ABSTRACT

PERRY, AYANNA DARA-FRANKLIN. Equitable Spaces in Early Career Mathematics Classrooms. (Under the direction of Dr. Hollylynne Lee).

The study characterizes equitable classroom spaces in early career high school mathematics teachers’ accelerated and academic courses. Equitable classroom spaces are observable parts of classroom instruction resulting from interactions within a community of learners focused on particular content with normative ways of being aimed at reducing the barriers for participation and engagement for all students. Participation and engagement are described as students’ opportunities to learn (OTL) in high school mathematics classes which consists of opportunities to (a) engage with high cognitive demand tasks using prior knowledge, (b) connect procedural and conceptual knowledge, and (c) justify and explain mathematical thinking. To investigate these opportunities, teachers’ tasks and task implementations in both academic and accelerated classes were examined. Through a collective case study of six high school mathematics courses, three academic and three accelerated, the researcher describes how teachers’ choice and implementation of mathematical tasks support students’ OTL.

The participants were three early career high school mathematics teachers who taught two courses, one academic and one accelerated. Each teacher participated in six semi-structured interviews and five classroom observations for each course. Field notes and classroom artifacts such as homework, classwork, and assessment tasks were collected.

The theoretical framework is a modified Teaching Cycle (Simon, 1995). Task analysis was conducted at the course level, which yielded six cases of tasks. Tasks were numerically coded using the Potential of the Task Rubric (Boston, 2012c) and statistically
analyzed to investigate students’ opportunities to learn. Then, similarities in task collections showed that students with persistent opportunities to learn frequently engage with high demand tasks on homework, classwork, and assessments, while moderate and limited opportunities provide less frequent opportunities to engage with these tasks.

Task implementation analysis was conducted to examine how teachers’ classroom practice supported equitable classroom spaces. Equitable classroom spaces depend on students’ opportunities to use appropriate resources to engage with high cognitive demand tasks with clear expectations, share written work publicly, justify and explain thinking, observe models of high level thinking, and use resources to answer their own questions. These analyses were conducted by analyzing lessons, which resulted in 2 separate cases. The findings showed emergent, moderate, and prominent equitable classroom spaces. Emergent spaces provide students opportunities to use appropriate resources to engage with high cognitive demand task that have clear expectations. Moderate spaces provide students opportunities to share their written work and justify and explain their mathematical thinking in addition to the features of emergent spaces. Finally, prominent spaces include the opportunities included in moderate spaces and provide students opportunities to observe models of high-level reasoning and act as resources for themselves and others.

These findings suggest that early career mathematics teachers can support students’ opportunities to learn through the use of common classroom practices. The features of task collections and task implementations that support students’ opportunities to learn described in this study may provide a framework for early career teacher support and instruction of prospective teachers.
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Equitable Spaces in Early Career Teachers’ Mathematics Classrooms

by
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DEDICATION

I dedicate this dissertation to my family, especially my husband Jason Perry, my parents Mr. Glenister Franklin and Mrs. Joy Franklin, and sisters both biological and those forged by experience. God blessed me with the best support system ever!
BIOGRAPHY

Ayanna Dara-Franklin Perry was born on December 30, 1983 in Newark, New Jersey. She moved to Durham, North Carolina in 1993. She attended Charles E. Jordan High School in North Carolina where she was a member of the marching band for four years. After graduating from high school, she attended and then graduated magna cum laude from Shaw University, a HBCU in Raleigh, NC in May 2005 with a Bachelor of Science degree in Mathematics. Upon graduation, Ayanna attended North Carolina Central University, a HBCU in Durham, NC. She graduated in July 2007 with a Master of Science in Mathematics, and then moved to Laurinburg, NC to teach for two years in the Scotland High School of Business, Finance, and Marketing. During her two years as a teacher, she taught Algebra II, Geometry, Technical Mathematics, and Advanced Function and Modeling.

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Upon receiving her degree, Ayanna will be working with a national foundation whose focus is the support of inservice mathematics and science teachers. She also plans to continue pursuing her research interests in classroom practices that support equity.
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To my Mom, Dad, sisters, and extended family, you have been my steady foundation, my listening ear, and my shoulder to lean on. Thanks for always having my back.

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CHAPTER 1: INTRODUCTION

Background

For over 30 years, equity in mathematics classrooms has been identified as one of the major goals in mathematics education. The National Council for Teachers of Mathematics (NCTM), one of the largest professional organizations for mathematics teachers in the U. S., has been the source of one stance on equity in the mathematics classroom. In the *Agenda for Action*, NCTM (1980) recommends more mathematics study be required for all students because careers, both vocational and technical, require mathematical proficiency. The *Assessment Standards for School Mathematics*, published in the mid-90s, defines equitable assessment practices as those which raise academic expectations, clarify the purpose and meaning of mathematics, and help all students learn (NCTM, 1995). Subsequently, the *Principles and Standards for School Mathematics* (NCTM, 2000) composed the equity principle which states “All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study—and support to learn—mathematics” (NCTM, 2000, para. 1).

More recently, the Common Core State Standards for Mathematics (CCSSM) asserted “all students must have the opportunity to learn and meet the same high standard if they are to access the knowledge and skills necessary in their post-school lives” (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010a, p. 4). While these documents are written in reference to practicing teachers, teachers may not have a clear conception of what it means to provide support and opportunities for students to study and learn mathematics. This study aims to examine classroom practices
and develop guidelines for, and descriptions of, equitable classroom spaces that can be used to focus teachers’ attention on equitable practices in their classrooms.

In this study, classroom spaces are defined as observable parts of classroom instruction resulting from interactions within a community of learners focused on particular content with normative ways of being (Hiebert & Grouws, 2007; Sherin, Mendez, & Louis, 2004; Yackel & Cobb, 1996). The phrase *community of learners* is used to evoke the idea that learners must be active and engaged in the learning process and afforded opportunities to reflect on their learning (Sherin et al., 2004). The normative ways of being refer to classroom norms, which are accepted behaviors and actions within a classroom. The joint presence of particular content and classroom norms reference content related norms (i.e. social norms and sociomathematical norms), which are accepted behaviors and actions that are specific to the interactions among students and the teacher during lessons (Yackel & Cobb, 1996). Because equity can be conceived as accommodations that promote access and attainment for all students (Diversity in Mathematics Education (DiME) Center for Learning and Teaching, 2007; Gutiérrez, 2007; NCTM, 2000; Yackel & Cobb, 1996), equitable classroom spaces are *observable parts of classroom instruction resulting from interactions within a community of learners focused on particular content with normative ways of being aimed at reducing the barriers for participation and engagement for all students, especially those who have been systematically undervalued in these settings.*

An important step towards helping teachers understand and support equitable classroom spaces is to conduct research aimed at describing these spaces (Hiebert & Grouws, 2007). These descriptions will contribute to increased confidence in the knowledge base on
teaching as more detailed descriptions of teachers’ classroom practices help to create a “richer and more coherent body of literature on teacher practice” (Hiebert & Grouws, 2007, p. 395). This research will also provide opportunities for teachers to describe, discuss, and reflect on their practice, which may lead to improvements in classroom practice. Ball, Cohen, and colleagues (Ball, 1988, 2009; Cohen, 1988, 1990; Cohen, Raudenbush, & Ball, 2003) suggest that helping teachers reflect on their teaching may result in a desire to learn more effective practices to support student learning.

Descriptions of equitable classroom spaces will be developed from an analysis of teachers’ task choice and task implementation. Task choice refers to the mathematical tasks that are selected to use with students, and task implementation refers to parts of enacted lessons focused on mathematical tasks. Using task choice and implementation as a way to examine equitable classroom spaces supports teachers’ daily practice of posing tasks and expecting all students to engage with them (Stein & Smith, 1998; TeachingWorks, 2012). Though there are myriad ways that tasks are discussed in literature, the tasks described in this study are characterized by the Opportunity to Learn (OTL) framework. The use of the OTL framework informs classroom norms and characterizes equitable classroom spaces.

OTL is not a new framework for describing classroom practice. OTL initially referred to the amount of time allocated to a learner for the learning of a specific task or process (Carroll, 1963). It has since been used to describe the relationship between what is tested and what is taught, the types of activities that students engage in during class, and how students take advantage of opportunities in the classroom (Husen, 1967; K. Jones & Byrnes,
2006; Stevens, 1993; Wang, 2010). The current study aims to describe classroom spaces using tasks as a lens into students’ OTL at the high school level.

When teachers choose mathematical tasks, they need to consider the learning opportunities that students will have based on the tasks (Stein & Smith, 1998). Research suggests that students’ experiences of mathematics are closely tied to the tasks that teachers choose for them to engage with, both inside and outside of class (Hiebert et al., 1997). Further, students’ engagement with high cognitive demand tasks can provide them opportunities for mathematical reasoning and explanation of mathematical concepts (Boston & Smith, 2009; Boston & Wolf, 2006; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010a). Another important consideration around tasks is how students are supported during task implementation to justify and explain their thinking in both written and verbal formats. In order for students to benefit from high cognitive demand tasks, teachers need to know how to provide opportunities for student expression about mathematics during engagement with tasks (Boaler & Brodie, 2004; Henningsen & Stein, 1997). Finally, all students need access to the same opportunities and targeted supports so they can take advantage of these opportunities.

**Purpose of the Study**

To incorporate the constructs of equitable classroom spaces and students’ opportunities to learn, the overarching question guiding this study was “How do early career high school mathematics teachers’ classroom practices support students’ opportunities to learn?” This study focuses on early career high school mathematics teachers’ (ECMTs) equitable practices because there is a lack of research on how the classroom practices of
ECMTs promote equity (Bianchini & Brenner, 2009). Also there is documented evidence that the practices of early career teachers differ from those of more experienced teachers (Borko & Livingston, 1989). The three goals of this study are to 1) describe ECMTs’ instructional practices around choosing homework, classwork and assessment tasks, 2) describe practices around task implementation that can provide students opportunities to contribute and participate publicly, and 3) explore the similarities and differences among ECMTs’ instructional practices when teaching courses at different academic levels. Based on these goals, two research questions were answered.

1. How do the tasks that ECMTs choose support students’ opportunities to engage with high cognitive demand tasks in class, outside of class, and on assessments?
   a. How do the task choices of an ECMT differ when teaching academic versus accelerated courses?

2. How do ECMTs’ implementations of mathematical tasks support equitable classroom spaces?
   a. How do the task implementations of academic and accelerated courses differ?

**Significance of the Study**

The significance of this study applies in several areas. First, early career high school mathematics classrooms represent an understudied context. Conducting more research in these settings may provide insights into how ECMTs support their students’ learning. Second, this study focuses on the use of tasks and task implementation to describe students’ opportunities to learn. This provides an accessible framework for describing teachers’ practice. Use of the OTL framework may provide guidelines for how teachers can best
support their students’ learning. Third, this study seeks to compare teachers’ instructional practice in academic (regular or college preparatory) and accelerated (honors and advanced placement) courses. This comparison may provide examples of how students in these different course types are provided opportunities to learn, and help teachers consider what changes can be made in their practice to provide similar opportunities for students in different courses. Finally, this study provides portraits of task implementations, which can be used as exemplars of different levels of equitable classroom spaces that support students’ OTL.
CHAPTER 2: LITERATURE REVIEW

This chapter serves three purposes. The first is to offer a brief discussion of equity in mathematics education and how Opportunity to Learn (OTL) can be used as a framework to describe equity. The second is to use teachers’ mathematical task choice and implementation as a lens to describe OTL. The third is to discuss competent teaching and the classroom practices of beginning teachers. Literature relevant to the specific research questions for the study is presented.

A Brief Treatment of Equity

Equity is a difficult concept to discuss because the ways that equity is treated in research are based on the worldviews and contexts of particular studies, which cannot be generalized to different contexts. In spite of the many ways to approach research on equity, there is consensus on how the concept of equity is discussed in research; however before describing what equity is, we should state what equity is not. Equity is not equality. Equality refers to sameness that can be quantitatively measured (Esmonde, 2009). For example, students who have access to the same teachers and books and who receive identical instruction experience equality in the classroom (Bishop & Forgasz, 2007; Gutiérrez, 2007; NCTM, 2000). Instead, equity demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students (Alder, 2001; Gutiérrez, 2007; NCTM, 2000).

The equity principle posed by NCTM speaks generally about equity as “high expectations and strong support for all students” and references a specific goal that “all students, regardless of their personal characteristics, backgrounds, or physical challenges,
must have opportunities to study- and support to learn- challenging mathematics (NCTM, 2000, para. 1). Gutiérrez (2007) further elaborates on this notion of equity when she describes equity as: “the inability to predict mathematics achievement and participation of students based solely upon student characteristics such as race, class, ethnicity, sex, beliefs, and proficiency in the dominant language” (p. 2). This definition suggests that equity is about creating and supporting counter-stories to the achievement gap discourse (Stinson, 2008). These counter-stories foreground the opposite of falsely perpetuated ideas in mathematics, such as females and African Americans are not capable of success in mathematics (DiME, 2007).

