ABSTRACT

KHARBASH, SHAWQI QAED. Modeling and Analysis of Wireless Network Reliability. (Under the direction of Dr. Wenye Wang.)

Network reliability concerns with the capability of the underlying network to provide successful communication between set of nodes, i.e., sources and destinations, which is an issue of concern whenever there is a need of assurance of conveying critical data within the network. Network reliability analysis has been an important area of research in wired networks, but a little research have been accomplished in wireless networks. Most of the existing works have considered networks as graphs with perfect nodes and edges of identical reliabilities, but wireless networks are quite different from wired networks because nodes are prone to failure and there is a large variance in links reliabilities. Wireless networks have several aspects that make them more susceptible to failures and loss of connectivity such as fluctuating medium characteristics and the properties of wireless devices. For instance, the broadcast nature of wireless communication links makes them unique in their vulnerability to loss of connectivity due to interference, weather conditions, terrain effects and security breaches. Additionally, wireless mobile nodes have limited power supplies, limited transmission range and ability to change their locations. Because of these impediments, there have been a paradigm shift from the traditional reliability analysis for wired network to wireless networks.

In this doctoral study, we first investigated the problem of computing the two-terminal reliability in mobile Ad hoc networks. We presented how the mobility patterns affect the two-terminal reliability in mobile Ad hoc networks and how the imperfect wireless nodes affect the network performance parameters. Then, we pursued a better understanding of reliability analysis in the context of wireless network by exploring the statistical characteristics of wireless medium and exploring how the variation of nodes transmission power impact the all-terminal network reliability. Based on this understanding, we proposed a power-control algorithm which takes into account the statistical variation of the Signal-to-Interference Ratio at receiving nodes and optimally allocate nodes’ transmission powers to maximize both of the links reliabilities and all-terminal network reliability. Then, we focused on the deployment of relaying nodes in hierarchical hybrid wireless network such that the network reliability is maximized. To that end, we first defined a generic hierarchical hybrid wireless network architecture which is suitable for modeling variety
of networking scenarios (i.e., hybrid cellular/ad hoc networks, large-scale sensor nets and networked distributed power system) where different radio access technologies can be used for each tier to comply with performance requirements and constraints imposed by scenario being addressed. This generic architecture consists of three tiers of radio nodes: low-power terminal nodes (TNs) at the lowest tier, higher powered radio relaying nodes (RNs) that support multihop routing at the second tier, and wired Gateway nodes (GNs) at the third and highest tier. Then, we presented an algorithm to find the RNs locations such that every TN in the network is connected to an RN and the sub-cluster reliabilities are maximized. After that, we utilized the information of RNs locations in maximizing the cluster reliability, via controlling the RNs transmission powers to GN.

Finally, we presented a reliability influence ranking algorithm to identify the most reliability influenced nodes to be considered in reliability importance measures evaluation. We view node reliability influence ranking as an initial and crucial step in reducing the computation complexity associated with reliability calculation in reliability importance measures evaluation. By identifying node reliability influence ranking on network reliability, we can focus the reliability importance measures evaluation on fewer nodes and hence we can prioritize the reliability improvement activities efficiently.
Modeling and Analysis of Wireless Network Reliability

by
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To my parents.
BIOGRAPHY

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Chapter 1

Introduction

1.1 Motivation

In the last few years, wireless multi-hop networks have gained great attentions in the realm of research and telecommunication communities. A wireless multi-hop network is an infrastructureless network, where all nodes equipped with radio transceivers are capable to move and communicate via radio channels. In these networks, each node may operate not only as a host but also as a router for discovering and maintaining routes to other nodes. Owing to the limited transmission range, an end-to-end communication is carried out dynamically through a number of intermediate relaying nodes, which is thus called multi-hop forwarding. The goal of a wireless multi-hop network is to provide a rapidly and easily deployable means of interconnection between wireless nodes, which rely on no fixed infrastructures and are carried out without a centralized control (i.e., self-forming). The absence of the infrastructure and self-forming nature make a wireless multi-hop network an attractive choice for a variety of applications such as military operations (e.g., tactical communication in the battlefield), field surveillance (e.g. monitoring roadside weather conditions, Bio-sensing and machinery prognostics), interactive information sharing (e.g., electronic classrooms, convention centers and construction cites), Internet access and home networking [2].

The deployment of wireless networks implies providing users with ubiquitous and continuous ability to share information and access computing resources. However, guaranteeing such abilities is challenged by the fluctuating characteristics of the wireless medium that affect the signal strength and propagation. Moreover, the broadcast nature
of wireless communication links makes them unique in their vulnerability to loss of connectivity due to interference, weather conditions, terrain effects and security breaches. All the previous aspects of the medium characteristic and wireless nodes properties make wireless networks more susceptible to failures and connectivity fluctuations. Node failures and connectivity fluctuations hinder wireless networks from evolving and degrade the quality of services that wireless networks can provide. With the growing dependency of many applications on wireless networks, there is an increasing demand for more reliable networks. The degree to which a network is able to provide the required services needs to be quantitatively assessed by defining measurable quantities. These measurable quantities are the network reliability measures.

Network reliability concerns with the capability of the underlying network to provide successful communication between set of nodes, i.e., sources and destinations, which is an issue of concern whenever there is a need of assurance of conveying critical data within the network. Network reliability has long been a practical issue, and will remain so for years, since variety of realistic applied problems in IP networks, mobile phone networks, and cognitive radio networks set the successful connection rate of 99.999% [3][4][5] as an essential objective for communication network providers, and premium services may only be deployed if the connection reliability is close enough to unity. For the network to be functional, its components (nodes and links) must be fit for service. Unfortunately, this is not always the case, because network components are not always reliable[6][7][8][9][10][11][12] and networks failures become a norm rather than an expectation [13]. Reliability is therefore a crucial parameter in the design and analysis of the various types of networks which pervading our everyday life.

Network reliability analysis has been an important area of research in wired networks, but a little research have been accomplished in wireless networks [14][15][16]. However, the tremendous growth of wireless networks as a substitute and an extension for the traditional wired network called for a revise in the assumptions, evaluations and assessments of network reliability[17][18] [19] [20]. Most of the existing studies have considered networks as graphs with perfect nodes ($p_{n} = 1$) and edges of identical reliability. However, wireless networks are more prone to failures and loss of access due to nodes failure, weak transmission power, terrain effects and interference. The extensive experimental study carried out by Aguayo et al. [21][22] on wireless networks have concluded that (1) there is a large variance in the links reliabilities (measured as the probability of successful delivery of packet over a specific period of time), (2) long term variation (in the order of hours)
in the reliability of a link is common, and (3) physical conditions such as obstacles and radio interference can have strong and unpredictable effects on link reliability. Therefore, assumption of a wireless network with edges of identical reliability does not capture the effects of propagation losses [23][24][25][26][27] and interference[23][28][29][30][31][32][33]. Because of these impediments, there have been a paradigm shift from the traditional reliability analysis for wired network to wireless networks [17][18].

1.2 Research Objectives

Motivated by the demand of reliable wireless multi-hop networks and the limitation of the current research, we devote this doctoral study to the modeling, design, and analysis of the reliability of wireless multi-hop networks. More specifically, the main topic of this study is further integrated with the following research objectives:

- **Understanding the Impact of Mobility on Network Reliability**
  
  Most of the existing techniques for network reliability evaluation are based on assumptions that all the nodes are perfect and the communication links are static and irreplaceable, but this is not applicable for mobile Ad hoc networks. There are a rapid changes in connectivity and link characteristics in Ad hoc networks due to nodes mobility and nodes failures. In Chapter 2, we considered the computation of the two-terminal reliability in Ad hoc networks by extending the algorithm proposed by Rai *et al* [34] to handle imperfect nodes and the changing network connectivity. The effect of nodes failure rates and the mobility pattern on the two-terminal reliability are presented.

- **Power Allocation for All-Terminal Network Reliability Optimization in Fading Environment via a Cross Entropy Approach**

  In wireless networks, information needs to be conveyed reliably between communicating parties despite of the harsh nature of wireless medium and the randomness of the deployment. Therefore, reliability requirements of wireless networks should be rigorously assessed. Reliability analysis of a wireless network is concerned with measuring the probability of successful communication between a given set of nodes, i.e., sources and destinations, can occur. This explicitly implies the successful communication between intermediate nodes connecting these sources and destinations.
In wireless network, successful direct communication between neighboring nodes is affected by various factors including channel path loss, fading, additive noise, data rate and transmission power [35][36].

We observed that most of the proposed approaches for link-level and transport-level reliability are primarily based on common transmission control where an identical transmission power $P_{tx}$ is used by every node. These works only provided a guidance on the critical selection of common transmission power constrained by problem requirements, e.g. connectivity [37][38], capacity [39], coverage [37][38][39] and energy efficiency [37][38], to attain a specific reliability objective either on link or transport level. The common transmission power selection approaches do not take into account the effect of variation of transmission power on links and network reliabilities.

In this work, we pursued a better understanding of reliability analysis in the context of wireless network by exploring the statistical characteristics of wireless medium and exploring how the variation of nodes transmission powers impacts the all-terminal network reliability. Based on this understanding, we proposed a power-control algorithm which takes into account the statistical variation of the Signal to Interference Ratio (SIR) at receiving nodes and optimally allocate nodes’ transmission power to maximize both of the links reliabilities and all-terminal network reliability. The details of this work is elaborated in Chapter 3.

- **Reliability-Aware Relaying Node Deployment: Maximizing All-Terminal Network Reliability for Hierarchical Wireless Networks**

  Wireless Networks have been widely used for distributed control systems (DCS) to enhance the production and control efficiencies of industrial and commercial system such as power systems. The integration of geographically distributed assets through centralized control improves agility in responding to supply and demand fluctuations, reduces cost of operations and enables process efficiencies unachievable in the past. Since these systems assets and end-users are randomly distributed on large graphical area, there is a need for a flexible network structure that can assist the diverse reliability and connectivity requirement while lowering the cost for installation and commissioning. In this work, we presented a self-organized hierarchical hybrid wireless network structure that can accommodate the connectivity and reli-
ability needs for a distributed control system. A self-organized hierarchical hybrid wireless network structure consists of terminal nodes associated with system assets or end users, relaying nodes responsible for routing inquiries or data from users or assets toward the gateway nodes and gateway nodes can forward communication inquiries or collected data to a centralized control station within the network. In this network structure, we studied the problem of relaying nodes deployment in order to maximize all-terminal network reliability for hierarchical wireless network. Toward that end, we defined the reliabilities of on link, sub-cluster and cluster levels. Then, we presented an algorithm to find the relaying nodes locations such that every terminal node in the network is connected to a relaying node and the sub-cluster reliabilities, i.e., $\text{Rel}(RN_i)$, are maximized. Then, we utilized the information of relaying nodes locations in maximizing the cluster reliability, i.e., $\text{Rel}(GN)$, via controlling the relaying nodes transmission powers to the gateway node. Detailed description of proposed algorithms are presented in Chapter 4.

- **Nodes Reliability Influence Ranking**

Computing network reliability has been and continue to be a very important problem, but it is not the only problem in reliability analysis. One of the purposes of network reliability analysis is to identify the weakness of network, quantify the impact of the component failure and hence prioritize the reliability improvement activities at the minimum cost or effort. The so called "reliability importance measures" are used for this purpose. The importance measures has been extensively studied for both binary system [40][41][42][43][44], i.e., component has fully functioning or fully failed, and multi-state system [45][46][47][48][49]. However, we notice that there is a missing initial step in all these efforts in identifying which nodes are best candidates to be considered in reliability importance measures calculations. This initial step is crucial in reducing the computation complexity of reliability importance measures calculations by reducing the number of nodes used in reliability importance measures evaluation, since most of this reliability importance measures involve network reliability calculation which are NP-hard problems. Therefore, our initial step will specify the elite nodes whose reliability influences are superior to other nodes without the need of performing extensive reliability calculation. In our approach, we utilized the concept of node potential [50] from field theory to evaluate the node reliability influence on given network. Detailed
description of proposed approach is presented in Chapter 5.

1.3 Challenges

During the course of this doctoral research, there were quite a few non-trivial challenges we had to bear in mind and tackle with. For example,

- **Reliability Definition:** There are different forms of multi-hop wireless networks such as the multi-hop wireless network (Ad hoc), mesh wireless network (MWN), distributed wireless sensor network (DSN) and wireless networks (WLANs). With the availability of several wireless network platforms, it is intuitive to explore the reliability problem in each form. Thus, the network reliability analysis differs from one wireless network to the other. For example, the *end-to-end instantaneous two-terminal reliability* [15] and the *coverage-oriented reliability* have been introduced to resemble the two-terminal reliability in WLAN and DSN, respectively. Therefore, it is important to understand the reliability requirements for a wireless network and how the reliability is related to the network tasks (e.g. DSN task is to capture a phenomenon in a sense field). Based on understating of reliability requirement and the wireless networks task, the reliability can be modeled and analyzed.

- **Heterogeneous Network Parameters:** In traditional network reliability analysis, network parameters are assumed to be identical to facilitate the simulation and the analytical analysis. However, these assumptions are non-realistic because there are many factors affecting the successful communication between two nodes in a wireless network such as channel conditions, power transmission range, fading, interference and noise. Therefore, the existence of the link between two nodes is a probabilistically changing variable, unlike wired networks where links have fixed probability of operation. These differences will render the reliability analysis and make it more realistic and indeed more complicated.

- **Scalability:** Wireless networks lack the aggregation techniques used in wired networks where groups of nodes can be represented by a single node via the addressing or topological schemes. Therefore, each node in a wireless network might contribute to the network and should not be omitted. The increase in the number of nodes
leads into an increase in the simulation and the computation load of network reliability measures.

- **Computational Complexity:** The computation of network reliability is, generally, NP-hard problem. All the known computation algorithms for these problems have an exponential time complexity [51][52]. Therefore, the computation of the exact values is limited to small-sized networks. On the other hand, reliability estimations is more preferable for moderate and large-sized networks.

This dissertation is organized as follows. Chapter 2 presents an algorithm for computing two-terminal reliability in mobile Ad hoc networks. This algorithm is used in understanding the impact of mobility on two-terminal reliability. Chapter 3 presents a power-control algorithm which takes into account the statistical variation of the Signal to Interference Ratio at receiving nodes and optimally allocate nodes’ transmission powers to maximize both of the links reliabilities and all-terminal network reliability. Chapter 4 studies the problem of relaying nodes deployment in order to maximize all-terminal network reliability for hierarchical wireless network. Chapter 5 presents an algorithm for ranking node based on their reliability influence in given network. Finally, Chapter 6 summarizes the research results and suggests set of problems for future investigation.
Chapter 2

Understanding the Impact of Mobility on Two-Terminal Reliability

2.1 Motivation and Related Work

Wireless Ad hoc networks are self-organizing, rapidly deployable and required no fixed infrastructure. They consist of wireless nodes, which can be deployed anywhere and must cooperate with each other to dynamically establish communication using limited management and administration. These networks have a wide range of applications such as disaster relief operations, military operations, fields surveillance, interactive information sharing, Internet access and home networking [2]. Ad hoc networks operate under severe constraints such as mobility of nodes, limited power for data transmission, limited memory on mobile devices for packet buffering and bandwidth restrictions of the wireless medium. Therefore, the wireless links and nodes fail frequently and abruptly which impacts the paths between the communicating parties. The effects of failures can be realized on the network performance metrics such as the packet loss and end-to-end delay. The key of making Ad hoc networks viable is the ability of providing the mobile users with reliable services with the present of fluctuating medium characteristics and limited-capabilities wireless devices. To ensure that future wireless networks retain the level of reliability that we have come to expect, we must develop a deeper understanding of the relationship between dynamics of Ad hoc networks and their reliability.
Networks reliability problem is an important issue in the design and evaluation of networks. Various reliability measures can be used to assess the robustness of network against the components’ failures and links’ disconnections. The typical network reliability problem is to calculate the probability that a certain set of nodes can communicate with each other for a given period of time. There are three main communication patterns in networks: unicasting, broadcasting and multicasting communication. Based on these forms of communication, there are three main formulations of the network reliability problem: two-terminal, \( K \)-terminal and all-terminal reliabilities \[11\]. In this work we are concerned with evaluating the two-terminal reliability in mobile Ad hoc networks. The *two-terminal reliability* is defined as the probability that there exists at least one operating path between a source and a destination nodes. The two-terminal reliability problem has been studied extensively for wired networks with unreliable links under assumptions that the nodes are fault-free, static and their locations are known \[34\][53][54]. In addition, links connecting the nodes are assumed to be irreplaceable. Mathematically, a wired network is modeled by a probabilistic graph, \( G(V, E) \), where communication links are represented by edges and communicating devices are represented by nodes. Each edge has an operational probability of \( p_e \), whereas each node has an operational probability of \( p_n = 1 \) (i.e., perfect nodes).

On the other hand, the two-terminal reliability problem in wireless networks is quite different from that for wired networks. Wireless networks have several aspects that make them more susceptible to failures and loss of connectivity. These aspects include the medium characteristics and the properties of wireless devices \[35\]. For instance, the broadcast nature of wireless communication links makes them unique in their vulnerability to loss of connectivity due to interference, weather conditions, terrain effects and security breaches. Additionally, wireless mobile devices have limited power supplies, limited transmission range and ability to change their locations. Thus, the reliability computation and modeling techniques developed for wired networks cannot directly be utilized in wireless networks.

In this work, we are primarily interested in the two-terminal reliability problem in Ad hoc wireless networks. The two-terminal reliability is the ability of two nodes, a source and a destination, to communicate with each other. The *two-terminal reliability* is defined as the probability that there exists at least one path for a given source and destination node pair of the network \[55\]. In ad hoc networks, routes change dynamically according to many factors such nodes mobility, failure of intermediate nodes and the fluctuating
characteristics of the wireless medium. Therefore, there are frequent changes in the nodes participation in the path between the source and the destination nodes. These changes alter the two-terminal reliability. Accordingly, the two-terminal reliability value changes with respect to time. None of the researches in wired network address this issue.

This work seeks a simple method to compute the two-terminal reliability in mobile wireless ad hoc network and examines the effect of network dynamics on the two-terminal reliability. The remaining sections are organized as follows: Section 2.2 presents the preliminaries for two-terminal reliability evaluation for a given networks. Section 2.3, describes algorithm used for computing two-terminal reliability in ad hoc networks. Section present simulation environment and results. Section contains the conclusion.

2.2 Preliminaries

2.2.1 Two-Terminal Reliability Definition

A mobile ad hoc network can be modeled by undirected time-varying graph $G(V(t), E(t))$, where $V(t)$ and $E(t)$ are, respectively, the set of nodes (vertices) and the set of links (edges) in the network at a given time $t$. Each node has an operation probability of $p_n$. The problem is to compute the probability that there exists an operational path between source node, $n_s$, and destination node, $n_d$, which is denoted as $R_{n_s,n_d}(G)$. All nodes, but source and destination nodes, are allowed to move freely according to a known mobility model. Therefore, the two-terminal reliability is a function of time and changes frequently due to nodes’ movements, nodes’ failures and edges’ failures.

Each edge $e \in E$ has an operational probability, $p_e$, which depends on the operational probabilities of nodes the edge is connecting. Therefore, $p_e$ of edge $e$ connecting nodes $n_i$ and $n_j$ can be expressed as $p_e = Pr(e \text{ exists} \mid n_i \text{ and } n_j \text{ are operating})$. Since each edge, $e$, can have one of two states, operating or failed, the state of the network can be represented using a vector $S(t) = [S_1(t), S_2(t), \ldots, S_E(t)]$. The $e^{th}$ component of $S(t)$ equal to 1 if edge $e$ is operating and 0 otherwise. Thus, the probability of a given state $S(t)$ is

$$Pr\left(S(t)\right) = \prod_{e=1}^{E} p_e^{S_e(t)}(1 - p_e)^{1 - S_e(t)}.$$  (2.1)

The states are examined using function $\Psi_{n_s,n_d}(\cdot)$. This function checks if there exists
at least one path between the $n_s$ and $n_d$. If state $S(t)$ contains one or more paths between the two nodes, then $\Psi_{n_s,n_d}(S(t)) = 1$, otherwise $\Psi_{n_s,n_d}(S(t)) = 0$. Then, the two-terminal reliability can be formulated as follows:

$$\text{Rel}_{n_s,n_d}(G(t)) = \sum_{all \ S(t)} \Psi_{n_s,n_d}(S(t)) \Pr(S(t)).$$

(2.2)

### 2.2.2 Node, Link and path Characteristics

Wireless nodes are susceptible to failures due to internal or external problems. Ad hoc networks are deployed in diverse environments including hostile areas. In these environments, weather conditions, interference effects and security breaches may result in nodes malfunction. In addition, nodes may have internal hardware or software defects that cause failures. Moreover, nodes are subject to power depletion due to power consumption in receiving and forwarding packets. Therefore, modeling the reliability of the nodes must be of increasing failure rate, i.e., as the time passes, the length of the node’s life decreases due to power consumption. Therefore, we utilize the Weibull distribution in modeling the reliability of a wireless node. Then, the reliability of a node $i$ is at time $t$ can be expressed as:

$$R_i(t) = e^{-\left(\frac{t}{\lambda(t)}\right)^{\beta(t)}}.$$  

(2.3)

where $\beta(t) > 1$ is a positive shape parameter, $\lambda(t) > 0$ is called the characteristic life. These two parameters are dependent on many factors such as the power depletion, hardware/software failures and malicious attacks [56][57]. If all the wireless nodes have the same hardware/software characteristics and their network duties are the same then their $\beta(t)$ and $\lambda(t)$ are statistically identical. Nevertheless, heterogeneous nodes with different hardware plate-forms and different network duties may have different scales and characteristic life functions. For simplicity we consider a plain ad hoc network in which all the nodes are equal in term of their hardware/software capabilities (i.e., battery, antenna orientation, processing power and so on), their network duties (i.e., routing, route maintenance and so on) and their prone to malicious attacks.

Now, we describe how two nodes can establish a wireless link with each other. Suppose a node is transmitting a signal with power $P_t$ and another node is receiving the signal with $P_r$. If the received signal power exceeds some threshold power called the receiver sensitivity $P_{r,th}$, a link is established between the two nodes in one direction from the
transmitted node to the received node. Assuming all the nodes have the same $P_t$ and $P_r$ and using omnidirectional antennas, two nodes can establish a bi-directional link if the distance between them is less than or equal to

$$R = (\frac{P_t}{P_{r,th}})^{1/\alpha}, \quad (2.4)$$

where $\alpha$ is an environment-dependent parameter, ranging from 2 (free space) to 4 (indoor) [35].

For two nodes to communicate with each other, there should be at least one operating path between them. An operating path implies that all the intermediate nodes and links are operating. A node is operating if and only if it preforms its indicated functions. A wireless link is operating if and only if it allows communication from its initial node to its terminal node. At any moment of time, the components of the network (i.e., nodes and links) can be either in operational or failed state. Once a node fails, it stays in the failed state. On the other hand, wireless links can be brought back to the operational state if the two mobile nodes come close again in each other transmission range and satisfy the SINR requirement.

### 2.3 Exact Two-Terminal Reliability Computation

In this section, we present an efficient algorithm for exact computation of two-terminal reliability in mobile ad hoc networks. Then, we walk through a simple example to explain how the algorithm works.

#### 2.3.1 Algorithm Description

We propose a computation method based on the algorithm in [34] in order to compute the instantaneous two-terminal reliability in mobile ad hoc networks. In this algorithm, there are three main operations: constructing connection matrix, substitutions, and derivation and evaluation of the reliability expression, which are described in the subsequent sections.

1. Constructing connection matrix:

   The existence of the links is based on the locations of the nodes with respect to each other at any given time. A bidirectional radio link exists between two nodes if they
are located within the transmission range of each other. Therefore, a connection matrix $M[c](t)$ represents connection relations between different nodes at any given time. Each element in $M[c](t)$ represents a radio link between two communicating nodes. An element of $M[c](t)$ is labeled as $X_{i,j}$, where $i$ and $j$ are the nodes at the ends of the radio link. The index $i$ is always lower than the index $j$, thus the element in the $i^{th}$ row and $j^{th}$ column is similar to the element in $j^{th}$ row and $i^{th}$ column (i.e., bidirectional links). This labeling method is useful in the evaluation phase, because it indicates the nodes participating in creation of a wireless link. The source node and destination nodes have the indexes of 1 and $N$, respectively. $M[c](t)$ of ad hoc network in Figure 2.1 is

$$ M[c](t) = \begin{pmatrix}
  0 & x_{1,2} & x_{1,3} & 0 & 0 \\
  x_{1,2} & 0 & 0 & x_{2,4} & 0 \\
  x_{1,3} & 0 & 0 & x_{3,4} & x_{3,5} \\
  0 & x_{2,4} & x_{3,4} & 0 & x_{4,5} \\
  0 & 0 & x_{3,5} & x_{4,5} & 0
\end{pmatrix} \quad (2.5) $$

Figure 2.1: Wireless Multi-Hop Network With Five Nodes.

