ABSTRACT

DICK, LARA KRISTEN. Preservice Student Teacher Professional Noticing Through Analysis of their Students’ Work. (Under the direction of Paola Sztajn and Temple Walkowiak).

Research on preservice teacher learning has often focused on coursework and course related field experiences, and not as much on the learning that takes place during student teaching field experiences. Since preservice student teachers are engaged with children on a daily basis, attention to their own students’ thinking should be viewed as a means of development. Jacobs, Lamb, and Philipp (2010) proposed a construct of professional noticing of children’s mathematical thinking that can be used to evaluate teachers’ decision-making processes when analyzing students’ verbal and/or written responses. Analysis of student work has a strong potential benefit for preservice student teachers. This design research study focuses on preservice student teachers’ learning to professionally notice their students’ mathematical thinking through analysis of their students’ multi-digit addition and subtraction work samples.

Four preservice student teachers placed in first grade and three preservice student teachers placed in second grade participated in this study. A sequence of three professional learning tasks (PLTs) focused on preservice student teacher analysis of student work was developed. Data analysis led to an extension of the professional noticing framework proposed by Jacobs and colleagues to the field of preservice teacher education (2010). Results show specialized content knowledge, a subset of mathematical knowledge for teaching, as an integral part of the professional noticing framework components. The results suggest that preservice student teachers can be taught how to professionally notice their students’ mathematical thinking during their preservice student teaching field experience.
© Copyright 2013 Lara Kristen Dick
All Rights Reserved
Preservice Student Teacher Professional Noticing Through Analysis of their Students’ Work

by
Lara Kristen Dick

A dissertation submitted to the Graduate Faculty of
North Carolina State University
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

Mathematics Education

Raleigh, North Carolina

2013

APPROVED BY:

__________________________________________________________
Paola Sztajn
Committee Chair

__________________________________________________________
Temple Walkowick
Committee Co-Chair

__________________________________________________________
H.T. Banks

__________________________________________________________
Karen Keene

__________________________________________________________
Vicki Jacobs
DEDICATION

For my grandfather, Amos T. Stroud, Jr., a retired mathematics educator who taught me to value education and supported me during my doctoral studies.
BIOGRAPHY

Lara Kristen Stroud was born on December 28, 1980 in New Bern, North Carolina. She grew up as a child of two public-school teachers. Lara graduated from New Bern High School in 1999. She attended Meredith College in Raleigh, NC and graduated summa cum laude in May 2003 with a Bachelor of Science degree in mathematics and a Bachelor of Arts degree in religion. Upon graduation, Lara enrolled in graduate school at North Carolina State University (NCSU) where she studied applied mathematics. In December 2003, Lara married Jonathan Harland Dick, a high school mathematics teacher. Lara graduated from NCSU with her Master of Science degree in applied mathematics in May 2005. She then spent two years teaching high school mathematics, Algebra I and AP Calculus AB, at Needham B. Broughton High School in Raleigh, NC.

In August 2007, Lara returned to Meredith College as an instructor in the Mathematics and Computer Science Department. In that role, she taught mathematics courses and served as the Assistant Director of the learning center. In the fall of 2008, Lara chose to take a mathematics education PhD course as a post-baccalaureate student at NCSU. The course confirmed for Lara that she wanted to pursue a doctoral degree, thus Lara began working part-time towards her PhD in Mathematics Education. As Lara continued in her PhD coursework, and increased her exposure to mathematics education, her job at Meredith College shifted towards that of a mathematics teacher educator. Lara developed and taught two new mathematics content courses for preservice elementary teachers. She advised secondary preservice mathematics majors and served as the college supervisor for preservice secondary mathematics student teachers. Lara also served as an undergraduate research
advisor for an elementary preservice mathematics major, a North Carolina Teaching Fellow completing her Honors Thesis. In recognition of her accomplishments, Lara was approved for promotion to Assistant Professor in March 2013.

In addition to her work at Meredith College, Lara was asked to serve as a higher education consultant for the Teachers and Administrators Partnering for Mathematics Learning project. In that role, Lara helped with development and led professional development sessions for elementary and middle school teachers throughout North Carolina. During Lara’s tenure at Meredith, she gave birth to two daughters, Claire Elizabeth Dick and Irene Kristen Dick. May 2013, Lara left Meredith College and returned to NCSU to work as a research assistant on the All Included in Mathematics (AIM) professional development project. Lara plans to continue working with AIM post-graduation and eventually hopes to return to a university faculty teaching position.
ACKNOWLEDGMENTS

My deepest thanks to:

My advisor and dissertation co-chair, Dr. Paola Sztajn for her time, feedback, continued support and guidance on this dissertation, and for providing the opportunity to work on the AIM team.

Members of my committee: Dr. Temple Walkowiak, Dr. Karen Keene, Dr. H. T. Banks, and Dr. Vicki Jacobs for their leadership, thoughtfulness, support and insight.

My teachers and fellow graduate students, both past and present, at NC State for their encouragement: Dr. Allison McCulloch, Dr. Hollylynne Lee, Dr. Jere Confrey, Ayana Perry, Dr. Andrew Corley, William Hall, Carrie Ritter, Zuhal Yilmaz, Marielle Myers, Dr. Cynthia Edgington, Aaron Trocki, Tracy White, Marggie Gonzalez-Toldedo, Nadia Monrose, Christine Taylor, Drs. Jon & Stacey Ernstberger, Dr. Sarah Muccio, Dr. Jimena Davis, and Dr. Tom Braun.

My mathematics education colleagues for believing in me and inspiring me with their commitment to teaching: Dr. Timothy Hendrix, Jeane Joyner, Dr. P. Holt Wilson, Dr. Jane Gleason & Dr. Toni Parker.

My mentors and friends at Meredith College for listening and supporting: Carmen Christopher Caviness, Dr. Emily Burkhead, Dr. Jennifer Hontz, Dr. Cammey Manning, and Dr. Jackie Dietz.

My undergraduate advisor and longtime mentor: Dr. Charles Davis.

My former students at Meredith College for instilling in me a passion for teaching and learning.
The preservice teachers and college supervisors who volunteered to participate in my study.

My Bible study who keeps me grounded: Amber & Steve Dunn, Heather & Jonathan Hefner, Meghan Kent, Sarah Blackmon, Mike & Jessica Ellington, Megan Chesser and Linda Anstee.

My friends and journey mates through life: Kelli Billingsley, Jennifer Hare, Kathy Mitchell, Kristen Vincint, Tara Dew, Christy Orthner, Amy Wilkins, Elizabeth Desmond and becca Simmons.

My extended family for their love and support: Crystal & Bryan, Angela & Andres, Aubrey & Jacob, Yia Yia, Amos, Nan, Angie and Kaky.

My family for their constant love and sacrificial time as I worked on this degree: Mom, Dad, Maggie, Sally & Avery, Bill, Val, and Rebecca.

My husband and best friend: Jonathan, without whose patience, love and support this would not have been possible.

My amazing daughters: Claire and Irene, who bring joy into my life every day.

God: who is my reason for living.
# TABLE OF CONTENTS

LIST OF TABLES ..................................................................................................................... xi
LIST OF FIGURES ................................................................................................................ xiii
CHAPTER 1 ............................................................................................................................... 1
  Purpose of the Study & Research Question ...................................................................... 3
  Overview of Design Research Methodology ...................................................................... 4
  Outline of the Dissertation ............................................................................................... 4
CHAPTER 2 ............................................................................................................................... 6
  Theoretical Framework ...................................................................................................... 6
  Preservice Teachers .......................................................................................................... 7
  Content Based Field Supervision ..................................................................................... 10
  Analysis of Student Work ................................................................................................. 11
    Knowledge needed for successful analysis .................................................................... 13
    Known benefits of analyzing student work ..................................................................... 17
    Initial obstacles ............................................................................................................... 20
    Shifts in analysis & classroom practice following interventions focused on analysis of 
      student work ................................................................................................................... 23
  Multi-digit Addition and Subtraction ............................................................................... 27
  Conceptual Framework ..................................................................................................... 32
  Research Questions .......................................................................................................... 34
CHAPTER 3 ................................................................................................................................ 36
  Introduction .......................................................................................................................... 36
  Layout of the Study ............................................................................................................ 36
  Hypothetical Learning Trajectory ..................................................................................... 39
    Hypothetical learning conjectures for PLT #1 ............................................................... 40
    Hypothetical learning conjectures for PLT #2 ............................................................... 41
    Hypothetical learning conjectures for PLT #3 ............................................................... 41
  Professional Learning Tasks .............................................................................................. 42
  Participants ......................................................................................................................... 44
  Data Sources & Collection ............................................................................................... 46
    PLTs: Preservice student teachers’ written reflections of their collected student work. 46
    PLTs: video and audio-taped observations of reflection sessions with transcriptions & 
      researcher field notes. ................................................................................................. 47
    Student work brought to the reflection sessions ............................................................ 48
    Demographic and beliefs survey information .................................................................. 48
    Final questionnaire ........................................................................................................... 49
  Data Analysis ...................................................................................................................... 49
    Ongoing data analysis ..................................................................................................... 49
    Retrospective data analysis ............................................................................................ 52
  Trustworthiness .................................................................................................................. 61
    Internal validity ............................................................................................................... 62
    Reliability ......................................................................................................................... 63
First-grade group ................................................................. 108
Second-grade group .......................................................... 109
Emergent themes ................................................................. 110
  Interpret, identify and decide: Collective analysis leading to increased focus on
  students’ mathematical thinking ........................................... 111
  Identify: Levels of sophistication of strategy ............................ 114
  Decide: Importance of questioning students ............................ 116
  Other: Role of specialized content knowledge .......................... 119
Changes to the hypothetical learning conjectures for PLT #3 .............. 119
Discussion of emergent themes ............................................... 121
Summary of Changes to the Hypothetical Learning Trajectory ............. 121
Future Hypothetical Learning Trajectory .................................... 123
Redesigned Professional Learning Tasks .................................... 125
CHAPTER 5 ........................................................................... 130
Introduction to the Results ....................................................... 130
Research Question #1 ......................................................... 131
  Development of the ways in which preservice student teachers attend to their students’
  mathematical thinking ......................................................... 132
  Defining levels of attend ...................................................... 132
    Level A0 ................................................................. 133
    Level A1 ................................................................. 134
    Level A2 ................................................................. 134
  Attend over time ............................................................... 136
  Development of the ways in which preservice student teachers interpret their students’
  mathematical thinking ......................................................... 139
  Defining levels of interpret .................................................... 139
    Level R0 ................................................................. 141
    Level R1 ................................................................. 142
    Level R2 ................................................................. 143
  Interpret over time .............................................................. 145
  Development of the preservice student teachers’ understanding and ability to identify
  levels of sophistication of their students’ mathematical thinking ....... 148
  Defining levels of identify .................................................... 148
    Level I0 ................................................................. 149
    Level I1 ................................................................. 150
    Level I2 ................................................................. 151
  Identify over time .............................................................. 152
  Development of preservice student teachers’ ability to decide on appropriate next steps
  in instruction ................................................................. 155
  Defining levels of decide ..................................................... 155
    Level D0 ................................................................. 156
    Level D1 ................................................................. 156
    Level D2 ................................................................. 157
LIST OF TABLES

Table 1. Initial designed PLT ................................................................. 43
Table 2. Data by Participant ................................................................ 47
Table 3. Exemplars for attend, interpret, identify and decide .................. 51
Table 4. Number of idea units for each of the six PLT sessions ................. 54
Table 5. Mathematical Tasks Requiring SCK as Related to Professional Noticing 56
Table 6. Examples of discourse exchanges coded as R2 ............................. 58
Table 7. Complete coding information for second-grade PLT session #2 idea units 13 & 1461
Table 8. PLT Session #1 ........................................................................ 69
Table 9. Revisions to the PLT #1 hypothetical learning conjectures .......... 84
Table 10. Revisions to PLT Session #2 ....................................................... 86
Table 11. Revisions to the PLT #2 hypothetical learning conjectures .......... 89
Table 12. Re-revised hypothetical learning conjectures for PLT session #2 .... 104
Table 13. Revisions to PLT session #3 ....................................................... 106
Table 14. Revisions to the hypothetical learning conjectures for PLT session #3 108
Table 15. Re-revised hypothetical learning conjectures for PLT session #3 .... 120
Table 16. Emergent themes throughout the PLT sessions ......................... 121
Table 17. Descriptors for attend ............................................................... 133
Table 18. Attend for idea units dealing with student work throughout the PLT sessions—first grade .............................................................................. 137
Table 19. Attend for idea units dealing with student work throughout the PLT sessions—second grade ...................................................................... 137
Table 20. Combined attend for all idea units for both groups throughout the PLT sessions................................. 139
Table 21. Descriptors for interpret ............................................................. 140
Table 22. Interpret for idea units dealing with student work throughout the PLT sessions—first grade ........................................................................ 146
Table 23. Interpret for idea units dealing with student work throughout the PLT sessions—second grade ..................................................................... 146
Table 24. Combined interpret for all idea units for both groups throughout the PLT sessions ......................... 147
Table 25. Descriptors for identify .............................................................. 149
Table 26. Identify for idea units dealing with student work throughout the PLT sessions—first grade ........................................................................ 153
Table 27. Identify for idea units dealing with student work throughout the PLT sessions—second grade .................................................................... 153
Table 28. Combined identify for all idea units for both groups throughout the PLT sessions ......................... 154
Table 29. Descriptors for decide ............................................................... 156
Table 30. Decide for idea units dealing with student work throughout the PLT sessions—first grade ........................................................................ 158
Table 31. Decide for idea units dealing with student work throughout the PLT sessions—
second grade........................................................................................................... 158
Table 32. Combined decide for all idea units for both groups throughout the PLT sessions.
.................................................................................................................................... 160
Table 33. Professional noticing codes for idea units dealing with student work throughout the
PLT sessions—first grade ......................................................................................... 161
Table 34. Professional noticing codes for idea units dealing with student work throughout the
PLT sessions—second grade .................................................................................. 161
Table 35. Professional noticing codes for idea units that contain evidence of SCK throughout
the PLT sessions. ........................................................................................................ 170
Table 36. Professional noticing codes for idea units that contain collective influence
throughout the PLT sessions ...................................................................................... 175
LIST OF FIGURES

Figure 1. Ball and colleagues’ mathematical knowledge for teaching framework, as depicted by Goggins (2007, p. 59) ................................................................. 15
Figure 2. Mathematical tasks of teaching (Hill, Ball & Schilling, 2008, p. 10) ......................... 16
Figure 3. Professional noticing framework derived from Jacobs et. al. (2010) ......................... 34
Figure 4. Hypothetical learning trajectory ............................................................................. 42
Figure 5. Cycle of ongoing analysis ...................................................................................... 50
Figure 6. Student work samples from second-grade PLT session #2 idea unit 13 .......... 57
Figure 7. Donna's student work sample for attend: surprise ............................................. 74
Figure 8. Lacey's student work sample for attend: surprise ................................................. 75
Figure 9. Kelli's student work sample for interpret: plausible interpretation ..................... 76
Figure 10. Tara's student work sample for interpret: difficult to interpret .......................... 78
Figure 11. Tammy's student work sample #1 for interpret: influenced interpretations .... 79
Figure 12. Tammy's student work sample #2 for interpret: influenced interpretations .... 79
Figure 13. Ashton's board work example for other: other influences .............................. 81
Figure 14. Tara's student work sample for other: role of specialized content knowledge 83
Figure 15. Ashton's student work sample for interpret: collective analysis ...................... 94
Figure 16. Adding on example .............................................................................................. 97
Figure 17. Make-a-ten example: Leah's student work sample identify: levels of sophistication of strategy .................................................................................................. 97
Figure 18. Tara's student work sample for decide: importance of questioning students .... 98
Figure 19. Lacey's student work number line samples for other: role of SCK .................... 101
Figure 20. Kelli's first student work examples for interpret, identify and decide: collective analysis ............................................................................................................. 112
Figure 21. Lacey's student work examples for interpret, identify and decide: collective analysis ............................................................................................................. 113
Figure 22. Kelli's second student work examples for interpret, identify and decide: collective analysis ............................................................................................................. 114
Figure 23. Donna's student work decide: importance of questioning students ............... 117
Figure 24. Tara's student work decide: importance of questioning students .................... 118
Figure 25. Redesigned PLTs with implementation suggestions ........................................ 126
Figure 26. Student work sample from (20,3,1). ............................................................... 135
Figure 27. Student work sample from (13,2,1). ............................................................... 136
Figure 28. Student work sample from (6,2,2). ................................................................. 141
Figure 29. Student work sample from (9,2,1). ................................................................. 142
Figure 30. Student work sample from (8,1,1) .................................................................. 143
Figure 31. Student work sample from (1,2,1). ............................................................... 144
Figure 32. Student work sample from (6,3,2). ................................................................. 145
Figure 33. Student work samples from (18,2,2) .............................................................. 166
Figure 34. Student work sample from (16,3,2). .............................................................. 168
Figure 35. Student work sample from (18,2,2). .............................................................. 174
Figure 36. Student work sample from (13,2,1). .............................................................. 176
CHAPTER 1

Learning is an integral part of the teaching profession. Teachers continue to learn from their students’ thinking and use what they have learned to make instructional decisions in their classrooms. When teachers base their instructional decision-making on their students’ mathematical understandings, students benefit (Wilson & Bearne, 1999). Thus, teachers need to learn to analyze and interpret their students’ mathematical thinking, starting at the preservice level. Bartell, Webel, Bowen & Dyson called for further research on “how to further support PSTs [preservice teachers] in their analysis of children’s thinking” (2012, p. 18).

This design research study investigates how preservice student teachers learn to notice and make instructional decisions based on their students’ mathematical thinking. Jacobs, Lamb, and Philipp’s (2010) construct of professional noticing is used to examine preservice student teachers’ learning around a series of professional learning tasks focused on the practice of analyzing students’ work. According to Ball (2011), noticing is an essential practice for teachers because “teachers attend to and must notice important aspects of learners’ thinking, experience and resources” (p. xxi).

Research has shown that exposure to children’s mathematical thinking helps preservice teachers develop their mathematical knowledge for teaching. (Crespo, 2000; Goggins, 2007; Mewborn, 2000; Philipp et al., 2007, Rovengo, 1992; Wolf, 2003). Yet, research on the development of mathematical knowledge for teaching for preservice teachers has traditionally focused on course-related rather than on student-teaching field experiences. Preservice student teachers are by definition engaged with children and children’s
mathematical work on a daily basis. Thus, the student teaching field experience provides an advantageous means for developing preservice teachers’ mathematical knowledge for teaching. I consider that as preservice teachers have opportunities to learn to professionally notice their students’ mathematical thinking through purposeful engagement with student work during their student teaching field experience, they develop their mathematical knowledge for teaching. As Blanton, Berenson, and Norwood (2001) expounded, student teaching is the “optimal setting in which knowledge of content and pedagogy coalesce in the making of a teacher” (p. 177).

Researchers agree on the need for practice to be at the center of all aspects of teacher preparation (Ball & Bass, 2000; Boyle-Baise & McIntyre, 2008; NCATE, 2010). Ball and Bass (2000) further proposed that mathematics teacher educators ground teachers’ mathematical knowledge for teaching in practice. One area of practice that has been a focus of research is analysis of student work (Fernandez, Llimares, & Valls, 2013; Goldsmith & Seago, 2011; Jacobs & Philipp, 2004; Kazemi & Franke, 2004; Little, 2004). Analysis of student work has been shown to be a beneficial professional development activity for both pre and in-service teachers (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Jacobs & Philipp, 2004; Kazemi & Franke, 2004; Little, 1994; Little, 2003). Little (1994) explained “looking at student work exemplifies efforts to root teacher learning more consistently and deeply in dilemmas of practice” (p. 96). Analysis of student work is an area of weakness for many preservice student teachers due to lack of experience working with real students (Jacobs et al., 2010). Preservice student teachers, therefore, need practice learning how to analyze their own students’ mathematical thinking and how to respond to their students’
work (Crespo, 2000; Philipp, 2008). Little work has been done to look specifically at how analysis of student work can affect preservice student teachers’ professional noticing of their students’ mathematical thinking (Bartell et al., 2012; Cameron et al., 2009; Crespo, 2000; Jacobs & Philipp, 2004). Analysis of student work has a strong potential to be beneficial for preservice student teachers, but they cannot do it alone. For the benefit to occur, preservice student teachers require focused support on how to learn from analysis of their students’ work.

Elementary preservice student teachers receive guidance from their college supervisors during their student teaching field experience. College supervisors play a multifaceted role; one of their responsibilities is to help student teachers develop knowledge for teaching each subject. Yet, many elementary college supervisors are not trained in mathematics education. Thus, elementary college supervisors may not know how to engage their student teachers with mathematics and assist them in developing their professional noticing of children’s mathematical thinking. College supervisors need to be able to help their preservice student teachers discover how to learn during their student teaching field experience (Blanton et al., 2001; Duquette, 1997; NCATE, 2010; Wolf, 2003). Mathematics educators who partner with college supervisors may be able to work with the supervisors to help preservice student teachers further develop their professional noticing of children’s mathematical thinking during the preservice student teaching field experience.

**Purpose of the Study & Research Question**

In 2011, Jacobs, Philipp, & Sherin stated, “Noticing is a critical component of mathematics teaching expertise and thus better understanding of noticing could become a
better tool for improving mathematics teaching and learning” (p. xxvii). In light of this need, and of the studies previously cited, the primary purpose of this design research study is to explore how elementary preservice student teachers learn to notice from analysis of their students’ work. In particular, this study examines how preservice student teachers develop, through analysis of their students’ work, their capacity to professionally notice their students’ mathematical thinking. Specifically, this study investigates the following overarching research question: In what ways and to what extent do elementary preservice student teachers learn to notice through the practice of analyzing their students’ work?

**Overview of Design Research Methodology**

Design research is used in education to study the process of learning as it occurs in a real-time setting. The purpose of design research studies is to provide “systematic and warranted knowledge about learning and to produce theories to guide instructional decision-making” (Confrey, 2006). The developed theories often address complex scenarios and interactions found in the field of education. This design research study was conducted with preservice student teachers taking part in collaborative reflection sessions based on the professional learning tasks developed by the researcher and facilitated by college supervisors. These professional learning tasks were designed to foster the preservice student teachers’ development of their ability to notice their students’ mathematical thinking and to make instructional decisions based on their students’ mathematical understandings.

**Outline of the Dissertation**

This dissertation is organized around six chapters. Chapter One provides an introduction to the problem area of the study and a description of the research’s focus.
Chapter Two reviews the research literature pertaining to the problem area. It focuses on the domains of elementary preservice teachers and mathematics, professional development designed around analysis of student work, and multi-digit addition and subtraction. It concludes with a discussion of the professional noticing conceptual framework that is used to guide this study followed by a presentation of the refined research questions. Chapter 3 outlines and justifies the methodology, and describes the participants, context, sources of data, and methods of analysis used. Chapter 4 presents the ongoing analysis of the data while Chapter 5 presents the retrospective analysis of the data. Finally, Chapter 6 discusses the findings from the study including the study’s contributions, implications, and future research related to the study.
CHAPTER 2

In this chapter, I discuss the theoretical framework that underlies my position as a researcher, and I present a review of relevant literature to situate the study. First, I briefly review the literature on elementary preservice teachers and mathematics, as well as the literature on content-based supervisors for preservice teachers completing field experiences. Then, I examine empirical research focused on the outcomes of professional development designed around analysis of student work. I describe specialized content knowledge as related to analysis of student work and discuss what is known about practicing and in-service teachers’ initial tendencies when analyzing student work. Next, I discuss how pre- and in-service teachers develop their ability to analyze student work, and use their newfound knowledge to move toward evidence-based instructional decisions. Finally, I briefly review the literature on different strategies for multi-digit addition and subtraction. Following the literature review, I describe the conceptual framework on which this proposed study is based. I conclude this chapter by presenting my research questions.

Theoretical Framework

This is a study concerned with student teacher learning. I approach this study from a social constructivist perspective of learning in line with the Vygotskian tradition. I believe that students learn as they work to understand and create meaning out of their experiences situated within a social setting. Knowledge is not something that is constructed alone, but rather a co-construction taking place in a specific context. Vygotsky (1986) viewed learning as occurring first on a social level and then on an individual level. Thus, approaching research on learning from a Vygotskian perspective “requires an interweaving of different
aspects of development, involving the individual and the cultural-historical as well as the interpersonal, and focusing on the processes of development themselves” (Hogan & Tudge, 1999, p. 40). Individuals do not construct knowledge in a vacuum, they are influenced and their knowledge is shaped by a multitude of factors. Vygotsky viewed individuals’ beliefs, physical location, knowledge and past experiences as a means of influence for their interpretations of tasks and activities (Crawford, 1996). Individuals interact with tasks or activities; the interactions themselves influence the development of their knowledge.

Another aspect of Vygotsky’s theory of learning is what he termed, the zone of proximal development (ZPD). He defined the ZPD as “the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers” (Vygotsky, 1978, p. 86). Under Vygotsky’s theory, it is the interactions between peers and a more competent other, often a teacher, which mediates development. These more competent others serve to advance learning by supporting less mature ideas of others. For this study, I examined how preservice student teachers construct their learning through interactions with professional learning tasks (PLT), both with each other and with their college supervisors acting as facilitators. I designed the PLTs based on the belief that development occurs through collaboration with peers and also through facilitation led by a more competent other.

**Preservice Teachers**

Ball (1990) completed a longitudinal content knowledge comparison study with 252 secondary preservice math majors and elementary preservice teachers; she found that many
of the preservice teachers’ lacked conceptual understandings of mathematics content and procedures. Both sets of preservice teachers answered questions using mostly rules and memorized techniques; they “believed rules were explanations” (p. 460). Ball used her results to argue against what were then common assumptions about what is needed in order to learn to teach mathematics: (1) that mathematics content is easy, (2) that traditional K-12 education includes most of what teachers need to know about mathematics, and (3) that mathematics majors possess the necessary subject matter knowledge.” (p. 449). She concluded her paper with the following statement:

Attending seriously to the subject matter preparation of elementary and secondary math teachers implies the need to know much more than we currently do about how teachers can be helped to transform and increase their understanding of mathematics, working with what they bring and helping them move toward the kinds of mathematical understanding needed in order to teach mathematics well (p. 464).

Defining what is necessary to build this kind of understanding has since been the subject of numerous research studies.

In light of Ball’s work, for the past twenty years researchers have asked what types of mathematics preservice teachers should be exposed to while in college. Do they need to take more college level mathematics? Researchers do not think so; traditional mathematics instruction is not inquiry oriented and does not include conceptual understandings for mathematics concepts and standard procedures. (Ball, 1990; Monk, 1994; Schram, Wilcox, Lanier, & Lappan, 1988). If more upper level mathematics courses are not the answer, then do we need to strengthen mathematics methods courses? As Manouchehri (1997) explained,
“Modifying deeply rooted conceptions of mathematics and its teaching in the short period of a course in methods of teaching is difficult“ (p. 198). Since neither of these two types of courses, college level mathematics or mathematics methods, have proven to be sufficient in developing preservice elementary teachers’ mathematical knowledge for teaching, the focus has moved toward developing elementary mathematics content courses.

Developing and accessing the effectiveness of these mathematics content courses has proven to be difficult. Ball (2000) discussed this quandary,

To improve our sense of what content knowledge matters in teaching, we would need to identify core activities of teaching, such as figuring out what students know; choosing and managing representations of ideas; appraising, selecting, and modifying textbooks; and deciding among alternative courses of action, and analyze the subject matter knowledge and insight entailed in these activities (p. 244).

While the current emphasis on elementary mathematics content courses is to be applauded, they are not grounded in the practice of teaching actual students.

Boyd, Grossman, Lankford, Loeb, & Wyckoff (2009) completed a study in which they explored the relationship between features of teacher preparation programs and student achievement. They found that experiences focused on work in the elementary classroom to be the one aspect of preparation programs that consistently related to positive student outcomes. No specific type of field experience was found to be most effective; rather a variety of practice type experiences seemed to be imperative (Boyd et al., 2009). Again, what is important is that preservice teachers be exposed to experiences that are based on the work of teaching students (Ball & Forzani, 2009). This study seeks to address this void in the
literature by focusing on the teaching practice of analysis of student work concurrent with the preservice student teaching field experience.

**Content Based Field Supervision**

College supervisors must possess strong content knowledge so they can foster preservice student teachers’ development of their content knowledge (Blanton et al., 2001; Slick, 1998; Wolf, 2003). Slick (1998) discussed a case study completed with a graduate student serving as the college supervisor for secondary social studies preservice teachers. The supervisor had no background in social studies, and was therefore not shown respect by either the cooperating teacher or the education department. The college supervisor’s lack of content knowledge was a hindrance to the preservice student teacher’s growth. In contrast, Blanton et al.’s (2001) and Wolf’s (2003) case studies illustrated the powerful role that a supervisor’s content knowledge can play in helping preservice student teachers learn content during their student teaching field experience. The college supervisor in Wolf’s (2003) study spent countless hours with the student teacher, Maria, helping her to plan lessons, learn subject matter, and understand her students’ thinking. Their conversations helped Maria “realize that she could learn mathematics through her own teaching” (p. 95). All college supervisors should teach their student teachers how to learn content during student teaching (Duquette, 1997). To do so, college supervisors must possess strong content knowledge.

While content knowledge is imperative, knowledge itself is not sufficient. College supervisors must be able to use their content knowledge to guide student teachers’ in their own content knowledge development (Blanton et al., 2001; NCATE, 2010; Wolf, 2003). The cooperating teacher in Blanton et al.’s (2001) study challenged her student teacher’s beliefs.
about mathematics and ultimately helped the student teacher increase her mathematics content knowledge. The college supervisor did this through use of questions that drew on the preservice student teachers’ “sense making” (p. 188). Through their discussion, “what ultimately became the focus of supervision, namely, how to verbally engage students in mathematical problem solving, was seen as intrinsically bound to her [the student teacher’s] knowledge about the nature of mathematics” (p. 191). It was necessary for the college supervisor to diagnose the student teacher’s weaknesses and to guide her toward development.

Many colleges of education do not have content specific elementary college supervisors, nor does the institution from which the participants of this study hail. This study seeks to provide an example of a way to incorporate content specific guidance and development into the preservice student teaching field experience.

**Analysis of Student Work**

Philipp (2008) discussed the need to construct settings for preservice teachers to grapple with mathematics while in the context of working with real students. One of the ways mathematics educators have chosen to address this need is through having prospective teachers analyze students’ work; although, simply looking at student work does not ensure that preservice teacher learning will occur (Ball & Cohen, 1999; Bartell et al., 2012). Tasks and reflections should be designed in such a manner as to address preservice teachers’ struggles and to help them move toward actually learning from their analysis.

Prior research focusing on preservice teachers’ analysis of student work during preservice coursework (Bartell et al., 2012; Crespo, 2000; Hiebert, Morris, Burk & Jansen,
2007; Jansen & Spitzer, 2009; Philipp, 2008; Spitzer, 2010) showed that preservice teachers need assistance learning how to analyze student work and that such assistance should continue beyond preservice teacher coursework. Hammerness, Darling-Hammond, & Shulman (2005) wrote, “The knowledge, skills, and attitudes needed for optimal teaching are not something that can be fully developed in teacher education programs” (p. 358). Because the skill of analyzing student work cannot be fully developed during preservice coursework, other studies have also looked at ways to assist practicing teachers develop their ability to analyze their students’ work (Cameron, Loesing, Rorvig, & Chval, 2009; Franke, Carpenter, Levi, & Fennema, 2001; Goldsmith & Seago, 2011; Kazemi & Franke, 2003; Little, 1994; Little, 2003).

These studies show a need for teachers to learn how to both learn from their students’ work and also to make instructional decisions based on their analysis. For example, Little (2003) completed a set of case studies of teacher work groups in four classrooms in three different schools. She sought to identify specific practices employed by teachers who participated in collaborative sessions in which they examined student work. Through analysis of the data, Little found three dilemmas that teachers faced during the sessions: 1) the teachers felt the need to comfort their colleagues and to justify their own teaching practices, 2) the teachers had a difficult time keeping their discussions focused on student work; their talk often shifted toward other aspects of their teaching, and 3) the teachers had difficulty deciding what to “look at” in their students work. In another study, Goldsmith & Seago (2011) completed a mixed-methods study with middle and high school teachers in two different professional development programs. They developed and used a scoring rubric to
code pre- and post-video analysis and pre and post-written analysis of student work and used their data to complete a hierarchical generalized linear model (HGLM) with measures over time. They also transcribed and coded transcripts of the professional development discussions. Goldsmith & Seago (2011) found teachers learned how to interpret artifacts, but their success was largely dependent on the role of the facilitator.

None of the research studies mentioned above considered the role of the preservice student teaching field experience as a means of helping preservice student teachers continue to develop their mathematical knowledge for teaching and their ability to analyze and learn from their students’ work. To address this void, this study looks at how preservice student teachers learn through professional learning tasks focused on analysis of their students’ work.

**Knowledge needed for successful analysis.** The types of knowledge necessary for pre and in-service teachers to successfully analyze student work are in question. Some studies discussed the necessity of preservice teachers possessing a special type of content knowledge in order to be able to connect what they learn from the analysis of student work to actual instruction that goes on in the classroom (Bartell et al., 2012; Ferendez, Llimares & Valls, 2013; Hiebert et al., 2007; Jacobs et al., 2010). I consider this special content knowledge to be in line with Ball and colleagues concept of Specialized Content Knowledge (SCK) (Ball, 2000). Hiebert et al. (2007) explained that making connections between analysis of student work and instructional practice “requires a set of competencies or skills that draw directly on subject matter knowledge combined with knowledge of student thinking” (p. 52). They discussed how teachers must: a) observe and predict types of strategies students will use to solve a problem; and b) know what a particular response
implies about the student’s thinking. Similarly Jacobs et al. (2010) explained, “to interpret children's understandings, one must not only attend to children's strategies but also have sufficient understanding of the mathematical landscape to connect how those strategies reflect understanding of mathematical concepts” (p. 195). These two examples serve as exemplars of specialized content knowledge as defined by Hill, Ball & Shilling (2008). Pre and in-service teachers must draw upon specialized content knowledge when analyzing student work. A detailed discussion of specialized content knowledge follows.

In 2000, Ball identified three concerns that needed to be addressed before successful merging of mathematics content and pedagogy could occur in elementary preservice courses. She explained “the [first] problem concerns identifying the content knowledge that matters for teaching, the second regards understanding how much knowledge needs to be held, and the third centers on what it takes to learn to use such knowledge in practice” (p. 244). During the past decade, Ball and her colleagues have worked to develop a comprehensive framework for describing mathematical knowledge as it is used in teaching that seeks to address these problems. Their framework defines mathematical knowledge for teaching (MKT) and breaks it into two main domains, pedagogical content knowledge which is comprised of knowledge of content and students and knowledge of content and teaching, and subject matter knowledge which is comprised of specialized content knowledge and common content knowledge (Ball, Thames & Phelps, 2008; Hill, Ball & Schilling, 2008). See Figure 1.
Knowledge of Content and Students includes the ability to predict how students will respond to mathematical topics, what they will find interesting and which topics will be the most difficult for them to grasp. Knowledge of Content and Teaching includes making appropriate choices for examples or representations, knowing how to best sequence a topic in order to develop students’ understandings, and being able to guide classroom discussions. Common Content Knowledge is comprised of the ability to solve mathematical problems, provide definitions of terms, and compute correct answers; this type of knowledge is not specific to teachers. Specialized Content Knowledge is the knowledge teachers draw upon when evaluating students’ invented definitions and interpreting their developed algorithms. Knowledge of Content and Students (KCS) is the knowledge teachers draw upon when placing students’ strategies along a projected trajectory of development. Specialized Content Knowledge (SCK) is the knowledge teachers draw upon when analyzing student work. Since the focus of this study is analysis of student work, research on SCK is reviewed below.
According to Ball, Thames, & Phelps (2008), specialized content knowledge is content knowledge unique to teachers. “Teachers need to understand different interpretations of the operations in ways that students do not…They also need to know features of mathematics that they may never teach to students, such as a range of non-standard methods or the mathematical structure of student errors” (p. 8). Ball, Thames & Phelps (2008) provided readers with a chart detailing teaching tasks that require specialized content knowledge beyond common content knowledge (Figure 2).

![Figure 2. Mathematical tasks of teaching (Hill, Ball & Schilling, 2008, p. 10)](image)

For this study (see chapter 3), professional learning tasks focused on analyzing student work were developed in order to assist preservice student teachers’ development, among
other things, of specialized content knowledge. As listed in Figure 2, these PLTs required the preservice student teachers to “evaluate the plausibility of students’ claims,” “evaluate mathematical explanations,” and “critique mathematical notation and language.” The goal of the professional learning tasks was focused on analysis of student work to not only to help preservice student teachers with the previous three aspects of teaching, but also to learn how to “ask productive mathematical questions” and finally to draw upon these experiences to choose the most appropriate instructional next step for their students.

