_{+r} (t) \) and \( T_{-r} (t) \) (corresponding to the SMA\(_+\) and SMA\(_-\) tendons, respectively), Figure 3.16. These reference temperatures are estimated by the inverse of a hysteretic recurrent neural network (HRNN), while SMA temperatures \( T_{+s} (t) \) and \( T_{-s} (t) \) are estimated using a Luenberger observer.
3.3.2.2 Sliding Mode Control Laws

Because the system has two inputs and is modeled with two decoupled state equations, two augmented integral sliding surfaces are defined

\[ s_+ = \lambda \int (T_+ (t) - T_{+r} (t)) dt + (T_+ (t) - T_{+r} (t)) \]

\[ s_- = \lambda \int (\hat{T}_- (t) - T_{-r} (t)) dt + (\hat{T}_- (t) - T_{-r} (t)) \]

where \( T_+ (t) \) and \( T_- (t) \) are the temperatures of the SMA+ and SMA- tendons, respectively, \( T_{+r} (t) \) and \( T_{-r} (t) \) are the reference SMA temperatures, and the integral gain \( \lambda \) is chosen for desired tracking performance. First-order SMA temperature dynamics (detailed in Section 3.3.2.3) are assumed for both tendons

\[ \dot{T}_+ (t) = \hat{f} (T_+ (t)) + \hat{g} u_+ (t) \]

\[ \dot{T}_- (t) = \hat{f} (T_- (t)) + \hat{g} u_- (t) \]

where \( \hat{f}(T_{+r} (t)) \) and \( \hat{g} \) are models of the actual temperature dynamics \( f (T_{+r} (t)) \) and \( g \). Because the temperature states are decoupled, two independent control laws are derived. The equivalent control laws are
\[ u_{eq}(t) = \frac{\dot{T}_e(t) - \dot{f}(T_c(t)) - \lambda(T_c(t) - T_e(t))}{\hat{g}} \tag{3.35} \]

\[ u_{-eq}(t) = \frac{\dot{T}_e(t) - \dot{f}(T_c(t)) - \lambda(T_c(t) - T_e(t))}{\hat{g}} \tag{3.36} \]

and the switching terms are

\[ u_{sw} = -k \text{ sat} \left( \frac{s}{\varphi} \right) \tag{3.37} \]

\[ u_{-sw} = -k \text{ sat} \left( \frac{s}{\varphi} \right) \tag{3.38} \]

where \( \varphi \) is the boundary layer thickness. The switching gain

\[ k \geq \hat{g} \left( F + \eta \right) + (G - 1)\left| u_{-sw} \right| \tag{3.39} \]

is chosen to guarantee stability in the presence of model uncertainty, where \( \eta > 0 \) determines the attractiveness to the sliding surface, and \( F \) and \( G \) represent additive and multiplicative bounds, respectively, on the uncertainty between the modeled temperature dynamics \( \dot{f}(T_{c,e}(t)) \) and \( \hat{g} \) and the actual temperature dynamics \( f(T_{c,c}(t)) \) and \( g \).

The complete SMC laws combine the equivalent control terms (3.35)-(3.36) with the switching terms (3.37)-(3.38)

\[ u_e(t) = \frac{\dot{T}_e(t) - \dot{f}(T_c(t)) - \lambda_e(T_c(t) - T_{c,e}(t))}{\hat{g}} \]

\[ -k \text{ sat} \left[ \frac{\lambda_e \int (T_c(t) - T_{c,e}(t)) dt + (T_c(t) - T_{c,e}(t))}{\varphi} \right] \tag{3.40} \]
\[ u_*(t) = \frac{\dot{T}_r(t) - \bar{f}(T_r(t)) - \lambda_p(T_r(t) - T_{r-}(t))}{k \text{ sat} \left[ \lambda_r \int (T_r(t) - T_{r-}(t)) \, dt + (T_r(t) - T_{r-}(t)) \right]} \]  

(3.41)

where the integral gain \( \lambda \) in (3.31)-(3.32) is replaced with dual gains \( \lambda_p \) and \( \lambda_i \) that allow for independent control of proportional and integral tracking errors, respectively. Note however that the condition for stability (3.39) as derived is only valid when the gains are equal.

The controller parameters \( k, \phi, \lambda_p, \) and \( \lambda_i \) can be chosen to optimize the tradeoff between response time and overshoot; for this system an iterative tuning process resulted in gains of \( k = 30, \phi = 40, \lambda_p = 0, \) and \( \lambda_i = 3. \)

### 3.3.2.3 SMA Temperature Model

The first-order temperature models (3.33)-(3.34) are assumed to be identical and independent of the opposing tendon temperature. The temperature model derived for a single tendon is applied to each antagonistic tendon.

\[ \dot{T}_r(t) = -h(T_r(t) - T_u) + \gamma V_r(t) \]  

(3.42)

\[ \dot{T}_{r-}(t) = -h(T_{r-}(t) - T_u) + \gamma V_{r-}(t) \]  

(3.43)

where the heat loss coefficient \( h = 0.637 \) and power coefficient \( \gamma = 2.08 \) were experimentally determined previously for the single tendon case, the latter being adjusted to account for differences in circuit resistance. In standard form, the temperature dynamics become

\[ \dot{T}_r(t) = \bar{f}(T_r(t)) + \bar{g}u_r(t) \]  

(3.44)

\[ \dot{T}_{r-}(t) = \bar{f}(T_{r-}(t)) + \bar{g}u_{r-}(t) \]  

(3.45)

where \( \bar{f}(T_{r-}(t)) = -h(T_{r-}(t) - T_u), \bar{g} = \gamma, \) and \( u_{r-}(t) = V_{r-}(t) \).
3.3.2.4 SMA Temperature Observer

The observer dynamics of this multiple-input, single output system are based on the difference between measured $\theta_\alpha$ and observed $\theta_\alpha$ bending angle:

$$
\begin{bmatrix}
\dot{T}_{+,\alpha}(t) \\
\dot{T}_{-,\alpha}(t)
\end{bmatrix} =
\begin{bmatrix}
-h & 0 \\
0 & -h
\end{bmatrix}
\begin{bmatrix}
T_{+,\alpha}(t) - T_{+,\alpha} \\
T_{-,\alpha}(t) - T_{-,\alpha}
\end{bmatrix} +
\begin{bmatrix}
y & 0 \\
0 & \gamma
\end{bmatrix}
\begin{bmatrix}
Y^+ (t) \\
Y^- (t)
\end{bmatrix} + L(\theta_\alpha(t) - \theta_\alpha(t))
$$

(3.46)

where $T_{+,\alpha}(t)$ and $T_{-,\alpha}(t)$ are the observed temperatures of the SMA$_+$ and SMA$_-$ tendons, respectively. The observer gain matrix is initialized to $L = [2000 \ -2000]'$ to ensure the observed temperatures track the actual (though unmeasured) SMA temperatures during actuation.

3.3.2.5 Hysteretic Recurrent Neural Networks

At each time step, the sliding mode control laws (3.40)-(3.41) require a pair of SMA reference temperatures $(T_{+,\alpha}, T_{-,\alpha})$ that are predicted to achieve the reference bending angle $\theta_r$. Additionally, the observer (3.46) requires bending angle $\theta_\alpha$ as a function of observed tendon temperatures $(T_{+,\alpha}, T_{-,\alpha})$. When only one tendon is heated, the correlation between tendon temperature and bending angle is highly nonlinear and hysteretic. When both tendon temperatures vary, a known bending angle can result from infinite combinations of antagonistic SMA temperatures. For simplicity, the mapping is decoupled, and two single input, single output hysteretic mappings are used: one between SMA$_+$ temperature $\hat{T}_{+}$ and the bending it produces $\hat{\theta}_+$, and one between SMA$_-$ temperature $\hat{T}_{-}$ and the bending it produces $\hat{\theta}_-$. The total bending angle $\hat{\theta}$ is assumed to be a linear combination of the independent mapping outputs

$$
\hat{\theta} = \hat{\theta}_+ - \hat{\theta}_- .
$$

(3.47)
A two-phase hysteretic recurrent neural network (HRNN) is used to map each of these two decoupled relationships [56]. Each network consists of interconnected neurons with conjoined activation functions, whose output at time step \( q \) is

\[
f_{i(+)}(\hat{T}_{i+}(q)) = \frac{1 - f_{i(-)}(\hat{T}_{i-/}(q-1))}{1 + \exp\left(\left(\frac{T_{F,i} - \hat{T}_{i-/}(q)}{\chi_i}\right)\right)} + \frac{f_{i}(\hat{T}_{i-/}(q-1))}{1 + \exp\left(\left(\frac{T_{R,i} - \hat{T}_{i-/}(q)}{\chi_i}\right)\right)}
\]

where \( \chi_i > > 1 \) and the forward and reverse transition temperatures \( T_{F,i} \) and \( T_{R,i} \) are the same for both tendons. Each network output

\[
\hat{\theta}_{i-}(q) = \sum_{i=1}^{N} w_{i,i} f_{i(+)}(\hat{T}_{i-}(q))
\]

is a weighted combination of these activation functions, where the network weights \( w_{i,i} \) are optimized through network training subject to the constraint

\[
\sum_{i=1}^{N} w_{i,i}^2 = 1.
\]

Each HRNN is independently trained (as detailed in Section 3.2.2.5) using open-loop data acquired from the physical plant.

The tendon temperature pair \((T_{+}, T_{-})\) that theoretically achieves the reference bending angle \( \theta \) must be computed at each time step, requiring the system mapping (3.47)-(3.49) to be inverted. Because the individual components of (3.47), namely \( \theta_{+,} \) and \( \theta_{-,} \), are not known, the inverse is ill-defined. Alternatively, a generalized inverse based on the sign of the bending angle error is computed (Figure 3.17). If the reference bending angle \( \theta_{+} \) exceeds the measured bending angle \( \theta_{+} \) (resulting in positive tracking error), the SMA+ tendon is considered to be “active”. The “inactive” tendon’s reference bending angle and temperature are set to its observed values: \( \theta_{-,} = \theta_{-,} \) and \( T_{-,} = T_{-,} \). The required bending angle for the active tendon \( \theta_{+,} \) can then be computed from (3.47). At this point, the single
input, single output mapping between $T_{r\theta}$ and $\theta_{r\theta}$ is inverted using a bisection algorithm. A similar procedure is employed if the tracking error is negative. Because one tendon’s reference temperature is set equal to its observed temperature, this generalized inverse method simplifies system control (step changes in temperature error occur for only one tendon at a time). However, if the computed reference bending angle for the active tendon (i.e. $\theta_{r\theta}$) exceeds the system’s achievable bending range, the inactive tendon’s bending angle (i.e. $\theta_{r-\theta}$) must be reduced, requiring calculation of both reference temperatures $T_{r\theta}$ and $T_{r-\theta}$ (illustrated in the outer two columns of Figure 3.17). A complete block diagram of the antagonistic indirect intelligent sliding mode controller (IISMC) is shown in Figure 3.18.