The connection matrix is modified by adding it to the identity matrix $I_N$ (i.e., $N=5$ for network in Figure 2.1) and removing the first column and last row which correspond to the source and destination nodes connections with other nodes, re-
spectively. The ones in $I_N$ gives hints on the connection between nodes for a certain path as we are going to see in the next section.

$$M[c](t)_{\text{modified}} = \begin{pmatrix} x_{1,2} & x_{1,3} & 0 & 0 \\ 1 & 0 & x_{2,4} & 0 \\ 0 & 1 & x_{3,4} & x_{3,5} \\ x_{2,4} & x_{3,4} & 1 & x_{4,5} \end{pmatrix} \quad (2.6)$$

II. Substitutions:

The procedures for computing the two-terminal reliability consists of two main substitution operations: *link-drop operation* and *link-add operation*. We present these two operations as follows:

1. Link-drop Operation:

   This operation basically resembles removing the radio link from the wireless network. Link-drop operation substitutes 0 value in the non-zero element of the first row of the modified connection matrix. For example, given the modified version of $M[c](t)$ in (2.6), the first row contains the following non-zero elements: $x_{1,2}$ and $x_{1,3}$. Applying the link-drop operation on the element $x_{1,2}$ yields the following matrix:

   $$A_{\text{new}}[x] = \begin{pmatrix} 0 & x_{1,3} & 0 & 0 \\ 1 & 0 & x_{2,4} & 0 \\ 0 & 1 & x_{3,4} & x_{3,5} \\ x_{2,4} & x_{3,4} & 1 & x_{4,5} \end{pmatrix} \quad (2.7)$$

2. Link-add Operation:

   This operation resembles discovering a path between two nodes by move one step (i.e., one link) in path toward the destination node. Link-add operation is preformed as follow:

   - The value of 1 is substituted in a non-zero element of the first row of the connection matrix.
   - In the column corresponding to the 1 substitution, the row $r$ with entry of 1 (i.e., from $M[c](t) + I[N]$ modification) is extracted and the following
operation is carried out:
row( first row ) ← row( first row ) + row(r)
- The column corresponding to 1 substitution and row r are removed.

Applying the link-add operation to element $x_{1,2}$ in the modified version of the connection matrix in (2.6) yields:

$$A_{new}[x] = \begin{pmatrix} x_{1,3} & x_{2,4} & 0 \\ 1 & x_{3,4} & x_{3,5} \\ x_{3,4} & 1 & x_{4,5} \end{pmatrix} \quad (2.8)$$

III. Derivation and Evaluation:

The reliability expression is the summation of all the possible paths between the source and destination nodes. These paths are obtained by transversing the resultant tree of matrices from the root to the pendant vertexes of non-zero variables as shown in the example in Figure 2.2. The variables in the rectangular boxes represent one link in path between the source and the destination nodes. There are two types of variables $X_{i,j}$ and $\overline{X_{i,j}}$. $X_{i,j}$ indicates that the path between node $i$ and node $j$ is operational. On the other hand, $\overline{X_{i,j}}$ indicated that the path between node $i$ and node $j$ is failed. For each path, we consider the variables of type $X_{i,j}$ because they represent the reliability of the different components along that path. Then, variables in the reliability expression can be substituted with corresponding nodes and link operational probability. The operational probability of a variable $X_{i,j}$ consists of three components: operational probability of node $i$, operational probability node $j$ and operational probability of link $e$ connecting them. Since most of the wireless network routing protocols utilize a single path during the communication process. We calculate the two-terminal reliability such that one path is used at any given time.

Next, we present the algorithm used in obtaining the reliability expression for a given source and destination nodes.

1. Set time $t = 0$.

2. The $M[c](t)$ is constructed and modified by adding it to the identity matrix, $I_N$, where $N$ is the total number of operational nodes in the network. Then, the first column and the last row are removed.
3. The level is set to zero \((l \leftarrow 0)\).

4. The non-zero element of the first row are extracted (e.g. \(X_{1,2}, X_{1,3}, X_{1,4}, \ldots\)) and arranged according to following steps:
   - Elements whose corresponding columns contain no ones come first.
   - The rest of entries are arranged in ascending order based on their indexes.

5. Link-drop and link-add operations are preformed as follow:
   - A substitution logic is defined as: 1, 01, 001, ..., 000 ... 01 for non-zero elements of the first row of the connection matrix (e.g. the first term corresponds to \(X_{1,2} = 1\) followed by \(X_{1,2} = 0, X_{1,3} = 1\) and so on).
   - The resultant matrix of each substitution will be a new connection matrix in next level, i.e., \(l + 1\).
   - If the entry of 1 does not exists in same column where link-add operation is performed, the substitution result is 1 instead of a matrix.

6. \(l\) is incremented and procedures at step 5) are repeated for each matrix in \(l\).

7. Step 6) is repeated until the substitution operations result is either 1, 0 or branch with variable, i.e., \(X_{i,N}\).

8. The derivation and evaluation procedure is carried out to obtain the two-terminal reliability for time \(t\).

9. \(t\) is incremented. If the end of simulation is reached, skip the next step and exit.

10. The nodes’ location at time \(t + 1\) are obtained. Then, algorithm is started again from step 2).

At any given time, a tree structure of matrices is constructed as shown in Figure 2.2. In this structure, the variables in the rectangular boxes represent the used variables in the substitution operations to obtain new reduced connection matrices. At the last level, all the non-zero variables in the parentheses indicate a potential paths between the communicating nodes. Then, two-reliability can be found by proper substitution of variables as discussed in Derivation and Evaluation procedure. This algorithm has a
complexity of $O(n^3)$ for fully connected network (i.e., all the nodes are connected directly to each other), however this is the worst case for complexity since in a typical wireless network we need more than one hop to reach the destination.

2.3.2 Walk-Through the Algorithm by an Example

In this section, we walk through an example to demonstrate how the two-terminal reliability is computed. The computing algorithm is applied to the wireless network in Figure 2.1. The two-terminal reliability is calculated for the source node $s = 1$ and the destination node $d = 5$. Figure 2.2 shows how the algorithm was carried out recursively. In this example, there are 3 levels (e.g. $N - 2$ where $N$ is total number of nodes). In the first level (i.e., level 0), there is only one matrix with two elements in the first row $X_{1,2}$ and $X_{1,3}$. Therefore, the substitution operations 1 and 01 are applied with correspondence to $X_{1,2}$ and $X_{1,2}X_{1,3}$, respectively. Thus, there are two possible paths toward the destination node. One path through link $X_{1,2}$ and other one through $X_{1,3}$ if $X_{1,2}$ link is not available (i.e., $X_{1,2}X_{1,3}$). In the second level, there are two matrices to be manipulated. The same discovering procedure is repeated for each matrix and new link is added to the path between $s$ and $d$. When the recursive procedures finish, all the paths with non-zero variables at the last vertex (i.e., variables between parentheses) are summed to construct the reliability expression. The resultant expression for the instantaneous two-terminal reliability between $s = 1$ and $d = 5$ at time $t_0$ $Rel_{s=1,d=5}[G(t_0)]$ is shown in equation 2.9. Although this network is relatively small (i.e., number of nodes is small), the number of states is quite high because the number of possible wireless links in this network is six which leads to 64 possible states. Therefore, obtaining two-terminal reliability expression implies manipulating all 64 states. On the other hand, the used algorithm only implies manipulating seven terms with the expense of manipulating six matrices as shown in Figure 2.2.

$$Rel_{1,5}[G(t_0)] = X_{1,2}X_{1,3}X_{3,5} + X_{1,2}X_{1,3}X_{3,5}X_{2,4}X_{4,5} + X_{1,2}X_{1,3}X_{3,5}X_{2,4}X_{3,4}X_{4,5}$$
$$+ X_{1,2}X_{1,3}X_{2,4}X_{4,5} + X_{1,2}X_{1,3}X_{2,4}X_{4,5}X_{3,4}X_{3,5} + X_{1,2}X_{1,3}X_{3,5}$$
$$+ X_{1,2}X_{1,3}X_{3,5}X_{3,4}X_{4,5}$$

(2.9)
2.4 Evaluations of Two-Terminal Reliability in Mobile Environment

2.4.1 Two-Terminal Reliability Under Non-uniform and Uniform Nodes Distribution

We considered the movement of the nodes under two mobility models: Random way point (RWP) [58] and Smooth Mobility Model (SMM) [59]. RWP and SMM lead, respectively, to non-uniform and uniform nodes distribution within the simulation area.

In RWP model, nodes initially stay stall (i.e., paused) for a certain time. Then, they start moving within the area of simulation with a known average speed for a given time. After the nodes reach their destination, they stay stall in their position for some random time, i.e., pause time. After that, nodes again choose another random destinations in the simulation field and move towards them. The whole process is repeated again and again until the simulation ends. If a node hits the simulation edge during its movement, it bounces back to the simulation area with the same speed and with an angle equal to the one it hits the border with. RWP leads to non-uniform nodes distribution within the simulation area. On the other hand, SMM maintains a uniform nodes distribution.
within the observed area. SMM model follows the physical law of the smooth motion, each nodes’ movement will have three phases: speed-up phase $\alpha$, middle-smooth phase $\beta$ and slow-down phase $\gamma$. For each movement, a node selects a target direction $\theta$ and a target speed $v$. In $\alpha$-phase, a node uniformly increases its speed until it reaches the target speed $v$. After that, the node’s maintain its speed and direction around the value of the target $v$ and $\theta$ during the $\beta$-phase. In $\gamma$-phase, the node decreases its speed in $\gamma$ steps until it completely stops. After each movement, the node stay still in its position for a random time i.e., pause time. After, pause time expires, the node picks a new direction and new speed and the three-phases movement is repeated.

We considered Ad hoc networks consists of 11 nodes. The transmission range of the wireless nodes was chosen to be $30m$ and the source and destination nodes are placed permanently at $(x_0 = 0, y_0 = 50)$ and $(x_{11} = 100, y_{11} = 50)$, respectively. Therefore, four hops at least are needed to create a path between the source node the destination node. Since each node has a transmission range of $30m$, the total areas covered by 11 is three times the observed area. All nodes, except source and destination nodes, are placed randomly in an area of dimensions $100m \times 100m$ at the beginning of the simulation. The source and destination nodes have a deterministic fixed locations throughout the simulation time. Once the nodes starting moving, it is expected that the two-terminal reliability will change accordingly. In this simulation, we will show how the two-terminal reliability is affected by the mobility pattern of nodes. For each simulation scenario, the simulation time is 500 seconds, and the results are obtained via averaging values from 100 different runs with different seeds. We assumed that all the nodes have an identical hardware plate-form and perform the same networking duties, i.e., routing, exchanging hello messages, and so on. Therefore, all the nodes have the same reliability behavior with respect to time. We assumed that, a link between any two nodes have an operating probability of 0.9, independent of the distance between the nodes. The simulation environments and parameters of the Ad hoc network are shown in Table 2.1:
Table 2.1: Parameters and Constants Used in the Simulations for Network in Figure 2.6.

<table>
<thead>
<tr>
<th>Field space</th>
<th>100 x 100 flat space</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of nodes</td>
<td>11</td>
</tr>
<tr>
<td>Average node speed</td>
<td>10 and 20 (m/s)</td>
</tr>
<tr>
<td>Node mobility</td>
<td>RWP and SMM</td>
</tr>
<tr>
<td>Simulation run time</td>
<td>500 sec</td>
</tr>
<tr>
<td>Node pause time</td>
<td>5 sec</td>
</tr>
<tr>
<td>Transmission range</td>
<td>30 m</td>
</tr>
</tbody>
</table>

We found that the uniform nodes distribution outperforms the nonuniform node distribution. The nonuniform node distribution leads to a concentration of the node at certain regions of the observed area, at the center for network, which lead to fewer disjoint paths between the communicating nodes. On the other hand, the uniform distribution allows more disjoint paths between the source and destination node and that boost the network reliability against failures. Moreover, mobility models which maintains a uniform node distribution results in better network reliability as shown in Figure 2.3 and Figure 2.4.

Figure 2.3: Comparison of Two-Terminal Reliability Under RWP and SMM with Speed=10 m/s and Pause Time= 5 sec.
It is obvious that the mobility pattern (i.e., speed and pause time) does influence the network reliability. First, the relationship between the mobility metrics and the two-terminal reliability can be observed via the effect of these metrics on the connectivity parameters of the network. There is a clear correlation between average degree of node, average relative speed, average link duration and reliability of the network. With the same spatial nodes distribution under a given mobility model, if mobility pattern has a high relative speed, the nodes might move from each other transmission range more quickly. Thus, lower link duration are experienced more frequently, which reduces the number of disjoint paths which in turn lower two-terminal reliability between the source and destination nodes.

This influence is less severe under SMM due to the physical constraint imposed on the node movement under SMM. The velocity of a mobile node changes gently instead of abruptly, so the current node’s velocity is dependent on the previous velocity. Accordingly, nodes location with respect to each other will not experience tremendous changes in short time intervals under SMM. Thus, an existing link between two nodes is expected to remain stable for a longer period of time as the nodes are likely to be within the transmission range of each other for longer time. On the other hand, the velocities of a
node at two different time instants are independent under RWP movement. Thus, nodes locations under RWP change abruptly with respect to each other for any consequent time instants. These smooth and abruptly changes in the nodes location will impose their effects upon the two-terminal reliability.

Figure 2.3 and Figure 2.4 show that reliability under SMM movement changes smoothly between two consequent time instants, because the resultant uniform nodes distribution imposes spatial dependency among the nodes such that the number of links between any two hops keeps fluctuating around an average value. On the other hand, RWP leads to sharp changes in the reliability values between consequent time instants, because the nonuniform nodes distribution makes the nodes rushing to the middle of the simulation area most of the time. Thus, the spatial dependency between nodes is location dependent parameter, so hops which are closed to the network center have higher number of links from the ones close to the boundaries. Accordingly, the number paths which are available between two consequent hops changes sharply from one time instant to the other.

2.4.2 Effect of Node Failure Rates on Two-Terminal Reliability and Network Performance Metrics

In this simulation, we first study the effect of different failure rates of nodes on the networks performance parameters such the packet lost rate and number of control messages and the end-to-end delay. Then, we present the effect of network dynamics on the two-terminal reliability.

We have consider ad hoc networks of 6, 11, 18 and 27 nodes placed in 600m × 600m plane to construct a grid structure as shown in Figure 2.5 to Figure 2.8. Grid structure was selected to ensure that high level of reliability can be obtained in each case. The transmission range of the wireless nodes was chosen to be 250 m with two-ray ground propagation model [35].

![Figure 2.5: Ad Hoc Network with Nodes Initially Placed in Grid Structure: 6 Nodes.](image)
Figure 2.6: Ad Hoc Network with Nodes Initially Placed in Grid Structure: 11 Nodes.

Figure 2.7: Ad Hoc Network with Nodes Initially Placed in Grid Structure: 18 Nodes.

Figure 2.8: Ad Hoc Network with Nodes Initially Placed in Grid Structure: 27 Nodes.
Random way point mobility model [58] is used to resemble the movement pattern of the wireless nodes. In this simulation, we consider different values for the average speed and pause time. The simulation environments and parameters of the Ad hoc networks are shown in Table 2.2

Table 2.2: Parameters and Constants Used in the Simulations for Networks in Figure 2.5 to Figure 2.8.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field space</td>
<td>600 x 600 flat space</td>
</tr>
<tr>
<td>Number of nodes</td>
<td>6, 11, 18, 27</td>
</tr>
<tr>
<td>Average node speed</td>
<td>5, 10, 15, 20, 25, 30 (m/s)</td>
</tr>
<tr>
<td>Node mobility</td>
<td>Random waypoint model [58]</td>
</tr>
<tr>
<td>Simulation run time</td>
<td>500 sec</td>
</tr>
<tr>
<td>Node pause time</td>
<td>5, 10, 15, 20, 25, 30 sec</td>
</tr>
<tr>
<td>MAC layer type</td>
<td>IEEE 802.11</td>
</tr>
<tr>
<td>Transmission range</td>
<td>250 m</td>
</tr>
<tr>
<td>Number of packet</td>
<td>1000 packet</td>
</tr>
<tr>
<td>Size of the packet</td>
<td>1000 byte</td>
</tr>
<tr>
<td>Interval between packets</td>
<td>0.5 sec</td>
</tr>
<tr>
<td>Routing Protocol</td>
<td>AODV [60]</td>
</tr>
</tbody>
</table>

The failure of a wireless node implies the failure of all wireless links which are incident from it. Therefore, the failure of the nodes alters the network topology and affects the reliability of the network. We investigated the effect of nodes failure rates on some network performance parameters such packet loss and control message load. As the failure rate increases, the network is flooded with control messages (i.e., ADOV messages) and the packet loss increases dramatically as shown in Figure 2.9 to Figure 2.12. The routing protocol will try to cope for the failure of the nodes by discovering new paths between the remaining set of nodes. For example, the failure rate of 0.7 in Figure 2.11 leads to 34.4% increase in control messages load and 63.2% packet loss. The packet loss effect is reduced slightly as the density of the nodes increases. As an example, the packet loss due to 0.2 failure rate is 18.7%, 13.5%, 13.7% and 8.8% as shown in Fig 2.9 to Fig 2.12, respectively.
Figure 2.9: Effect of Nodes’ Failure Rate and Node Density on Network Performance (Control Messages vs. Received Messages) for Network in Figure 2.5.

Figure 2.10: Effect of Nodes’ Failure Rate and Node Density on Network Performance (Control Messages vs. Received Messages) for Network in Figure 2.6.
Figure 2.11: Effect of Nodes’ Failure Rate and Node Density on Network Performance (Control Messages vs. Received Messages) for Network in Figure 2.7.

Figure 2.12: Effect of Nodes’ Failure Rate and Node Density on Network Performance (Control Messages vs. Received Messages) for Network in Figure 2.8.
For the two-terminal reliability, with slow speeds and large pause times, the reliability of the network show better stability. This is due to the stability of the network routes for longer time. As the average speed of the mobile nodes increases, more links are broken and that result in less paths between the source and the destination nodes as shown in Figure 2.13 and Figure 2.14. The two-terminal reliability obtained from nodes moving at 5m/s with 5 sec pause time outperforms the one obtained from nodes moving with 20-30 m/s by 60% on average. On the other extreme, nodes with pause time greater than 20 sec are not affect much by increasing the movement speed because all nodes are relatively static to each most of the time. We notice that the failure characteristic of the components in a homogeneous wireless networks have an effect in shaping and scaling the overall network reliability. The reliability of a node decays exponentially as time passes as shown in Equation (2.3) due the batteries power depletion. Therefore, it is expected that the overall reliability will have similar decaying behavior.

![Reliability calculations pause time= 5 sec different speeds (5,10,15,20,25,30)](image)

Figure 2.13: Effect of Node Movement Patterns on the Two-Terminal Reliability: Different Node Speeds.
Figure 2.14: Effect of Node Movement Patterns on the Two-Terminal Reliability: Different Pause Times.

2.5 Summary

In this chapter, we investigate the problem of computing the two-terminal reliability in mobile Ad hoc networks. We assumed that the nodes are subject to failures with an exponential distribution. We present how the mobility affect the two-terminal reliability in mobile Ad hoc networks and how the imperfect wireless nodes affect the network performance parameters. With smaller pause times, the speed of the nodes have great impact on the two-terminal reliability. When the speed is increased from 5 m/s to more than 20 m/s there is 60% , on average, loss in the two-terminal reliability value. On the other hand, as the pause time exceeds 20 sec, there is no significant gain in the two-terminal reliability and the effect of increasing the nodes’ speed is negligible. We also showed the effect of non-uniform and uniform node distribution effect on two-terminal reliability of a given network.
Chapter 3

Power Allocation for All-Terminal Network Reliability Optimization in Fading Environment via a Cross Entropy Approach

3.1 Introduction

Reliability analysis of a wireless network is concerned with measuring the probability of successful communication between a given set of nodes, i.e., sources and destinations. This explicitly implies the successful communication between intermediate nodes connecting these sources and destinations. In this work, we are concerned with the all-terminal network reliability which is defined as probability that each node can communicate with all other nodes in network. In wireless network, successful direct communication between neighboring nodes is affected by channel fluctuations (i.e., channel path loss, fading and additive noise) and resource constraints (i.e., data rate and transmission power)[35][36].

To overcome resource constraints and wireless network fluctuations, network reliability has been addressed within two levels in wireless networking: link-level and transport-level. At the link-level, reliability is concerned with the successful one-hop reception of transmitted packets. Link-level reliability can be achieved via two main approaches: retransmission of packets until they are successfully delivered, i.e., automatic repeat request (ARQ)[61][62] [63] [64] and transmission of a set of coded packets to enable receiver
to accurately recover or correct lost and corrupted packets, i.e., forward error correction (FEC). These two approaches suffer from degradation of throughput rate, latency and overall system instability. For instance, in ARQ, the randomness of the channel errors introduce a random delay due to the possible need for multiple retransmissions. Many applications have latency requirements, which cause the data to be useless if it is not delivered within a specified delay. On the other hand, FEC is transmitting, together with original data, redundant copies of it so that these copies can be used when the original data is lost, in case the copies themselves are not also lost. This redundancy utilizes scarce and costly wireless resources. Therefore, hybrid combinations of both ARQ and FEC are used to give a better performance[65][66][67][68]. Hybrid ARQ (HARQ) protocols make use of incremental channel codes to achieve reliable transmission over wireless channels using fewer packets transmissions while maintaining an acceptable level of throughput at reasonable latency which is suitable for real-time applications[62][69].

While link-level reliability can significantly improve hop-by-hop reliability, these approaches operate independently of higher layer protocols. The main concern about link-level reliability is the possibility of an adverse effect on the reliability at the transport-level. It might lead to a great deal of inefficiency in network performance, i.e., throughput and end-to-end delay[70]. A well-known example is the TCP over-wireless performance degradation phenomenon, in which numerous studies[70] were conducted in attempt to mitigate the shortcoming of techniques used to improve the link-level reliability at lower layers.

The aforementioned reliability studies in link and transport levels focus mainly on the design of mechanisms facilitating successful communications between nodes in wireless network without concerning about the different aspects of wireless channel and nodes’ characteristics, i.e., transmission power, that lead into communication failure. Moreover, these link and transport levels approaches are concerned about the hop-by-hop and point-to-point reliabilities, respectively. Therefore, their ability of improving all-terminal network reliability is inconclusive because their reliability scope is limited to portions of network at a given time as shown in Figure 3.1.

For multi-hop wireless network, reliability of network’s portions need to be integrated together to improve all-terminal network reliability. However, the aforementioned approaches for link and transport reliability are reactive in nature and hence fail to analyze reasons of links failures in underlying network. In wireless network, a link is categorized as failed link when the Signal-to-Interference-Ratio (SIR) falls below a certain threshold
and hence received signal can not be recognized. The statistical model that relates the
received transmitted signal to the received interference depends on nodes transmission
power, channel path loss, fading and additive noise. In this statistical model, nodes’
transmission powers are the only adjustable parameters. Transmission power control is a
highly effective technique for minimizing effect of interference, channel path loss, fading
and additive noise and hence improving links reliabilities. For instance, link reliability
can be improved by increasing the transmission power. In fact, the bit error proba-
bility, $P_e$, decreases when the ratio $(E_b/N_0)$ increases, where $E_b$ is the received energy
per bit and $N_0$ is the noise power spectral density[71]. Therefore, most of the proposed
approaches for link-level and transport-level reliability have common transmission con-
trol where the same transmission power $P_{tx}$ is used by every node. In these works, a
guidance is provided on the critical selection of common transmission power constrained
by problem requirements, e.g. connectivity [37][38], capacity [39], coverage [37][38][39]
and energy efficiency [37][38], to attain a specific reliability objective either on link or
transport level. The common transmission power selection approaches do not take into
consideration the effect of variation of transmission power on links and network reliabil-
ities. Moreover, the effect of nodes’ transmission powers variation on link and network
reliabilities is not capture by typical reliability analysis where a network is represented
as a probabilistic graph with vertexes and links operating with identical probabilities
$p_v$ and $p_e$, respectively [72][73]. Unfortunately, these probabilities do not capture the
failures induced by constrained node energy and error-prone wireless links. Few recent
published works tried to address these issues by relating the nodes reliabilities to the
probabilistic expenditure of nodes residual energies [74][75], while links probabilities are
related to the probabilistic of link characteristics such as link life time [76][77], percent of packet loss and retransmission rate over links [78]. However, little research efforts were made to characterize links reliabilities based on channel status and nodes’ transmission powers.

It have been shown that one-hop reliability cannot be improved by random common transmission power selection in deterministic channel, log-normal shadowing channel and super-imposed log-normal shadowing and Rayleigh fading channel[79]. Moreover, nodes are randomly deployed in multihop wireless network and hence the channel conditions affecting the links connecting these nodes differ due to the variation of distances between the nodes and the level of interference every node is exposed to. Hence, each node requires a precise power level to improve it’s out going links’ reliabilities and on same time not to be an interference that degrade other nodes’ links reliabilities. To do so, we have to intelligently allocate nodes transmission powers such that links and network all-terminal reliabilities are maximized. Therefore, we propose a power-control algorithm which takes into account the statistical variation of the Signal-to-Interference Ratio, $SIR$, at receiving nodes and optimally allocate nodes’ transmission powers to maximize the links reliabilities and all-terminal network reliability.

The remainder of the chapter is organized as follows. Section 3.2 presents an overview of the problem to be solved including a system description, assumptions, and problem inputs. Section 3.3 presents, in details, the dynamics of the algorithm used for allocating transmission power to nodes in a given network in order to maximize the all-terminal network reliability. Lastly, Section 3.4 concludes the chapter.