**Known benefits of analyzing student work.** Research studies have shown two potential benefits associated with an increased ability to analyze student work—content knowledge benefits and pedagogical benefits (Jacobs & Philipp, 2004; Kazemi & Franke, 2003; Little, 1994). Cameron et al. (2009) cited insufficient content knowledge as one feature preventing teachers from effectively using student work. Yet as Bartell et al. (2012) explained, “content knowledge alone does not support [preservice teachers] in looking beyond surface features contained in children’s responses” (p. 15). For this reason, some studies exploring the benefits of having pre and in-service teachers analyze student work have focused on ways to further develop content knowledge during this experience. Kazemi & Franke (2003) completed a professional development study focused on collective analysis of student work with eleven K-4 in-service teachers. They found that student work offered teachers a means to explore non-standard, student generated mathematical procedures and to make sense of students’ mathematical thinking. Similarly, Cameron et al. (2009) found that their professional development focused on analysis of student work fostered communication
between teachers “regarding important mathematical ideas needed to effectively teach mathematics” (p. 493).

Other studies have identified positive pedagogical changes for teachers who have participated in professional development focused on analysis of student work (Ball & Cohen, 1999; Cameron et al., 2009; Jacobs & Philipp, 2004; Kazemi & Franke, 2003; Little, 1994; Little, 2003). Kazemi & Franke (2003) “contend that when teachers engage in ongoing study of student work, it can create a cycle of experimentation and reflection” in their classrooms (p. 34). Cameron et al. (2009) completed a professional development study with K-5 teachers at two different institutions. The professional development was focused on using analysis of student work to understand progressions of students’ addition strategies. They found that the experience changed the ways the teachers’ approached mathematics instruction. At the end of the professional development the teachers felt more comfortable facilitating whole class discussions and were more capable of supporting individual students with specific feedback.

In order for these benefits to occur, the research sheds light on factors that are most conducive to pre and in-service teacher learning both content and pedagogy. Some of these factors include analysis of work from teachers’ own classrooms, collective analysis, and the effectiveness of the facilitator. Professional development focused on analyzing student work does not always employ student artifacts from teachers own students. Yet, Kazemi & Franke (2003) and Goldsmith & Seago (2011) discussed the positive affordances associated with teachers analyzing work from their own classrooms. The teachers are apt to be more engaged because the students are familiar and the challenge of assisting them is real. Goldsmith & Seago (2009) explained that using student work from teachers’ own classrooms is
“ecologically valid” because they are situated within actual practice with actual students (p. 170). For this reason, the student work analyzed in this study was collected from the preservice student teachers’ own students in their field placement classrooms.

Reviewed research studies that looked at analysis of student work did so in collaborative learning settings. Crespo (2000) had her math methods preservice teachers collaborate with 4th grade students on a letter writing exchange where the preservice teachers proposed challenging mathematical tasks to their assigned students. She found the experience to be beneficial for her students because they had the opportunity to act as a teacher in an environment that was safe and collaborative while under the guidance of an experienced mathematics teacher educator. This idea of designing a safe space for collective analysis emerged from Kazemi & Franke’s (2003) study as well. They explained, “By collectively engaging in the study of student work, teachers can make public their own assumptions about teaching and learning, and deliberate differences they see in the ways their practices affect students’ thinking” (Kazemi & Franke, 2003, p. 6). Thus, working together forced discussion and deliberations about student work that may have been overlooked had the teachers been working alone. For this study, during the PLT sessions, the preservice student teachers worked together in small, grade-level groups to collectively analyze their student work samples.

The professional development research associated with analysis of student work most often involves a facilitator who works with a group of teachers. The facilitator should be knowledgeable about the subject, be skillful at directing conversation, and be cognizant of potential areas of difficulty (Goldsmith & Seago, 2011; Jacobs, Lamb & Philipp, 2010;
Kazemi & Franke, 2003; Little, 2003). Specifically, with respect to students’ mathematics work samples, Goldsmith & Seago (2011) believe facilitators must “have clear goals for directing teachers’ attention to mathematically important elements of the artifacts” (p. 184). Their study was a success partially “because facilitators actively promoted teachers’ increasingly targeted and sophisticated analysis of students’ thinking” (p. 184). College supervisors assigned to elementary preservice student teachers will be the facilitators for this study. The researcher chose to partner with college supervisors known to possess mathematics specialized content knowledge in the hope that they could effectively direct the preservice student teachers’ learning during the guided PLT sessions based on analysis of their students’ work.

**Initial obstacles.** The benefits associated with learning to analyze student work do not immediately occur; this expertise must be learned (Jacobs, Lamb & Philipp, 2010). When preservice and in-service teachers begin to analyze student work, they face many obstacles that are detailed in the literature. These initial obstacles include, but are not limited to 1) not knowing how to choose which students’ work to analyze, 2) describing the work as right or wrong without attending to the mathematics present, 3) drawing on personal experience and/or knowledge to fill in gaps in students’ work, and 4) not knowing what to do with non-standard or surprising solutions. The literature related to each of these known obstacles in initial experiences analyzing student work is discussed below.

It is not always easy for teachers to choose which samples of their students’ work to bring to collective analysis sessions (Heibert et al., 2007; Little, Warren & Gearhart, 2003). Questions including how many, which student(s), and which assignment are not easy for
teachers to answer. It is also difficult for teachers to articulate their reasons for which work samples they chose to bring. To overcome this obstacle, it is imperative that the facilitators help teachers understand the trade-offs they have to make in choosing which artifacts to bring to collective analysis sessions (Heibert et al., 2007).

Most pre and in-service teachers initial descriptions of their students’ work are very general and are focused on the correctness of the solutions; there is a tendency to quickly judge the work based on procedural fluency and to make conclusive claims (Crespo, 2000; Goldsmith & Seago, 2011; Jansen & Spitzer, 2009; Kazemi & Franke, 2003; Little et al., 2003; Spitzer, 2010). Facilitators must help teachers move beyond surface features of students’ work. One way they can do this is to help teachers understand the role of questioning their students’ logic either while the students are completing the task or when teachers return to their classroom after analyzing student work (Franke et al., 2001; Goldsmith & Seago, 2011; Kazemi & Franke, 2003; Jacobs & Philipp, 2004). Teachers must not jump to conclusions or be too hasty in evaluating their students’ responses. Turning to the students to gather their descriptions of their thought process helps teachers move toward a more complete analysis of their students’ work.

Another initial obstacle for teachers when they begin to learn how to analyze student work is their tendency to draw upon their own mathematical understandings and experiences to artificially fill in missing pieces in students’ solutions (Bartell et al., 2012; Goldsmith & Seago, 2011; Spitzer, 2010). Spitzer (2010) found that his preservice teachers often over-estimated their students understanding. Goldsmith & Seago (2011) found that teachers possessed a need to use the student work “to fashion a plausible story line about students’
thinking” (p. 182). The teachers read into students’ solutions, projecting their own understandings onto students’ work. Bartell et al. (2012) completed a pre and post-test for preservice teachers to compare their content knowledge with performance on different pedagogical tasks, one of which was a task focused on analyzing student work. They found the preservice teachers to be overly critical of student work and explained, “an overly critical stance is a reflection of [preservice teachers] drawing on their experiences and projecting those onto their analyses of children’s mathematical thinking” (p. 18). This result may seem contrary to those of Goldsmith & Seago (2011) and Spitzer (2010), but preservice teachers are often critical of their own mathematical thinking and therefore Bartell et al. (2012) found that they tended to do the same for student work. Regardless of the direction of the projection of their own understanding, teachers require assistance in learning how to analyze student work for what it actually says, not what they would like for it to say.

The final obstacle surfacing from an analysis of the research is the tendency for teachers to be surprised by non-standard responses (Bartell et al., 2012; Crespo, 2000; Kazemi & Franke, 2003; Kazemi & Franke, 2004; Goldsmith & Seago, 2011). Research shows that non-standard responses are causes for intrigue and force teachers to delve more deeply into the mathematics and reasoning behind the solutions. In Bartell et al.’s (2012) study one preservice teacher noted, “[this response] was all wording, none of it was numbers, so I really had to look more deep into it” (p. 18, [] in original). Taking a deeper look is not always easy for teachers. Jacobs, Lamb & Philipp (2010) found that preservice teachers struggled with interpreting children’s mathematical thinking. Less than 1/5 of the preservice teachers in their study “provided robust evidence of interpreting children’s understandings”
Yet, struggle can be beneficial. Crespo (2000) found that “it was the contradictions and surprises preservice teachers found in students’ work that challenged their conclusive and evaluative claims” (p. 172). Thus, once again, facilitators need to be able to assist teachers in their analysis of non-standard or surprising approaches and help them move toward deeper analysis of all student work.

**Shifts in analysis & classroom practice following interventions focused on analysis of student work.** As pre and in-service teachers begin to engage in recurring analysis of student work, their analysis shifts and they begin to apply their learning to classroom practice. These shifts include, but are not limited to: 1) more detailed and less conclusive descriptions of student work; 2) a focus on children’s mathematical thinking and the mathematical meanings behind solutions; 3) an ability to anticipate students’ responses and correctly identify levels of sophistication of students’ work; and 4) an ability to make instructional decisions based on their students’ strategies. The literature related to each of these documented shifts in analysis is discussed below.

With exposure to collaborative analysis of student work, research has shown a change in teachers’ descriptions of students’ work (Crespo, 2000; Goldsmith & Seago, 2011; Kazemi & Franke, 2004; Spitzer, 2010). Goldsmith & Seago (2011) found that teachers’ analysis shifted to a more intentional searching for logic behind students’ solutions even if students approached their solutions from a manner differently than the teacher. Over time, Crespo’s (2000) preservice teachers’ journals became more analytical and included speculative questions. Spitzer (2010) found his preservice teachers learned how to disregard irrelevant evidence and focus more on the mathematics presented in the solutions. All three of these
studies indicate the beneficial shifts in teachers’ descriptions of student work that can occur through carefully designed reflections on student work.

Exposure to collective analysis of student work has proven to be an optimal setting for helping pre and in-service teachers learn from children’s mathematical thinking (Crespo, 2000; Franke et al., 2001; Goldsmith & Seago, 2011; Kazemi & Franke, 2003; Kazemi & Franke, 2004). In Kazemi & Franke’s (2003) study, as the teachers struggled with students’ invented algorithms, they began to view the strategies as a new way to think about doing mathematics. The researchers explained, “the group was attending to the various flexible ways children used their knowledge of place value to break apart numbers” (p. 28). The teachers learned mathematics from the children’s thinking. Similarly, as the semester progressed, Crespo’s (2000) preservice teachers focused on the students’ mathematical thinking. When exploring mathematical meanings of students’ solutions, the underlying mathematics surfaces. Learning from children’s mathematical thinking involves returning to the mathematics itself (Jacobs & Philipp, 2004; Kazemi & Franke, 2003). Kazemi and Franke (2003) found that making sense of student work provided the teachers with a means to delve into mathematics content. Making sense of students’ work is one of the tasks that Ball, Thames, & Phelps (2008) describe as drawing upon specialized content knowing. Thus, I claim that through analyzing students’ work and evaluating their solutions for mathematical understandings, pre and in-service teachers are engaged in development of their specialized content knowledge.

Through exposure to tasks and reflections focused on analyzing student work, both pre and in-service teachers learn how to anticipate students’ solutions (Jacobs & Philipp,
2004; Kazemi & Franke, 2003). Jacobs & Philipp (2004) explained, “We want teachers to grapple with the mathematics and to consider how children might make sense of the mathematical situation” (p. 3). As teachers are exposed to students’ invented strategies, they begin to anticipate how their students may approach mathematical problems. Their anticipation of students’ solutions deepens as teachers develop the ability to analyze vastly different student solutions and to differentiate between differing levels of sophistication (Cameron et al., 2009; Philipp, 2008). In Cameron et al.’s (2009) study, the K-5 teachers analyzed their students’ work on addition tasks appropriate for their different grade levels. The teachers were asked to sort the students’ work based on common strategies and then created a gallery walk so they could watch the progression of levels of sophistication. Because the teachers represented all of the grade levels, as the teachers explained their students’ strategies, the group of teachers was exposed to addition strategies at each stage of development. Following the intervention, the teachers expressed the belief that the professional development experience increased their knowledge about appropriate mathematical strategies children use in their respective grade levels (Cameron et al., 2009).

Following interventions focused on learning how to analyze student work, in-service teachers are often better equipped to make instructional decisions as to how to respond to their students (Cameron et al., 2009; Goldsmith & Seago, 2011; Hiebert et al., 2007; Jacobs, Lamb & Phillip, 2010; Jacobs & Philipp, 2004; Little; 1994). Once teachers develop specialized content knowledge associated with explaining students’ strategies, they are better able to make informed decisions about instruction. Goldsmith & Seago (2011) contended, “Teachers’ careful analysis of the cognitive, mathematical, and pedagogical features of
artifacts will help them develop the disposition to attend more closely to the mathematical thinking of their own students and the skills needed to make instructional decisions that will advance their students’ thinking” (p. 170). While some studies have shown gains in this area for in-service teachers, both Hiebet et al. (2007) and Jacobs, Lamb & Philipp (2010) found little increases preservice teachers’ ability to choose how to respond via proposed instruction. Hiebert et al.’s (2007) paper described a lesson study in which only one component was analysis of student work. They found the preservice teachers’ ability to make instructional decisions based on analysis of collected student work to be lacking. They cited the following as a reason for their difficulty, “subject matter knowledge of a special kind is needed to formulate appropriate hypotheses. Understanding the demands that students’ responses make on different kinds of knowledge and what instructional cues might have triggered particular kinds of thinking requires knowing the subject deeply” (Hiebert et al., 2007, p. 55).

Specialized content knowledge once again appears as a necessity. In order for preservice teachers to be able to make instructional decisions and respond appropriately to students’ solutions, they must possess strong specialized content knowledge. Both Hiebert et al. (2007) and Jacobs et al. (2010) called for preservice preparation programs to focus more on helping preservice teachers develop the knowledge and skills necessary to be able to make instructional decisions and respond to students’ work. This study seeks to address this apparent deficiency by assisting preservice student teachers in developing these skills through collaborative PLTs focused on analyzing their students’ work.
Multi-digit Addition and Subtraction

For this study, the student work samples that the preservice student teachers were asked to bring to the PLT sessions were multi-digit addition and subtraction story problems. Researchers are in agreement about the importance of allowing children to invent their own strategies for multi-digit addition and subtraction problems (Carpetner et al., 1998; Fuson, 2003; NRC, 2001). Children can and do invent their own algorithms for solving problems, but they must be provided opportunities to do so. According to Fuson (2003), the emphasis must be on children’s understandings. Allowing children to invent their own strategies often increases their confidence with regards to mathematics as well as their conceptual understanding of the mathematics (NRC, 2001). Yet many researchers stress the importance of teachers’ facilitation of their children’s strategies, especially with multi-digit numbers (Fuson, 2003; Hiebert & Wearne, 1994; NRC, 2001; Verschaffel, Greer, & De Corte, 2007).

In order for teachers to be master facilitators, they must possess specialized content knowledge. They must be aware of different strategies their children are likely to use and of the progression of levels of sophistication of those strategies. Teachers also need to be aware of the different types of story problems and how the problem type affects their students’ choice of strategy.

There have been studies that compare students’ performance on problems based on the type of multi-digit addition and subtraction instruction they received. Hiebert & Wearne (1994) completed a longitudinal comparison study in which they followed first-grade students receiving two different types of instruction through 2nd and 3rd grade. The students received either traditional textbook instruction focused on procedures or alternative
instruction focused on the students’ development and critique of self-generated procedures; students in both instructional settings were taught the standard algorithm in second grade. Significant differences were found between the groups only at the end of third grade; the third-grade students in the alternative instruction groups performed significantly higher than the other groups. The authors concluded, “the instructional approach may have influenced the way in which the standard algorithm for subtraction was acquired” (Hiebert & Wearne, 1994, p. 271). Carpenter, Franke, Jacobs, Fennema, & Empson (1998) completed a study with groups of students in 1st, 2nd and 3rd grade; the students received instruction based on Cognitively Guided Instruction which encouraged children’s invented strategies (Carpenter, Fennema & Franke, 1996). Carpenter et al. (1998) found the children who chose to use student-generated strategies developed knowledge of base-ten number concepts earlier than children who relied more on standard algorithms. Developing their own strategies helped the students conceptually understand what they were doing; the standard algorithms did not by themselves provide a conceptual basis for the operations. These two examples from research illustrate Fuson’s conclusion from her research synthesis on whole numbers, “Research clearly indicates that nontraditional approaches can help children carry out, understand, and explain methods of multi-digit addition and subtraction rather than merely carry out a method” (2003, p. 84).

Children approach mathematics from their reality, thus working with mathematics is best done by story problems that children can act out. In their publication, Adding It Up, the National Research Council explained, “it is in solving word problems that young children have opportunities to display their most advanced levels of counting performance and to
build a repertoire of procedures for computation” (2001, p. 183). Children solve problems by acting out the situation at hand, thus differently worded problems, or different problem types, are solved differently by children (Fuson et al., 1997; Fuson, 2003; Hiebert & Wearne, 1994; NRC, 2001). For example “Mark has 11 toy cars, 3 are blue and the rest are red. How many are red?” is often approached differently than “Mark has 11 toy cars. He gave away 3 of them. How many does he have left?” For adults, both of these problems seem like an easy subtraction problem of 11-3, but for children, they can be understood quite differently. For the first problem, children tend to count up from 3 to 11; while for the second problem, children tend to count down 3 from 11.

Both of these types of problems have formal names, but researchers use different terminology. Since this research study uses the Common Core State Standards for Mathematics (CCSS-M), their language will be used throughout this literature review (NGACBP, 2010). The CCSS-M table with different problem types can be found in Appendix A. Not all types of multi-digit addition and subtraction problems follow the action (see compare problems in the problem type table), so it is vastly important for teachers to be aware of the different problem types so they can anticipate how their students may solve different problems (Fuson, 2003).

Researchers agree that there are general levels through which students progress when developing strategies for multi-digit addition and subtraction, but once again there is a wide range of terminology. The terminology presented in this literature review is based on the CCSS-M (NGACBP, 2010) and was used with the preservice student teachers during their PLT sessions. Children’s invented multi-digit addition and subtraction strategies stem from
their single-digit strategies (NRC, 2001). Fuson (2003) discussed the tendency of children in the United States to view multi-digit numbers as single-digit numbers placed side-by-side. This mistake is made when children do not possess a strong understanding of place value. Children who do not yet possess a strong understanding of place value often operate under the first level of sophistication when solving multi-digit addition and subtraction problems. Level 1 strategies involve directly modeling the situation using concrete manipulatives to act out the situation (Carpenter et al., 1998; Fuson et al., 1997; NRC, 2001). It is through their concrete work with manipulatives, that children begin to count and to develop their own, more efficient strategies. The NRC explained, “students invented procedures can be constructed through progressive abstraction of their modeling strategies” (2001, p. 199).

As children move from direct modeling, they enter into Level 2 of sophistication which includes different counting strategies (Carpenter et al., 1998; Fuson, 2003; NRC, 2001). For addition, level 2 includes counting on and beginning to develop counting strategies that do not increase by one, i.e. by twos or tens. For subtraction, level 2 includes counting up and counting down. The operations are completed sequentially, thus for all of these, the sum is kept as a running total. Children may choose to represent their strategies with hundreds boards, open number lines or using base-ten pictures (Fuson, 2003; Verschaffel et al., 2007).

The third level of sophistication includes a variety of derived number fact strategies. Within this level, children draw upon their knowledge of foundational whole number concepts, such as properties of operations and place value understandings (NRC, 2001). The strategies that children develop at this highest level are varied and are not necessarily
developed or taught in a particular order. Strategies at level 3 include chunking or jump strategies, decomposition strategies and compensation strategies (Carpenter et al., 1998; Fuson, 2003; NRC, 2001). Jump strategies often utilize open number lines and make use of children’s knowledge of ten. Decomposition strategies are used to break numbers apart in order to make addition and subtraction easier. Often, decomposition strategies are used to add/subtract like units. Compensation strategies are used to change both numbers (i.e. subtraction’s minuend and subtrahend) by giving some amount to the other number, but preserving the difference. Both decomposition and compensation strategies can be completed using tools such as open number lines or can be completed more abstractly. In general, decomposition strategies are easier for students because they are easily generalized; in contrast, compensation strategies are not easily generalized (Fuson, 2003).

While children tend to follow these documented levels, their choice of strategy may depend on instruction and their own personal choice. Carpenter et al.’s (1998) study showed most children were capable of using different strategies interchangeably. Children must be given the opportunity to develop strategies on their own. Teachers must possess both specialized content knowledge and knowledge of content and students related to levels of sophistication of strategy in order to facilitate their students’ development along different levels of sophistication. One of the design foci of this study was the development of preservice student teachers’ specialized content knowledge. The professional learning tasks that the preservice student teachers’ completed were designed around different aspects of SCK relating to multi-digit addition and subtraction.
Conceptual Framework

The conceptual framework for this study draws upon the professional noticing of children’s mathematical thinking framework developed by Jacobs, Lamb & Philipp (2010). The basic premise behind the framework is that novices in any profession must learn to notice in ways unique to the profession. This framework was chosen because of its focus on children’s mathematical thinking and its application to collective analysis of student work. In this section, I describe the framework, introduce an additional component to the framework that I am considering in this study and then discuss the aspects of the framework that are used to inform the current study.

Jacobs et al.’s (2010) professional noticing construct can be used to evaluate the decision-making process teachers use when evaluating students’ responses. The authors conceptualize professional noticing of children’s mathematical thinking as comprised of three interrelated skills: attending to children’s strategies, interpreting children’s understandings, and deciding how to respond on the basis of children’s understandings (Jacobs et al., 2010, p. 172). For mathematics, the first of the three skills, attending to children’s strategies looks at the extent to which teachers attend to the mathematics found in children’s strategies (Jacobs et al., 2010). Goldsmith & Seago (2011) looked specifically at student work and detailed how the professional noticing framework can be applied. They explained how professional noticing of student work “involves attending to both the mathematical content of the task and students’ mathematical thinking” (p. 170). Thus attending to children’s strategies when examining student work, involves noticing mathematically significant details.
The second skill is interpreting children’s understandings. This involves the teacher’s ability to describe their students’ strategies and compare their descriptions with known research on children’s mathematical development (Jacobs et al., 2010). For analysis of student work, the professional noticing framework involves teachers “generating plausible interpretations of students’ work” (Goldsmith & Seago, 2011, p. 170) which draws upon specialized content knowledge. The final skill presented by Jacobs et al. (2010) is deciding how to respond in light of student’s mathematical thinking. This skill involves synthesizing what was learned and making educated decisions about how to proceed with instruction based on the previous analysis.

The professional noticing of children’s mathematical thinking framework emphasizes teachers’ mathematical interpretation of student responses. Jacobs et al. (2010) explain, “Professional Noticing of children’s mathematical thinking requires not only attention to children’s strategies but also interpretation of the mathematical understandings reflected in those strategies” (p. 184). In light of this comment, I have added a fourth skill as a part of the framework: identifying levels of sophistication of strategies (See Figure 3). This skill draws upon multi-digit addition and subtraction specialized content knowledge relating to non-standard strategies and knowledge of content and students regarding students’ strategies usage general progression, and is placed as the third skill in my conceptual framework. As teachers interpret the work, they should be able to identify their students’ strategies’ levels of sophistication prior to deciding how to respond instructionally.
This study utilizes preservice student teachers’ engaged in professional learning tasks focused on analysis of their students’ multi-digit addition and subtraction work. By focusing on how the PLTs help the preservice teachers learn to attend to, interpret, identify and finally, decide on appropriate next steps for instruction, the preservice student teachers were developing in these four areas. In summary, Jacobs, Lamb & Philipp’s (2010) professional noticing framework serves as a progression of development for the preservice student teachers as they learn how to professionally notice from analysis of their students’ work.

Research Questions

In light of the literature that has been discussed and the conceptual framework that has been presented, I have further elaborated the overarching research question to consider specific areas of learning that I expect to take place during this study. The initial question of how do preservice student teachers learn to notice from the practice of analyzing their students’ work has been refined into three distinct questions looking at the factors that
influence and the ways in which the preservice student teachers learn to notice through interactions with the PLTs focused on analysis of their students’ work.

1. Through their experience with a set of guided, professional learning tasks focused on analysis of their students’ multi-digit addition and subtraction work, in what ways do elementary preservice student teachers:

   • Develop the ways in which they **Attend** to their students’ mathematical thinking,
   • Develop the ways in which they **Interpret** their students’ mathematical thinking,
   • Develop an understanding of and an ability to **Identify** levels of sophistication of their students’ mathematical thinking,
   • Develop the ability to **Decide** appropriate next steps in instruction?

2. How does development of specialized content knowledge relate to the elementary preservice student teachers’ learning to professionally notice through their experience with a set of guided, professional learning tasks focused on analysis of their students’ multi-digit addition and subtraction work?

3. What factors, other than SCK, influence the elementary preservice student teachers’ learning to professionally notice through their experience with a set of guided, professional learning tasks focused on analysis of their students’ multi-digit addition and subtraction work?
CHAPTER 3

Introduction

This study follows a design research methodology. In this chapter, I describe the layout of the study, its context and sample, as well as the study’s data sources, data collection and data analysis processes. The trustworthiness of the study is discussed. A subjectivity statement is included, along with a discussion of ethical considerations and limitations of the study.

Layout of the Study

This study uses a design research methodology. Design research is used in education to study the process of learning as it occurs in real-time settings. Confrey (2006) defined design studies as “an extended investigation of educational interactions provoked by use of a carefully sequenced and typically novel set of designed curricular tasks studying how some conceptual field, or set of proficiencies and interests, are learned through interactions among learners with guidance” (p. 135). The purpose of design studies is to provide “systematic and warranted knowledge about learning and to produce theories to guide instructional decision-making” (Confrey, 2006, p.135). The developed theories often address complex scenarios and interactions found in the field of education.

In many design studies previously completed within education, research was conducted in classrooms with researchers partnering with classroom teachers to study student learning (Bakker, 2004; Cobb, Stephan, McClain, & Gravemeijer, 2001; Confrey & Lachance, 2000). For example, Cobb et al. partnered with a first-grade teacher and conducted a design research study in her classroom. The main objective of the study was that “the
activity of measuring by iterating a tool along the physical extent of an object would come to signify the accumulation of distance” (2001, p. 132). During the intervention, two of the researchers followed five students as they completed different individual and small group activities. After these activities, the researchers met with the classroom teacher to plan the ensuing whole class discussion. As a part of their design methodology, the researchers recorded their ongoing conjectures about the five students’ measurement reasoning.

Bakker (2004) completed a design research study in which he partnered with an 8th grade teacher during the implementation of two instructional activities that he had designed. Bakker analyzed students’ learning processes in order to learn more about their development of key statistical concepts. As a part of his methodology, Bakker presented a hypothetical learning trajectory that he tested during analysis. His results led to the revision of the instructional materials and to a revised hypothetical learning trajectory of how students learn measures of central tendency through the instructional designs.

Differently than the previously discussed studies, this design research study investigates preservice student teacher learning in the context of teacher education. Parallel to the studies just described in which the researchers partnered with teachers to study learning of K-5 students, I partnered with two selected college supervisors to study learning of preservice student teachers. As the design component, a set of carefully sequenced professional learning tasks (PLTs) focused on analysis of the preservice student teachers’ students’ work on multi-digit addition and subtraction problems was developed. Each of the PLTs included a reflection session facilitated by the college supervisors. This intervention was designed because there is no content-based supervision for elementary preservice student
teachers at the participants’ institution. In line with design research, a hypothetical learning trajectory that described the conjectured preservice student teacher learning through their experience with the PLTs was developed, analyzed and refined.

Cobb et al. (2003) identified five features unique to design research studies. First, design research studies develop a theory comprised of both the process of learning and the means used to support learning in a particular context. Second, design studies are interventions and should be thought of as “test-beds for innovation” (p. 10). Third, design studies have prospective and reflective components. They are prospective in that they begin with hypothetical learning processes as to how learning will take place and what means of support will be necessary to foster learning. They are reflective because the initial hypothetical learning processes are constantly in question, and alternatives may arise that then must be considered. Fourth, design studies are iterative in that they utilize prior research and experimental results to inform initial design, revisions, and subsequent iterations of the study. Finally, the theories generated from design studies “must do real work” (p. 10), that is, theories developed from design studies must have a practical contribution to the field. diSessa (1991) explained that the theoretical outcome of design studies is based on research within a specific environment (e.g. a particular school or classroom) but has a more broad influence beyond the experimental environment.

This study is in line with the aforementioned five elements of design research. It develops a theory for preservice student teacher learning to notice their students’ mathematical thinking and the means to support their learning through grade-level based reflections guided by college supervisors. The designed professional learning tasks (PLTs)
serve as interventions for the preservice student teachers. The study has both a prospective and reflection component. It is prospective because it begins with a hypothetical learning trajectory as to how preservice student teachers learn to notice. It is reflective because the initial hypothetical learning trajectory is constantly under review. This study utilized prior research and experimental results to inform the design of the intervention and on-going revisions. Further, the PLTs went through two iterations as I used them with two groups of preservice student teachers. The developed theory of learning has both practical and theoretical contributions for the field of preservice student teacher mathematics education. A theory for preservice teacher professional noticing is the theoretical outcome of the study. A set of refined PLTs to support preservice student teacher learning is the practical instructional outcome.

**Hypothetical Learning Trajectory**

In light of the review of literature presented in chapter two, a hypothetical learning trajectory that encompasses the researcher’s expectations for preservice student teachers’ learning to professionally notice throughout their participation in this study was developed. Simon (1995) introduced the term hypothetical learning trajectory and defined it as a “prediction as to the path by which learning might proceed” (p. 135). The learning trajectory is hypothetical in that the actual learning trajectory may differ. The researcher analyzes whether the actual learning corresponds with what was conjectured. This then leads to new insights regarding the participants’ thinking. These new insights form the basis for a revised hypothetical learning trajectory for the remaining instructional interventions. Hypothetical learning trajectories illustrate the impossibility of separating learning from interventions.
For this study, as the preservice student teachers progressed through the three professional learning tasks, they were developing their ability to notice. The hypothetical learning trajectory for this study is comprised of PLT-specific hypothetical learning conjectures that are presented below. (Note: the bolded words serve to highlight the professional noticing framework within the hypothetical learning trajectory. See Figure 4 for a visual depicting the relationship between the hypothetical learning trajectory, the professional learning tasks and the hypothetical learning conjectures.

**Hypothetical learning conjectures for PLT #1.**

- The preservice student teachers have a difficult time choosing and articulating their choices as to which student work to bring to the reflection session.
- They initially **attend** to their student work with descriptions, written and verbal, focused on procedural correctness and/or surprise at non-standard solutions.
- When asked to describe the mathematics behind their students’ solutions, the preservice student teachers **interpret** their students’ work in light of their personal experiences, beliefs, and mathematical knowledge. The preservice student teachers recognize their tendency to project themselves onto their students’ solutions and begin to understand how to **interpret** the work based on the mathematics actually present.
- Through the discussion facilitated by the college supervisors, the preservice student teachers draw on their newfound SCK regarding problem types to **interpret** their students’ mathematical thinking. The discussion helps the preservice student teachers
recognize the importance of asking their students probing questions and taking notes in order to assist them in interpreting their students’ mathematical thinking.

**Hypothetical learning conjectures for PLT #2.**

- The preservice student teachers have an improved grasp on the addition and subtraction tasks they gave and the specific examples of student work they are analyzing.
- Their **interpretations** of their students’ work are partially based on the probing questions they asked of their students during the implementation of the task.
- Their **interpretations** are focused more on the mathematics behind their students’ thinking and less on correctness.
- The preservice student teachers **identify** differing levels of sophistication of multi-digit addition and subtraction strategies throughout their collective student work.
- Their discussion serves as a catalyst for improving their ability to **decide** on an appropriate next step in instruction based on their students’ mathematical thinking.

**Hypothetical learning conjectures for PLT #3.**

- The preservice student teachers note differences in their students’ strategies and **interpret** their students’ strategies based on **identification** of levels of sophistication. They use their knowledge to analyze their students’ success at achieving the goals of the task they assigned.
• The preservice student teachers’ **interpretations** of their students’ work are focused on the logic behind their mathematical thinking.

• Their discussion helps the preservice student teachers draw upon their SCK & KCS regarding levels of sophistication of strategy to **decide** appropriate next steps in instruction for their students.

![Hypothetical Learning Trajectory](image)

*Figure 4. Hypothetical learning trajectory*

**Professional Learning Tasks**

Multi-digit addition and subtraction was the content focus for the professional learning tasks. The first component of each PLT consisted of pre-session directions for the preservice student teachers. These directions included examples of the types of multi-digit addition and subtraction problems to pose to their students and prompts for a brief written reflection. The second component of each PLT was comprised of a session guide with questions and directions for the college supervisors. Table 1 contains the specific directions and questions for both the preservice student teachers and college supervisors for each of the three reflections sessions.
Table 1.

Initial designed PLT.

<table>
<thead>
<tr>
<th>PLT</th>
<th>Preservice Student Teacher Directions</th>
<th>Facilitator Guide</th>
</tr>
</thead>
</table>
| One | Choose two story problems: one that lends itself to an addition strategy and one that lends itself to a subtraction strategy. Ask students to solve them in two ways:  
1. Solve the task yourself and write out ways you anticipate your students will solve the task.  
2. Administer the task and monitor your students as they are working.  
3. Collect all of your students’ work.  
4. Choose six student responses that you would like to bring to the reflection session. Briefly describe their solutions. | Begin by having the preservice student teachers share their task and reasons behind their choices as to which of their students’ work to bring to the session.  
1. What strategies did your students use to solve the task?  
2. What did you find surprising or unexpected in your students’ work?  
   Lead a discussion about their initial anticipation of the ways their students would approach the problems.  
3. What is the mathematics embedded in each of their strategies?  
   Lead a discussion about different types of addition/subtraction problems and differing levels of complexity depending on the type of time, begin discussion levels of sophistication of strategies.  
4. What questions could you ask to help your student reflect on their strategy?  
   Lead a discussion on how to probe student thinking without guiding their work and how to describe student work without projecting their knowledge onto the solution. Suggest that the student teachers take notes while monitoring their students as they complete tasks. |
| Two | Choose two story problems: one that lends itself to an addition strategy and one that lends itself to a subtraction strategy. Ask students to solve them in two ways:  
1. Solve the task yourself and write out ways you anticipate your students will solve the task.  
2. Administer the task and monitor your students as they are working.  
3. Question your students as they are working and take notes.  
4. Collect all of your students’ work.  
5. Choose six student responses that you believe represent different mathematical approaches to the problem. Briefly describe their solutions. | Begin by having the preservice student teachers share their task and reasons behind their choices as to which of their students’ work to bring to the session.  
1. What strategies did your students use to solve the task?  
2. What is the mathematics embedded in each of their strategies?  
3. What questions could you ask to help your students reflect on their strategy?  
4. What questions might encourage your students to consider a more efficient strategy?  
   Lead a discussion on levels of sophistication of strategies and how to respond to students’ different levels of thinking. |
| Three | Choose two story problems: one that lends itself to an addition strategy and one that lends itself to a subtraction strategy. Ask students to solve them in two ways:  
1. Solve the task yourself and write out ways you anticipate your students will solve the task.  
2. Briefly describe what you want your students to learn from the task.  
3. Administer the task and monitor your students as they are working.  
4. Question your students as they are working and take notes.  
5. Collect all of your students’ work.  
6. Choose six student responses that you believe represent a range of levels of mathematical sophistication to bring to the reflection session.  
7. Brainstorm possible next problems to give your students based on their responses. | Begin by having the preservice student teachers share their task and reasons behind their choices as to which of their students’ work to bring to the session.  
1. What strategies did your students use to solve the task?  
2. What did you find surprising or unexpected in your students’ work?  
3. What is the mathematics embedded in each of their strategies?  
4. What did your students learn on their task and how does it compare with what you wanted them to learn?  
5. On the basis of your students’ individual understandings, what problem might you pose next and how might your student solve it?  
   Lead a discussion on choosing next steps in instruction and have students share what they have learned from their participation in this study. |
During the ongoing data analysis, the PLTs were constantly revised based both on the results of analysis and changes to the hypothetical learning trajectory. The revised versions of Table 1 are presented and discussed in Chapter 4.

**Participants**

Seven elementary preservice student teachers at a small, single-sex, liberal arts college in the Eastern United States took part in the study. Participant selection began with purposefully choosing two grade levels for the two iterations of the study. At this college, the grades that had the highest numbers of assigned preservice student teachers were first and second grade; there were a total of four preservice student teachers assigned to first grade and four preservice student teachers assigned to second grade. All eight preservice student teachers were invited to participate in the study; one preservice student teacher assigned to second grade chose not to participate. Thus for the study, there were two groups of participants: four first grade preservice student teachers and three second grade preservice student teachers. The groups met separately with their assigned college supervisor.

The preservice student teachers’ participation was not a requirement of their teacher education program nor was it a part of the regular student teaching field experience. The four participants completing their preservice student teaching experience in first-grade were Dona, Kelli, Tammy and Leah; the three participants completing their preservice student teaching experience in second grade were Lacey, Tara and Ashton. All of the aforementioned names are pseudonyms. One of the second-grade participants, Ashton, withdrew from her student teaching experience and this study prior to the third PLT session; her data remained a part of the analysis for the first two PLT sessions. For the purpose of knowing my
participants before the project started, the participants were asked to complete a demographic and beliefs questionnaire (See Appendix B). The questionnaire served to build a profile of each of the preservice student teachers. A subset of the participants’ responses to the questionnaire is located in Appendix C.