![Figure 3.17: Flowchart for computing the generalized inverse of the hysteretic mapping between SMA tendon temperatures and bending angle.](image-url)
Because the inverse HRNN solution is not unique, the observed temperatures may achieve the measured bending angle without tracking the actual SMA temperatures. While maintaining a minimum level of power in the inactive tendon can prevent slack and decrease response time [51], structural plant damage can occur if both tendons are actuated simultaneously. To address this tradeoff, a dual voltage limit $V_{dual}$ is imposed. When both tendon voltages exceed this limit, they are both decreased proportionally until the lower voltage drops below $V_{dual}$.

### 3.3.3 TISO Experimental Methods

Experimental validation of the antagonistic IISMC was conducted on the flexible beam system shown in Figure 3.15a. The system construction is identical to the one described in Section 3.2.3.2, but includes an additional (antagonistic) SMA tendon. For real-time implementation, the antagonistic IISMC was programmed in Visual Studio C++ (Microsoft Corporation, Redmond, WA) and run at 50 Hz. During operation, control commands for
both tendons are sent through serial communication to a microcontroller which regulates the voltage sent from the power supply (Agilent E3620A, Agilent Technologies, Santa Clara, CA) to the SMA tendons using a custom pulse width modulation (PWM) amplifier. Electromagnetic position sensing (trakSTAR 3D Magnetic Tracking System, Ascension Technology Corporation, Burlington, VT) at the base and tip of the flexible beam system is used for real-time feedback. A schematic of the experimental setup is shown in Figure 3.19.

Bending angle data for the SMA+ and SMA− tendons was independently acquired for HRNN training using the plant inputs of Figure 3.20. SMA tendon temperatures were calculated using (3.42) and (3.43). The HRNNs were then trained to optimize network weights; weights were then imported into the control code.
3.3.4 TISO Experimental Results

The HRNN weights for both tendons were optimized using 3213 equally-spaced neurons (\( 0 \leq T_{r,j} \leq 1, \ -1 \leq T_{s,j} \leq 1 \), and using only neurons where \( T_{r,j} \geq T_{s,j} \)). The training and testing data along with the trained HRNN predictions are shown in Figure 3.21. The optimal solution reduced initial training and testing costs of approximately 0.5 to training costs of \( 1.63 \times 10^{-4} \) and \( 4.67 \times 10^{-5} \) and testing costs of \( 1.01 \times 10^{-3} \) and \( 3.02 \times 10^{-4} \) for the SMA+ and SMA− tendons, respectively. The constraint equation on network weights (3.50) was satisfied to within 0.02 for the SMA− tendon and 0.01 for the SMA+ tendon.

Figure 3.20: Plant inputs to the active tendon for generating HRNN training and testing data for the antagonistic controller.
Controller performance was evaluated using sinusoidal and triangular reference trajectories of varying amplitude, Figure 3.22, to verify HRNN minor-loop tracking capabilities. The observed temperatures tracked the reference temperatures well, Figure 3.22c, resulting in precise regulation of the reference trajectory, Figure 3.22a. The inactive tendon voltage, Figure 3.22b, was controlled to the dual voltage level $V_{\text{dual}} = 1.1V$ while the active tendon voltage remained unsaturated, ensuring fast response times when reference direction changed.
For comparison, a proportional-integral (PI) controller was implemented and tuned using a modified Ziegler-Nichols method [64]. The gains were then manually adjusted for enhanced performance, resulting in a proportional gain of $K_p = 5$ and an integral gain of $K_i = 16$. Figure 3.23 compares antagonistic IISMC and PI controller performance for sinusoidal bending angle reference trajectories of various frequencies. This figure demonstrates superior tracking performance for the IISMC over the PI controller. This is especially evident when the reference bending angle changes sign. The IISMC controller immediately compensates for residual temperatures in the opposing tendon while the PI controller does not, resulting in tracking error. Additionally, maintaining a low voltage level on the non-active tendon helps prevent tendon slack and thus allows faster response to
direction changes. Computing the RMS tracking error over one period, Table 3.3, the IISMC resulted in an average of 32.7 ± 17.7 % less error than the PI controller for the three reference trajectories tested. The IISMC resulted in 11.38 ± 1.3 % more RMS control than the PI controller, which is a direct consequence of applying voltage to both tendons simultaneously. However, minimizing tracking error, not control effort, is the primary objective here, and thus increased control costs are justified by shorter response times.

Figure 3.23: Experimental antagonistic IISMC and PI controller tracking performance against sinusoidal references of 0.033, 0.050, and 0.067 Hz frequencies.
Table 3.3: Performance measures comparing antagonistic IISMC and PI controller for sinusoidal reference trajectories of various frequencies.

<table>
<thead>
<tr>
<th>Reference Trajectory</th>
<th>RMS Tracking Error (deg)</th>
<th>RMS Control (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IISMC</td>
<td>PI</td>
</tr>
<tr>
<td>Sinusoidal, 0.033 Hz</td>
<td>0.44</td>
<td>0.80</td>
</tr>
<tr>
<td>Sinusoidal, 0.050 Hz</td>
<td>0.57</td>
<td>0.96</td>
</tr>
<tr>
<td>Sinusoidal, 0.067 Hz</td>
<td>0.99</td>
<td>1.13</td>
</tr>
</tbody>
</table>

An additional set of experiments was conducted to demonstrate IISMC robustness to disturbances and parameter variations. First, a small mass (1 g, 1 cm$^3$) was added to the tip of the flexible beam system resulting in a 33% increase in effective system mass. The added mass significantly lowered the beam’s natural frequency, resulting in large oscillations which the IISMC effectively attenuated to maintain precise trajectory tracking (Figure 3.24d-f). The PI controller failed to attenuate these oscillations near $\theta = 0$ (see Figure 3.24e at 10 s and 30 s). As before, the IISMC methodology maintains voltage on both tendons, shown here to increase its responsiveness to reversals in tendon role (active versus non-active).

Next the mass was removed and a small blower (JMC Products, Austin, TX, Model 97332611-3-2, 0.0123 m$^3$/s, placed 40 cm away) was directed toward the flexible beam system. The blower not only introduced a disturbance force, it also provided active cooling for the SMA tendons, which changed the heat loss coefficient $h$ used in the temperature model for the IISMC. The results (Table 3.4, Figure 3.24g-i for air flow normal to the bending plane and Figure 3.24j-l for air flow opposing positive bending) demonstrate IISMC robustness to variations in temperature model parameters.
Figure 3.24: IISMC and PI controller disturbance rejection results: (a-c) no disturbance, (d-f) added mass, (g-i) active air cooling normal to beam actuation plane, and (j-l) active air cooling opposing positive bending.

Table 3.4: Performance measures comparing antagonistic IISMC and PI controller for disturbance rejection tests using a 0.05 Hz sinusoidal reference trajectory.

<table>
<thead>
<tr>
<th>Disturbance</th>
<th>RMS Tracking Error (deg)</th>
<th>RMS Control (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IISMC</td>
<td>PI</td>
</tr>
<tr>
<td>No disturbance</td>
<td>0.57</td>
<td>0.96</td>
</tr>
<tr>
<td>Added mass</td>
<td>0.71</td>
<td>1.14</td>
</tr>
<tr>
<td>Active air cooling normal to the beam actuation plane</td>
<td>1.08</td>
<td>1.50</td>
</tr>
<tr>
<td>Active air cooling opposing positive bending</td>
<td>1.52</td>
<td>1.72</td>
</tr>
</tbody>
</table>
Not only does the antagonistic SMA configuration enable bi-directional actuation, it offers increased bandwidth compared to the single tendon configuration. To demonstrate this performance benefit, both configurations were experimentally evaluated using offset triangle wave reference trajectories. Tracking results are presented in Figure 3.25 for increasing reference frequencies (0.125 Hz, 0.25 Hz, and 0.5 Hz) and peak-to-peak amplitudes (20° and 30°). At 0.125 Hz (Figure 3.25a-b), tracking differences between the two systems are negligible. However, at 0.25 Hz (Figure 3.25c-d), the bandwidth advantages of antagonistic actuation are evident. The single tendon response develops phase lag between the reference and measured bending angles, and the range of motion is compromised. The antagonistic configuration accommodates changes in bending direction more effectively, achieving a broader range of motion and less tracking error. At 0.5 Hz (Figure 3.25e-f), both systems fail to track the trajectory without significant phase lag and amplitude loss; however, the antagonistic system does reach higher peaks than the single tendon system. This performance benefit would be more significant if the SMA tendons were enclosed, as Joule heating (of an antagonistic tendon) tends to be significantly faster than conductive/convective cooling of a single enclosed tendon.
3.4 Conclusions

This chapter discussed the development of an indirect intelligent sliding mode controller (IISMC) for single tendon and antagonist SMA bending actuators. The IISMC incorporates a hysteretic recurrent neural network (HRNN) for hysteresis compensation to track the bending angle of a flexible beam system. Single input, single output and multivariable implementations were presented. The controller effectively compensated for system hysteresis, precisely tracking reference trajectories of varying shape, amplitude, and frequency for both the single and antagonist configurations. The IISMC was compared to an optimized PI controller, demonstrating superior tracking performance in all cases. For the antagonistic case, the largest difference in tracking performance is seen when the non-active tendon became active. The IISMC was able to quickly and accurately respond by reducing or eliminating tendon slack and by accounting for residual forces in the opposing tendon.
Finally, the IISMC demonstrated robustness in the presence of external disturbances and environmentally-induced parameter variations.
Chapter 4

Single Segment Catheter Control

4.1 Introduction

The SMA-actuated catheter is a direct extension of the antagonistic bending actuator described in Chapter 3: it consists of two bending segments each actuated by two antagonistic SMA tendon pairs. However, the robotic catheter proved to be significantly more difficult to control than the reduced-order system. This can be attributed to several factors, which are detailed in Chapter 2 through an analysis of open-loop catheter behavior. First, the SMA-actuated catheter is an eight input, four output multivariable system. Coupling exists between both the segments and the individual tendons. The system exhibits very low bandwidth actuation and highly nonlinear, hysteretic behavior, with steep, wide hysteresis curves (a direct result of using larger diameter SMA tendons with increased slack compared to the reduced-order system). Additionally, the system is uncertain, time-varying, and not fully modeled. In vivo position sensing has limited resolution, bandwidth, and accuracy, further complicating control. Finally, the catheter must overcome endocardial hemodynamic disturbances while not over-stressing cardiac tissue.