### 3.2 Preliminaries and Problem Formulation

In this section, we determine the influence of wireless environment on the links’ reliabilities. To do so, we introduce the Rayleigh fading environment, then we describe how to calculate the links’ reliabilities in this environment. Then, we present the formal method of calculating all-terminal reliability of a given wireless network. Next, we present the problem formulation of power allocation and all-terminal reliability optimization.
3.2.1 Network Model

A wireless network is modeled as a graph $G = (V,E)$ in which $V$ and $E$ are the set of nodes and links, respectively. The graph is assumed to have directional links and there are no parallel (i.e., redundant) links. A link is denoted as $e_{ij}$ and it is connecting node $v_i$, the transmitter, and node $v_i$, the receiver. In this work, the existence of $e_{ij}$ implies the existence of $e_{ji}$ in order to ensure the bidirectional communication between any pair of communicating nodes. Failures are assumed to be independent and components can only take one of the two states, operational or failed. If a link in failed state, it cannot be repaired.

3.2.2 Rayleigh Fading Environment

In wireless network, the nodes forming these networks are deployed randomly and left unattended to and are expected to be able to communicate each other properly and efficiently. As a result of this random deployment, nodes may not have a direct line-of-sight communication signal. Instead, a transmitted signal is reflected along multiple paths before it is received. This situation occurs when wireless network is deployed in dense urban area where none of the nodes has any direct line of sight communication with each other[26][27]. Rayleigh fading can be a useful signal propagation model in heavily built-up city urban area where there is no line of sight between the transmitter and receiver and many buildings and other objects attenuate, reflect, refract, and diffract the signal.

Therefore, we consider both desired and interference signals are subject to Rayleigh fading, hence we refer to this channel model as Rayleigh/Rayleigh fading environment [25][26][27]. In this environment, the receiving node has no direct line of sight signal component, either from the transmitting or the interfering nodes. The strength of the received signal in a Rayleigh/Rayleigh fading environment is dependent on the following factors: channel gain, fading and nodes’ transmission powers. Next we describe how the aforementioned factors affect the received power at a given node.

Consider a wireless network consisting of $n$ nodes labeled $v_1, \ldots, v_n$. Each node can act as a transmitter and as a receiver at the same time utilizing different frequency channels, codes, or antenna beams in an antenna array. A vector $P^{tx} = (P^{tx}_1, \ldots, P^{tx}_n)$ represents the transmission powers associated with nodes $v_1, \ldots, v_n$, respectively. The received power at $v_j$ from $v_i$ is:
\[ P_{ij}^{rx} = G_{ij} F_{ij} P_{i}^{tx}, \]  

(3.1)

where \( G_{ij} \) represents the channel gain (not including fading) from \( v_i \) to \( v_j \). We assume that \( G_{ij} \) is constant and represents the distance dependent power attenuation [80]. \( F_{ij} \) represents the Rayleigh fading induced by node \( v_i \) and received by node \( v_j \). \( F_{ij} \)s are assumed to be independent exponential random variables with unit mean. Thus, the received power at \( v_j \) from \( v_i \) is an exponentially distributed random variable with a mean of \( E[G_{ij} F_{ij} P_{i}^{tx}] = G_{ij} P_{i}^{tx} \) [25]. We assume that the interference from other transmitting nodes is much larger than the white noise at the receiving nodes, therefore, we neglect the impact the noise at the receivers. Next, we utilize expression of received power in Equation (3.1) to calculate the link reliability.

### 3.2.3 Link Reliability Definition

Successful communication over a given link is dependent, in addition to channel attenuation, upon the level of interference induced by other nodes in the network. If the level of interference is higher than the receive signal, the receiver will not be able to recognize or decode the received signal. Therefore, the first step in finding the expression for computing the link reliability is to specify the statistical model that related received transmitted signal to the received interference from the transmissions of the rest of nodes in the network. In Rayleigh/Rayleigh fading environment, the Signal-to-Interference-Ratio (SIR) is defined as follows:

\[ SIR_j = \frac{G_{ij} F_{ij} P_{i}^{tx}}{\sum_{k \neq i} G_{kj} F_{kj} P_{k}^{tx}}, \]  

(3.2)

where the term in the numerator denotes the received power at \( v_j \) from \( v_i \) and the denominator denotes the total interference due to all other nodes’ received power at \( v_j \). For a successful communication over a given link, the SIR should be greater than a threshold called acceptable reception threshold, \( SIR_{th} \). Note that the \( SIR_{th} \) falls in the range of -80 dBm (10 pW) to -60 dBm (1000 pW) which is the typical range received signal power over wireless 802.11x networks. While typical wireless router transmission power falls in the range of 15 dBm (32 mW) to 20 dBm (100 mW).

For \( v_j \), if its successfully receiving threshold is \( SIR_{th} \), then link reliability is the probability of successful communication between \( v_i \) and \( v_j \). Link reliability is defined as
follows:

\[
Rel_{ij} = 1 - Pr(SIR_j \leq SIR_{th}) = 1 - Pr\left(P_{ij}^{rx} \leq SIR_{th} \sum_{k \neq i} G_{kj} F_{kj} P_{tx}^k \right),
\]

(3.3)

where \(Pr\left(P_{ij}^{rx} \leq SIR_{th} \sum_{k \neq i} G_{kj} F_{kj} P_{tx}^k \right)\) can be interpreted as the probability that the communication between \(v_i\) and \(v_j\) experiences a failure due to fading. This probability has been defined as link outage probability \([25][26][27]\).

**Result 1** For a wireless network with \(n\) nodes operating in a Rayleigh/Rayleigh fading environment, it was proven in \([26]\) that the probability of successful communication between any pair of nodes \(v_i, v_j\) where \(i, j \in n\) is obtained as follows:

\[
Rel_{ij} = \prod_{k \neq i} \frac{1}{1 + \frac{SIR_{th} G_{kj} P_{tx}^k}{G_{ij} P_{tx}^i}}.
\]

(3.4)

From the expression in Equation (3.4), we can interpret that a node may select to increase its transmission power to improve the reliability of the node’s outgoing links by overcoming the interference and attenuation effects. However, the arbitrary increase of a node’s transmission power may diminish the other links’ reliabilities by introducing higher interference on their ongoing communication and hence diminish the all-terminal network reliability. In order to find the optimal nodes’ transmission powers that improve both links and network reliabilities, we need to present how the all-terminal network reliability is computed and how links’ reliabilities affect such computation.

### 3.2.4 Computing All-Terminal Network Reliability

All-terminal network reliability is defined as the probability that all nodes can communicate with all other nodes. Computing all-terminal reliability entitles: defining the all network states in which all nodes are connected, defining links status (i.e., operational or fail) in each network state, computing links reliabilities and computing all-terminal reliability for all possible network states. Next, we present how all-terminal network reliability is calculated.
Let $|V| = n$ be set of terminal nodes in the network. For a fully connected network, the number of links is $|E| = n^2 - n$. A vector $e = (e_{12}, \ldots, e_{ij}, \ldots, e_{nn-1})$ denotes the directed links between nodes. In this work, the existence of $e_{ij}$ implies the existence $e_{ji}$ in order to ensure the bidirectional communication between any pair of communicating nodes. A vector $P_{tx} = (P_{tx1}, \ldots, P_{txn})$ represents the transmission powers associated with nodes. The selection of $P_{txi}$ and $P_{txj}$ controls links reliabilities, $Rel_{ij}$ and $Rel_{ji}$, of links connecting $v_i$ and $v_j$.

We identify the operational links using a link state $y_{ij}$. If a link $e_{ij}$ is functioning, then its state $y_{ij} = 1$ and $y_{ij} = 0$ otherwise. Thus, a vector $y = (y_{12}, \ldots, y_{ij}, \ldots, y_{nn-1})$ is called the state vector. Each state vector represents a network topology. Thus, the set of all possible state vectors represents the set of all possible topologies that can be constructed on a given network. The set of possible state vectors is denoted as $\Upsilon$.

For a given network topology, i.e., state vector $y$, let $\Phi(\cdot)$ be an indicator function that map state vector into network state:

$$\Phi(y) = \begin{cases} 1 & \text{if a reliability condition is met}, \\ 0 & \text{otherwise}. \end{cases}$$

(3.5)

For all-terminal reliability, $\Phi(y) = 1$ if and only if all nodes are connected. All-terminal network reliability is computed as

$$Rel(y) = \sum_{y \in \Upsilon} \Phi(y) \prod_{i=1}^{n} \prod_{j=1, j \neq i}^{n} Rel_{ij}^{y_{ij}}(1 - Rel_{ij})^{1-y_{ij}}.$$

(3.6)

As stated above, the selection of $P_{txi}$ and $P_{txj}$ controls links reliabilities, $Rel_{ij}$ and $Rel_{ji}$, of links connecting $v_i$ and $v_j$. By increasing $P_{txi}$ and $P_{txj}$, the reliabilities of links connecting $v_i$ and $v_j$ will improve in case all other nodes keep their transmission power at same level. However, we cannot predict if all-terminal network reliability will improve as well due to adverse effect of increasing $P_{txi}$ and $P_{txj}$ on the other links’ reliabilities. Therefore, we need to intelligently adjust nodes’ transmission powers such that both links and all-terminal reliability are maximized. Next, we formulate the problem of power allocation and network reliability optimization in a wireless network.
3.2.5 Problem Formulation

For a given network topology, our goal is to maximize the network reliability by intelligently allocating nodes’ transmission powers. In practice, wireless node may not have the capability of controlling transmission power continuously, but rather support only finite set of power levels. Thus, we consider power assignment in discrete power domain. The power levels that a wireless node can support are $\{\Delta P_{tx}, 2 \Delta P_{tx}, \ldots, L \Delta P_{tx}\}$ where $L$ is the number of power levels and $\Delta P_{tx} = P_{tx}^{max}/L$, where $P_{tx}^{max}$ is the maximum transmission power allowed for a node’s transmission.

Then, the problem of network reliability optimization (NRO) can be formulated as follows:

\[
\begin{align*}
\text{maximize} & \quad \text{Rel}(\mathbf{y}), \\
\text{subject to} & \quad 0 \leq P_{tx}^{i} \leq P_{tx}^{max}, \quad \forall P_{tx}^{i} \in \mathbf{P}_{tx}, \\
& \quad P_{tx}^{i} \in \{\Delta P_{tx}, 2 \Delta P_{tx}, \ldots, L \Delta P_{tx}\}. \\
\end{align*}
\] (3.7)

We observe that the all-terminal reliability function, $\text{Rel}(\mathbf{y})$, in Equation (3.6) is homogeneous, i.e., if we scale all the nodes’ transmission powers by any positive scale factor, $\text{Rel}(\mathbf{y})$ remains the same. $\text{Rel}(\mathbf{y})$ depends only on the ratios of the nodes’ transmission powers. Since the nodes’ transmission powers, $P_{tx}^{i}$’s, are also homogeneous, we observe that if $\mathbf{P}_{tx}^{*}$ is optimal power allocation vector that yields the maximum all-terminal network reliability, then so is $\beta \mathbf{P}_{tx}^{*}$ for any integer $\beta > 0$. Therefore, an identical adjusting of nodes transmission power, i.e., increasing/decreasing all nodes’ transmission powers with same factor, will have no affect on the all-terminal network reliability.

The optimization problem in Equation (3.7) entitles two NP-hard problems: the first NP-problem is to determine optimal nodes’ transmission powers that attain the maximum all-terminal reliability. The second NP-hard problem is to calculate the all-terminal network reliability for a given power allocation.

For a network with $n$ nodes, $l$ links and $L$ power levels, the search space of optimal nodes’ transmission allocation is $(L + 1)^n \times 2^l$ (i.e., first NP-problem). In addition, the exact calculation of network reliability in Equation (3.6) entitles excessive computational effort growing exponentially with the number of nodes and links in the network (i.e., second NP-problem). To overcome the burden of exact reliability calculation \[81, 82, 83, \]
and excessive search of the huge search space of possible solutions, we have to develop an algorithm which not only calculate efficiently all-terminal network reliability but also be able efficiently to find nodes’ transmission powers allocation that attain the maximum all-terminal reliability. Finding the optimal nodes’ power transmission allocation in huge search space is considered a form of finding the occurrence of a rare but an important event in large events space. Rare-event simulation involves estimating extremely small but important probabilities [85][86][87]. A well known simulation method in field of rare event simulation is Cross-Entropy (CE) method. CE has been used to accurately estimate rare events in various applications such: buffer allocation problem [85], vehicle routing problem [88], and network planning problem [89].

In this study, a simulation method based on Cross-Entropy method (CE) has been developed to not only to calculate all-terminal network reliability but also to determine the optimal nodes’ transmission powers allocation that attain the maximum all-terminal reliability. Next, we present in details our solution to optimize power allocation to maximize all-terminal network reliability for a wireless network.

3.3 Solution Algorithm For NRO Problem

In this section, we present the main CE algorithm for allocating transmission power to nodes in a given the network in order to maximize the all-terminal network reliability defined in Section 3.2.5

3.3.1 Cross-Entropy Method: an Overview

For this work to be self-contained, in this section we present an overview of the usage of CE method in solving an optimization problem. We borrow part of this overview from the work by Allon et. al [85] and we refer to [86][87] for extended details.

CE method is an adaptive importance sampling derived from the associated stochastic problem (ASP) of a given optimization problem for estimating the probability of a rare-event occurrence. The estimation of this probability is determined using a log-likelihood estimator govern by a parametrized probability distribution. CE method adaptively estimates the parameters of the probability distribution which produces a random variable in the neighborhood of the globally optimal solution by minimizing cross entropy. CE method has two phases:
1. Generating candidate solutions sampled from some parametrized probability distribution.

2. Updating the parameters of the probability distribution based upon the current best solutions to bias the future search to obtain samples closer to the optimal solution.

In order to maximize an objective function $S(x)$ over a finite set of $\chi$, i.e., $x \in \chi$, assume that $S(x)$ has a unique maximizer $x^*$, i.e., $\gamma^* = S(x^*)$, then

$$S(x^*) = \gamma^* = \max_{x \in \chi} S(x).$$  \hspace{1cm} (3.8)

The first step in applying CE method is to associate an estimation problem with the optimization problem in Equation (3.8). Thus, a collection of functions $\{H(\cdot; \gamma)\}$ on $\chi$ are defined for each $x \in \chi$ and threshold $\gamma \in \mathbb{R}$ as follow:

$$H(x; \gamma) = \begin{cases} 
1 & S(x) \geq \gamma, \\
0 & S(x) < \gamma. 
\end{cases}$$  \hspace{1cm} (3.9)

Next, let $\{f(\cdot; v)\}$ be a family of probability mass functions (pmf’s) on $\chi$, parametrized by real-valued parameter vector $v$. The problem in Equation (3.8) is associated with the problem of estimating:

$$\ell_v(\gamma) = \mathbb{P}_v(S(X) \geq \gamma),$$
$$= \sum_x H(x; \gamma)f(x; v) = \mathbb{E}_v H(X; \gamma),$$  \hspace{1cm} (3.10)

where $\mathbb{P}_v$ is a probability measure under which the random state $X$ has probability mass function $f(\cdot; v)$, and $\mathbb{E}_v$ denotes the corresponding expectation. The problem in Equation (3.10) is the associated stochastic problem (ASP). To indicate how (3.10) is associated with (3.8), suppose for example that $\gamma$ is equal to $\gamma^*$. Thus, $\ell_v = f(x^*; v)$ will be a very small number. Therefore, Importance Sampling technique (IS) [86] is used to estimated such small number. Using IS, a random sample $X^{(1)}, \ldots, X^{(N)}$ is taken from a different pmf $g$ on $\chi$, then $\ell_v(\gamma)$ is estimated as follows:

39
\[ \ell_v(\gamma) = \frac{1}{N} \sum_{k=1}^{N} H(X^{(k)}; \gamma) \frac{f(X^{(k)}; \nu)}{g(X^{(k)})}. \quad (3.11) \]

Optimal \( \gamma^* \) occurs when \( g \) assigns all its probability mass to select \( x^* \). Therefore, as \( g \) assigns most of its probability mass close to \( x^* \), the obtained \( \gamma \) is close to \( \gamma^* \). CE method will efficiently estimate \( \ell_v(\gamma) \) in Equation (3.10) by controlling \( g \) and hence generation of state \( x \). The optimal way to estimate \( \ell_v(\gamma) \) is to use the change of measure with pmf:

\[ g^*(x) = \frac{H(x; \gamma) f(x; \nu)}{\ell_v(\gamma)}. \quad (3.12) \]

Note that \( g^* \) in Equation (3.12) depends on unknown parameter \( \ell_v \). To overcome this problem, an optimal pmf \( f(\cdot, v^*) \) is chosen such that the distance between this pmf and \( g^* \) is minimal. The distance between two pmf’s \( g \) and \( h \) is measured using Kullback-Leibler distance, i.e., cross-entropy[90] which is

\[ K(g, h) = E_g \log \frac{g(X)}{h(X)} = \sum_x g(x) \log g(x) - \sum_x g(x) \log h(x). \quad (3.13) \]

For estimating \( \ell_{v^*}(\gamma) \) in Equation (3.10), parameter \( v^* \) is chosen such that \( K(g^*, f(\cdot, v^*)) \), with \( g^* \) in Equation (3.12), is minimal. Thus, \( v^* \) should be selected such that (3.14) is maximal.

\[ E_v H(X; \gamma) \log f(X; v^*). \quad (3.14) \]

In CE method, the components of \( v \) can be calculated analytically in order to drive change of measure \( g \) toward \( g^* \) to achieve \( \gamma^* \). For discrete random vectors \( X, v \) is

\[ v = \frac{E_v H(X; \gamma) I_{\{X \in A\}}}{E_v H(X; \gamma) I_{\{X \in B\}}}, \quad (3.15) \]

where \( I_{\{X \in A\}} \) and \( I_{\{X \in B\}} \) are indicator random variables and \( A \subset B \subset \chi \). The vector \( v \) in Equation (3.15) is estimated as follows:

\[ v = \frac{\sum_{k=1}^{N} E_v H(X^{(k)}; \gamma) I_{\{X^{(k)} \in A\}}}{\sum_{k=1}^{N} E_v H(X^{(k)}; \gamma) I_{\{X^{(k)} \in B\}}}, \quad (3.16) \]

where \( X^{(1)}, \ldots, X^{(N)} \) is a random samples from the pmf \( f(\cdot; v) \).

Now CE method will construct a sequence of parameters \( v_0, v_1, \ldots \) and performance
measures $\gamma_0, \gamma_1, \ldots$ such that $v_k$ converges to a parameter vector whose corresponding pmf assigns high probability mass to the collection of states that enforces $\gamma_k$ to converge to the optimal $\gamma^\ast$. CE method procedures are summarized in Algorithm 3.1. These procedures can be adopted to solve any combinatorial optimization problem.

**Algorithm 3.1 CE Algorithm for Combinatorial Optimization**

START with some $v_0$. Let $k = 0$.

REPEAT

1: Draw a random samples $X^{(1)}, \ldots, X^{(N)}$ from $f(\cdot, v_k)$.
2: Calculate $S(X^{(i)})$ for all $i$, and arrange them in descending order, $S_1 \geq \ldots \geq S_N$.
   Let $[\rho N]$ be the integer part of $\rho N$. Define $\gamma_k = S_{[\rho N]}$.
3: Define $v_{k+1}$ as the estimate of the optimal $v^\ast$ in Equation (3.14) with $v = v_k$. Thus,
   the components of $v_{k+1}$ are found from (3.16). Increase $k$ by 1.

UNTIL convergence is reached.

Note that only stopping criterion, the initial state $v_0$, the sample size $N$ and the number $\rho$ (i.e., $0.01 \leq \rho \leq 0.1$) have to be specified in advance. The procedures in Algorithm 1 can be applied to any maximization problem. Nevertheless, each problem is unique in term of generating solution candidates and updating CE parameters to converge to an optimal solution.

CE method has been used in solving many optimization problems. Examples are Buffer Allocation Problem [85], Vehicle Routing Problem [88], and Network Planning Problem [89]. We refer to [86][87] and references therein for more details about the CE method properties and its applications.

### 3.3.2 Network Reliability Optimization (NRO) Algorithm

In order to apply CE to solve NRO we need to specify how to generate random power assignment and how to update the parameters of parametrized probability distribution used in generating the power assignment at each iteration. Let each possible transmission power assignment be presented by a vector $x = (x_0, \ldots, x_{n-1})$ in the set $X = \{ x = (x_0, \ldots, x_{n-1}) : x_i \in \{0, \Delta P^{tx}, \ldots, L \Delta P^{tx}\} \}$. Define a function $S$ on $X$ such that $S(x)$ denotes the all-terminal network reliability. Then, we can obtain the optimal power assignment by solving the this problem:
maximize \( S(\mathbf{x}) \) over \( \mathbf{x} \in X \). \hspace{1cm} (3.17)

The CE method requires a random mechanism to generate the power level assignment for the nodes. A simple method to generate a random power assignment \( \mathbf{x} = (x_0, \ldots, x_{n-1}) \) in \( X \) is to first draw a \( x_0, \ldots, x_{n-1} \) independently, each \( x_i \) according to a \((L + 1)\)-point discrete distribution \((p_{i,0}, \ldots, p_{i,L}), i = 1, \ldots, n\) and accept the sample only if \( \Phi(y) = 1 \), i.e., to ensure that all-terminal reliability condition, i.e., all nodes are connected, is considered while assigning the transmission powers for each node. Note that \( p_{ij} \) represents the probability that a node \( i \) is assigned a transmission power level of \( j \triangle P_{tx} \). Thus, a matrix \( \mathbf{P} := (p_{ij}) \) with dimension of \( n \times L + 1 \) determines which power level should be assigned to which node. Note that each row of \( \mathbf{P} \) sums up to one. The pmf \( f(\cdot; \mathbf{P}) \) of \( X \) is thus parameterized by matrix \( \mathbf{P} \) and given by

\[
f(\mathbf{x}; \mathbf{P}) = \prod_{i=0}^{n-1} \sum_{j=0}^{L} p_{ij} \mathbf{1}_{\{\mathbf{x} \in X_{ij}\}}, \hspace{1cm} (3.18)
\]

where \( X_{ij} = \{ \mathbf{x}_{ij} \in X : x_i = j \triangle P_{tx} \} \), i.e., \( X_{ij} \) is the \( i \)-th coordinate of \( \mathbf{x} \) is \( j \triangle P_{tx} \).

Starting from some initial value matrix \( \mathbf{P}_0 \), the CE algorithm updates elements \( \mathbf{P} := (p_{ij}) \) to converge to \( \mathbf{P}^* \), such that at each row, \( i \), of \( \mathbf{P}^* \) contains only one element, \( j \), equals (or close) to one and the remaining equal (or close) to zero, i.e., the power assignment \( j \triangle P_{tx} \) to node \( i \). The non-zero elements define the power assignment which will maximize the all-terminal network reliability. The updating rules for NRO is derived from the maximization of (3.14) with the condition that the sum of each row is one. Applying Lagrangian optimization to (3.14), the following maximization problem is obtained:

\[
\max_{\mathbf{P}, \lambda_0, \ldots, \lambda_{n-1}} \left[ \mathbb{E}_{\mathbf{P}} H(\mathbf{X}; \gamma) \log f(\mathbf{X}; \mathbf{P}) + \sum_{i=0}^{n-1} \lambda_i \left( \sum_{j=0}^{L} p_{ij} - 1 \right) \right], \hspace{1cm} (3.19)
\]

where \( \lambda_0, \ldots, \lambda_{n-1} \) are Lagrange multipliers. By differentiating with respect to \( p_{ij} \):

\[
\mathbb{E}_P \frac{H(\mathbf{X}; \gamma) I_{\{\mathbf{x} \in X_{ij}\}}}{p_{ij}} + \lambda_i = 0, \hspace{1cm} \forall j = 0, \ldots, L. \hspace{1cm} (3.20)
\]

Summing over \( j = 0, \ldots, L \) yields \( \mathbb{E}_P H(\mathbf{X}; \gamma) = -\lambda_i \) so that

\[
p_{ij} = \frac{\mathbb{E}_P H(\mathbf{X}; \gamma) I_{\{\mathbf{x} \in X_{ij}\}}}{\mathbb{E}_P H(\mathbf{X}; \gamma)}. \hspace{1cm} (3.21)
\]
For $N$ samples, (3.21) can be estimated as follows:

$$ p_{ij} = \frac{\sum_{k=1}^{N} I_{\{S(x^{(k)}) \geq \gamma\}} I_{\{x^{(k)} \in X_{ij}\}}}{\sum_{k=1}^{N} I_{\{S(x^{(k)}) \geq \gamma\}}} \tag{3.22} $$

This estimator count how many of the $x^{(i)}$ obtains value greater, i.e., reliability, than $\gamma$, let this number be $B$, and of those have their $i$-th coordinate equal to $j$, let this number be $A$. Dividing $A$ by $B$ gives the updates of $p_{ij}$, i.e., similar to (3.16). Algorithm 3.2 is used to generate power level assignment to each node using (3.22).