In addition to the inability to predict students’ achievement based on physical or social characteristics, Gutiérrez (2012) includes access to physical resources, measurable outcomes of student learning, inclusion of students’ identities in curricula, and experiences of social transformation as doers of mathematics as features of equity. Still others define equity as “obliterating the differential and socially unjust outcomes in mathematics education” (Gutstein & Secada, 2000, p. 26); “a fair distribution of opportunities to learn or opportunities to participate” (Esmonde, 2009, p. 1010); and the opportunity to “participate substantially in all phases of mathematics lessons (e.g., individual work, small group work, whole class discussion), but not necessarily in the same ways” (Jackson & Cobb, 2010, p. 3).

Regardless of how equity has been defined, a focus on equity within mathematics education has resulted from evidence of bias or favoritism in education. Many researchers have discussed how underrepresented groups in science, technology, engineering, and mathematics (STEM) fields; such as women, African Americans, Latino/a students, and
students from poor socioeconomic backgrounds, have been undervalued with respect to the resources they bring to the classroom or held to low expectations in mathematics classrooms (e.g. Bishop & Forgasz, 2007; Carraher, Carraher, & Schliemann, 1985; DiME, 2007; Gutiérrez, 2007; Ladson-Billings, 1995a; McGee & Martin, 2011; Moody, 2001). This favoritism or bias may adversely affect student achievement and reduce opportunities for historically marginalized groups to participate in careers that require mathematics proficiency (DiME, 2007; Martin, Gholson, & Leonard, 2010; McKown & Weinstein, 2008; Moses & Cobb, 2001). One documented instance of favoritism is the ranking of theoretical or abstract knowledge over empirical knowledge (Code, 1991; Scribner & Cole, 1973). In mathematics education, this ranking of knowledge may result in a deficit model of what students learn outside of the classroom (Civil, 1998; Moschkovich, 2012), and in a positioning of students who can express concepts abstractly above those who struggle with such abstract expressions (DiME, 2007; Esmonde, 2009). The social and cultural inequities that may present themselves in the classroom setting suggests that there is more work to be done (Gates & Jorgensen-(Zevenbergen), 2009; Strutchens et al., 2011). However, in order to decide what should be done; there should be a consensus of what is already known.

Researchers have looked at creating equitable opportunities in instruction in various ways. Critical race theory (CRT) and situated or sociocultural learning theory (SLT) challenge “the common sense understandings, routine practices, policies, and forms of scholarship that intentionally or unintentionally dehumanize, depersonalize, and oppress Black people [and other marginalized people groups] in symbolic and material ways” (Martin & Gholson, 2012, p. 205). CRT asserts that racism is a normal part of American society that
affects all aspects of society, even learning in schools. This theory values stories expressing individual experiences that expose, analyze, and challenge the consistent stories of racial privilege (Ladson-Billings & Tate, 1995). Within the classroom setting, CRT requires the foregrounding of race and attending to how classroom practices or teachers’ perceptions of students may perpetuate an imbalance in the opportunities for all students to learn (Stinson, 2008).

SLT suggests learning is grounded in situations, needs to be linked to practical needs, and takes place in complex social environments (Reder, Anderson, & Simon, 1996). Application of SLT in classrooms focuses teachers’ attention on students as participants in a classroom community whose purpose is to contribute to and develop mathematical knowledge and practices. Teachers should also consider how mathematical content connects to the outside world and how classroom experiences improve students’ abilities to perceive interactions with content and act on their perceptions (Cobb & Bowers, 1999; Gee, 2008; Yackel & Cobb, 1996).

Each theory supports a shift in the ways that learning is characterized in order to value and redefine the identities and contributions of all participants and to change societal practices once they are exposed (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Gutiérrez, 2012; Ladson-Billings, 1995a; Martin et al., 2010; Strutchens et al., 2011). With respect to mathematics education, these theories suggest that the social activity of teaching requires a commitment to equal inclusion of all students. This commitment to inclusion should contribute to shifts in (a) the power structures of the classroom and (b) the influence, value, and positioning of classroom participants (Lave, 1996). Ultimately, these theories
suggest that teaching mathematics should enable students to see mathematics as a tool to spark social consciousness and change (Mukhopadhyay & Greer, 2001).

This commitment to challenging the common and discriminatory ways of being has also been addressed by frameworks for teaching. For example, Gutiérrez’s (2007, 2012) dimensions of learning and political conocimiento (knowledge) for teaching mathematics require teachers to attend to issues of access, achievement, identity, and power as components of classroom teaching. Culturally relevant or social justice pedagogy and multicultural instruction are classroom structures that also share these foci (Ladson-Billings, 1995a, 1995b; Leonard, Brooks, Barnes-Johnson, & Berry III, 2010). One example of culturally relevant pedagogy was presented by Smith and Stiff’s (1993) use of student originated contexts to teach mathematics. Smith and Stiff attended to students’ cultural realities and used students’ experiences as the basis for instruction in the classroom. That is, they mathematized students’ everyday experiences. To a lesser extent, curricula such as Carpenter and colleagues’ Cognitively Guided Instruction (1989) encourages teachers to investigate the problem types that students are able to solve as a basis for moving forward with instruction which shifts the focus from what students cannot do to the resources that all students bring to the classroom.

Though these theories and approaches have merit and can lead to improvements in students’ classroom experiences, there are barriers to widespread implementation. Theories like CRT and SLT, while meaningful worldviews, do not prescribe approaches to help with practical implementation. That is, once a teacher notices how race might be affecting students’ learning in his classroom or lopsidedness in the participation of students in the
classroom community, there is no clear explanation of what to do next. When considering the frameworks for teaching, teachers must have a substantial understanding of the mathematical knowledge base in order to mathematize seemingly mathless contexts (L. B. Smith & Stiff, 1993).

Culturally relevant pedagogy or multicultural instruction advocates allowing students to be the originators of the instructional material used in class which might highlight the limited knowledge that some teachers have on how mathematics connects with the outside world (Ladson-Billings, 1995a; Sowder, 2007). To implement these pedagogies, teachers also need to be dedicated to teaching as a tool for challenging the status quo, which may be overwhelming for teachers, especially early career teachers in traditional educational settings (Kelly & Brandes, 2001; Sowder, 2007). In addition to the depth of knowledge that teachers must have to connect students’ experiences to mathematical content, teachers must also have in-depth knowledge of their students’ cultural contexts to create relevant pedagogy for them (Leonard et al., 2010). To acquire this in-depth knowledge of students, researchers suggest teachers visit and learn from the families of students; an highly emotional and time intensive endeavor (c.f. Civil, 1998).

This review of research on equity may paint a bleak picture for pedagogies or teaching frameworks that have been applied to equitable instruction; however, the intent is to show the need for additional frameworks that provide teachers with approaches to classroom practice that attend to issues of equity and can be applied on a larger scale. An Opportunity to Learn (OTL) framework can meet this need. "The circumstances that allow students to engage in and spend time on academic tasks such as working on problems, exploring
situations and gathering data, listening to explanations, reading texts, or conjecturing and justifying have been labeled opportunity to learn” (National Research Council, 2001b, pp. 333-334, emphasis in original). OTL frameworks highlight practices that have been described as good teaching in classrooms using multicultural instruction (Ladson-Billings, 1995a), and these circumstances can be targeted as a way to support equitable instruction without a major focus on challenging societal injustices. The OTL framework also incorporates themes found in culturally relevant pedagogy such as the creation of equitable opportunities to learn in classroom settings, inclusive discourse practices, and inclusive assessment practices that expand conceptions of mathematical ability (DiME, 2007).

Before discussing the circumstances that support students’ OTL, two assumptions must be articulated (Harris & Anderson, 2012): students’ learning evolves over time and builds on prior knowledge and interactions with curriculum. These assumptions are also tenets of constructivist learning theories (Confrey, 1990), as well as, social justice and culturally relevant pedagogy; pedagogies that encourage implementing curriculum based on what students know and learn both inside and outside of the classroom (Gutiérrez, 2007; Ladson-Billings, 1995a). The DiME Center for Learning and Teaching posits that OTL applied as a framework allows researchers to consider access to engagement in mathematics classrooms for diverse student groups while considering the social and cultural resources that students bring to the classroom (2007). The next section will give a history of OTL and will discuss OTL as a framework for measuring classroom instruction.
Opportunity To Learn (OTL)

The earliest known applications of OTL are Carroll (1963) and Husen (1967). Carroll describes OTL as the amount of time allocated to a learner for the learning of a specific task or process acknowledging a difference between the time needed for learning, time of student engagement, and OTL (Carroll, 1963), while Husen describes it as the amount of overlap between what is taught and what is assessed (Husen, 1967; Porter, 1993). Over time, OTL has been used to describe more varied classroom practices. Stevens (1993) uses a four variable OTL framework to reanalyze data from a multi-case study conducted with elementary teachers during reading instruction in 10 urban schools. The variables used assess content coverage, content exposure, content emphasis, and quality of instruction delivery. Content coverage variables measure whether or not students are exposed to critical subject matter for a specific grade while content exposure variables measure the depth of instruction offered during structured instructional time. Content emphasis variables refer to how topics and students are paired and the quality of instructional delivery variables measure how classroom pedagogical strategies affect students’ academic achievement. Stevens’ (1993) work shows a difference in opportunities to learn across the 10 schools. This difference was based on inconsistencies in the number of reading lessons across the schools and in the expression of lesson objectives when reading lessons were implemented. Inconsistencies in the number of reading lessons are described as low content exposure for some students. The absence of learning objectives or misalignment between objectives and lesson activities in some of the schools during reading instruction is described as low quality of instruction due to a lack in coherence of lessons. As a result of the inconsistencies in the
data, Stevens suggests that the students in the 10 schools do not have the same opportunities to learn.

Wang (2010) measures OTL variables such as mathematics instructional time, instructional method, and instructional emphasis as a way to describe OTL in kindergarten classes. Using pre and post-tests to investigate kindergarten students’ mathematics skills and surveys to learn about teachers’ mathematics teaching activities, Wang investigates connections between students’ OTL and teachers’ instructional emphasis. Wang (2010) finds that kindergarten students with more concentrated instructional time; more access to manipulatives, worksheets, and textbooks; and more access to direct instruction and direct modeling of correct measuring practices have higher mathematics achievement. Harskamp and Suhre (1994) use log books to measure OTL, where teachers in Dutch primary schools record the number of lessons per week, the amount of time spent on lessons per week, and pacing (the number of exercises presented per week) as a way to measure factors that influence student achievement on assessments. Their work shows that the amount of lesson time and pacing both affect students’ achievement test results and ultimately students’ opportunities to learn.

When OTL is used at the secondary level, the focus is on how students take advantage of opportunities to learn within the classroom (K. Jones & Byrnes, 2006). Jones and Byrnes (2006) suggest that opportunities to learn are only as effective as students’ willingness to take advantage of these opportunities. They assumed that access to a high quality teacher as defined by the researchers, high socioeconomic backgrounds, homogeneity in the race of participants, and the same previous courses result in similar opportunities to
learn; therefore, their research focus was on how differences in student achievement could be explained by differences in student motivation.

Previous applications of OTL have focused on time-on-task and the overlap of coverage and assessment (e.g. Carroll, 1963; Husen, 1967; Stevens, 1993; Wang, 2010), however, these approaches to students’ learning opportunities do not take into consideration whether students are engaging with demanding curricula and neglect the fact that assessments generally test basic skills, not overarching ideas (Tate, 1995). Further, though students do play a part in their learning (Carroll, 1963; K. Jones & Byrnes, 2006), the QUASAR (Quantitative Understanding: Amplifying Student Achievement and Reasoning) project is based on the premise that students’ failures may be a consequence of a lack in opportunities to participate in meaningful experiences versus a lack of ability or potential on the part of the students (Henningsen & Stein, 1997). Tate (2001) seconds this notion in his discussion of tracking of students by curricula and ability. He states that students who do not have opportunities to take important prerequisites are prohibited from taking higher-level courses that support the learning of challenging curricula (Tate, 2001). One way to investigate students’ opportunities to engage in meaningful tasks is to focus on the tasks that their teachers select for use during instruction. The following section will discuss OTL and mathematical tasks.