Two of the college supervisors assigned to the participating preservice student teachers were asked to partner with the researcher and to facilitate the PLTs for the two groups. The college supervisors were chosen based on their familiarity with elementary mathematics and their commitment to helping both preservice and inservice teachers focus on children’s mathematical thinking. The college supervisor who facilitated the first-grade PLT sessions was a former elementary principal who implemented Cognitively Guided Instruction (CGI) school-wide during her tenure (Carpenter et al., 1999). This was the third year she served as a supervisor for elementary student teachers. The college supervisor who facilitated the second-grade PLT sessions has been a professor in the Education department at the same institution as the participants for over 20 years. She has her doctorate in Mathematics Education and teaches K-5 mathematics methods courses in addition to supervising elementary student teachers. Throughout the course of the study, the college supervisors met with the researcher prior to each of the PLT sessions to review the PLT questions and discuss the goals of the session. Following each PLT session, each college supervisor met with the researcher via conference call to debrief the session. During these debrief meetings, the initial hypothetical learning conjectures for the session was examined for potential revisions. In addition, emergent themes from the session were discussed and negotiated. During the debriefings, the college supervisors also provided suggestions for
revisions to the next PLT. In these ways, the college supervisors fulfilled their role as research partners.

**Data Sources & Collection**

As with most qualitative studies, this study uses rich, thick descriptions of the phenomenon under study obtained from multiple sources (Creswell, 2007; Merriam, 1998). The primary sources of data from the study include the preservice student teachers’ written reflections from the PLTs, the six video and audio-taped PLT sessions with transcriptions, copies of the student work that the preservice student teachers brought to each of the PLT sessions, and researcher field notes for each session. Secondary sources of data include demographic and beliefs survey information obtained from the preservice student teachers, and a final questionnaire completed by the preservice student teachers. See Table 2 for a list of data by participant. All electronic data was stored on the researcher’s computer and backed up on an external hard drive in the researcher’s office. Hard copies were stored in a locked filing cabinet. The following section provides a description of collection for each of these sources of data.

**PLTs**: Preservice student teachers’ **written reflections of their collected student work**. The preservice student teachers were asked to complete a brief written reflection on their chosen student work prior to attending the reflection sessions. For each of the PLT sessions, a subset of the participants submitted written reflections. The reflections were analyzed during the ongoing analysis phase of data analysis and were used as a means of triangulation for emergent themes. All communication between the researcher and the participants took place via an online course Blackboard site.
Table 2.

*Data by Participant.*

<table>
<thead>
<tr>
<th>Participant</th>
<th>PLT #1</th>
<th>PLT #2</th>
<th>PLT #3</th>
<th>Other Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dona</td>
<td>Student Work Written Reflection Attended Session</td>
<td>Student Work Written Reflection Attended Session</td>
<td>Student Work Written Reflection Attended Session</td>
<td>Initial Questionnaire Final Questionnaire</td>
</tr>
<tr>
<td>Kelli</td>
<td>Student Work Written Reflection Attended Session</td>
<td>Student Work Attended Session</td>
<td>Student Work Written Reflection Attended Session</td>
<td>Initial Questionnaire Final Questionnaire</td>
</tr>
<tr>
<td>Tammy</td>
<td>Student Work Written Reflection Attended Session</td>
<td>Attended Session</td>
<td>Student Work Attended Session</td>
<td>Initial Questionnaire Final Questionnaire</td>
</tr>
<tr>
<td>Leah</td>
<td>Student Work Written Reflection Attended Session</td>
<td>Student Work Written Reflection Attended Session</td>
<td>Student Work Written Reflection Attended Session</td>
<td>Initial Questionnaire Final Questionnaire</td>
</tr>
<tr>
<td>Lacey</td>
<td>Student Work Written Reflection Attended Session</td>
<td>Student Work Written Reflection Attended Session</td>
<td>Student Work Written Reflection Attended Session</td>
<td>Initial Questionnaire Final Questionnaire</td>
</tr>
<tr>
<td>Tara</td>
<td>Student Work Attended Session</td>
<td>Student Work Written Reflection Attended Session</td>
<td>Student Work Written Reflection Attended Session</td>
<td>Initial Questionnaire Final Questionnaire</td>
</tr>
<tr>
<td>Ashton</td>
<td>Attended Session</td>
<td>Student Work Written Reflection Attended Session</td>
<td>N/A</td>
<td>Initial Questionnaire</td>
</tr>
</tbody>
</table>

**PLTs: Video and audio-taped observations of reflection sessions with transcriptions & researcher field notes.** Each of the PLT sessions lasted for approximately one hour and took place on the college campus in the afternoon on days when the preservice student teachers were required to be on campus for a credit bearing colloquium related to their student teaching field experience. The college supervisors facilitated the PLT sessions with their respective group of preservice student teachers. Each of the PLT sessions was audio-taped for transcription purposes. In addition, the PLT sessions were videotaped using a camera placed on a tripod situated in the back of the room. For each session, a videographer manned the tripod with the goal of keeping all participants in view and zooming in on
discussed student work samples. Each of the six PLT reflection sessions was transcribed verbatim.

As the researcher, I assumed the role of participant observer during each of the six PLT sessions held with the college supervisors and the preservice student teachers. According to Yin (1998) participant observation involves the researcher as an active participant while he or she is simultaneously observing (p. 115). As a participant observer, I sometimes interjected comments or questions when the conversation veered off course or when my expertise was needed in analyzing a student work sample. During each of the PLT sessions, I took field notes using a field note observational guide based on my conceptual framework (See Appendix D).

**Student work brought to the reflection sessions.** The first component of each of the PLTs included directions for the preservice student teachers regarding the types of multi-digit addition and subtraction problems to pose in their classroom and collect from their students. The preservice student teachers were to choose a specified number of work samples to bring to the PLT sessions. For the chosen samples, they were instructed to black out student names and were asked to scan and upload their samples to the course Blackboard site prior to the PLT sessions. For those participants who did not have access to a scanner, the researcher scanned their chosen work samples on the day of the PLT sessions, just before the sessions began. The images of the student work samples acted as a source of data for the analysis process. All original documents were returned to the preservice student teachers.

**Demographic and beliefs survey information.** Demographic and beliefs information was collected from each of the participants prior to the beginning of the study.
The beliefs and survey information was used to build a profile of the participants in order to help me get to know them prior to the beginning of the study. See Appendix B for a copy of the demographic and beliefs survey completed by the preservice student teachers.

**Final questionnaire.** Each of the preservice student teachers completed a final questionnaire which asked them to detail their experience with the study. The questionnaire served as a triangulation of data for the results of the study. It can be found in Appendix F.

**Data Analysis**

**Ongoing data analysis.** In line with design research methodology, the data was analyzed in two stages: ongoing and retrospective (Cobb, Confrey, DiSessa, Lehrer, & Schauble, 2003). The goal of the ongoing analysis was to compare results with the hypothetical learning trajectory. This was completed by identify themes relating to the professional noticing framework as well as any additional emergent themes. The ongoing stage of data analysis occurred during data collection. Each PLT session was analyzed after the session in order to inform both the design of the upcoming PLT, and revisions to the session’s hypothetical learning conjectures. The cycle of ongoing analysis depicted in Figure 5 was developed based on Cobb (1999)’s design.

During the ongoing analysis stage, both open coding and pre-determined coding were used. All coding took place using the coding software, Atlas TI. The four pre-determined codes were based on the study’s professional noticing framework and research questions. The codes attend, interpret, identify and decide were applied whenever an instance showed evidence of either of the four; during the ongoing analysis stage, these codes were not
leveled. It was possible for instances to be coded with multiple codes simultaneously. Table 3 includes examples of instances that were coded with each of the predetermined codes.

Figure 5. Cycle of ongoing analysis

While open-coding, many codes were developed based on emergent themes. The constant comparative method was used to confirm and build more refined categories (Merriam, 1998; Strauss & Corbin, 1998). According to Merriam (2001),

“The constant comparative method involves comparing one segment of data with another to determine similarities and differences…data are grouped together on a similar dimension. This dimension is tentatively given a name; it then becomes a category. The overall object of this analysis is to seek patterns in the data” (p. 18).

As themes emerged, categories were developed and refined. For example, during the initial pass of open-coding of PLT session #1, both “inadequate description” and “literal description” were codes. After the second pass of open-coding, I realized that literal description rarely occurred, thus it was merged with “inadequate description.” Inadequate
description included instances where the preservice student teachers had a difficult time describing details of their students’ mathematical thinking. As open-coding continued, the code “hard-to-tell” emerged. Hard-to-tell included instances where the preservice student teachers noted difficulty in interpretation. As data saturation was reached during final passes of open-coding, hard-to-tell was merged with inadequate description and was renamed “difficult to interpret” since both codes dealt with preservice student teacher difficulty. Difficult to interpret became a theme.

Table 3.
Exemplars for attend, interpret, identify and decide.

<table>
<thead>
<tr>
<th>Attend</th>
<th>“I was hoping if they used the ten frame they would see that it’s a make-a-ten strategy, they would use that. But most of them were just counting on”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpret</td>
<td>“And I think it was because it just said plus one in the problem so they assumed it meant doubles plus one, not like 5+4. That’s the doubles plus one strategy. They would have to use counting on for like 8+1 or 6+1.”</td>
</tr>
<tr>
<td>Identify</td>
<td>“So it is subtracting instead of counting up, which is more sophisticated, than if she was drawn 62 and then added 10 sticks, but it’s still the same process. It’s still, it’s not the most higher level thinking in terms of how you would solve a subtraction problem.”</td>
</tr>
<tr>
<td>Decide</td>
<td>“I might ask, ‘What made you do a number line?’…it looks like instead of using the number line to go up they went down, so maybe why did they do that. I know why they did that, but I would want them to explain to me why they did that.”</td>
</tr>
</tbody>
</table>

As another example of this process, the code “outside influence” was created at the end of the analysis of PLT session #1. Outside influence incorporated previous codes of “mentor teacher influences” and “assessment pressures.” As ongoing analysis continued with
PLT session #2 and PLT session #3, the code “outside influence” also represented district requirements placed on the preservice student teachers and parent influences. Towards the end of the ongoing open coding process, this code became a theme and was renamed “other influences.” The coding process was detailed in memos created in Atlas TI which served as an audit trail for the data analysis. For each of the PLT sessions, once data saturation was reached, emergent themes were identified and revisions to the both the set of PLTs and the hypothetical learning conjectures were completed. This process occurred three times. See Appendix G for a list of the emergent codes following each of the PLT sessions. At the end of the ongoing analysis for PLT session #3, both the hypothetical learning trajectory and the PLTs were revised for future iterations of this study.

**Retrospective data analysis.** Following the conclusion of the ongoing analysis of PLT session #3, the retrospective data analysis phase considered the data as a whole, focusing mostly on pre-service teachers’ discourse as they engaged in the professional learning tasks. The main goal of the retrospective analysis phase was to develop a theory for preservice teachers’ learning to notice. I was interested in answering my research questions regarding how preservice student teachers develop their professional noticing through interactions with the designed intervention focused on developing the preservice student teachers’ specialized content knowledge regarding analysis of their students’ multi-digit addition and subtraction.

In order to code the participants’ discourse during the PLT sessions, participants’ talk was divided into “idea units.” Jacobs, Yoshida, Stigler, & Fernandez (1997) defined idea units as “a distinct shift in focus or change in topic.” The choice was made to code discourse
idea units rather than individual talk turns because collective analysis had emerged as an important aspect of what occurred during the PLT sessions. Further, this was in line with my social constructivist perspective of learning as occurring through interactions with tasks and each other. The preservice student teachers learned as they worked together during the PLT sessions to create meaning out of their students’ mathematical thinking. Their understandings were influenced by each other, thus their professional noticing was analyzed collectively. Idea units allowed me to consider many talk turns, by various preservice teachers, as part of the same idea unit in which they discussed one topic.

As a first step in the retrospective analysis, all of the discourse during the three sessions for each of the two grade level groups was parsed into idea units. For this study, most of the idea units were centered on discussion around particular student work samples. Other idea units included discussions about pedagogical ideas in the classroom and discussions on the preservice student teachers’ development of specialized content knowledge. For example, idea unit 13 from first-grade PLT session #2 began with a discussion about a student work sample. As the group of preservice student teachers worked to interpret the students’ understanding, one of the preservice student teachers drew on evidence from the students’ work on another problem to claim that the student understood tens and ones. A discussion followed related to this idea. All this conversation was considered part of the same idea unit. The groups’ talk eventually shifted away from interpreting the students’ understanding and towards the wording of the problem. At this point the conversation became labeled as idea unit 14 which now focused on the problem and
not the student. Table 4 shows the number of idea units for each of the three PLT sessions for each of the two iterations of the PLTs.

Table 4.
Number of idea units for each of the six PLT sessions.

<table>
<thead>
<tr>
<th>Idea Units</th>
<th>PLT Session #1</th>
<th>PLT Session #2</th>
<th>PLT Session #3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st Grade</td>
<td>2nd Grade</td>
<td>1st Grade</td>
</tr>
<tr>
<td>24</td>
<td>28</td>
<td>22</td>
<td>22</td>
</tr>
</tbody>
</table>

For the retrospective analysis, both open coding and pre-determined coding were used for coding each of the idea units. As with the ongoing analysis, the constant comparative method was employed to confirm and build more refined coding categories. For the pre-determined codes, I returned to the professional noticing codes: attend, interpret, identify and decide. I examined the various instances of these codes that emerged from the ongoing analysis to determine three levels (0, 1, 2) for each of the four components of the professional noticing framework. These developed levels were based on Jacobs’ professional noticing coding scheme (personal communication, April 3, 2013). The results of the ongoing analysis were used to define what the levels meant for the preservice student teachers’ analysis of their student work samples. The characterization of the various levels of the coding scheme for preservice teacher learning is presented as part of the retrospective analysis’ findings in chapter 5.

All idea units, when applicable, received a level for attend (A), interpret (R), identify (I) and/or decide (D). For example, idea unit 13 was coded as A2, R2, I1; there was no code
for decide because the idea unit did not contain discussion around instructional decisions. Idea unit 14 was coded as A2 because in the exchange the preservice student teachers were attending to mathematically significant details in general but they were not discussing students’ work and therefore did not interpret, identify or decide. It was also possible for an idea unit to receive no codes relating to the professional noticing components. For example, idea unit 11 from second-grade PLT session #2 contained pedagogical discourse about students using the standard algorithms for multi-digit addition and subtraction. Their discourse was not related to students’ mathematical thinking, but to pedagogy. This idea unit did not receive any codes for noticing. The data table containing all of the data’s codes has a mark under pedagogical to indicate the content of the discourse exchange for the idea unit (See Table 6 & Appendix I). The additional discourse contents indicated in the table are “discussing student work,” “discussing student thinking,” “development/discussion of SCK,” and “other.”

Research question #2 is focused on specialized content knowledge (SCK), thus evidence of SCK was another predetermined code. For each of the four noticing components, within each of the levels of coding, the designation “with evidence of specialized content knowledge,” was used to highlight instances where the idea unit provided explicit evidence of the preservice student teachers applying their SCK to any of the four components of the professional noticing framework. I used Hill, Ball & Schilling’s (2008) table of mathematical tasks of teaching that draw on SCK (see Figure 2, p. 16) as a guide while coding each of the professional noticing components for SCK. For example, the mathematical task “critique notations and language” was considered part of attend since it deals with noticing.
mathematically significant details. Table 6 contains the subset of Hill, Ball & Schilling’s (2008) table of mathematical tasks items that I coded for evidence of each of the components of SCK.

Table 5.

Mathematical Tasks Requiring SCK as Related to Professional Noticing.

<table>
<thead>
<tr>
<th>Mathematical Tasks Requiring SCK</th>
<th>Related Professional Noticing Component(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critique notation and language</td>
<td>Attend</td>
</tr>
<tr>
<td>Evaluating plausibility of student claims</td>
<td>Interpret</td>
</tr>
<tr>
<td>Evaluate math explanations</td>
<td>Interpret</td>
</tr>
<tr>
<td>Evaluate students’ mathematical algorithms</td>
<td>Interpret, Identify</td>
</tr>
<tr>
<td>Know non-standard methods &amp; common errors</td>
<td>Identify</td>
</tr>
<tr>
<td>Ask productive math questions</td>
<td>Decide</td>
</tr>
</tbody>
</table>

Idea unit 13 contained evidence of SCK in both the preservice student teachers’ attention to the mathematics and their interpretation of the students’ mathematical thinking, thus both A and R received the designation “with SCK,” which was denoted with an asterisk in the coding table (See Table 7 below). In the following excerpt from idea unit 13, two of the preservice student teachers attended to and interpreted the reason for the students’ different approaches to subtraction problems (See Figure 6).

K: And there we can see he took away the eight by adding…by drawing out the ones.
D: Um hmm. But it’s interesting that he did it there are not up there.
K: I know.
D: I mean I guess four is a smaller number so you could probably just count back.
Attending to and recognizing that students’ strategies can depend on the size of the numbers requires SCK, thus this exchange was labeled as evidence of SCK for both attend and interpret. The professional noticing component identify considers the preservice student teachers’ identification of the level of sophistication of their students’ strategies. Identify draws upon both SCK regarding non-standard multi-digit addition and subtraction strategies as well as KCS regarding the progression of levels of sophistication. Instances where Identify was coded as with evidence of SCK were instances where the preservice student teachers drew on their SCK regarding multi-digit addition and subtraction strategies when identifying the level of sophistication of the strategy in question. For idea unit 13, the preservice student teachers did not exhibit SCK in their identification of the levels of sophistication of the students’ work samples (occurred in the idea unit earlier than the exchange presented above), thus no asterisk is provided with identify. Idea unit 14 did not contain any exchanges with evidence of SCK and therefore did not receive the designation.

Ball et. al. (2008) described evaluating students’ strategies and explanations as teaching tasks that require SCK. Thus there most likely were instances where the preservice student teachers drew on their SCK but they did not explicitly talk about it in the discourse.
exchanges. The decision to code for SCK required that idea units contained explicit evidence of the preservice student teachers’ drawing on SCK to be labeled as “with SCK.” Table 6 contains examples of three exchanges coded as R2; the first is an interpretation where there is no evidence of the preservice student teachers’ SCK, the second is an exchange where SCK may have been present, but it was not explicit and therefore was not marked. The third is an exchange where SCK is explicit.

Table 6.

*Examples of discourse exchanges coded as R2.*

<table>
<thead>
<tr>
<th>No Evidence of SCK</th>
<th>T: This child did a subtraction problem and I think they used the counting on strategy because they still drew out all the numbers. (Comment from a correctly answered work sample for 65 – 34.)</th>
</tr>
</thead>
</table>
| No Explicit Evidence of SCK                                                      | A: It doesn’t make sense to do 100 minus 60 minus 3 minus 4. You wouldn’t get 41. 

  

  T: He noticed there wasn’t a 100 and a 63, he just…he jumped to the tens place and said that was 60. But at least he is getting that 60, not 6. 

  A: Yeah, also I liked that he was getting that 6 is 6 tens, so it’s 60, it’s not just a six. (Exchange around an incorrectly answered work sample for 104 – 63) |
| Explicit Evidence of SCK                                                        | T: Yes, with an unknown number and he used addition so he separated the tens and ones and did a tens stick and one circles and just counted up. So, I guess it’s still…it’s kind of counting but it’s also decomposing in a way. 

  K: When I looked at this, I thought it was interesting that he…So he did the 34 here. He knew that he had to get to 65 so instead of counting up to 34 to 50, he knew…the way he did it was interesting, like he went in and did tens first. 

  D: So he just held 34 in his head and did…Like 34, 54, 64 (Exchange around a correctly answered work sample for 65 – 34) |
For the first example, the student had drawn out 65 circles and crossed out 34. The preservice student teacher developed a plausible interpretation of the students’ strategy, but the interpretation lacked evidence that the preservice student teacher drew on any SCK. Interpreting the work sample did not require any expert teacher knowledge, thus, the preservice teacher could be drawing solely on common content knowledge for her interpretation. In the second example it is unclear as to whether the preservice student teachers were using SCK around place value to interpret the students’ thinking. They did not explicitly discuss place value. Thus, while they may have drawn on their understanding of place value to interpret the students’ work, it was not explicit and therefore this exchange was not coded as “with SCK.” In the final example, the preservice student teachers interpreted a students’ counting up strategy. Their discussion begins with one of the preservice student teachers connecting the strategy to the concept of decomposition. Decomposition was one of the strategies discussed during the PLT sessions; an understanding of decomposition requires SCK. Because the preservice student teacher explicitly connected her interpretation of the students’ mathematical thinking to her SCK regarding decomposition this exchanged was labeled as “with SCK.” To reiterate, explicit evidence was required in order for an idea unit to be labeled as “with SCK.”

During the ongoing analysis phase, the importance of collective analysis emerged and I realized that sometimes the collective analysis led to a change in levels of noticing within a talk exchange; I termed this concept “collective influence.” I considered this to be a possible answer for research question #3 and hence, was interested in analyzing it further. Thus, for the retrospective analysis, collective influence became a predetermined code. The code
“collective influence” was used for instances where the idea unit contained evidence of the preservice student teachers developing a deeper ability to notice via interactions with each other. Differently from the designation “with SCK,” “collective influence” was not tied to individual components of the professional noticing framework.

Idea unit 13 was coded as collective influence because at the beginning of the talk exchange, one of the preservice student teachers indicated difficulty in understanding her students’ strategy, but as the exchange progressed, and specifically due to a comment about tens and ones from another preservice student teacher, the group developed a plausible interpretation of the students’ mathematical thinking. The highest level of professional noticing achieved by the group was the code assigned to the idea unit. The entire idea unit was designated as collective influence which meant that sometime during the idea unit exchange, collective influence occurred. Idea unit 14 did not contain any evidence of collective influence and thus did not receive the designation. Table 7 contains all the coding information for the two idea unit examples discussed above and serves as a visual depiction of the data analysis table created for all idea units (See Appendix I). Idea units designated as containing “collective influence” were colored burgundy. Tables such as this one will be presented throughout chapter 5 as part of the study’s findings.

In addition to the coding around the professional noticing components, codes were developed based on emergent themes as they related to research question #3 which considered the factors that influenced the preservice student teachers’ noticing. As themes emerged, categories were developed and refined. For example, characteristics of the student work samples emerged as an important factor possibly influencing the preservice student
teachers’ noticing. Once its importance emerged, I went back to each idea units and noted the characteristics of the student work samples being discussed. I then compared the characteristics to the coded levels of professional noticing and with instances of collective influence. For example, the discussion from idea unit 13 was based on student work for which the uniqueness of the students’ approach interested the preservice student teachers.

Table 7.
*Complete coding information for second-grade PLT session #2 idea units 13 & 14*

<table>
<thead>
<tr>
<th>Idea Unit</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attend</td>
<td>2*</td>
<td>2</td>
</tr>
<tr>
<td>Interpret</td>
<td>2*</td>
<td></td>
</tr>
<tr>
<td>Identify</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Decide</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discussing Student Work</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Discussing Students’ Thinking</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pedagogical</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Development/Discussion of SCK</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Evidence of SCK

Collective Influence

All of the aforementioned coding processes are detailed via memos in Atlas TI. They serve as an audit trail for the data analysis. Once data analysis saturation was reached, the answers to the research questions were formulated and my theory for preservice teacher noticing was developed.

**Trustworthiness**

For the purposes of this study, trustworthiness is discussed using Merriam’s (2001) categories of internal validity, reliability and external validity. She defines internal validity as
the extent to which research findings match reality, reliability as the extent to which findings can be replicated and external validity as the extent to which findings are generalizable (Merriam, 2001).

**Internal validity.** Internal validity can be met by spending an extended time in field, triangulating data, discussing researcher bias, completing an external audit of ones work, peer debriefing and/or member checking. Creswell (2007) suggested using at least two of these processes in qualitative studies. In order to ensure internal validity for this study, the methods of triangulation, discussion of researcher bias and peer-debriefing were utilized.

Triangulation is using multiple data sources as a means to ensure the data collected is saying the same thing. During analysis, I looked across my data sources and triangulated using student work, preservice student teachers’ written PLT reflections, and transcripts of the PLT sessions in order to ensure the emerging findings were consistent. I also clearly stated my biases and beliefs as a researcher which served to situate the findings of this study. The two college supervisors that I partnered with served as peer-debriefers during the ongoing data analysis phase. During the post-session debriefings with the college supervisors, the initial hypothetical learning conjectures for the session were examined for potential revisions. In addition, emergent themes from the session were discussed and negotiated. During the debriefings, the college supervisors also provided suggestions for revisions to the next PLT. In addition, since this is a dissertation study, my committee also served as peer-debriefers. Throughout the study, I had regular meetings with my committee co-chairs. They read drafts and provided input to my descriptive coding scheme, revised hypothetical learning trajectory, and revised PLT designs. Also, during the duration of my
study and analysis, I met periodically with other members of my committee to discuss my progress and obtain feedback.

**Reliability.** According to Merriam (2002), the best way to increase the reliability of research findings is to make sure the results are consistent with the data. This can be done using triangulation of data, peer examination, and creating an audit trail (Merriam, 2002). As mentioned above, I triangulated my data and had peer reviewers. In addition, I created an audit trail of my data analysis process using memos in Atlas TI (Bernard & Ryan, 2010). The memo tool in Atlas TI, helped me keep track of my analysis as I progressed through the iterations of the study by maintaining time stamps for the memos and all created codes.

**External validity.** The question of how to generalize qualitative research is one debated between many researchers. Stake (1978) referred to naturalistic generalization as the results leading the reader to expect that something similar will occur under similar circumstances with similar participants; he warns that this is only possible if procedures for internal validity and reliability are in place. Similar results were found for the two iterations of this study which serves to increase naturalistic generalization. Merriam (2002) defined reader generalizability as when “readers themselves determine the extent to which findings from a study can be applied to their context” (p. 29). Both Merriam (2001) and Creswell (2007) discussed the use of rich, thick descriptions as a means of increasing generalizability. Thus, I have employed rich, thick descriptions throughout the results sections of this study.

**Subjectivity Statement**

I served both as the designer and as a participant observer for this research project. I define myself as a social constructivist in line with Vygotsky. At the time of the study, I was
a mathematics faculty member at the same institution as both the participants and college supervisors. At the institution, students cannot major in elementary education. They may major in any subject and obtain K-6 licensure in addition to the major. Due to this organizational structure, content educators are housed in content departments. Thus, my role in the mathematics and computer science department was both to teach traditional mathematics courses but also to work with and teach licensure students. Since the focus of my doctoral studies is on preparing elementary teachers to teach mathematics, during my six-year tenure at the institution, my role in the department shifted to that of an elementary mathematics teacher educator. In that capacity, I developed and taught a sequence of mathematics content courses for prospective elementary teachers focused on development of specialized content knowledge. Two of the seven preservice student teacher participants in this study took the first course in the sequence with me (they were in different classes from each other).

**Ethical Issues**

Consent for research was obtained from both the North Carolina State University Institutional Review Board (IRB) and the Meredith College IRB prior to the implementation of this study. Each participant signed a legal informed consent form before they participated in the study. There were no risks anticipated for participation in the study. All participants were assigned pseudonyms for use with written, audio, or video data. All data was stored on my work computer. Written documents were securely stored in my office in the Science & Math Building at my institution. Back up files will be stored on an external hard drive also located in my office.
Limitations of the Study

While the goal is to minimize limitation, I recognize that limitations are unavoidable in any research study. This study is subject to limitations related to the role of the researcher, issues relating to time & iterations of the design, data-analysis, and generalizability of the findings. I was the course instructor for the preservice mathematics content knowledge course taken by two of the preservice student teachers who participated in the study, Ashton and Tammy; both of whom were high performers in the class. A limitation is that their responses may have been influenced by their perceptions about my beliefs. While it is hoped that the participants indeed shared openly, this cannot be known for sure. In order to minimize this problem, I made sure that the participants understood that their participation in the study in no way jeopardized their standing within the School of Education or with any grades associated with their student teaching field experience.

The amount of time spent with the preservice student teachers is another limitation of this study. Three hour-long sessions was not a large amount of time. Had the preservice student teachers been available to meet more often, the results of study may have been different. In addition, since this study was not a required aspect of the preservice teachers’ student teaching field experience, there was little incentive for them to fully complete and follow directions for the PLTs. This limited the implementation fidelity of the designed PLTs. Relating to time, Cobb et. al. (2003) explained that design research studies should be iterative. Since this is a dissertation study, there was not time to iterate the redesigned intervention. However, the intervention used in this study was completed twice in the sense
that there were two groups of participants completing the designed intervention separately. The results were similar, which adds to the trustworthiness of the findings.

The lack of iterrater coding is another limitation of this study. This study was a graduate research study completed individually. In order to increase reliability of the results, the data could be coded by another coder to establish iterrater reliability of a desired percentage. Another possible limitation deals with the participants themselves. The participants are all of the same gender and all hail from the same institution of higher education. This study may have had different results with preservice student teachers from another institution. This limitation relates to the overall limitation of generalizability of the findings. While the very specific nature of the study limits the amount of generalizability that can be claimed, I claim naturalistic generalization as defined by Stake (1978). I expect that something similar will occur under similar circumstances with similar participants. Rich, thick descriptions, transparent data analysis and other aspects of trustworthiness as discussed above work to ensure naturalistic generalization could occur.
CHAPTER 4

Introduction to the Results

I report the findings from this design study in two chapters. In Chapter 4, I detail the implementation of the professional learning tasks, and accompanying ongoing analyses including the evolution of revisions to both the PLTs and the hypothetical learning trajectory as they unfolded during implementation. Chapter 5 details findings from the retrospective analysis of the study organized around my research questions. These two chapters combine to provide a foundation for an explanation of elementary preservice student teachers’ learning to professionally notice that is discussed in Chapter 6. Examples included in Chapters 4 and 5 were selected to exemplify findings from the analysis. They are representative of the two groups of participating preservice student teachers, though some exemplars were more articulate and more clearly illustrated the essence of the ideas.

Implementation of the Professional Learning Tasks

The hypothetical learning trajectory presented in chapter 3 was based on a review of the literature and my professional experience in working with elementary preservice teachers. Professional Learning Tasks were designed and implemented with the goal of assisting the preservice student teachers as they moved along the hypothetical learning trajectory. As the PLTs were conducted, the preservice student teachers’ responses were reviewed in light of the hypothetical learning conjectures associated with each PLT. Oftentimes, these observations led to revisions to the PLT design and ultimately to the revision of the hypothetical learning trajectory. The following sections discuss the ongoing analysis affecting the implementation the PLTs and the evolution of the hypothetical learning
trajectory guiding the study. For each of the PLTs, I describe the data resulting from implementation of the PLTs that led to changes to the design of the PLTs. I then draw conclusions linked to the hypothetical learning trajectory of the study, and describe how these influenced the remainder of the PLTs.

**Professional Learning Task #1**

**Preparation and design.** The initial design of the PLTs was completed prior to participant selection. Following the choice of first and second grade preservice student teachers as participants for the two iterations, the PLTs were updated to reflect multi-digit addition and subtraction, a common topic in both first and second grade that the preservice student teachers indicated they would be teaching during their student teaching field experiences. In order to support the preservice student teachers as they developed their professional noticing, each of the PLTs focused on one aspect of specialized content knowledge as it related to multi-digit addition and subtraction. I conjectured that by developing the preservice student teachers’ SCK, they would be better able to interpret their students’ mathematical thinking and to identify levels of sophistication of their students’ strategies. PLT #1’s specialized content knowledge focus was on different types of addition and subtraction story problems. I conjectured that the preservice student teachers would not recall different types of addition and subtraction story problems and/or how problem type affects students’ choices of strategies. Along with the directions seen in Table 8, the preservice student teachers were provided sample story problems from their district textbook.

The PLTs’ college supervisor guide reflected the specialized content knowledge focus on multi-digit addition and subtraction (See Table 8). The preservice student teachers had
previously been exposed to different terminology associated with problem types: Beckman’s (2011) terms, those from Cognitively Guided Instruction (Carpenter et al., 1999), and the Common Core State Standards for Mathematics’ terminology (CCSS-M) (NGACBP, 2010). The preservice student teachers were completing their student teaching field experience under the second year of their state’s implementation of the CCSS-M, so I chose to use the CCSS-M terminology. I adapted the CCSS-M problem type table (NGACBP, 2010, p. 88) to include all of the aforementioned terminology (See Appendix A). This problem type table was given to the college supervisors at their pre-PLT session #1 meeting and to each of the preservice student teacher participants during their first PLT session.

Table 8.

*PLT Session #1.*

<table>
<thead>
<tr>
<th>PLT</th>
<th>Preservice Student Teacher Directions</th>
<th>College Supervisor Guide</th>
</tr>
</thead>
</table>
| One | Choose two story problems: one that lends itself to an addition strategy and one that lends itself to a subtraction strategy. Ask students to solve them in two ways.  
1. Solve the task yourself and write out ways you anticipate your students will solve the task.  
2. Administer the task and monitor your students as they are working  
3. Collect all of your students’ work  
4. Choose six student responses that you would like to bring to the reflection session. Briefly describe their solutions. | Begin by having the student teachers share their task and reasons behind their choices as to which of their students’ work to bring to the session.  
1. What strategies did your students use to solve the task?  
2. What did you find surprising or unexpected in your students’ work?  
*Lead a discussion about their initial anticipation of the ways their students would approach the problem.*  
3. What is the mathematics embedded in each of their strategies?  
*Lead a discussion about different types of addition/subtraction problems and differing levels of complexity depending on the type.*  
*If time, begin discussion levels of sophistication of strategies.*  
4. What questions could you ask to help your student reflect on their strategy?  
*Lead a discussion on how to probe student thinking without guiding their work and on how to describe student work without projecting their knowledge onto the solution. Suggest that the student teachers take notes while monitoring their students as they complete tasks.* |
Prior to the implementation of PLT session 1, I developed a handout for the college supervisors (See Appendix E) and met with them both together to discuss the layout and goals of the session. The only feedback received from them was some concerns about timing. I suggested that they spend less time on #3 if necessary, but asked them to be sure to move to #4 before the end of the session.

**Overview of PLT session #1.**

**First-grade group.** All four of the first-grade preservice student teachers were present at the first PLT session. Neither Donna nor Kelli followed the PLT directions about task implementation in their classrooms. Donna had not started story problems with her students so she brought in a worksheet that she had designed that asked students to look at single-digit addition problems and label the most appropriate strategy out of: “make-a-ten,” “counting on,” and “doubles plus one.” At Kelli’s school, students are separated by ability groups and provided instruction by one of the first-grade team members. Kelli’s mentor teacher is assigned to one of the “on-grade level” groups. For this task, Kelli pulled out six of her students and had them complete the task.

None of the first-grade preservice student teachers completed the written reflection. Since prior reflection had not occurred, the first-grade college supervisor used the first few minutes of PLT session #1 for individual, written reflection. After the reflection time, the college supervisor asked if there was anything the preservice student teachers wanted to share. They then had a discussion about factors that they believed to have influenced their students during the implementation of the tasks in their classrooms. Following that discussion, the college supervisor proceeded through the questions as outlined in the PLT
guide. The preservice student teachers took turns discussing the work samples they brought to the session. In all, the first-grade group discussed 10 student work samples, some from each of the four preservice student teachers. The group was not able to spend as much time as was intended discussing problem types or levels of sophistication of strategies. The last few minutes of the first-grade group’s session were spent discussing the importance of questionings students. The first grade preservice student teachers indicated their intention to question their students while implementing their task for PLT session #2.

Second-grade group. All three of the second-grade preservice student teachers attended the first PLT session. Only Lacey completed the written reflection. Ashton had not yet started teaching mathematics, and though I told her she could bring student work from her mentor teacher, she said she left it at home. As a result of only having two participants with student work samples, the college supervisor for the second-grade group structured her session differently than the first-grade college supervisor. She had each of the preservice student teachers present their first story problem that they gave to their students and then went through questions #1-4 from the supervisor guide individually with that student. This structure meant that the preservice student teachers waited until they were called on to speak, but this also meant that the session stayed focused on analysis of the student work. During the second-grade session, 15 student work samples were analyzed. As with the first-grade group, there was not much time for discussing problem types or levels of sophistication. Like the first-grade group, the second-grade preservice student teachers recognized the need for questioning their students and indicated their intention to do so while implementing their task for PLT session #2.
**Emergent themes.** The themes that emerged during PLT session #1 are discussed below. The themes are organized based on the professional noticing framework. For PLT session #1, three themes emerged that related to Attend: articulated choice, affect, and surprise. Three themes emerged that related to Interpret: plausible interpretations, difficult to interpret, and influenced interpretations. There were no emergent themes associated with Identify. There was one emergent theme that related to Decide: importance of questioning. In addition to the themes based on the professional noticing framework, there were three other emergent themes: other influences, reliance on the standard algorithm & multiple strategies, and the role of SCK. Instances could be coded with multiple themes, for example affect and other influences. Many of the samples provided below may illustrate multiple themes, but the samples were chosen because they clearly evidenced the specific theme under explanation.