**Algorithm 3.2 Random Power Assignment Algorithm**

1: For a given $P=p_{ij}$, generate a random permutation $(\pi_0, \ldots, \pi_{n-1})$ of $\{0, \ldots, n-1\}$.
2: for $i = 0$ to $n-1$ do
3: \hspace{1cm} $t = \sum_{j=0}^{L} P_{\pi_i,j}$
4: \hspace{1cm} for $j = 0$ to $L$ do
5: \hspace{2cm} $P_{\pi_i,j} = \frac{P_{\pi_i,j}}{t}$
6: \hspace{1cm} end for
7: \hspace{1cm} Generate $X_{\pi_i}$ according to $(p_{\pi_i,0}, \ldots, p_{\pi_i,L})$
8: end for

The remaining unspecified parameters of the CE algorithm are the initial matrix $P_0$, the stopping criterion and $N$. For $P_0$, we simply take all elements equal to $\frac{1}{L+1}$. The stopping criterion is the convergence parameter matrix $P$ to $P^*$, such that at each row, $i$, of $P^*$ contains only one element, $j$, equals (or close) to one and the remaining elements equal (or close) to zero. The convergence of $P$ is associated with the convergence of the all-terminal reliability to a certain value, i.e., $\gamma^*$. Thus, CE algorithm is terminated if for some integer $d$ and $k \geq d$, $\gamma_k^* = \gamma_{k-1}^* = \ldots = \gamma_{d}^*$. $N$, the sample size, can be derived from the matrix $P$’s dimensions, e.g. $N = \tau n (L + 1)$ where $\tau$ is an integer greater than 0.

Rubinstein et al [86, 85, 87] introduced a smoothing procedure for updating the $P_k$ to $P_{k+1}$ in Equation (3.23) as follows:

$$ P_{k+1} = \alpha P_{k+1}^{\text{temp}} + (1 - \alpha) P_k, \tag{3.24} $$

where $P_{k+1}^{\text{temp}}$ is obtained via Equation (3.23). It was found that selecting $\alpha$ in the range
Algorithm 3.3 Main CE Algorithm for NRO

START with some \( P_0 = \frac{1}{L+1} \). Let \( k = 1 \).

REPEAT

1: Draw a random sample of power allocations \( X^{(1)}, \ldots, X^{(N)} \) according to Algorithm 3.2, with \( P = P_{k-1} \).
2: Calculate the all-terminal network reliability \( S(X^{(i)}) \) for all \( i \) using Algorithm 3.4, and arrange them in descending order, \( S_1 \geq \ldots \geq S_N \). Let \( \lfloor \rho N \rfloor \) be the integer part of \( \rho N \). Define \( \gamma_k = S_{\lfloor \rho N \rfloor} \).
3: Using the same sample, calculate \( P_{k+1} = (P_{k+1,ij}) \)

\[
P_{k+1,ij} = \frac{\sum_{k=1}^{N} I\{S(x^{(k)}) \geq \gamma\} I\{x^{(k)} \in \mathbf{X}_{ij}\}}{\sum_{k=1}^{N} I\{S(x^{(k)}) \geq \gamma\}}.
\]

(3.23)

Increase \( k \) by 1.

UNTIL For some \( k \geq d \), \( \gamma_k = \gamma_{k-1} = \ldots = \gamma_{k-d} \).

0.7 to 0.9 gives the best results [85, 86, 87].

3.3.3 Estimating All-Terminal Network Reliability

The exact calculation of all-terminal network reliability is an NP-hard problem, with computational effort growing exponentially with the number of nodes and links in the network. Therefore, we utilize a Monte Carlo (MC) simulation approach to estimate the all-terminal network reliability. MC simulation approaches have been extensively used to attain an accurate estimation of the network reliability [91, 92, 93, 94, 95, 96]. Basically, the MC simulation proceeds as follows: given a network topology, \( T \), power assignment vector \( X^{(i)} \) and the distances between nodes, i.e., \( G \), this approach calculates links’ reliabilities, i.e., \( \text{Rel}_{ij} \), using (3.4). Then, this approach utilizes \( \text{Rel}_{ij} \) in randomly generating a state vector \( y = (y_{12}, \ldots, y_{1n}, y_{23}, \ldots, y_{2n}, \ldots, y_{ij}, \ldots, y_{nn-1}) \). Then, using breadth search first (BSF), if \( y \) contains a spanning tree, i.e., all nodes are connected, we first check if \( y \) has been selected before, if not the \( \text{Rel}(y) \) is calculated using (3.25). The generation and evaluation of a randomly state vector \( y \) is repeated for a specific number of times, i.e., SampleSize. In the last step, the all-terminal network reliability is calculated as the summation of the reliabilities of all successful state vectors, i.e., containing a spanning tree. The pseudo code of the all-terminal network reliability estimation is shown in Algorithm 3.4.
Algorithm 3.4 All-terminal Reliability Estimation Algorithm

1: Given $X^{(i)}$, $SIR^{th}$, $G_{ij}$, and Network topology $T_{ij}$. Calculate links reliability $Rel_{ij}$'s using (3.4). Set $k = 0$ and $Rel_{est} = 0$.

2: while $k < \text{SampleSize}$ do
3:  Generate state vector
4:  
5:  for $i = 1$ to $n$ do
6:   
7:   if $T_{ij} == 0$ then
8:     $y_{ij} = 0$
9:   else
10:     Generate a random number $\text{rand} \in [0, 1]$
11:     if $\text{rand} < Rel_{ij}$ then
12:       $y_{ij} = 1$
13:     else
14:       $y_{ij} = 0$
15:     end if
16:   end if
17:  end for
18:  Check if $y$ forms a spanning tree using $BSF(y)$
19:  if $BSF(y) == 1$ then
20:    if $y$ have been selected, skip to step 24.
21:    Calculate $Rel(y)$ as using
22:    
23:    
24:  end if
25:  $k = k + 1$
26: end while
27: All-terminal network reliability estimation is $Rel_{est}$

3.3.4 NRO Complexity

In this section we present the complexity of the algorithms used in NRO. First, the first step the random power assignment algorithm, Algorithm 3.2, requires $O(n)$ operations where $n$ is the number of nodes in the network. Since $N$ random power allocations are generated, the complexity of random power generation step is $O(n \times N)$. The second
step, the calculation of all-terminal reliability requires \textit{SampleSize} evaluations as shown in Algorithm 3.4. Each evaluation requires the determination if the state vector \( y \) contains a spanning tree, i.e., all nodes are connected (all-terminal reliability condition). The existence of spanning tree can be evaluated in \( O(l) \) operations where \( l = n^2 - n \) is the number of edges in fully connected directed network. That means second step requires \( O(\text{SampleSize} \times N \times l) \) operations to evaluate all the generated samples from the first step. The third step, parameters updates and solution discovery, initially required \( O(N \log(N)) \) operations for sorting the power allocation samples, \( N \), in descending order. The updates of the \( P_{k+1,ij} \) parameters requires \( O(N^2 \times n \times (L + 1)) \) operations where \( L \) is the total number of discrete power levels. Therefore, the third step requires total of \( O(N \log(N) + N^2 \times n \times (L + 1)) \) operations. Finally, all these three steps are repeated \( k \) times.

### 3.3.5 Illustrative Example

Consider a 5-nodes fully connected network as shown in Figure 3.2 which have been studied in [97][98] in maximizing all-terminal network reliability subject to links costs. Different from [97][98], we utilize the link cost as distance metric between the nodes. The network is functioning if all the nodes are connected by the functioning links, i.e., all-terminal reliability condition. We used the following CE parameters \( N = 2n(L+1) = 160 \), \( \rho = 0.1 \), \( \alpha = 0.7 \) and, we took \( P_0 = (\frac{1}{L+1}) = \frac{1}{21} \forall p_{ij} \in P_0 \), where \( L \) is number of power levels as described in Section 3.3. For the network parameters, \( SIR^{th} = 0 \) dBm (1 mW) for illustration purpose, \( P_{max}^{tx} = 20 \) dBm (32 mW) and the number of power levels is \( L = 20 \), where \( \Delta P^{tx} = 1 \) dBm.

![Figure 3.2: Illustrative Example: 5 Nodes Network.](image-url)
At each iteration $N = 2n(L + 1)$ random power assignments are generated and the parameters of $P_k$ matrix are updated according to the smoothed updating procedure in Equation (3.24) with $\alpha = 0.7$. Figure 3.3 shows the dynamics of the updating matrix $P_k$. Starting from $P_0$ with the elements $\frac{1}{1+L} = 0.0476$. Algorithm 3.3 stopped after 9 iterations assigning the transmission power levels 10, 10, 11, 11, and 8 (in dBm) to nodes $v_1, v_2, v_3, v_4,$ and $v_5$, respectively. Note each row in $P_k$ sums to one and at the 9th iteration only one element in each row has much greater probability than any other elements. This probability is approximately 1. Table. 3.1 shows the assigned transmission power levels at the end of each iteration and the associated all-terminal reliability with such assignment.

![Figure 3.3: The Dynamics of Updating the Parameter Matrix $P_k$.](image)

Note that the solution obtained is based on sampling, thus the accuracy of NRO depends on the sample size and efficiency of updating procedure for matrix $P_k$. The efficiency of updating procedure can be enhanced by controlling the smoothing parameter as shown in [86][85] [87]. On the other hand, the obtained power assignments can be
enhanced by either increasing the sample number $N$ or generating and averaging several independent solutions under relatively small sample number, i.e., $N = 2n(L + 1)$. We select the later approach to generate more accurate power assignment levels. For network in Figure 3.2, we generate ten independent solutions then averaging these solution using $\left[\frac{1}{10} \sum_{i=1}^{10} \text{alloc}(v_j)_i\right]$, $\forall v_j, j = 1, \ldots, n$, where $[\cdot]$ is the integer part. Table 3.2 shows ten optimized power allocations for network in Figure 3.2.

Table 3.2: Optimal Power Assignment for 10 Independent Runs of NRO on the Network in Figure 3.2.

<table>
<thead>
<tr>
<th>Sol. #</th>
<th>All-Terminal Reliability</th>
<th>Power (dBm)</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.86028</td>
<td>9</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>0.86028</td>
<td>10</td>
<td>10</td>
<td>11</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>0.86028</td>
<td>10</td>
<td>10</td>
<td>11</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>0.86028</td>
<td>9</td>
<td>9</td>
<td>10</td>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>0.86028</td>
<td>10</td>
<td>10</td>
<td>11</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>0.86028</td>
<td>11</td>
<td>11</td>
<td>12</td>
<td>12</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
<td>0.86028</td>
<td>10</td>
<td>10</td>
<td>11</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>0.86028</td>
<td>10</td>
<td>10</td>
<td>11</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>0.86028</td>
<td>11</td>
<td>11</td>
<td>12</td>
<td>12</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>0.86028</td>
<td>10</td>
<td>10</td>
<td>11</td>
<td>11</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 3.1: Performance of NRO Algorithm for Solving Power Assignment for the Network in Figure 3.2.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>All-Terminal Power (dBm)</th>
<th>Reliability</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.8513660</td>
<td>18</td>
<td>17</td>
<td>17</td>
<td>16</td>
<td>14</td>
</tr>
<tr>
<td>1</td>
<td>0.8577314</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>0.8588672</td>
<td>11</td>
<td>10</td>
<td>12</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>0.8596982</td>
<td>8</td>
<td>8</td>
<td>9</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>0.8602836</td>
<td>10</td>
<td>10</td>
<td>11</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>0.8602836</td>
<td>11</td>
<td>11</td>
<td>12</td>
<td>12</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>0.8602836</td>
<td>10</td>
<td>10</td>
<td>11</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>0.8602836</td>
<td>10</td>
<td>10</td>
<td>11</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>0.8602836</td>
<td>10</td>
<td>10</td>
<td>11</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>0.8602836</td>
<td>10</td>
<td>10</td>
<td>11</td>
<td>11</td>
<td>8</td>
</tr>
</tbody>
</table>

\[
\left[\frac{1}{10} \sum_{i=1}^{10} \text{alloc}(v_j)_i\right], \forall v_j, j = 1, \ldots, n\]
3.3.6 Numerical Results

In this section, 7 different network problems are used to show the simplicity and efficiency of the proposed assignment algorithm. The first five problems originally used by Dengiz et al.[99][97], and AbiElFotoh et al.[98] to minimize the network design cost subject to a known constraint on the all-terminal reliability whereas links’ costs and reliabilities are priori known. Different from these works we utilize the link cost as a distance metric between nodes. Moreover, links’ reliabilities are not fixed and subject to change with respect to changes in the nodes’ transmission power levels.

The performance of the proposed algorithm is compared against Simulated Annealing (SA) and Genetic Algorithm (GA). Two performance metrics are used to judge the effectiveness and the efficiency of the proposed algorithm: number of searched solutions and attained all-terminal network reliability. We assumed that the allowable number of solutions searched to be at most 10% of the search space. This corresponds to upper bound and not necessarily to the actual number of analyzed solutions. A brief description of the network problems is given in Table. 3.3. A detailed description of the network problems is given in Appendix 3.5.3. For each network problem, 10 independent runs have been conducted.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Network Specification</th>
<th>Search Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>l</td>
<td>$(L + 1)^n \times 2^l$</td>
</tr>
<tr>
<td>1</td>
<td>5 10  Fully connected mesh</td>
<td>4.0841E06 × 1.204E3</td>
</tr>
<tr>
<td>2</td>
<td>7 21  Fully connected mesh</td>
<td>1.8011E09 × 2.097E6</td>
</tr>
<tr>
<td>3</td>
<td>8 28  Fully connected mesh</td>
<td>3.7823E10 × 2.684E8</td>
</tr>
<tr>
<td>4</td>
<td>9 36  Fully connected mesh</td>
<td>7.9428E11 × 6.872E10</td>
</tr>
<tr>
<td>5</td>
<td>10 45  Fully connected mesh</td>
<td>1.6680E13 × 3.518E13</td>
</tr>
<tr>
<td>6</td>
<td>16 24  4 × 4 grid</td>
<td>1.4306E21 × 1.678E7</td>
</tr>
<tr>
<td>7</td>
<td>16 24  4 nodes-cluster</td>
<td>1.4306E21 × 1.678E7</td>
</tr>
</tbody>
</table>
3.3.7 NRO Algorithm Performance Comparison

This section illustrates the results of 7 problems which optimal solutions can be achieved via enumerating and analyzing all the solution in the solution search space. As discussed, in order to test the effectiveness of the proposed algorithm in comparison with widely used simulated annealing (SA) and genetic algorithm (GA), it was assumed that at most 10% of the solution space would be analyzed. To our knowledge, existing SA and GA algorithms explored network reliability optimizations problems with fixed link reliabilities which are priori known, and none of the works consider the dynamics of links reliabilities due to the variant transmission power. Therefore, we developed SA and GA approaches to find the optimal all-terminal network reliability by controlling the nodes transmission power (see appendix 3.5.1 and appendix 3.5.2 for more details). Table 3.4 shows the specifications used for NRO, GA and SA algorithms in numerical analysis of the 7 network optimization problems.

Table 3.4: Specifications of NRO, GA and SA Algorithms.

<table>
<thead>
<tr>
<th>NRO</th>
<th>GA</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power levels (L) = 20</td>
<td>Power levels (L) = 20</td>
<td>Power levels (L) = 20</td>
</tr>
<tr>
<td>$\Delta P_{tx} = 1 \text{ dBM}$</td>
<td>$\Delta P_{tx} = 1 \text{ dBM}$</td>
<td>$\Delta P_{tx} = 1 \text{ dBM}$</td>
</tr>
<tr>
<td>Sample Size (N) = $20 \times n \times L$</td>
<td>Selection rate = 0.5</td>
<td>initial temperature = 300</td>
</tr>
<tr>
<td>Smoothing factor ($\alpha$) = 0.9</td>
<td>Mutation rate = 0.2</td>
<td>beta = 0.9999</td>
</tr>
<tr>
<td>$SIR_{th} = -30 \text{ dBM}$</td>
<td>Population size = 1024</td>
<td>$SIR_{th} = -30 \text{ dBM}$</td>
</tr>
</tbody>
</table>

Table 3.5 lists the searched space averaged over 10 runs for each problem. It is an important feature of the optimization method to converge to the optimal solution via searching small portion of the search space because the calculation of all-terminal reliability put computation burden on extensive search of the all the problem space for large size problems.
Table 3.5: Comparison of Search Efforts for NRO, GA and SA Algorithms.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Solutions Searched</th>
<th>NRO</th>
<th>GA</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>12549</td>
<td>25269</td>
<td>59527</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>21403</td>
<td>26189</td>
<td>46500</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>26405</td>
<td>22302</td>
<td>40826</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>34396</td>
<td>24655</td>
<td>69343</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>40731</td>
<td>27519</td>
<td>454087</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>93100</td>
<td>27110</td>
<td>45466</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>85771</td>
<td>26189</td>
<td>39264</td>
</tr>
</tbody>
</table>

The GA method examine a consistent number of solutions regardless of the size of total search space because GA examines a fixed size population (i.e. generation) where mutation and reproduction rate are fixed and used to filter out solutions and keep only best candidate solutions that maximize the network reliability. Nevertheless, GA requires pre-warming experimental stage to determine initial first population and tuning parameters such as selection rate, mutation rate and the populations size. After conducting pre-warming experiment, we select the fixed the GA tuning parameters as shown in Table 3.4. Although the GA examine a fixed number of samples regardless of the size network problem, it yields near-optimal solutions. However, this performance is conditioned on having a pre-existing initial phase. On the other hand, NRO had shown better performance in achieving better solutions with expense of increasing the sample size with respect to the increase of the network size. The main advantage of NRO is the absence of the need of pre-tuning the algorithm parameter and absence of the necessity to create an initial population samples. SA algorithm has shown the worst performance among the three algorithms in term of the number of searched solution and the attained all-terminal network reliability for the given network problems. The results behind the inefficiency of SA is the dependency of SA of the initial starting point since SA sequentially test candidate solutions and hence it can be trapped in sub-optimal regions. Unlike GA and GA, SA only considers one sample at a time compared to N size sample and population of fixed size for NRO and GA, respectively.

NRO outperformed GA and SA in attaining near optimal solution with much lower variance as shown in Table 3.6. The very small variance attained by NRO indicates its ability of finding a consistent optimal solution at each run of the algorithm. For example, the ratio of the variance of GA and SA to NRO is in order of 320 and 1E6, respectively.
It is evident that the proposed algorithm provides a better more efficient approach than GA and SA for solving problem with medium-sized search space. It also should be noted that the number of times GE successfully achieved the optimal allocation is higher than GA and SA as shown in Table 3.6.

Table 3.6: Comparison of the Accuracy of NRO, GA and SA Algorithms in Attaining Optimal Power Allocation for Networks in Table 3.3.

<table>
<thead>
<tr>
<th>#</th>
<th>Results of NRO</th>
<th>Results of GA</th>
<th>Results of SA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Best</td>
<td>Mean</td>
<td>Variance</td>
</tr>
<tr>
<td>1</td>
<td>0.99985</td>
<td>0.99985</td>
<td>4.55E-21</td>
</tr>
<tr>
<td>2</td>
<td>0.99931</td>
<td>0.99931</td>
<td>1.37E-32</td>
</tr>
<tr>
<td>3</td>
<td>0.92678</td>
<td>0.92675</td>
<td>7.53E-10</td>
</tr>
<tr>
<td>4</td>
<td>0.72659</td>
<td>0.72633</td>
<td>2.94E-08</td>
</tr>
<tr>
<td>5</td>
<td>0.88702</td>
<td>0.88689</td>
<td>2.86E-08</td>
</tr>
<tr>
<td>6</td>
<td>0.99956</td>
<td>0.99956</td>
<td>3.95E-21</td>
</tr>
<tr>
<td>7</td>
<td>0.99977</td>
<td>0.99977</td>
<td>0</td>
</tr>
</tbody>
</table>

3.4 Conclusion

This work presents an optimization algorithm, NRO, based on the cross-entropy optimization approach to solve the problem of maximizing all-terminal reliability via controlling the nodes’ transmission powers hence controlling the SIR of every receiving node. The proposed algorithm is compared with the widely used optimization methods in reliability analysis: simulated annealing and genetic algorithm. As illustrated by the results, the proposed approach is superior to aforementioned methods in term the speed of convergence and attaining maximum all-terminal reliability for different network sizes. Moreover, NRO shows the tendency of attaining the optimal results at every run with very small variance and without the need for pre-warming or initial stage to tune its parameters.
3.5 Appendix

3.5.1 Simulated Annealing Algorithm Overview

Simulated annealing (SA) is a randomized generic search and optimization technique proposed by S. Kirkpatrick et al. [100] for improving local optimization algorithms. There are many different variants, but in its basic form, the technique, when applied to our power-allocation design problem, involves the following steps:

1. Generate, randomly, a power allocation $X^{(k+1)}$ for the given network from the current power allocation $X^k$. If $Rel(X^{(k+1)}) > Rel(X^{(k)})$, accept $X^{(k+1)}$ as the new power allocation; otherwise accept $X^{(k+1)}$ only with probability $e^{\Delta/T}$ where $\Delta = Rel(X^{(k+1)}) - Rel(X^{(k)})$ and $T$ is the temperature.

2. Reduce the temperature using a pre-specified cooling scheme.

There are many ways to generate a new power allocation $X^{(k+1)}$. Since there is no preference of selecting a specific transmission power at a given node, each node randomly and uniformly chooses its transmission power from a discrete values $0, \ldots, j \Delta P_{tx}, \ldots, L \Delta P_{tx}$ where $L$ is maximum number of power levels and $\Delta P_{tx}$ is transmission power, i.e., 1 dBm in this study.

The temperature decrement at each iteration is very important in obtaining the optimal power allocation for the network. For example, if the temperature decreases very quickly, the algorithm may be trapped in sub-optimal region. On the other hand, if the temperature decreases very slowly, it will slow down the convergence and as a result will increase a computational time.

Numerous methods have been used to decrease the temperature[100][101]. In this work, we adopted the geometric decay method to update the temperature as follows:

$$T = \beta T,$$

(3.26)

where the decay parameter $\beta$ is a constant less than 1.

3.5.2 Genetic Algorithm Overview

Genetic Algorithm (GA) is search technique used in computing exact or approximating solutions to optimization and search problems. GA has been used as heuristic optimiza-
tion in many reliability design problems (e.g. [102, 103, 104, 105]) due to its flexibility, robustness and ability to find optimal or near-optimal solutions for these reliability design problems which are classified as NP-hard problems. GA is a meta-heuristic inspired by the biologic paradigm of evolution. It was pioneered by Holland [106], De Jong [26] [107], and Goldberg[108] in the context of continuous non-linear optimization, and later extended to combinatorial problems[109][110].

In GA, the search space is composed of candidate solutions to the problem, each represented by a string, termed a chromosome. Each chromosome has an objective function value, termed fitness. A set of chromosomes together with their associated fitness is the population. This population, at a given iteration of the GA, is a generation. GA has three main steps which are repeated until the solution is attained:

1. Select strings from the current generation to be parents of the next generation, with preference for fitter strings. The selected strings are used for reproduction.

2. Combine two selected strings to generate new children strings, which is called crossover. Probabilistically, components of a chromosome are perturbed while generating a child. This process is mutation. Together, crossover and mutation comprise reproduction.

3. Compute the fitness using the objective function of each new solution.

There are many different variants of GA, but in its basic form, the technique, when applied to our power-allocation design problem, involves the following:

1. Initial Population: Generate a set of random power allocations \( X^{(1)}, \ldots, X^{(N)} \), where \( X^{(i)} = \{P_{tx}^1, \ldots, P_{tx}^n\} \).

2. Chromosome Encoding: Convert each \( X^{(i)} = \{P_{tx}^1, \ldots, P_{tx}^n\} \) into binary vector where each \( P_{tx}^j \) is encoding by four bits as follows:

\[
\text{Chromosome} = \{000000000, \ldots, 11111111\}
\]

In our problems \( P_{tx}^j \) falls in the range 00000 to 10100, i.e., 0 dBm to 20 dBm.

3. Fitness Function: Calculate the all-terminal network reliability for each chromosome using Algorithm 3.4.
4. Mutation:
   (a) Ranking and Selection: First, the chromosomes are ranked from lowest reliability to highest reliability. Then, only the best are selected to continue, while the rest are deleted using a selection rate, i.e., $SelectRate = 0.5$.
   
   (b) Weighted Random Pairing. The probabilities assigned to the chromosomes in the mating pool are proportional to their reliability. A chromosome with the highest reliability has the greatest probability of mating, while the chromosome with the smallest reliability has the lowest probability of mating. A random number determines which chromosome is selected.
   
   (c) Mating: Creating two offspring from the parents selected in the pairing process and add them to the population using mutation rate of 0.15.
   
   (d) Population Re-evaluation: Calculate the all-terminal network reliability for each chromosome using Algorithm 3.4. Find the maximum all-terminal reliability and mark the chromosome.
   
5. Convergence Check: terminate If the number of generations exceeds a pre-specified value, i.e. $GenerationMax$, or the improvement in the all-terminal network reliability is negligible (e.g., less than 1E-7). Otherwise, go to step (4).

The choice of parameters for GA can affect the performance of the algorithm. Therefore, we run a set of experiments to establish parameters values which work well and to gauge the sensitivity of the GA to alterations in those values. For this study, the initial population, N, is 1024, the selection rate, $SelectRate$ is 0.5 and mutation rate is 0.20.