Controller performance is evaluated based on several criteria. First, stability is a critical requirement; an unstable catheter is unsafe for the patient. The controller must maintain catheter stability regardless of reference trajectory, which is unknown a priori. Second, tracking performance is critical to ensure safe endocardial maneuverability and precise ablation lesion placement. Robustness in the presence of model uncertainties,
parameter variations, and sensing inconsistency is required. Additionally, accurate control in the presence of hemodynamic and contact disturbances is necessary. Finally, ease of implementation is considered, to minimize both computational requirements and data collection requirements for generating models.

This chapter discusses the development, implementation, and evaluation of several strategies for controlling the distal bending segment of the SMA-actuated catheter (control will be extended to two segments, accounting for tendon coupling, in Chapter 5). All controllers (except the baseline PID controller) include a model for hysteresis compensation and utilize a control law based on the decoupled SMA temperature model derived in Section 2.5.3.1. Four initial controllers are experimentally tested, with their performance evaluated against the aforementioned criteria. Finally, a fifth controller is derived, combining the best features of each initial controller, and results are presented.

4.2 Initial Controllers

4.2.1 Controller Synthesis

A closed-loop controller was developed and implemented to track catheter reference angles. The bending and orientation angles of each segment \((\theta, \phi)\) are decoupled (Figure 4.1) into two planar bending angles \((\theta_x = \theta \cos \phi, \theta_y = \theta \sin \phi)\), each controlled by an antagonistic SMA tendon pair: \((SMA_{x+}, SMA_{x-})\) and \((SMA_{y+}, SMA_{y-})\). The complete system block diagram is shown in Figure 4.2. The following sections describe several potential control strategies for each antagonistic controller block.
Figure 4.1: Decoupling catheter bending and orientation angles into perpendicular planar bending angles for control by each antagonistic SMA actuator pair.

Figure 4.2: Single segment catheter block diagram showing decoupling for antagonistic SMA pair control and temperature-dependent saturation of controller-generated voltages.
4.2.1.1 Proportional+Integral+Derivative Controller

A proportional+integral+derivative (PID) controller was implemented as a baseline comparison; any model-based controller must perform better than this industry standard to justify implementation. The PID output voltage is

\[ V_{PID}(t) = k_p e(t) + k_i \int_0^t e(\tau) d\tau + k_d \frac{de(t)}{dt} \]  

(4.1)

where \( e(t) = \theta_r(t) - \theta_m(t) \). Because the control law is synthesized for an antagonistic SMA tendon pair, voltage is applied either to the SMA\(_+\) or SMA\(_-\) tendon, depending on the sign of \( V_{PID}(t) \).

4.2.1.2 Feedforward Controller

The first model-based approach considered combines a feedforward (FF) controller with a PID (4.1) feedback loop:

\[ V(t) = V_{FF}(t) + V_{PID}(t) \]  

(4.2)

The feedforward term \( V_{FF}(t) \) is an estimate of the steady-state voltage required to maintain the reference angle. From the temperature dynamics (2.11)

\[ V_{FF}(t) = \frac{h}{\gamma} \left( T_r(t) - T_\infty \right) \]  

(4.3)

where \( T_r(t) \) is the SMA temperature corresponding to the reference bending angle.

The relationship between SMA temperature and bending angle (Figure 2.9b) is highly nonlinear and hysteretic. Several models have been proposed to map this relationship [55], [65], [66], [67], [68], however most are computationally intensive, require extensive experimental data, and/or are limited in accuracy. Recalling the aforementioned model uncertainties, hysteresis loop variations, and unmodeled dynamics, it is unlikely these highly
complex models would be accurate locally, and they certainly would not be accurate over the entire range of motion. Therefore, their sophistication does not justify the implementation challenges.

Practically, a simple model which navigates the catheter to the general area, and compensates for hysteresis, should suffice; the added PID term will ensure robustness to modeling errors. Simple linear models, one for heating \( h \) and one for cooling \( c \), are used for this purpose

\[
T_{b,r} = \frac{\theta}{m} + \alpha \\
T_{c,r} = \frac{\theta}{m} + \beta
\]  

where \( m \) is the slope of the outer hysteresis loop (Figure 4.3a), and \( \alpha \) and \( \beta \) are the temperature axis intercepts for the heating and cooling lines, respectively. Minor hysteresis loops are ignored. Alternatively, a critical temperature \( T_c \) is computed using the outer-loop relationships (4.4) and the tracking error \( e(t) \)

\[
T_c(t) = \beta + \frac{\alpha - T_{c,\text{prev}}}{1 + \exp \left[ \chi (\varepsilon - e(t)) \right]} + \frac{T_{c,\text{prev}} - \beta}{1 + \exp \left[ \chi (-\varepsilon - e(t)) \right]}
\]  

Parameters \( \varepsilon \) and \( \chi \) establish smooth transitions between the outer-loop heating and cooling curves. The critical temperature is used to calculate the reference temperature for (4.3).

\[
T_r(t) = \frac{\theta_r(t)}{m} + T_c(t)
\]  

Each time a new tendon is activated, the heating constant \( \alpha \) is updated online to account for hysteresis curve variations. Assuming constant hysteresis width \( \delta \), the cooling constant \( \beta = \alpha - \delta \) is also updated. Fast response is emphasized through the initial condition
(4.7) $$\alpha_0 = \frac{\gamma}{h} V_{\text{max}}^2 + T_x$$

where $V_{\text{max}}$ is the power supply voltage limit.

The antagonistic feedforward controller block diagram is shown in Figure 4.3b. The feedforward voltage (4.3) is calculated for both antagonistic tendons, and the PID term is added (or subtracted). The maximum voltage is applied; the opposing tendon is deactivated.

Because the feedforward term controls the system to the general location, the PID term can work locally, with gains derived to correct only small tracking errors. Additionally, this controller does not rely on the integral gain to hold the required steady-state voltage, and therefore limits can be placed on the integral term contribution to prevent unwanted windup in the presence of large contact disturbances.

4.2.1.3 Indirect Intelligent Sliding Model Controller

The indirect intelligent sliding mode controller (IISMC), described in Chapter 3, was extended for evaluation on the catheter. This method utilizes a complete hysteresis model (using the HRNN to map both major and minor hysteresis loops), but incorporates a variable
structure control architecture to account for model uncertainties and parameter variations. The control architecture, described in detail in Chapter 3, was modified slightly (see Figure 4.4b) to require only one HRNN for each antagonistic controller, reducing computational burden.

The HRNN activation function (3.16), developed in [56], was modified to accommodate antagonistic training data (Figure 4.4a)

\[
f_i(\hat{T}(q)) = \frac{2}{1+\exp(f(q-1)\chi)} \left[ \frac{-f_i(\hat{T}(q-1))}{1+\exp\left((T_{F,j}-\hat{T}(q))\chi\right)} \right]
\]

\[
+ \frac{2}{1+\exp(-f(q-1)\chi)} \left[ \frac{-f_i(\hat{T}(q-1))}{1+\exp\left((\hat{T}(q)-T_{R,j})\chi\right)} \right] + f_i(\hat{T}(q-1))
\]

(4.8)

where the inputs and outputs are now signed values (positive for the \(SMA_+\) tendon and negative for \(SMA_-\) tendon).

![Figure 4.4: Antagonistic IISMC controller: (a) catheter input-output hysteresis curves modeled by the HRNN; (b) IISMC architecture modified to require only one HRNN](image-url)
4.2.1.4 Adaptive Controller

A self-tuning adaptive controller (ADAPT) [59] was developed using the HRNN, which is adapted online to account for hysteresis loop variations.

The control laws are derived from the SMA temperature dynamics (2.11)

\[
V_+ (t) = \sqrt{\frac{1}{\gamma} \left[ \dot{T}_{\text{e}} (t) + h (T_{\text{e}} (t) - T_\infty) \right]}
\]

\[
V_- (t) = \sqrt{\frac{1}{\gamma} \left[ \dot{T}_{\text{r}} (t) + h (T_{\text{r}} (t) - T_\infty) \right]}
\]

(4.9)

Similar to the IISMC, an inverse HRNN computes reference temperatures. However, now the transition temperatures \( T_e \) and \( T_r \) are adapted online

\[
\hat{\theta} = \text{HRNN} \left( \hat{T}, T_F + \eta, T_R + \eta \right)
\]

(4.10)

The adaptive variable \( \eta \) is updated using the model prediction \( \hat{\theta} \) and the measured angle \( \theta_m \)

\[
\dot{\eta} = p \left( \hat{\theta} - \theta_m \right)
\]

(4.11)

The block diagram for the self-tuning adaptive controller is shown in Figure 4.5.

---

**Figure 4.5:** Adaptive controller block diagram showing inverse HRNN to predict SMA temperature and adaption of the hysteresis model
4.2.2 Experimental Results

Figure 4.6, Figure 4.7, and Figure 4.8 compare catheter performance using the four control architectures. The best step response (Figure 4.6, Table 4.1) was seen using the feedforward controller; settling time (6.3 sec), rise time (3.0 sec), overshoot (1.5°), and steady-state error (<1°) were all desirable. The best controller for dynamic tracking (Figure 4.7, Figure 4.8) is less obvious. Figure 4.9 compares average error and maximum tracking error (or maximum overshoot for step changes) for eight reference trajectories. The feedforward controller demonstrated the lowest average tracking error, while the PID and IISMC controllers demonstrated the lowest maximum tracking error. However, all controllers demonstrated unacceptable maximum tracking error (up to 30-60°).

![Figure 4.6: Single segment catheter step response](image)

Figure 4.6: Single segment catheter step response: (a) polar plot showing location of reference, and (b) measured bending and orientation angle for each controller.