**OTL and Mathematical Tasks**

This study uses the National Research Council’s definition of OTL—circumstances that allow students to engage with academic tasks (2001b). Academic tasks are defined as structures that have a specific goal, a set of resources present in the situation, and operations
that can be used to reach an end goal (Doyle & Carter, 1984). Because mathematical tasks shape students’ ideas about what mathematics is and what it means to be doing mathematics (Hiebert et al., 1997), a focus on the choice and use of mathematical tasks can be used as a way to describe high school students’ opportunities to learn (OTL). A mathematical task is “a classroom activity, the purpose of which is to focus students’ attention on a particular mathematical idea” (Stein, Grover, & Henningsen, 1996, p. 460). The mathematical tasks that students engage in can either perpetuate a conception of mathematics as a rigid collection of rules and procedures or as a collection of generalizable and connected processes that can be reasoned about, questioned, and explored (Henningsen & Stein, 1997).

Research investigating mathematical tasks may not explicitly refer to students’ OTL, but much of the research conducted on the choice and use of mathematical tasks does provide insight on how students’ opportunities to learn may be influenced by the types of tasks that they engage with in mathematics classrooms. For example, Arbaugh and Brown (2006) conducted a study on how professional development aimed at task analysis could influence teachers’ thinking about and choice of tasks because they believe that the types of tasks that students engage with influence the ways that students think and learn mathematically. To describe how task choices influence students’ OTL, three components of task choice will be discussed: choosing tasks that support learning goals, build on prior knowledge, and link procedural and conceptual knowledge. A discussion of task implementation as an indication of OTL will be discussed in the following section.

**OTL and learning goals.** In order to choose appropriate mathematical tasks for students, first teachers must clearly articulate learning goals (Stein, Smith, Henningsen, &
There are two levels of learning goals in each classroom: long-term and short-term. Long-term learning goals help teachers sequence tasks over time to aid in the planning of sequential mathematical tasks to be used in a classroom (Hiebert et al., 1997). Attending to long-term goals reinforces the belief that student learning is gradual and comes from interaction with mathematical objects and other mathematics learners over time (Confrey, 1990). Short-term learning goals help teachers sequence items within a task and tasks within a class to assist students in moving toward greater understanding of mathematical concepts within the space of a lesson or class period (Hiebert, Morris, Berk, & Jansen, 2007; Stein, Engle, Smith, & Hughes, 2008).

Gradual building of student knowledge requires that tasks be related; that is tasks should allow students to consider the same idea from different points of view or to discover new solutions building on previous strategies. Hiebert and colleagues (1997) suggest that relationships between mathematical tasks support students’ notion that mathematics is coherent, can be reasoned about, and builds on prior knowledge. To ensure that students see the coherence in the tasks used during class, teachers must consider students’ needs when creating learning goals.

Considering students’ needs moves teachers from just thinking about the materials to be covered to considering how students may interact with planned tasks and how these tasks may support mathematical learning. Simon (1995) describes a teacher’s plan for how students might interact with mathematical tasks over time as a hypothetical learning trajectory (HLT) (see Figure 1). This concept was also the basis for Confrey’s (2006) model of learning which is a visual of a hypothesized learning process (see Figure 2).
Figure 1: Hypothetical Learning trajectory (Simon, 1995, p. 137)

Figure 2: Model of Learning (Confrey, 2006, p. 146)
Together these models of how teachers conceive their students’ learning provide insight into how important learning goals are to guiding learning. The HLT suggests that teachers constantly reevaluate how students are interacting with mathematical tasks and make adjustments to the lesson while still assisting student in moving from prior knowledge to the learning of new material (Simon, 1995). In order to evaluate students’ learning, teachers have to continually consider their learning goals. Confrey’s visual of the model of learning presents a conceptual corridor that represents the range of ways that a teacher may choose to represent mathematical ideas, and presents prior knowledge as a tool used to introduce or orient students to the corridor (Confrey, 2006). In this setting, the learning goal directs the path that the class will traverse to move from prior knowledge to learned ideas. Having discussed the purpose of learning goals, the next discussion will describe how learning goals may influence students’ opportunities to learn.

Again, considering students’ OTL as the circumstances that support engagement with challenging mathematics, learning goals set by teachers for students directly influence the circumstances in which students learn. Research suggests that teachers set learning goals for students for various reasons. Szajn (2003) reports that the instructional strategies that teachers choose may be a result of beliefs that teachers hold about the needs of the students they are teaching. There is also consensus that what teachers believe about the nature of and purpose for learning mathematics can shape learning goals and consequently students’ opportunities to learn mathematics (National Research Council, 2001a). In order for students to have ample opportunities to learn, teachers must set learning goals with a somewhat dual focus: curricula and student needs. Teachers should always have an eye toward the
mathematics that students need to understand to become participants in the mathematical discourse both inside and outside of schools (Gutiérrez, 2012). Also, teachers should have an eye towards the academic supports that students might need in order to traverse the HLT constructed for the class. A practical way to link the mathematics and the supports that students need to learn it is to focus on students’ prior knowledge as a starting point for instruction.

**OTL and prior knowledge.** Assuming that students are not blank slates and that prior knowledge heavily interacts with students’ understanding of new mathematical concepts (Elby, 2000), teachers need to learn about students’ prior knowledge and use it as a basis for instruction (Confrey, 2006). Tasks can be used to learn about students’ prior conceptions of mathematical content. By choosing tasks that relate to previously learned skills or procedures and allowing students to interact with these tasks, teachers can learn about students’ conceptions of previously learned mathematics. For example, students who learned about transformations of figures during geometry instruction may have a more robust understanding of transformations of functions if they are given the opportunity to link those two experiences.

The opportunity to link previously learned information to new situations through the selection of coherent and related tasks can support students in restructuring and/or linking previously disjointed mathematics ideas, which may result in an articulate, unified, coherent concept. In addition to supporting the development of coherent mathematics knowledge, complex tasks based on prior knowledge can support students’ perseverance and progress (Stein et al., 1996) and provide relevance for students as previously learned information is
applied to novel situations. When students have the necessary tools based in prior knowledge to troubleshoot incomplete or alternative solution strategies, they may be more likely to continue grappling with tasks. Further, students’ “ability to articulate their mathematical understanding in the context of prior knowledge and experience gives their teachers insights into [their] conceptual understandings as well as their lives outside of the classroom” (Merritt, Rimm-Kaufman, Berry, Walkowiak, & McCracken, 2010, p. 243).

Tasks with real-world scenarios can also provide students opportunities to learn using prior knowledge. Problems with real-world scenarios that relate to students’ out of school experiences support the mathematization of settings students may perceive as mathless (L. B. Smith & Stiff, 1993). Real-world scenarios can also support students’ reasoning because prior knowledge of the scenario can support students in reasoning mathematically (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013). Though there is a benefit to having tasks that relate to students’ prior experiences in mathematics classes and outside of school, the tasks that students engage with need to provide opportunities for mathematical reasoning. Using tasks that link procedural and conceptual knowledge is one way to support students’ mathematical reasoning because students cannot blindly apply procedures. Students must consider the underlying mathematical concepts that support the procedures (M. S. Smith & Stein, 1998).

**OTL and linking procedural and conceptual knowledge.** Tasks that link or require students to draw on both procedural and conceptual knowledge have been classified as high cognitive demand tasks (Stein & Smith, 1998). Cognitive demand is “the kind and level of thinking required by students in order to successfully engage with and solve a task”
High cognitive demand tasks are tasks that require students to solve genuine problems, reason, justify, make connections, or make sense of mathematical ideas while low demand tasks require students to perform rote procedures or recall previously memorized material (Boston, 2012b). There are a few considerations when classifying mathematical tasks as high or low cognitive demand. To properly classify a mathematical task, teachers must consider the features of the task and the students who will be presented with the tasks. For tasks to elicit a high level of cognitive demand, they must be appropriately challenging for the students who will engage with them, require students to justify their reasoning process, and allow a level of flexibility with the ways that students can arrive at a solution or have multiple solutions.

Stein and Smith (1998) developed the Task Analysis Guide to rank mathematical tasks (Figure 3). This guide provides descriptors for tasks that rank low demand and tasks that rank high demand. Low demand tasks are categorized as memorization or procedures without connections to understanding, meaning, or concepts tasks (procedures without connections). Memorization tasks are described as tasks that only require students to recall previously learned facts. These types of tasks commonly do not have procedures for finding solutions or are not allotted enough time for students to use known procedures. Drill assignments, such as having students recite or record multiplication facts over a short period of time, would fall under this category. Though memorization tasks are useful in class settings, they should not be the sole type of task that students engage with because this limits students’ opportunities to use learned procedures and explain their reasoning process.
The second low cognitive demand task category is procedures without connections. The mathematical tasks ranked in this category are algorithmic. Students are generally told what procedure to use to solve the task or know what procedure to use based on the type of task they receive. An example of a task that engages students in procedures without connections is, *Use synthetic division to solve the following problem* \((5x^3 + 7x^2 - 8) : (x + 4)*. Tasks can also fall into this category if students have learned only one strategy to reach the solution. Again, tasks that provide students opportunities to use and practice with procedures are necessary in the classroom setting to help students gain facility with procedures but students also need opportunities to make connections between the procedures they use and the mathematical concepts that support those procedures.
<table>
<thead>
<tr>
<th>Lower-level Demands</th>
<th>Higher-level Demands</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Memorization</strong></td>
<td><strong>Procedures With Connections</strong></td>
</tr>
<tr>
<td>• Involve either reproducing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory.</td>
<td>• Focus students’ attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts.</td>
</tr>
<tr>
<td>• Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.</td>
<td>• Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.</td>
</tr>
<tr>
<td>• Are not ambiguous-such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated.</td>
<td>• Usually are represented in multiple ways (e.g. visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning.</td>
</tr>
<tr>
<td>• Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.</td>
<td>• Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Procedures Without Connections</strong></th>
<th><strong>Doing Mathematics</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Are algorithmic. Use of the procedure is either specifically called for or is evident based on prior instruction, experience, or placement of the task.</td>
<td>• Require complex and nonalgorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked out example).</td>
</tr>
<tr>
<td>• Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.</td>
<td>• Require students to explore and understand the nature of mathematical concepts, processes, or relationships.</td>
</tr>
<tr>
<td>• Have no connections to the concepts or meaning that underlie the procedure being used.</td>
<td>• Demand self-monitoring or self-regulation of one’s own cognitive processes.</td>
</tr>
<tr>
<td>• Are focused on producing correct answers rather than developing mathematical understanding.</td>
<td>• Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.</td>
</tr>
<tr>
<td>• Require no explanations, or explanations that focus solely on describing the procedure that was used.</td>
<td>• Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.</td>
</tr>
</tbody>
</table>

Figure 3: Task Analysis Guide (Stein et al., 2000, p. 16)
There are also two categories of high demand tasks: procedures with connections to understanding, meaning, or concepts (procedures with connections) and doing mathematics. Procedures with connections tasks require students to connect the procedures that are being use to the underlying mathematical concepts. Procedures with connections tasks may have multiple representations that students must interpret and connect to reach a solution. The purpose of these tasks is to help students to gain understanding of mathematical concepts through the use of procedures. An example of a procedures with connections task is, Use synthetic division to solve the following problem: \((5x^3 + 7x^2 - 8) : (x + 4)\) Explain how this process relates to long division of polynomials. This task affords students the opportunity to use what may be a new procedure to explain a previously learned procedure and requires students to see the similarity between two strategies for division of polynomials. Because much of the mathematics that is taught today is based in mathematical procedures (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010a), students need regular opportunities to connect the procedures they use to underlying mathematical concepts.

Doing mathematics, the highest level of cognitive demand of tasks, characterizes complex tasks that require nonalgorithmic thinking. These types of tasks provide students opportunities to enact relevant knowledge and experiences to solve mathematical tasks. The purpose of doing mathematics tasks is to help students consider the limitations and assumptions that must be expressed to solve nonroutine tasks. An example of a doing mathematics task is, How many Ping-Pong balls can fit into a cube with side length 6. State your assumptions and provide an explanation of your solution. This task asks students to
measure the volume of a cube with Ping-Pong balls and to explain how they arrive at their solution. Tasks that provide students opportunities to create and explain their own solution strategies can support the building of a cohesive network of mathematical concepts. As these types of nonroutine tasks may not align with teachers’ daily curriculum, students may not have regular opportunities to do these types of tasks, but they should have some opportunities to grapple with mathematics on this level.

Researchers at the Learning Research and Development Center developed another tool to rank mathematical tasks. The Instructional Quality Assessment (IQA) toolkit is used to measure classroom practice (Junker et al., 2006). Early in its development the IQA contained 20 rubrics organized into three groups. The rubrics were created around four principles of learning: academic rigor, clear expectations, self-management of learning, and accountable talk. Currently, there are two collections of rubrics: rubrics rating academic rigor in mathematics and rubrics rating accountable talk (Boston, 2012b, 2012c, 2012d; Boston & Smith, 2009; Boston & Wolf, 2006). The rubric discussed here is the Potential of the Task rubric.