3.5.3 Network Problems

1. Problem 1: Fully-Connected Mesh (5 Nodes, 10 Arcs)

\[
\begin{bmatrix}
0 & 32 & 54 & 62 & 25 \\
- & 0 & 34 & 58 & 45 \\
- & - & 0 & 36 & 52 \\
- & - & - & 0 & 29 \\
- & - & - & - & 0
\end{bmatrix}
\]
2. Problem 2: Fully-Connected Mesh (7 Nodes, 21 Arcs)
\[
\begin{bmatrix}
0 & 125 & 150 & 125 & 150 & 150 & 130 \\
- & 0 & 75 & 100 & 150 & 200 & 250 \\
- & - & 0 & 75 & 90 & 250 & 200 \\
- & - & - & 0 & 75 & 100 & 150 \\
- & - & - & - & 0 & 75 & 200 \\
- & - & - & - & - & 0 & 75 \\
- & - & - & - & - & - & 0 \\
\end{bmatrix}
\]

3. Problem 3: Fully-Connected Mesh (8 Nodes, 28 Arcs)
\[
\begin{bmatrix}
0 & 59 & 19 & 98 & 77 & 35 & 40 & 93 \\
- & 0 & 68 & 39 & 16 & 48 & 12 & 81 \\
- & - & 0 & 17 & 41 & 24 & 89 & 41 \\
- & - & - & 0 & 60 & 23 & 72 & 45 \\
- & - & - & - & 0 & 23 & 51 & 84 \\
- & - & - & - & - & 0 & 54 & 1 \\
- & - & - & - & - & - & 0 & 33 \\
- & - & - & - & - & - & - & 0 \\
\end{bmatrix}
\]

4. Problem 4: Fully-Connected Mesh (9 Nodes, 36 Arcs)
\[
\begin{bmatrix}
0 & 37 & 77 & 61 & 97 & 58 & 41 & 63 & 3 \\
- & 0 & 40 & 30 & 4 & 53 & 61 & 37 & 63 \\
- & - & 0 & 56 & 63 & 71 & 13 & 90 & 34 \\
- & - & - & 0 & 33 & 70 & 39 & 7 & 35 \\
- & - & - & - & 0 & 89 & 55 & 97 & 65 \\
- & - & - & - & - & 0 & 23 & 57 & 88 \\
- & - & - & - & - & - & 0 & 2 & 70 \\
- & - & - & - & - & - & - & 0 & 77 \\
- & - & - & - & - & - & - & - & 0 \\
\end{bmatrix}
\]

5. Problem 5: Fully-Connected Mesh (10 Nodes, 45 Arcs)
\[
\begin{bmatrix}
0 & 24 & 26 & 69 & 25 & 48 & 3 & 82 & 45 & 98 \\
- & 0 & 12 & 75 & 22 & 33 & 82 & 54 & 4 & 82 \\
- & - & 0 & 30 & 8 & 75 & 38 & 21 & 79 & 23 \\
- & - & - & 0 & 67 & 18 & 64 & 50 & 78 & 12 \\
- & - & - & - & 0 & 72 & 92 & 94 & 21 & 96 \\
- & - & - & - & - & 0 & 5 & 81 & 18 & 84 \\
- & - & - & - & - & - & 0 & 19 & 37 & 34 \\
- & - & - & - & - & - & - & 0 & 7 & 98 \\
- & - & - & - & - & - & - & - & 0 & 50 \\
\end{bmatrix}
\]
6. Problem 6: Cluster Network (16 Nodes, 24 Arcs)

Figure 3.4: Problem 6: Cluster Network (16 Nodes, 24 Arcs).

7. Problem 7: Grid Network (16 Nodes, 24 Arcs)

Figure 3.5: Problem 7: Grid Network (16 Nodes, 24 Arcs).
Chapter 4

Reliability-Aware Relaying Node Deployment: Maximizing All-Terminal Network Reliability for Hierarchical Wireless Networks

4.1 Introduction

Modern industrial and commercial systems tend to integrate computing, communication and control into different levels of manufacturing operations and information processes [111]. The automation in process industry has been interested in the utilization of the so-called networked distributed control systems (NDCS) approach to enhance the production and control efficiencies. For instance, the integration of geographically distributed assets through centralized control improves agility in responding to supply and demand fluctuations, reduces cost of operations and enables process efficiencies unachievable in the past. For example, the concept for intelligent distributed power system automation [112] indicates that systems controlling distributed infrastructure for generating, transmitting, distributing, storing, and utilizing energy are no longer isolated. For substations and houses located in remote regions, network-based access reduces operational costs by enabling remote monitoring, debugging and maintenance. Moreover, the ability to perform data gathering and audit-report generation from headquarters is crucial for keeping overhead costs of regulatory requirements in check. A fundamental need in power indus-
tries is for instantaneous access to current operational data. Accessing such operational
data can promptly enhance the response time to system failure and improve the utiliza-
tion of energy resources. Supervisory Control and Data Acquisition (SCADA) system
[113] is introduced as distributed control system approach to enhance the production,
distribution and control of electrical power systems. SCADA systems [113] have evolved,
in parallel with the growth and sophistication of modern computing and communication
technology, from monolithic SCADA systems to distributed SCADA systems and finally
to networked SCADA systems. In monolithic SCADA systems processing and comput-
ing were done by mainframe computers with generally non-existent networking. On the
other hand, distributed SCADA systems processing and computing are shared among
multiple stations which are connected through proprietary LAN and they share information in real time. External communications networks in distributed SCADA systems are
limited to vendor-proprietary protocols and are not available for other types of network
traffic. The current SCADA systems, i.e., networked SCADA system, use open system
architecture rather than a vendor-controlled proprietary environment, so processing functionalities are distributed across an open WAN rather than a proprietary-LAN. Also, it
is easier to connect third party peripheral devices with different networking interfaces
such as Zigbee [114], 802.11a/b/g/n, WCDMA/3G [115] or WiMax/IEEE 802.16 [116].
The move toward utilizing open communication protocols standards and geographically
distributed assets in SCADA networked system implies the need of new self-organized
network structure that can accommodate the hierarchical functionality of monitoring and
controlling different assets.

Self-organized hierarchical hybrid wireless networks have emerged as a new and promis-
ing communication paradigm for networked distributed control systems, compared to
wired networks and flat ad hoc networks, as it offers more flexibility and lower cost for
installation and commissioning. Unlike flat ad hoc wireless network, wireless links in
hierarchical hybrid wireless networks can be used for long-range communication in dis-
tributed systems in addition to short-range communication usage for data acquisition
and controlling devices on last mile, i.e., sensors field, automated industrial machines, or
a distributed power system. However, in order to extend wireless networks for remote
control, coordination and applications involving emergency alerts, strict requirements on
network connectivity reliability have to be satisfied in order to meet application require-
ments and industrial standards.

Network reliability concerns with the capability of the underlying network to provide
successful communication between set of nodes, i.e., sources and destinations, which is an issue of concern whenever there is a need of assurance of conveying critical data within the network. Network reliability has long been a practical issue, and will remain so for years, since variety of realistic applied problems in IP networks, mobile phone networks, and cognitive radio networks set the successful connection rate of 99.999% [3][4][5] as an essential objective for communication network providers, and premium services may only be deployed if the connection reliability is close enough to unity.

Network reliability depends on two factors: network connectivity and links reliabilities which vary with respect to the interplay between nodes in a given network. For the first factor, connectivity is highly dependent on nodes locations with respect to each other and links connecting these nodes. In many wireless network researches, i.e., sensor networks and ad hoc networks, there are common assumptions that nodes are randomly deployed where each node can act as data source and traffic relay. However, Wang [117] pointed out that there are variety of application scenarios where a careful manual placement of nodes is a necessity for successful operating, monitoring and controlling these application scenarios [118][119]. An example of these application scenarios is the deployment a large number of accelerometers, thermometers, and strain sensors on structural building, e.g., Guangzhou New TV Tower [118] in Guangzhou, China and TsingMa Bridge [119] in Hong Kong, for real-time data monitoring and analyzing in order to fulfill civil engineering requirements.

Relaying node deployment problem has been investigated for wireless networks from various aspects [120][121][122][123][124]. From connectivity prospective, the relaying node deployment problem was first formulated in [121] and had been proven to be NP-hard. Then, Lin and Xue [121] proposed an approximation algorithm based on steinerization which assigns all relaying nodes with the same distance on each edge. This problem was further generalized for k-connectivity and called the survivability problem for \( k \geq 2 \) in [125]. Misra et. al. [120] extended the problem by considering relaying nodes can only placed at some given locations. From the network lifespan prospective, there were several works explicitly considering the relaying node placement to prolong network lifetime [122][124]. For instance, Xu et. al. [122] focused on massive random relaying node deployment while Hou et. al. [124] emphasized on using energy provisioning and giving each relaying node different energy budget to achieve better performance.

In this work we focus on the deployment of relaying nodes in hierarchical hybrid wireless network such that the network reliability is maximized. In a typical deployment
of hierarchical hybrid wireless network, there are three types of nodes: terminal nodes (TNs), relaying nodes (RNs) and gateway nodes (GNs) as shown in Figure 4.1. The GNs and TNs locations are predetermined and fixed by service provider and end-users needs or application requirements, respectively. While the geographical locations of RNs are managed and controlled by the network planner. Therefore, by controlling RNs deployment and their coverage area, a network planner can control how many TNs node are connected to each RN and how many RN needed to cover an area served by a GN node. Therefore, the first part of network reliability analysis of a hierarchical hybrid wireless network is to determine the optimal RN deployment such that every TN is connected to one RN and reliability of TNs to RN, i.e., sub-cluster reliability in Section 4.2.5, is maximized.

![Figure 4.1: Three Tiers Hierarchical Wireless Network.](image)

For the second factor, i.e., link reliabilities, link is considered a failed link when the Signal-to-Interference-Ratio (SIR) falls below a certain threshold and hence received signal can not be recognized. SIR at a given node depends on transmission power of the transmitter node, channel path loss, fading, additive noise and Interference induced by nodes other than the transmitter. Transmission power control is a highly effective tech-
nique for minimizing effect of interference, channel path loss, fading and additive noise and hence improving links reliabilities. For instance, link reliability can be improved by increasing the transmission power. In fact, the bit error probability, $P_e$, decreases when the ratio $(\frac{E_b}{N_0})$ increases, where $E_b$ is the received energy per bit and $N_0$ is the noise power spectral density[71]. Therefore, most of the proposed approaches for link-level reliability have common transmission control where the same transmission power $P_{tx}$ is used by every node. In these works, a guidance is provided on the critical selection of common transmission power constrained by problem requirements, e.g. connectivity [37][38], capacity [39], coverage [37][38][39] and energy efficiency [37][38], to attain a specific reliability objective either on link. The common transmission power selection approaches do not take into consideration the effect of variation of transmission power on links and network reliabilities. Moreover, the effect of nodes’ transmission powers variation on link and network reliabilities is not capture by typical reliability analysis where a network is represented as a probabilistic graph with vertexes and links operating with identical probabilities $p_v$ and $p_e$, respectively [72][73]. Unfortunately, these probabilities do not capture the failures induced by constrained node energy and error-prone wireless links.

It have been shown that one-hop reliability cannot be improved by random common transmission power selection in deterministic channel, log-normal shadowing channel and supper-imposed log-normal shadowing and Rayleigh fading channel[79]. The channel conditions affecting the links connecting these nodes differ due to the variation of distances between the nodes and the level of interference every node is exposed to. Hence, each node requires a precise power level to improve its out going links’ reliabilities and on same time not to be an interference that degrade other nodes’ links reliabilities. To do so, we have to intelligently allocate nodes transmission powers such that links and network reliabilities are maximized. Therefore, we utilize a power-control algorithm proposed in [126] which takes into account the statistical variation of the Signal-to-Interference Ratio, $SIR$, at receiving nodes and optimally allocate nodes’ transmission powers to maximize the links reliabilities and network reliability.

We first define a generic architecture which is suitable for modeling variety of networking scenarios (i.e., hybrid cellular/ad hoc networks, large-scale sensor nets and networked distributed power system) where different radio access technologies can be used for each tier to comply with performance requirements and constraints imposed by scenario being addressed. Then, we model wireless link failure due to the Signal to Interference Ratio (SNR) falling below the required power requirement of the receiver, $SIR^{th}$ due to fading
of the signal. We utilize link reliability model to calculate sub-cluster and cluster reliabilities in Section 4.2. Then, we present an algorithm to find the RNs locations such that every TN in the network is connected to an RN and the sub-cluster reliabilities, i.e., $\text{Rel}(RN_i)$, are maximized. Finally, we utilize the information of RN locations in maximizing the cluster reliability, i.e., $\text{Rel}(GN)$, via controlling the RN transmission power to GN.

The remainder of the chapter is organized as follows. In Section 4.2, system model, problem formulation and reliabilities measures that will be used in later sections. In Section 4.3, we present the algorithms used for relaying node deployment and controlling their transmission power in order to maximize network reliability. We present numerical results in Section 4.4 and conclude the work in Section 4.6.

### 4.2 Preliminaries and Problem Formulation

In this section we provide an overview of a typical deployment of hierarchical hybrid wireless network. This overview details the characteristics of each network entity and its duties within architecture. Then, we also present a network planner model which describes the connections among different nodes. After that, we present the link, sub-cluster and cluster reliabilities in hierarchical hybrid wireless network. Next, we present the problem formulation of finding the locations of RN and their transmission power such as the overall network reliability is optimized.

#### 4.2.1 System Model Overview

As shown in Fig. 4.1, a typical deployment of hierarchical hybrid wireless network consists of three tiers of radio nodes: low-power terminal nodes (TNs) at the lowest tier, higher powered radio relaying nodes (RNs) that support multihop routing at the second tier, and wired Gateway nodes (GNs) at the third and highest tier. The GNs and RNs use a self-organizing discovery protocol to form a multihop routed wireless infrastructure network. TNs in this network are simply connect to the nearest available (i.e., strongest signal) RN in order to conserve power and are thus not required to carry any intermediate multihop routed traffic.

This architecture is suitable to variety of networking scenarios (i.e., hybrid cellular/ad hoc networks, large-scale sensor nets and networked distributed power system) where
different radio access technologies can be used for each tier to comply with performance requirements and constraints imposed by scenario being addressed. For example, the new trend in networked distributed power system implies the need for continuous data acquisition from distributed devices which can utilize low power access protocol such as Zigbee [114] to allow TNs to communicate with RN and RN can utilize higher power protocols such as 802.11b for establishing links as part of local mesh WLAN to cover a certain neighborhood/block or even use WCDMA/3G [115] or WiMax/IEEE 802.16 [116] for wide area communication cover a whole city. Note that this hierarchical network allow the use of different radio technologies throughout its tiers, therefore, RNs as the second tier must be equipped with multiple radio interfaces to be able to route packets among themselves and to the RNs and TNs in the other two tiers.

Each of the network entities in the proposed system are defined in further detail as follows:

- **Terminal Node (TN)** is a end-user device (such as a sensor, mobile device) at the lowest tier of the network. TN attaches itself to one or more nodes at the higher tiers of the network in order to obtain service using a discovery protocol. For instance, TN may use a single 802.11a, b, or g radio operating in ad hoc mode to communicate with the point(s) of attachment. As an end user node, TN is responsible for generate data or respond to queries from top tiers, so TN is not required to route data/queries from/to other nodes. TN is battery-operated thus node energy gradually depletes and eventually runs out. Therefore, TNs have strict energy constraints to prolong their lifespan.

- **Relaying Node (RN)** is a fixed or mobile intermediate radio relaying node capable of routing multihop traffic to and from all three tiers of the networks hierarchy. RN is not responsible for generating or collecting any data in the structure, its sole mission is to route the transit packet among different nodes in the three tiers. RN may have two radio cards, one for RN-TN transmissions and the other for intra-RN and RN-GN traffic flows which typically carried on different frequencies. The interconnections of RN at each of three tiers is highly dependent on the specific requirements of the deployed scenario and the used discovery routing protocol that optimize hierarchical distributed topology. The RN is typically a compact radio device that can be plugged into an electrical outlet, but in certain scenarios (e.g. mesh WLAN networks), may also be a battery-powered device. Thus, the RN is
also energy-constrained, but the cost of power is typically assumed to be an order of magnitude lower than that of the one defined for TN.

- **Gateway Node (GN)** is a fixed radio access node at the highest tier of the network, with both a radio interface (e.g., 802.11, WCDMA/3G or WiMax/IEEE 802.16) and a wired interface to the Internet. GN’s communication is restricted to GNs or RNs within range but not to TNs at lowest tier. GN is responsible for routing traffic locally within the lower tier RNs and TNs withing range as well as to and from the Internet. Since the GN is a wired node, it is usually associated with an electrical outlet, and energy cost is thus considered negligible.

### 4.2.2 Network Planner Model

We consider a hybrid hierarchical wireless network (HHWN) consisting of TNs, RNs, and GNs. We assume that all TNs have communication range \( r > 0 \) and that all RNs have two communication range: \( r > 0 \) to communicate with TN within its sub-cluster and \( R > r \) to communicate with its GN in a given cluster, where \( r \) and \( R \) are given constants. We also assume that the GNs are powerful enough so that their communication range is much greater than \( R \), and that any two GNs can communicate directly with each other.

Since the objective of this work is to place the RNs to meet connectivity and reliability requirements, our assumption simplifies notations without losing any generality. We use \( d(u, v) \) to denote the Euclidean distance between two points \( u \) and \( v \) in the plane. We will also use \( s_i \) to denote the location of a node \( v_i \).

Two nodes and can communicate directly with each other if and only if \( d(u, v) \) is less than or equal to the smaller of the communication ranges of the two nodes. In other words, a \( u \) TN node can communicate directly with a \( v \) RN node or if and only if \( d(u, b) \leq r \). A \( u \) RN node can communicate directly with another \( v \) node (which could be a RN or a GN) if and only if \( d(u, v) \leq R \). Similarly, any pair of GNs can communicate directly with each other. Following these rules, the TNs, the RNs, and the GNs, together with the values of \( r \) and \( R \), collectively induce a hybrid hierarchical communication graph (HHCG), a cluster in hybrid hierarchical communication graph (CHHCG) and a sub-cluster in hybrid hierarchical communication graph (sCHHCG) are defined in the following:
Definition 1 Let $G$ be a set of GNs, $T$ be a set of TNs, $Z$ be a set of RNs, and $R \geq r \geq 0$ be the respective communication ranges of RNs and TNs. The hybrid hierarchical communication graph $HHCG(r, R, G, T, Z)$ induced by the 5-tuple $(r, R, G, T, Z)$ is an undirected graph with node set $V = G \cup T \cup Z$ and edge set $E$ defined as follows. For any two GNs $g_i, g_j \in G$, $E$ contains the undirected edge $(g_i, g_j) = (g_j, g_i)$. For a RN $z \in Z$ and a node $g \in G$, $E$ contains the undirected edge $(g, z) = (z, g)$ if and only if $d(z, g) \leq R$. For a TN $t \in T$ and a node RN node $z \in Z$, $E$ contains the undirected edge $(t, z) = (z, t)$ if and only if $d(z, g) \leq r$.

Definition 2 A cluster in a hybrid hierarchical communication graph $CHHCG(r, R, G, T, Z)$ induced by the 5-tuple $(r, R, G, T, Z)$ is an undirected graph with node set $V = G \cup T \cup Z$ where there a single GN nodes $G = \{g\}$, $Z$ be a set of RNs that have one-hop connection with $g$, $T$ a set of TNs in which each $t_i$ have a one-hop connections with at most one of $z_i \in Z$ giving that $d(t_i, z_i) \leq r$, $R \geq r \geq 0$ be the respective communication ranges of RNs and TNs, and edge set $E$ defined in same manner in Definition 1.

Definition 3 A sub-cluster in a hybrid hierarchical communication graph $sCHHCG(r, R, G, T, Z)$ induced by the 5-tuple $(r, R, G, T, Z)$ is an undirected graph with node set $V = G \cup T \cup Z$ where there a single GN nodes $G = \{g\}$, a single RN node $Z = \{z_i\}$, $T$ a set of TNs that have one-hop connection with $z_i$, $R \geq r \geq 0$ be the respective communication ranges of RNs and TNs, and edge set $E$ defined in same manner in Definition 1.

We suppose in every cluster there are one GN at the center of the observing area. TNs are independently and uniformly distributed, while RNs can be placed at defined locations. Each TN is associated with the nearest RN via direct transmission (assuming there is always at least one RN within its transmission range), and each RN is associated with its nearest GN via one-hop transmissions. After associations, all nodes in network form nonoverlapping clusters, each of which consists of one GN, its associated RNs, and RNs’ associated TNs. Each RN and its associated TNs form a sub-cluster as shown in Figure 4.2.

The wireless channel is divided into two subchannels: one is carried on frequency $f_L$, (i.e., lower tier TN to/from RN), and the other on $f_H$, (i.e., higher tier intra-RN, RN to/from GN). Each RN utilizes two radios, so it can route packets among the other
nodes. All nodes working on one frequency have a common transmission range, but the range could be different from transmissions on the other frequency. From network planner prospective, the deployment locations and transmission ranges for TNs in lower layer are inaccessible because these nodes are usually under end-user control. Thus, we assumed that the transmission range for TNs is identical, i.e., $r$ and cannot be adjusted by network planner. Also, TNs are assumed to be independently and uniformly distributed and network planner cannot change their locations. On the other hand, a network planner will have full control on deployment locations of RNs nodes and full control of adjusting RNs transmission range via controlling their transmission power. Thus, a network planner can control network structure and hence connectivity by controlling: RNs locations with respect to TNs and RNs transmission power.

The network model of a planar layout is shown in Figure 4.2

![Analytical System Model for Three Tiers Hierarchical Wireless Network](image)

**Figure 4.2: Analytical System Model for Three Tiers Hierarchical Wireless Network.**

In our hierarchical wireless networks, TNs are at the lowest tier and responsible for generating data, e.g., computing, sensing or actuation. TNs does not forward packet to others, but send and receive their own. RNs are responsible for forwarding packet to other nodes, but donot generate any traffic of their own. GNs interconnected with other GNs via infrastructure network or Internet.
4.2.3 Rayleigh Fading Environment

In wireless network, the nodes forming these networks are deployed randomly and left unattended to and are expected to be able to communicate each other properly and efficiently. As a result of this random deployment, nodes may not have a direct line-of-sight communication signal. Instead, a transmitted signal is reflected along multiple paths before it is received. This situation occurs when wireless network is deployed in dense urban area where none of the nodes has any direct line of sight communication with each other[26][27]. Rayleigh fading can be a useful signal propagation model in heavily built-up city urban area where there is no line of sight between the transmitter and receiver and many buildings and other objects attenuate, reflect, refract, and diffract the signal.

Therefore, we consider both desired and interference signals are subject to Rayleigh fading, hence we refer to this channel model as Rayleigh fading environment [25][26][27]. In this environment, the receiving node has no direct line of sight signal component, either from the transmitting or the interfering nodes. The strength of the received signal in a Rayleigh fading environment is dependent on the following factors: channel gain, fading and nodes’ transmission powers. Next we describe how the aforementioned factors affect the received power at a given node.

We consider a wireless network consisting of $|V| = n$ nodes labeled $v_1, \ldots, v_n$. A vector $P^{tx} = (P^{tx}_1, \ldots, P^{tx}_n)$ represents the transmission powers associated with nodes $v_1, \ldots, v_n$, respectively. The received power at $v_j$ from $v_i$ is:

$$P_{ij}^{rx} = G_{ij} F_{ij} P_{i}^{tx}, \quad (4.1)$$

where $G_{ij}$ represents the channel gain (not including fading) from node $v_i$ to node $v_j$. In Rayleigh model, we assume that $G_{ij}$ is constant and represents the distance dependent power attenuation[80]. $F_{ij}$ represents the Rayleigh fading induced by node $v_i$ and received by node $v_j$. $F_{ij}$s are assumed to be independent exponential random variables with unit mean. Thus, the received power at $v_j$ from $v_i$ is an exponentially distributed random variable with a mean of $E[G_{ij} F_{ij} P_{i}^{tx}] = G_{ij} P_{i}^{tx}[25]$. We assume that the interference from other transmitting nodes is much larger than the white noise at the receiving nodes, therefore, we neglect the impact the noise at the receivers. Next, we utilize expression of received power in Equation (4.1) to calculate the link reliability.
4.2.4 Computation of the Link Reliability

Successful communication over a given link is dependent, in addition to channel attenuation, upon the level of interference induced by other nodes in the network. If the level of interference is higher than the receive signal, the receiver will not be able to recognize or decode the received signal. Therefore, the first step in finding the expression for computing the link reliability is to specify the statistical model that related received transmitted signal to the received interference from the transmissions of the rest of nodes in the network. In Rayleigh fading environment, the Signal-to-Interference-Ratio (SIR) is defined as follows:

\[
SIR_j = \frac{G_{ij} F_{ij} P_{tx}^i}{\sum_{k \neq i} G_{kj} F_{kj} P_{tx}^k},
\]  

(4.2)

where the term in the numerator denotes the received power at node \( v_j \) from node \( v_i \) and the denominator denotes the total interference due to all other nodes’ received power at node \( v_j \). For a successful communication over a given link, the SIR should be greater than a threshold called acceptable reception threshold, \( SIR^{th} \). Note that the \( SIR^{th} \) falls in the range of -80 dBm (10 pW) to -60 dBm (1000 pW) which is the typical range received signal power over wireless 802.11x networks. While typical wireless router transmission power falls in the range of 15 dBm (32 mW) to 20 dBm (100 mW) [127][128].

For a node \( v_j \), if its successfully receiving threshold is \( SIR^{th} \), then link reliability is the probability of successful communication between node \( v_i \) and node \( v_j \). Link reliability is defined as follows:

\[
Rel(i \rightarrow j) = 1 - Pr( SIR_j \leq SIR^{th} ) = 1 - Pr \left( P_{rx}^{ij} \leq SIR^{th} \sum_{k \neq i} G_{kj} F_{kj} P_{tx}^k \right),
\]  

(4.3)

where \( Pr \left( P_{rx}^{ij} \leq SIR^{th} \sum_{k \neq i} G_{kj} F_{kj} P_{tx}^k \right) \) can be interpreted as the probability that the communication between node \( v_i \) and node \( v_j \) experiences a failure due fading. This probability has been defined as link outage probability [25][26][27].