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>FF</td>
<td>3.0</td>
<td>1.5</td>
<td>6.3</td>
<td>0.97</td>
</tr>
<tr>
<td>IISMC</td>
<td>4.5</td>
<td>0.36</td>
<td>9.5</td>
<td>0.74</td>
</tr>
<tr>
<td>PID</td>
<td>4.6</td>
<td>-</td>
<td>15</td>
<td>1.8</td>
</tr>
<tr>
<td>ADAPT</td>
<td>2.9</td>
<td>2.5</td>
<td>14</td>
<td>7.6</td>
</tr>
</tbody>
</table>
Figure 4.7: Single segment tracking results for a variety of reference trajectories: (a), (c), and (e) polar plots showing reference trajectories; (b), (d), and (f) corresponding tracking results for each controller.
Figure 4.8: Single segment tracking results for a circular reference: (a) polar plot showing reference trajectory, (b) tracking results, and (c) control effort for each architecture.
Results presented to this point were conducted in an open tank; no physical limitations impeded catheter tracking. In a surgical environment, the joystick-supplied reference may be unattainable (e.g. the reference sends the catheter into an atrial wall). When this occurs, the catheter must not over-stress the atrial tissue. Additionally, the catheter must quickly respond when the reference changes direction. Control in the presence of disturbances was evaluated using a plastic heart model, Figure 4.10a. Figure 4.10b shows catheter response to an unattainable reference followed by an attainable reference. The impeded path (0-10 seconds) cannot be tracked using any of the controllers (as expected). However, because the PID controller relies heavily on integral control for catheter stabilization, windup occurs at the disturbance (Figure 4.10c) which both over-stresses the heart wall and increases response times to direction changes. The unimpeded path response (10+ seconds) shows that nearly 15 seconds is required for the PID controller to recover the
windup before the catheter reaches the new reference. The catheter response using the feedforward controller was quickest, required only 5 seconds to reach the new reference.

Figure 4.10: Control in the presence of disturbances: (a) catheter inside plastic pig heart used to mimic in vivo disturbances, (b) tracking results, and (c) control effort

The four controllers were rated in several categories (e.g. tracking, robustness, disturbance rejection) to compare overall performance (Table 4.2). The feedforward controller demonstrates fast response times, superior tracking in the presence of disturbances, and requires only a simple model; however, its main limitation is robustness to switches between active tendons. The IISMC is robust and accommodates low bandwidth actuation; however, the control law is based on a discontinuous reference temperature and modeling is
complex. The PID controller is simple, robust, and requires no model; however, it demonstrates slow response times and poor tracking in the presence of disturbances. The adaptive controller responded well to some references; however it failed to demonstrate good global tracking performance.

All controllers presented had advantages, but no one controller exhibited desired performance in every category. The next section presents a hybrid control approach which combines the best features of each of these initially-evaluated controllers.

<table>
<thead>
<tr>
<th></th>
<th>FF</th>
<th>IISMC</th>
<th>PID</th>
<th>ADAPT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tracking Performance</td>
<td>Good</td>
<td>Okay</td>
<td>Okay</td>
<td>Okay</td>
</tr>
<tr>
<td>Steady-State Accuracy</td>
<td>Okay</td>
<td>Good</td>
<td>Good</td>
<td>Poor</td>
</tr>
<tr>
<td>Responsiveness</td>
<td>Good</td>
<td>Okay</td>
<td>Poor</td>
<td>Okay</td>
</tr>
<tr>
<td>Robustness</td>
<td>Okay</td>
<td>Good</td>
<td>Good</td>
<td>Poor</td>
</tr>
<tr>
<td>Stability</td>
<td>Okay</td>
<td>Okay</td>
<td>Good</td>
<td>Okay</td>
</tr>
<tr>
<td>Contact Disturbance Rejection</td>
<td>Good</td>
<td>Okay</td>
<td>Poor</td>
<td>Poor</td>
</tr>
<tr>
<td>Hemodynamic Disturbance Rejection</td>
<td>Good</td>
<td>Good</td>
<td>Good</td>
<td>Okay</td>
</tr>
<tr>
<td>Ease of Implementation</td>
<td>Okay</td>
<td>Poor</td>
<td>Good</td>
<td>Poor</td>
</tr>
</tbody>
</table>

4.3 Hybrid Control Strategy

4.3.1 Controller Synthesis

The results of Section 4.2 were used to develop a fifth controller, which combines the best features of each initially-evaluated controller. Each antagonistic SMA pair is controlled using a hybrid approach combining three terms: a feedforward (FF) term for hysteresis compensation, a proportional+integral+derivative (PID) term for robustness, and a proportional cubed (P3) term to decrease response time (Figure 4.11b):

\[ V(t) = V_{FF}(t) + V_{PID}(t) + V_{P3}(t) \]  \hspace{1cm} (4.12)
The feedforward term $V_{FF}(t)$ is an estimate of the steady-state voltage required to maintain the reference angle. Considering only outer-loop hysteresis data, Figure 4.11a, two analytical functions can be derived, one for heating and one for cooling

$$V_{ss,h} = a_{h1} \tan \left[ a_{h2} \left( \theta - a_{h3} \right) \right] + a_{h4}$$
$$V_{ss,c} = a_{c1} \tan \left[ a_{c2} \left( \theta - a_{c3} \right) \right] + a_{c4}$$

(4.13)

where the coefficients $a_{hi}$ and $a_{ci}$ are optimized to fit open-loop experimental data. For simplicity, minor hysteresis loops are ignored. Alternatively, the feedforward voltage is computed using the outer-loop relationships (4.13) and the tracking error $e(t) = \theta_r(t) - \theta_m(t)$

$$V_{FF}(t) = V_{ss,c} + \frac{V_{ss,h} - V_{FF,prev}}{1 + \exp \left( \chi (e(t)) \right)} - \frac{V_{ss,c} - V_{FF,prev}}{1 + \exp \left( \chi (-e(t)) \right)}$$

(4.14)

Parameters $\chi$ and $\epsilon$ establish smooth transitions between the outer-loop heating and cooling curves. This model balances the tradeoff between model complexity, robustness, and accuracy. The model is simple, requires only outer-loop hysteresis data, provides a smooth transition between heating and cooling and between tendons, and is reasonably accurate.

Figure 4.11: Hybrid control approach: (a) outer-loop hysteresis curve relating steady-state voltage to bending angle for an antagonistic SMA tendon pair; (b) antagonistic controller architecture
Due to the presence of unmodeled dynamics, parametric uncertainties, external disturbances, and measurement errors, feedforward control alone cannot provide the required tracking performance. For robustness, a PID term is added

\[
V_{PID}(t) = k_p e(t) + k_i \int_0^t e(\tau) d\tau + k_d \frac{de(t)}{dt} \tag{4.15}
\]

Additionally, a proportional cubed term [26] is incorporated to decrease response times by rapidly heating tendons when tracking error is large

\[
V_{p3}(t) = k_3 e(t)^3 \tag{4.16}
\]

Because the control law is synthesized for an antagonistic SMA tendon pair, voltage is applied to either the \( \text{SMA}_+ \) or \( \text{SMA}_- \) tendon, depending on the sign of \( V(t) \).

### 4.3.2 Experimental Results

SMA-actuated catheter control via the hybrid approach was demonstrated using several reference trajectories. A step response, Figure 4.12a, shows fast response times, low overshoot, and stable steady-state behavior. Tracking results, Figure 4.12b-c, show fast response times, stable regulation, and low error. Robustness to reference trajectories that are unknown \textit{a priori} (and can vary at any rate) was demonstrated.
Figure 4.12: Hybrid control strategy experimental tracking results: (a) step response, (b) mixed reference tracking, and (c) circle tracking at increasing rates (left to right: 120s, 60s, 30s, 15s).

4.4 Conclusions

This chapter detailed the development of several nonlinear control strategies for the SMA-actuated catheter distal segment. Three initial controllers were developed and implemented, each designed to compensate for system hysteresis and based on an SMA temperature model. A fourth, PID controller was also implemented for baseline comparison. The controllers were evaluated against each other and against several performance requirements. While all
the controllers had advantages, no single controller was found to satisfy all performance goals.

A fifth controller was developed which incorporates the best features of each initial controller. This hybrid approach includes a feedforward term which provides a nominal steady-state voltage, a PID term for robustness, and a P3 term for decreased response times. Experimental results demonstrated robust and accurate control for a wide variety of reference trajectories.

The hybrid control approach presented in this chapter utilizes an input-output hysteresis model derived from steady-state voltage and bending angle data; it is assumed that steady-state voltage and steady-state temperature are directly related. In the next chapter, control is extended to two segments, temperature coupling between adjacent tendons is considered, and the hysteresis model is modified accordingly. Additionally, the feedforward term is modified to account for force coupling between bending segments.
Chapter 5

Dual Segment Catheter Path Optimization and Control

5.1 Introduction

Chapter 4 detailed the development of several nonlinear control strategies for the SMA-actuated catheter distal bending segment, treating the proximal segment as passive. In this chapter, the full two segment catheter is considered. This introduces new control challenges, but also significantly increases catheter maneuverability and allows for increased control over catheter position and orientation.

The ability to create continuous, transmural lesions is the fundamental performance objective of this SMA-actuated catheter. This requires not only precise position control of the catheter tip, but also sufficient surface contact, most easily achieved through maintaining the catheter tip perpendicular to the atrial tissue. Catheter tip position \((x_D, y_D, z_D)\) (Figure 2.4) is kinematically dependent on the proximal \((P)\) and distal \((D)\) bending \((\theta_P, \theta_D)\) and orientation \((\phi_P, \phi_D)\) angles. Multiple combinations of these angles can achieve a given tip position (Figure 5.1); this redundancy enables limited control of catheter tip orientation. Optimization can be used to determine the most advantageous tip position and orientation to create transmural lesions in minimal time.
Previous closed-loop control methods relied on user-defined reference bending and orientation angles (the only controllable quantities in the single segment case); now reference trajectories can be provided in terms of desired catheter tip positions. This chapter presents a trajectory optimization and control strategy for the dual segment SMA-actuated catheter, with emphasis on the efficient creation of continuous, transmural lesions in atrial tissue during endocardial radiofrequency ablation. The proposed clinical procedure for creating lesions is as follows:

I. Using the catheter’s joystick, the physician navigates the catheter tip and specifies three critical locations on the endoatrial surface.
II. An algorithm computes an ablation path through these physician-specified locations.
III. Optimal bending and orientation angles are generated for several locations along the path.
IV. Real-time control algorithms ensure adequate tracking of the reference trajectory during ablation.

Steps I and II are discussed in Section 5.2.1. For Step III, the optimization problem is presented in Section 5.2.3 and the solution method is detailed in Section 5.2.4.
For real-time control, the hybrid approach presented in Section 4.3 is extended to accommodate force coupling between bending segments. Additionally, a temperature-based hybrid control approach is derived which accounts for thermal coupling between adjacent tendons. Path optimization and control results are presented for the dual segment catheter.