The Potential of the Task rubric is based on the Task Analysis Guide (Stein et al., 2000, see Figure 3) and assigns numerical ranks to the categories of low and high cognitive demand tasks. However, this rubric includes additional task descriptors, which explicitly refer to students’ opportunities to connect conceptual and procedural knowledge and explain and justify their thinking. Tasks are ranked from 0 to 4. Tasks ranked between 0 and 2 inclusive are low cognitive demand and tasks ranked 3 or 4 are high cognitive demand. The
highest potential of reasoning present in the tasks corresponds to the rank assigned (Boston & Smith, 2009).

Providing students opportunities to express their thinking in verbal and written formats can bolster OTL - the circumstances that support students’ engagement with mathematical tasks. Tasks that have the potential to encourage mathematical reasoning and complex thinking, and require students to express their thinking help students to verbalize and form cohesive, coherent theories about mathematics (Hiebert et al., 1997). Also, this level of engagement on worthwhile tasks is what can assist students in seeing themselves as doers of mathematics (DiME, 2007; Esmonde, 2009).

The previous sections described how mathematical tasks used during instruction support students’ OTL. The next sections describe how the mathematical tasks that students interact with that are not used for in-class instruction, summative assessments and homework, support students’ OTL as well.

**OTL and Assessments**

Assessment of student learning is a large part of classroom instruction (Wiliam, 2007). Teachers generally use tests, quizzes, and various projects as summative assessments to learn about what students know and know how to do (Senk, Beckmann, & Thompson, 1997). Summative assessments have previously been the subject of students’ OTL. Husen (1967) describes students’ OTL as the overlap between what is taught and what is assessed. Building on Husen’s conception of opportunity to learn, teachers must consider assessment tasks when they construct learning goals. In order for a teacher to assess students’ learning he/she must first clearly articulate learning goals and then think about what type of
assessment is best used to assess that learning. NCTM (1995) describes effective mathematical assessment as "activities that provide all students with opportunities to formulate problems, reason mathematically, make connections among mathematical ideas, and communicate about mathematics" (The Mathematics Standard, para. 3). In this view of assessment, procedural and factual knowledge are tools used to solve worthwhile mathematical tasks. In order for teachers to accurately select assessments that require students to reason mathematically and make connections among mathematical ideas, their learning goals must coincide with this view of assessment (Stein et al., 2000).

Assessments as a tool to support students’ mathematical reasoning align with literature on mathematical tasks. The assessments require students to justify and explain their thinking and use procedures to support mathematical reasoning, and therefore are described as high cognitive demand tasks.

**OTL and Homework**

Though there have been longstanding debates about the necessity of and reasons for homework (Gill & Schlossman, 2000), homework is still a part of many teachers’ instructional plans. Research suggests that worthwhile homework assignments may positively affect high school students’ grades and achievement on standardized tests (Marzano & Pickering, 2007; Trautwein, 2007). This effect is based on the amount of homework assignments that are completed (Trautwein, 2007). Homework helps to extend student learning beyond the structure of the school day, which increases their learning opportunities. Also, well-structured homework assignments may encourage student completion, which supports students’ OTL. Research states that appropriate homework
assignments are purposeful and efficient, aesthetically appealing, and promote ownership and competence (Carr, 2013; Marzano & Pickering, 2007).

Purposeful homework assignments are meaningful. They have clear expectations and provide teachers with feedback on students’ understanding. In order for teachers to assign homework tasks that are purposeful, teachers must think about homework assignments through the lens of learning goals. As was the case in assessment, homework tasks must also advance the learning goals. Efficiency refers to homework assignments that require students to think but do not require an exorbitant amount of time to complete (Carr, 2013).

The promotion of ownership and competence are features of effective homework assignments that align with the literature on mathematical tasks. Helping students to understand the utility of homework and connecting assignments to students’ interests can support ownership, while differentiating homework assignments to ensure that students with different prior knowledge can successfully engage with them promotes competence (Carr, 2013; Henningsen & Stein, 1997). Research shows that assigning tasks that are too difficult for students can reduce motivation, so homework assignments should be developed at the appropriate level to challenge students but not frustrate them (Henningsen & Stein, 1997). These features also align with the literature on mathematical tasks because high cognitive demand tasks are those that have multiple entry points and encourage students to develop their own solution strategies. Finally, homework assignments that are visually appealing can support students’ motivation to complete them (Carr, 2013).

There are similarities in the features of high demand mathematical tasks and purposeful homework assignments. Teachers who are thoughtful about the mathematical
tasks that students engage with during class should apply those same considerations to the work that students complete outside of the classroom to ensure that students have comparable opportunities to learn and are motivated to take advantage of those opportunities.

**Teachers’ Role in OTL**

A discussion of students’ OTL would be incomplete without discussing the part that teachers play in supporting students’ OTL. Having challenging or high demand tasks alone cannot support students’ OTL (Stein et al., 2000). Teachers must carefully consider the needs of students when planning instruction to facilitate student learning during instruction. In considering students’ needs, teachers must be aware of and sensitive to student cultures and experiences outside of class (Gutiérrez, 2007; Hiebert et al., 1997; Ladson-Billings, 1995b; Stein et al., 2000). Teachers must also structure their classrooms in a way that supports the participation and expression of all students (Yackel & Cobb, 1996). There are infinitely many things that teachers may consider when planning how to support students, but those things fall into what might be described as different facets of teacher knowledge.

Recall that Simon’s (1995) *HLT* is informed by teachers’ knowledge of mathematics content, mathematical tasks, how students learn, and hypotheses of students’ prior knowledge (see Figure 1).

Teachers’ knowledge of mathematics supports the development and articulation of specific short term and long term learning goals. When teachers have a grasp on how mathematics knowledge develops over time, they can support students in engaging with meaningful mathematics (Stein et al., 2000). Teacher knowledge of tasks supports the use of mathematical tasks that engage students at the appropriate cognitive level (Yeo, 2007).
Teachers’ hypotheses about students’ prior knowledge may support the selection of tasks better suited for use in their classrooms if teachers have knowledge of the types and levels of tasks that can support their learning goals (Hiebert et al., 1997; Stein et al., 2000; Yeo, 2007). Teacher knowledge of how students learn influences the ways that teachers structure their classrooms and the flow of information (Hiebert et al., 1997). Knowledge of tasks and hypotheses of students’ prior knowledge may be most useful in the lesson planning phase of teacher practice, while knowledge of how students learn may be most useful for task implementation (Stein et al., 2008). Knowledge of mathematical content is an essential support for both lesson planning and task implementation (Hiebert et al., 2007; Stylianides & Stylianides, 2008).

**OTL and task implementation.** Teachers’ beliefs about and knowledge of how students learn heavily influences how teachers structure their classrooms (Franke, Kazemi, & Battey, 2007; Philipp, 2007). If teachers believe that learning is “social and shared, where teachers and students bring histories and identities to the interaction, [and] where participation is the focus” (Franke et al., 2007, p. 228), then teachers will work to include students as active participants in the class. In order for learning to be social and shared, students and the teacher need to ask questions of each other, challenge each other’s mathematical explanations, and work publicly toward a shared meaning for mathematical content (Hiebert et al., 1997; Yackel & Cobb, 1996). This conception of learning supports students’ opportunities to explain and justify their thinking because teachers who hold this conception of learning value discourse. Teachers who consider learning a social and shared activity will also provide their students opportunities to work in small group and individual
settings with the intention of providing opportunities for students to share their individual knowledge with each other and share small group knowledge with the whole group.

The IQA also contains a rubric that assesses task implementation, Rubric 2: Implementation of the Task rubric (see Figure 4). This rubric is to be used in conjunction with the Mathematics Lesson Checklist (See Figure 5). Together these rubrics assist researchers in measuring student engagement as a result of a teacher’s task choices and guidance during a lesson. The Implementation of the Task rubric ranks a teacher’s implementation of tasks based on what classroom observers see students do during the lesson. Task implementations that rank at level 4 show students justifying and explaining or illustrating mathematics concepts, processes, or relationships, while those at level 3 show students engaged in complex thinking but with incomplete expressions of reasoning. Implementations at level 2 show students engaged in using procedures that are specifically called for and those at level 1 show students providing facts or previously memorized information. To help classroom observers rank task implementations the Mathematics Lesson Checklist was developed. This checklist is designed to be used during the viewing of observation videos to describe the opportunities available to students during task implementation.
### RUBRIC 2: Implementation of the Task

At what level did the teacher guide students to engage with the task in implementation?

<table>
<thead>
<tr>
<th>Score</th>
<th>Description</th>
</tr>
</thead>
</table>
| 4     | Students engaged in exploring and understanding the nature of mathematical concepts, procedures, and/or relationships, such as:  
· Doing mathematics: using complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example);  
OR Procedures with connections: applying a broad general procedure that remains closely connected to mathematical concepts. There is explicit evidence of students’ reasoning and understanding.  
For example, students may have:  
· solved a genuine, challenging problem for which students' reasoning is evident in their work on the task; developed an explanation for why formulas or procedures work;  
· identified patterns, formed and justified generalizations based on these patterns;  
· made conjectures and supported conclusions with mathematical evidence;  
· made explicit connections between representations, strategies, or mathematical concepts and procedures; and/or followed a prescribed procedure in order to explain/illustrate a mathematical concept, process, or relationship. |
| 3     | Students engaged in complex thinking or in creating meaning for mathematical concepts, procedures, and/or relationships. However, the implementation does not warrant a “4” because:  
· there is no explicit evidence of students’ reasoning and understanding.  
· students engaged in doing mathematics or procedures with connections, but the underlying mathematics in the task was not appropriate for the specific group of students (i.e., too easy or too hard to sustain engagement with high-level cognitive demands);  
· students identified patterns but did not form or justify generalizations;  
· students used multiple strategies or representations but connections between different strategies/representations were not explicitly evident;  
· students made conjectures but did not provide mathematical evidence or explanations to support conclusions. |
| 2     | Students engaged in using a procedure that was either specifically called for or its use was evident based on prior instruction, experience, or placement of the task. There was little ambiguity about what needed to be done and how to do it. Students did not make connections to the concepts or meaning underlying the procedure being used. Focus of the implementation appears to be on producing correct answers rather than developing mathematical understanding (e.g., applying a specific problem solving strategy, practicing a computational algorithm).  
OR There is evidence that the mathematical content of the task is at least 2 grade-levels below the grade of the students in the class. |
| 1     | Students engage in memorizing or reproducing facts, rules, formulae, or definitions. Students do not make connections to the concepts or meaning that underlie the facts, rules, formulae, or definitions being memorized or reproduced. |
| 0     | Students did not engage in mathematical activity. |
| N/A   | The students did not engage with a mathematical task. |

Figure 4: Implementation of the Task Rubric (Boston, 2012c, p. 20)
<table>
<thead>
<tr>
<th>Mathematics Lesson Checklist: Check each box that applies.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A: The Lesson provided opportunities for students to engage with high-level cognitive demands:</strong></td>
</tr>
<tr>
<td>Students</td>
</tr>
<tr>
<td>o engaged with the task in a way that addressed the teacher’s goals for high-level thinking and reasoning.</td>
</tr>
<tr>
<td>o communicated mathematically with peers.</td>
</tr>
<tr>
<td>o had appropriate prior knowledge to engage with the task.</td>
</tr>
<tr>
<td>o had opportunities to serve as mathematical authority in classroom</td>
</tr>
<tr>
<td>o had access to resources that supported their engagement with the task.</td>
</tr>
<tr>
<td>Teacher</td>
</tr>
<tr>
<td>o supported students to engage with the high-level demands of the task while maintaining the challenge of the task</td>
</tr>
<tr>
<td>o provided sufficient time to grapple with the demanding aspects of the task and for expanded thinking and reasoning.</td>
</tr>
<tr>
<td>o held students accountable for high-level products and processes.</td>
</tr>
<tr>
<td>o provided consistent presses for explanation and meaning.</td>
</tr>
<tr>
<td>o provided students with sufficient modeling of high-level performance on the task.</td>
</tr>
<tr>
<td>o provided encouragement for students to make conceptual connections.</td>
</tr>
<tr>
<td><strong>B: The lesson did not provide opportunities for students to engage with high-level cognitive demands:</strong></td>
</tr>
<tr>
<td>The task</td>
</tr>
<tr>
<td>o expectations were not clear enough to promote students’ engagement with the high-level demands of the task.</td>
</tr>
<tr>
<td>o was not rigorous enough to support or sustain student engagement in high-level thinking</td>
</tr>
<tr>
<td>o was too complex to sustain student engagement in high-level thinking (i.e., students did not have the prior knowledge necessary to engage with the task at a high level).</td>
</tr>
<tr>
<td>The teacher</td>
</tr>
<tr>
<td>o Allowed classroom management problems to interfere with students’ opportunities to engage in high-level thinking.</td>
</tr>
<tr>
<td>o provided a set procedure for solving the task.</td>
</tr>
<tr>
<td>o shifted the focus to procedural aspects of the task or on correctness of the answer rather than on meaning and understanding.</td>
</tr>
<tr>
<td>o Gave feedback, modeling, or examples that were too directive or did not leave any complex thinking for the student.</td>
</tr>
<tr>
<td>o Did not press students or hold them accountable for high-level products and processes or for explanations and meaning.</td>
</tr>
<tr>
<td>o Did not give students enough time to deeply engage with the task or to complete the task to the extent that was expected.</td>
</tr>
<tr>
<td>o Did not provide students access to resources necessary to engage with the task at a high level.</td>
</tr>
<tr>
<td><strong>C: The Discussion provides opportunities for students to engage with the high-level demands of the task.</strong></td>
</tr>
<tr>
<td>Students:</td>
</tr>
<tr>
<td>o Use multiple strategies and make explicit connections or comparisons between these strategies, or explain why they choose one strategy over another.</td>
</tr>
<tr>
<td>o use or discuss multiple representations and make connections between different representations or between the representation and their strategy, underlying mathematical ideas, and/or the context of the problem.</td>
</tr>
<tr>
<td>o identify patterns or make conjectures, predictions, or estimates that are well grounded in underlying mathematical concepts or evidence.</td>
</tr>
<tr>
<td>o generate evidence to test their conjectures. Students use this evidence to generalize mathematical relationships, properties, formulas, or procedures.</td>
</tr>
<tr>
<td>o (rather than the teacher) determine the validity of answers, strategies or ideas.</td>
</tr>
</tbody>
</table>