**Result 2** For a wireless network with \( n \) nodes operating in a Rayleigh fading environment, it was proven in [26] that the probability of successful communication between any

69
pair of nodes \( v_i \) and \( v_j \) where \( v_i, v_j \in V \) is obtained as follows:

\[
\text{Rel}(i \rightarrow j) = \prod_{k \neq i} \frac{1}{1 + \frac{\text{SIR} G_{kj} P_{tx}^k}{G_{ij} P_{tx}^i}}.
\]  

(4.4)

This expression is used for calculating the link reliability at lower tier from \( TN_i \) node to its sub-cluster head \( RN_j \) i.e., \( \text{Rel}(TN_i \rightarrow RN_j) \), and the link reliability at upper tiers from \( RN_j \) to its cluster head \( GN \) i.e., \( \text{Rel}(RN_j \rightarrow GN) \). From the expression in Equation (4.4), we can interpret that link reliability is affected by the interference induced by transmission powers of other nodes in the network, i.e., the term \( G_{kj} P_{tx}^k \). For the upper tiers, we only consider the interference induced by the RNs who are members of the cluster, i.e., associated with the GN. On the other hand, there are two types of TNs with respect to a given sub-cluster in the lower tier: members TNs nodes who act as signal and interference transmitters and non-member TNs nodes who are close enough to \( RN \) such that they interfere with \( RN \) connections. In this work, we consider both types of TNs nodes, i.e., sub cluster members and non-members, to insure that their effect is reflected on link reliability and sub-cluster reliability. Utilizing this link reliability model, we introduce the sub-cluster reliability and cluster reliability measures in the next sections.

### 4.2.5 Computation of Sub-Cluster Reliability

Sub-cluster reliability is defined as the probability that all TNs nodes can communicate successfully with their RN, the sub-cluster head. Computing this reliability entitles: defining the location of RN, the sub-cluster head, the locations of TNs nodes who are members of the sub-cluster, the locations of TNs nodes who are not members of the sub-cluster and interfering with RN connection, defining links status (i.e., operational or fail) in each network state, computing links reliabilities and computing reliability for all possible network states that meet the reliability connectivity condition, i.e., all TNs are successfully connected to RN. Next, we present how sub-cluster network reliability is calculated.

Let \( |V| = n \) be set of TN nodes in a sub-cluster network. TNs are independently and uniformly distributed, therefore we have no control in their positions. However, RN nodes can have defined locations with respect of TN in sub-cluster and GN in the cluster.

For a connected sub cluster network, the number of links is \( |E| = n \). A vector
\( \mathbf{e} = (e_1, \ldots, e_i, \ldots, e_n) \) where \( e_i = (t_i, z) \), \( t_i \) is TN node and \( z \) is RN node defined in 1, i.e., the directed links between TNs nodes and their RN sub-cluster head. A vector \( \mathbf{P}^{tx} = (P_1^{tx}, \ldots, P_n^{tx}) \) represents the transmission powers associated with TN nodes. The selection of \( P_i^{tx} \) affect links reliabilities, \( \text{Rel}(TN_i \to RN) \forall 1 < i < n \) either as a desired signal or as an interference.

We identify the operational links using a link state \( y_i \). If a link \( e_i \) is functioning, then its state \( y_i = 1 \) and \( y_i = 0 \) otherwise. Thus, a vector \( \mathbf{y} = (y_1, \ldots, y_i, \ldots, y_n) \) is called the state vector. Each state vector represents a network state. Thus, the set of all possible state vectors represents the set of all possible network states that can be constructed on a given sub-cluster network. The set of possible state vectors is denoted as \( \Upsilon \).

For a given network, i.e., state vector \( \mathbf{y} \), let \( \Phi(\cdot) \) be an indicator function that map state vector into network connectivity state:

\[
\Phi(y) = \begin{cases} 
1 & \text{if a reliability condition is met,} \\
0 & \text{otherwise.} 
\end{cases} \quad (4.5)
\]

For sub-cluster reliability, \( \Phi(y) = 1 \) if and only if all TNs nodes are connected to their RN cluster head. Network reliability is computed as

\[
\text{Rel}(RN_j) = \sum_{\mathbf{y} \in \Upsilon} \Phi(y) \prod_{i=1}^{n} \left( \text{Rel}(TN_i \to RN_j)^{y_i} \right) \left( 1 - \text{Rel}(TN_i \to RN_j) \right)^{1-y_i}. \quad (4.6)
\]

For our sub-cluster network reliability, there is only one network state in which all TNs nodes are connected to the sub cluster head, RN. Thus, sub cluster reliability can be expressed as follows:

\[
\text{Rel}(RN_j) = \prod_{i=1}^{n} \left( \text{Rel}(TN_i \to RN_j) \right). \quad (4.7)
\]

Notice that TNs who are not members of sub-cluster are introduced as interferences at RN in link reliability calculation as stated in section 4.2.4.
4.2.6 Computation of the Cluster Reliability

Cluster reliability is defined as the probability that all sub-clusters heads, RNs nodes, can communicate successfully with their GN, the cluster head. Computing this reliability entails: defining the location of RNs, the sub-cluster heads, the location of GN node who is head of the cluster, defining links status (i.e., operational or fail) in each network state connecting RNs to GN, computing links reliabilities, sub-cluster reliabilities and computing reliability for all possible network states that meet the reliability connectivity condition, i.e., all RNs are successfully connected to GN. Utilizing the same procedures for calculating sub-cluster reliability in section 4.2.5, cluster network reliability is calculated as follows:

\[
\text{Rel}(\text{GN}) = \prod_{i=1}^{m} \left( \text{Rel}(\text{RN}_i \rightarrow \text{GN}) \ast \text{Rel}(\text{RN}_i) \right).
\]  

(4.8)

where \text{Rel}(\text{RN}_i \rightarrow \text{GN}) is the reliability of the link connecting sub-cluster head RN$_i$ to cluster head GN and \text{Rel}(\text{RN}_i) is the reliability of sub-cluster RN$_i$.

4.2.7 Problem Statement and Assumptions

We consider problem of maximizing the reliability of a given cluster in a self-organized hierarchical hybrid wireless network that consists of TNs, RNs and RN nodes as described in the network planner model in Figure 4.2. The maximization problem entails two aspects: (1) finding the geographical locations for RNs such that sub-clusters reliabilities are maximized with each TN node in the network is connected to an RN and (2) intelligently controlling RNs transmission powers such that the reliability of links connecting these RNs to GN and cluster reliability are maximized.

In the first step, our goal is to find the set of RN locations such that every TN node is connected to RN and the sub-clusters reliabilities, \text{Rel}(\text{RN}_i)’s, are maximized. In our network planner model, TNs are independently and uniformly distributed and their locations are known. The following are formal definition for the relaying node placement problem:

**Definition 4** Let \( R \geq r \geq 0 \) be the respective communication ranges for RNs and TNs. Let \( \mathcal{G} \) be a set of GNs, \( \mathcal{T} \) be a set of TNs, and \( \mathcal{W} \) be a set of candidate locations where RNs can be placed. A set of RNs \( z \in \mathcal{Z} \) is said to be a feasible connected relaying node place-
ment (denoted by $\mathcal{F}$-RNP) for $(r, R, G, T, Z)$ if the cluster graph $CHHCG(r, R, G, T, Z)$ is connected. $\mathcal{F}$-RNP is said to be a maximum reliability connected relaying node placement (denoted by MRNP) for $(r, R, G, T, Z)$ if every sub-cluster $sCHHCG(r, R, G, T, Z)$ has maximum sub-clusters reliabilities $\text{Rel}(\text{RN})$ for $z \in Z$ in $\mathcal{F}$-RNP).

**Definition 5** Let $R \geq r \geq 0$ be the respective communication ranges for RNs and TNs. Let $\mathcal{G}$ be a set of GNs, $T$ be a set of TNs, and $W$ be a set of candidate locations where RNs can be placed. The relaying node placement for $(r, R, G, T, Z)$, denoted by $RNP(r, R, G, T, Z)$, seeks a MRNP for $(r, R, G, T, Z)$.

The potential deployment locations of RNs, i.e., $W := w_1, \ldots, w_m$ are assumed to be the locations of TNs $S := s_1, \ldots, s_n$. This assumption has been used in [125] [129] [123] by promoting or upgrading the TN to act like a RN node in order to improve the order of connectivity in a given network. The potential deployment locations have the prospective to be RN locations due the feasibility of deployment which is concerned of the location compliance for the deployment of RN despite the terrain structure and man made obstacles. Since TNs can be deployed at these locations, we can assume that there are no environment or man made obstacles that may hinder the RN deployment. Moreover, from clustering design prospective, nodes tend to select a head cluster based of information they exchange with each other about their locations, connectivity, transmission power or any other designed criterion. Therefore, by choosing these locations we have two advantages: ability to utilize these information in evaluating the links reliabilities at a given potential RN location while ensure that every TN is connected to one RN and restrict the space search for RN locations that can maximize the sub-clusters reliability. RN deployment should maximize the following problem:

\[
\text{maximize} \quad \text{Rel}(RN_i), \quad \forall i \in m,
\]
\[
\text{such that} \quad TN_j, \forall j \in n \text{ is connected to one } RN_i, \quad (4.9)
\]

\[
m \ll n.
\]

After defining the RNs locations and their associated reliability, i.e., $\text{Rel}(RN_i)$ from (4.9), we utilize these information in the second step for maximizing the cluster reliability. In the second step, our goal is to maximize the cluster reliability a given cluster topology by controlling RNs’ transmission powers. We assume that RNs nodes may not have the
ability of controlling transmission power continuously, but rather support only finite set of power levels. Thus, we consider power control in discrete power domain. The power levels that a RN node can support are \( \{\triangle P_{tx}, 2\triangle P_{tx}, \ldots, L\triangle P_{tx}\} \) where \( L \) is the number of power levels and \( \triangle P_{tx} = \frac{P_{tx}^{max}}{L} \), where \( P_{tx}^{max} \) is the maximum transmission power allowed for a RN node transmission. Then, the problem of cluster reliability optimization can be formulated as follows:

\[
\begin{align*}
\text{maximize} & \quad \text{Rel}(GN), \\
\text{subject to} & \quad 0 \leq P_{tx}(RN_i) \leq P_{tx}^{max}, \quad \forall P_{tx}(RN_i) \in P_{tx}(RN), \quad (4.10) \\
& \quad P_{tx}(RN_i) \in \{0, \triangle P_{tx}, 2\triangle P_{tx}, \ldots, L\triangle P_{tx}\}.
\end{align*}
\]

The optimization problem of determining the optimal RNs’ transmission powers in Equation (4.10) is NP-hard. For cluster network with \( n \) RN nodes with one hop connection to the head and \( n \) links, there are total of \( (L + 1)^n \) possible combinations that satisfy connectivity condition that all RNs are connected to GN. To overcome the excessive search of the huge search space of possible solutions, we developed an algorithm which efficiently find RNs nodes transmission powers allocation that attain the maximum cluster reliability. Finding the optimal RNs’ optimal power transmission in huge search space is considered a form of finding the occurrence of a rare but an important event in large events space. Rare-event simulation involves estimating extremely small but important probabilities [85][86][87]. A well known simulation method in field of rare event simulation is Cross-Entropy (CE) method. CE has been used to accurately estimate rare events in various applications such: vehicle routing problem [88] and network planning problem [89]. In this study, we utilize the a simulation method based on Cross-Entropy method (CE) which has been developed in [126] to determine the optimal RNs transmission powers allocation that attain the maximum cluster reliability.

Next, we present in details the algorithms used to maximize a cluster reliability in a self-organized hierarchical hybrid wireless network.

### 4.3 Cluster Reliability Maximization Algorithms

In this section, we give details of algorithms used to maximize a given cluster reliability in a hybrid hierarchical wireless network. First, we show how to relaying nodes are placed
to such that the resultant sub-clusters have are the maximal sub-cluster reliabilities. Then, we describe the algorithm used for maximizing cluster reliability via controlling the relaying nodes transmission power to gateway node.

### 4.3.1 RN Reliability-Aware Deployment Algorithm

In this section we present relaying node reliability aware deployment algorithm that select relaying node locations such as the sub-cluster reliabilities are maximal in a given cluster. First, we defined the RNs potential locations $W := w_1, \ldots, w_m$ as the locations of TNs whose euclidean distances from the GN are not greater than $R$, i.e., $w_i - s_0 < R$, so that any deployed RN will have the ability to communicate to GN. Utilizing $W$, we find TNs whose euclidean distances from a given $RN_i$ at $w_i$ are not greater than $r$, i.e., $w_i - s_i < r$, so that TNs have a communication link to $RN_i$ at $W_i$. For every $RN_i$ in a given sub-clusters, there are two types of TNs with $w_i - s_i < r$: members of sub-cluster and non-members of the sub-clusters. The sub-cluster TNs members participate as signal and interference transmitters to the sub-cluster head, i.e., RN node, while sub-cluster TNs non-members act as interference on RN and thus affect the links reliabilities in a given sub-cluster.

Algorithm 4.1 starts with inputs of TN locations, GN location, RN potential locations, transmission ranges $r$ and $R$ for TN and RN, and TN membership flag. A membership flag, e.g. $f_i$ indicates if TN node is a member of a sub-cluster. Once a node is selected to be a member of a sub-cluster, it cannot join another sub-cluster nor its location can be used as a deployment location for RN.

For a given RN feasible location, $w_i \in W$, we find the locations of TNs $s_i \in S$ which can affect the reliability, i.e., as signal transmitter or an interferer, of the received transmission at RN at $w_i$. This is done by finding euclidean distance between $s_i$ and $w_i$ for every $s_i \in S$, if a TN at $s_i$, $w_i - s_i < r$ and is a non-member of any sub-cluster, i.e., flag membership $f_i = 0$, TN is selected as member of sub-cluster with RN at $w_i$ and its reliability and interference effect will be considered in Equation (4.7). Initially, all TNs are in the non-member status, however if a TN at $s_i$, $w_i - s_i < r$ and is a member of sub-cluster other than one with RN at $w_i$, i.e., flag membership $f_i = 1$, TN acts as an interference and its effect will be considered in Equation (4.7). Other TN nodes with $s_i$ such that $w_i - s_i > r$ will not have any effect on $Rel(RN_i)$ at $w_i$. After finding all TN nodes who are contributing to sub-cluster reliability $Rel(RN_i)$ at $w_i$, we calculate
Algorithm 4.1 RN Reliability-Aware Deployment Algorithm

Input
GN location: \( s_0 \), TNs locations: \( S := s_1, \ldots, s_n \),
TNs membership flag: \( F := f_1 = 0, \ldots, f_n = 0 \),
RNs potential locations: \( W := w_1, \ldots, w_m \),
RN coverage: \( r \)

Output
RNs locations: \( Z := z_1, \ldots \)
RN Reliability: \( Rel(RN) := Rel(RN_1), \ldots \)

1: while \( (s_i \in S \text{ and } f_i = 0) \) do
2: for all \( w_i \in W \) do
3: Find TNs with \( w_i - s_i < r \) and \( f_i = 0 \).
4: Find TNs with \( w_i - s_i < r \) and \( f_i = 1 \).
5: Calculate \( Rel(RN_i) \) using (4.7).
6: end for
7: Select \( z_i = w_i \) from \( W \) who has the largest \( Rel(RN_i) \)
8: Mark all TNs with \( w_i - s_i < r \) and \( f_i = 0 \) as selected and set \( f_i = 1 \).
9: end while
10: return \( Z, Rel(RN) \)

\( Rel(RN_i) \). We repeat the previous steps for every potential location \( w_i \in W \) and find the \( w_i \) whose \( Rel(RN_i) \) is the maximum among \( w_i \in W \). Once \( w_i \) with maximal value is determined, we mark the location as RN deployment location \( z_i \in Z \) and flag all TNs, i.e., change their \( f_i \) status to 1, whose \( w_i - s_i < r \) and initial flag \( f_i = 0 \) as members of sub-cluster with \( RN_i \) at \( w_i \). After the removal for \( RN_i \) and its associated TN sub-cluster members from our search set, we repeat the aforementioned procedures to find the locations of RNs with maximum sub-cluster reliability, \( Rel(RN_i) \) at each iteration. The algorithm terminate when all TNs are flagged, in other words, belong into a sub-cluster in cluster under study.

4.3.2 Power-Aware Cluster Reliability Optimization Algorithm

In this section we present a power aware cluster reliability algorithm that intelligently assign RNs transmission power in such that the cluster reliability is maximized. As we stated in section 4.2.7, finding optimal power assignment in given network is rare event problem which can be solved using a cross-entropy simulation method (CE method
CE method is an adaptive importance sampling derived from the associated stochastic problem (ASP) of a given optimization problem for estimating the probability of a rare-event occurrence. The estimation of this probability is determined using a log-likelihood estimator govern by a parametrized probability distribution. CE method adaptively estimates the parameters of the probability distribution which produces a random variable in the neighborhood of the globally optimal solution by minimizing cross entropy. CE method has two phases:

1. Generating candidate solutions sampled from some parametrized probability distribution.

2. Updating the parameters of the probability distribution based upon the current best solutions to bias the future search to obtain samples closer to the optimal solution.

We refer to [85][86][87] and references therein for more details about the CE method properties and its applications.

In order to apply CE to solve cluster reliability optimization problem in Equation (4.10), we need to specify how to generate random power assignment and how to update the parameters of parametrized probability distribution used in generating the power assignment at each iteration. Let each possible transmission power assignment be presented by a vector $\mathbf{x} = (x_0, \ldots, x_{n-1})$ in the set $X = \{ \mathbf{x} = (x_0, \ldots, x_{n-1}) : x_i \in \{0, \Delta P_{tx}, \ldots, L \Delta P_{tx}\} \}$. Define a function $S$ on $X$ such that $S(\mathbf{x})$ denotes the cluster reliability. Then, we can obtain the optimal power assignment by solving the this problem:

$$\maximize \quad S(\mathbf{x}) \text{ over } \mathbf{x} \in X. \quad (4.11)$$

The CE method requires a random mechanism to generate the power level assignment for the nodes. A simple method to generate a random power assignment $\mathbf{x} = (x_0, \ldots, x_{n-1})$ in $X$ is to first draw $x_0, \ldots, x_{n-1}$ independently, each $x_i$ according to a $(L + 1)$-point discrete distribution $(p_{i,0}, \ldots, p_{i,L}), i = 1, \ldots, n$ and accept the sample only if $\Phi(\mathbf{y}) = 1$, i.e., to ensure that all-terminal reliability condition, i.e., all nodes are connected, is considered while assigning the transmission powers for each node. Note that $p_{ij}$ represents the probability that a node $i$ is assigned a transmission power level of
jΔPt. Thus, a matrix \( P := (p_{ij}) \) with dimension of \( n \times L + 1 \) determines which power level should be assigned to which node. Note that each row of \( P \) sums up to one. The pmf \( f(\cdot; P) \) of \( X \) is thus parametrized by matrix \( P \) and given by

\[
f(x; P) = \prod_{i=0}^{n-1} \sum_{j=0}^{L} p_{ij} 1_{X \in X_{ij}}, \tag{4.12}
\]

where \( X_{ij} = \{ X_{ij} \in X : x_i = j \Delta Pt \} \), i.e., \( X_{ij} \) is the \( i \)-th coordinate of \( x \) is \( j \Delta Pt \).

Starting from some initial value matrix \( P_0 \), the CE algorithm updates elements \( P := (p_{ij}) \) to converge to \( P^* \), such that at each row, \( i \), of \( P^* \) contains only one element, \( j \), equals (or close) to one and the remaining equal (or close) to zero, i.e., the power assignment \( j\Delta Pt \) to node \( i \). The non-zero elements define the power assignment which will maximize the cluster reliability.

For \( N \) samples, the updating rule is estimated as follows[126]:

\[
p_{ij} = \frac{\sum_{k=1}^{N} I\{S(x^{(k)} > \gamma)\} I\{X^{(k)} \in X_{ij}\}}{\sum_{k=1}^{N} I\{S(x^{(k)}) \geq \gamma\}}. \tag{4.13}
\]

This estimator count how many of the \( x^{(i)} \) obtains value greater, i.e., reliability, than \( \gamma \), and of those have their \( i \)-th coordinate equal to \( j \). By taking the ratio of the counts of \( x^{(i)} > \gamma \) to ones with have their \( i \)-th coordinate equal to \( j \), we update of \( p_{ij} \). Algorithm 4.2 is used to generate power level assignment to each node using Equation (4.13).

**Algorithm 4.2 Random Power Assignment Algorithm**

1: For a given \( P = p_{ij} \), generate a random permutation \( (\pi_0, \ldots, \pi_{n-1}) \) of \( \{0, \ldots, n-1\} \).
2: for \( i = 0 \) to \( n-1 \) do
3: \( t = \sum_{j=0}^{L} P_{\pi_{i,j}j} \)
4: for \( j = 0 \) to \( L \) do
5: \( P_{\pi_{i,j}} = P_{\pi_{i,j}} / t \)
6: end for
7: Generate \( X_{\pi_i} \) according to \( (p_{\pi_i,0}, \ldots, p_{\pi_i,L}) \)
8: end for

The remaining unspecified parameters of the CE algorithm are the initial matrix \( P_0 \), the stopping criterion and \( N \). For \( P_0 \), we simply take all elements equal to \( \frac{1}{L+1} \). The
stopping criterion is the convergence parameter matrix $P$ to $P^*$, such that at each row, $i$, of $P^*$ contains only one element, $j$, equals (or close) to one and the remaining elements equal (or close) to zero. The convergence of $P$ is associated with the convergence of the all-terminal reliability to a certain value, i.e., $\gamma^*$. Thus, CE algorithm is terminated if for some integer $d$ and $k \geq d$, $\gamma^*_k = \gamma^*_{k-1} = \ldots = \gamma^*_{k-d}$. $N$, the sample size, can be derived from the matrix $P$’s dimensions, e.g. $N = \tau n(L + 1)$ where $\tau$ is an integer greater than 0.

\textbf{Algorithm 4.3 Main CE Algorithm for Power-Aware Cluster Reliability Optimization}

\textbf{START} with some $P_0 = \frac{1}{L+1}$. Let $k = 1$.

\textbf{REPEAT}

1: Draw a random sample of power allocations $X^{(1)}, \ldots, X^{(N)}$ according to Algorithm 4.2, with $P = P_{k-1}$.

2: Calculate the cluster reliability $S(X^{(i)})$ for all $i$ using (4.8), and arrange them in descending order, $S_1 \geq \ldots \geq S_N$. Let $[\rho N]$ be the integer part of $\rho N$. Define $\gamma_k = S_{[\rho N]}$.

3: Using the same sample, calculate $P_{k+1} = (P_{k+1,ij})$

\begin{equation}
P_{k+1,ij} = \frac{\sum_{k=1}^{N} I\{S(x^{(k)}) \geq \gamma\} I\{X^{(k)} \in X_{ij}\}}{\sum_{k=1}^{N} I\{S(x^{(k)}) \geq \gamma\}}.
\end{equation}

Increase $k$ by 1.

\textbf{UNTIL} For some $k \geq d$, $\gamma^*_k = \gamma^*_{k-1} = \ldots = \gamma^*_{k-d}$.

Rubinstein et al [86, 85, 87] introduced a smoothing procedure for updating the $P_k$ to $P_{k+1}$ in Equation (4.14) as follows:

\begin{equation}
P_{k+1} = \alpha P_{k+1}^{\text{temp}} + (1 - \alpha) P_k,
\end{equation}

where $P_{k+1}^{\text{temp}}$ is obtained via Equation (4.14). It was found that selecting $\alpha$ in the range 0.7 to 0.9 gives the best results [85, 86, 87].
4.4 Numerical Results

In this section, we consider a $100m \times 100m$ area which contains 900 TN nodes which are distributed randomly. GN is located in the middle of the observed area while the locations of RN will be determined via the placement Algorithm 4.1. GN and RN nodes has transmission range, $R$, of 100 m while TN has different transmission range, $r$, of 20m to 70m. Ten independent runs have been conducted to for each TN range, $r$.

Figures 4.3-4.5 show the RN nodes deployment for a network with three RN coverage ranges at lower tier, i.e., $r = 40$, $r = 50$ and $r = 60$, and same $R$ coverage range at higher tiers, i.e., $R = 100$. As $r$ gets larger, the number of nodes affecting a certain RN’s location as interferers of signal transmitters seldom changes or does not change at all. Therefore, similar RN deployment pattern is observed for the network under study as shown in Figures 4.3-4.5.

![Figure 4.3: Cluster1: Relaying Node Deployment for $r=40$ and $R=100$.](image)

Figure 4.3: Cluster1: Relaying Node Deployment for $r=40$ and $R=100$. 
Figure 4.4: Cluster2: Relaying Node Deployment for r=50 and R=100.

Figure 4.5: Cluster3: Relaying Node Deployment for r=60 and R=100.
Nevertheless, we observed that the sub-clusters and cluster reliabilities maintain averages of 0.87 and 0.77 as shown in Figure 4.6, respectively. When we considered a wider range of $r$, i.e., 20m to 70m, we notice that cluster reliability average around 0.77 as $r$ value grows larger than 45m. We can interpret the reason or this to be that the effective node participation as desire signal or interference becomes steady and greater coverage range for $r$ will have less influence due to the weak effect of the received signal or interference at the RN.