5.2 Path Optimization

5.2.1 Ablation Path

The methods described here begin with three physician-specified points on the atrial wall \((A, B, \text{ and } C)\), expressed in global \(XYZ\) coordinates (the global coordinate system is located at the base of the proximal segment, Figure 5.2a). An ablation path is constructed on the plane containing these points. This plane is defined using the surface normal

\[
\mathbf{n} = \mathbf{r}_{AB} \times \mathbf{r}_{AC}
\]

\[
= \left[ (y_B - y_A)(z_C - z_A) - (y_C - y_A)(z_B - z_A) \right] \mathbf{i}
\]

\[
- \left[ (x_B - x_A)(z_C - z_A) - (x_C - x_A)(z_B - z_A) \right] \mathbf{j}
\]

\[
+ \left[ (x_B - x_A)(y_C - y_A) - (x_C - x_A)(y_B - y_A) \right] \mathbf{k}
\]

\[
= n_x \mathbf{i} + n_y \mathbf{j} + n_z \mathbf{k}
\]

where \(\mathbf{r}_{AB}\) and \(\mathbf{r}_{AC}\) are vectors directed from point \(A\) to points \(B\) and \(C\), respectively. The ablation plane is constructed such that:

\[
n_x (x - x_A) + n_y (y - y_A) + n_z (z - z_A) = 0
\]

This ablation plane defines a new local \(\hat{X}\hat{Y}\hat{Z}\) coordinate system, shown in Figure 5.2. The following three rotation matrices describe the transformation from \(\hat{X}\hat{Y}\hat{Z}\) to \(XYZ\) coordinates:
where

\[
\begin{align*}
\alpha &= \tan^{-1}\left(\frac{y_C - y_A}{x_C - x_A}\right) \\
\beta &= -\tan^{-1}\left(\frac{z_C - z_A}{\sqrt{(x_C - x_A)^2 + (y_C - y_A)^2}}\right) \\
\psi &= \cos^{-1}\left(\frac{z' \cdot n}{n}\right)
\end{align*}
\]

and where \( z' = \cos(\alpha)\sin(\beta)i + \sin(\alpha)\sin(\beta)j + \cos(\beta)k \). Defining the total rotation matrix \( \mathbf{R}_{XYZ} = \mathbf{R}_Z(\alpha)\mathbf{R}_Y(\beta)\mathbf{R}_X(\psi) \) gives the transformation matrix

\[
\begin{bmatrix}
x_A \\
y_A \\
z_A
\end{bmatrix} = \begin{bmatrix}
x_{\tilde{X}} \\
y_{\tilde{Y}} \\
z_{\tilde{Z}}
\end{bmatrix} = \begin{bmatrix}
c_a c_\beta & -s_a c_\beta & s_\alpha s_\beta + c_\alpha s_\psi \\
s_a s_\beta + c_a s_\psi & c_a c_\beta & s_\alpha s_\psi + c_\alpha s_\beta c_\psi \\
-s_\beta & c_\beta s_\psi & c_\beta c_\psi
\end{bmatrix}
\begin{bmatrix}
x_A \\
y_A \\
z_A
\end{bmatrix}
\]

(5.5)

The inverse transformation is then used to compute the local coordinates:
where

\[
\begin{bmatrix}
\hat{x} \\
\hat{y} \\
\hat{z} \\
1
\end{bmatrix} = ^{xyz}T_{\hat{xyz}}^{-1} \begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} \tag{5.6}
\]

Two ablation paths, from \( A \) to \( B \) \((q = 1)\) and from \( B \) to \( C \) \((q = 2)\), are created using cubic splines based on the distance \( r_q(k) \) from the midpoint \( M \) of physician-specified points \( A \) and \( C \) (Figure 5.2b) to the \( k^{th} \) path point:

\[
\begin{align*}
\hat{x}(k) &= \hat{x}_M - r_q(k) \cos(\varphi_q(k)) \\
\hat{y}(k) &= r_q(k) \sin(\varphi_q(k)) \\
\end{align*} \tag{5.8}
\]

where \( \varphi_A \leq \varphi_1(k) \leq \varphi_B \), and \( \varphi_B \leq \varphi_2(k) \leq \varphi_C \). The physician-specified points expressed in transformed coordinates \((\hat{x}_A, \hat{y}_A, \hat{z}_A)\), \((\hat{x}_B, \hat{y}_B, \hat{z}_B)\), and \((\hat{x}_C, \hat{y}_C, \hat{z}_C)\) are used to define boundary conditions.
\[ r_A = \hat{x}_M = \frac{\hat{x}_C}{2}, \quad r'_A = 0, \quad \varphi_A = 0 \]

\[ r_B = \sqrt{\left(\hat{x}_M - \hat{x}_B\right)^2 + \left(-\hat{y}_B\right)^2}, \quad r'_B = 0, \quad \varphi_B = \tan^{-1}\left(\frac{\hat{y}_B}{\hat{x}_M - \hat{x}_B}\right) \quad (5.9) \]

\[ r_C = \hat{x}_C - \hat{x}_M, \quad r'_C = 0, \quad \varphi_C = \pi \cdot \text{sign}(\varphi_B) \]

which are used to calculate the coefficients of (5.8):

\[ a_1 = \frac{2(r_A - r_B)}{\varphi_B^3} \]

\[ b_1 = -\frac{3}{2} \frac{\varphi_B a_i}{\varphi_C} \]

\[ c_1 = 0 \]

\[ d_1 = r_A \]

\[ a_2 = -\frac{c_2}{2 \varphi_B \varphi_C} \]

\[ b_2 = -\left(\frac{\varphi_B + \varphi_C}{2 \varphi_B \varphi_C}\right) c_2 \]

\[ c_2 = \frac{r_B - r_C}{-\frac{\varphi_B^2}{6 \varphi_C} + \frac{\varphi_C^2}{6 \varphi_B} + \varphi_B - \frac{\varphi_C}{2}} \]

\[ d_2 = r_C + \left(\frac{\varphi_C^2}{6 \varphi_B} - \frac{\varphi_C}{2}\right) c_2 \quad (5.10) \]

The reference ablation path is specified by transforming from \((\hat{x}(k), \hat{y}(k), \hat{z}(k))\) to \((x(k), y(k), z(k))\) coordinates using (5.5).
5.2.2 Catheter Kinematics

The catheter tip measurements and the reference ablation path are specified in global Cartesian coordinates \((x, y, z)\), while control is based on the system’s generalized coordinates \((\theta_p, \phi_p, \theta_D, \phi_D)\), requiring conversion between the two coordinate systems. First, inverse kinematics between the tip coordinates of the proximal segment \((x_p, y_p, z_p)\) and the generalized coordinates \((\theta_p, \phi_p)\) are derived. Because the proximal segment’s base establishes the origin of the global coordinate system, and because proximal segment bending does not exceed 90°, a straightforward trigonometric relationship exists (Figure 5.3):

\[
\theta_p = \tan^{-1}\left( \frac{2z_p\sqrt{x_p^2 + y_p^2}}{z_p^2 - x_p^2 - y_p^2} \right)
\] (5.11)
\[ \phi_p = \tan^{-1}\left( \frac{y_p}{x_p} \right) \]  

(5.12)

Figure 5.3: Conversion from proximal tip global Cartesian coordinates to proximal segment generalized coordinates.

For the distal segment, the global coordinates \((x_D, y_D, z_D)\) must be transformed to local coordinates \((\tilde{x}_D, \tilde{y}_D, \tilde{z}_D)\), with origin at the base of the distal segment (the tip of the proximal segment, Figure 5.4):

\[
\begin{bmatrix}
\tilde{x}_D \\
\tilde{y}_D \\
\tilde{z}_D \\
1
\end{bmatrix} = \begin{bmatrix} x_D \\ y_D \\ z_D \\ 1 \end{bmatrix} \mathbf{T}^{-1}_{XYZ}^{ZYX} \mathbf{T}_{XYZ}^{XYZ}
\]

(5.13)

The rotation matrices required for the transformation matrix \(\mathbf{T}_{XYZ}^{XYZ}\) are
\[
R_Z(\phi_p) = \begin{bmatrix}
\cos \phi_p & -\sin \phi_p & 0 \\
\sin \phi_p & \cos \phi_p & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
\[
R_Y(\theta_p) = \begin{bmatrix}
\cos \theta_p & 0 & \sin \theta_p \\
0 & 1 & 0 \\
-\sin \theta_p & 0 & \cos \theta_p
\end{bmatrix}
\]
\[
R_Z(\xi - \phi_p) = \begin{bmatrix}
\cos (\xi - \phi_p) & -\sin (\xi - \phi_p) & 0 \\
\sin (\xi - \phi_p) & \cos (\xi - \phi_p) & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

where \(\xi\) is the offset angle between the proximal and distal tendons. Letting \(\eta = \xi - \phi_p\), the distal transformation matrix becomes

\[
^\text{XYZ}T_{^\text{ZZ}} = \begin{bmatrix}
^\text{XYZ}R_{\phi_p}^T & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
^\text{XYZ} \begin{bmatrix}
x_p \\
y_p \\
z_p
\end{bmatrix} = ^\text{XYZ} \begin{bmatrix}
x_p \\
y_p \\
z_p
\end{bmatrix} + ^\text{XYZ} \begin{bmatrix}
\eta \xi \phi \eta \\
\eta \xi \phi \eta \\
\eta \xi \phi \eta
\end{bmatrix}
\]

where \(^\text{XYZ}R_{\phi_p} = R_Z(\phi_p)R_Y(\theta_p)R_Z(\xi - \phi_p)\). The inverse transformation matrix is therefore

\[
^\text{XYZ}T_{^\text{ZZ}}^{-1} = \begin{bmatrix}
c_{\phi_p}c_{\theta_p}c_{\eta} - s_{\phi_p}s_{\eta} & s_{\phi_p}c_{\theta_p}c_{\eta} + c_{\phi_p}s_{\eta} & -s_{\phi_p}c_{\eta} \\
-c_{\phi_p}c_{\theta_p}s_{\eta} - s_{\phi_p}c_{\eta} & -s_{\phi_p}c_{\theta_p}s_{\eta} + c_{\phi_p}c_{\eta} & s_{\phi_p}c_{\eta} \\
c_{\phi_p}s_{\theta_p} & s_{\phi_p}s_{\theta_p} & -c_{\theta_p}
\end{bmatrix}
\]

\[
^\text{XYZ} \begin{bmatrix}
x_p \\
y_p \\
z_p
\end{bmatrix} = ^\text{XYZ} \begin{bmatrix}
x_p \\
y_p \\
z_p
\end{bmatrix} + \left(\begin{bmatrix}
\eta \xi \phi \eta \\
\eta \xi \phi \eta \\
\eta \xi \phi \eta
\end{bmatrix} - \begin{bmatrix}
\eta \xi \phi \eta \\
\eta \xi \phi \eta \\
\eta \xi \phi \eta
\end{bmatrix} \begin{bmatrix}
x_p \\
y_p \\
z_p
\end{bmatrix} - \begin{bmatrix}
\eta \xi \phi \eta \\
\eta \xi \phi \eta \\
\eta \xi \phi \eta
\end{bmatrix} \begin{bmatrix}
x_p \\
y_p \\
z_p
\end{bmatrix}
\]