**Attend: Articulated choice.** Differently from what was expected, both sets of preservice student teachers were able to articulate their choices as to how they chose which samples of student work to bring to the PLT session. Two main sub-themes emerged from their discussion. The preservice student teachers chose their work samples based on wanting to bring a variety of responses and/or choosing samples of student work that surprised them. Both Kelli and Tammy indicated choosing their responses based on the variety of the responses. Tammy stated, “I just wanted a wide range of students.” Leah, Lacey, and Tara indicated choosing responses that surprised them by being different from what they had anticipated. Donna chose responses that represented a surprising mistake that many of her students made.
Attend: Affect. As a part of the professional noticing framework, attend deals with attending to mathematically significant details. However, there were instances where the preservice student teachers attended to their students’ affect, not the mathematics. The first-grade preservice student teachers discussed what they believed to have influenced their students during the implementation of the tasks in their classrooms. Kelli discussed her students’ stress and their urgency to complete the problem. Leah shared that her cooperating teacher told the students that they were completing an assignment for Leah’s college. Leah attributed some of her students’ incorrect solutions to stress associated with having to complete college work. Donna believed that her students rushed during their assignment because she posed the task just before “free choice” centers. From their exchange, it was clear that the preservice student teachers initially attended to their students’ work, not based on the mathematics, but on affect. They blamed incorrect solutions on affective factors rather than their students’ mathematical understanding.

Attend: Surprise. Both groups of preservice student teachers attended to their students’ work based on surprise. The first-grade preservice student teachers were surprised at their students’ preferred choice of strategy and their non-standard solutions. Donna shared the worksheet of her only student to correctly identify the best strategy for each problem. The student had written dots under the numbers. Donna noticed that his written responses conflicted with the drawn dots. She approached the student and discovered that like the majority of her students, this student had actually used counting on to get the answers and then gone back to label the strategy he believed Donna wanted him to say. However, for the
first problem that was supposed to be a “make a 10” strategy, the student solved the problem by compensating the 4 to make a ten with the 8 (See Figure 7). Donna noticed,

“Instead of doing the tens frame, he said 8 + 2 at the top is 10 and he took away 2 from the 4 and then he wrote his new equation, which was 10 + 2. And so I was like, ‘Okay, well that works.’”

Donna was surprised both at her student’s preferred choice of strategy and at her student’s application of the tens frame manipulative to symbolic notation. It is important to note that none of the other preservice student teachers indicated an awareness of these strategies mentioned by Donna. No one else referred to them by name.

All of the second-grade preservice student teachers expected their students to use the standard algorithm to solve their problems. What surprised the second-grade preservice student teachers was their students’ choice of alternate strategies. Some of Lacey’s students used number lines to solve their problems, which she did not anticipate because she had not covered number lines during her tenure as the students’ teacher. As she expected, many of her students used sticks and dots to solve the subtraction problem, but she was surprised at one of her students’ use of expanded form and another’s ones and tens algorithm. Figure 8 shows another of Lacey’s student’s addition solutions. Lacey says,
“I thought this one was interesting because they added their ones and then added their
tens. I haven’t seen anyone do that…this student said, ‘Well 9 + 5 is 14 so I’m gonna
write 14, and then 60 + 20 is 80 and I’m gonna add 14 and 80 and that gives me 94’.”

Lacey shows that she is able to interpret her student’s strategy, but this strategy was new to
her and she did not seem to consider it as a strategy to be shared amongst her students.

\[
\begin{array}{c}
+65 \\
\hline
29 \\
\hline
14 \\
\hline
80 \\
\hline
94
\end{array}
\]

*Figure 8.* Lacey's student work sample for attend: surprise.

*Interpret: Plausible interpretations.* Some of the first-grade preservice student
teachers developed plausible interpretations of their students' mathematical thinking. One of
Kelli’s students invented his own strategy for solving his problems (Figure 9). Kelli noticed
the uniqueness of his strategy and asked him to explain the strategy while she audio-recorded
his explanation. The student explained,

“\begin{quote}
I had 18 and 29 so I subtraction 2 from 29 and it equaled 7, then I plussed that 2 for
the 18 and equaled 20 and I kept doing the same thing and the same thing over and
over and I finally got the answer and I found out that the strategy was working really
good.
\end{quote}

Kelli noticed that if the student continued to use this strategy, it could be quite inefficient.
She stated,
“I knew that he was gonna get stuck on the next one ‘cause the next answer was 71, so I was like, ‘How is he going to do this when we get to the 45 oak leaves and 26 maple leaves?’ So I just left him alone for a little bit.”

Her student ended up making an error subtracting 3 when he wrote to subtract 2 which caused him to get the incorrect answer. Kelli did not catch the mistake, and instead of helping her student to find it, she pushed him toward an alternate tens and ones strategy. Through this exchange, it is evident that Kelli attended to the mathematics behind her student’s solution and developed a plausible interpretation of his surprising strategy.

Figure 9. Kelli’s student work sample for interpret: plausible interpretation.

**Interpret: Difficult to interpret.** Both groups often experienced difficulty when trying to interpret their students’ mathematical thinking. The following exchange between Leah and the college supervisor illustrates both Leah’s surprise at her students’ approach to solving what Leah believed to be a subtraction problem, and her difficulty at interpreting her students’ mathematical understanding.

Leah: So it’s like they kind of got confused on it just because it wasn’t worded, I guess exactly how most of theirs are. It was kind of…it could have gone
either way, as addition or subtraction, just depending on the way they wrote the problem.

CS: Can you read it?

Leah: Yes. It says, “John has 15 buttons; 8 are red, the rest are blue. How many blue buttons does John have?” So a lot of them added 15 and 8 and then got 23, instead of doing 15, take away 8 is 7.

CS: Um hmm. Okay. And if they had turned it into an addition problem, because you had said some of them did turn it into an addition problem.

Leah: Um hmm. Some of them did.

CS: So how did that…

Leah: Some of them did 8 + and then they left a blank, equals 15, and then they did their math mountain and drew the dots up from 8 to see how many were left.

Leah exhibited difficulty interpreting the mathematical understandings that would lead students to add instead of subtract for this problem or what would lead them to use an add-up strategy.

Tara had a difficult time interpreting some of her students’ solutions. She was mainly focused on the correctness of the strategy and not on the mathematics behind her students’ work. In Figure 10, Tara’s student is trying to use a place value chart to organize the addition problem: 189 + 245 + 26. Tara did not recognize the potential understanding present in the values in the final row. 80 + 40 does give one 12 tens. The college supervisor prompted her many times to see if Tara understood the place value understanding present in an answer of 12, but Tara persisted in seeing the entry as a mistake, because the rest of the work does not lead to the correct answer (even though the correct answer is present via the standard algorithm in method 1). Her student’s breakdown in logic can be seen in the top right circle. The student added up the hundreds, tens and ones as if they were all ones. Again, Tara does not interpret this correctly.
Interpret: Influenced interpretations. Both groups showed evidence of their personal mathematical understandings and beliefs influencing their interpretations of their students’ mathematical thinking. Leah had a difficult time interpreting the thinking of her student that used a number line to count backwards. She said she would have counted up to solve the problem if using a number line. She thought the student was checking his work, not actually solving the problem with a strategy that would produce an answer. To Leah, number lines were only to be utilized in increasing order.

Tammy indicated surprise at one of her non-native speaker’s apparent grouping by fives (See Figure 11).

“She didn’t speak English at the very beginning of the year, so this is like a huge… how to group like this in fives. But um, so she’s…she may be getting that from somewhere else but I thought that was neat how she did that, even though she got the answer wrong. But, she’s trying to understand grouping, I think, from what I’ve seen.”

Tammy noticed that the strategy was different than the way most of her students directly represented their solutions. While Tammy was attending to the mathematics, she drew on her
personal knowledge of mathematics to draw a conclusion that her student understood grouping.

Figure 11. Tammy's student work sample #1 for interpret: influenced interpretations.

Tammy also drew on her mathematical experience when interpreting the work in Figure 12. She stated, “18 + 5 = and there was a question mark and it was question mark equals 23, so it was kind of like he was starting to do a little algebra there.” She did not notice that she told the students to use a symbol to represent the unknown as part of her directions.

Figure 12. Tammy's student work sample #2 for interpret: influenced interpretations.

**Decide: Importance of questioning.** As anticipated, in both groups, the theme of the importance of questioning their students arose. Both groups realized that oftentimes they cannot develop a full interpretation of their students’ mathematical thinking without
returning to the students. The first-grade student teachers realized that they were reading into some of their students’ solutions and that they sometimes needed to take notes and ask questions while they were observing their students’ work. The first-grade college supervisor had the preservice student teachers develop generic questions they could ask their students to probe their thinking, but she did not tie the questions directly to any of their student work samples. By the end of the session, all of the first-grade preservice student teachers indicated that they would be sure to question their students and have them explain their thinking, and that they would take notes while their students were working.

Throughout the PLT session, the second-grade college supervisor asked the preservice student teachers what questions they might be able to ask their students to obtain a better understanding of their children’s mathematical thinking. Some of the questions the preservice student teachers came up with were vague, but others were good probing questions. For example, Leah wanted to know, “Why did you choose to do that in the first problem but not in this problem? What…what about this problem made you choose that separate strategy?” She was interested in why her students chose particular strategies over other ones. Other questions involved specific mathematics they noticed, usually relating to an incorrect answer. The second-grade preservice student teachers realized that questioning their students could assist them when interpreting their students’ mathematical thinking.

**Other: Other influences.** The theme of other influences emerged from both groups of preservice student teachers. As mentioned before, at Kelli’s school, mathematics is taught to students based on ability grouping. This factor is outside of Kelli’s control, but it possibly influenced the variety of samples she was able to bring to the PLT session. For the second-
grade group, all three of the preservice student teachers were in classrooms where the standard addition and subtraction algorithms had been presented to the students prior to the beginning of their field experience. This was an instructional influence. While Ashton did not bring in any student work samples, she shared a problem she was having with her students on the board (See Figure 13). She explained,

“They only used the standard algorithm. And when I always prompt them like, ‘Try and use another one,’ and they’ll actually like write out like, ‘Okay, 34 circles plus 24 circles equals,’ and then they count it. That’s like, you know, the strategies are really limited. But one thing that a lot of them did was when they read out the problem, they had put like 24 + 3 like that. And then they would add 4 and then 2 + 3, 5 and then they’d say 54.”

Ashton explained how her cooperating teacher may have influenced the students’ responses because the cooperating teacher was opposed to manipulatives and only taught the standard algorithm.

![Figure 13. Ashton's board work example for other: other influences.](image)

**Other: Reliance on the standard algorithm and multiple strategies.** It was previously mentioned that all three of the second-grade preservice student teachers were in classrooms where the standard algorithms for multi-digit addition and subtraction had been taught prior to their teaching the class. The preservice student teachers named this fact as a reason that their students relied on the algorithm and struggled to develop alternative
strategies. Tara provided her students with space for 4 methods of solving her problems, so she had the largest variety of responses. She noticed that her students most often began with the standard algorithm and then filled in the other three boxes. For the most part, they came up with multiple strategies for the sake of filling in the spaces, not actually trying to come up with methods that worked. Tara noticed that her students would do one of two things. Either they would find the answer with the standard algorithm and stick with that solution throughout the rest of their strategies, even if the strategy gave them a different solution, or they would get different answers with different strategies and not notice. The purpose behind developing multiple strategies was lost for the second-grade students in the preservice student teachers’ classes because of their reliance on the standard algorithms.

**Other: Role of specialized content knowledge.** As a part of the PLT #1 design, the college supervisors focused part of their discussion time around developing specialized content knowledge surrounding problem types. This exchange from the second-grade group occurs at the very end of the session so there was not enough time for the group to have a discussion about how the problem type affects students’ choice of solution strategy (Figure 14).

CS: How could you help the student that did the 4 minus the 8? Do you have ideas? I’m assuming that’s a mistake that many students make. That’s a mathematically…how can you help that student?

Tara: I can just ask him, um, “What do you notice about the location of these numbers and which number are you taking away from?” instead of saying, “What is the difference between 84 and 8,” say, “What number are you taking away from instead of…” cause, I mean, I think it’s just the way you phrase the question. If you phrase it by saying, “It’s 284, take away 338. Why would you go back…backwards just for the units place?” So I think if I just sat down and explained to him how he took away 3 from 8 but he took
away 4 from 8 for the units only. Maybe that would help him realize that he…he can’t like flip flop and the way is to subtract. So…

CS: But is this a…a removal subtraction problem? And that’s…that’s a question I would ask too. What type of problem is this?

Tara: Yeah. It’s not a takeaway.

Figure 14. Tara's student work sample for other: role of specialized content knowledge.

The college supervisor helped Tara realize that the type of problem may have affected her student’s choice of strategy. After the first-grade group’s discussion on problem types, Leah specifically mentioned a goal of focusing on the problem types when choosing which problems to give to her students. These examples illustrate how PLT #1’s focus on problem type assisted the preservice student teachers in developing SCK.

Changes to the hypothetical learning conjectures. The results of the analysis of PLT session #1, led to changes in the hypothetical learning conjectures for PLT session #1 (See Table 9). The first conjecture did not prove to be true. The preservice student teachers were able to articulate their reasons for choosing their work samples; they chose based on a variety of responses and/or responses that surprised them. The second conjecture was partially realized. The preservice student teachers attended to their student work with descriptions, written and verbal, focused on procedural correctness. Their surprise
encompassed more than just non-standard solutions; they were also surprised at their students’ preferred choices of strategies and the mathematical features of their students’ work.

Table 9.

*Revisions to the PLT #1 hypothetical learning conjectures.*

<table>
<thead>
<tr>
<th>PLT #1 Hypothetical Learning Conjectures</th>
<th>Revised PLT #1 Hypothetical Learning Conjectures</th>
</tr>
</thead>
<tbody>
<tr>
<td>The preservice student teachers have a difficult time choosing and articulating their choices as to which student work to bring to the reflection session.</td>
<td>When making choices about what student work to bring to the reflection session, the preservice student teachers choose a variety of responses and/or surprising responses.</td>
</tr>
<tr>
<td>They initially attend to their student work with descriptions, written and verbal, focused on procedural correctness and/or surprise at non-standard solutions.</td>
<td>They initially attend to their student work with descriptions, written and verbal, focused on procedural correctness, surprise at non-standard solutions &amp; students’ preferred choices of strategies, and/or mathematical features of their students’ work.</td>
</tr>
<tr>
<td>When asked to describe the mathematics behind their students’ solutions, the preservice student teachers interpret their students’ work in light of their personal experiences, beliefs and mathematical knowledge. The preservice student teachers recognize their tendency to project themselves onto their students’ solutions and begin to understand how to interpret the work based on the mathematics actually present.</td>
<td>When asked to describe the mathematics behind their students’ solutions, the preservice student teachers address mathematics significant features but their interpretations often draw on personal beliefs and experiences. The preservice student teachers’ interpretations alternate between plausible explanations and noted difficulty at describing their students’ work.</td>
</tr>
<tr>
<td>Through the discussion facilitated by the college supervisors, the preservice student teachers draw on their newfound SCK regarding problem types to interpret their students’ mathematical thinking.</td>
<td>Through the discussion facilitated by the college supervisors, the preservice student teachers begin to merge SCK about problem types and different strategies with their interpretations of their students’ work.</td>
</tr>
<tr>
<td>The discussion helps the preservice student teachers recognize the importance of asking their students probing questions and taking notes in order to assist them in interpreting their students’ mathematical thinking.</td>
<td>The discussion helps the preservice student teachers recognize the importance of asking their students probing questions and taking notes in order to assist them in interpreting their students’ mathematical thinking.</td>
</tr>
</tbody>
</table>
The preservice student teachers’ interpretations often focused on mathematically significant details, but they used their personal mathematical experiences to read into their students’ thinking. They did not always realize their tendency to read into the solutions, as had been conjectured. The preservice student teachers did not reach the level of interpretation as anticipated in the hypothetical learning conjectures based on their newfound SCK about problem types. They were at the beginning stages of applying problem type SCK to their interpretations of their students’ mathematical thinking. They began to draw on their SCK regarding not only problem types but also knowledge of different strategies that could be used to solve multi-digit addition and subtraction problems. Oftentimes, the preservice student teachers noted difficulty in interpreting their students’ thinking, usually due to lack of questioning their students. The need to question was a major theme of the session, as had been conjectured.

These results to the ongoing analysis of PLT Session #1 also led to changes in the hypothetical learning conjectures for PLT Session #2 and ultimately to changes in the design of PLT Session #2. See the next two sections for descriptions of the changes to PLT Session #2.

**PLT Session #2**

**Preparation and design.** After the analysis of both PLT #1 sessions, PLT session #2 was redesigned (See Table 10). The preservice student teachers were given specific directions as to which types of multi-digit addition and subtraction problems to pose to their students. This change was made in order to help the preservice student teachers note how their students’ strategies related to the problem type. The only other change in the preservice
Table 10.

Revisions to PLT Session #2.

<table>
<thead>
<tr>
<th>PLT</th>
<th>Preservice Student Teacher Directions</th>
<th>College Supervisor Guide</th>
</tr>
</thead>
</table>
| Two  | Choose two story problems: one that lends itself to an addition strategy and one that lends itself to a subtraction strategy. Ask students to solve them in two ways.  
1. Solve the task yourself and write out ways you anticipate your students will solve the task.  
2. Administer the task and monitor your students as they are working  
3. Question your students as they are working and take notes  
4. Collect all of your students’ work  
5. Choose six student responses that you believe represent different mathematical approaches to the problem. Briefly describe their solutions. | Begin by having the preservice student teachers share their task and reasons behind their choices as to which of their students’ work to bring to the session.  
1. What strategies did your students use to solve the task?  
2. What is the mathematics embedded in each of their strategies?  
3. What questions could you ask to help your students reflect on their strategy?  
4. What questions might encourage your students to consider a more efficient strategy?  
Lead a discussion on levels of sophistication of strategies and how to respond to students’ different levels of thinking. |
| Revis | Choose two story problems: one Put Together/Take Apart and one Compare problem. Ask students to solve them in two ways without using the standard algorithm.  
1. Solve the task yourself and write out ways you anticipate your students will solve the task.  
2. Administer the task and monitor your students as they are working  
3. Question your students as they are working and take notes  
4. Collect all of your students’ work  
5. Choose six student responses that you believe represent different mathematical approaches to the problem. Briefly describe their solutions. |  
1. What changed from the last time that you came to this time? (i.e. What things have you done differently either with implementation of the tasks, questioning your students and/or choosing what samples to bring?)  
Have the preservice student teachers briefly share their task and reasons behind their choices as to which of their students’ work to bring to the session.  
2. What strategies did your students use to solve the task?  
3. What did you find surprising or unexpected in your students’ work?  
4. What is the mathematics embedded in each of their strategies?  
Lead a discussion about similarities/differences in their students’ approaches between sessions. Discuss how the problem type affected students’ choice of strategies.  
Lead a discussion about levels of sophistication of strategies. Have students work together to identify student’s levels.  
5. What questions might encourage your students at each level of sophistication to consider a more efficient strategy?  
Lead a discussion on how to respond to students’ different levels of thinking. |

Note: bolded words highlight changes.
student teacher directions was to have their students develop two strategies none of which could be the standard algorithm. This change was made in hopes that the second-grade preservice student teachers would have a variety of strategies to interpret.

Changes to the college supervisor guide for PLT session #2 included an opening question about classroom changes that the preservice student teachers made in the month since PLT session #1. This opening question was suggested by the first-grade college supervisor during the PLT session #1 debriefing. Question #3 was added in order to capture the theme of surprise that arose during the first PLT session. I decided to focus the 2nd PLT session on developing the preservice student teachers’ specialized content knowledge and knowledge of content and students (KCS) around levels of sophistication of strategy. The directions listed under the revised question #4 outlined the intended discussion/development of SCK and KCS for this session. The initial questions #4 and #5 were merged into the new #5, again with the focus on the importance of taking level of sophistication of strategy into account when questioning students. The only other design change instituted had been suggested by the college supervisors—the addition of a document camera. I switched the PLT session location to a conference room with a permanent projector, screen, and document camera so the preservice student teachers would be able to see the student work samples under discussion at all times. As before, I created a handout for the supervisors that was discussed with them during the PLT #2 pre-session meetings (See Appendix E). The handout included copies of the North Carolina Department of Public Instruction unpacking documents for the CCSS-M for their respective grade levels (NCDPI, 2012).
**Proposed changes to the hypothetical learning conjectures.** In light of the revisions to the PLT session #1 learning conjectures and the changes in design of PLT session #2, the PLT session #2 hypothetical learning conjectures were revised (See Table 11). I expected the preservice student teachers to demonstrate an improved grasp on how different types of multi-digit addition and subtraction problems lend themselves to different strategies. I expected to see their interpretations of their students’ work heavily influenced by the probing questions they asked their students and the notes they took during their classroom implementations. Partially due to the increased questioning and the PLT session #2 design changes, I expected their interpretations to be focused not only on the mathematics and less on correctness, but also less on surprise and the standard algorithm.

Through the discussion generated during PLT session #2, I expected the preservice student teachers to identify and explain the levels of sophistication of strategy associated with their student work samples. Finally, through the discussion, I expected to see evidence of improved questioning abilities that led to the preservice student teachers deciding on an appropriate next step in instruction for their individual students based on their interpretations of their students’ mathematical thinking and their identified levels of sophistication of strategies.
Table 11.

Revisions to the PLT #2 hypothetical learning conjectures.

<table>
<thead>
<tr>
<th>Initial PLT #2 Hypothetical Learning Conjectures</th>
<th>Revised PLT #2 Hypothetical Learning Conjectures</th>
</tr>
</thead>
<tbody>
<tr>
<td>The preservice student teachers have an improved grasp on the addition and subtraction tasks they gave and the specific examples of student work they are analyzing.</td>
<td>The preservice student teachers have an improved grasp on how different types of multi-digit addition and subtraction problems lend themselves to different strategies.</td>
</tr>
<tr>
<td>Their interpretations of their students’ work are partially based on the probing questions they asked of their students during the implementation of the task.</td>
<td>Their interpretations of their students’ work are partially based on the probing questions they asked of their students and notes they took during the implementation of the task.</td>
</tr>
<tr>
<td>Their interpretations are focused more on the mathematics behind their students’ thinking and less on correctness.</td>
<td>Their interpretations are focused more on the mathematics behind their students’ thinking and less on surprise, standard procedure and correctness.</td>
</tr>
<tr>
<td>The preservice student teachers identify differing levels of sophistication of multidigit addition and subtraction strategies throughout their collective student work.</td>
<td>The preservice student teachers identify and explain differing levels of sophistication of multidigit addition/subtraction strategies throughout their collective student work.</td>
</tr>
<tr>
<td>Their discussion serves as a catalyst for improving their ability to decide on an appropriate next step in instruction based on their students’ mathematical thinking.</td>
<td>Their discussion serves as a catalyst for developing probing questions for their students and ultimately their ability to decide on an appropriate next step in instruction based on their analysis of their students’ mathematical thinking.</td>
</tr>
</tbody>
</table>

Overview of PLT Session #2.

First-grade group. All four first-grade preservice student teachers were present at PLT session #2. All of the preservice student teachers except Tammy brought student work samples to the session. Only Donna and Leah completed their written reflection. The school system in which the participants were completing their student teaching field experiences has both traditional-calendar schools and year-round, multi-track schools. Both Tammy and Donna were completing their student teaching field experiences in year-round schools and
were tracked out for three of the four weeks between PLT sessions #1 and #2. Track outs were the reasons provided as to why Tammy did not bring student work samples and why Donna did not implement her task with the whole class.

The first-grade college supervisor began the session by asking the preservice student teachers if they had made any changes in their classrooms as a result of PLT session #1. The college supervisor followed the PLT session guide throughout the rest of the session. The first-grade preservice student teachers analyzed a total of 18 different samples, some from each of the three preservice student teachers who brought student work samples to the session. At the end of their PLT session, the college supervisor referred back to the NCDPI (2012) Unpacking Document and encouraged the preservice student teachers to work with their students to help them move past the counting-on and concrete make-a-ten strategies. She also encouraged them to question their students so as to be better prepared to interpret their mathematical thinking; the second-grade preservice teachers once again indicated their intention to question their students during their task implementation of PLT session #3.

**Second-grade group.** All three of the second-grade preservice student teachers came to PLT session #2 and brought student work samples. Only Lacey and Ashton completed the written reflections. Only Lacey followed the directions for PLT session #2 and set the requirement that her students not use the standard multi-digit addition and subtraction algorithms to solve their problems. Ashton asked her students to show two ways to solve her tasks, but she allowed for the use of the standard algorithm because as she wrote in her reflection, “for some, that’s all they know.” Instead of allowing her students to choose their own strategies, Tara chose two strategies—math mountains (Fuson, 2009a; Fuson, 2009b)
and the standard algorithm—and had her students complete a graded assignment on multi-digit addition and subtraction in which they were required to use both strategies.

During their session, each of the second-grade preservice student teachers shared two of their students’ solutions to one or more problems, for a total of 15 student work samples analyzed. The college supervisor followed the questions as listed on the PLT session guide, but the session felt very rushed at the end. The college supervisor did not have time to look at the NCDPI (2012) Unpacking Document with the second-grade preservice student teachers. Therefore, the second-grade preservice student teachers were not exposed to the alternate strategies mentioned in the document and did not have time for a full discussion of levels of sophistication of strategy. During the session, the second-grade preservice student teachers once again discussed the importance of questioning their students and indicated their intention to do so with their task implementation for PLT session #3.

**Emergent themes.** The themes that emerged during PLT session #2 are discussed below. As with PLT session #1, the themes are organized based on the professional noticing framework. For PLT session #2, no major themes emerged that related to Attend. In contrast with PLT session #1, the preservice student teachers did not get stuck on their surprise at their students’ work. They often noted surprise, but moved past their surprise and continued to interpret their students’ thinking. This was evident as Donna shared one of her student work samples for her put together/take apart problem. As expected, Donna was surprised by her student’s approach to the problem, but she still showed an ability to interpret the work. She explained, “So it was technically supposed to be a subtraction problem because they had needed help with subtraction within 20 and this student did the addition because that’s just
how, it’s easier for him to do the addition. He found the missing partner.” This illustrates how the preservice student teachers came to PLT session #2 attending to mathematically significant details in their students’ work.

One main theme emerged that related to Interpret: collective analysis leading to increased focus on their students’ mathematical thinking. One theme emerged that related to Identify: levels of sophistication of strategies. As with PLT session #2, the same theme emerged relating to Decide: importance of questioning their students. Three other emergent themes arose: other influences, reliance on the standard algorithm & multiple strategies, and role of specialized content knowledge. Instances could be coded with multiple themes, for example collective analysis and role of SCK. Many of the samples provided below may illustrate multiple themes, but the samples were chosen because they clearly evidenced the specific theme under explanation.

*Interpret: Collective analysis leading to increased focus on students’ mathematical thinking.* In PLT session #1, most of the analysis and discussion was between the preservice student teachers and the college supervisor, but for both groups, during PLT session #2, the preservice student teachers held discussions with each other. They asked each other questions and oftentimes influenced each other’s interpretations of the student work samples in question. Their collective discussions helped them to focus on the mathematics and often moved them toward a deeper interpretation. This was a major change between PLT sessions #1 and #2, possibly attributed to the addition of the document camera. Thus, the theme of collective analysis emerged leading the preservice student teachers towards an increased focus on their students’ mathematical thinking.
The exchange below, illustrates a discussion when the first-grade preservice student teachers worked together to make sense of a student’s thinking. The student used sticks and dots (tens and ones) to solve the first subtraction problem in which regrouping was not necessary. In the second problem, the student drew out all 46 dots, instead of using tens sticks. The preservice teachers discussed why the student would solve two similar problems differently. Through their discussion, Kelli, whose student work this is referring to, is able to comprehend more of her student’s thinking.

Kelli: He didn’t do his tens and ones, um, there, but he did up at the top so that’s confusing. He chose to move back to ones. Um…but he drew the 20 apples first and took 2 out and then he drew the 30 apples and took 2 out.

Donna: I just think it’s interesting that they’re both two 10’s numbers and he decided to write 20 circles and 30 circles.

Kelli: I do too.

CS: Instead of making 10’s.

Kelli: Tens. It would have been so much…that’s what I was talking about earlier.

CS: Especially with the tens in the front…top problem.

Kelli: I know. He, um…is, uh, distracted easily, as you can tell just with the drawings and the thick line. Um…so I’m not real sure if he just wasn’t really thinking and just did it or…

Leah: Maybe that’s how he knew that he could take away the two one’s.

Kelli: Two ones. Yeah.

Leah: Because if he had the sticks, then he couldn’t…

Kelli: That’s true.

Leah: …it wouldn’t be as easy.

Donna: Or even do like two sticks or one stick and ten ones or two sticks and…

Kelli: Right.

Donna: Ten ones.

Kelli: Yeah.

Kelli had not made the connection between the necessity of regrouping and her student’s choice of strategy. Through the collective analysis, she was better able to interpret her student’s thinking.
In the following exchange between the second-grade group, the preservice student teachers are discussing the strategy on the right in Figure 15. Ashton had multiple students represent their work in the manner, but she had not taught it this way and was confused about what her students may be thinking. The exchange begins with Sarah asking Ashton a question and ends with all three of the preservice student teachers exhibiting an understanding of the students’ mathematical thinking.

Figure 15. Ashton's student work sample for interpret: collective analysis.

Sarah: Is that a strategy that you taught to line it up like that or is that something they’re just doing?
Ashton: No. That’s…that was interesting why they all decided to do it that way. But that’s for the next one though. Let me see if any of them did that one but did it correct. No of course not.
CS: Well we talked about…what do you think…what does the zero represent after the 60?
Tara: There was no hundreds.
Ashton: Or no tens.
Tara: Or there was no tens. Oh, with the 104, there’s no tens in the 104.
Ashton: Oh yeah.
Lacey: That’s where he got that zero from.
Ashton: And then the ones.
Lacey: Yeah. But why is he subtracting it? He confused the 60 as the hundred for…to line it up with the 100 from the 104.
Ashton: But he lined it up right there.
Lacey: Yeah. Or he did hundreds and then the 60 had no hundreds and then…
Tara: So what he could have done for it to be correct would have been 100 minus zero.
Ashton: Um hmm.
Tara: And then 60 minus...
Ashton: He couldn’t have done that. He should have done…Yeah.
Tara: So that must have thrown him off.
Ashton: Yeah. ‘Cause what we had done was, um, we did it as, you know, like the place value would be 100 plus 0 plus 4 so that’s what he’s trying to do. 100 plus 0 plus 4…
Lacey: Yeah.
Ashton: And then 60 plus 3.
Tara: So he has the numbers in there…
Ashton: Yeah.
Tara: But they’re just in the wrong line.
Ashton: It depends on how you subtract them, so like you, you know, what makes up 63, you know. Like so that’s where…now I see where I made…the disconnected point in what I was teaching also. So…it’s that they saw only like subtract the whole way through.
Lacey: Was it expanded form that you were teaching?
Ashton: Um hmm.

The preservice student teachers worked together to interpret the students’ thinking and to help Ashton figure out where in her classroom instruction her students may have become confused.

Identify: Levels of sophistication of strategy. The SCK and KCS focus for PLT session #2 was on identification of levels of sophistication of strategy. Therefore, it was not surprising that level of sophistication of strategy was one of the emergent themes. The first-grade college supervisor pressed the preservice student teachers in her group to name their students’ strategies. Two main strategies were present. The first-grade students mostly used either counting on (See Figure 16) or making a ten (See Figure 17). The preservice student teachers discussed the sophistication of each of these strategies. When looking at Leah’s student’s solution in Figure 16, Donna noted, “I’m glad that they know how to do the hold the thirteen and then just count on from there instead of drawing all eighteen. I’ve been
trying to get my kids to do that and they won’t do it.” The group realized that counting on is more sophisticated than counting all. The make-a-ten strategy was seen by all of the preservice student teachers. Leah noted that in her class they are doing, “double digit, plus double digit now and that’s the way we’re teaching them to do it. To make their…to draw their ten sticks and their one circles and then to make new tens.” The preservice student teachers appropriately saw this as a higher level of sophistication than the counting strategies.

Tammy also discussed levels of sophistication that she had taught and seen in her classroom. She explained, “So if they have like 43 + 18, they’ll do 40 + 10 and then the 3 + 8.” The other three preservice student teachers had never seen this strategy so Tammy took time to make up an example that illustrated the flexibility of the method. Donna exclaimed, “Oh, that is a beautiful way!” Again, this place value strategy with adding tens and then ones was appropriately identified as a high level of sophistication. Through these examples, it is evident that the discussion focused on levels of sophistication helped the first-grade preservice student teachers begin to develop specialized content knowledge associated with levels of sophistication of strategies for multi-digit addition and subtraction problems. This development of SCK with regards to levels of sophistication was not as prevalent with the second-grade group, though it was present. See the emergent theme, Other: SCK, for an example from the second-grade group.
Decide: Importance of questioning students. One of the main expectations I had entering PLT session #2 was that the preservice student teachers would have questioned their students and made notes while they were working; yet none of them, in either group, had done so. Once again, therefore, the theme of importance of questioning their students arose. When discussing the solution of one of her students, in which a picture was drawn and some sort of counting strategy was used, Kelli noted, “because this was whole group instruction, and so I wasn’t able to sit and watch him do it. But I think that would be a benefit of asking him…” Tara had a student that solved a put together/take apart problem as is seen in Figure 18. The student’s work on the paper does not allow for an interpretation without assumptions being made. Tara conjectured that her student added up in his head, but the other preservice student teachers were quick to ask her how she could know without asking him. There were...
many instances like these of students’ solutions that the preservice student teachers could not fully interpret due to lack of student questioning.

*Figure 18.* Tara's student work sample for decide: importance of questioning students.

**Other: Other influences.** The theme of other influences emerged again for both groups of preservice student teachers. As the participant observer with the first-grade group, I asked if any of the preservice student teachers had used number lines with their students. None of them had, despite the fact that they are mentioned in the CCSS-M. Just like with the method of adding tens before adding ones, if the preservice student teachers are not aware of multiple strategies, they do not press their students to use alternate methods. Similarly, the second-grade group had a conversation about the possibly having their students journal about their solutions or have them present their solutions in front of the class. I consider these examples instructional influences. Outside influences also emerged. The preservice student teachers had a discussion about parents showing their students the standard multi-digit addition and subtraction algorithms. As was mentioned previously, Kelli’s forced ability grouping of her students may have limited the strategies she was able to see from her
students. These other influences are often outside of the scope of the preservice student teachers’ control but can greatly affect their students’ work.

**Other: Reliance on standard algorithm and multiple strategies.** From the second-grade group, both Tara and Ashton’s students relied heavily on the standard algorithm. Like Tara had in PLT session #1, Ashton provided her students with room to show four different solutions for their problems. Most of her students chose to do the standard algorithm first and then used other methods. Many of her students did not check to ensure their other methods provided them with the same answer as the standard algorithm (Figure 15, p. 93). The students were filling in the strategy blocks, not actually trying to determine the answer with a different strategy. Once again, the purpose behind developing multiple strategies was lost for the second-grade students in the preservice student teachers’ classes. Lacey commiserated with Ashton saying that her students almost always chose to do the standard algorithm first. Ashton made the point that under the CCSS-M, the standard algorithm is not a focus in second-grade. Tara then stated, “So I’m almost wondering if we should even teach the algorithm in second grade.” The college supervisor then shared, “Well, your observation is supported by a lot of math educators, is that once children learn the algorithm, that they have blinders on to any other solution strategies, and recommend holding off on that.” The group then had a discussion about parents showing their children the algorithm and how outside influences affect their instruction in the classroom. It was evident that the second-grade students’ reliance on the standard algorithm and lack of understanding the purpose behind multiple strategies was impacted by outside influences.
**Other: Role of specialized content knowledge.** Throughout both groups’ sessions, there were instances of the preservice student teachers exhibiting SCK and also exhibiting a lack of SCK. The following exchange illustrates both. In the exchange, the preservice student teachers are comparing solutions of two of Lacey’s students on a put together/take apart task for 82 - 53. The students both used number lines; one did so by counting up and one by counting down (See Figure 19). Lacey’s interpretation of her students’ work is based on her personal experiences with mathematics and how she would solve the problem herself by counting down. Tara encourages Lacey to see that counting up makes more sense based on the problem type. The group then discussed the level of sophistication of each of the strategies, and Ashton exhibited SCK as she made a keen observation about what the counting up student could have done to increase the level of sophistication of the strategy.

Lacey: And it’s so interesting because when I saw his work, the first one up there that did the minus tens, minus ten, I was like, “Oh, I understand this, I understand exactly why they did that.” And then I looked at the other one and it took me a few minutes. I was just kind of like…

Tara: It looks like he started with 53.

Lacey: 53 and counted up.

Tara: He went 53, 63, 73. He can’t go to 83 so he tried 5. And then he counted the rest of the 1’s.

Ashton: But then he didn’t notice that when you got to 83, that’s only one less. So then… [multiple people talking 46:29]

CS: Right. That’s it.

Ashton: But it’s like if you added 30 then it’s 83. But that was…

CS: Which of these two do you think is most sophisticated or more efficient? Those are two different things, I know.

Tara: I think the second one.

CS: And why do you think that?

Ashton: Yeah the second one.

Tara: Because he didn’t use…when he was counting up, he used the unknown number to get there. Like the first one he used the two numbers that were given in the problem.
Lacey: Yeah. I could see that. But then, because I would have done the other one, I would have…I said that one would have been more efficient.

Ashton: Did you teach them how to do the number line or is this something they just used?

Lacey: This is something they already knew.

Ashton: Okay. So it’s carried over.

Lacey: So, it’s something that they’ve already done in the past. I just find it interesting because his approach made perfect sense to me. And this one makes sense to me but I feel like the first one - the top one - is almost more efficient. I mean that’s just my…that’s personal opinion. But…in your own personal mind. But I see how the one on the bottom is more sophisticated because like Sarah said, it’s the…he’s counting up to the unknown

Tara: The unknown number.