Figure 5: Mathematics Lesson Checklist (Boston, 2012c, p. 21)
Each item on the checklist describes opportunities for students to engage with a high
cognitive demand task. Box A describes the ways that teachers can support students in
engaging with high cognitive demand tasks. These supports include giving students enough
time to engage with tasks, holding students accountable for high-level products, consistently
pressing students for explanations of strategies, and providing encouragement to make
conceptual connections. Box B describes the ways that teachers may struggle in providing
students supports for engaging with high cognitive demand tasks. These include providing
set procedures for solving the task, shifting focus to procedures during implementations,
modeling examples or giving feedback that removes opportunity for complex thinking, and
not providing enough time to engage with tasks. The final box, Box C describes the ways
that class discussion should support students’ opportunities to engage with high demand
tasks. These supports include opportunities to discuss multiple representations and make
connections between them, opportunities to express conjectures and test them, and
opportunities to validate solutions and solution strategies. Lessons that rank 4 on the rubric
for task implementation are those that can be linked to the practices described in Box A and
Box C. Lessons that rank 3 do not have strong links to the practices in Box C but do
exemplify those practices described in Box A. Lessons that rank 2 align with the practices
described in Box B and those lessons that rank 1 only provide students opportunities to share
previously learned facts or rules.

The features of classes that support or prohibit students’ engagement with high
demand tasks align with Henningsen and Stein’s (1997) factors that support students’
engagement with high cognitive demand tasks. The five features that support students’
engagement in high demand tasks are: 1) tasks that build on prior student knowledge, 2) scaffolding, 3) appropriate amount of time, 4) modeling of high-level performance, and 5) sustained press for explanation and meaning. Scaffolding refers to the simplification of a task while maintaining the cognitive demand. Modeling of high-level performance refers to the explicit modeling of processes and thinking strategies by either the teacher or students. Finally, the sustained press for explanation and meaning refers to the teacher’s questioning, comments, and feedback that require students to provide explanations and meaning after providing solutions.

Another framework that describes a teacher’s practice during task implementations are the five practices for facilitating mathematical discussions around cognitively demanding tasks (Stein et al., 2008). The five practices are

1. anticipating likely student responses to cognitively demanding mathematical tasks
2. monitoring students’ responses to the tasks during the explore phase,
3. selecting particular students to present their mathematical responses during the discuss-and-summarize phase,
4. purposefully sequencing the student responses that will be displayed, and
5. helping the class make mathematical connections between different students’ responses and between students’ responses and key ideas (Stein et al., 2008, p. 321).

Stein et al. (2008) posit that anticipation of students’ likely responses takes place during a teacher’s lesson planning. The teacher uses the lesson plan to construct possible
student solutions and to flesh out the correct solution and common misconceptions that students might have. Teachers’ knowledge of how students learn and hypotheses about students’ prior knowledge directly inform teachers’ anticipation of students’ responses to a task. Knowledge of students’ prior knowledge and how students learn can support teachers in considering common misconceptions and planning to address these misconceptions.

Monitoring students’ responses to tasks involves paying attention to students’ mathematical thinking during engagement with tasks (Stein et al., 2008). Teachers’ knowledge of mathematics informs this practice. Knowledge of mathematics allows teachers to see the sophistication of and connections among students’ strategies. Once teachers see the sophistication of and connections among students’ strategies, students are asked to share their written work as a way to help students make connections among their work and the work of their peers. The five features that support students’ engagement with high cognitive demand tasks in conjunction with the five practices that facilitate discussion around high cognitive tasks describe task implementations from task launch to discussion.

Summary

This portion of the literature review 1) briefly described how equity has been studied in mathematics classrooms and 2) discussed students’ opportunities to learn as a lens for examining teachers’ classroom practice around task choice and task implementation. This literature shows how a focus on students’ opportunities to learn might provide researchers and teachers a framework to analyze practices that encourage equitable spaces without requiring the adoption of new curricula or worldviews. Further, OTL frameworks include clear guidelines for considering how classroom practices support students’ opportunities to
learn. For example, requiring students to provide justifications for their mathematical strategies, engaging students in high cognitive demand tasks that relate to their prior knowledge, and providing students ways to share their knowledge publicly in verbal and written formats can provide all students opportunities to learn. To situate the current study, the next section discusses literature that describes the classroom practices of beginning mathematics teachers.

**Beginning Mathematics Teachers’ Classroom Practices**

Task choice and implementation are important facets of teachers’ classroom practices. Teachers’ classroom practices refers to the actions that teachers take in class to support students’ learning. These are distinct from lesson planning which should happen before in class interactions (Lederman, 1999). As described in the previous section, teachers’ hypotheses of what students know and knowledge of mathematics, mathematical tasks, and how students learn inform what teachers do in the classroom (Simon, 1995). Research suggests that these knowledge constructs evolve over time as teachers engage with students in their own classrooms (Hiebert et al., 2007) which implies that beginning teachers’ classroom practices may be different from those of more experienced teachers (Borko & Livingston, 1989; Darling-Hammond, 1995). This review seeks to discuss what research describes as competent teaching and how beginning mathematics teachers engage in instructional practices in the classroom.

**Competent Teaching**

There has been much research on effective teaching and how beginning teachers’ practices compares to those of expert or veteran teachers (Bennett, 2010; Borko &
Livingston, 1989; Reynolds, 1992; Roehrig, Turner, Grove, Schneider, & Liu, 2009). This section reviews features of competent teaching related to lesson planning and lesson implementations, and beginning teachers’ lesson planning and lesson implementation practices. Competent teaching is a composite developed by Reynolds (1992) which reviews and links literature on learning to teach with literature on effective teaching practices. Ambitious teaching or teaching for understanding literature highlights similar effective teaching practices (Silver, Mesa, Morris, Star, & Benken, 2009); hence the difference is in the linking of teaching for understanding with what literature describes as common beginning teaching.

Lesson planning and beginning teachers. Lesson planning is a feature of competent teaching. Whether it is written or mental, teachers should have a plan for the ways that they intend to instruct their students. Education has conceptualized planning in multiple ways and has compared novice teachers to expert teachers in an attempt to share with the field what effective planning looks like. Lesson planning has been conceptualized as dual-focused structures that attend to classroom activities (e.g. location, duration, content, materials) and classroom routines (e.g. questioning, classroom management, coordination of the activity) (Sawyer, 2011), and as agendas, which are plans of operations that supplement more formal lesson plans. Lesson planning agendas include learning goals and the specific actions that will be used to achieve them (Leinhardt, 1989). Regardless of the way that lesson plans have been conceptualized, research has come to some consensus about the features of effective lesson planning. Effective lesson plans include activities with clear goals, have a logical structure and progression through content, and include connections
between students’ preconceptions or misconceptions and the current instruction (Borko & Livingston, 1989; Leinhardt, 1989; Stronge, 2007b; Zahorik, Halbach, Ehrle, & Molnar, 2003).

Setting clear expectations for students’ engagement with tasks and social behavior is another feature of competent teaching. Clarity of expectations for students can reduce the amount of time spent addressing discipline issues (Stronge, 2007a) and support the use of classroom time for instructional tasks (Stronge, 2007b). Effective instruction includes students knowing “what work they are accountable for, how to get help when they need it, and what to do when they are finished with their assignment” (Reynolds, 1992, p. 9). Students may learn these expectations through teachers’ verbal expression or through teachers modeling of the expected behavior.

With respect to lesson planning, research shows that the lesson planning practices of novice and expert teachers are different. Leinhardt (1989) found that novice teachers’ agendas were less developed than those of expert teachers, which resulted in an inability to describe how students would engage in class during instruction. Novice teachers in Leinhardt’s study also struggled to connect mathematical concepts for students during instruction. Novice teachers did not help students to connect previously learned material to new content, while expert teachers reminded students of prior lessons and described how the new and previous material was related. This difference is also evident in research conducted by Borko and Livingston (1989) who found that novice teachers’ planning was short-term, which was partially attributed to the time investment required for long-term planning. Novice teachers also struggled to identify the big ideas for a lesson and separate those ideas
from more peripheral concepts or ideas. Finally, novice teachers struggled to find appropriate examples to present content to students. In contrast, expert teachers rarely use written lesson plans, but they practiced long-term planning, keyed in on big ideas for instruction, and had multiple examples that could be used to explain mathematical concepts to students. This difference in lesson planning may be contributed to experience because teachers with more experience might have a better grasp on what students may understand and struggle with related to specific content, a better grasp on what tasks are available to teach a particular concept, and more experience teaching similar content (Borko & Livingston, 1989; Stronge, 2007b). The next feature of effective teaching that will be discussed is the use of appropriate and engaging tasks.

**Mathematical tasks and beginning teachers.** Just as constructing effective lesson plans is a feature of competent teaching, choosing appropriate and engaging tasks is also a feature of competent teaching. The literature on choosing mathematical tasks is vast (e.g. Boston & Smith, 2009; Doyle & Carter, 1984; Hiebert et al., 1997; M. S. Smith & Stein, 1998; TeachingWorks, 2012) but is generally based on Smith and Stein’s conception of cognitive demand of mathematical tasks. Given that students benefit from engaging with tasks that connect procedural and conceptual knowledge, provide opportunities to justify and explain thinking, and have multiple approaches to gain a correct solution, beginning teachers need to choose these types of tasks for instruction (Hiebert et al., 1997; Stein et al., 1996; Stein & Smith, 1998).

With respect to beginning teachers’ choice of tasks, Borko and Livingston (1989) found that beginning teachers with strong content knowledge were able to choose tasks that
supported students’ learning of content but struggled to produce examples on the fly to aid students’ understanding. However, they attribute the inability to produce examples on the spot to a lack of familiarity with the content and the amount of time required to prepare for new class assignments. Though there is not much research on the task choices of early career teachers, Ball and her colleagues (TeachingWorks, 2012) define choosing mathematical tasks as a high leverage teaching practice, which is a research-based practices projected to support students’ learning. The next section described competent teaching within the context of lesson implementation.

**Lesson implementation and beginning teachers.** Competent lesson implementation includes providing students opportunities to engage with high demand mathematical tasks at a high level and structuring an inclusive and inviting classroom. Competent teachers establish rapport with their students, express expectation for engagement in the class, and spend the majority of their time on instruction (Hiebert et al., 1997; Reynolds, 1992; Yackel & Cobb, 1996). Each of these features of competent teaching supports students’ opportunities to learn and engage with mathematics.

As a strategy to support student engagement, both novice and expert teachers identify students who are likely to impede the instructional process (Reynolds, 1992). The difference is that expert teachers differentiate between students who choose not to participate and those who become frustrated and unmotivated; and have strategies to encourage each type of student to work (Reynolds, 1992). Research also shows that expert and novice teachers attend to different things when monitoring classroom instruction and therefore; develop different interpretations of what is being observed (Sabers, Cushing, & Berliner, 1991).
suggests that even while teaching, novice teachers interpret classroom events differently than experienced teachers might, which may lead to the use of different instructional strategies.

**Summary**

This review of the instructional strategies of beginning teachers shows that the practices of beginning teachers differ from those of more experienced teachers. This study aims to add knowledge about beginning high school mathematics teachers to the conversation about beginning teachers’ classroom practice.