101
Because the distal segment can bend up to 180°, two distinct kinematic cases must be considered. For $\theta_D < 90^\circ$, after conversion to local coordinates, the inverse kinematics are equivalent to the proximal segment case (5.11)-(5.12). However, for $90^\circ \leq \theta_D \leq 180^\circ$, a slight modification is required for the bending angle calculation (see Figure 5.5):

$$\theta_D = \pi - \tan^{-1} \left( \frac{2z_D \sqrt{x_D^2 + y_D^2}}{x_D^2 + y_D^2 - z_D^2} \right)$$

(5.17)

Because $\theta_D$ is unknown, however, the choice of algorithm must be based on the Cartesian coordinates:

Figure 5.4: Global $XYZ$ and local $\tilde{X}\tilde{Y}\tilde{Z}$ coordinate systems and coordinates for the distal segment, related by the proximal segment angles $\theta_p$ and $\phi_p$. 
\begin{align*}
\theta_D &= \begin{cases} 
\tan^{-1}\left(\frac{2\hat{z}_D \sqrt{\hat{x}_D^2 + \hat{y}_D^2}}{\hat{z}_D^2 - \hat{x}_D^2 - \hat{y}_D^2}\right) & \text{if } \hat{z}_D^2 \geq \hat{x}_D^2 + \hat{y}_D^2 \\
\pi - \tan^{-1}\left(\frac{2\hat{z}_D \sqrt{\hat{x}_D^2 + \hat{y}_D^2}}{\hat{x}_D^2 + \hat{y}_D^2 - \hat{z}_D^2}\right) & \text{else}
\end{cases} 
(5.18)
\end{align*}

$$
\phi_D = \tan^{-1}\left(\frac{\hat{y}_D}{\hat{x}_D}\right) 
(5.19)
$$

Figure 5.5: Conversion from distal tip local Cartesian coordinates to distal segment generalized coordinates for \(\theta_D > 90^\circ\)
The forward kinematics, illustrated in Figure 5.6 for the proximal segment, are:

\[
\begin{align*}
x_p &= \frac{L_p}{\theta_p} (1 - \cos \theta_p) \cos \phi_p \\
y_p &= \frac{L_p}{\theta_p} (1 - \sin \theta_p) \sin \phi_p \\
z_p &= \frac{L_p}{\theta_p} \sin \theta_p
\end{align*}
\]  

(5.20)

where \( L_p \) is the length of the proximal segment.


Figure 5.6: Conversion from proximal segment generalized coordinates to proximal tip global Cartesian coordinates

As noted previously, the forward kinematics for the distal segment must consider two cases based on distal bending angle. The conversion for \( \theta_d < 90^\circ \) is
\[\tilde{x}_D = \frac{L_D}{\theta_D} (1 - \cos \theta_D) \cos \phi_D\]
\[\tilde{y}_D = \frac{L_D}{\theta_D} (1 - \cos \theta_D) \sin \phi_D\]
\[\tilde{z}_D = \frac{L_D}{\theta_D} \sin \theta_D\]  \hspace{1cm} (5.21)

and for 90° ≤ θ_D ≤ 180° is

\[\tilde{x}_D = \frac{L_D}{\theta_D} (1 - \cos (\pi - \theta_D)) \cos \phi_D\]
\[\tilde{y}_D = \frac{L_D}{\theta_D} (1 - \cos (\pi - \theta_D)) \sin \phi_D\]
\[\tilde{z}_D = \frac{L_D}{\theta_D} \sin (\pi - \theta_D)\]  \hspace{1cm} (5.22)

where \(L_D\) is the length of the distal segment. These local coordinates are then transformed to global coordinates:

\[
\begin{bmatrix}
x_D \\
y_D \\
z_D \\
1
\end{bmatrix} = \begin{bmatrix}
\tilde{x}_D \\
\tilde{y}_D \\
\tilde{z}_D \\
1
\end{bmatrix} = x_{YZ} T_{XYZ} \begin{bmatrix}
x_D \\
y_D \\
z_D \\
1
\end{bmatrix} \hspace{1cm} (5.23)
\]

where \(x_{YZ} T_{XYZ}\) is given in (5.15).

### 5.2.3 Optimization Problem

Accurate ablation path tracking requires determination of reference angles \((\theta_{p,r}, \phi_{p,r}, \theta_{D,r}, \phi_{D,r})\) which achieve desired catheter tip positions \((x_{D,r}, y_{D,r}, z_{D,r})\). By combining (5.15) and (5.23), the relationship between these generalized and Cartesian coordinates is:
where \( (x_{D,r}, y_{D,r}, z_{D,r}) \) and \( (\tilde{x}_{D,r}, \tilde{y}_{D,r}, \tilde{z}_{D,r}) \) are functions of \( (\theta_{D,r}, \phi_{D,r}, \theta_{D,r}, \phi_{D,r}) \) given by (5.20) and (5.21)-(5.22), respectively.

Because (5.24) is three equations and four unknowns, additional performance criteria can be considered, including catheter tip orientation and path smoothness. An optimization problem is constructed to compute reference angles at each point \( k \) along the path. The cost function contains two key terms that penalize deviation from the path generated in Section 5.2.1:

\[
J_1(k) = \sqrt{\left(\tilde{x}_{D,r}(k) - \hat{x}(k)\right)^2 + \left(\tilde{y}_{D,r}(k) - \hat{y}(k)\right)^2}
\]

\[
J_2(k) = \sqrt{\left(\tilde{z}_{D,r}(k) - \hat{z}(k)\right)^2}
\]

where \( J_1(k) \) ensures planar ablation path adherence and \( J_2(k) \) ensures tissue contact via normal penetration.

A third cost function term can be added to penalize deviation of catheter tip orientation (represented by unit vector \( \mathbf{u}(k) = u_x(k)\mathbf{i} + u_y(k)\mathbf{j} + u_z(k)\mathbf{k} \)) from surface normal \( \mathbf{n} \) (5.1)

\[
J_3(k) = \sqrt{\left(\frac{\mathbf{n}}{||\mathbf{n}||} - \mathbf{u}(k)\right)^T \left(\frac{\mathbf{n}}{||\mathbf{n}||} - \mathbf{u}(k)\right)}
\]

where, in distal coordinates, the unit vector components are
\[\begin{align*}
\tilde{u}_x(k) &= \sin(\theta_{D,r}(k)) \cos(\phi_{D,r}(k)) \\
\tilde{u}_y(k) &= \sin(\theta_{D,r}(k)) \sin(\phi_{D,r}(k)) \\
\tilde{u}_z(k) &= \cos(\theta_{D,r}(k))
\end{align*}\] (5.28)

which are transformed to the global system using the rotation matrix \(\mathbf{R}_{XYZ}\) given in (5.15)

\[\begin{bmatrix}
\tilde{u}_x(k) \\
\tilde{u}_y(k) \\
\tilde{u}_z(k)
\end{bmatrix} = \mathbf{R}_{XYZ} \begin{bmatrix}
\tilde{u}_x(k) \\
\tilde{u}_y(k) \\
\tilde{u}_z(k)
\end{bmatrix}\] (5.29)

A fourth cost function can be used to penalize large variations in reference angles between adjacent path points

\[J_4(k) = \sqrt{(\theta_{p,r}(k) - \theta_{p,r}(k-1))^2 + (\phi_{p,r}(k) - \phi_{p,r}(k-1))^2}
+ (\theta_{D,r}(k) - \theta_{D,r}(k-1))^2 + (\phi_{D,r}(k) - \phi_{D,r}(k-1))^2}\] (5.30)

This function helps ensure that reference trajectories remain within the system bandwidth, and reduces the range of angles the catheter is required to track. The latter is critical during endocardial ablation, where contact with tissue limits the catheter’s range of motion.

Combining (5.25), (5.26), (5.27), and (5.30), the cost function becomes

\[J(k) = pJ_1(k) + qJ_2(k) + rJ_3(k) + sJ_4(k)\] (5.31)

where \(p, q, r,\) and \(s\) are weighting constants for each individual cost function. The optimization problem is thus to minimize \(J(k)\) subject to constraints on the reference angles:

Minimize:

\[J(k) = f(\theta_{p,r}(k), \phi_{p,r}(k), \theta_{D,r}(k), \phi_{D,r}(k))\] (5.32)
Subject To:

\[
0 \leq \theta_{p,r}(k) \leq \frac{\pi}{2}
\]
\[
0 \leq \phi_{p,r}(k) \leq 2\pi
\]
\[
0 \leq \theta_{d,r}(k) \leq \pi
\]
\[
0 \leq \phi_{d,r}(k) \leq 2\pi
\]

\[
\theta_{p,r}(k) \leq \max \left\{ \frac{\pi}{2} + \frac{r_{\text{stiff}}}{\cos \phi_{p,r}(k)} \theta_{d,r}(k) \cos (\phi_{d,r}(k) + \xi), \right. \\
\left. \frac{\pi}{2} + \frac{r_{\text{stiff}}}{\sin \phi_{p,r}(k)} \theta_{d,r}(k) \sin (\phi_{d,r}(k) + \xi) \right\}
\]

(5.33)

The first four constraints ensure the reference angles are within the catheter’s range of motion: 90° and 180° of omnidirectional bending for the proximal and distal segments, respectively. The final constraint accounts for coupling between the distal and proximal segments. Because the distal pull wires run through the proximal segment, actuation of the distal SMA tendons causes significant bending in the proximal segment:

\[
\theta_{p}^{(D)} = r_{\text{stiff}} \theta_{D}
\]

(5.34)

where \(r_{\text{stiff}}\) is the bending stiffness ratio between the distal and proximal segments. The actual bending in the proximal segment is assumed to be a linear combination of the bending caused by the proximal SMA tendons \(\theta_{p}^{(P)}\) and the distal SMA tendons \(\theta_{p}^{(D)}\):

\[
\theta_{p} = \theta_{p}^{(P)} + \theta_{p}^{(D)}
\]

(5.35)

Accounting for the direction of bending \((\phi_{p}, \phi_{d})\), the final constraint is derived to ensure the proximal SMA tendons can meet the actuation requirements of the reference angles.
5.2.4 Parallel Genetic Algorithms

Because genetic algorithms [69] are well-established methods for solving multimodal, nonlinear, and multivariate optimization problems, they are used here to minimize (5.32)-(5.33). To increase computational efficiency and ensure that a global minimum is found, a parallel approach [70] is employed by subdividing the design space. Partitioning the proximal and distal orientation angle space \((\phi_p, \phi_d)\) into four quadrants, \((0-90^\circ, 90^\circ-180^\circ, 180^\circ-270^\circ, \text{and} 270^\circ-360^\circ)\) gives 16 subpopulations. Genetic algorithms are run in parallel to find the optimal design for each subpopulation, from which the global optimum is selected.