Lacey: Like not knowing how many he needs to have until he gets to...

Ashton: Yeah. 82.

Tara: I think it would be more efficient if they did recognize that it was one over and then count back one.

Lacey lacked the SCK necessary to understand that her student’s counting up strategy matched the problem type. Ashton made a keen observation based on her SCK of different strategies about how the student could have decomposed with the number line. Tara understood Ashton’s point and brings it up again at the end of the exchange. This exchange also exhibits other emergent themes discussed previously. It illustrates collaborative analysis and levels of sophistication of strategy. This exchange also highlights the emergent theme of personal experiences with mathematics from PLT session #1.; while present in PLT session #2, it was no longer a recurring theme.

Figure 19. Lacey’s student work number line samples for other: role of SCK.
During the first-grade PLT session #2, the group had a discussion about Tammy’s students work on a story problem for 13+5. Donna made the comment that she was happy to see Tammy’s students “hold the thirteen, and count on from there.” I asked the preservice student teachers if based on the student work they saw for 13+5, if they could come up with different strategies for 13+8. This led to a discussion about making a new 10. Donna shared her knowledge of students’ wanting to work from 10. For example, she discussed a student who when adding 9+8, changed the problem to 10+7. She was able to apply her SCK relating to her student’s strategy to this new problem 13+8. This led to a whole group discussion about different compensation strategies in which some of the preservice student teachers exhibited further SCK and others a lack of SCK. For another example, when discussing her decision to have her students use math mountains to help them understand how to solve different problem types, Tara exhibits SCK. She was able to articulate how story problems with unknowns in different places can be organized via math mountains (Fuson, 2003a; Fuson, 2003b). Through analysis of these examples, it is evident that specialized content knowledge can both positively and negatively impact the student teachers’ interpretations and identification of levels of sophistication of strategy.

**Changes to the hypothetical learning conjectures.** The results of the analysis of both PLT #2 sessions led once again, to changes in the hypothetical learning conjectures for PLT session #2 (See Table 12). The first conjecture was realized; the preservice student teachers exhibited evidence that they recognized different types of multi-digit addition and subtraction story problems and how the different problem types influence their students’ choice of strategy. The second conjecture did not prove to be true. The preservice student
teachers did not question their students during their classroom implementations of their tasks. During the PLT session, they were reminded of the importance of asking their students probing questions and of taking notes as a means to assist their interpretations of their students’ thinking. The third conjecture proved to be true, but an emergent finding from the session was the importance of collective analysis in influencing interpretations of student thinking. Therefore, the word “collective” was added to the 3rd conjecture. In both sessions, some identification of levels of sophistication of strategy was present, but it was at a beginning level. Thus, the word “begin” was added to the 4th conjecture in the trajectory. Finally, the preservice student teachers did not follow directions and question their students while they worked, thus, the final conjecture was not realized.

The results to the ongoing analysis of PLT session #2 also led to changes in the hypothetical learning conjectures for PLT session #3 and ultimately to changes in the design of PLT #3. See the next two sections for descriptions of the changes to PLT #3.

**PLT Session #3**

**Preparation and design.** At the culmination of the analysis of PLT session #2, PLT session #3 was redesigned (See Table 13). During PLT sessions #1 and #2, there was not enough time to allow each of the preservice student teachers to share all of their collected student work samples, thus a decision was made to have the preservice student teachers pose only one problem and to only choose three samples to bring. No directions regarding the type of problem was given since the preservice student teachers had provided evidence of their understanding different problem types. The specific directions that the three responses they bring should represent what they believe to be different levels of sophistication was from the
original design. The requirement that their students solve the problem with two strategies, none of which could be the traditional algorithm was once again an addition to the directions.

Table 12.
*Re-revised hypothetical learning conjectures for PLT session #2.*

<table>
<thead>
<tr>
<th>Revised PLT #2 Hypothetical Learning Conjectures</th>
<th>Re-Revised PLT #2 Hypothetical Learning Conjectures</th>
</tr>
</thead>
<tbody>
<tr>
<td>The preservice student teachers have an improved grasp on how different types of multi-digit addition and subtraction problems lend themselves to different strategies.</td>
<td>They are once again reminded of the importance of asking their students probing questions and taking notes in order to assist them in interpreting their students’ mathematical thinking.</td>
</tr>
<tr>
<td>Their interpretations of their students’ work are partially based on the probing questions they asked their students and notes they took during the implementation of the task.</td>
<td>Their collective interpretations are focused more on the mathematics behind their students’ thinking and less on surprise, standard procedure and correctness.</td>
</tr>
<tr>
<td>Their interpretations are focused more on the mathematics behind their students’ thinking and less on surprise, standard procedure and correctness.</td>
<td>The preservice student teachers begin to identify and explain differing levels of sophistication of multi-digit addition/subtraction strategies throughout their collective student work.</td>
</tr>
<tr>
<td>The preservice student teachers identify and explain differing levels of sophistication of multi-digit addition/subtraction strategies throughout their collective student work.</td>
<td>The preservice student teachers have an improved grasp on how different types of multi-digit addition and subtraction problems lend themselves to different strategies.</td>
</tr>
<tr>
<td>Their discussion serves a catalyst for developing probing questions for their students and ultimately their ability to decide on an appropriate next step in instruction based on their analysis of their students’ mathematical thinking.</td>
<td></td>
</tr>
</tbody>
</table>

The only other change in preservice student teacher directions was #6. The preservice student teachers did not exhibit evidence that they were ready to choose appropriate next problems to pose to their students. They were, however, at the point where they could start to think about how questions posed to their students might help their students move toward a
a higher level of sophistication of strategy choice. Question #6 reflects this change of focus. Additionally, a few days before the scheduled PLT session #3, I sent the preservice student teachers Jacob & Ambrose’s (2008) *Making the Most of Story Problems* article focused on questioning of students and asked the preservice student teachers to read prior to attending the session.

Changes to the college supervisor guide included instructions with specific pages to show the preservice student teachers from the NCDPI (2012) *Unpacking Documents* with the goal of both pinpointing examples of different multi-digit addition and subtraction strategies their students could use and identifying the level of sophistication of each of the strategies. During the post-PLT session #2 debriefing, the first-grade college supervisor had suggested having the preservice student teachers share their chosen work samples in a round robin fashion. I decided to have the round robin sharing occur and decided to ask the preservice student teachers to make notes on both the mathematics they noticed and to identify levels of sophistication of strategy for each of the samples. The first three discussion questions on the college supervisor guide remained the same as in the original design, but question #4 was removed and question #5 was reworded to focus on questioning their students’ based on the goal of helping them move toward the use of strategies with a higher levels of sophistication. Questioning students based on interpretation and identification of levels of sophistication of strategy was the SCK and KCS focus for PLT session #3.

Once again, I created a handout for the supervisors (See Appendix E), and made copies of the Jacobs & Ambrose (2008) article mentioned above. We discussed the goals of the session. Neither of the college supervisors had lingering questions or concerns.
<table>
<thead>
<tr>
<th>PLT</th>
<th>Preservice Student Teacher Directions</th>
<th>College Supervisor Guide</th>
</tr>
</thead>
</table>
| **Three Initial** | Choose two story problems: one that lends itself to an addition strategy and one that lends itself to a subtraction strategy. Ask students to solve them in two ways.  
1. Solve the task yourself and write out ways you anticipate your students will solve the task.  
2. Briefly describe what you want your students to learn from the task.  
3. Administer the task and monitor your students as they are working  
4. Question your students as they are working and take notes.  
5. Collect all of your students’ work  
6. Choose six student responses that you believe represent a range of levels of mathematical sophistication to bring to the reflection session.  
7. Brainstorm possible next problems to give your students based on their responses. | Begin by having the student teachers share their task and reasons behind their choices as to which of their students’ work to bring to the session.  
1. What strategies did your students use to solve the task?  
2. What did you find surprising or unexpected in your students’ work?  
3. What is the mathematics embedded in each of their strategies?  
4. What did your students learn on their task and how does it compare with what you wanted them to learn?  
5. On the basis of your students’ individual understandings, what problem might you pose next and how might your student solve it? Lead a discussion on choosing next steps in instruction and have students share what they have learned from their participation in this study. |
| **Three Revised** | Choose one story problem that can be solved with either an addition or a subtraction strategy. Ask your students to solve it in two ways without using the standard algorithm.  
1. Solve the task yourself and write out ways you anticipate your students will solve the task.  
2. Administer the task and monitor your students as they are working  
3. Question your students as they are working and take notes.  
4. Collect all of your students’ work  
5. Choose three student responses that you believe represent a range of levels of mathematical sophistication to bring to the reflection session.  
6. Brainstorm possible questions to ask the three students to assist them in moving towards a high level of sophistication.  
Read Making the Most of Story Problems (Jacobs & Ambrose, 2008) prior to attending the session. | Begin by reviewing levels of sophistication of strategies. Look through the DPI Unpacking the CCSS-M documents for examples of different strategies.  
Then have student teachers collectively share all of their chosen pieces of student work. Have the student teachers make notes on the mathematics they notice and identify levels of sophistication of strategies.  
Discussion questions:  
1. What strategies did your students use to solve the task?  
2. What did you find surprising or unexpected in your students’ work?  
3. What is the mathematics embedded in each of their strategies?  
4. On the basis of your students’ individual understandings, what question(s) might you pose next to help your student move towards a high level of sophistication of strategy? Lead a discussion on questioning as a next step in instruction. |

Note: bolded words highlight changes
Changes to the hypothetical learning conjectures. The hypothetical learning conjectures for PLT session #3 were revised based on the previously discussed changes to the learning conjectures for PLT sessions #1 and #2, as well as changes to the design of both PLT sessions (See Table 14). I maintained my expectation that the preservice student teachers would demonstrate an understanding of differences between their students’ strategies, and based on the collective analysis that occurred during PLT session #2, I now expected the preservice student teachers to collectively interpret their students’ work. I expected their collective interpretations to focus on levels of sophistication. The two main changes from the initial hypothetical learning conjectures to this revised version, was a shift away from the focus on instructional implications. I no longer expected that the preservice student teachers would be able to analyze the task they assigned based on their students’ work. I also no longer expected the preservice student teachers’ interpretations and levels of identification of sophistication of strategy to lead them toward choosing an appropriate next task to pose their students. During both PLT sessions #1 and #2, the preservice student teachers illustrated a need to learn how to develop appropriate questions to ask their students; thus, I expected that the design of PLT session #3 would assist the preservice student teachers in their development of questioning of their students based on their collective interpretations and identification of sophistication of strategy.
Table 14.

Revisions to the hypothetical learning conjectures for PLT session #3.

<table>
<thead>
<tr>
<th>Initial PLT #3 Hypothetical Learning Conjectures</th>
<th>Revised PLT #3 Hypothetical Learning Conjectures</th>
</tr>
</thead>
<tbody>
<tr>
<td>The preservice student teachers note differences in their students’ strategies and interpret their students’ strategies based on levels of sophistication. They use their knowledge to analyze their students’ success at achieving the goals of the task they assigned.</td>
<td>The preservice student teachers note differences in their students’ strategies and collectively interpret their students’ strategies based on levels of sophistication.</td>
</tr>
<tr>
<td>The preservice student teachers’ interpretations of their students’ work are focused on the logic behind their mathematical thinking.</td>
<td>The preservice student teachers’ collective interpretations of their students’ work are focused on the logic behind their mathematical thinking.</td>
</tr>
<tr>
<td>Their discussion helps the preservice student teachers to draw upon their SCK and KCS regarding levels of sophistication of strategy to decide appropriate next steps in instruction for their students.</td>
<td>Their discussion helps the preservice student teachers draw upon their SCK &amp; KCS regarding levels of sophistication to choose appropriate probing and/or extending questions to ask their students.</td>
</tr>
</tbody>
</table>

Overview of PLT session #3.

First-grade group. All four of the first-grade preservice student teachers came to PLT session #2 and brought in student work samples as directed. However, only Donna and Kelli submitted their written reflections. None of the first-grade preservice student teachers had read the Jacobs and Ambrose (2008) article. As was designed, at the beginning of the PLT session, the college supervisor directed the preservice student teachers to the NCDPI (2012) Unpacking Document and opened the session with a discussion about different strategies and their levels of sophistication. A handout was provided to the preservice student teachers that labeled three levels of sophistication of strategies and provided the questioning tables from Jacobs & Ambrose (2008) article (see Appendix I for a copy of the preservice student teacher handout). The levels of sophistication as provided follow:
Levels of Sophistication of Strategies

1. Direct Modeling
2. Counting
3. Number Fact Strategies: making a ten, decomposition, creating equivalent but easier problems—all draw on knowledge of place value, properties of operations and/or relationship between addition and subtraction

During their conversation, the first-grade preservice student teachers discussed the strategies they were seeing in their classroom. They were able to correctly name the different strategies their students were utilizing. The first-grade group’s students were mostly at levels 1 & 2, with some examples of level 3.

Following their discussion about strategies and levels of sophistication, the first-grade college supervisor had her group take turns sharing their work samples. She incorporated the SCK component of pressing/probing questions into each of the preservice student teachers’ discussions of the student work samples. The first-grade preservice student teachers brought a combined 15 samples of student work to the session, but time only allowed for 10 of them to be discussed with the whole group. The college supervisor ended the session by asking what the preservice student teachers would do next based on what they had learned during all three PLT sessions. The first-grade preservice student teachers indicated a desire to work on the relationship between addition and subtraction as well as focus on questioning their students.

Second-grade group. Only Lacey and Tara were present at PLT session #3. Ashton withdrew from her student teaching experience and therefore the study just before the 3rd session. Only Tara submitted her written reflections and only Lacey had read the Jacobs & Ambrose (2008) article on questioning. However, both Lacey and Tara followed the PLT
directions and required that their students not use the standard multi-digit addition and subtraction algorithms when solving their task.

As was designed, at the beginning of PLT session #3, the second-grade college supervisor showed the preservice student teachers the NCDPI (2012) Unpacking Document and began the session with a discussion about different strategies and their levels of sophistication. During the ensuing discussion, the preservice student teachers considered the strategies they were seeing in their classroom. The second-grade group’s students were mostly at levels 1 & 2. They discussed the resistance they received from their students when trying to move their students away from the standard algorithm.

Following their discussion about different strategies and levels of sophistication, the round robin sharing began. The second-grade college supervisor structured her session different than was planned. She made the decision to project a student work sample and have the opposite preservice student teacher interpret first. This made for a truly collective analysis and also showed areas where questioning of their students was highly important. After the second-grade preservice student teachers had shared all six of their student work samples, their college supervisor rushed to formally discuss questioning. For the sake of time, the second-grade college supervisor had the preservice student teachers choose one of their student work samples and decide on appropriate probing and/or extending questions to ask that student. Due to time constraints, there was no formal conclusion to the PLT session nor the PLT sessions as a whole.

**Emergent themes.** The themes that emerged during PLT session #3 are discussed below. As with the previous PLT sessions, the themes are organized based on the
professional noticing framework. Like PLT session #2, no major themes emerged that related
to Attend. The emergent theme of collective analysis seen in PLT session #2 relating to
Interpret, was once again present. However, during PLT session #3, both groups of
preservice student teachers’ collective analyses of each other’s student work samples led to
increased focus on their students’ mathematical thinking which exhibited itself under
Interpret, Identify, and Decide. These three will be discussed together. All of the other
emergent themes during PLT session #3 were also seen in previous PLT sessions. Collective
analysis was the only emergent theme under Interpret. In addition to collective analysis,
under Identify, the theme levels of sophistication of strategies once again emerged. The
theme, importance of questioning, once again appeared under Decide. The other emergent
theme exhibited in PLT session #3 was the role of specialized content knowledge. To
reiterate, instances could be coded with multiple themes, for example collective analysis and
role of SCK. Many of the samples provided below may illustrate multiple themes, but the
samples were chosen because they clearly evidenced the specific theme under explanation.

**Interpret, identify and decide: Collective analysis leading to increased focus on
students’ mathematical thinking.** Both groups of preservice student teachers collectively
analyzed each other’s student work samples. The questions they asked each other and the
input they provided oftentimes led the two groups of preservice student teachers toward
greater interpretations of their students’ mathematical thinking, increased ability to correctly
identify levels of sophistication of strategies, and increased ability to decide appropriate
questions to ask their students. During the following exchange, the first-grade preservice
student teachers are discussing Kelli’s student’s work seen in Figure 20.
Donna: The problem that I’ve seen with my kids is that when they do have to break down their numbers like this, it’s great when the ones is the highest that it goes in that line, but when it gets to like $20 + 40 = 60$ and they’ve got like 6 and 7 and that’s 13, then you have $16 + 13$ and they’re like, “Okay….,” and then they draw up 13 circles and they don’t understand it’s a 10 and a 3…

Tammy: I like how the student used…that’s what I try to do with my students, is add the tens first, then add the ones so that it’s getting away from the standard algorithm…it’s level 3.

Kelli: …I wonder if she knew that she was gonna have to make a new 10 here so she chose to do it this way ‘cause she made a new 10.

Donna: I think I would agree with that too ‘cause when my kids know that they have to make a new 10, they’ll normally just go ahead and do tens and ones and then they’ll circle their new ten rather than doing the decomposing method.

Through their collective analysis, they come to realize that while Kelli’s student exhibits a higher level of sophistication of strategy for the first problem, she still needs help applying her strategy to problems requiring making a new ten.

![Figure 20. Kelli's first student work examples for interpret, identify and decide: collective analysis.](image)
The following segment also illustrates the theme of collective analysis leading to deeper interpretation and identification of levels of sophistication. The second-grade group is discussing the student work samples for a compare problem for 101 – 62 seen in Figure 21.

Tara: It looked like she had 101. She just had the 101 and then she broke it up into 3, 4, 5, 6…Let me see.
Lacey: She knew she needed some tens so she broke it up in the ten sticks.
Tara: Then she broke up one of the tens into ones and then she could subtract the ones and had them left over.
CS: So she was...Is that sort of the same strategy and just a different representation from the first one or is it entirely different do you think?
Lacey: From the expanded one?
CS: Yeah.
Lacey: It’s similar because it’s the same process but it is a little bit, I think, less abstract because it is a representation of the concrete models.
CS: She’s still removing 62 like she was removing 62.
Lacey: Right. So it is subtracting instead of counting up, which is more sophisticated than if she had drawn 62 and then added ten sticks. But it’s still the same process. It’s still…it’s not the most higher level thinking in terms of how you would solve a subtraction problem.

Through analyzing this exchange, it is evident that the preservice student teachers are interpreting the students’ thinking and identifying the levels of sophistication. Unlike in PLT sessions #1 and sessions #2, Lacey realizes that for this problem, counting up on a number line would follow the problem type and therefore is labeled as more sophisticated.

Figure 21. Lacey’s student work examples for interpret, identify and decide: collective analysis.
The first-grade group discussed the work sample for \(67 + 34\) shown in Figure 22. They collectively discussed how the student made a new 10 and then recognized that ten 10s make one hundred. They go on to discuss possible questions to ask this student.

Kelli: I think beforehand I would want to maybe ask him what he knows about the problem, like what’s the problem asking him to do. And then afterwards I guess I might would say, “Explain your strategy. Why do you have all these numbers here?” and, again, “Why do you have separate columns? What’s your thought process in that?” And a number sentence to go with the problem so I could see what he was thinking.

Donna: It’d also be interesting to see too if he could use another strategy besides that one every time. ‘Cause that’s sophisticated. I haven’t really seen a child do that.

This exchange illustrates how the group’s collective analysis led to an increased focus on the student’s mathematical thinking which in turn led to an increased ability to decide on appropriate questions to ask the student.

Figure 22. Kelli's second student work examples for interpret, identify and decide: collective analysis.

**Identify: Levels of sophistication of strategy.** Throughout both sessions, the college supervisors attempted to maintain a focus on levels of sophistication of strategies. The
preservice student teachers correctly identified the level of sophistication of strategies throughout the PLT #3 sessions. Tammy shared an instance of one of her students exhibiting a higher level of sophistication with decomposition. She provided the following example,

“one of my students came up after...like after the lesson she said, ‘I have another way to solve it,’ and she went down like...we’re counting by two digit numbers and she’s adding 10, 20, and down the hundreds board and then she moved one more ten. She’s like, ‘I know that it’s one less ‘cause...’ it was like 39 or something, she went, ‘I know it’s one less,’ and she goes...she went back one.”

Tammy not only understood the mathematics behind her student’s strategy, but she was able to correctly identify its level of sophistication. For the second-grade preservice student teachers, having the requirement of no standard algorithm meant most of the strategies their students used were at a lower level of sophistication. When discussing one of her student’s work samples, Lacey explained, “So it is subtracting instead of counting up, which is more sophisticated than if she had drawn 62 and then added ten sticks, but it’s still the same process. It’s still not the most higher level of thinking in terms of how you would solve a subtraction problem.”

Despite their increased ability to identify levels of sophistication, when looking at some of the number fact strategies presented in the NCDPI (2012) Unpacking Document, both Kelli and Donna indicated skepticism about their students being able to work at the higher levels. Kelli questioned the group,

“If a child is solving problems and getting them correct by counting but it takes them a longer time, should you encourage them to move to the number facts strategy and then when they grow the number facts strategy they don’t...like they just don’t get it...I just don’t know like how far I should push them, you know, if they’re getting it right every time.”
The college supervisor chose not to continue this conversation. While this belief did not emerge for the second-grade preservice student teachers, they did not have a chance to discuss it. It is unclear whether or not they believed their students to be capable of working at higher levels of sophistication.

**Decide: Importance of questioning students.** Unlike with PLT sessions #1 and #2, for PLT session #3, all of the preservice student teachers questioned during implementation of their task. One of Donna’s students solved the problem 100 – 54 in a manner that would have been very difficult to interpret without questioning him (See Figure 23). During the session, Tammy noted that she was not sure what the student was doing. Donna was able to explain the student’s thinking in light of her questioning of him and was able to focus her interpretation on his mathematical thinking.

“I didn’t ask him about that during but afterwards when I looked at it, I said, “Why did you draw five on top and on bottom?” and he said, “Well there were 54 so I drew 54 first” and then he said, “And then I realized that I needed to get to 100 so I was drawing…I was making partners on top to make sure they added up,” and then he realized that 50 and 50 was 100. So he said, “And I knew then that I needed less than 50 to make my problem true.” And he can count back and not many of my students can count back, especially with two-digit numbers. So that really surprised me.”

The other preservice student teachers collectively discussed this student’s solution and collectively identified the level of sophistication of the strategy as a level 3. Kelli explained her reasoning, “I would think so. Since he could explain it. Looking at this I couldn’t really tell his work but he could explain it. I mean I think explaining it is the hardest part. And the fact that he knew he needed less than 50 because he already had over 50.” Kelli’s comment is focused on the mathematics and clearly illustrates her realization of the importance of questioning this student in order to interpret his thinking.
During PLT session #2 Tara recognized the importance of having her students explain their thinking so she added “Explain how you got your answer” to the bottom of her task worksheets. She explained, “We’ve been practicing it ever since we talked about it in here. I just had a little explaining what you’re doing. Explain what you’re thinking. So I modeled a couple of those for them and they’ve been doing it for pretty much everything I have them do independently and that helps me see what they’re doing and what they’re thinking.” Having her students provide an explanation helped Tara interpret the student work sample for 111 – 89 seen in Figure 24. An initial look at the student work, may lead to an interpretation that the student had counted down, but the student wrote, “I got my answer by counting up.” Tara, Lacey, the second-grade college supervisor, and I then had a discussion about what the student may have done if he indeed had counted up. So while having her students provide explanations proved to assist in understanding the students’ mathematical thinking, further
questioning was necessary. The group’s discussion ended with a decision that Tara would need to ask the student to explain more in order for a full interpretation to be made.

Figure 24. Tara's student work decide: importance of questioning students.

In looking at one of Lacey’s chosen samples, Lacey and Tara discussed possible questions to ask the student. The student had used two different counting up strategies when solving the problem.

Lacey: So what I would want to push the student to do is instead of counting up to see this problem as a problem where subtraction could be an option for them because it seems like both of the strategies that this student used involved counting up or adding on to the base number.

Tara: And then I also…just to check their work instead of having to rewrite using the algorithm. Algorithm to check their work, does the answer make sense? Who is supposed to have more at the end? Does the answer reflect what the question is asking or does it match what the question is asking? So that also kind of helps them see, “Does the answer make sense? Does the right person have more? Does the right person have less?”
Both Lacey and Tara showed evidence of thinking about the mathematics behind both the problem and the solution to determine appropriate questions. Overall, the preservice student teachers showed evidence of an increased ability to determine questions to probe or extend their students’ thinking. PLT session #3 ended with Tammy discussing her main take-away thoughts dealing with questioning, “Asking questions before they solved it. I normally ask what problem is this but I don’t ask…I don’t know. Just thinking about the before instead of just the after.”

**Other: Role of specialized content knowledge.** Throughout both groups’ sessions, there were once again instances of the preservice student teachers exhibiting SCK and also exhibiting a lack of SCK. In looking at the student work sample and exchange presented under the collective analysis theme (p. 111), Donna exhibited evidence of her SCK when she noted that students often choose addition strategies based on the numbers in the problem. Tara exhibited a lack of SCK when she was discussing the solution strategy of one of her students for 111-89. She explained that the student “went 9+2 gives you 11 and then 8+2 gives you [10].” Tara did not use terminology associated with SCK. She should have said 9 tens + 2 tens gives you 11 tens or something to that effect. Once again, it is evident that specialized content knowledge can both positively and negatively impact the student teachers’ interpretations.

**Changes to the hypothetical learning conjectures for PLT #3.** The results of the analysis of both PLT #3 sessions led to changes in the hypothetical learning conjectures for PLT session #3 (See Table 15). The first two conjectures were realized. The preservice student teachers showed evidence of understanding differences in their students’ strategies
and collectively interpreted their students’ mathematical thinking based on levels of sophistication of strategy. The third conjecture was altered with the addition of “begin” and “interpret their students’ thinking.”

Table 15.
Re-revised hypothetical learning conjectures for PLT session #3.

<table>
<thead>
<tr>
<th>PLT #3 Revised Hypothetical Learning Conjectures</th>
<th>PLT #3 Re-Revised Hypothetical Learning Conjectures</th>
</tr>
</thead>
<tbody>
<tr>
<td>The preservice student teachers understand differences in their students’ strategies and collectively interpret their students’ strategies based on levels of sophistication.</td>
<td>The preservice student teachers understand differences in their students’ strategies and collectively interpret their students’ strategies based on levels of sophistication.</td>
</tr>
<tr>
<td>The preservice student teachers’ collective interpretations of their students’ work are focused on the logic behind their mathematical thinking.</td>
<td>The preservice student teachers’ collective interpretations of their students’ work are focused on the logic behind their mathematical thinking.</td>
</tr>
<tr>
<td>Their discussion helps the preservice student teachers draw upon their SCK and KCS regarding levels of sophistication to choose appropriate probing and/or extending questions to ask their students.</td>
<td>Their discussion helps the preservice student teachers begin to draw upon their SCK and KCS regarding levels of sophistication to both interpret their students’ thinking and to choose appropriate probing and/or extending questions to ask their students.</td>
</tr>
</tbody>
</table>

Although the first-grade preservice student teachers exhibited multiple instances of drawing on SCK to interpret and choose questions based on their students’ levels of sophistication, the word “begin” was added because the second-grade preservice student teachers showed only two instances. They were still at the beginning level, possibly because their student work samples all exhibited lower levels of sophistication. For the instances of SCK that were recorded, they involved interpretation as well as questioning based on identification of sophistication.
**Discussion of emergent themes.** The themes that emerged during the ongoing analysis for all three PLTs are summarized in Table 16. This table shows how themes emerged over time and also how the themes varied with relation to the two groups of preservice student teachers. In chapter 5, this progression over time will be discussed as it relates both to the research questions and the professional noticing framework.

Table 16.

*Emergent themes throughout the PLT sessions.*

<table>
<thead>
<tr>
<th>PLT #1</th>
<th>PLT #2</th>
<th>PLT #3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Attend</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Articulated Choice</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Affect *</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surprise</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Interpret</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plausible Interpretations *</td>
<td>Collective Analysis with Focus on Students’ Math Thinking</td>
<td>Collective Analysis with Focus on Students’ Math Thinking</td>
</tr>
<tr>
<td>Difficult to Interpret</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Influenced Interpretations</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Identify</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Levels of Sophistication of Strategies</td>
<td>Collective Analysis with Focus on Students’ Math Thinking</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Levels of Sophistication of Strategies</td>
</tr>
<tr>
<td><strong>Decide</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Importance of Questioning Students</td>
<td>Importance of Questioning Students</td>
<td>Collective Analysis with Focus on Students’ Math Thinking</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Importance of Questioning Students</td>
</tr>
<tr>
<td><strong>Other</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other Influences</td>
<td>Other Influences</td>
<td>Role of Specialized Content Knowledge</td>
</tr>
<tr>
<td>Reliance on SA &amp; Multiple Strategies *</td>
<td>Reliance on SA &amp; Multiple Strategies ^</td>
<td>Role of Specialized Content Knowledge</td>
</tr>
<tr>
<td>Role of Specialized Content Knowledge</td>
<td>Role of Specialized Content Knowledge</td>
<td></td>
</tr>
</tbody>
</table>

Regular Text: Both groups; *: First-grade only; ^: Second-grade only

**Summary of Changes to the Hypothetical Learning Trajectory**

Many aspects of the hypothetical learning trajectory were not realized. The elementary preservice student teachers did not lack the ability to articulate their decisions as to which samples of student work they brought to PLT session #1. They chose their samples based on the desire to have a variety of responses and to discuss samples that surprised them. The elementary preservice student teachers’ initially attended to the mathematics behind their
students’ work with interpretations focused on procedural correctness and surprise at either non-standard solutions or their students’ strategy preferences.

I had not anticipated the importance of surprise, nor had I expected to see the elementary preservice student teachers so quickly addressing mathematically significant details of their students’ work. While they did address mathematically significant details, their interpretations alternated between plausible descriptions and noted difficulty due to lack of questioning their students. Yet, as I anticipated, the elementary preservice student teachers’ interpretations often drew upon their personal beliefs and experiences with mathematics. The elementary preservice student teachers began to develop specialized content knowledge about different types of multi-digit addition and subtraction problems.

After PLT session #1, as anticipated, the elementary preservice student teachers showed progress in their ability to anticipate the ways in which their students would solve different types of multi-digit addition and subtraction problems. Despite the fact that both groups of elementary preservice student teachers discussed the importance of questioning their students during PLT session #1, they did not question their students before coming to PLT session #2 as I had anticipated. The elementary preservice student teachers once again exhibited some difficulty interpreting their students’ mathematical thinking due to lack of questioning. While they did not question their students, both their interpretations and identification of levels of sophistication were influenced by their collective analysis of each other’s students’ mathematical thinking which I had not anticipated. Instead of questioning their students, their collective analysis of each other’s students’ mathematical thinking provided the elementary preservice student teachers with a basis for interpreting their
students’ mathematical thinking and for beginning to identify levels of mathematical sophistication.

As was projected in the initial hypothetical learning trajectory, the elementary preservice student teachers continued developing specialized content knowledge relating to multi-digit addition and subtraction throughout the PLT sessions. Through their experiences with the first two professional learning tasks, the elementary preservice student teachers learned how to focus on the mathematical meaning and logic behind their students’ thinking. Another change to the hypothetical learning trajectory was the addition of the term “collective” as a descriptor of the analysis that occurred during the PLT sessions. The elementary preservice student teachers’ collective interpretations of their students’ work shifted from evaluative toward identification of levels of mathematical sophistication.

A major changed to the hypothetical learning trajectory was the ways in which the elementary preservice student teachers made the connections to their classroom. They did not reach the point where they could choose the most appropriate instructional next step for their students. Instead, they realized the importance of questioning their students and used their newly developed SCK and KCS surrounding levels of sophistication and extending/probing questions to attend to, interpret, and identify levels of sophistication of their students’ mathematical thinking. There was evidence of their beginning to draw upon SCK while working through all aspects of the professional noticing framework.

**Future Hypothetical Learning Trajectory**

If I were to complete this study a second time, I would begin with the following hypothetical learning trajectory. This trajectory is not session specific, thus would need to be
adapted based on the number of PLT sessions that would occur. The hypothetical learning conjectures comprising the hypothetical learning trajectory follow:

- At the onset of their experience with guided professional learning tasks focused on collective analysis of their students’ mathematical thinking, elementary preservice student teachers focus on a variety of and/or surprising responses.

- They initially attend to the mathematics behind their own students’ work with interpretations focused on procedural correctness, and surprise at either non-standard solutions or students’ strategy preferences.

- Elementary preservice teachers address mathematically significant features of their students’ work but their interpretations alternate between plausible descriptions and noted difficulty due to lack of questioning their students. Their interpretations often draw upon their own mathematical experiences, understandings and/or beliefs.

- Through their experience in the guided professional learning tasks, elementary preservice student teachers begin to develop specialized content knowledge.

- Elementary preservice student teachers begin to anticipate how their students’ may solve problems.

- Elementary preservice student teachers’ collective analysis of each other’s students’ mathematical thinking provides the preservice student teachers with a basis for interpreting their students’ mathematical thinking and for beginning to identify levels of mathematical sophistication of their students’ strategies.

- Through their continued experiences in guided professional learning tasks, elementary preservice student teachers continue developing specialized content
knowledge. They learn how to focus on the mathematical meaning and logic behind their students’ thinking.

- Elementary preservice student teachers’ collective interpretation of their students’ work shifts from evaluative toward identification of levels of mathematical sophistication.

- As a result of their experiences with guided professional learning tasks, elementary preservice student teachers connect their collective analysis of their students’ work to their classroom practice. They recognize the importance of taking notes while their students are working and of asking their students probing and/or extending questions.

- They begin to draw upon their specialized content knowledge when attending to, interpreting and identifying levels of sophistication of their students’ mathematical thinking, and finally deciding appropriate next questions to ask their students.

**Redesigned Professional Learning Tasks**

In light of the analysis of the PLT design, a final set of three PLTs was developed. Changes include: allowing 1.5 hours for each PLT session, including these sessions as part of required coursework during the student teaching field experience, including the requirement of no standard algorithm from the onset, and providing more specific directions to the college supervisors regarding the structure of the sessions. I also suggested the addition of a 4th PLT session not focused on SCK, but that would serve as a repeat of session #3 with the preservice student teachers bringing in a larger amount of student work samples. The preservice student teachers would focus on interpreting their students’ mathematical thinking, identify levels of sophistication of strategies and finally deciding on appropriate
probing and/or extending questions based on the students’ mathematical thinking. The revised PLTs and detailed suggestions for their use follow in Figure 25.
Figure 25. Redesigned PLTs with implementation suggestions.
<table>
<thead>
<tr>
<th>PLT</th>
<th>Preservice Student Teacher Directions</th>
<th>College Supervisor Guide</th>
</tr>
</thead>
</table>
| One | Choose two story problems: one that lends itself to an addition strategy and one that lends itself to a subtraction strategy. Ask students to solve them in two ways without using the standard algorithm.  
1. Solve the task yourself and write out ways you anticipate your students will solve the task.  
2. Administer the task and monitor your students as they are working.  
3. Collect all of your students’ work.  
4. Choose four student responses (two for each problem) that you would like to bring to the reflection session.  
Briefly describe their solutions.  
Note: The number of samples to bring to the session should change depending on how many are participating. Four is appropriate for a group of 3 or 4. | Lead a discussion about their initial anticipation of the ways their students would approach the problems. Ask them to share how they chose their six responses to bring to the session.  
Begin student work sample sharing. Ask each of the following questions and have the participants take turns displaying work samples on a document camera as they answer each question. Encourage the participants to collectively analyze the work samples:  
1. What strategies did your students use to solve the task?  
2. What did you find surprising or unexpected in your students’ work?  
3. What is the mathematics embedded in each of their strategies?  
Lead a discussion about different types of addition/subtraction problems and differing levels of complexity depending on the type if time, begin discussion levels of sophistication of strategies.  
Lead a discussion on how to probe student thinking without guiding their work and on how to describe student work without projecting their knowledge onto the solution. Suggest that the preservice student teachers take notes and question their students while they are monitoring. Ask the following question. Once again, have the participants take turns displaying work samples:  
1. What questions could you ask to help your student reflect on their strategy? |
| Two | Choose two story problems: one Put Together/ Take Apart and one Compare problem. Ask students to solve them in two ways without using the standard algorithm.  
1. Solve the task yourself and write out ways you anticipate your students will solve the task.  
2. Administer the task and monitor your students as they are working.  
3. Question your students as they are working and take notes.  
4. Collect all of your students’ work.  
5. Choose four student responses (two from each problem) that you believe represent different mathematical approaches to the problem. Briefly describe their solutions. | Have the preservice student teachers briefly share their task and reasons behind their choices as to which of their students’ work to bring to the session.  
Begin student work sample sharing. Ask each of the following questions and have the participants take turns displaying work samples on a document camera as they answer each question. Encourage the participants to collectively analyze the work samples:  
1. What strategies did your students use to solve the task?  
2. What did you find surprising or unexpected in your students’ work?  
3. What is the mathematics embedded in each of their strategies?  
Discuss how the problem type affected students’ choice of strategies. Then, lead a discussion about levels of sophistication of strategies (direct modeling, counting and number facts). Have students work together to identify levels of sophistication for different work samples. |
| Three | Choose one story problem that can be solved with either an addition or a subtraction strategy. Ask your students to solve it in two ways without using the standard algorithm.  
1. Solve the task yourself and write out ways you anticipate your students will solve the task.  
2. Administer the task and monitor your students as they are working  
3. Question your students as they are working and take notes.  
4. Collect all of your students’ work  
5. Choose four student responses that you believe represent a range of levels of mathematical sophistication to bring to the reflection session.  
6. Read Jason & Ambrose’s (2008) article on developing questions based on student work.  
7. Brainstorm possible questions to ask the three students to assist them in moving towards a higher level of sophistication.  

|---------------------------------------------------------------|

| Begin by reviewing levels of sophistication of strategies. Have the participants come up with examples at each of the three levels: direct modeling, counting, and number facts.  
|---------------------------------------------------------------|

| Have the participants collectively share their student work samples in a round-robin session. Have the student teachers make notes on the mathematics they notice and identify levels of sophistication of strategies.  
|---------------------------------------------------------------|

| Ask each of the following questions and have the participants take turns displaying work samples on a document camera as they answer each question. Encourage the participants to collectively analyze the work samples.  
1. What strategies did the students use to solve the task?  
2. What is the mathematics embedded in each of their strategies?  
3. What level of sophistication is exhibited by each of the students’ strategies?  
|---------------------------------------------------------------|

| Lead a discussion on questioning as a next step in instruction. Discuss the Jacobs & Ambrose (2008) article. Ask the following question. Once again, have the participants take turns displaying work samples.  
1. On the basis of the students’ individual understandings, what question(s) might you pose next to help your students move towards a higher level of sophistication of strategy?  
|---------------------------------------------------------------|

Notes:

- Have the participants submit their written reflections and copies of their chosen work samples a few days prior to the sessions. The facilitators will need to review responses.  
- 90 minutes should be allotted for each of the sessions.  
- If time allows, I suggest having a 4th session. This session should not be focused on development of SCK, but should instead be a repeat of session #3 with the preservice student teachers bringing in a larger amount of samples. Then have them interpret the students’ mathematical thinking, identify levels of sophistication of strategies and finally decide on appropriate probing and/or extending questions based on the students’ mathematical thinking.  
- These sessions can be adjusted based on grade level and mathematical topic of focus.  
- Any of the areas of topic specific SCK development could be replaced based on the topic of focus.
CHAPTER 5

Introduction to the Results

The preceding chapter described findings from the ongoing analysis of the designed professional learning tasks and the evolution of the study’s hypothetical learning trajectory. This chapter details findings from a retrospective analysis of the data collected. I examine data related to the research questions. The results lead to a theory of preservice student teacher professional noticing presented in chapter six.