**Chapter Summary**

This chapter served three purposes. A brief review of the literature on equity was conducted to situate this work. Though there are many different theories and pedagogies that may be applied to classroom practice as a way to influence equitable classroom spaces, they are not structured to support beginning teachers in knowing what to do to support students’ learning. Using the OTL framework may support beginning teachers in considering specific classroom practices that can support students’ learning opportunities. Finally, the field’s view of beginning teacher practice was described to provide a basis of exploration of early career teachers’ classroom practice. The conceptual framework (Figure 6), described in the following chapter, shows how these constructs work together to guide this work.
CHAPTER 3: METHODS

As previously stated, the overarching question guiding this study is, “How do early career high school mathematics teachers’ classroom practices support students’ opportunities to learn?” The purpose of this study is to describe early career high school mathematics teachers’ (ECMTs) instructional practices around mathematical task choice and implementation. The three goals of this study are to 1) describe ECMTs’ instructional practices around homework, classwork and assessment tasks, 2) describe practices around task implementation as a way to support equitable spaces, and 3) explore the similarities and differences among instructional practices used in courses of different academic levels.

In this study, two potentially ambiguous phrases are used in the posing of the research questions: accelerated or academic courses and tasks. Accelerated versus academic courses are theoretically “differentiated by the rigor of their courses and the nature of their instruction” (Mickelson, 2011, p. 220). In this study, an accelerated course is a course described as honors or AP (advanced placement) where the curriculum is more rigorous (as defined by detail, pace, etc.) than the curriculum in an academic course. An academic course is a course described as college preparatory or regular. A mathematical task is defined as “a classroom activity, the purpose of which is to focus students’ attention on a particular mathematical idea.” Further “an activity is not classified as a different or new task unless the underlying mathematical idea toward which the activity is oriented changes” (Stein et al., 1996, p. 460). In this study, mathematical tasks are also delineated by changes in the mathematical idea of focus and/or transitions in the lesson. That is, if the mathematical focus of a lesson is consistent for the duration of a class period, then the task is said not to have
changed during the class period: all of the mathematics items that students engage with are described as one task. If there is a transition in the lesson from note-taking to independent seatwork, for example, then a new task is presented because the classroom activity that students were engaging in has changed. The desired outcome from these investigations is constructed cases that illustrate examples of instruction on the continuum of equitable classroom spaces in high school mathematics classroom settings.

Based on the aforementioned goals, two research questions were investigated.

1. How do the tasks that ECMTs choose support students’ opportunities to engage with high cognitive demand tasks in class, outside of class, and on assessments?
   a. How do the task choices of an ECMT differ when teaching academic versus accelerated courses?

2. How do ECMTs implementations of mathematical tasks support equitable classroom spaces?
   a. How do the task implementations of academic and accelerated courses differ?

To carefully study how teachers’ choices and implementations of tasks may support students’ opportunities to learn, an instrumental multi-case study design is used (Creswell, 2007; Stake, 1995). The case study is situated within the context of early career mathematics teachers from the Robert Noyce Scholarship Program, briefly described below. Then study design, conceptual framework, data collection and analysis is described. Validity and reliability issues are also addressed.
Robert Noyce Scholars Program

The Robert Noyce Scholarship (Noyce) Program serves as the context that influences the selection of participants for this study. The Noyce Program is a National Science Foundation (NSF) grant program that aims to “respond to the critical need for K-12 teachers of science, technology, engineering, and mathematics (STEM) by encouraging talented STEM students and professionals to pursue teaching careers in elementary and secondary schools” (AAAS, 2012, para 1). The program encourages students and STEM professionals to teach by providing programmatic and financial support to scholarship recipients. The program also aims to improve the quality of teachers who teach in high need school districts, because high need districts have difficulty recruiting and retaining qualified teachers (Berry, Rasberry, & Williams, 2007; Darling-Hammond, 2010). Ultimately, the Noyce program aims to provide competitive scholarships and professional supports at local universities as a way to recruit individuals with strong STEM backgrounds to the profession of teaching.

Local Noyce Goals

The Noyce scholars participating in this research are part of a local Noyce program at a large Southeastern urban university, and are committed to teaching in high need districts. Undergraduate scholars in this program earn a double degree in mathematics or statistics and mathematics education and graduate scholars have a degree in a STEM field and earn a Master of Arts in Teaching or a Masters of Education in secondary mathematics education. Locally, the Noyce program provides programmatic and professional development supports through conference attendance, allocation of funds to purchase technology for learning and classroom use, a school-based mentor, a university-based mentor, and social meetings to
encourage the building of a community. During the 2012-2013 school year, the local Noyce program had scholars employed in 11 different high-needs school districts in four states. An invitation to participate in the current study was extended to the scholars teaching in a local large school district.

**Study Design**

To investigate the classroom practices of ECMTs, a qualitative research design was selected. Bogdan and Biklen (2007) define qualitative research by highlighting five common features. Though these features may not be equally represented in all qualitative research, the authors state that generally qualitative research is naturalistic, descriptive, focused on process, inductive, and concerned with participant perspectives.

Research described as naturalistic is conducted in the natural environment where the phenomenon or participant is found. Further, the data that is collected by the researcher is word rich and nontrivial. All that is observed is recorded as meaningful in order to help readers to experience the natural setting in which the participant is being studied. Creswell (2007) states that this “up-close information gathered by actually talking directly to people and seeing them behave and act within their context is a major characteristic of qualitative research” (p. 37).

A focus on process is also a very important characteristic of qualitative research because the purpose of the research is to understand how something takes place and not simply to document the result of what has taken place. The process of and the reasoning attached to an activity are held in higher esteem than the final results. Being able to
understand a participant’s thought process, struggles, and ultimate decision can give others insight into the daily happenings of the participant.

Qualitative research is inductive in nature. That is, researchers establish themes that grow out of data analysis opposed to collecting data to prove or disprove previously formed hypotheses. Finally, qualitative research is concerned with participant perspectives. This research is conducted to understand how participants make sense of ideas and dilemmas and the research works to express this sense making in the form of rich descriptive results. These research features aimed at understanding teacher practice around task choice and implementation allowed the researcher to gain a greater understanding of teachers’ instructional practices and the factors that influence these practices. Because a deeper understanding of teacher practice was pursued, case study was the best form of qualitative research to answer these questions.

**Why Case Study?**

Yin (2012) defines a case study as:

> An empirical inquiry about a contemporary phenomenon (e.g. a “case”), set within its real-world context, especially when the boundaries between phenomenon and context are not clearly evident (Yin, 2009, p.18). Thus…case study assumes that examining the context and other complex conditions related to the case(s) being studied are integral to understanding the case(s) (italics in original, p. 4).

Based on Yin’s definition, case study should be conducted in settings where understanding the context is necessary to understand the case. Classrooms exemplify these complex environments because it can be difficult to investigate teachers’ instructional practices
without a level of understanding about classroom norms and characteristics. Further, because the instructional strategies that teachers choose may be a result of the students they are teaching (Sztajn, 2003), case study could provide insight into how students influence teachers’ classroom practice as well. Ultimately, research on teaching should happen in classroom settings to best understand teachers’ choices, practices, and support of their students.

**Definition of the case.** This is a multi-case study (Creswell, 2007). The multi-case design allows for generalization across cases because “by comparing sites or cases, the researchers can establish the range of generality of a finding or explanation and at the same time, pin down the conditions under which that finding will occur” (Borman, Clarke, Cotner, & Lee, 2006, p. 123). A case was defined differently for each of the research questions. Because the first research question addressed the task choices of individual ECMTs in different level courses, a case was defined as a teacher’s individual course. Each course was observed for five lessons; therefore, the tasks used during these five lessons, homework assigned, and subsequent summative assessments were used to inform a case. This resulted in six cases because each of the three participants was observed teaching five lessons in an academic Geometry course and in an accelerated course. The tasks collected during observations in each course were the primary source of data for these cases.

There were two cases for the second research question addressed in this study. The cases consisted of the lessons taught in the two levels of courses observed in this study: the academic Geometry course and the accelerated courses. Each of the three participants was observed teaching five lessons in an academic Geometry course. Each was also observed
teaching five lessons in an accelerated course, either Honors Algebra II or Advanced Placement Statistics. This resulted in 15 lessons in each case. Defining cases at the course level in each instance allowed for investigation of course specific teaching contexts and practices and allowed for within-case and cross-case analysis, which supports generalization as previously described.

Individual cases were bounded by time (Spring Semester 2013) and location (three senior high schools in a large, high-need, southeastern school district). Multiple data sources allowed the researcher to provide descriptions of teachers’ mathematical task choices including choices of classwork, homework, and assessment tasks; and teachers’ support of equitable classroom spaces with a focus on students’ opportunities to engage with high demand tasks and participate publicly. Public participation refers to participation that can be observed by others in the class. These data sources included audiotaped pre-lesson and post-lesson interviews, classroom observations, and artifacts from the classroom observations. Field notes were also recorded during each interview and observation because audio recordings failed to capture classroom context.

**Conceptual Framework**

An adaptation of Simon’s (1995) mathematics teaching cycle that incorporates elements from research on opportunity to learn serves as the conceptual framework for this study (see Figure 6). Constructs of teachers’ knowledge, such as knowledge of mathematics content, students’ prior knowledge, mathematical activities, and student learning trajectories, inform the teaching cycle, which is lesson planning and task choice, mathematical task implementation, and summative assessment. The arrows in the framework attempt to model
the highly cyclical nature of task choice and implementation and the effects of teacher knowledge on this cycle. There are no arrows linking the teacher knowledge constructs to summative assessment because the major foci of this study are mathematical task choice and implementation. The arrows from each of the knowledge constructs to task choice and instruction mirror the arrows in Simon’s original diagram of the Mathematical Teaching Cycle. Lesson planning, instruction, and summative assessment all influence students’ opportunity to learn (OTL). Finally, classroom norms provide context for implementations.
Figure 6: Mathematical Teaching Cycle and Students’ OTL
Data collection and analysis also follow the framework. In the mathematical task choice portion of the framework, homework, classwork, and assessment tasks were collected from each class and analyzed for potential level of cognitive demand and alignment. As tasks were used in the classroom, transitions in the lesson plan bounded a task. Classroom tasks were numerically coded for statistical analysis (Caracelli & Greene, 1993). These analyses were used to form descriptors of classroom practices that supported students’ opportunities to learn. Task implementations were also analyzed with respect to students’ opportunities to learn. In the case of implementation, descriptors were used to frame equitable classroom spaces with respect to students’ opportunities to engage with high cognitive demand tasks, participate publicly, and develop understanding of sociomathematical norms. These descriptors were used to frame the types of instruction that were common among the participants, as well as, determine students’ opportunities to justify and explain their thinking.

**Participant Selection**

**Schools.** One high-needs school district in a large southeastern state that employs a large subset of Noyce Scholars was identified to be included in the study based on convenience. The decision about which school district to include in the study was based on the amount of scholars employed in the district and proximity to the researcher. This district is the 16th largest district in the US and contains 169 schools, 25 of which are high schools. The district contains rural, urban, and suburban schools. Student demographics in high schools in this district are 5% Asian, 26% African American, 13% Hispanic, 50% White, 4% Multi-Race, and less than 1% American Indian and Native Hawaiian.
Six Noyce Scholars are employed in this district and all six were invited to participate in this study. The only requirement for participation was the teaching of both academic and accelerated courses during the time of the study; therefore, all were eligible for the study. Three teachers volunteered to participate, which resulted in three high schools being represented in this study. The free and reduced lunch rates at these particular schools range from 26% to 32% and the teacher turnover rates ranged from 8% to 16% in 2012.

Each teacher was observed in classes taught on a block schedule, meaning students enrolled in four 90-minute classes per semester. The study took place during the second semester of the academic year. There were three participants in this study: Mr. Nimrick, Mrs. Moreland, and Mrs. Kaiser, all pseudonyms.

Mr. Nimrick. Mr. Nimrick was an undergraduate scholar in the local Noyce program and earned a B.S. in Mathematics and a B.S. in Mathematics Education. He was in his second year of teaching high school mathematics at the time of the study. During the study he taught Geometry and Advanced Placement Statistics. During his pre-lesson interviews, he reported that he had taught these courses before.

Mrs. Moreland. Mrs. Moreland, a graduate scholar in the local Noyce program, has a B.S. in Mathematics, a B.A. in Communication, and a M.S. in Mathematics Education. She was in her third year of teaching high school mathematics at the time of the study. During the study she taught Geometry and Advanced Placement Statistics. During her pre-lesson interviews, she reported that she had taught these courses before.

Mrs. Kaiser. Mrs. Kaiser, a graduate scholar, has a B.S. in Mathematics, a M.S. in Applied Mathematics and a M.Ed. in Mathematics Education. She was in her fourth year of
teaching high school mathematics at the time of the study. During the study she taught Geometry and Honors Algebra II. During her pre-lesson interviews, she reported that she had taught these courses before.