Subpopulations (of size 40) are randomly initialized within the appropriate regional design variable bounds. Each generation, candidate designs are sorted by fitness and the top 20% are automatically advanced to the next generation. Crossover is performed on the top 80% of candidate designs using fitness proportionate selection to pair parent designs and the BLX-\(\alpha\) [71] method to generate children designs. A mutation rate of 5% is applied to the subpopulation. The process is repeated until the minimum subpopulation fitness converges.

The fitness function \(F(k)\) to evaluate candidate designs combines the cost function \(J(k)\) (5.31) with a penalty term \(P(k)\) on constraint violations

\[
F(k) = J(k) + r_p P(k)
\]

\[
P(k) = \sum_{j=1}^{9} \max\left[g_j(k), 0\right]^2
\]

where \(g_j(k) \leq 0\) are the eight regional design variable bound constraints and the nonlinear constraint (5.33). The penalty weight increases linearly with generation number:

\[
r_p = 0.01 \times n_{\text{gen}}.\]

This ensures the entire design space is searched by allowing some constraint violations initially, but also ensures that the final solution is feasible by later rejecting constraint violations.
5.3 Dual Segment Controller

A closed-loop controller was developed and implemented to track the optimal reference angles generated in Section 5.2.4. As in the single segment case, the bending and orientation angles of each segment \((\theta, \phi)\) are decoupled into two planar bending angles \((\theta_X = \theta \cos \phi, \theta_Y = \theta \sin \phi)\), each controlled by an antagonistic SMA tendon pair: \((SMA_{X+}, SMA_{X-})\) and \((SMA_{Y+}, SMA_{Y-})\). The complete system block diagram is shown in Figure 5.7. The remainder of this section describes controller synthesis for each antagonistic controller block.

![Figure 5.7: Dual-segment catheter closed-loop controller showing decoupling to regulate voltage in antagonistic SMA actuator pairs](image-url)
Each antagonistic SMA pair is controlled using a hybrid approach, initially detailed in Section 4.3, and extended here for the dual segment catheter. As before, the output is a combination of three terms: a feedforward term for hysteresis compensation, a proportional-integral-derivative (PID) term for robustness, and a proportional cubed (P3) term to decrease response time:

\[ V(t) = V_{FF}(t) + V_{PID}(t) + V_{P3}(t) \]  

The second and third terms are identical to the single segment case; however the model-based feedforward term must be modified to account for force coupling between bending segments and thermal coupling between adjacent tendons.

### 5.3.1 Voltage-Based Hybrid Control Strategy

First, only force coupling will be considered and thermal coupling will be neglected. In this case, the voltage-based model developed in Section 4.3.1, (4.13)-(4.14), can be utilized. No modifications are required for the two antagonistic SMA tendon pairs of the distal bending segment. However, as noted previously, bending in the distal segment results in additional deflection in proximal segment. To account for this, proximal segment model-based control terms (4.13) utilize \( \theta_{r}^{(P)} \) (5.35) instead of \( \theta_{r} \).

### 5.3.2 Temperature-Based Hybrid Control Strategy

To increase model accuracy, thermal coupling is now considered. Recalling Section 2.5.2, the actuation of a tendon is significantly affected by heating in adjacent tendons. This indicates that antagonistic steady-state voltage-bending angle hysteresis curves will not be accurate for the dual segment catheter. Because individual tendon actuation results directly from temperature-induced phase transformations, it is necessary in this case to utilize temperature-bending angle hysteresis curves for modeling.
As before, two analytical functions can be derived, one for heating and one for cooling:

\[
T_{ss,h} = b_{h0} \tan \left[ b_{h1} (\theta_r - b_{h2}) \right] + b_{h3} (\theta_r - b_{h2}) + b_{h4}
\]

\[
T_{ss,c} = b_{c0} \tan \left[ b_{c1} (\theta_r - b_{c2}) \right] + b_{c3} (\theta_r - b_{c2}) + b_{c4}
\]

(5.39)

where the coefficients \( b_{hi} \) and \( b_{ci} \) are optimized to fit open-loop temperature-bending angle data from the catheter. Because temperature is not measured, it is calculated, using the applied steady-state voltage \( V_{i,ss} \), from the coupled temperature model (2.13):

\[
T_{i,ss} = \frac{\gamma}{h} V_{i,ss}^2 + \frac{\kappa}{h} (T_{i-1,ss} - T_\infty + T_{i+1,ss} - T_\infty) + T_\infty
\]

(5.40)

for \( i = 1,8 \).

Again, for simplicity minor hysteresis loops are ignored and the feedforward temperature is computed online using the outer-loop relationships (5.39) and the tracking error \( e(t) = \theta_r(t) - \theta_m(t) \)

\[
T_{FF}(t) = T_{ss,c} + \frac{T_{ss,h} - T_{FF,prev}}{1 + \exp \left[ \chi (\varepsilon - e(t)) \right]} - \frac{T_{ss,c} - T_{FF,prev}}{1 + \exp \left[ \chi (-\varepsilon - e(t)) \right]}
\]

(5.41)

which results in a smooth transition between the outer-loop heating and cooling curves. The feedforward voltage for the active tendon \( q \) is calculated from the feedforward temperature using the coupled temperature model (2.13) and the online estimated temperature of adjacent tendons \( \hat{\theta}_{q-1} \) and \( \hat{\theta}_{q+1} \):

\[
V_{FF,q}(t) = \sqrt{\frac{h}{\gamma} \left( T_{FF,q}(t) - T_\infty \right) - \frac{\kappa}{\gamma} \left( \hat{\theta}_{q-1}(t) - T_\infty + \hat{\theta}_{q+1}(t) - T_\infty \right)}
\]

(5.42)
5.4 Results

5.4.1 Simulated Path Optimization Results

Simulations were conducted in MATLAB (The MathWorks, Inc., Natick, MA) to optimize genetic algorithm parameters (population size, number of subpopulations, crossover rate, and mutation rate) and cost function weighting factors (5.31). The three specified points and computed path used for optimization simulations are shown in Figure 5.8. The cost function weighting factors (5.31) were initially chosen as $p_0 = 1$, $q_0 = 1$, $r_0 = 10$, and $s_0 = 10$, which specifies approximately equal emphasis on each term. By independently amplifying each weighting factor, individual cost function contributions were demonstrated and compared (Table 5.1). First, planar path adherence was emphasized, followed by normal penetration (a measure of tissue contact force). Next catheter tip normality was emphasized. Finally, large variations in adjacent reference angles were heavily penalized. The final weighting factors $p = 2$, $q = 5$, $r = 5$, and $s = 5$ were determined by iterative simulation to optimize the tradeoff between path adherence and lesion quality, as shown in Figure 5.9.

Figure 5.8: Selected points and computed path used for optimization simulations
Table 5.1: Simulated cost functions associated with independently magnifying each function weight.

<table>
<thead>
<tr>
<th>Weights</th>
<th>Planar Deviation ($J_1$) [mm]</th>
<th>Normal Penetration ($J_2$) [mm]</th>
<th>Deviation from Normal ($J_3$) [deg]</th>
<th>Average Change in Angles ($J_4$) [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_0, q_0, r_0, s_0$</td>
<td>0.01</td>
<td>4.09</td>
<td>42.5</td>
<td>10.6</td>
</tr>
<tr>
<td>$10 p_0, q_0, r_0, s_0$</td>
<td>0.01</td>
<td>5.43</td>
<td>40.6</td>
<td>23.7</td>
</tr>
<tr>
<td>$p_0, 10 q_0, r_0, s_0$</td>
<td>0.61</td>
<td>0.00</td>
<td>54.3</td>
<td>14.5</td>
</tr>
<tr>
<td>$p_0, q_0, 10 r_0, s_0$</td>
<td>2.94</td>
<td>9.85</td>
<td>1.0</td>
<td>34.5</td>
</tr>
<tr>
<td>$p_0, q_0, r_0, 10 s_0$</td>
<td>38.20</td>
<td>0.41</td>
<td>13.6</td>
<td>5.7</td>
</tr>
<tr>
<td>$2 p_0, 2 q_0, \frac{1}{2} r_0, \frac{1}{2} s_0$</td>
<td>0.00</td>
<td>0.08</td>
<td>55.1</td>
<td>20.1</td>
</tr>
</tbody>
</table>

Figure 5.9: Simulated optimal ablation trajectory: (a) generated ablation path and catheter orientation rendering at each optimized path point, (b) performance factors including planar tracking error, normal penetration, and tip normality, and (c) optimal reference angles.

5.4.2 Experimental Path Optimization and Control Results

Real-time optimization and control algorithms were programmed in Visual Studio C++ (Microsoft Corporation, Redmond, WA) for experimental evaluation on the SMA-actuated robotic catheter. The experimental setup is as described in Section 2.3, with the exception of the feedback kinematics, which utilize the derivations from Section 5.2.2. Electrode readings at the base of the proximal segment are used to set the reference coordinate system. Then real-time electrode readings at the tip of the proximal and distal segments are transformed to
the global coordinate system at the proximal base, and the measured bending and orientation angles are computed from (5.11), (5.12), (5.18), and (5.19).