As detailed in the chapter 3, the retrospective analysis of this study’s data involved identification of idea units and the coding of each of the idea units using the developed descriptive coding scheme. Because this scheme emerged from my data, I begin by discussing it. Each of the four components of the professional noticing framework was divided into levels that described how the preservice student teachers attended, interpreted, identified, and decided. Within each of the levels of coding, the designation “evidence of specialized content knowledge” was added for instances where the idea unit provided evidence of the preservice student teachers applying their SCK to any of the four components of the professional noticing framework. Similarly, within each of the levels of coding the designation “collective influence” was added for instances where the idea unit provided evidence of the preservice student teachers’ collective analysis leading the group towards a higher level on any of the four components of the professional noticing framework. A discussion of each of these levels with exemplars is provided as part of the discussion of the research questions.
Research Question #1

Through their experience with a set of guided, professional learning tasks focused on analysis of their students’ multi-digit addition and subtraction work, in what ways do elementary preservice student teachers:

- Develop the ways in which they **Attend** to their students’ mathematical thinking,
- Develop the ways in which they **Interpret** their students’ mathematical thinking,
- Develop an understanding of and an ability to **Identify** levels of sophistication of their students’ mathematical thinking,
- Develop the ability to **Decide** appropriate next steps in instruction?

Each of the four bullets from this research question directly addresses the four components of the conceptual professional noticing framework that drove the study (see chapter 2). The preservice student teachers’ development on each of the four components is first discussed separately and then briefly considered as a whole.

This study’s developed descriptive coding scheme was based on Jacob’s professional noticing scheme (personal communication, April 3, 2013). As part of the scheme, each of the four components of professional noticing was broken into three levels: level 0: attention lacking; level 1: limited attention; and level 3: robust attention. The study’s phase of ongoing analysis informed the development of the descriptors for each of the levels. The descriptors are presented individually for each of the four noticing components. Information about idea units, session numbers and grade will provided as follows (*idea unit, session #, grade level*); for example (14,1,2) refers to idea unit 14 from PLT session #1 with the second-grade
preservice student teachers. Exemplars are often segments of talk from idea units, not the entire idea unit itself.

**Development of the ways in which preservice student teachers attend to their students’ mathematical thinking.** Attend considers how and the extent to which the preservice student teachers mention mathematically significant details about their students’ mathematical thinking. This research study was focused on preservice student teachers’ analysis of their students’ work; thus, the preservice student teachers generally attended to the mathematics behind their students’ thinking.

**Defining levels of attend.** In order to define levels of attend, I analyzed all attend codes from the ongoing data analysis. For the preservice student teachers, lacking evidence (A0) was described as failing to attend to the mathematics behind their students’ thinking. A0 included the ongoing analysis emergent theme of affect—instances where the preservice student teachers attended to affect instead of the mathematics. Limited evidence (A1) was described as mentioning mathematically significant details of their students’ thinking without specifics or with incorrect details. Robust evidence (A2) was described as correctly discussing some mathematically significant details; to receive a code of A2, not all mathematical details present had to be discussed. Both A1 and A2 included the theme of surprise that emerged from the ongoing analysis—instances where the preservice student teachers attended to their students’ work samples with surprise. The preservice student teachers’ articulation of their surprise as it related to the mathematics behind the students’ strategies determined whether a code of A1 or A2 was assigned. Each of the three levels for of attend could be described as with or without evidence of SCK; SCK refers to research
question #2 and is discussed in the next section. Table 17 includes a summary table of the descriptors for levels of coding for attend; a discussion with exemplars follows.

Table 17.

Descriptors for attend.

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Robust Evidence</td>
</tr>
<tr>
<td></td>
<td>- Mentions CORRECT specifics of mathematics they notice</td>
</tr>
<tr>
<td></td>
<td>- Discussion of strategy type</td>
</tr>
<tr>
<td>1</td>
<td>Limited Evidence</td>
</tr>
<tr>
<td></td>
<td>- Mentions some INCORRECT specifics of mathematics they notice</td>
</tr>
<tr>
<td></td>
<td>- General mention of mathematics including naming strategy</td>
</tr>
<tr>
<td>0</td>
<td>Lacking Evidence</td>
</tr>
<tr>
<td></td>
<td>- Missed opportunity for mentioning mathematics</td>
</tr>
</tbody>
</table>

**Level A0.** Throughout the PLT sessions there were only two instances of A0. Both occurred with the first-grade preservice student teachers. For example, the preservice student teachers discussed factors relating to affect, not mathematics. Leah stated,

“And even I saw that some of the kids that are like really high in math usually in the class, they messed up on one of the problems that was something that they’ve been doing exactly the same in math recently but they were all like, “We have to do college work? This is too hard.” But it’s the same stuff that they were doing, so I think that kind of…kind of stressed them out too.”

(1,1,1)

There were no instances of A0 with the second-grade group. The design of the PLTs was such that the preservice student teachers focused on the mathematics from the beginning. They were interested in looking at their students’ mathematical thinking.
Level A1. There were many instances of limited evidence for attend throughout the PLT sessions. For example, Tara discussed one of her student’s work samples,

“And then some of them, like for one student sample, he did the sticks and dots and he had them broken up into hundreds, tens, and ones but he did them backwards. He put the hundreds on the right side and then tens and ones on the left side but he still got the same answer, he knew what he was doing. So it…I mean I didn’t really see why it wouldn’t be okay to do that, just to…just for him to get the right answer. It kind of made me realize that he knows how to do it.”

This example illustrates Tara’s limited evidence for attending to her student’s mathematics. She mentioned the strategy name, sticks and dots, but was focused on correctness, not on the specifics present in the student’s work. Similarly, Lacey discussed her belief that a student solved a problem incorrectly because the student added-on and did not take-away. She stated, “I considered it subtraction because I feel like most people would subtract 110 minus 41 to figure out how many” (3,1,2). This instance shows Lacey’s incomplete understanding of subtraction and was therefore coded as A1. While focused on the mathematics, her response was general and provided limited evidence of her full understanding of what it means to subtract. The idea units coded as A1 lacked the specifics and discussion characteristics of A2’s robust evidence of attend.

Level A2. For most of the idea units in PLT sessions #2 and #3, the preservice student teachers showed robust evidence for attending to mathematically significant details. In the exchange below Lacey described her students’ strategy for 82 – 53 and began to delve into the students’ thinking (See Figure 26). She mentioned specifics about the mathematics she noticed.
Lacey: This student used the base ten and ungrouped, and then they also used the number line, um, going backwards, and they took 82 apples and then they subtracted, um, 53 from it.

CS: Explain what you learned as a result of the students’ work on the first one, what he or she knows.

Lacey: Well, I can tell they understand what they’re doing, so if they would have done the standard algorithm and done 82 minus 53, I can tell that they understand that they need to ungroup a 10 into ten ones and then take away 3 of them...

Figure 26. Student work sample from (20,3,1).

For another example, the following excerpt refers to the student work shown in Figure 27. The students were answering the following story problem, “While at the orchard, Billy picked 20 apples. Natasha picked 30 apples. Both parents eat two apples each from the apples that Billy and Natasha picked. How many apples do they have altogether?” In this exchange, the preservice student teachers collectively attended to the mathematics behind the student’s mathematical thinking. They mentioned specifics regarding the mathematics, and their attention led them toward an interpretation of the student’s thinking.

Kelli: We can just talk about this one at the bottom, even though it is wrong. The way he chose to solve it was interesting. Um…so he did the 30 + 20 first. He combined the apples into one big group and then took away four, which should have been eight from the top. I mean from the whole group. I thought that was interesting.

CS: Is there anything else you notice about that though?
Leah: He wrote the numbers with the two taking away [inaudible 24:23]
CS: Yeah. The eighteen…
Kelli: The 18 and the 28.

Kelli: He probably counted backwards. I can’t…I should ask him.

Kelli: I don’t know if it’s math thinking, but that he has to work in steps. That he chose to do the 30 + 20 first. Like chunked it.
Leah: Because that’s what the problem said. Like he probably knew after reading that that’s what…he knew he needed to add first.

(13,2,1)

Figure 27. Student work sample from (13,2,1).

Attend over time. In order to provide a snapshot of the preservice student teachers’ development of the ways in which they attend to mathematically significant details, I created a table that shows the coded levels for attend throughout all of the idea units for all three PLT sessions. Table 8 and Table 19 show the subset of the full results that includes the preservice student teachers’ attend levels for the idea units that included discussions about their student work samples. For example, idea unit (7,3,1) contains a discussion about students’ understanding of number lines; this idea unit does not appear in Table 18 because it does not include a discussion about a particular piece of student work. However, the idea unit was
coded as A2 because the group was attending to mathematically significant details. I chose to focus on the subset of idea units dealing with student work since student work is the focus of this study. Note: blank entries are for idea units that included discussion of student work samples, but the discussion did not lend itself towards the particular component of professional noticing. For example, when the college supervisor asked Tara to generalize levels of sophistication of her students’ strategies, Tara equated multiple methods with higher levels of sophistication (12,1,2). The conversation did not lend itself toward her specific attention to mathematically significant details, thus attend was not coded and was left blank; however, the idea unit did receive a code for identify.

Table 18.

*证据 of SCK Collective Influence* Attend for idea units dealing with student work throughout the PLT sessions—first grade.

<table>
<thead>
<tr>
<th>1st Grade</th>
<th>Session1</th>
<th>Session2</th>
<th>Session3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Idea Unit</td>
<td>1 6 8 9 10 11 12 13 19</td>
<td>1 2 4 5 6 7 8 9 10 11 12 13 15 16 17 12 13 14 16 17 18 19 20</td>
<td></td>
</tr>
<tr>
<td>Attend</td>
<td>0 2 1 1 1 1 2 1 2* 2 1 1 2 2 2 1 2 2 2* 1 1 2* 2 2 2*</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 19.

*证据 of SCK Collective Influence* Attend for idea units dealing with student work throughout the PLT sessions—second grade.

<table>
<thead>
<tr>
<th>2nd Grade</th>
<th>Session1</th>
<th>Session2</th>
<th>Session3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Idea Unit</td>
<td>1 2 1 1</td>
<td>1 2 2 1 2 2 1 1 2 1</td>
<td>2 2 2 1 2 2 2 2 2 2 2 2</td>
</tr>
<tr>
<td>Attend</td>
<td>1 2 1 1 1 1 2 1</td>
<td>1 2 2 1 2 1 2 2 2 2 2 2</td>
<td></td>
</tr>
</tbody>
</table>

*Evidence of SCK Collective Influence*
Both groups of preservice student teachers focused on their students’ mathematical thinking, thus the preservice student teachers attended to mathematically significant details. Through analysis of the results, it is clear that the preservice student teachers’ development of their ability to attend to their students’ mathematical thinking moved from limited evidence to robust evidence as they progressed through the PLTs; they shifted from mentioning incorrect mathematics and naming strategies to mentioning correct mathematics and discussing specific strategy types. I attribute this growth both to the preservice student teachers’ experiences with the PLTs in which they developed SCK, and to their experiences in their classrooms where they had the opportunity to attend to their students’ mathematical thinking on a daily basis.

Table 20 contains the combined summary results for attend for both groups of preservice student teachers and all idea units, not just those dealing with student work samples. As part of the design of the PLTs, the preservice student teacher directions were designed to focus the preservice student teachers on mathematically significant details exhibited in their students’ work. In response to this design, the preservice student teachers began PLT session #1 with an understanding that they would be expected to discuss mathematics. In considering the data as a whole for both groups, as the preservice student teachers progressed through the PLTs, they quickly developed their ability to attend to their students’ mathematical thinking. Their attention to mathematically significant details increased throughout the sessions. The total number of idea units in which the preservice student teachers attended to mathematically significant details remained somewhat constant throughout the three PLTs; however, the percentage of A2 grew while A1 decreased. This
once again shows that the design of the intervention allowed the preservice student teachers to focus on attending to mathematically significant details of their students’ thinking.

Table 20.
*Combined attend for all idea units for both groups throughout the PLT sessions.*

<table>
<thead>
<tr>
<th>Attend</th>
<th>PLT 1</th>
<th>PLT 2</th>
<th>PLT 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>16</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>21</td>
<td>32</td>
</tr>
<tr>
<td>Total</td>
<td>33</td>
<td>31</td>
<td>35</td>
</tr>
</tbody>
</table>

**Development of the ways in which preservice student teachers interpret their students’ mathematical thinking.** Interpret considers what the preservice student teachers indicate that they know about their students’ mathematical thinking; it is looking at the extent to which the preservice student teachers develop plausible interpretations of their students’ thoughts and mathematical understandings.

**Defining levels of interpret.** In order to define levels of interpret, I analyzed all interpret codes from the ongoing data analysis. For the preservice student teachers, lacking evidence (R0) was described as missing an opportunity for mathematical interpretation. Limited evidence for interpret (R1) was described as providing evidence when interpreting students’ mathematical thinking but developing a vague interpretation. R1 included the ongoing analysis emergent themes of difficult to interpret and influenced interpretations—instances where the preservice student teachers had difficulty interpreting their students’
mathematical thinking or where the preservice student teachers interpreted their students’ mathematical thinking based on their own personal experiences. Robust evidence (R2) was described as developing a coherent and plausible interpretation of the students’ mathematical thinking. This code reflected the ongoing analysis emergent theme of plausible interpretations. For interpret, the codes do not refer to a “best” interpretation, just whether or not the preservice student teachers’ reasoning was consistent with the specific mathematical details found within their students’ strategies. As with attend, each of the three levels could be described as with or without evidence of SCK; SCK refers to research question #2 and is discussed in the next section. Table 21 includes a summary table of the descriptors for levels of coding for interpret; a discussion with exemplars follows.

Table 21.
*Descriptors for interpret.*

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Robust Evidence</td>
</tr>
<tr>
<td></td>
<td>• Draws on evidence when giving a plausible interpretation what a student understands</td>
</tr>
<tr>
<td></td>
<td>○ Coherent discussion of students’ mathematical thinking</td>
</tr>
<tr>
<td>1</td>
<td>Limited Evidence</td>
</tr>
<tr>
<td></td>
<td>• Draws on some evidence when interpreting what a student understands</td>
</tr>
<tr>
<td></td>
<td>○ Making assumptions</td>
</tr>
<tr>
<td></td>
<td>○ Implausible interpretation</td>
</tr>
<tr>
<td></td>
<td>○ Interpretation hard to follow (vague and/or incomplete)</td>
</tr>
<tr>
<td>0</td>
<td>Evidence Lacking</td>
</tr>
<tr>
<td></td>
<td>• Missed opportunity for mathematical interpretation</td>
</tr>
<tr>
<td></td>
<td>• Interpretation lacking mathematical evidence</td>
</tr>
</tbody>
</table>
Level R0. Both groups of preservice student teachers exhibited instances where they lacked evidence that they could interpret their students’ mathematical thinking. For example, Ashton made the following observation about her student’s work for 104 – 63 shown in Figure 28, “it was interesting that he set it up like a subtraction problem and put the bar under...like he was subtracting them instead of like crossing them out like you usually see with, um, with using this...the drawings” (6,2,2). This exchange was coded as R0 because the preservice student teachers missed an opportunity to interpret the mathematics behind the student’s thinking. They did not consider how the student solved the problem given the setup or what the student’s approach told them about the student’s mathematical understanding.

![Figure 28. Student work sample from (6,2,2).](image)

Another example of R0 can be seen in the following exchange. Tammy’s interpretation of her student’s work is focused on correctness. Tammy described the student’s strategy, but she did not conjecture as to what the student may have been thinking.

Tammy: Like I know that this person just continued on with their circles. They didn’t draw a line to separate their 18 and their 5 but they continued on in making that 23.
CS: Okay. But do you think that hindered their ability to solve the problem or was that helpful or did it matter or…
Tammy: In this case it helped ‘cause they got it right.

(19,1,1)

Level R1. Limited evidence of the preservice student teachers’ interpretations of their students’ mathematical thinking was present throughout all three PLT sessions. The preservice student teacher’s interpretation of the student’s mathematical thinking for the problem seen in Figure 29 was incomplete. In looking at the student’s work, it was unclear whether the student counted on or combined tens and ones. The preservice student teachers made an assumption that it was counting on; Kelli stated, “I think he counted the tens first for 10, 20, 30, and then counted all the 1’s” (9, 2, 1). She goes on to note that she should have asked him. The group does not consider other possible interpretations which is why this exchange was considered incomplete.

Figure 29. Student work sample from (9,2,1).

Another example is when Tammy made assumptions about her student’s understanding of grouping (See Figure 30). Tammy stated, “this is like a huge, how to group like this in fives…she’s trying to understand grouping, I think, from what I’ve seen” (8,1,1).
While she provided evidence, her interpretation lacked a discussion of what she meant by grouping or what the student may have been thinking, thus this exchange was coded as R1.

*Figure 30. Student work sample from (8,1,1)*

In the following exchange, Lacey’s interpretation of her student’s work was incomplete and coded as R1. Lacey was surprised by her student’s approach to the problem and was more focused on how she expected her student to solve the problem, than what her student’s approach said about the student’s mathematical thinking.

Lacey: she actually drew out 29 additional dots. Um, that surprised me just because knowing this student and knowing what she is capable of, it surprised me that she went back that far to the one to one correspondence. She’s the one that I mentioned before that is in between. Um…But she’s more on grade level. So…it surprised me that she went back that far and didn’t use the traditional algorithm at all because this problem is one that we have been working with before so she should know, um, how to do it with the traditional algorithm.

CS: So she didn’t use a traditional algorithm at all?

Lacey: I don’t think…No. She didn’t at all. And see, the first thing she did, it looks like if she did this first, I would assume since it’s at the top, was to go through and write 65 and then draw 29 additional circles and then make tens to figure out how many more she had.

(18,1,2)

**Level R2.** All three PLTs contained examples of R2. For example, Donna interpreted her student’s mathematical thinking for the work sample seen in Figure 31 as follows,

“Well he’s using ones, like just ones. But he made the new ten here. Can you see…like he knew…he knew that I had 9 red apples and so he wrote nine little circles
for ones and then he wrote how ever many more he needed to get to 17 but he made his new ten. 9 and 1, so he knew that that’s 10 + 7 was my 17.”

(1,2,1)

Donna’s interpretation was robust in that she provided evidence for a plausible interpretation.

Figure 31. Student work sample from (1,2,1).

In the following exchange, the preservice student teachers’ collective analysis led to a robust interpretation of the student’s mathematical thinking. During this exchange, the preservice student teachers were discussing the student’s work for 101 – 62 shown in Figure 32.

Lacey: Well I said I thought that the expanded form she just made a little mistake there but I think it was great that she recognized that that wasn’t the correct answer and by doing it the second way, and I thought it was interesting that she started with 101 and then went backwards on her number line instead of starting with 62 and going up, which compared to some of the other number lines that I saw with this problem, I thought that was a little bit more sophisticated thinking than going up from 62.

Lacey: After she realized that she was one off because if you look at what she did, she just put…took the 10 and didn’t add it back in with the 1 so it was 10 - 2 instead of 11 - 2 and she recognized that, but instead of changing it she just circled it and put, “Not right,” and did a different way.

Tara: I think that’s more sophisticated then that she did recognize it wasn’t right.

Lacey: Which I was happy that she recognized that it wasn’t right.

(6,3,2)
As Lacey stated, the student made a mistake with borrowing in the circled part of the student’s solution. The student solved the problem in two other ways, both subtracting by taking away. The student believed her answer to be 39 and while the student did not correct her work in the circle, the student herself indicated its incorrectness. The preservice student teachers and the college supervisor continued to discuss the differences between the two other solutions and came to the conclusion that they both illustrate the same mathematical understanding, that of subtraction as “removing.” The evidence present in the student work, marking out and jumping back on the number line, helped the preservice student teachers develop their robust interpretation of the student’s mathematical thinking.

**Interpret over time.** As was done with attend, I created a table that shows the coded levels for interpret throughout all of the idea units for all three PLT sessions. Table 22 and Table 23 show the subset of the full results that includes the preservice student teachers’ interpret levels for the idea units that included discussions about their student work samples.
Appendix I contains the table with the results for all idea units and all levels of all of the components of professional noticing.

Table 22.

*Interpret for idea units dealing with student work throughout the PLT sessions—first grade.*

<table>
<thead>
<tr>
<th>Idea Unit</th>
<th>Session 1</th>
<th>Session 2</th>
<th>Session 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpret</td>
<td>2 1 0 1 1</td>
<td>0 2 2 1 1</td>
<td>1 0 1 2 1</td>
</tr>
</tbody>
</table>

*Evidence of SCK Collective Influence

Table 23.

*Interpret for idea units dealing with student work throughout the PLT sessions—second grade.*

<table>
<thead>
<tr>
<th>Idea Unit</th>
<th>Session 1</th>
<th>Session 2</th>
<th>Session 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpret</td>
<td>0 1 0 1 1</td>
<td>1 2 1 2 1</td>
<td>0 2 2 0 2</td>
</tr>
</tbody>
</table>

*Evidence of SCK Collective Influence

The preservice student teachers began PLT session #1 exhibiting variation in their levels of interpretation of their students’ mathematical thinking; all three levels were present. During PLT session #2, the variation of levels of interpret was once again evident; there were instances of all three levels. For PLT session #3, both groups of preservice student teachers showed more R2s, than R1s. No R0s were present during PLT session #3. Table 24 contains the combined summary results for interpret for both groups of preservice student teachers and all idea units, not just those dealing with student work samples.
Instances of R0 were few and decreased over time. For most of the idea units, the preservice student teachers provided limited or robust evidence when interpreting their students’ mathematical thinking. The preservice student teachers were mostly at least trying to understanding their students’ mathematical thinking. While the preservice student teachers did not reach a point when they were only exhibiting R2s, they did show progress. Throughout the PLT sessions, the preservice student teachers began to learn how to draw on evidence when interpreting their students’ mathematical thinking. Their ability to interpret moved from a mix of missed opportunities and vague interpretations to a mix of vague and plausible interpretations. By the end of the PLTs, their interpretations oscillated between vague and/or implausible, and coherent and/or plausible. The possible factors influencing the preservice student teachers’ mixed results for interpret will be discussed as a part of the results for the third research question. As seen with attend, the total number of idea units in which the preservice student teachers interpreted their students’ mathematical thinking remained somewhat constant throughout the three PLT sessions. This once again shows that
the design of the intervention allowed for the preservice student teachers to focus on analyzing, and hence interpreting, their students’ mathematical thinking.

**Development of the preservice student teachers’ understanding and ability to identify levels of sophistication of their students’ mathematical thinking.** Identification of levels of sophistication of strategies was my addition to Jacobs et. al. (2010)’s professional noticing framework. Identification refers to identifying children’s multi-digit addition and subtraction strategies’ level of sophistication. One of the foci for this study was developing the preservice student teachers’ SCK and KCS about levels of sophistication of strategies. The college supervisors asked the preservice student teachers to discuss levels of sophistication during all three PLT sessions. Recall the levels of sophistication presented in this study:

<table>
<thead>
<tr>
<th>Levels of Sophistication of Strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Direct Modeling</td>
</tr>
<tr>
<td>2. Counting</td>
</tr>
<tr>
<td>3. Number Fact Strategies: making a ten, decomposition, creating equivalent but easier problems—all draw on knowledge of place value, properties of operations and/or relationship between addition and subtraction</td>
</tr>
</tbody>
</table>

**Defining levels of identify.** As with the previously presented components of the professional noticing framework, I analyzed all identify codes from the ongoing data analysis as a means to define levels of identify. For the preservice student teachers, lacking evidence (I0) was described as missing an opportunity for identification of levels of mathematical sophistication. Limited evidence (I1) was described as providing evidence but misidentifying levels of mathematical sophistication. Robust evidence (I2) was described as providing evidence and correctly identifying levels of mathematical sophistication. All three of these levels incorporated the ongoing analysis theme of levels of sophistication of strategies. The
coded level depended on the evidence provided relating to identification of levels of sophistication. As with the previous components, each of the three levels could be described as with or without evidence of SCK; SCK refers to research question #2 and is discussed in the next section. Table 25 includes a summary table of the descriptors for levels of coding for identify; a discussion with exemplars follows.

Table 25.

Descriptors for identify.

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Identification Lacking</td>
</tr>
<tr>
<td>1</td>
<td>Identification Lacking</td>
</tr>
<tr>
<td>2</td>
<td>Robust Identification</td>
</tr>
</tbody>
</table>

- Limited Identification
  - Draws on specifics but incorrectly identifies strategy or LOS

Level 0. Throughout the PLT sessions, both groups of preservice student teachers were provided opportunities in which they could have identified levels of sophistication but either missed the opportunity or incorrectly identified levels of mathematical sophistication. There were multiple instances of 0 in the second-grade PLT session #1. Most of these instances illustrated the preservice student teachers’ reliance on the standard algorithm. In the following exchange, Tara responded to the college supervisor’s question about the sophistication of one of her student’s four different strategies.
CS: And in terms of, um, sophistication of strategies, can you make any generalizations from what you see? Any methods or students that are generally, um, more or less sophisticated than others?

Tara: Um, I think the students that did four different methods as opposed to like this one did all three methods. So I would say the ones that did use the algorithm and did more...did four methods were the more sophisticated students and the ones that got the answer each time...

CS: Do you ever, um, have conversations with your class about, um, which of these solution strategies are most sophisticated or most efficient with him?

Tara: Um, well we have talked about how the algorithm is the quickest way to do it, but the visual, like the sticks and dots show how the numbers work together when you’re adding them together. So as far as sophistication, I think if they were able to do the basic algorithm then they would already know what the numbers mean after they’ve done something more visual, so this would be the more sophisticated.

(12,1,2)

Tara believed that the standard algorithm was the most sophisticated. This belief persisted into PLT session #2, but by PLT session #3, the second-grade preservice student teachers realized that the standard algorithm was hindering their students’ ability to think about the mathematics and also their ability to diagnose their students’ mathematical thinking.

Other instances labeled as I0 were for instances where the preservice student teachers were asked about levels of sophistication but did not respond. In the first-grade PLT session #2, the college supervisor asked the group to comment on levels of sophistication while they presented their student work. Many times, the preservice student teachers would be eager to move to the next sample of student work and would skip the identification of level of sophistication.

*Level I1.* Limited evidence for identification of level of sophistication of strategy occurred thought the PLT sessions. In the following exchange, the preservice student teachers discussed the sophistication of a student’s counting up strategies.
Lacey: Which one do I feel like is more sophisticated?
CS: And how are they alike and different?
Lacey: They’re alike because they’re both using counting up. I would say the number line is a little bit more sophisticated than the base ten because I see that more as a transitional. I guess you could go either way but I see that more as a transitional strategy than a concrete or a representational…it’s more symbolic than…
Tara: Than having the actual numbers.
Lacey: Right. Having actual numbers in it. I guess you could argue that either way
Tara: Because 89 is 89. It’s not representing anything else.
Lacey: Exactly. It’s not a base ten 89 so I see it more as a transitional strategy.

In this example, Tara and Lacey discussed how they believed number lines to be more sophisticated than counting up with sticks and dots. This exchange was labeled as I1 because the preservice student teachers drew on evidence to label the sophistication, but their discussion did not describe what was meant by “transitional” or how transitional related to the three presented levels of sophistication.

**Level I2.** Not until PLT session #3, did the college supervisors focus on naming and identifying levels of sophistication. It is not surprising that most of the coding of I2 occurred during PLT session 3; focusing on the CCSS-M NCDPI (2012) Unpacking Document provided the preservice student teachers with vocabulary that they could use to describe their students’ strategies. In the following exchange, the preservice student teachers exhibited evidence of language development.

Kelli: So, this student used the…I guess we call it a decomposition method; in our classroom we call it the building method. So she started out by breaking down the 20 and the 6…when we originally learned this method, we did that on paper but she already knew that in her head so she went ahead and said 20 and 40 ‘cause that’s how many tens that are in each number and then 6 and 3 ‘cause that’s how many ones are in each number to add them together to get 69. So I guess I just would ask her why, just to see what she says. You know, ask her, “Well, why did you do 20+40 and then 6+3?” just to see if she just
speak, "'Cause I already knew that 20 and 6 was broken down." I guess I could 
ask her if there was another way she could solve it or if she would check it, 
how she would check it to make sure she was right.

CS: Anybody else see anything or have a contribution to the kinds of questions or 
as a teacher how you might respond to that?

Tammy: I like how the student used…that’s what I try to do with my students, is add 
the tens first, then add the ones so that it’s getting away from the standard 
algorithm…I have one boy who does it this way all…he does it going down 
but it works every time and I will give him the hardest problems I can think 
of, where it is like three and four digit ones. [inaudible 31:50] so he goes all 
the way down and it really just aligns the…it keeps them as tens and it keeps 
the right place value when you separate it out like this.

CS So what level of sophistication would this be?

Tammy: Level 3.

CS: Yeah, it would be a Level 3. Right. A higher level.

Kelli: And it’s decomposition but it’s also creating equivalent but easier problems 
because it’s easier to add 20, 40, and a 6 and 3 together. I mean even I 
personally use this method when I’m checking their work in my head because 
I know that it’s pretty efficient.

Donna: We weren’t taught this. We developed that for ourselves and realized…I 
showed my mentor teacher this before I even knew this was going to be in my 
unit. She was like, “Oh, that’s so much easier.” Like I know.

(13,3,1)

In the exchange, the preservice student teachers interpreted the student’s strategy and used 
CCSSM (2010) language to correctly identify the level of sophistication.

**Identify over time.** Once again, I created a table that shows the coded levels for 
identify throughout all of the idea units for all three PLT sessions. Table 26 and Table 27 
show the subset of the full results that include the preservice student teachers’ identify levels 
for the idea units that included discussions about their student work samples. Appendix I 
contains the table with the results for all idea units and all levels of all of the components of 
professional noticing.
Table 26.

*Identify for idea units dealing with student work throughout the PLT sessions—first grade.*

<table>
<thead>
<tr>
<th>1st Grade</th>
<th>Idea Unit</th>
<th>Identify</th>
<th>Session 1</th>
<th>Session 2</th>
<th>Session 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 6 8 9 10 11 12 13 14 15 16 17 18 19 20</td>
<td></td>
<td>1 6 8 9 10 11 12 13 14 15 16 17 18 19 20</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Identify</td>
<td></td>
<td>1 0 2 1 0 1 0 0 0 0 0 0 0 0 0 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Evidence of SCK</em></td>
<td>Collective Influence</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 27.

*Identify for idea units dealing with student work throughout the PLT sessions—second grade.*

<table>
<thead>
<tr>
<th>2nd Grade</th>
<th>Idea Unit</th>
<th>Identify</th>
<th>Session 1</th>
<th>Session 2</th>
<th>Session 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3 4 5 6 7 9 10 12 14 15 16 18 19 20 21 22 23 24 25 26 6 8 9 11 14 15 16 17 18 19 20</td>
<td></td>
<td>3 4 5 6 7 9 10 12 14 15 16 18 19 20 21 22 23 24 25 26 6 8 9 11 14 15 16 17 18 19 20 21</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Identify</td>
<td></td>
<td>1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>Evidence of SCK</em></td>
<td>Collective Influence</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

During PLT session #1 & #2, the preservice student teachers’ identification of levels of sophistication of strategies was mostly I0 or I1. The preservice student teachers often missed opportunities to identify levels of sophistication, discussed sophistication without providing evidence, or drew on evidence but incorrectly labeled levels of sophistication. As the PLT sessions progressed, the preservice student teachers began to draw upon evidence to correctly identify levels of sophistication of strategies. This robust identification, I2, mostly occurred during PLT session #3, but for both groups during PLT session #3, all three levels of identification remained.

Table 28 contains the combined summary results for identify for both groups of preservice student teachers and all idea units, not just those dealing with student work.
samples. By design, both the 2\textsuperscript{nd} and 3\textsuperscript{rd} PLTs focused on developing SCK and KCS regarding level of sophistication of strategies. These foci assisted the preservice student teachers as they developed their ability to identify the levels of mathematical sophistication of their students’ strategies. The total number of idea units in which the preservice student teachers had the opportunity to identify level of sophistication of their students’ mathematical thinking remained constant between PLT session #1 and PLT session #2. Differently than with attend or interpret, the number of idea units coded as identify greatly increased for session #3. This growth can be attributed to the design and implementation of the PLT session, but also to the preservice student teachers developing SCK and KCS which allowed them to discuss levels of sophistication of strategies for multi-digit addition and subtraction.

Table 28.

*Combined identify for all idea units for both groups throughout the PLT sessions.*

<table>
<thead>
<tr>
<th>Identify</th>
<th>PLT 1</th>
<th>PLT 2</th>
<th>PLT 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>17</strong></td>
<td><strong>16</strong></td>
<td><strong>26</strong></td>
</tr>
</tbody>
</table>

It is once again interesting to look at the number of missed opportunities or incorrect identification of levels of sophistication of strategies. The number of I0s remains somewhat constant, which shows the preservice student teachers still have room for growth in this component of professional noticing. They were making progress and moving toward more
I2s, but identifying their students’ mathematical thinking’s level of sophistication was not easy. It takes practice and continual development of SCK and KCS.

**Development of preservice student teachers’ ability to decide on appropriate next steps in instruction.** The design and implementation of each of the PLTs greatly affected the coding of Decide. As was discussed in chapter 4, the preservice student teachers had a difficult time developing questions to ask their students. For the preservice student teachers, therefore, deciding on an appropriate next step in instruction was about questioning.

**Defining levels of decide.** To define levels of decide, I analyzed all decide codes from the ongoing data analysis. For the preservice student teachers, lacking evidence (D0) was described as failing to develop a potential question to probe or extend students’ thinking. Limited evidence for decide (D1) was described as choosing a vague question without specific details. Robust evidence (D2) was described as drawing on specifics to develop a potentially useful probing or extending question. All three of these levels incorporated the ongoing analysis theme of importance of questioning. The coded level depended on evidence provided relating to development of potentially useful probing or extending questions. For decide, the codes do not refer to a “best” question, just whether the developed questions were based on evidence of the students’ mathematical thinking. Table 29 includes a summary table of the descriptors for levels of coding for interpret; a discussion with exemplars follows.
Table 29.

Descriptors for decide.

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
</table>
| 2     | Robust Evidence  
|       | - Draws on specifics to develop a potentially useful probing or extending question |
| 1     | Limited Evidence  
|       | - Develops a probing or extending question  
|       |   - Question not useful or vague  
|       |   - Question potentially useful but not drawing on specifics |
| 0     | Decision Lacking  
|       | - No question developed  
|       | - No mention of students’ mathematical thinking |

**Level D0.** The code D0 was only coded twice, both with the first-grade group during PLT session #3. First, the code was assigned to an instance when the preservice student teachers were asked to formulate a potential question and did not, so the college supervisor interjected a question of her own (22,3,1). Second, the code was assigned to an instance where I, as the participant observer, asked about questioning a student. In the talk exchange following my comment, a preservice student teacher mentioned a hundreds board, at which point the conversation shifted to hundreds board strategies and no answer about questioning was provided (19,3,1).