**Data Collection Methods**

Data collection occurred from February 2013 through April 2013. Teacher interviews (pre/post observation and assessment) and classroom observations were documented using video and/or audio recordings. Verbatim transcripts of interviews were created. Other artifacts included classwork, homework, and assessment tasks and field notes. A complete overview of data collection, data analysis tools, and matching of data sources to research questions may be found in Appendix A. The triangulation of data from multiple data sources provided vigorous evidence to support claims.

**Teacher interview data.** Pre-lesson, post-lesson, and assessment interviews were audio recorded and followed an interview protocol consisting of main questions and probes used to ensure attention to the specific details of the participant’s instructional plan (Appendix B). These protocols were adapted from the protocols found in the ITC Teacher Interview Protocol (Horizon Research Inc, 2003).

Pre-lesson interview data provided evidence of teachers’ lesson goals and how those goals may have interacted with task choices. Because teachers rarely had time to write detailed lesson plans, pre-lesson interviews generally took place during planning or after school the day before classroom observations took place. Post-lesson interviews allowed teachers to express their feelings about preparedness and surprises, and to explain any phenomena that were observed during the classroom observation. These interviews also
allowed teachers to address any changes made to the lesson and provide reasons for those changes. In the instances where teachers were unable to participate in in-person interviews, interviews were conducted via email before or after classroom observations.

**Classroom observation data.** Each teacher was observed teaching two courses. One course was academic and one course was accelerated. The academic courses in this study are labeled Class A and the accelerated courses are labeled Class B. Classroom observation data were collected three times during the semester for each teacher. In order to capture a more accurate picture of teachers’ classroom practice, an initial observation and two modified teaching sets, as defined below, were conducted. Both the initial observation and the first modified teaching set were conducted in February 2013 near the start of a new semester. The second modified teaching set was conducted in April 2013 near the middle of the semester after teacher and students had time to learn each other. Between the first and second teaching set, preliminary analysis of the observation data was conducted. This is described in more detail in the next section.

The initial observation day was used to gather information about classroom norms including common procedures for communication, movement around the classroom, and sharing of materials. Though this information would also be observed during the subsequent teaching sets, the initial observation allowed the researcher time for acclimation to the classroom environment.

The modified teaching sets followed the initial observation day. Simon, Tzur, Heinz, Kinzel, and Smith (2000) define a teaching set as,
two classroom observations and three interviews: a prelesson interview with the teacher about the first lesson to be observed, an observation of the first mathematics lesson, a second interview in which the teacher was asked about the first lesson and about plans for the second lesson, an observation of the second lesson, and an interview about the second lesson (Simon et al., 2000, p. 583).

Each of the modified teaching sets used in this study consisted of two classroom observations for Class A and Class B, (8 total observations across the two teaching sets) and three interviews. These modified teaching sets began with a pre-lesson interview with the teacher about the two consecutive lessons to be observed for Class A. An identical interview was conducted for Class B. Then an observation of each course was completed during two consecutive school days. After observations, a post-lesson interview was conducted about the lessons observed for Class A and an identical interview was conducted for Class B. Finally, an assessment interview was conducted to discuss the summative assessment that students took on the material presented during the observations. Figure 7 provides a diagram of the classroom observation process.
Because permissions for video recording were not granted at all schools, each observation was audio recorded and pre/post observation interviews were audio recorded. Audio recordings and field notes from the observations documented teachers’ instructional practices during task implementations, as well as, classroom norms and structure. Audio from the observations was transcribed when necessary but the majority of the analysis of classroom observations was conducted on the audio data.

Data Analysis Methods

Task and item analysis. Three types of tasks were analyzed in this study: homework tasks, classroom tasks, and assessment tasks. The homework tasks analyzed were tasks that students were expected to complete after the lesson concluded; even though it was common practice for students to have time to begin homework during the 90-minute class period. The classwork tasks analyzed were tasks that were part of the lesson. This included the
bellwork/warm up, tasks completed during the notes or discovery portion of the class, and any guided practice or independent seatwork. Though it was common for teachers to review the previous night’s homework in class, these items were not included in the classwork task analysis. Finally assessment tasks were analyzed. These tasks were on summative assessments given to students to assess mathematics taught during classroom observations.

The IQA Academic Rigor: Mathematics rubric for Potential of the Task, described in the literature review (see Figure 8), was used to analyze each mathematical task (Boston & Smith, 2009). A rank of 0 indicates the absence of mathematics in a task. A rank of 1 indicates a recall or memorization task. A rank of 2 indicates mathematical tasks that require use of procedures disconnected from conceptual reasoning. A rank of 3 indicates mathematical tasks that may use procedures, but also could support complex thinking or reasoning. A rank of 4 indicates mathematical tasks that require student to devise and apply a unique solution strategy or tasks that explicitly prompt for justification of procedures used to find solutions. The highest potential of reasoning present in the tasks corresponds to the rank assigned (Boston & Smith, 2009).
<table>
<thead>
<tr>
<th>RUBRIC 1: Potential of the Task</th>
</tr>
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</table>
| 4 | The task has the potential to engage students in exploring and understanding the nature of mathematical concepts, procedures, and/or relationships, such as:  
   · Doing mathematics: using complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example); OR Procedures with connections: applying a broad general procedure that remains closely connected to mathematical concepts.  
   The task must explicitly prompt for evidence of students’ reasoning and understanding.  
   For example, the task MAY require students to:  
   · solve a genuine, challenging problem for which students’ reasoning is evident in their work on the task;  
   · develop an explanation for why formulas or procedures work;  
   · identify patterns; form and justify generalizations based on these patterns;  
   · make conjectures and support conclusions with mathematical evidence; make explicit connections between representations, strategies, or mathematical concepts and procedures; follow a prescribed procedure in order to explain/illustrate a mathematical concept, process, or relationship. |
| 3 | The task has the potential to engage students in complex thinking or in creating meaning for mathematical concepts, procedures, and/or relationships. However, the task does not warrant a “4” because:  
   · the task does not explicitly prompt for evidence of students’ reasoning and understanding.  
   · students may be asked to engage in doing mathematics or procedures with connections, but the underlying mathematics in the task is not appropriate for the specific group of students (i.e., too easy or too hard to promote engagement with high-level cognitive demands);  
   · students may need to identify patterns but are not pressed to form or justify generalizations;  
   · students may be asked to use multiple strategies or representations but the task does not explicitly prompt students to develop connections between them; students may be asked to make conjectures but are not asked to provide mathematical evidence or explanations to support conclusions. |
| 2 | The potential of the task is limited to engaging students in using a procedure that is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. There is little ambiguity about what needs to be done and how to do it.  
   The task does not require students to make connections to the concepts or meaning underlying the procedure being used. Focus of the task appears to be on producing correct answers rather than developing mathematical understanding (e.g., applying a specific problem solving strategy, practicing a computational algorithm).  
   OR There is evidence that the mathematical content of the task is at least 2 grade-levels below the grade of the students in the class. |
| 1 | The potential of the task is limited to engaging students in memorizing or reproducing facts, rules, formulae, or definitions. The task does not require students to make connections to the concepts or meaning that underlie the facts, rules, formulae, or definitions being memorized or reproduced. |
| 0 | The task requires no mathematical activity. |
| N/A | Students did not engage in a task |

Figure 8: IQA Rubric for Potential of the Task (Boston, 2012c, p. 9)

Some tasks consist of a series of mathematics items such as a homework assignment that requires the completion of a certain number of exercises. Other tasks consist of a single
mathematics problem. Figure 9 shows a multi-item task and a single item task. To determine the level of cognitive demand of the tasks, a numerical ranking system was used (Boston & Smith, 2009).

<table>
<thead>
<tr>
<th>Multi-item Task</th>
<th>Single item Task</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Developing Proof:</strong> A rhombus is a parallelogram with four congruent sides. Write a plan for the following proof that uses SSS and a property of parallelograms.</td>
<td><strong>Developing Proof:</strong> A rhombus is a parallelogram with four congruent sides. Write a plan for the following proof that uses SSS and a property of parallelograms.</td>
</tr>
<tr>
<td><strong>Given:</strong> Rhombus ABCD with diagonals AC and BD intersecting at E.</td>
<td><strong>Given:</strong> Rhombus ABCD with diagonals AC and BD intersecting at E.</td>
</tr>
<tr>
<td><strong>Prove:</strong> AC \perp BD</td>
<td><strong>Prove:</strong> AC \perp BD</td>
</tr>
</tbody>
</table>

Figure 9: Sample Multi-item and Single item mathematical tasks (Pearson Education Inc, 2010, p. 152)

The IQA Mathematics Rubric for Potential of the Task was also used to rank the individual items that constituted tasks. This is an extension of the use of this rubric from prior research studies. The ranks were assigned on the item level to more closely examine
the potential for students to reason within mathematical tasks. Because a multi-item task
may be comprised of items ranked at different levels, this technique can provide a more fine-
grained analysis.

Table 1 provides an example of how a mathematical task and its items are ranked
using the Potential of the Task rubric. The task consists of 5 items. Each item is focused on
the mathematical topic of angle measures of polygons. Though the task received a rank of 4
based on students’ highest opportunities to justify and explain their mathematical thinking
within the complete task, the task items are ranked based on students’ opportunities to
explain or justify on the item level.
Table 1

*IQ A Ranks of sample mathematics items*

<table>
<thead>
<tr>
<th>Task Rank</th>
<th>Task Items</th>
<th>Item Rank</th>
<th>Cognitive demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>What is the sum of the measures of the exterior angles of a pentagon?</td>
<td>1</td>
<td>Recall</td>
</tr>
<tr>
<td>2.</td>
<td>Using the Omega formula [ \Omega = (n-2) \times 180 ], find the sum of the measures of the interior angles for a pentagon.</td>
<td>2</td>
<td>Procedures without connections</td>
</tr>
<tr>
<td>3.</td>
<td>Given that 60, 80, ( x ), ( x ), ( x ), and ( x ) are the measures of the interior angles of a convex polygon, find ( x ).</td>
<td>3</td>
<td>Procedures with connections</td>
</tr>
<tr>
<td>4.</td>
<td>Why does the formula [ \Omega = (n-2) \times 180 ] provide the sum of the measures of the interior angles of any convex polygon? Use diagrams and words to justify your answer.</td>
<td>4</td>
<td>Doing mathematics</td>
</tr>
<tr>
<td>5.</td>
<td>Write a brief letter to a friend telling him/her why it is important to study angle measures of polygons.</td>
<td>0</td>
<td>Non-mathematical</td>
</tr>
</tbody>
</table>

The ranks of 0, 1, and 2 given to items 5, 1, and 2, respectively are a result of the following: item 5 does not include mathematics and items 1 and 2 can be completed by recalling previously learned facts or by using the provided formula. Item 3 requires students to identify the type of polygon that has six angles, find the sum of the measures of the interior angles of the hexagon, realize that the repeated \( x \) measures represent four congruent angles, and construct an equation or other solution method to solve for \( x \). This type of item allows students to engage in complex thinking about the meaning of a problem as a way to
support mathematical reasoning and allows them to connect procedures and their underlying concepts. Finally, item 4 asks students to explain why the Omega formula provides the sum of the interior angle measures for any convex polygon. This allows students to use illustrations and words to provide an informal proof of this formula. Students must show their understanding of this formula, which provides opportunities for mathematical reasoning and the connection of procedural and conceptual knowledge.

Once each task and its items were ranked within a course across the five observation days, Chi-squared tests were used to compare the proportion of high-level (i.e. score of 3 or 4) and low-level (i.e. score of 0, 1, or 2) tasks in each type of coursework (3 x 2; Homework / Classwork / Assessment by High / Low), to determine whether the potential of the tasks to support student justification were different across the three coursework types, as evidenced by p-values compared to a significance level of 0.05 (Boston & Smith, 2009). The null hypothesis for these tests is that there is no difference in the distribution of high and low demand tasks and task items across the three coursework types. Therefore, the alternative hypothesis is that there is a difference in the number of high demand and low demand tasks and tasks items when comparing homework, classwork, and assessment tasks. A non-significant difference implies alignment across coursework types in the level of cognitive demand of tasks and task items. The teacher’s learning goals were then analyzed to investigate how closely the level of tasks used met the expressed learning goals. Then, learning goals, cognitive demand, and alignment of tasks across coursework types were used to place collection of tasks at the course-level (e.g., all tasks posed in a teacher’s Class A) on a continuum of tasks that support students’ opportunities to engage with meaningful tasks.
and reason mathematically. Finally, to test for differences between the distribution of task ranks in Class A and Class B for each teacher, the Mann-Whitney U test was used (D. Jones & Tarr, 2007). For the Mann-Whitney U tests, the null hypothesis is that the distribution of task ranks across all coursework types in a teacher’s academic course is the same as the distribution of task ranks in the same teacher’s accelerated course. The alternative hypothesis is that there is a difference in the distribution of task ranks of a teacher’s academic and accelerated courses. Each statistical test, Chi-Squared and Mann-Whitney U, used an alpha level of .05.