First, experimental performance of the path optimization strategy was quantified using the SMA-actuated catheter in a saline-filled tank. The catheter was navigated to three arbitrary points, which were stored and used to compute the optimal reference path. Path optimization took approximately 20 seconds to complete. The optimization process was repeated four times (using different specified points); optimal reference trajectories are shown in Figure 5.10. These results demonstrate efficient real-time generation of continuous ablation paths, and performance indices $J_1$ and $J_2$ emphasize path adherence to produce relatively arc-like curves. However, contributions from $J_3$ and $J_4$ (which emphasize tip normality and limit large variations in reference angles, respectively) result in optimal ablation trajectories that are not perfectly smooth arcs.

![Figure 5.10: Optimal ablation paths (solid curves) generated from experimentally recorded catheter tip locations (dots)](image)

Next, closed-loop control of the SMA-actuated catheter was demonstrated for one optimized ablation path. Because clinical ablation procedures require a series of discrete, overlapping lesions, the optimized path was discretized into 16 specific ablation locations,
which were implemented as step references every 40 seconds. Experimental results reveal an average response time of approximately 2 seconds with less than 10° overshoot. However, average steady-state tip tracking error (Figure 5.11) was 5.5 mm (approximately twice the diameter of the ablation catheter tip). Although average tracking error was larger than expected, and unacceptably large for clinical use, much of it can be traced back to the sensing limitations discussed in Section 2.3.2.

Figure 5.11: Experimental results showing reference and average catheter orientations and tip locations.

5.4.3 Temperature-Based Control Results

For simplicity, the voltage-based hybrid control approach developed in Section 5.3.1 was utilized to obtain the results in the previous section, where only low bandwidth step tracking was required. For general movement in the heart (when not ablating), much higher system bandwidth is desirable. Improving modeling accuracy has the potential to allow more
aggressive controller gains, without risking instability, causing faster system response and lower tracking error.

This section presents dual segment catheter tracking results using the temperature-based hybrid control approach, developed in Section 5.3.2, which accounts for temperature coupling between tendons. Performance was evaluated using reference trajectories which required continually changing the orientation of both catheter segments simultaneously, Figure 5.12a. This combination of reference angle trajectories incorporates all the control challenges of the two segment catheter: there are references where both segments bend in the same direction, references where the segments must bend opposite directions, times when heat transfer between adjacent tendons is significant, and times when tendon actuation is unaffected by adjacent tendons. The controller compensates for all these situations; robust and accurate tracking using the SMA-actuated catheter is shown (Figure 5.12b).
Figure 5.12: Two segment catheter experimental results using the temperature-based hybrid control approach: (a) two loop tip reference trajectory and rendering of catheter orientations for the first loop; (b) tracking results for the four generalized angles; (c) control effort.

5.5 Conclusions

This chapter focused on path optimization and control of the dual segment, SMA-actuated catheter. First, a path optimization and control approach for generating effective cardiac ablation lesions was detailed. Simulated and experimental results validate the potential of the
proposed ablation path optimization approach. The method proved to be computationally efficient (requiring on 20 seconds to optimize), producing continuous ablation paths and optimal reference trajectories through any physician-specified points. Additionally, the physician can easily modify the conditions for optimality by adjusting weights associated with each cost function.

The voltage-based hybrid control approach developed in Chapter 4 was modified for dual-segment control. Closed-loop control of the SMA-actuated catheter along the optimized path was efficient; step responses to several discrete path locations demonstrated acceptable response times with low overshoot. However, steady-state tracking error was larger than expected. The 5.5 mm average tracking error can be attributed to real-time sensing limitations of the EnSite NavX system. Individual catheter electrode measurements lack precision, varying on average 1.6 mm peak-to-peak, even when the catheter is held steady. Because catheter tip position is calculated from the four generalized angles, which depend on three different electrode measurements, this error has a tendency to propagate through the forward kinematic equations (5.23). Future work will focus on modifying the controller to compensate for measurement variation and ensure low tracking error and stability.

In an effort to improve controller performance and bandwidth, a temperature-based hybrid control approach was developed. This derivation incorporated the coupled temperature model developed in Chapter 2 to account for heat transfer between adjacent SMA tendons. Improved modeling accuracy permitted more aggressive controller gains, causing tracking bandwidth to increase. Chosen reference trajectories required simultaneous tracking of both segments; results demonstrated fast and accurate control.
Chapter 6

Conclusions

The overall objective of this work was to develop a viable control system for a teleoperated cardiac ablation catheter actuated by shape memory alloy (SMA) tendons. SMA actuators are attractive options for surgical robots due to their relatively large specific energies and strain recoveries; however their highly hysteretic behavior and low bandwidth actuation (resulting from temperature-induced phase transformations) make fast and accurate control difficult. Control is further complicated when multiple tendons are incorporated into a highly uncertain, flexible catheter. This dissertation focused on developing and experimentally evaluating nonlinear control strategies for a SMA-actuated catheter, incorporating hysteresis and heat transfer models to increase system bandwidth, tracking accuracy, and stability.

The SMA-actuated ablation catheter was integrated with hardware and software to develop the complete closed-loop control system. Real-time, in vivo, catheter position sensing was incorporated and feedback kinematics were derived. A custom C++ software program was developed for catheter control, and a user interface was designed. The complete system was used for both laboratory testing and clinical evaluations (live pig trials).

Open-loop catheter behavior was analyzed, and control challenges were highlighted. A series of experimental tests and finite element simulations enabled heat transfer between SMA tendons to be quantified. Models were developed to estimate SMA temperature online and for incorporation into model-based control laws.
First, an indirect intelligent sliding mode controller (IISMC) was developed and experimentally evaluated on a more tractable system: a flexible beam actuated by a single, or antagonistic pair of, offset SMA tendon(s). The IISMC successfully incorporates a hysteretic recurrent neural network for hysteresis compensation and utilizes a control law based on SMA temperature dynamics to account for slow heating and cooling. Results demonstrated fast response times, precise tracking, and controller robustness to model uncertainties and parameter variations.

Four nonlinear control strategies (including IISMC) were developed for the SMA-actuated catheter and were experimentally tested on the distal bending segment. Their performance was compared, and results were used to develop a hybrid control approach. This controller incorporates a model to compensate for SMA hysteresis, accommodates actuator bandwidth limitations, and is robust to model parameter variations, unmodeled dynamics, and sensing limitations. Experimental results demonstrated stable and accurate tracking of a wide variety of reference trajectories. Additionally, the controller maintains tracking performance in the presence of both hemodynamic and contact disturbances.

Finally, a control scheme for the full two segment catheter was developed. A method for automatically creating a continuous ablation path through physician-specified points was detailed. A parallel genetic algorithm optimization approach was implemented for real-time generation of controller reference angles. The approach proved to be globally valid, generating optimal reference trajectories regardless of selected points. The hybrid control approach was extended to account for force coupling between catheter bending segments. Ablation path tracking was demonstrated using the dual segment catheter through step responses to several discrete path locations. Finally, the hybrid control approach was further modified to account for heat transfer between adjacent tendons and dynamic path tracking was demonstrated. Closed-loop control results demonstrated fast response times, low overshoot, and stable behavior.
6.1 Future Work

The SMA-actuated catheter has been evaluated both in the laboratory and in live pig trials. However, before human clinical trials are considered, additional methods to potentially improve catheter safety and performance should be thoroughly evaluated. Three potential areas for improvement are discussed here: system bandwidth, sensing, and haptic feedback.

The work presented here incorporated several strategies to increase system bandwidth, which is inherently low due to temperature-induced SMA actuation. Imposing safe continuous voltage limits, the tendons require several seconds to fully actuate. Dynamic voltage limits (based on estimated SMA temperature) were implemented to decrease response times; however further improvements can be made. First, improving temperature model accuracy would allow more aggressive dynamic voltage limits. Similarly, more accurate system models (e.g. thermal, hysteresis, coupling) could be used to create more aggressive control algorithms; increased model accuracy decreases risk of overshoot when quickly heating tendons. Eliminating tendon slack would also increase system bandwidth. In Chapter 3, a control strategy was proposed which applies voltage to both antagonistic tendons simultaneously, ensuring quick response to reference direction changes. While this was not implemented on the catheter in an effort to reduce power requirements and model complexity, incorporating an anti-slack method could significantly reduce response times.

The previous discussion focused on increasing closed-loop control bandwidth. While control is the topic of this dissertation, it should also be mentioned that improvements to catheter design have the potential to increase inherent system bandwidth. Design optimization can be used to develop a catheter which requires less actuation force, and thus utilizes smaller diameter SMA tendons which heat and cool quicker. Additionally, active cooling could be implemented on each individual SMA tendon (instead of the bulk coolant method described here). Coolant could then be independently regulated to minimize heat transfer away from an activating tendon.

The previous two paragraphs focused on increasing system bandwidth, which helps ensure physician-supplied joystick movements are met with immediate visual feedback of
catheter bending. Now, patient safety and lesion quality are considered. Catheter ablation procedure success requires accurate tip positioning relative to cardiac anatomy and adequate tissue contact during ablation. To further address these requirements, position sensing and haptic feedback are discussed.

*In vivo* visualization and sensing has long been a challenge in the medical industry. Literature is replete with research on medical imaging; several journals publish original contributions regularly (i.e. Journal of Medical Imaging and Radiation Sciences, Journal of Medical Imaging and Radiation Oncology, IEEE Transactions on Medical Imaging). However, visual feedback is not sufficient for closed-loop control of robotic instruments; real-time, *in vivo* sensing is required. The EnSite NavX System (St. Jude Medical, St. Paul, MN) is presently the best available option for endocardial feedback, and was therefore chosen for the SMA-actuated catheter. However, because sensing accuracy and precision of the EnSite NavX are low, laboratory control results presented showed significant tracking error. To address this, future control algorithms should compensate for the known sensing nonlinearities and imprecision. Alternatively, more accurate and precise sensing technologies should be explored.

Haptic feedback (or lack thereof) is another major limitation of minimally-invasive procedures [72], [73], particularly compared to open heart procedures where tissue contact force is easily transmitted back to the surgeon’s hand. An accurate estimate of tissue contact force could prevent tissue damage and ensure transmurality of ablation lesions. One method to estimate contact force, without requiring design modifications, is to incorporate an observer into the controller. Successful implementation would require an accurate system model, such that model and actual responses could be compared to estimate force. This method has limited use because it gives a measure of tangential force (caused by bending), not normal tip force. Alternatively, the catheter could be redesigned to incorporate a strain gage or other force measurement device. Several researchers have proposed such devices [74], [75], [76], [77], however practical implementation on small-scale surgical instruments is currently a challenge. Nonetheless, 3D force measurement at the catheter tip would be
highly advantageous, as the feedback could be incorporated into control laws to increase the safety and effectiveness of catheter ablation procedures.
REFERENCES