![Figure 4.6: Sub-Clusters and Clusters Reliabilities for Network in Figures 4.3-4.5.](image-url)
Figure 4.7: Clusters Reliabilities for RN Nodes Deployment for $r=20m$ to $70m$.

The limitation of achieved sub-clusters and cluster reliability is not only due the effect of nodes locations with respect to each other but also of due of the effect of the ratio of nodes transmission powers on links reliabilities in the sub-clusters and cluster. Figure 4.8 shows the ability of power-aware reliability algorithm to improve the network reliability by 15 % to 22 %. Network reliability can be further enhanced via controlling the transmission power of the terminal nodes within every the sub clusters.

Figure 4.8: Clusters Reliabilities for Network in Figures 4.3-4.5 With and Without Power Optimization.
4.5 Future Renewable Electric Energy Delivery and Management Communication Architecture: An Application of Hybrid Hierarchical Network Architecture

In this section, we present an application of the proposed hybrid hierarchical network communication architecture for power system. We acknowledge reliable and secure communication (RSC) subthrust [1][130] for their detailed description of the FREEDM communication structure which we utilized at this section.

The Future Renewable Electric Energy Delivery and Management (FREEDM) Systems Center has recently been created by the NSF to develop technologies to incorporate distributed renewable resources of energy and storage devices into the existing power distribution systems [1]. The FREEDM center goal is to develop power electronic technologies to improve the existing power distribution (delivery) system to allow the plug and play operation of any Distributed Renewable Energy Resource (DRER), Distributed Energy Storage Device (DESD) or load. The new delivery system will also allow for bi-directional energy flow on the power grid using an Intelligent Energy Management system (IEM) and will be distributed along the electricity distribution grid. Because of their large number, they need to be controlled autonomously (or intelligently) using a distributed agent based architecture with each agent controlling the devices at a site. Agents will also be needed to actively monitor the grid and take corrective action using an Intelligent Fault Management System (IFM). This would ensure to stability of the grid and its insulation from any faults at the user level. The intelligent agents controlling energy devices need to communicate with each other for energy management, with loads or DRERs or DESDs for control and to a control center for monitoring. Communication will also be required to support the critical IFM functionality. The distributed nature of the FREEDM system makes the support for reliable communication very challenging. Hence a reliable communication architecture is required to support FREEDM applications.

Figure 4.9 depicts the proposed FREEDM power system architecture is essentially an improved version of electricity distribution system. Thus, the structure shown in the Figure 4.9 is replicated all along the electricity distribution feeder as Subdivision Loops. All the devices are distributed all along subdivision loops, which form the traditional 12kV
distribution grid, also called the Primary Circuit. The agents controlling the DRERs, DESDs and load devices are also attached to the 120V residential electricity network, also called the Secondary Circuit. They are called the Intelligent Energy Management (IEM) devices in the FREEDM system. These devices are responsible for efficient two-way energy conversion and exchange between the primary and secondary circuits using a solid state transformer. Similarly, agents for active management, fault detection and isolation are called the Intelligent Fault Management (IFM) devices. The IFM and IEM devices have built-in intelligence (DGI) and communication capability. In Figure 4.9, the DRERs are the energy resources such as wind turbine, dam turbine and solar energy panels, and the DESDs represent storage sources such as the batteries to store solar power. A central monitoring center (named in the Figure 4.9 as user-interface) is also required to monitor and control the IEM or IFM operations. It is referred to as a control center (CC). The FREEDM system is connected to the legacy grid through a FREEDM substation.

Figure 4.9: Proposed FREEDM Power System Architecture [1].
The control strategy for the FREEDM system dictates its communication requirements and hence the communication architecture. The control strategy for the FREEDM system is hierarchical in nature and uses agent technology. Reliable and Secure Communication (RSC) subthrust was focused on the communication requirements, architecture and communication protocols to be used within the FREEDM system. RSC subthrust defined three levels of communication[1][130]:

1. **Wide Area Communication**: Wide Area communication is required to support the following interactions within the FREEDM system.

   - **IEM-IEM**: The IEM devices exchange information with each other for energy management. For example currently attached load, surplus power from DRER.
   - **IFM-IFM**: The IFM devices responsible for fault management need to communicate with each other to exchange protection data and commands.
   - **IEM-CC or IFM-CC**: All communicating devices send monitoring information using standardized communication protocols (SNMP or SCADA) to a centralized control center (CC) which may be co-located with the FREEDM substation. Also, CC can inquires or send control commands to IEM/IFM units to maximize and optimize the utilization of the power grid and the DR-ERs, DESDs and load.

The wide area communication allows IEMs and IFMs which are controlling DRERs, DESDs and loads which are distributed all over the power grid to communicate with each others and with the control center to exchange information for energy management, protection data and inquiry commands to optimize the overall utilization of power grid system. Different networking technologies can be used for wide area communication such as WCDMA/3G [115] or WiMax/IEEE 802.16 [116]. These networking technologies are used due their ability to cover larger distance, provide high throughput and meet strict timing constraints needed by the FREEDM system.

2. **Local Area Communication**: The intelligent power management algorithms at the IEM/IFM devices require local communication to interact with the local loads, energy resources and storage devices. The interaction includes the commands
initiated by the IEM device to local loads, DRERs and DESDs, and monitoring information obtained from the energy devices. Local loads, energy resources and storage can utilize low power access protocol such as Zigbee \cite{114} to respond to continuous data acquisition to IEM/IFM and IEM/IFM can utilize higher power protocols such as 802.11b for establishing links as part of local mesh WLAN to cover a certain neighborhood/block.

3. **Substation Level Communication**: This consists of a SCADA interface between the legacy grid and FREEDM substation. Information aggregated from within the FREEDM system will be sent to the utility using SCADA protocols. The FREEDM substation will be connected to the control center of the utility providing electricity to the substation. It may also be connected to peer substations for inter-substation communication.

Figure 4.10 shows that the hierarchical hybrid wireless network presented in the work can be utilized to represent the different communication level in FREEDM power system.

![Substation Level Communication Diagram](image)

Figure 4.10: FREEDM Power System Communication Architecture.

In this section, we detailed an application of the purposed hybrid hierarchical network architecture for FREEDM power system. The hybrid hierarchical network architecture
components definitions, duties, and characteristics matched the needs and duties for different communication levels in FREEDM power system.

4.6 Conclusion

In this work, we presented present a self-organized hierarchical hybrid wireless network structure that can accommodate the connectivity and reliability need for a distributed control system. We developed two algorithms: reliability-aware deployment algorithm and a reliability power-aware optimization. The reliability-aware deployment algorithm select the relaying nodes location such that sub-clusters reliabilities are maximized in a given cluster while ensuring that all terminal nodes in the network are connected to relaying nodes. The reliability power-aware optimization intelligently assign relaying nodes transmission power such that the cluster reliability is maximized. We present network deployment examples and show the ability of the proposed algorithms to enhance the all-terminal reliability of the network. Finally, we present a detailed application of the purposed hybrid hierarchical network architecture for FREEDM power system.
Chapter 5

Nodes Reliability Influence Ranking

5.1 Introduction

Computing network reliability has been and continue to be a very important problem, but it is not the only problem in reliability analysis. One of the purposes of network reliability analysis is to identify the weakness of network, quantify the impact of the component failure and hence prioritize the reliability improvement activities at the minimum cost or effort. The so called ”reliability importance measures” are used for this purpose. The importance measures have been extensively studied for both binary system[40][41][42][43][44], i.e., component are fully functioning or fully failed, and multi-state system[45][46][47][48][49].

Vasseur and Llory [44] reviewed Reliability Achievement Worth (RAW), Reliability Reduction Worth (RRW), Fussell-Vesely (FV) and Birnbaum as the most valuable importance measure of binary systems. These measures assist in evaluating the most important components with respect to the overall system reliability. For instance, the RAW quantifies the maximum percentage increase in systems reliability generated by a particular component, the RRW evaluates the potential damage caused to a system by a particular component, FV considers the probability that a component state vector has a corresponding cut (path) that makes the system failure (function) and the Birnbaum considers the probability that a component is critical for the system. For multi-state system reliability, research efforts have been focused on generalizing frequently used binary importance measures to accommodate the multi-state behavior [49]. These efforts characterize, for a given component, the most important component state with regard to its impact on
system reliability. For instance, Griffith [45] formalized the concept of multi-state system performance, and studied the impact of component improvement on the overall system reliability behavior. Moreover, Griffith [45] introduced the concept of reliability importance vector for each system component. Through this concept, a generalization of the binary Birnbaum importance measure can be extended to the multi-state case. Levitin and Lisnianski [46] proposed importance and sensitivity measures for multi-state systems with binary capacitated components. These measures account for both the multi-state system performance, which is caused by the capacitated components, and stochastic system demand. Importance measures are obtained through the universal generating function. Zio and Podofillini [48] presented multi-state extensions for RAW, RRW, FV, and Birnbaum for multi-state system with multi-state components. Their results emphasized the importance of individual component state levels. Monte-Carlo simulation methods are used to imitate the stochastic nature of the multi-state components, and generate the proposed importance measures. Levitin, et al. [47] proposed similar importance measures as those presented by Zio and Podofillini [48]. Their evaluation method is performed via the universal generating function method. Ramierz-Maraquez and Coit [49] presented two sets of composite importance measures (CIM) for multi-state system with multi-state components. The first set extends frequently used binary importance measures to the multi-state case, while the second set is introduced to consider state probabilities.

Despite the extensive works in evaluating different importance measures for binary and multi-state system, there are two main problems within the context of importance measures analysis: computation of importance measures and initial identification of the important components within a system or a network. For computation problem, researchers utilized analytical, numerical and simulation approaches which are designed on the basis of the properties of particular importance measures to overcome the computation burden associated with these importance measures. Dutuit and Rauzy [131] proposed an efficient methodology based on binary decision diagrams (BDD), and Chang et al. [132] extended it to an algorithm based on ordered BDD to compute system failure frequency and the importance measures including the Birnbaum, RAW, and RRW for system risk evaluation. Chang et al. [133] later extended their ordered BDD algorithm to handle multi-state systems. Shrestha et al. [134] presented an analytical method based on multi-state multivalued decision diagrams for multi-state component importance analysis, which is a natural extension of BDD for the multivalued case. Monte Carlo methods
have been used to calculate different importance measures for binary and multi-state system. Zio and Podofillini [48] used Monte-Carlo simulation methods imitate the stochastic nature of the multi-state components, and generate the proposed importance measures. Gertsbakh and Shpungin [135] presented a Monte-Carlo model based on systems reliability gradient vectors for evaluating the Birnbaum reliability importance of nonidentical components as well as a Monte-Carlo model based on network combinatorial spectrum for the Birnbaum reliability importance of identical components.

Throughout the literature on importance measures, the research efforts focused on how to quantify and compute the importance of a given component with respect to the overall system reliability, but there is an absence of the initial step of identifying which components are best candidates to be considered in importance measures analysis of a given system. For instance, all importance measures involve extensive reliability calculation for a single component and reliability calculation problems are NP-hard problems. In addition, some efforts tried to avoid extensive reliability calculation via using ranking [136] in which two or more components are compared analytically without the extensive calculations. Unfortunately, ranking approach is applicable for small scale networks. Therefore, in this work we aim at developing an approach that select most valuable candidates, i.e., components, to be considered in importance measures analysis of a given system.

The value of node importance depends on its location and interactions with other nodes in the network. The positions of nodes in the network are inequality and obviously different in term number of connection and distances from other nodes, thus contribution of each node to overall network reliability differ from a node to another. Early works in social networks by Freeman [137] and other scholars pointed out that the importance of node is equivalent to the significance with which the node connects with other nodes. The methods of evaluating node importance are essentially derived from graph theory and graph-based data mining [138]. A variety of measures have been proposed to determine the significance of node as a vertex within a graph such as degree centrality ($DC(\cdot)$), betweenness centrality ($BC(\cdot)$) and closeness centrality ($CC(\cdot)$) [139]. These measures characterize the node importance from different angles.

The degree centrality describes the degree information of each node [140][139] according to the idea that more important nodes are more active and therefore should have more connections. The degree centrality for a node $v$ is calculated as follows: $CD(v) = \text{degree}(v)/(n-1)$, where $n$ is the number of nodes in the network. Calculating
the degree centrality for all nodes in a dense graph has a time complexity $O(n^2)$, which becomes $O(m)$ in a sparse graph, where $n$ is the number of nodes and $m$ is the number of edges in the network. Degree centrality only measures the immediate influence among the nodes by computing the number of the immediate neighbors of nodes. Thus, this measure cannot distinguish the node importance with respect to the operation of the whole network. For instance, nodes A and B in Figure 5.1 would both have the same degree centrality. However, they are not equally important to the graph. For instance, B would have much greater control over the entire network than A with respect to the flow information between different nodes and hence any reliability measure. For example, if we consider all-terminal reliability of the network which is defined as the probability that node $v_i$ can communicate with node $v_j$ for all pair $(v_i, v_j)$ where $i \neq j$, we notice that the number of paths connecting different nodes in the network which are passing through node B is greater than the paths that are passing through node A.

![Diagram](image)

Figure 5.1: Node Importance Using Degree Centrality.

The betweenness centrality ($BC(\cdot)$) describes the frequencies of nodes in the shortest paths between indirectly connected nodes [139]. It is based on the idea that if more nodes are connected via a node, then the node is more important. The betweenness centrality for node $v$ can be calculated as follows:

$$BC(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}} \frac{\sigma_{st}}{(n-1)(n-2)}, \quad (5.1)$$

where $\sigma_{st}$ is the number of the shortest path from node $s$ to node $t$, $\sigma_{st}(v)$ is the number of the shortest paths from node $s$ to node $t$ that pass through node $v$ and $n$ is the number
of nodes in the network. Time complexity of calculating the betweenness centrality is \( O(n^3) \) since the shortest paths between each pair of nodes in a graph can be found by FloydWarshall algorithm [141]. This measure has been valuable for a number of graph analysis applications, but fails to capture the robustness or reliability of a graph. Assuming that all links have identical reliabilities, a vertex that lies on a number of paths whose length is just one greater than the shortest path receives no additional value compared to a vertex with an equally large number of shortest paths, but few paths of length one greater. Moreover, shortest paths are not necessarily the most reliable paths. For example, a path consisting of three links where each link has a reliability of 0.9 is more reliable that a path with two links where each link has a reliability of 0.1, i.e., \( 0.9 \times 0.9 \times 0.9 > 0.1 \times 0.1 \). Therefore, the importance of node to the reliability of a network cannot be accurately captured from this measure.

The closeness centrality describes the efficiency of information propagation from one node to the others [139]. The closeness centrality can be regarded as a measure of the time to spread information from a node to other reachable nodes in the network. The closeness centrality is defined as the mean shortest path between a node \( v \) and all of the nodes reachable from \( v \). The closeness centrality for node \( v \) can be calculated as follows:

\[
CC(v) = \frac{(n - 1)}{\sum_{t \in V, v \neq t} \text{dist}(v, t)},
\]

(5.2)

where \( \text{dist}(v, t) \) denotes the distance between the node \( v \) and node \( t \), that is, the length of a shortest path between \( v \) and \( t \). Calculating the closeness centrality for each node in the graph has time complexity \( O(n^3) \) [139]. As mentioned before, shortest paths are not most reliable ones. Unlike betweenness centrality, this measure cannot indicate how many paths a given node is participating in as hop.

All the aforementioned measures are typically focused on graph-theoretic importance features of a node with respect to the nodes in the network, i.e., how many immediate nodes that a given node can influence, how many times a given node is participating in shortest path between other nodes in the network, or how far is a node from all the other nodes in the networks. However, when considering the node influence on the network reliability, these measures fall short of what is needed. For instance, degree centrality as explained in Figure 5.1 fails to distinguish the importance of two nodes with respect to the network operation and structure. Betweenness centrality cannot distinguish the
importance of a node on the edge of the network with a node that lies on paths whose of length is one greater than shortest path. In addition, all the aforementioned measures are purely graph-theoretic, which are seldom associated directly with reliability of nodes, links and paths. Therefore, these measures cannot help in characterizing the node reliability influence in a network. Node reliability influence should consider the nodes, links and paths reliabilities in order to characterize the impact of nodes on the reliability of the network.

Therefore, in this work we aim at utilizing the concept node potential [50] from field theory to evaluate the node reliability influence on given network. We use the field theory to describe the interactions of all nodes of a given network in a reliability context. To do so, we first introduce the model and definitions of node influence in a given network in Section 5.2. Then, we propose an algorithm for ranking nodes based on their reliability influence in Section 5.3. Then, we present numerical results comparing node reliability influence with degree centrality ($DC(\cdot)$), betweenness centrality ($BC(\cdot)$) and closeness centrality ($CC(\cdot)$) in Section 5.4. Finally, Section 5.5 concludes this work.

## 5.2 Model and Definitions

In this section, we introduce the field theory into the network topology for determining the reliability influence at given node within a wireless network.

According to the classic theory of fields [50], a field is a mode description of the interaction of material particles, that is instead of saying a particle acts on another, we say that a particle creates a field around itself and a certain force acts on every other particle located in this field. The potential of point in a data field is function of the position, which is inversely proportional of the distance between that point and the particle and is directly proportional to the magnitude of the particle mass and charge [50]. In data field, a function can be used to define the potential in field if the following rules are presented: given a data object in space $\Omega$, let $\Phi_x(y)$ be the potential at point $y \in \Omega$ in the data field produced by $x$, then $\Phi_x(y)$ must meet all the following conditions:

1. $\Phi_x(y)$ is continuous, smooth, and finite function in space $\Omega$.
2. $\Phi_x(y)$ is isotropic in nature.
3. $\Phi_x(y)$ monotonically decreases in the distance $|x-y|$. When $|x-y| = 0$, it reaches maximum, but does not go to infinity and when $|x-y| \to \infty$, $\Phi_x(y)$ goes to zero.

For example, the central force field of a single nucleon, in which the central potential at any point in space can be calculated by the following potential function:

$$\Phi_x(y) = m \times e^{-\frac{|x-y|}{\sigma}}^k,$$

where the $m$ is the strength of interaction of node $x$ with $y$, $\sigma$ is the influence or impact factor that indicates the range of interactions, $|x-y|$ is the distance between $x$ and $y$ and $k$ is the order of distance term, e.g., $k = 1$ gives exponential-like distance and $k = 2$ gives Gaussian-like distance.

Inspired from the above physical field idea, we introduce the theory of fields into network to describe the relationships among nodes being linked by edges and to reveal the general characteristic of underlying node reliability influence measure.

A wireless network is modeled as a graph $G = (V, E, R)$ in which $V = \{v_1, v_2, \ldots, v_n\}$, $E = \{e_{12}, \ldots, e_{ij}, \ldots, e_{nn-1}\}$ and $R = \{r_{e12}, \ldots, r_{eij}, \ldots, r_{enn-1}\}$ are the set of nodes, links and links reliabilities respectively. The graph is assumed to have no parallel links, i.e., there is at most one edge between any nodes in $G$. Links reliabilities between two nodes $v_i$ and $v_j$ are equal in both directions, i.e., $r_{ij} = r_{ji} : \in [0,1]$ for simplicity. Notice that we assume that reliability of any edge connecting two node is calculated by multiplying the reliabilities of nodes at the end of the edge by the reliability of link connecting them. This assumption is based on that the reliability of an edge connecting two nodes in wireless network depends on ability of sending node of successfully transmitting signal, the ability of wireless medium of successfully carrying the signal through, and the ability of the receiving node of successfully decoding the received signal.

Each node in the network can be viewed as particle which creates a potential field around itself by transmitting its signal isotropically and the interactions of all nodes form a potential field in the network. Each node's interaction with another node will decrease as distance increases. Moreover, directly coupled nodes usually exert a stronger interaction on each other, while this interaction tends to weaken for indirectly linked nodes due to intermediaries. We view this interaction as reliability between two nodes which is a function of two nodes reliabilities and reliability of link connecting them. For directly coupled nodes, the reliability interaction can be converted into distance by taking logarithmic of link reliability multiplied by -1 , i.e., $d_{ij} = -\log(r_{ij})$, assuming prefect
nodes. The shorter the distance between nodes, the stronger the reliability interaction between them. The reliability of path connecting indirectly connected two nodes is the product of links reliabilities connecting these nodes, assuming that nodes are prefect, i.e., node reliability equals one. To find the highest reliability interaction between indirectly linked nodes, we want to find a path from \(v_i\) to \(v_j\) such that \(\prod_{r_{xy} \in R_{ij}} r_{xy}\) is maximized, where \(R_{ij}\) represents sequence of links reliabilities from \(v_i\) to \(v_j\). This is equivalent to maximizing \(\log(\prod_{r_{xy} \in R_{ij}} r_{xy})\) which is in turn equivalent to minimizing \(\sum_{r_{xy} \in R_{ij}} -\log(r_{xy})\). Thus, if we assign weights \(d_{ij} = -\log(r_{ij})\) for all the links in the path joining nodes \(v_i\) and \(v_j\), we have the shortest path problem. The most reliable path between two nodes is defined as follows:

**Definition 6** \(R_{ij}\) denotes sequence of links in paths from node \(v_i\) to node \(v_j\). The most reliable path which has the largest reliability of all paths between node \(v_i\) and node \(v_j\), assuming prefect nodes, is denoted as \(D_{ij} = \min \sum_{r_{xy} \in R_{ij}} -\log(r_{xy})\).

The reliability influence is a continuous, smooth, and finite function in space. This finite function monotonically decreases in the distance when the separation distance goes to zero, the function reaches maximum but does not go to infinity and when separation distance goes to infinity the function goes to zero. Therefore, we can utilize the potential function introduced in data field [50] to determine the reliability influence of node in a given network. The formula for calculating node \(v_i\) potential in a network is given in Definition 7.

**Definition 7** Given a network \(G = (V, E, R)\) with \(n\) nodes and \(m\) edges, where \(V = \{v_1, v_2, \ldots, v_n\}\) denote set of nodes, \(E = \{e_{12}, \ldots, e_{ij}, \ldots, e_{nm-1}\}\) denotes a set of links and \(R = \{r_{e12}, \ldots, r_{eij}, \ldots, r_{enn-1}\}\) denotes a set of links reliabilities. According to the potential function definition [50], the potential at node \(v_i\) \(\in V\) can be expressed as

\[
\Phi(v_i) = \sum_{j=1, j \neq i}^{n} \sum_{d_{xy} \in D_{ij}} (s_x s_y \times e^{-(d_{xy}/\sigma)^2}),
\]

where \(D_{ij}\) denotes the most reliable path connecting \(v_i\) and \(v_j\), \(D_{ij} = \min \sum_{r_{xy} \in R_{ij}} d_{xy}\). \(d_{xy} = -\log(r_{xy})\) denotes the logarithmic of link reliability \(r_{xy} \in D_{ij}\) multiplied by -1 used convert \(r_{xy}\) into distance measure as mentioned before. Impact factor \(\sigma\) control the effected area of each node, \(s_x, s_y > 0\) denote the magnitude interaction of node \(v_x\) and node \(v_y\) in the path \(D_{ij}\). \(s_x, s_y\) are defined in Definition 8.
In the context of network reliability, we view the magnitude of interaction, \( s_x \), of node \( v_x \) with any other node in the network as a form of reliability betweenness. This magnitude indicates how many times a given node is participating in the most reliable paths connecting different nodes in the network as source, destination or relaying node. \( s_x \) is defined in Definition 8. The greater the value of \( s_x \), the more interactions this node has with other nodes in the network in reliability context.

**Definition 8** \( s_x \) denotes the ratio of the most reliable paths that node \( v_x \) participates in as source, destination and hop to the total number of most reliable paths in the network.

The formula in (5.5) shows reliability influence calculation at node 1 in the network the Figure 5.2.

\[
\Phi(v_i) = s_{12}e^{-\left(\frac{d_{12}}{\sigma}\right)^2} + s_{12}e^{-\left(\frac{d_{12}}{\sigma}\right)^2} + s_{23}e^{-\left(\frac{d_{23}}{\sigma}\right)^2} + s_{34}e^{-\left(\frac{d_{34}}{\sigma}\right)^2}.
\] (5.5)

Figure 5.2: A Simple Line Network Used in Demonstrating Calculation of Reliability Influence at Node 1, \( \Phi(v_1) \).

Now we can utilize the potential function from field theory to measure node reliability influence in a given network. Next, we will present an algorithm for node influence ranking in a given network \( G = (V,E,R) \).