**Level D1.** Examples of D1, limited evidence include the following exchange where the college supervisor asked Donna what she would do to extend her student’s thinking. Donna responded, “I mean I would want to talk with him through it so like him reflecting and explaining his own thinking was I didn’t tell him to take out a 10 and translate that to ten ones. That was his own doing” (14,3,1). This idea unit was labeled as D1 because Donna
vaguely referred to having her student explain his thinking, but did not develop a specific question to ask her student.

**Level D2.** The following exchange was coded as D2 because Ashton drew on specifics both from the problem she posed to her students as well as from the student’s answer to choose questions that would extend the student’s thinking toward sense making.

Ashton stated,

“I would definitely ask them if it makes sense. Like go back to reading the problem because if there’s 24…I don’t even know if it was tigers or lions…some kind of big cat. Um…and then three elephants. You know. “Okay, will you do it on your fingers?” You know. “Show me how that would work without even writing it out.” You know. “How many would there be? It couldn’t be?” You know. And kind of talk through it that way like it couldn’t be 54. “

The following exchange also illustrates the code D2; the exchange refers to the student work previously shown in Figure 22 (p. 113). In the exchange, the preservice student teachers discussed how the student made a new 10 and then recognized that ten 10s makes one hundred.

Kelli: I think beforehand I would want to maybe ask him what he knows about the problem, like what’s the problem asking him to do. And then afterwards I guess I might would say, “Explain your strategy. Why do you have all these numbers here?” and, again, “Why do you have separate columns? What’s your thought process in that?” And a number sentence to go with the problem so I could see what he was thinking.

Donna: It’d also be interesting to see too if he could use another strategy besides that one every time. ‘Cause that’s sophisticated. I haven’t really seen a child do that.
In their exchange, the preservice student teachers drew on evidence from the student’s work and considered the types of probing and extending questions they discussed previously in the session to decide on appropriate, potentially useful questions to pose to this student.

**Decide over time.** The coded levels for decide throughout idea units for all three PLT sessions are located in Table 30 and Table 31. These tables show the subset of the full results that include the preservice student teachers’ decide levels for the idea units that included discussions about their student work samples. Appendix I contains the table with the results for all idea units and all levels of all the components of professional noticing.

Table 30.

*Decide for idea units dealing with student work throughout the PLT sessions—first grade.*

<table>
<thead>
<tr>
<th>Idea Unit</th>
<th>1st Grade</th>
<th>Session 1</th>
<th>Session 2</th>
<th>Session 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decide</td>
<td>1 6 8 9 10 11 12 13 19 1 2 4 5 6 7 8 9 10 11 12 13 15 16 17 12 13 14 16 17 18 19 20</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 31.

*Decide for idea units dealing with student work throughout the PLT sessions—second grade.*

<table>
<thead>
<tr>
<th>Idea Unit</th>
<th>2nd Grade</th>
<th>Session 1</th>
<th>Session 2</th>
<th>Session 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decide</td>
<td>3 4 5 6 7 9 10 12 14 15 16 18 19 20 21 22 23 24 25 26 6 8 9 11 14 15 16 21 5 6 7 9 11 12 14 15 16 17 18</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Evidence of SCK: Collective Influence*

The design focus and implementation of the PLTs greatly affected the coding of decide. In the second-grade PLT session #1, the college supervisor pressed the preservice student teachers to develop questions for their students anytime the preservice student
teachers had difficulty interpreting their students’ mathematical thinking. Some of the
preservice student teachers’ responses lacked evidence for their chosen question while some
provided limited evidence. In contrast, the first-grade college supervisor did not focus on
questioning during PLT session #2, which is why only one idea unit received a decide code.
That one instance was during an exchange where a preservice student teacher independently
provided a question she wished she had asked her student. PLT session #2 was not designed
with a focus on questioning because I had expected the preservice student teachers to
question their students during implementation of their tasks for PLT #2. This lack of focus on
questioning is evident in the fact that only two idea units were given a code for decide in
either group’s PLT sessions. In order to assist the preservice student teachers in their
development of their ability to decide, selecting appropriate questions for probing and/or
extending became the focus for PLT session #3. For PLT session #3, the first-grade group
alone had 11 instances of decide codes dealing with student work samples. The supervisor
focused on questioning throughout the session.

Table 32 contains the summary results for decide. Limited evidence was present for
decide in all three PLT sessions; yet, there was a noticeable increase in the amount of D2s in
PLT session #3. The preservice student teachers were learning how to develop appropriate
probing and extending questions to ask their student, albeit with support and sample
questions provided. In further considering the results in the table, it is important to note that
the reason for no D0s in PLT sessions 1 and 2 was the lack of focus on questioning. Once the
preservice student teachers were asked to question in PLT session #3, there were instances
where they failed to do so.
Table 32.

**Combined decide for all idea units for both groups throughout the PLT sessions.**

<table>
<thead>
<tr>
<th></th>
<th>PLT 1</th>
<th>PLT 2</th>
<th>PLT 3</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decide</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>Total</td>
<td>7</td>
<td>2</td>
<td>15</td>
<td>24</td>
</tr>
</tbody>
</table>

Overall analysis of decide for the PLT sessions shows the importance of helping preservice student teachers focus on questioning as an instructional step in their classroom. Interpretation of students’ mathematical thinking was not sufficient, nor was identification of sophistication of strategies. The preservice student teachers needed to learn how to synthesize the information that they gathered from interpreting and identifying, and apply it to decisions they made about how to teach their students. While the preservice student teachers showed evidence of growth in the area of developing questions to ask their students, this growth was because of the PLTs’ design and the college supervisors’ questioning of the preservice student teachers. Questioning was not an immediate step for the preservice student teachers.

**Development of Professional Noticing.** While the research question asked how the preservice student teachers developed in their separate ability to attend, interpret, identify and decide, it is helpful to analyze how the preservice student teachers progressed through the professional noticing framework as a whole. Tables 33 and 34, once again contain the subset of the coding results for all four components of the professional noticing framework for
which discussions focused on student work samples (See Appendix I for the table with all results; the patterns and relationships discussed for the tables below also apply to the data when analyzed as a whole).

Table 33.

Professional noticing codes for idea units dealing with student work throughout the PLT sessions—first grade.

Table 34.

Professional noticing codes for idea units dealing with student work throughout the PLT sessions—second grade.

For the most part, the preservice student teachers learned to notice over time, but their growth can be seen in two directions in the tables. The horizontal line represents the change from limited evidence toward robust evidence for each of the professional noticing
components, visible in all four rows of the table. The diagonal line represents the preservice student teachers’ development in their overall ability to notice (all four components). It is important to note that the results are relatively similar for both groups of preservice student teachers, i.e. both iterations of the PLT design with different students and different facilitators yielded very similar results. Analysis of these tables provides important results to be considered:

- While the patterns can be seen, they do not show strictly linear growth. While students progressed from level 0 to level 2, once they reached a level 2 it did not mean that they did not return to a lower level. There is an in-between period because professional noticing is fluid and in-the-moment. One should not expect for preservice teachers to master noticing during such a short intervention.

- The attend level was always equal to or greater than the labels for the rest of the professional noticing components. Thus, attention proved to be a pre-requisite for further professional noticing. Without focusing on mathematically significant details, interpretations, identifications, and decisions about questioning were always limited or lacking. As robust attending increased, the other components on the professional noticing framework did so as well.

- Interpret, identify and decide did not always occur at the same level simultaneously. The preservice student teachers were constantly developing their professional noticing and did not reach a final stage.
The preservice student teachers sometimes provided evidence for identify or decide before mastering interpret. It was possible to have limited evidence for interpret but to still be able to provide robust evidence for identify or decide (ex. 18,3,1)

**Research Question #2**

*How does development of specialized content knowledge relate to the elementary preservice student teachers’ learning to professionally notice through their experience with a set of guided, professional learning tasks focused on analysis of their students’ multi-digit addition and subtraction work?*

Each of the three PLT sessions was designed with a goal of developing a specific aspect of specialized content knowledge. PLT session #1 focused on different types of multi-digit addition and subtraction problems, PLT session #2 focused on levels of sophistication of strategies (which also included development of KCS), and PLT session #3 focused both on levels of sophistication of strategies and deciding on appropriate probing and/or extending questions. As part of this study, I was interested in how the focused development of specialized content knowledge related to the preservice student teachers’ learning to notice. As was discussed in chapter 3, within each of the levels of coding of each of the components of professional noticing, the designation of “with evidence of specialized content knowledge” was added for instances where the idea unit provided explicit evidence of the preservice student teachers’ applying their SCK to any of the four components of the professional noticing framework. Before presenting the results of the SCK analysis, I will discuss the explicit instances of SCK as they appeared throughout the progression of the PLTs.
Prior specialized content knowledge was exhibited by some of the preservice student teachers. Donna and the college supervisor had an exchange about the task Donna gave to her students. Donna had created a worksheet that asked her students to look at single-digit addition problems and label the most appropriate strategy out of: “make-a-ten,” “counting on,” and “doubles plus one.” Donna’s attention to mathematically significant details and her interpretations of her students’ mathematical thinking showed evidence of SCK. For example, in part of the idea unit, she stated, “I found that they really don’t have a concept of like making a ten or using the double strategy or the doubles plus one, but they all can count on. Like if they have 5 + 4, they go: 1, 2, 3, 4, 5, 6, 7, 8, 9 like that, that’s how they do it. They don’t see that it’s 5+5 or that it’s 4+4+1 or doubles plus one. They just count on” (emphasis in original) (2,1,1). She went on to discuss individual conferences she had with her students following the assignment. She asked them, “You know, counting on does work but it’s not going to work every time for you. What’s a more efficient strategy for you to use?” During the exchange, Donna showed explicit evidence of her SCK regarding different strategies for single-digit addition. She was able to evaluate her students’ explanations and recognize areas where she wanted to push her students to use a more efficient strategy. Donna possessed this SCK prior to the PLT session, and she used it to assist in her interpretation of her students’ mathematical thinking.

Kelli also drew on her previously held SCK; Kelli’s exchange was presented in chapter 4 (Figure 9, p. 75). Kelli illustrated her SCK through both her interpretation of her student’s mathematical thinking and her recognition of the inefficiency of his strategy. The idea unit was coded as I0* because Kelli mentioned inefficiency of the strategy; however,
she did not label the sophistication in relation to other strategies nor was she asked to do so. There were no instances of evidence of SCK with the second-grade group during PLT session #1. I believe that was mostly due to the reliance on the standard algorithm present in the student work samples and due to the preservice student teachers’ lack of knowledge of alternate strategies.

PLT session #1 focused on different types of multi-digit addition and subtraction problems; it also provided the preservice student teachers the opportunity to be exposed to a variety of different strategies. The preservice student teachers in both groups drew on both of these areas of SCK during PLT session #2. At the start of the session, Donna began sharing the two different types of story problems she posed to her students: an addend unknown put together/take apart problem, and a bigger unknown compare problem. She explained how finding the missing partner using addition was easier for her students than her traditional view of subtracting as take-away. For the compare problem, Donna also explained how the word “fewer” affected her students’ choice of strategy; many of her students were confused as to whether or not to add or subtract for the compare problem (1,2,1). When the preservice student teachers discussed the two different number line strategies for 82 – 53 shown in Figure 33, they discussed how the problem type lent itself to a counting up versus a counting down strategy (18,2,2).
In the following exchange, Lacey explained her plans to help one of her students whose student work the group had previously analyzed,

“So what I would want to push the student to do is instead of counting up to see this problem as a problem where subtraction could be an option for them because it seems like both of the strategies that this student used involved counting up or adding on to the base number. So one of the things that I would work with them on is using the comparison bars to just help them see who has more, who has less, and how many more or less they have and talking about, “How could you figure out how many more Marie has?” and talking…maybe showing them an equation that they could write for this problem: 101 – 62 and talking about our subtraction strategies and how we could solve it that way. And talking about the word ‘more.’ Like how many more and what that means. Because my kids automatically think ‘more’ oftentimes means ‘add.’ “Oh, addition.” So a lot of times they’ll choose even though they know that you’re trying to find out how many more they have, they’ll automatically choose to add on. So talking about the word ‘more’ in some cases, like in this case, could also mean that you could use subtraction as a strategy to solve the problem. So I would push this student to do all of those things to maybe help them move to a higher level of understanding of the problem.”

(22,3,2)

Following Lacey’s remark, the group discussed problem types using the term “fewer” and then had a discussion about the importance of number sentences to assist the students in making sense out of compare problems. Throughout the exchange, the preservice student teachers were applying their SCK regarding problem types to instruction in their classroom.
Specialized content knowledge regarding different types of multi-digit addition and subtraction strategies was the most prevalent type seen throughout the PLT sessions. The preservice student teachers drew on their SCK about different strategies when evaluating their students’ claims and in evaluating efficiency of strategies. The preservice student teachers’ sometimes drew on their SCK to evaluate the flexibility of students’ strategies. For example, the first-grade preservice student teachers discussed adding 10 and subtracting 1 compared with adding 9 (2,3,1). They used their SCK to attend, interpret and identify the level of sophistication of the strategy. Similarly, when the preservice student teachers discussed using open number lines, Ashton identified a strategy to increase the level of sophistication (21,2,2). During PLT session #1 and session #2, the group had some idea units coded as lack of SCK regarding use of number lines, but the second-grade group’s SCK regarding number lines as a strategy grew throughout the PLT sessions. By PLT session #3, the second-grade preservice student teachers showed evidence that they could use their SCK relating to number line strategies to correctly interpret a students’ “backwards” number line for 101 – 62, discuss what the drawing showed about the student’s mathematical thinking, and decide on appropriate probing questions to ask the student (See Figure 34). Their exchange follows:

Tara: So it doesn’t really matter which way it goes but I think a student if they were…if they didn’t know the conventional way of writing numbers, they always go smaller and then they write left to right. So if they were to notice, ‘Hmm, I wonder why these are going smaller, they usually go bigger.’ But…
CS: Do you have any thoughts on that? Do you agree?
Lacey: I mean I kind of…yeah, I mean I see what she is saying. I do agree in some ways but it’s more like I feel like, Gosh, is it just because that’s the way I was always taught to do it? Because I mean, if you really look at sample 2, I mean what he did isn’t necessarily wrong given the context of the problem. I mean
now if he wrote me a number line and put… But then necessarily if it was going all the right way, it wouldn’t necessarily be wrong. It’s just the convention of…

Tara: It’s just the conventional way of writing the numbers
Lacey: We always do it that way, the other way. So I mean I wouldn’t ever say anything like ‘Oh, you have to write it this way,’ as long as they were still understanding the numbers and concepts. I might say, ‘Why did you choose to write it that way?’ or ‘Why did you choose to have the numbers going that way?’

Tara: I think what she said earlier, that just make him aware of it. You can just show him a ruler or show him something and say, ‘Do you notice your numbers are going this way?’
Lacey: They go up instead of down
Tara: So you can just make him aware. I don’t think it would be a stressing factor.

(16,3,2)

Figure 34. Student work sample from (16,3,2).

PLT session #3 was designed to focus on helping the preservice student teachers develop their ability to determine appropriate probing and/or extending questions to ask their students in order to probe and/or extend their students’ mathematical thinking. Both groups of preservice student teachers exhibited evidence of drawing on this SCK during PLT session #3. For example, as the preservice student teachers decided which questions to ask the student, Tara stated,

“So I think maybe pushing them to do more figure out what you’re looking for; figure out what the two numbers you know and write it using what you know and what you
need to find and then maybe seeing how they can use the relationship between what they have and what they need to find with subtraction.”

Tara’s statement showed her SCK related to the importance of number sentences in helping the students make sense out of the problem they are asked to solve. The preservice student teachers drew on their developing SCK regarding questioning to begin to relate their interpretations of their students’ mathematical thinking to questioning in their classrooms.

Table 3 contains all of the idea units that contained explicit evidence of SCK. The preservice student teachers’ exhibited increased evidence of SCK as the PLT sessions progressed. In PLT session #1, only the first-grade group showed evidence of SCK. In PLT session #2, the first-grade group showed evidence of SCK in four idea unit exchanges and the second-grade group in two. By PLT session #3, the first-grade group showed evidence of SCK in 10 exchanges which was just under half of the 23 total idea units. In contrast, the second-grade group showed evidence of SCK in 5 idea unit exchanges. PLT session #3 was the first session that the second-grade group had all non-standard algorithm samples to analyze. They were just recognizing the importance of developing SCK regarding different strategy types, and they did not have as high level of sophistication of strategies to analyze as did the first-grade group. Thus, when considering the groups of preservice student teachers separately, evidence of SCK increased over time.

The preservice student teachers’ specialized content knowledge assisted them throughout all four components of the professional noticing framework. The PLTs were designed with the goal of developing specific areas of SCK for multi-digit addition and subtraction. For the idea units where the preservice student teachers exhibited SCK, their
SCK led to greater levels of either their attention to their students’ mathematical thinking, their interpretations of their students’ mathematical thinking, their identification of levels of sophistication of strategies, and/or their decisions as to which questions to ask their students as a next step in instruction. Of the 34 documented instances showing evidence of SCK, 27 of them were for the highest level of professional noticing. Specialized content knowledge was related to an increase in the preservice student teachers’ professional noticing.

Table 35.

*Professional noticing codes for idea units that contain evidence of SCK throughout the PLT sessions.*

<table>
<thead>
<tr>
<th>Idea Unit</th>
<th>PLT Session #1</th>
<th>PLT Session #2</th>
<th>PLT Session #3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st Grade</td>
<td>1st Grade</td>
<td>2nd Grade</td>
</tr>
<tr>
<td>Attend</td>
<td>2</td>
<td>12</td>
<td>1</td>
</tr>
<tr>
<td>Interpret</td>
<td>1*</td>
<td>2</td>
<td>2*</td>
</tr>
<tr>
<td>Identify</td>
<td>0*</td>
<td>1</td>
<td>1*</td>
</tr>
<tr>
<td>Decide</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Research Question #3**

*What factors, other than SCK, influence the elementary preservice student teachers’ learning to professionally notice through their experience with a set of guided, professional learning tasks focused on analysis of their students’ multi-digit addition and subtraction work?*

Both the ongoing and retrospective analyses resulted in emergent themes that are considered factors that influenced the preservice student teachers learning to notice throughout this intervention. The four major factors that emerged were the design &
implementation of the PLT, collective influence, characteristics of the student work samples that the preservice student teachers brought to the PLT sessions, and external factors.

**Design & Implementation of PLT.** The overarching goal of the study was to help the preservice student teachers learn how to focus on the mathematics behind their students’ work; thus, the PLT sessions were designed around the preservice student teachers’ analysis of their student work samples. When the preservice student teachers were invited to participate in the study, they were made aware of the goal of the study. They came to PLT session #1 ready to focus on their students’ work. I attribute their enthusiasm to the fact that they were working with *their* own students’ work samples; the PLTs were relevant to their teaching practice. While the first-grade group attended to affect before attending to the mathematics during PLT session #1, both groups reached the point where they were focused on the mathematical thinking of their students. By designing the PLT sessions so that the preservice student teachers continued to focus on their students’ mathematical thinking regarding multi-digit addition and subtraction, the preservice student teachers showed growth in their ability to notice.

Other than the overarching design goal, there were other aspects of the design that played a role in the preservice student teachers’ learning to notice. The fact that the participants volunteered to be a part of the study and that their work was not required meant that I had no way to ensure that the participants completed all parts of the PLTs. There were many missing reflections, the preservice student teachers did not always follow directions as to the tasks they posed to their students, and three preservice student teachers attended a PLT session without bringing student work samples. For example, the preservice student teachers’
lack of questioning their students as directed in PLT #2 potentially lowered their professional noticing. It was sometimes difficult for them to interpret their students’ work without having probed their students’ thinking. I believe the PLT sessions not being part of the regular student teaching field experience requirements hindered the preservice student teachers’ learning to notice.

As mentioned in chapter 4, after PLT session #1, the college supervisors suggested getting a document camera to show student work samples. I believe this change in implementation was one of the reasons for the shift from individual to collective analysis that emerged during PLT session #2. The preservice student teachers engaged in analyzing each other’s student work samples, not just their own. Other than the document camera, I believe this was also because the preservice student teachers were more comfortable with each other and came to PLT session #2 understanding the expectations of what would occur during the session.

A final component of the PLT design that may have influenced the preservice student teachers’ learning to notice was implementation fidelity and decisions made by the college supervisors. For example, in the second-grade PLT session #1, the college supervisor chose to ask all questions to one preservice student teacher at a time. This meant that all of the questions were asked, but there was no room for collective discussions. I believe this decision by the college supervisor influenced the preservice student teachers’ ability to interpret their students’ mathematical thinking because they did not have the opportunity to learn from their fellow preservice student teachers. In another example, for PLT session #3, the first-grade college supervisor asked the preservice student teachers to decide on
appropriate questions as they shared each of their samples. In contrast, the second-grade college supervisor had the preservice student teachers share all of their samples before she asked anything about questioning. Thus, the first-grade group spent more time developing appropriate questions which can be seen in Table 35 by the fact that they had more codes for Decide. In light of the analysis of the PLT design, a final set of three PLTs was developed. It was presented in chapter 4.

**Collective Influence.** The preservice student teachers helped each other develop greater levels of attend, interpret, identify, and decide. During these instances, their collective influence caused the whole group to reach a higher level along the professional noticing framework. An example of collective influence follows. Prior to the following segment, Tara and the college supervisor were discussing possible ways the student may have been thinking to solve the problem shown in Figure 35. Through the following discussion, all three of the preservice student teachers work together to develop a plausible interpretation of the student’s mathematical thinking.

Ashton: I guess but then to like you were saying, like to know that…to know that six plus four is ten, but then to know you mentally like…okay, now that’s a four and then now it’s another ten. Okay, now that one is a two so there’s four. But then opposite of adding up, he’s kind of like doing addition like switching back and forth.
CS: And checking. Right.
Ashton: Yeah.
Tara: He’s doing all the crossing out and carrying and all that stuff.
Ashton: Yeah. He really is visualizing…
Tara: Either that or he’s counting up in his mind. Yeah.
Lacey: Four more gives me 140.
Ashton: Not only with addition, you do subtraction when you’re doing that too.
Lacey: Yeah. And if you add sixty more then you get to 200.

(18,2,2)
Figure 35. Student work sample from (18,2,2).

Table 36 shows all of the idea units coded for collective influence. These idea units contain exchanges between the preservice student teachers during which a lower level of professional noticing was present, but during the exchange, in response to at least one of the preservice student teacher’s input, the group’s level of professional noticing increased. Through analysis of Table 35, it is clear that mostly robust evidence was provided during the idea units in which collective influence was present. Often, when the preservice student teachers worked together to attend, interpret, identify, and decide, they provided higher levels of evidence for all areas of the professional noticing framework. For example, only five idea units from second-grade PLT session #2 were coded as R2; four of the five were instances with documented collective influence. Working together to analyze student work proved to advance the preservice student teachers’ professional noticing.
Table 36.

*Professional noticing codes for idea units that contain collective influence throughout the PLT sessions.*

<table>
<thead>
<tr>
<th>Idea Unit</th>
<th>PLT Session #1</th>
<th>PLT Session #2</th>
<th>PLT Session #3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st Grade</td>
<td>2nd Grade</td>
<td>1st Grade</td>
</tr>
<tr>
<td>Attend</td>
<td>2 2 2 2 2 2 2 2 2 2 2 2 2 2</td>
<td>2 1 2 2 2 2 2</td>
<td></td>
</tr>
<tr>
<td>Interpret</td>
<td>2 2 2 2 2 2 2 2 2 2 2 2 2 2</td>
<td>2 1 2 2</td>
<td></td>
</tr>
<tr>
<td>Identify</td>
<td>0 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
<td>1 2 1 1 2 2</td>
<td></td>
</tr>
<tr>
<td>Decide</td>
<td>2 2 2 2 2 2 2 2 2 2 2 2 2 2</td>
<td>2 0 2</td>
<td></td>
</tr>
</tbody>
</table>

Further analysis of Table 36 shows a slight decrease in the number of instances of collective influence between PLT session #2 and PLT session #3. While I am not claiming that collective influence should increase over time, it is helpful to discuss potential reasons for the drop. For the first-grade group, I believe the drop was mostly because they were only discussing student work samples for 8 of the 22 idea units. Out of the 8 idea units, collective influence was present for 3 of them. In comparison, for PLT session #2, the first-grade group had 16 idea units focused on student work samples with 5 of them showing collective analysis. In looking at the ratios, then, the preservice student teachers continued to exhibit a high level of collective influence during PLT session #3. The drop for the second-grade group was not for the same reason; the second-grade only had 2 instances of collective influence out of 11 idea units focused on analysis of work samples. There were two connected reasons for the drop exhibited by the second-grade student teachers. First, there were only two preservice student teachers present during the session, and second, as a preservice student teacher displayed her student’s work, the college supervisor had the
opposite preservice student teacher interpret the strategy and mathematical thinking first. Thus, there was not an opportunity to capture collective influence that may have occurred. Regardless of the amount of instances of collective influence, collective analysis of each other’s students’ work facilitated the preservice student teachers’ development of professional noticing.

**Characteristics of Student Work Samples.** Another factor that influenced the preservice student teachers’ learning to notice was the characteristics of the student work samples that the preservice student teachers chose to bring to the PLT sessions. Student work samples that were hard to interpret due to the preservice student teachers’ lack of questioning their students often meant lacking or limited evidence for components of the professional noticing framework. The discussion presented under research question #1 on p. 140 was coded as A2, R1; Kelli was focused on mathematically significant details, but she had not questioned her student and therefore was unable to determine how he counted his direct modeling representation. Because the process the student took to solve the problem was unknown, the interpretation of the students’ mathematical thinking was limited.

![Figure 36. Student work sample from (13,2,1).](image)
Another characteristic of the student work samples that often led to lower levels of professional noticing were easy and expected responses, for simple problems. Leah brought in student work for a single-digit subtraction problem to first-grade PLT session #2. Of the three samples, two students had used direct modeling to count all or count on. All three of the idea units associated with her three student work samples for the problem were labeled as R1, limited evidence. There was not much to interpret; therefore, the preservice student teachers did not provide evidence when discussing the students’ solutions. In contrast, many times the student work samples that were the most difficult to interpret were those that were coded with robust evidence of professional noticing. These were also often the idea units showing collective influence; seven of the eleven coded instances of collective influence relating to student work samples were with respect to student work samples that the preservice student teachers indicated as difficult to interpret. For example, the exchange presented in the previous section was coded as R2 (p.171). The preservice student teachers worked together to determine a plausible interpretation drawing that drew on specific evidence from the student’s work.

Surprise was another characteristic of the student work samples that affected the preservice student teachers’ professional noticing—sometimes surprise led to lower levels of professional noticing while sometimes higher. In general, surprising work samples were coded as lower levels of professional noticing during the first PLT sessions, but for later sessions, surprising work samples were coded with higher levels of professional noticing. During idea unit #6 in second-grade PLT session #3, Lacey’s student used a number line strategy that surprised her, but the preservice student teachers had analyzed many number
lines throughout the sessions and thus were able to develop a robust interpretation of the student’s mathematical thinking. Overall, the student work samples that were interesting and somewhat difficult to interpret proved to be those that the preservice student teachers were most committed to interpreting and therefore were coded as higher levels of professional noticing.

**External Factors.** There were a variety of external factors that may have influenced the preservice student teachers professional noticing. The preservice student teachers were teaching in their cooperating teachers’ classroom and had to follow requirements set by their cooperating teachers. For some of the preservice student teachers, decisions were made as to how they could teach by the cooperating teachers; these decisions affected the student work brought to the session and hence, the analysis. For the second-grade preservice student teachers, the fact that their cooperating teachers had already taught their students the standard multi-digit addition and subtraction algorithms influenced the lack of variety of work samples brought to the PLT sessions. There were also school and district level factors affecting the preservice student teachers’ classroom teaching, e.g. ability grouping of students and strict pacing guides.
CHAPTER 6

In the previous two chapters, I reported results relating to the ways in which elementary preservice student teachers learned to notice through analysis of their students’ work on multi-digit addition and subtraction. I answered the research questions and discussed the findings from the analysis of data. In this concluding chapter, I present what I see as the study’s three main contributions: adapting design research to study preservice student teacher learning with researchers partnering with college supervisors, the revised set of professional learning tasks, and the developed theory for professional noticing of preservice teachers. I conclude with implications and next steps in research related to preservice student teacher professional noticing.

Contributions

Design Research Studying Preservice Student Teachers Partnering with College Supervisors. Design research methodology has become increasingly popular in the field of mathematics education, but it has predominately been used with researchers partnering with classroom teachers to examine K-12 student learning (Cobb, Zhao & Dean, 2009). In recent years, researchers have started using design research to study teacher learning in professional development settings (Sztajn, Wilson, Edgington, Myers & Dick, 2013). This study extended the use of this methodology to preservice teacher education.

For preservice teacher education, design research provides a methodology allowing for partnerships between researchers and college practitioners working together to design interventions and analyze conjectures regarding the learning of preservice teachers. As the researcher for this study, I partnered with college supervisors to study preservice student
teacher learning. Viewing the student teaching field experience as a continuance of the learning preservice teachers are exposed to in their education courses and studying how learning transpires may strengthen preservice preparation programs. Designing and revising interventions to help preservice teachers focus on how to learn while completing their student teaching field experiences provides them with a prototype for continued learning as they transition to the role of practicing teacher.

**Set of Professional Learning Tasks.** As a part of this design research study, the analysis of the hypothetical learning trajectory led to ongoing revisions of a set of designed professional learning tasks. The final set of three refined PLTs is now available for use by others. The redesigned PLTs were presented at the end of chapter 4. These PLTs can easily be adapted for use with preservice student teachers at different levels of licensure. The mathematical topic, along with the focus area for development of specialized content knowledge can be changed. Suggestions for how to make these changes are included at the end of chapter 4. In addition, these PLTs could be adapted to other fields of study (e.g., literacy or science). Implementing the PLTs with preservice student teachers supports their development of knowledge for teaching as well as their development of professional noticing.

**Theory of Professional Noticing for Preservice Student Teachers.** Professional noticing of children’s mathematical thinking includes how and the extent to which teachers notice children’s mathematical thinking (Jacobs et al., 2010). In 2011, Sherin, Jacobs, and Philipp (2011) called for research on how noticing of practicing teachers compares to that of preservice teachers. They asked, “what trajectories of development related to noticing
expertise exist for prospective and practicing teachers?” (Sherin et al., 2011, p. 11). The results from this study address this question as my research examined how preservice student teachers’ engagements in professional noticing of their students’ mathematical thinking changed over the course of an intervention focused on analysis of multi-digit addition and subtraction student work. The findings suggest a theory of professional noticing for preservice student teachers.

**Attend.** Attending to mathematically significant details in children’s strategies requires “skill in finding those mathematically significant indicators in children’s messy, often incomplete, strategy explanations” (Jacobs et al., 2010, p. 194). The PLTs intervention for this study was designed as a means to focus preservice student teachers’ attention on mathematically significant details found in their students’ multi-digit addition and subtraction work samples. The design of the study allowed for the preservice student teachers to purposefully attend to their students’ mathematical thinking. During the intervention, the preservice student teachers’ attention quickly shifted from mentioning affect, and inaccurate or incomplete mathematical specifics to providing correct specifics of mathematically significant details. The results showed the preservice teachers were capable of attending to at least some of the mathematically significant details present in their students’ work. Their growth required support via the PLTs facilitated by their college supervisors and discussed with their peers. Like others have shown, support is a necessity when developing preservice teachers’ ability to notice (Jacobs et al., 2010, Sherin & Van Es, 2009, Star & Strickland, 2007; Vondrova & Zalaska, 2013).
Attending to mathematically significant details in their students’ work proved to be extremely important for further professional noticing. When the preservice student teachers failed to focus on mathematically significant details, their interpretations, identifications, and decisions about questioning were always limited or lacking evidence. As their level of attending to mathematically significant details increased, the levels of the other three components of the professional noticing framework did so as well. It is important to note that this study was looking at preservice teacher noticing in a setting outside of the classroom.

The professional noticing framework was designed under the assumption that all components are integrated and occur simultaneously when responding to children’s mathematical thinking in the classroom (Jacobs et al., 2010). Like others, this study showed the interconnectability of the four professional noticing components, but also highlighted the importance of attending to mathematically significant details requires support of designed interventions.

**Interpret.** Interpretation of children’s mathematical thinking involves the teacher’s ability to describe their students’ strategies and to develop plausible interpretations of their students’ mathematical thinking. In this study, like Goldsmith & Seago’s study, the preservice teachers engaged in interpretation from the beginning of the PLT intervention, but “the change was in the degree to which these interpretations were close to, and warranted by, evidence” (2011, p. 177). Drawing upon evidence when interpreting their students’ work was
not easy for the preservice student teachers; often their students’ work samples were difficult to interpret partially due to lack of questioning their students during or after task implementation. Throughout the intervention, the preservice student teachers gradually began to draw upon evidence when interpreting their students’ work. In Jacobs et al.’s (2010) comparative study on professional noticing, without an intervention, half of their preservice teachers exhibited limited evidence of interpret; none exhibited robust evidence. The results from this study showed that with a designed intervention preservice teachers are capable of developing plausible interpretations of their students’ mathematical thinking.

Interpreting was challenging for the preservice teachers. Jacobs et al. (2010) explained that interpreting takes years to develop; they noted lack of mathematical knowledge as a possible reason for the difficulty. Like Ferendez, Llimares, & Valls’ (2013) and Vondrova & Zalaska’s (2013) studies, this study highlighted the role of mathematical knowledge for teaching in the development of the preservice teachers’ interpretations of their students’ work. Each of the three PLT interventions was designed with a focus on SCK; analysis of the results showed instances where the preservice teachers drew upon their newly developed SCK when interpreting their students’ strategies. Development of SCK takes time and practice; hopefully, it will continue to occur for preservice teachers once they become practicing teachers through professional development opportunities or in-house support (e.g. math coaches).

Franke et al. (2001) explained how teachers who learn by interpreting their students’ mathematical thinking continue to learn after designed interventions. Following the intervention, one of the preservice student teachers from this study explained,
As a result of my participation in this study, I began looking deeper at student work and trying to figure out why students did what they did instead of just looking and seeing what strategy they used or the mistakes they made. By thinking about why they did what they did when solving an equation I was able to help the students better.

Focusing on professional noticing during the preservice student teachers’ field experience gave the preservice teachers a basis for how to learn by interpreting their students’ work. Like others, this study showed that preservice student teachers can successfully increase their engagement with the interpret component of professional noticing if supported by an intervention (Sherin & Van Es, 2005; Star & Strickland, 2008; Vondrova & Zalaska, 2013).

**Identify.** Identification of levels of mathematical sophistication of their students’ strategies was my topic specific addition to the professional noticing framework. After teachers interpret their students’ multi-digit addition and subtraction work samples, they should be able to identify their students’ strategies’ levels of sophistication. Jacobs et al. (2010) noted that deciding how to respond requires knowledge about children’s mathematical development. Identification encompasses this idea and thus, can be considered as a part of interpret that comes before deciding how to respond. Like interpret, identification of mathematical sophistication draws upon SCK, but since it relates to children’s general strategy progressions, it also draws upon KCS. The PLTs intervention was designed around developing the preservice teachers’ SCK and KCS regarding multi-digit addition and subtraction strategies and the different strategies’ levels of sophistication. By the culminating PLT session, the preservice student teachers showed that they could sometimes draw upon evidence and correctly identify their students’ strategies’ level of mathematical
sophistication. One of the preservice teachers in this study explained, “The study helped me to see what level my students were on and [to] help them rise to a higher level.”

The preservice teachers did not automatically identify levels of sophistication; they had to be prompted to do so. Thus, as was seen with interpret, preservice teachers need time to develop their ability to identify levels of mathematical sophistication. As preservice teachers enter the classroom and increase their exposure to different multi-digit addition and subtraction strategies, their ability to correctly identify levels of sophistication should increase if they are provided support. Along with their increased ability to identify levels of sophistication, preservice teachers need to learn to use their interpretations and identifications of mathematical sophistication as a basis for making instructional decisions in their classrooms. This skill will require time, practice and support.

**Decide.** Basing instructional decisions on children’s mathematical thinking is the final component of the professional noticing framework, and for preservice teachers as with practicing teachers (Jacobs et al., 2010), it proved to be the most difficult. Choosing how to respond includes a range of instructional activities, from questioning individual students to designing a lesson (Jacobs et. al., 2011, Santagata, 2011, van Es & Sherin, 2002). For the preservice student teachers, deciding on an appropriate next step in instruction was about questioning in order to probe or extend students’ thinking relating to their strategies. During the first PLT session, the preservice student teachers discussed the importance of questioning their students and indicated their intention to do so for the following PLT. Jacobs, Lamb, Philipp, & Schappelle, stated “teachers are not likely to respond on the basis of children’s understandings without purposeful intention to do so” (2011, p. 99). The question is whether
teachers’ intentions manifest themselves in higher engagements with professional noticing. For this study, despite their stated intentions, none of the preservice student teachers questioned their students for the second PLT session. It became apparent that the preservice student teachers did not know how to choose appropriate questions to either probe or extend their students’ thinking. As Jacobs et al. (2011) discovered, purposeful intention was not enough.