### 5.3 Nodes Influence Ranking

In this section, the first part will present the main algorithm for node influence ranking for given the network utilizing the concept node potential from the field theory. In second part of this section, we present how to optimize the impact factor \( \sigma \).
5.3.1 Node Influence Ranking Algorithm

In this section, we present the main algorithm which is used for node influence ranking. First, we construct the connection reliability matrix, \( R = (r_{ij}) \), that describes the link reliabilities between every pair of nodes in the network. Since the potential approach in the field theory utilize the shortest path to measure the interactions between nodes, we modify our reliability connection matrix by taking the logarithm of each element of the \( R \), i.e., \(- \log(r_{ij})\). Thus, finding the most reliable path between any two node becomes equivalent to the finding of shortest path between two nodes. We utilize Dijkstra algorithm [142] to find the shortest path between every pair of nodes in the network. Utilizing this information, we calculate node magnitude of interaction as defined in Definition 8. Then, we optimize the impact factor \( \sigma \) using golden section search method [143] as shown in Section 5.3.2. After finding the optimized value for \( \sigma \), we use the formula (5.4) which we introduced in Definition 7 to calculate reliability influence of node \( \Phi(v_i) \) for all \( v_i \in V \). The final step is sorting nodes according to their reliability influence \( \Phi(v_i) \).

\begin{algorithm}
\caption{Node Influence Ranking Algorithm}
\begin{algorithmic}
\Input \( G = (V, E, R) \)
\Output Nodes Influence Ranking
1: Structure the reliability matrix \( R = (r_{ij}) \) of \( G \) and modified by taking \(- log(r_{ij})\) of each element, \( R_m = (- log(r_{ij})) \).
2: Compute the most reliable paths \( D_{ij} \) between node \( v_i \) and \( v_j \) \( \forall i = 1, 2, \ldots, n, j = 1, 2, \ldots, n \), by applying Dijkstra algorithm [142] on \( R_m \).
3: Compute nodes magnitude of interactions \( s_i, i = 1, 2, \ldots, n \).
4: Select an impact factor \( \sigma \).
5: Compute the influence of node \( \Phi(v_i) \) for all \( v_i \in V \) using Equation (5.4).
6: Sort nodes by comparing their \( \Phi(v_1), \ldots, \Phi(v_n) \).
7: Return nodes influence ranking.
\end{algorithmic}
\end{algorithm}
5.3.2 Optimizing Impact Factor $\sigma$

The impact factor $\sigma$ in (5.4) is controlling the effected area of each node. If $\sigma$ is too small, the range of influence between the nodes is very small and the potential function in (5.4) will go to zero, i.e., $\Phi(v_i) \mid_{\sigma \to 0} = 0$. This extreme means that there exists no influence of the nodes. On the other hand, if $\sigma$ is very large, there is a strong influence between the nodes and potential function in Equation (5.4) will go to $s_i s_j$, $\Phi(v_i \rightarrow j) \mid_{\sigma \to \infty} = s_i s_j$. It is clear that these two extremes cannot produce a useful insights of the effect influence of nodes in the underlying network. Therefore, the selection of the $\sigma$ should take into considerations the underlying network structure and the potential functions $\Phi(v_1), \ldots, \Phi(v_n)$ for all the nodes $V = \{v_1, v_2, \ldots, v_n\}$ in the network. The most visible approach to determine impact factor, i.e., $\sigma$, is the potential entropy which is defined as follows:

**Definition 9** Given a network $G = (V, E, R)$, the corresponding impact factor $\sigma$ and the nodes potential $\Phi(V) = \{\Phi(v_1), \ldots, \Phi(v_n)\}$ for nodes $V = \{v_1, \ldots, v_n\}$. The potential entropy for this network is

$$H = -\sum_{i=1}^{n} \frac{\Phi(v_i)}{Z} \log \left( \frac{\Phi(v_i)}{Z} \right),$$

where $Z = \sum_{i=1}^{n} \Phi(v_i)$ denotes a normalization factor.

For any $\sigma \in [0, +\infty]$, $H$ satisfies $0 \leq H \leq \log(n)$, and $H = \log(n)$ if and only if $\Phi(v_1) = \Phi(v_2) = \ldots = \Phi(v_n)$. The size of the potential entropy reflects the strength of uncertainty associated with system under consideration, with $\sigma$ increasing monotonically, the curve of potential entropy, $H$, reaches the maximum values at both ends, in the middle there will be a minimum [144]. Therefore, an optimal $\sigma$ can be obtained by minimizing $H$ using a simple one-dimensional optimization method. In this section, we utilize the golden section search method [143] with initial interval $[\sqrt[3]{2} \min D_{ij}, \sqrt[3]{2} \max D_{ij}]$, where $D_{ij}$ is defined in Definition 6. The detailed algorithm is shown below:

For the Algorithm 5.2, the total time complexity is $O(k \cdot n^2)$, where $k$ is the number of iterations. The complexity of this algorithm can be improved by adopting a random sampling technique with sample size $n_s \ll n$ to reduce computational cost.
Algorithm 5.2 Optimizing Impact Factor $\sigma$

**Input**
$G = (V, E, R)$

**Output**
optimized $\sigma$

1: Set $a = \frac{\sqrt{3}}{3} \min D_{ij}$, $b = \frac{\sqrt{3}}{3} \max D_{ij}$, and $\text{err}$ is an error threshold.
2: Set $\sigma_l = a + (1 - t)(b - a)$, $\sigma_r = a + t(b - a)$, where $t = \frac{\sqrt{5} - 1}{2}$.
3: Calculate $H_l = H(\sigma_l)$ and $H_r = H(\sigma_r)$ using (5.6).
4: while $|b - a| \geq \text{err}$ do
5: if $H_l < H_r$ then
6: Set $b = \sigma_r$, $\sigma_r = \sigma_l$ and $H_r = H_l$.
7: Calculate $\sigma_l = a + (1 - t)(b - a)$ and $H_l = H(\sigma_l)$.
8: else
9: Set $a = \sigma_l$, $\sigma_l = \sigma_r$ and $H_l = H_r$.
10: Calculate $\sigma_r = a + t(b - a)$ and $H_r = H(\sigma_r)$.
11: end if
12: end while
13: if $H_l < H_r$ then
14: $\sigma = \sigma_l$
15: else
16: $\sigma = \sigma_r$
17: end if
18: Return $\sigma$

5.4 Numerical Results

First we examine the difference between different centrality measures such as degree centrality, betweenness centrality, closeness centrality as presented in Table 5.1 for the network in Figure 5.3. The network nodes reliabilities are identical and has value of 0.9 while links reliabilities are equal to one, i.e., perfect links. Thus, all edges in the network has same weight, i.e., multiplication of nodes reliabilities and link connecting them. Since all edge has same weight, thus shortest paths are equivalent to the most reliable paths. Thus, we can utilize centrality measures such as betweenness centrality and closeness centrality in characterizing the importance of node and compare them with the proposed influence reliability metric.

To characterize the centrality measures, the degree centrality is a local centrality, and it only records the degrees of nodes without any global information. Nodes 1, 2, 3
and 6 have degree 3; Nodes 5, 7 and 8 have degree 2; and node 4 has degree 4. The degree centrality here is normalized by total number of nodes in the network, i.e., n-1. Notice that you cannot deduce any information about the location nor the importance of the node with respect to network. On the other hand, closeness centrality implies that nodes 4, 5 and 6 are the main efficient part of the network since they are closer to every other node in the network. The betweenness centrality reflects the frequencies of nodes occurring in the shortest paths between indirectly connected node pairs. Therefore, nodes 4, 5 and 6 can be called core nodes of the network where most the traffic pass through. However, betweenness centrality cannot indicate if the node with zero value are on the edge of the network or on a path at least one hop longer than the shortest paths in the network. For the node reliability influence metric ($RI(\cdot)$), we intentionally set all nodes and link reliabilities to be identical in order show which are most important nodes in the network from the reliability prospective. As we have seen from the aforementioned centrality measures nodes 4, 5 and 6 are the core of this network, our reliability influence measure was able to pick node 5 as most important node in the network. Our measure was able to indicates that node 5 is crucial to the reliability network. By looking at it centrality measures, node 5 has the highest betweenness and closeness, but we cannot depend on these centrality measures as an indication of node reliability importance in the network. For instance, node 6 has lower betweenness and closeness than the ones for of node 4, however its reliability influence measure ($RI(\cdot)$) is higher than node 4. As we mentioned before, the node reliability influence measure utilizes the number of reliable paths a node participates in as source, destination or relaying node, therefore nodes with high $RI(\cdot)$ is an indication that this node is in the center of the network with respect to reliability context and participate in many reliable paths. We can argue that this idealistic state where all element are identical can be very helpful in the design state of network where you can locate easily the most important nodes. On the hand, nodes with

Figure 5.3: An Examination Case for Comparing Different Centrality Measures.
lower $RI(\cdot)$ are usually located around the edges of the network and their improvement or removal might not harm the most reliable paths in the network. We utilize Reliability Achievement Worth (RAW) [44] which quantifies the maximum percentage increase in systems reliability generated by a particular component to test our proposed reliability influence ranking. The following equation characterizes RAW of a given node:

$$RAW(i) = \frac{Rel(v_i = 1) - Rel(base)}{Rel(base)} \times 100,$$

(5.7)

where $Rel(v_i = 1)$ is the all-terminal network reliability giving that node $v_i$ is perfectly reliable, $Rel(base)$ is the original all-terminal network reliability without improvement.

The last two columns of Table 5.1 indicate a match between reliability influence measure and RAW for the nodes in the network. On the other hand, all the other centrality measures failed to obtain such match. Moreover, these centrality measures cannot determine correctly which nodes are most effective ones in the all-terminal network reliability. For example, node 4, which is ranked the first with respect to the degree centrality, betweenness centrality and closeness centrality, is ranked third with respect to RAW. The all-terminal reliability of network in Figure 5.3 is calculated using all-state enumeration approach, i.e., enumerate all the possible combination states for the network. Thus, all-terminal reliability values are exact. It is noticeable that the 0.1 improvement in reliability of node 5, i.e., 0.9 to 1, leads to 23.46% improvement in all-terminal network reliability which makes this node very critical to reliability of the network. While the same improvement, i.e., 0.1, of nodes 1, 2, 3 nodes leads only to 1.68% improvement in the all-terminal network reliability. Thus, network designer can easily elevate the network reliability by targeting the key nodes in the network. For instance, if we improve reliability of node 4, 5 and 6 by 0.1, i.e., 0.9 to 1, the all-terminal network reliability will improve from 0.577 to 0.944 which is a 63.6% improvement. On the other hand, the improvement of nodes 1, 2, 3, 7 and 8 by 0.1, will improve the all-terminal network reliability from 0.577 to 0.649 which is 12.5% improvement.
Table 5.1: An Examination Case for Comparing the Degree Centrality (DC(v)), Closeness Centrality (CC(v)), Betweenness Centrality (BC(v)), Node Reliability Influence (RI(v)) and Reliability Achievement Worth (RAW(v)) for Network in Figure 5.3.

<table>
<thead>
<tr>
<th>Node</th>
<th>DC(v)</th>
<th>CC(v)</th>
<th>BC(v)</th>
<th>RI(v)</th>
<th>RAW(v) %</th>
</tr>
</thead>
<tbody>
<tr>
<td>v5</td>
<td>0.2857</td>
<td>0.5833</td>
<td>0.8571</td>
<td>1.1583</td>
<td>23.46</td>
</tr>
<tr>
<td>v6</td>
<td>0.4286</td>
<td>0.5000</td>
<td>0.7143</td>
<td>0.8895</td>
<td>17.29</td>
</tr>
<tr>
<td>v4</td>
<td>0.5714</td>
<td>0.5833</td>
<td>0.8571</td>
<td>0.8847</td>
<td>12.98</td>
</tr>
<tr>
<td>v7</td>
<td>0.2857</td>
<td>0.3684</td>
<td>0</td>
<td>0.3663</td>
<td>5.56</td>
</tr>
<tr>
<td>v8</td>
<td>0.2857</td>
<td>0.3684</td>
<td>0</td>
<td>0.3663</td>
<td>5.56</td>
</tr>
<tr>
<td>v1</td>
<td>0.4286</td>
<td>0.4375</td>
<td>0</td>
<td>0.3663</td>
<td>1.68</td>
</tr>
<tr>
<td>v2</td>
<td>0.4286</td>
<td>0.4375</td>
<td>0</td>
<td>0.3663</td>
<td>1.68</td>
</tr>
<tr>
<td>v3</td>
<td>0.4286</td>
<td>0.4375</td>
<td>0</td>
<td>0.3663</td>
<td>1.68</td>
</tr>
</tbody>
</table>

Table 5.2 shows the influence reliability calculation for network in Figure 5.3 for several $\sigma$ values. According to the discussion about the optimization of impact factor, $\sigma$ in Section 5.3.2, $\sigma_{opt}$ was found to be 0.2132. In our network example, we observe that as the value of $\sigma$ decreases, we lose the ability to distinguish the difference between nodes reliability influence and their values become inaccurate in reflecting their correct contribution in network reliability. Therefore, selecting the optimal $\sigma$ is crucial to the accuracy of the proposed ranking algorithm.

Table 5.2: Evaluation of Node Reliability Influence for Network in Figure 5.3.

<table>
<thead>
<tr>
<th>Node</th>
<th>Reliability Influence</th>
<th>$\sigma_{opt} = 0.2132$</th>
<th>$\sigma = 0.08$</th>
<th>$\sigma = 0.1$</th>
<th>$\sigma = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>v1</td>
<td>0.3663</td>
<td>0.0009</td>
<td>0.0115</td>
<td>0.8148</td>
<td></td>
</tr>
<tr>
<td>v2</td>
<td>0.3663</td>
<td>0.0009</td>
<td>0.0115</td>
<td>0.8148</td>
<td></td>
</tr>
<tr>
<td>v3</td>
<td>0.3663</td>
<td>0.0009</td>
<td>0.0115</td>
<td>0.8148</td>
<td></td>
</tr>
<tr>
<td>v4</td>
<td>0.8847</td>
<td>0.0023</td>
<td>0.0277</td>
<td>1.9682</td>
<td></td>
</tr>
<tr>
<td>v5</td>
<td>1.1583</td>
<td>0.0030</td>
<td>0.0363</td>
<td>2.5769</td>
<td></td>
</tr>
<tr>
<td>v6</td>
<td>0.8895</td>
<td>0.0023</td>
<td>0.0279</td>
<td>1.9789</td>
<td></td>
</tr>
<tr>
<td>v7</td>
<td>0.3663</td>
<td>0.0009</td>
<td>0.0115</td>
<td>0.8148</td>
<td></td>
</tr>
<tr>
<td>v8</td>
<td>0.3663</td>
<td>0.0009</td>
<td>0.0115</td>
<td>0.8148</td>
<td></td>
</tr>
</tbody>
</table>

Reliability influence metric is a useful tool in identifying the most important set of nodes for the network reliability. By controlling the reliability of these set of nodes, we can achieve a tremendous enhancement of the all-terminal network reliability. For a net-
work with nodes with identical reliabilities and perfect links, network reliability analyst can choose to improve only the nodes with highest reliability influence in order to enhance the all-terminal network reliability. For instance, Figure 5.4 shows a Erdors Renyi random graph of 12 nodes with operational reliability of 0.5 and 19 prefect links. The network reliability analyst may choose to improve only half of the nodes in network due to resources limitations, e.g., hardware improvement, installation time, etc, to optimize the utilization of such resources. Therefore, first step is to determine the best candidate nodes whose improvement will have a great impact on the all-terminal network reliability. Table 5.3 shows different centrality measures such as degree centrality $DC(\cdot)$, betweenness centrality $BC(\cdot)$, closeness centrality $CC(\cdot)$ compared with node reliability influence $RI(\cdot)$ of Network in Figure 5.4. The nodes in this table are ranked in descending order according to the node reliability influence metric. Thus, node 2, 3, 5, 6, 7 and 10 are the nodes with the highest node reliability influence in the network and the rest of nodes, i.e., 1, 4, 8, 9, 11 and 12 are with the lowest node reliability influence.

![Figure 5.4: Erdors-Renyi Random Graph with 12 Nodes and 19 Edges.](image)

We compare the improvement of all-terminal network reliability due to the improvement of nodes reliability in Figure 5.5. In this figure, we have two set of nodes: nodes with highest $RI(\cdot)$ and nodes with lowest $RI(\cdot)$. All the nodes reliabilities are originally
Table 5.3: An Examination Case for Comparing the Degree Centrality (DC(v)), Closeness Centrality (CC(v)), Betweenness Centrality (BC(v)) and Node Reliability Influence (RI(v)) for Network in Figure 5.4.

<table>
<thead>
<tr>
<th>Node</th>
<th>DC(v)</th>
<th>CC(v)</th>
<th>BC(v)</th>
<th>RI(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>v_7</td>
<td>0.3636</td>
<td>0.5789</td>
<td>0.2828</td>
<td>0.65172</td>
</tr>
<tr>
<td>v_2</td>
<td>0.5455</td>
<td>0.6471</td>
<td>0.4495</td>
<td>0.50849</td>
</tr>
<tr>
<td>v_{10}</td>
<td>0.2727</td>
<td>0.5238</td>
<td>0.0657</td>
<td>0.48288</td>
</tr>
<tr>
<td>v_6</td>
<td>0.3636</td>
<td>0.5790</td>
<td>0.2828</td>
<td>0.41777</td>
</tr>
<tr>
<td>v_3</td>
<td>0.2727</td>
<td>0.5238</td>
<td>0.0379</td>
<td>0.41509</td>
</tr>
<tr>
<td>v_5</td>
<td>0.2727</td>
<td>0.5500</td>
<td>0.0505</td>
<td>0.38644</td>
</tr>
<tr>
<td>v_{11}</td>
<td>0.5455</td>
<td>0.6471</td>
<td>0.2727</td>
<td>0.34216</td>
</tr>
<tr>
<td>v_8</td>
<td>0.2727</td>
<td>0.5500</td>
<td>0</td>
<td>0.31584</td>
</tr>
<tr>
<td>v_1</td>
<td>0.1818</td>
<td>0.4400</td>
<td>0</td>
<td>0.30163</td>
</tr>
<tr>
<td>v_4</td>
<td>0.0909</td>
<td>0.3793</td>
<td>0</td>
<td>0.26194</td>
</tr>
<tr>
<td>v_{12}</td>
<td>0.1818</td>
<td>0.4400</td>
<td>0</td>
<td>0.21933</td>
</tr>
<tr>
<td>v_9</td>
<td>0.0909</td>
<td>0.3793</td>
<td>0</td>
<td>0.21485</td>
</tr>
</tbody>
</table>

0.5 and we improve them to different reliability levels, i.e., 0.6, 0.7, 0.8 and 0.9. First, we improve the reliability of nodes in the set of nodes with highest $RI(\cdot)$ by setting their reliability to 0.6 while keeping all the nodes in the other set, i.e., nodes with lowest $RI(\cdot)$, at 0.5. Then, we calculate the all-terminal network reliability. We repeat this for the other nodes’ reliability setting 0.7, 0.8 and 0.9. Then, we preform all aforementioned procedures for second set, i.e., nodes with lowest $RI(\cdot)$, and we keep the nodes of first set, i.e., nodes with highest $RI(\cdot)$, at 0.5. We observed that the improvement of nodes with highest $RI(\cdot)$ always outperform the improvement of the nodes with the lowest $RI(\cdot)$ in enhancing all-terminal network reliability. Figure 5.5 shows that the all-terminal network reliability improvement due the improvement of nodes with highest $RI(\cdot)$ is double the improvement due the improvement of nodes with lowest $RI(\cdot)$ for nodes reliability of 0.6 and 0.7, and all-terminal network reliability improvement due the improvement of nodes with highest $RI(\cdot)$ is triple the improvement due the improvement of nodes with lowest $RI(\cdot)$ for nodes reliability of 0.8 and 0.9. Therefore, the intelligent selection of the best candidates nodes for reliability improvement is very crucial to the improvement of the all-terminal network reliability.
Figure 5.5: Comparison of All-Terminal Reliability Improvement due to the Improvement of Nodes with Highest $RI(v)$ and Nodes with Lowest $RI(v)$: Nodes with Identical Reliabilities.

5.5 Conclusion

This work presents a reliability influence ranking algorithm to identify most valuable candidate nodes to be considered in reliability importance measures calculation. We view this ranking procedure as an initial and crucial step in reducing the computation complexity associated with reliability calculation in reliability importance measures evaluation. By identifying node reliability ranking on network reliability, we can focus the reliability importance measures evaluation on fewer nodes and hence we can prioritize the reliability improvement activities efficiently.
Chapter 6

Conclusion and Future works

6.1 Conclusion

Here we summarize our contributions in the course of this dissertation:

- **Understanding the Impact of Mobility on Network Reliability** We investigated the problem of computing the two-terminal reliability in mobile Ad hoc networks. We assumed that the nodes are subject to failures with an exponential distribution. We presented how the mobility affect the two-terminal reliability in mobile Ad hoc networks and how the imperfect wireless nodes affects the network performance parameters. The mobility pattern (i.e., speed and pause time) does influence the network reliability. First, the relationship between the mobility metrics and the two-terminal reliability can be observed via the effect of these metrics on the connectivity parameters of the network. There is a clear correlation between average degree of node, average relative speed, average link duration and reliability of the network. With the same spatial nodes distribution under a given mobility model, if mobility pattern has a high relative speed, the nodes might move from each other transmission range more quickly. Thus, lower link duration are experienced more frequently, which reduces the number of disjoint paths which in turn lower two-terminal reliability between the source and destination nodes.

This influence is less severe under SMM due to the physical constraint imposed on the node movement under SMM. The velocity of a mobile node changes gently instead of abruptly, so the current node’s velocity is dependent on the previous
velocity. Accordingly, nodes location with respect to each other will not experience
tremendous changes in short time intervals under SMM. Thus, an existing link
between two nodes is expected to remain stable for a longer period of time as
the nodes are likely to be within the transmission range of each other for longer
time. On the other hand, the velocities of a node at two different time instants
are independent under RWP movement. Thus, nodes locations under RWP change
abruptly with respect to each other for any consequent time instants. These smooth
and abruptly changes in the nodes location will impose their effects upon the two-
terminal reliability.

• Understanding the Impact of Node Transmission Power on All-Terminal
  Network Reliability

In this work, presented an optimization algorithm, NRO, based on the cross-entropy
optimization approach to solve the problem of maximizing all-terminal reliability
via controlling the nodes’ transmission powers hence controlling the SIR of every
receiving node. The proposed algorithm is compared with the widely used optimi-
zation methods in reliability analysis: simulated annealing and genetic algorithm.
As illustrated by the results, the proposed approach is superior to aforementioned
methods in term the speed of convergence and attaining maximum all-terminal
reliability for different types of networks. Moreover, NRO shows the tendency of
attaining the optimal results at every run with very small variance and without the
need for pre-warming or initial stage to tune its parameters.

• Reliability-Aware Relaying Node Deployment: Maximizing All-Terminal
  Network Reliability for Hierarchical Wireless Networks

In this work, we presented a self-organized hierarchical hybrid wireless network
structure that can accommodate the connectivity and reliability needs for a dis-
tributed control system. We developed two algorithms: reliability-aware deploy-
ment algorithm and a reliability power-aware optimization. The reliability-aware
deployment algorithm selects the relaying nodes locations such that sub-clusters
reliabilities are maximized in a given cluster while ensuring that all terminal nodes
in the network are connected to relaying nodes. The reliability power-aware op-
timization intelligently assigns relaying nodes transmission powers such that the
cluster reliability is maximized. We presented several network deployment exam-
bles and showed the ability of the proposed algorithms to enhance the all-terminal reliability of the network.

- **Nodes Reliability Influence Ranking**
  
  In this work, we presented a reliability influence ranking algorithm to identify most candidate nodes to be considered in reliability importance measures calculation. We view this ranking procedure as an initial and crucial step in reducing the computation complexity associated with reliability calculation in reliability importance measures evaluation. By identifying node reliability influence ranking on network reliability, we can focus the reliability importance measures evaluation on fewer nodes and hence we can prioritize the reliability improvement activities efficiently.

6.2 **Future Research Directions**

We have examined the problem of network reliability with heterogeneous network parameters such as channel conditions, power transmission range, fading, interference, mobility and hierarchical topology structure. Our emphasis has been on understanding the effects of these heterogeneous network parameters on links reliabilities and hence on all over network reliability. Based on this understanding, we provide efficient heuristic algorithms that optimize link reliability and maximize the network reliability. Nevertheless, there are still many possible promising directions in the broad topic of reliability modeling, analysis, and design in networking context. In this section, we arise few questions that we will consider in our future research topics.

- **Reliability Analysis of Cloud Computing Model**
  
  Cloud computing has emerged as a new and promising model for enabling convenient, on-demand network access to a shared pool of configurable computing resources (e.g., networks, servers, storage, applications, and services) that can be rapidly provisioned and released with minimal management effort or service provider interaction [145]. Unlike traditional networking, i.e., wired and wireless networks, reliability concerns are more services-oriented than resources oriented. End users are more concerned with the services they are using and do not pay much attentions to either location of resources or the networking platform used to deliver these services. Therefore, reliability in cloud computing context can refer
to the probability that a system will offer failure-free service for a specified period of time within the bounds of a specified environment. There are many challenges associated with the reliability analysis in cloud computing environment. The first challenge is to define the types of failures encountered with cloud computing context. Secondly, reliability measurement is a function of an environment, and it may be hard to fully understand the entire environment in which a cloud is operating. Finally, cloud computing depends on network connectivity, so there is a network dependency which implies that every cloud service is a networked service. This dependency makes cloud service operation sensitive to network coverage, contention for local networking resources and efficiency of networking routing. By understanding and investigating these challenges, cloud computing providers might be able to guarantee high level of reliability to the end users and evaluate the ability cloud to meet a particular reliability requirement.

• Reliability Analysis of Social Networks

There is a growing interest in quantifying the individual influence and their interactions with others in a social network particularly in the field of analyzing clandestine organizations, on-line social marketing, politics, etc. In this context, reliability analysis can be useful in determining which are the most influence individuals in the network, quantifying how influence is passed among individuals to target most of the community and determining how to improve influence effect. To utilize reliability analysis in stoical network, a mapping of relationships between social network terminology and to that associated with network reliability problem is needed. For example, individuals can be represented as nodes and their interactions correspond to arcs between these nodes. Then, reliability measures such as node reliability importance measure, two-terminal, all-terminal reliability, etc, can enable better understanding of social network dynamics and help decision makers in taking the required actions accordingly.
REFERENCES


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