In order to assist the preservice student teachers in their development of their engagement with decide, selecting appropriate questions for probing and/or extending became the focus for PLT session #3. The first-grade college supervisor focused the session around interpretation and identification of mathematical sophistication of the student work samples. Following the preservice student teachers’ interpretations and identification, the college supervisor asked for appropriate questions to ask the student. When given sample questions and specifically asked to develop questions, the preservice student teachers proved they were sometimes able to do so. The second-grade college supervisor structured her session differently. Questioning was not a main focus, thus the preservice teachers did not exhibit many instances of choosing questions based on interpretations and identification of mathematical sophistication. These contrasting results highlight both the purposeful nature of decide, but also the need for developing SCK around pressing and probing questions (Kazemi, Elliot, Mumme, Carroll, Lesseig & Kelley-Petersen, 2011). Preservice teachers need to be taught and provided time to practice how to choose appropriate questions to ask their students based on their students’ mathematical thinking. If they are given the opportunity, it can affect their classroom instruction. One of the preservice teachers
explained, “This study allowed me to anticipate what type of mathematical learner many of my students are and what are some great questions to ask each student.”

**Summary.** The findings from this study show how analyzing student work in a setting outside of their classrooms and with guidance provided opportunities for preservice student teachers to increase their engagement with professional noticing of their students’ mathematical thinking. All four of the components of the professional noticing framework that guided the study: attend, interpret, identify, and decide proved to be related. Results showed that in general, preservice teachers can only progress in their professionally noticing by first attending to mathematically significant details in their students’ strategies. With intervention and support, they can then interpret their students’ mathematical understandings and draw upon SCK and KCS regarding levels of sophistication of strategies to choose appropriate questions to either probe or extend their students’ mathematical thinking. It is not expected that interpretation, identification, and deciding occur in order or simultaneously, but that the preservice student teachers’ engagement with all three of these professional noticing components increases throughout interventions.

**Implications**

The results from this study showed how preservice teachers develop their engagement with professional noticing of their students’ mathematical thinking through specially designed interventions during their student teaching field experience. The study’s findings have implications for education practitioners, researchers, and policy makers.

**Practitioners.** In this study, the student teaching field experience provided an optimal setting for developing both preservice student teachers’ engagement with professional
noticing and their mathematics specialized content knowledge. Colleges of education should design field experiences that include specific tasks for preservice teachers to complete; the tasks should be designed with a specific purpose in mind (Crespo, 2000; Feiman-Nemser, 2001; Lowery, 2002; Mewborn, 2000; NRC, 2010). For elementary preservice teachers, the goals of the designed mathematics tasks should include developing mathematical knowledge for teaching. By providing preservice teachers with purposeful, content-specific, field experiences, preservice teachers can learn how to continue to learn while in the classroom (Lowery, 2002). For the student teaching field experience, the goal of the designed tasks should include developing the preservice teachers’ professional noticing of children’s mathematical thinking. By integrating analysis of their own students’ mathematical thinking, preservice student teachers can learn how to learn from their own teaching practice.

The tasks designed for the preservice student teaching field experiences should involve well-trained and qualified college supervisors. College supervisors should possess specialized content knowledge so they can foster preservice student teachers’ development of their specialized content knowledge (Blanton et al., 2001; Slick, 1998; Wolf, 2003). College supervisors should teach their preservice student teachers how to learn from their own teaching and via analysis of their students’ mathematical thinking (Duquette, 1997). Part of teaching preservice student teachers how to learn from their own teaching is fostering the preservice student teachers’ development of and engagement with professional noticing of their students’ mathematical thinking. The National Council for the Accreditation of Teacher Education explained the importance of helping preservice teachers, “connect what they learn with the challenge of using it, while under the expert tutelage of skilled clinical educators.”
(2010, p. ii). This study highlights the important role that college supervisors can play in preservice student teachers’ development. College supervisors must fit the aforementioned criterion of being skilled clinical educators who possess both knowledge of content and pedagogy, as well as knowledge of how to help preservice teachers learn from their teaching practice.

**Research.** Design research is used to study learning. For this study, design research proved to be a useful methodology for studying the development of preservice student teachers’ learning. Researchers should continue completing design research studies with preservice student teachers in order to better understand preservice teacher learning in the context of preservice teacher education. A better understanding of preservice student teacher learning should lead to improvements in the development of preservice teacher education. Another aspect of design research methodology is that it allows for the development and testing of hypothetical learning trajectories in the context of practice. Sztajn et al. explained, “Design experiments promote researchers’ examinations of both the processes of learning and the means for supporting them” (2013, p. 10). For preservice student teachers, these means of support can be interventions designed around development of different aspects of preservice student teacher learning.

In addition to conducting design research with preservice teachers, researchers should also continue to study preservice teacher noticing. This study looked at preservice student teacher’s development of their professional noticing of their students’ mathematical thinking. The results showed that preservice student teachers could learn to engage with professional noticing of their students’ mathematical thinking. The results from the study suggested a
theory of preservice student teacher noticing. Interventions such as those designed for this study should continue to be developed by researchers looking at professional noticing. Research should compare preservice and in-service teachers’ engagement with professional noticing when exposed to the same interventions. Research such as the type mentioned above will further the knowledge of trajectories of development related to noticing expertise (Fernandez, Llinares, & Valls, 2013; Sherin, Jacobs, & Philipp, 2011).

**Policy.** Field experiences vary between teacher preparation programs. Preparation programs that focus on preservice teacher’s learning from field experiences, especially their student teaching field experience are needed (NCATE, 2010; Feiman-Nemser, 2001). These types of field experiences require time and planning, as well as support from colleges of education. The results of this study show that preservice student teachers can learn from their own teaching practice. Their learning occurred through collaboration with each other and under the supervision of their well-qualified, college supervisors. Field experiences must include opportunities for preservice teachers to collaborate and learn from each other (Hanuscin, 2004; Hanuscin & Musikul, 2007; Lowery, 2002).

Teaching is a complex profession that requires preservice teacher training. The challenge for preservice teacher education programs is ensuring that preservice student teaching field experiences are designed around helping preservice student teachers learn how to learn from teaching. Currently, not all teacher accreditation programs require preservice teachers to undergo a student teaching field experience. This study provided evidence of preservice teachers’ learning from their teaching practice during their student teaching field experience. Decisions by accreditation agencies and state school boards to not require these
types of preservice teacher field experiences deny preservice teachers opportunities to learn which affects their professional knowledge and hence, students in their classroom.

Next Steps

This study was a first attempt at designing interventions led by college supervisors and focused on developing preservice student teacher noticing during the preservice student teaching field experience. A thorough evaluation of the re-designed intervention occurring during the preservice student teaching field experience should inform the future development of interventions designed to increase preservice student teacher noticing. Future interventions could include preservice student teachers’ analysis of their students’ mathematical thinking based on clinical interviews or on classroom video clips. Interventions could be designed around different topics in mathematics or other subjects of interest. This study showed how preservice student teachers increased their engagement with professional noticing of their students’ mathematical thinking through designed professional learning tasks focused on analysis of their students’ multi-digit addition and subtraction work. Future research should continue to address the ways that preservice student teachers develop their professional noticing of their students’ mathematical thinking. Furthering researcher knowledge about preservice student teacher noticing has the potential to improve preservice teacher education.
REFERENCES


APPENDICES
Appendix A: Addition & Subtraction Problem Types with Different Terminology

<table>
<thead>
<tr>
<th>Table 1 Common addition and subtraction situations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Result Unknown</strong> [Unknown Totals (MI)]</td>
</tr>
<tr>
<td><em>Add to Join (CO)</em>*</td>
</tr>
<tr>
<td><em>Change Plus (MI)</em></td>
</tr>
<tr>
<td>Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?</td>
</tr>
<tr>
<td>*Take from Separate (CG!)</td>
</tr>
<tr>
<td><em>Change Minus (ME)</em></td>
</tr>
<tr>
<td>Five apples were on the table. I ate two apples. How many apples are on the table now?</td>
</tr>
<tr>
<td><strong>Put Together/ Take Apart</strong></td>
</tr>
<tr>
<td>*Part/Part/Whole (CG! &amp; B)</td>
</tr>
<tr>
<td><em>Collection (ME)</em></td>
</tr>
<tr>
<td>Three red apples and two green apples are on the table. How many apples are on the table?</td>
</tr>
<tr>
<td><strong>Compare</strong></td>
</tr>
<tr>
<td>*Version with “more”**: Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?</td>
</tr>
<tr>
<td>*Version with “fewer”**: Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie?</td>
</tr>
</tbody>
</table>

| **Change Unknown** [Unknown Partners (ME)]       |
| *Add to Join (CO)**                               |
| *Change Plus (MI)*                                |
| Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? | 2 + ? = 5 |
| *Take from Separate (CG!)                         |
| *Change Minus (ME)*                               |
| Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? | 5 − ? = 3 |
| **Put Together/ Take Apart**                     |
| *Part/Part/Whole (CG! & B)                        |
| *Collection (ME)*                                 |
| Five apples are on the table. Three are red and the rest are green. How many apples are green? | 3 + ? = 5, 5 − 3 = ? |
| **Compare**                                      |
| *Version with “more”**: Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? | 5 − 3 = 2 |
| *Version with “fewer”**: Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have? | 3 + 2 = 5 |

| **Start Unknown** [Unknown Partners (ME)]        |
| *Add to Join (CO)**                               |
| *Change Plus (MI)*                                |
| Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? | 7 + 3 = 5 |
| *Take from Separate (CG!)                         |
| *Change Minus (ME)*                               |
| Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? | 7 − 2 = 3 |

| **Total Unknown** [Whole Unknown (CG!)]          |
| *Add to Join (CO)**                               |
| *Change Plus (MI)*                                |
| Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? | 5 + 5 = 10 |
| *Take from Separate (CG!)                         |
| *Change Minus (ME)*                               |
| Grandma has five flowers. How many apples does she have? | 5 − 0 = 5 |
| **Put Together/ Take Apart**                     |
| *Part/Part/Whole (CG! & B)                        |
| *Collection (ME)*                                 |
| Five apples are on the table. How many apples are there in total? | 5 + 5 = 10 |
| **Compare**                                      |
| *Version with “more”**: Julie has five apples. Lucy has two apples. How many more apples does Lucy have? | 5 − 2 = 3 |
| *Version with “fewer”**: Lucy has three apples. Julie has five apples. How many fewer apples does Lucy have? | 5 − 3 = 2 |

| **Addend Unknown** [Part Unknown (CG!)]          |
| *Add to Join (CO)**                               |
| *Change Plus (MI)*                                |
| (Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? | 5 − 3 = 2 |
| *Take from Separate (CG!)                         |
| *Change Minus (ME)*                               |
| (Version with “fewer”): Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have? | 5 − 3 = 2 |
| **Put Together/ Take Apart**                     |
| *Part/Part/Whole (CG! & B)                        |
| *Collection (ME)*                                 |
| (Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? | 5 − 3 = 2 |
| (Version with “fewer”): Lucy has three fewer apples than Julie. Lucy has five apples. How many apples does Lucy have? | 5 − 3 = 2 |

| **Both Addends Unknown** [Part Unknown (CG!)]    |
| *Add to Join (CO)**                               |
| *Change Plus (MI)*                                |
| (Version with “more”): Julie has three more apples than Lucy. Lucy has five apples. How many apples does Lucy have? | 5 − 3 = 2 |
| *Take from Separate (CG!)                         |
| *Change Minus (ME)*                               |
| (Version with “fewer”): Lucy has three fewer apples than Julie. Lucy has five apples. How many apples does Lucy have? | 5 − 3 = 2 |

**Note:**

- **1st:** Problem types to be mastered by the end of the Kindergarten year.
- **2nd:** Problem types to be mastered by the end of the First Grade year, including problem types from the previous year(s). However, First Grade students should have experiences with all 12 problem types.
- **3rd:** Problem types to be mastered by the end of the Second Grade year, including problem types from the previous year(s).

- **MI:** Math Expressions (2008)
- **B:** Beckmann (2011)
Appendix B: Preservice Student Teacher Demographic & Beliefs Questionnaire

Demographic & Beliefs Questionnaire

Thank you for your willingness to participate in my dissertation study. In order to help me choose a representative sample, please complete the following questionnaire. Thank you!

* Required

Name*

Age*

Placement Information

Please answer the following questions about your student teaching placement.

School & Track if Year Round*

Grade Level*

College Supervisor *

Powered by Google Docs

Report Abuse Terms of Service Additional Terms
Demographic & Beliefs Questionnaire

* Required

Mathematics Background
Please answer the following questions about your previous experiences with mathematics.

Describe what you remember about your experiences with math in elementary school.*

Describe what you remember about your experiences with math in middle school.*

Describe what you remember about your experiences with math in high school.*

List the math classes you took in high school.*

Describe what you remember about your experiences with math in college.*
If any of these classes were taken outside of Meredith, please let me know.
List the math classes you took in college. Include any elementary math content courses and math methods courses.*
Beliefs About Teaching Mathematics
Please answer the following questions about your beliefs about teaching mathematics.

Describe your ideal mathematics classroom.*

Describe how you view your role as a mathematics teacher.*

Describe the strengths you will bring to your mathematics classroom.*

Describe any weaknesses you believe you possess with respect to mathematics teaching.*
Demographic & Beliefs Questionnaire

Thank You!
Thank you for completing this questionnaire. I will be in contact shortly to let you know if you were chosen for this study.
Mrs. Dick

Powered by Google Docs

Report Abuse-Terms of Service-Additional Terms
## Appendix C: Subset of Participant Responses to the Demographic & Beliefs Questionnaire

<table>
<thead>
<tr>
<th>Name (Age)</th>
<th>Grade Level</th>
<th>High School Math Classes</th>
<th>College Math Classes</th>
<th>Describe what you remember about your experiences with math in elementary school.</th>
<th>Describe how you view your role as a mathematics teacher.</th>
<th>Describe the strengths you will bring to your mathematics classroom.</th>
<th>Describe any weaknesses you believe you possess with respect to mathematics teaching.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Donna</td>
<td>7th Grade</td>
<td>Algebra II, Geometry Pre-Calculus</td>
<td>Calculus I, Calculus II, Calculus III, Statistics, Methods I, Methods II</td>
<td>Math was always presented to me that there was only one way to teach it. At the front of the class with everyone copying down the same math problems on the board and solving them the exact same way. It made math boring and mundane to endure and therefore made it difficult to understand. I know how to correctly answer and solve math problems, but what I don’t understand was why it was done a certain way if there was more than one way. Elementary was simple math: addition and subtraction. Then fourth grade hit hard when I had to learn long division. Multiplication was easy to grasp.</td>
<td>I want to use the right terminology and language to teach math so that the students will understand not only how to solve problems but why it makes sense to solve problems a certain way. I want my students to make mistakes and then learn from their mistakes.</td>
<td>I know what it is like to get frustrated with math. I enjoy making boring subjects fun and engaging.</td>
<td>I feel that I will not be comfortable using the correct language with the students. I grew up learning a particular math language that I now know is wrong and is one of the many reasons why math was difficult for me to learn.</td>
</tr>
<tr>
<td>Kelli</td>
<td>8th Grade</td>
<td>Geometry, Algebra I, Advanced Functions &amp; Modeling Pre-Calculus, Geometry</td>
<td>Algebra I, Statistics, Methods I, Methods II</td>
<td>I don’t remember much about math in elementary school besides worksheets. The only positive memory I have is a second-grade teacher dressing up as an Italian chef to teach us fractions using pizza.</td>
<td>My role as a mathematics teacher is to teach the students a concept or method and then show them how to practice using hands-on activities that are engaging and applicable. After assessing them they try the skill my role is to reteach or assist a student who seems stuck. If my teaching did not work then the next time I must teach in a different way. I believe teaching math is about building a strong foundation in skills to add onto as math gets more challenging. I want to be able to challenge my students and build upon students who believe they learned it.</td>
<td>The strengths I will bring to my mathematics classroom will be my willingness to try new things. I want to learn new and innovative teaching techniques. Also, a strength will be in providing fun activities that are interactive.</td>
<td>I believe a weakness I will possess in teaching mathematics will be not knowing how to describe something to a student after I have exhausted all methods. I feel as though this creativity will come with experience, but can be seen being a current weakness.</td>
</tr>
<tr>
<td>Leah</td>
<td>7th Grade</td>
<td>Algebra I, Geometry, Algebra I, Advanced Functions &amp; Modeling</td>
<td>Calculus, Algebra, Statistics, Methods I, Methods II</td>
<td>I don’t remember much about elementary school, but I’m thinking the kids in 5th grade were good. The only math I remember is the area of a triangle.</td>
<td>I view my role as a math teacher as that of the person that is responsible for getting the students excited about math and excited about learning. I feel that it is my responsibility to show students that math is important and that it will be used in everyday life.</td>
<td>I love math so I am very excited about teaching it. I like to teach math in ways other than simply lecturing or having all students sitting in their seats doing individual work.</td>
<td>Sometimes I find it difficult to explain math concepts well in many different ways so that all students understand.</td>
</tr>
<tr>
<td>Tammy</td>
<td>7th Grade</td>
<td>Geometry, Algebra II, Advanced Functions &amp; Modeling, Statistics, Discrete Mathematics</td>
<td>College Algebra, Statistics, Elementary Math I, Methods I, Methods II</td>
<td>I enjoyed math in elementary school. Many of my teachers let us use manipulatives which really helped me in making connections with the numbers however they also gave worksheets.</td>
<td>It is my job to scaffold children into learning about numbers, math, and how to solve problems. I feel that my job is to make the problems more clear by using different equations. Asking open-ended questions such as how could you solve this question? I will use correct vocabulary and try to get the children ready for future years in math by allowing opportunities for them to think on their own about how to solve the problems. I will also study the children’s work to learn more about how each child works through different equations.</td>
<td>I feel pretty confident in the math subject area. I can’t really think of any weaknesses in this area that I feel I have.</td>
<td></td>
</tr>
<tr>
<td>Grade</td>
<td>Course</td>
<td>Reason</td>
<td>Strength</td>
<td>Weaknesses</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>-------------------------</td>
<td>------------------------------------------------------------------------</td>
<td>---------------------------------------------------------------------------</td>
<td>-----------------------------------------------------------------------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td>Algebra I, II, Geometry, Trigonometry, Calculus, and ICM</td>
<td>I was on grade level but math was not my favorite subject.</td>
<td>My role is to understand the content well enough to explain the concepts that the students need to know in a way that they would understand.</td>
<td>The new standards require students to look at alternate ways of solving problems and explaining those ways. I fear that some of the explanations and strategies will be over my head.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3rd</td>
<td>Geometry, Algebra II, Advanced Functions in Math, Discrete Math, Calculus</td>
<td>In elementary school, I remember loving math! My teachers planned lots of fun activities for us to do. I also remember understanding math very well in elementary school. However, I do remember having less opportunities to use manipulatives than I would have liked.</td>
<td>As a teacher of mathematics, I feel that my role is to inspire the love of learning in my students. I want my students not just to know how to do a math problem, but also understand why they are doing and solving the problem in that manner. I want full understanding of mathematical concepts. I feel that it is my job to ensure that each student has a concrete and sound understanding of mathematical concepts.</td>
<td>One weakness I possess is an understanding of explaining a mathematical concept. I feel that I can see when a student does not understand a concept, but I do not know how to word it or explain something in a way that may help them. I try my best to explain a concept, but I feel that I could improve in my explanation of mathematical concepts.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AP</td>
<td>Calculus, Statistics, Methods I, Methods II</td>
<td>I went to a Montessori School, so most of my math experiences involved using hands-on manipulatives and math centers.</td>
<td>The use of technology is an ever-increasing method to teach all areas of study. I like using different mediums of technology and I love seeing students engaged in 21st century skills. As well, Manipulative and multiple ways of working out math problems is another idea I hope to have students understand.</td>
<td>I have never been very good at math, so understanding the content thoroughly enough to allow students to use multiple ways of solving problems is going to be a challenge for me.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Reflection Session:

<table>
<thead>
<tr>
<th>Anticipation</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Attend: mathematical details (ie. similarities, differences)</td>
<td></td>
</tr>
<tr>
<td>Interpret: describe/plausible interpretations of student work</td>
<td></td>
</tr>
<tr>
<td>Identify: levels of sophistication of strategies</td>
<td></td>
</tr>
<tr>
<td>Decide: instructional decisions</td>
<td></td>
</tr>
</tbody>
</table>
Appendix E: College Supervisor Guides for all PLT sessions

College Supervisor Handout—PLT Session 1

Agenda
- Discuss Design Research and talk through hypothetical learning conjectures for session one
- Discuss session one setup and reflection questions
- Review addition/subtraction story problem types with Common Core Language and discuss other terms that the student teachers may know
- Review levels of sophistication for addition/subtraction solution strategies

Facilitating Directions/Reflection Questions
Begin by having the student teachers share their task and reasons behind their choices as to which of their students’ work to bring to the session.

1. What strategies did your students use to solve the task?
2. What did you find surprising or unexpected in your students’ work?
   - Lead a discussion about their initial anticipation of the ways their students would approach the problems
3. What is the mathematics embedded in each of their strategies?
   - Lead a discussion about different types of addition/subtraction problems and differing levels of complexity depending on the type
   - If time, begin discussion levels of sophistication of strategies
4. What questions could you ask to help your student reflect on their strategy?

Lead a discussion on how to probe student thinking without guiding their work and on how to describe student work without projecting their knowledge onto the solution. Suggest that the student teachers take notes while monitoring their students as they complete tasks.

Session One Hypothetical Learning Conjecture
The student teachers will have a difficult time choosing and articulating their choices as to which student work to bring to the reflection session. Their initial descriptions, both written and verbal, of their students’ work will be focused on procedural correctness and/or surprise at non-standard solutions. When asked to describe the mathematics behind their students’ solutions, the student teachers will interpret their students’ work in light of their personal experiences, beliefs and mathematical knowledge. Through the discussion facilitated by the college supervisors, the student teachers will recognize their tendency to project themselves onto their students’ solutions and will begin to understand how to interpret the work for what it actually says. The discussion led by the college supervisors will also assist the student teachers in recognizing the importance of asking their students probing questions and taking notes in order to assist them in identifying their children’s mathematical thinking.
   - Possible New Conjecture: The student teachers will not be aware of different types of addition/subtraction problems and/or how the different types affect students’ solution strategies.

Levels of Sophistication of Strategies
1. Direct Modeling
2. Counting
3. Number Fact Strategies: making a ten, decomposition, creating equivalent but easier problems—all draw on knowledge of place value, properties of operations and/or relationship between addition and subtraction


210
College Supervisor Handout—PLT Session #2

Agenda
• Recap session one
• Discuss reflection questions and directions given to student teachers
• Review levels of sophistication for addition/subtraction solution strategies
• Discuss hypothetical learning trajectory

Reflection Questions
1. What changed from the last time that you came to this time? (ie. What things have you done differently either with implementation of the tasks, questioning your students and/or choosing what samples to bring?)
   Have the student teachers share their task and reasons behind their choices as to which of their students’ work to bring to the session.

2. What strategies did your students use to solve the task?

3. What did you find surprising or unexpected in your students’ work?

4. What is the mathematics embedded in each of their strategies?
   • Lead a discussion about similarities/differences in their students’ approaches between sessions. Discuss how the problem type affected students’ choice of strategies.
   • Lead a discussion about levels of sophistication of strategies. Have students work together to identify student’s levels.

5. What questions might encourage your students at each level of sophistication to consider a more efficient strategy?
   Lead a discussion on how to respond to students’ different levels of thinking.

Levels of Sophistication of Strategies
1. Direct Modeling
2. Counting
3. Number Fact Strategies: making a ten, decomposition, creating equivalent but easier problems—all draw on knowledge of place value, properties of operations and/or relationship between addition and subtraction
College Supervisor Handout—PLT Session #3

Agenda
- Recap sessions one and two
- Discuss Noticing Framework
- Recall Levels of Sophistication of Strategies
- Discuss Marking the Most of Story Problems article
- Discuss reflection questions and directions given to student teachers
- Discuss hypothetical learning trajectory

### Noticing Framework

![Noticing Framework Diagram]

### Levels of Sophistication of Strategies

1. Direct Modeling
2. Counting
3. Number Fact Strategies: making a ten, decomposition, creating equivalent but easier problems—all draw on knowledge of place value, properties of operations and/or relationship between addition and subtraction

   - **1st Grade Unpacking**
     - p. 13-14 (level 2)
     - p. 21-24 (levels 1 & 3)
   - **2nd Grade Unpacking**
     - p. 9-10 (levels 2 & 3)
     - p. 17-23 (levels 1 & 3)

### Reflection Questions

Choose one story problem that can be solved with either an addition or a subtraction strategy. Ask your students to solve it in two ways without using the standard algorithm.

1. Solve the task yourself and write out ways you anticipate your students will solve the task.
2. Administer the task and monitor your students as they are working
3. Question your students as they are working and take notes.
4. Collect all of your students’ work
5. Choose three student responses that you believe represent a range of levels of mathematical sophistication to bring to the reflection session.
6. Brainstorm possible questions to ask the three students to assist them in moving towards a higher level of sophistication.

### Questions for Session Three

*Begin by reviewing levels of sophistication of strategies. Look through the DPI Unpacking the CCSSM documents for examples of different strategies.*

Then have student teachers collectively share all of their chosen pieces of student work. *Have the student teachers make notes on the mathematics they notice and to identify levels of sophistication of strategies.*

Discussion questions:

1. What strategies did the students use to solve the task?
2. What did you find surprising or unexpected in the students’ work?
3. What is the mathematics embedded in each of the strategies?
4. On the basis of the students’ individual understandings, what question(s) might you pose next to help the student correctly solve the problem or move towards a higher level of sophistication of strategy?
Lead a discussion on questioning as a next step in instruction.

**Making the Most of Story Problems**

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Teacher Moves to Support a Child’s Thinking before a Correct Answer Is Given</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Category</strong></td>
<td><strong>Sample Teacher Moves</strong></td>
</tr>
<tr>
<td>Ensure that the child understands the problem.</td>
<td>Ask him to explain what he knows about the problem.</td>
</tr>
<tr>
<td></td>
<td>Rephrase or elaborate the problem.</td>
</tr>
<tr>
<td></td>
<td>Use a more familiar or personalized context in the problem.</td>
</tr>
<tr>
<td>Change the mathematics in the problem to match</td>
<td>Change the problem to use easier numbers.</td>
</tr>
<tr>
<td>the child’s level of understanding.</td>
<td>Change the problem to use an easier mathematical structure.</td>
</tr>
<tr>
<td>Explore what the child has already done.</td>
<td>Ask him to explain a partial or incorrect strategy.</td>
</tr>
<tr>
<td></td>
<td>Ask specific questions to explore how what he has already done relates to</td>
</tr>
<tr>
<td></td>
<td>the quantities and relationships in the problem.</td>
</tr>
<tr>
<td>Remind the child to use other strategies.</td>
<td>Ask him to consider using a different tool.</td>
</tr>
<tr>
<td></td>
<td>Ask him to consider using a different strategy.</td>
</tr>
<tr>
<td></td>
<td>Remind him of relevant strategies he has used before.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Teacher Moves to Extend a Child’s Thinking after a Correct Answer Is Given</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Category</strong></td>
<td><strong>Sample teacher moves</strong></td>
</tr>
<tr>
<td>Promote reflection on the strategy the child just completed.</td>
<td>Ask her to explain her strategy.</td>
</tr>
<tr>
<td></td>
<td>Ask specific questions to clarify how the details of her strategy are</td>
</tr>
<tr>
<td></td>
<td>connected to the quantities and mathematical relationships in the problem.</td>
</tr>
<tr>
<td>Encourage the child to explore multiple strategies and their connections.</td>
<td>Ask her to try any second strategy.</td>
</tr>
<tr>
<td></td>
<td>Ask her to try a second strategy connected to her initial strategy in</td>
</tr>
<tr>
<td></td>
<td>deliberate ways (e.g., more efficient counting or abstraction of work</td>
</tr>
<tr>
<td></td>
<td>with manipulatives).</td>
</tr>
<tr>
<td></td>
<td>Ask her to compare and contrast strategies.</td>
</tr>
<tr>
<td>Connect the child's thinking to symbolic notation.</td>
<td>Ask her to write a number sentence that “goes with” the problem.</td>
</tr>
<tr>
<td></td>
<td>Ask her to record her strategy.</td>
</tr>
<tr>
<td>Generate follow-up problems linked to the problem the child just completed.</td>
<td>Ask her to solve the same or a similar problem with numbers that are more</td>
</tr>
<tr>
<td></td>
<td>challenging.</td>
</tr>
<tr>
<td></td>
<td>Ask her to solve the same or a similar problem with numbers that are</td>
</tr>
<tr>
<td></td>
<td>strategically selected to promote more sophisticated strategies.</td>
</tr>
</tbody>
</table>
Appendix F: Participant Final Questionnaire

Final Questionnaire

Thank you so much for your participation in my dissertation study. As a final point of closure, I would like to have your thoughts and reflections on the experience in a format that I can use in my write-up and presentation. Please take a few minutes to answer the following questions. They are open ended, so please elaborate on whatever you want to share. Thanks again!

* Required

Name *

Describe what you learned about different strategies for multidigit addition and subtraction. *

Describe what you learned about analysis of student work. *

Describe any changes you made (if any) in your classroom as a direct result of your participation in this study.

Describe how you anticipate applying what you have learned as a direct result of your participation in this study in your first few years of teaching.
Is there anything else you want me to know? Please feel free to share positive and negative thoughts about this experience. Please, please be honest!

Submit

Never submit passwords through Google Forms.

Powered by Google Docs

Report Abuse - Terms of Service - Additional Terms
Appendix G: Ongoing Analysis Emergent Themes throughout the PLT Sessions

<table>
<thead>
<tr>
<th>Anticipation</th>
<th>PLT Session One</th>
<th>PLT Session Two</th>
<th>PLT Session Three</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choosing what to Bring</td>
<td>NEW</td>
<td>Change in Practice</td>
<td>Classroom Discourse</td>
</tr>
<tr>
<td>Evidence of SCK</td>
<td>Characteristic of Work Sample</td>
<td>Student Difficulty</td>
<td>Student Explanation</td>
</tr>
<tr>
<td>Hard to Tell</td>
<td>Choice: Surprise</td>
<td>MERGED: Reliance on Standard Algorithm &amp; Multiple Strategies</td>
<td></td>
</tr>
<tr>
<td>Importance of Questioning</td>
<td>Choice: Variety</td>
<td>Inadequate Description</td>
<td>Difficult to Interpret</td>
</tr>
<tr>
<td>Inadequate Description</td>
<td>Collective Analysis</td>
<td>Hard-to-tell</td>
<td></td>
</tr>
<tr>
<td>Instructional Influences</td>
<td>Collective Influence</td>
<td>Choice: Surprise</td>
<td>Articulated Choice</td>
</tr>
<tr>
<td>Instructional Steps</td>
<td>Questioning Each Other</td>
<td>Choice: Variety</td>
<td></td>
</tr>
<tr>
<td>Lack of SCK</td>
<td>Recognition of Personal Bias</td>
<td>Perceived Understandings</td>
<td>Influenced Interpretations</td>
</tr>
<tr>
<td>Level of Sophistication of Strategy</td>
<td></td>
<td>Evidence of SCK</td>
<td></td>
</tr>
<tr>
<td>Monitoring</td>
<td>MERGED: Instructional Influences</td>
<td>Lack of SCK</td>
<td>Role of SCK</td>
</tr>
<tr>
<td>Multiple Strategies</td>
<td>Instructional Influences</td>
<td>DROPPED: Change in Practice</td>
<td></td>
</tr>
<tr>
<td>Other Influences</td>
<td>Instructional Steps</td>
<td>Students Sharing</td>
<td>Questioning Each Other</td>
</tr>
<tr>
<td>Perceived Understanding</td>
<td>Monitoring</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Students at Different Levels</td>
<td>Students Sharing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student Affect</td>
<td>Surprise</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Students Sharing</td>
<td>Reliance on Standard Algorithm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surprise</td>
<td>Types of Story Problems</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Types of Story Problems | | | |
Appendix H: PLT Session #3 Preservice Student Teacher Handout

Levels of Sophistication of Strategies:
1. Direct Modeling
2. Counting
3. Number Fact Strategies: making a ten, decomposition, creating equivalent but easier problems—all draw on knowledge of place value, properties of operations and/or relationship between addition and subtraction.


<table>
<thead>
<tr>
<th>Table 1</th>
<th>Teacher Moves to Support a Child’s Thinking before a Correct Answer Is Given</th>
</tr>
</thead>
<tbody>
<tr>
<td>Category</td>
<td>Sample Teacher Moves</td>
</tr>
<tr>
<td>Ensure that the child understands the problem.</td>
<td>Ask him to explain what he knows about the problem.</td>
</tr>
<tr>
<td></td>
<td>Rephrase or elaborate the problem.</td>
</tr>
<tr>
<td></td>
<td>Use a more familiar or personalized context in the problem.</td>
</tr>
<tr>
<td>Change the mathematics in the problem to match the child’s level of understanding.</td>
<td>Change the problem to use easier numbers.</td>
</tr>
<tr>
<td></td>
<td>Change the problem to use an easier mathematical structure.</td>
</tr>
<tr>
<td>Explore what the child has already done.</td>
<td>Ask him to explain a partial or incorrect strategy.</td>
</tr>
<tr>
<td></td>
<td>Ask specific questions to explore how what he has already done relates to the quantities and mathematical relationships in the problem.</td>
</tr>
<tr>
<td>Remind the child to use other strategies.</td>
<td>Ask him to consider using a different tool.</td>
</tr>
<tr>
<td></td>
<td>Ask him to consider using a different strategy.</td>
</tr>
<tr>
<td></td>
<td>Remind him of relevant strategies he has used before.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Teacher Moves to Extend a Child’s Thinking after a Correct Answer Is Given</th>
</tr>
</thead>
<tbody>
<tr>
<td>Category</td>
<td>Sample Teacher Moves</td>
</tr>
<tr>
<td>Promote reflection on the strategy the child just completed.</td>
<td>Ask her to explain her strategy.</td>
</tr>
<tr>
<td></td>
<td>Ask specific questions to clarify how the details of her strategy are connected to the quantities and mathematical relationships in the problem.</td>
</tr>
<tr>
<td>Encourage the child to explore multiple strategies and their connections.</td>
<td>Ask her to try any second strategy.</td>
</tr>
<tr>
<td></td>
<td>Ask her to try a second strategy connected to her initial strategy in deliberate ways (e.g., more efficient counting or abstraction of work with manipulatives).</td>
</tr>
<tr>
<td></td>
<td>Ask her to compare and contrast strategies.</td>
</tr>
<tr>
<td>Connect the child’s thinking to symbolic notation.</td>
<td>Ask her to write a number sentence that “goes with” the problem.</td>
</tr>
<tr>
<td></td>
<td>Ask her to record her strategy.</td>
</tr>
<tr>
<td>Generate follow-up problems linked to the problem the child just completed.</td>
<td>Ask her to solve the same or a similar problem with numbers that are more challenging.</td>
</tr>
<tr>
<td></td>
<td>Ask her to solve the same or a similar problem with numbers that are strategically selected to promote more sophisticated strategies.</td>
</tr>
</tbody>
</table>
Appendix I: Retrospective Analysis Data with Codes for All Idea Units

<table>
<thead>
<tr>
<th>First Grade Session One</th>
<th>First Grade Session Two</th>
<th>First Grade Session Three</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Attend</strong></td>
<td><strong>Interpret</strong></td>
<td><strong>Identify</strong></td>
</tr>
<tr>
<td>0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
<td>1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
<td>1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
</tr>
<tr>
<td><strong>Decide</strong></td>
<td><strong>Discussing Student Work</strong></td>
<td><strong>Discussing Students’ Thinking</strong></td>
</tr>
<tr>
<td>1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
<td>X X X X X X X X X X X X X X X X X X X X X X X</td>
<td>X X X X X X X X X X X X X X X X X X X X X X X</td>
</tr>
<tr>
<td><strong>Other</strong></td>
<td><strong>Pedagogical Development/Discussion of SCK</strong></td>
<td><strong>Other</strong></td>
</tr>
<tr>
<td>X X X X X X X X X X X X X X X X X X X X X X X</td>
<td>X X X X X X X X X X X X X X X X X X X X X X X</td>
<td>X X X X X X X X X X X X X X X X X X X X X X X</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Second Grade Session One</th>
<th>Second Grade Session Two</th>
<th>Second Grade Session Three</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Attend</strong></td>
<td><strong>Interpret</strong></td>
<td><strong>Identify</strong></td>
</tr>
<tr>
<td>1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
<td>1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
<td>1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
</tr>
<tr>
<td><strong>Decide</strong></td>
<td><strong>Discussing Student Work</strong></td>
<td><strong>Discussing Students’ Thinking</strong></td>
</tr>
<tr>
<td>1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td>
<td>X X X X X X X X X X X X X X X X X X X X X X X</td>
<td>X X X X X X X X X X X X X X X X X X X X X X X</td>
</tr>
<tr>
<td><strong>Other</strong></td>
<td><strong>Pedagogical Development/Discussion of SCK</strong></td>
<td><strong>Other</strong></td>
</tr>
<tr>
<td>X X X X X X X X X X X X X X X X X X X X X X X</td>
<td>X X X X X X X X X X X X X X X X X X X X X X X</td>
<td>X X X X X X X X X X X X X X X X X X X X X X X</td>
</tr>
</tbody>
</table>