For mathematical tasks, the unit of analysis is the collections of tasks in each teacher’s course, which results in six separate collections of tasks. Once each collection of tasks was analyzed, four categories emerged that describe students’ OTL: limited, moderate, moderately persistent, and persistent. Limited, moderate, moderately persistent, and persistent refer to teachers’ support of students’ opportunities to learn based on the potential of tasks. These will be elaborated upon in Chapter 4.

Task Implementation. Between the first and second teaching sets, preliminary analysis of task implementation data was conducted. This early analysis allowed the researcher to revisit the research questions and ensure that the data being collected was sufficient to answer the questions. This preliminary work also served to focus the field notes that were collected. Because there were organizational similarities across the classrooms (seating arrangements, lesson structure) the researcher focused more on what features of the lessons and courses could be used to show how different instructional practices supported
students’ OTL. The result of these analyses was the decision to use the items on the IQA rubrics for task implementation as a guide for final analysis of the lessons.

When applicable, tasks used during three parts of classroom instruction were analyzed in this study: warm-ups/bellwork, tasks completed during notes, and independent or seatwork tasks. Recall, the focus for task implementations was support for student to engage with high cognitive demand tasks and students’ opportunities to participate publicly during implementations. The researcher focused on these aspects of task implementations as a way to investigate students’ opportunities to engage with high cognitive demand tasks using prior knowledge, connect procedural and conceptual knowledge, and justify and explain thinking. An adaptation of the IQA rubric for the Implementation of tasks (see Figure 4) and the Mathematics Lesson Checklist (see Figure 5) discussed in the previous chapter were used. Because this study’s focus is the work done by the teacher, the characteristics in the rubrics that describe teacher practice were used to analyze the task implementations. The following features of task implementation were attended to in order to assess how teachers supported students’ opportunities to learn:

1. Use of high demand tasks
2. Modeling of high-level thinking
3. Providing opportunities to justify and explain thinking in response to the teacher’s and other students’ questions
4. Giving clear expectations for engagement with tasks, and
5. Providing students with necessary resources to fully participate in lessons
Because the microphone was clipped to the teacher, the teacher was credited with letting students answer each other’s questions if he or she answered a question with a question and allowed students to respond. Another feature of task implementation that was included in this analysis was the opportunity for students to share their knowledge publicly in written form. Sharing written work publicly includes students writing their solutions on a board, presenting their work on posters, and any other form of having their work displayed during the lesson.

Preliminary analysis of task implementation data took place in March 2013. To analyze the task implementations, the researcher listened to or watched the classroom observations multiple times and made note of the presence of the aforementioned features. The researcher also reviewed field notes to provide context for class events that took place outside of the view of the camera and in the classes that did not allow video recording. For implementation, each lesson was taken as a unit of analysis to reinforce the idea that this study describes teachers’ classroom practice and not teachers. Analyzing each lesson as a unit also reinforces the idea that the same teacher can implement tasks very differently within the same course (different observation days) and in different courses. Once each of the 30 class implementations was analyzed, three categories emerged to describe equitable classroom spaces that support for students OTL: emergent, moderate, and prominent. These categories will be described in more detail in Chapter 5.

Other artifacts. Descriptive and reflective field notes were recorded for pre/post lesson interviews and classroom observations. These documents allowed the researcher to consider links between interview and observation data and provided a picture of what was
happening during classroom observations. The reflective field notes motivated minor adjustments to interview protocols and provided the researcher with a lens to focus what was captured by field notes in future observations. Descriptive field notes were necessary to help the researcher match physical teacher movement with classroom dialogue. They also served to clarify ambiguous phrases used during observations.

Internal Validity, Reliability, and External Validity

Researchers bear the burden of reporting valid and reliable research (Creswell, 2007). Merriam (2002) describes three types of validity. They are internal validity, reliability, and external validity. Strategies that were used in this study to support each of these types of validity are discussed here.

Internal validity. Internal validity is also commonly referred to as credibility and trustworthiness (Creswell, 2007). Merriam (2002) suggests using triangulation (multiple sources of data), member checks, peer review, extended observations, and articulation of researcher biases as ways to ensure internal validity. This study employs triangulation, extended observations, and bias disclosure through a subjectivity statement in order to increase validity.

Various forms of data were collected within the context of the study in order to triangulate participants’ classroom practices and influences on these practices. Teachers were observed for two teaching cycles during a semester. These repeated observations also increase validity (Merriam, 1998).

Reliability. Reliability refers to the notion that based on the data collected the results are logical (Merriam, 2002). Again, triangulation and articulation of biases all support
reliability in addition to the use of an audit trail (Creswell, 2007; Merriam, 2002). Audit trails serve as a place to record researcher reactions to data including analysis decisions and reflections on analysis choices, reflections of the researcher as an instrument of analysis, data collection issues, and data interpretation decisions. This study uses an audit trail to support reliability.

**External reliability.** Merriam (2002) suggests that the use of rich, thick descriptions provide external reliability. These descriptions will allow readers to decide how similar the study contexts are to the one(s) they are familiar with and make a decision about generalizability. As previously stated, this study will provide rich, thick descriptions of study contexts as well as detailed descriptions of the outcomes.

**Subjectivity Statement**

As a former high school mathematics teacher, I entered into graduate school to improve the ways that teachers interact with their students. This is still a research focus and has motivated the current study. My prior experiences as a mathematics student who always excelled in mathematics classes also affects my research. I was always aware of the small number of minorities that enrolled in the classes that I took and as a teacher I became increasingly aware of what appeared to be preferential treatment in terms of class assignments and learning opportunities of students who were not minorities. This noticing will influence what I attend to during classroom observations.

I hold a socioconstructivist view of learning. Learning happens at the intersection of personal interactions with concepts and interaction with other people around those concepts. This will affect the ways that I conduct my research and the ways in which I express my
findings. This view of pedagogy will influence the types of instructional activities that I evaluate as effective and the ways that I communicate with teachers around their practice. Allowing teachers to evaluate their practice and reflect on its effectiveness may be a better tool for instructional practice improvement compared to telling teachers what instructional strategies they should use.

Finally before the onset of the current study, I attended a training conducted by Melissa Boston on the IQA rubrics and had interacted with Noyce scholarship participants for over a year as a program manager. These experiences will support my analysis and may increase the level of comfort between the researcher and the participants.

**Ethical Issues**

Consent was obtained from the North Carolina State Institutional Review board (IRB) to conduct this study. Each participant signed a legal consent form before participating in this study (Appendix C). Notifications and permission slips were also approved by the district research offices and distributed and signed by the students in each class (Appendix D). Only one participant permitted video recording in a classroom. In that class, a standalone stable camera linked to a Bluetooth microphone that was attached to the teacher was aimed at the board. The camera was also aimed above students’ heads to keep from capturing their faces when they spoke to peers seated behind them. There was minimal risk anticipated for participation in this study as all participants were given pseudonyms that will be used when audio or written data is presented. All data were stored in the researcher’s computer, which is password protected. Back up files were stored on a double password
protected university based drive system and on a secure external hard drive. Written
documents were securely stored in a locked cabinet.
CHAPTER 4: ANALYSIS OF MATHEMATICAL TASKS AND FINDINGS

The purpose of this chapter is to address the first research question of this study:

1. How do the tasks that ECMTs choose support students’ opportunities to engage with high cognitive demand tasks in class, outside of class, and on assessments?

   a. How do the task choices of an ECMT differ when teaching academic versus accelerated courses?

For this question, there are six cases; each case is an individual course taught by each teacher. The purpose was to examine how early career mathematics high school teachers supported students’ learning opportunities through task choice. The source of data for analysis was the collection of tasks used in a course by each teacher for the duration of the study. These collections include the tasks used during the initial observation and the two teaching sets.

First, the potential ranks of the mathematical tasks used by individual teachers in each course are described to show the overall opportunity for high-level reasoning. Descriptive statistics are presented to show the distribution of high and low demand tasks within homework, classwork, and assessment tasks. The ranks of items within each coursework category (i.e. homework, classwork, and assessment) are described to show students’ opportunities to reason at a high level within tasks. To determine whether there is a significant difference between the distributions of high demand and low demand tasks within the types of coursework, Chi-squared tests for homogeneity are used. If the distributions of high demand and low demand tasks within the types of coursework are not significantly different, then they are described as aligned. Alignment implies that students may receive
comparable opportunities to engage with high demand and low demand tasks across homework, classwork, and assessments. Second, cross-case analysis within each teacher are presented using the Mann-Whitney U test to show whether there are significant differences in the distribution of the ranks of tasks between courses taught by the same teacher as a response to research question 1a. Third, cross-case themes that emerged during analysis are presented. Examples of the ranks assigned to different mathematical tasks are included in Appendix E along with coursework samples.

**Mr. Nimrick’s Class A**

Mr. Nimrick’s Class A is a Geometry course with a total of 16 students. Students receive unit overview sheets with all homework and classwork assignments and assessment dates for that unit (see Figure 10). During the observations, Mr. Nimrick taught lessons on parallelogram properties and proofs and circle properties. Mr. Nimrick used multiple textbooks, free worksheets from Internet websites (i.e. Kuta Software) and activities shared in his professional learning community as resources for tasks.

<table>
<thead>
<tr>
<th>Date</th>
<th>Lesson/Objective</th>
<th>Classwork/Activity</th>
<th>Homework</th>
<th>Checked</th>
</tr>
</thead>
</table>
| Tues. Nov 13 | 4.1 Congruent Figures  
Objective: To identify congruent figures | Pg. 221 #1-3, 10-38 even 
VISUAL ASSESSMENT 2 | Pg. 91 # 1-20      |           |
| Wed. Nov 14 | 4.2 Prove Triangles Congruent by SAS and SSS  
Objective: To use sides and angles to prove congruence | Pg. 230 #1-10      | Pg. 95 #1-12      |           |
| Thurs. Nov 15 | 4.2 Prove Triangles Congruent by SAS and SSS  
Objective: To use sides and angles to prove congruence  
QUIZ 4.1-4.2 | Pg. 231 #11-21, 24-31 | Pg. 96 #11-17 | Pg. 97 #1-5 |

Figure 10: Nimrick’s Class A Assignment schedule
**Homework**

Homework was generally assigned and checked for completeness each day. The number of mathematical tasks in each homework assignment varied. Recall, mathematical tasks are bound by mathematical focus. The researcher delineated mathematical tasks within homework assignments by grouping together items with the same mathematical focus. No mathematical tasks spanned more than one homework assignment. There were 20 mathematical tasks assigned for homework during the study. Table 2 shows the total number of homework tasks and items ranked at each level.

Table 2

*Task and Item Ranks for Nimrick's Class A Homework*

<table>
<thead>
<tr>
<th>Rank</th>
<th>Description</th>
<th>Tasks (%)</th>
<th>Items (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Doing Math or procedures with connections and justifications</td>
<td>4 (20%)</td>
<td>15(16%)</td>
</tr>
<tr>
<td>3</td>
<td>Procedures with connections but no justification</td>
<td>9 (45%)</td>
<td>40 (44%)</td>
</tr>
<tr>
<td>2</td>
<td>Procedures without connections</td>
<td>7 (35%)</td>
<td>35 (38%)</td>
</tr>
<tr>
<td>1</td>
<td>Memorization</td>
<td>0</td>
<td>1 (1%)</td>
</tr>
</tbody>
</table>

*Note: Percentages may not add to 100 due to rounding.*

These descriptive statistics show that a higher percentage of the homework in Class A were high demand (rank 3-4). Sixty-five percent of the tasks that students were assigned had the potential to engage students in complex thinking and/or exploring and understanding mathematics. As previously stated, a higher percentage of high demand homework tasks does not necessarily imply the same for task items because task ranks are based on the
highest potential reasoning level apparent in the task. However, in Mr. Nimrick’s Class A there were a higher percentage of high demand items as well with 60% of the items being ranked at level 3 or 4.

Classwork

This section describes the cognitive demand of the classwork tasks that students encountered based solely on task prompts. These rankings exclude any teacher moves that may have taken place during implementation. Implementation is discussed in the following chapter. Students generally experienced three types of classwork tasks during class. Students completed warm-ups, mathematics tasks during the notes or lecture portion of class, and independent or group seatwork. There were 23 mathematical tasks assigned for classwork during the study. Mr. Nimrick’s classwork tasks ranked in three of the four cognitive demand levels on the IQA task rubric (Table 3).

Table 3

<table>
<thead>
<tr>
<th>Rank</th>
<th>Description</th>
<th>Tasks (%)</th>
<th>Items (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Doing Math or procedures with connections and justifications</td>
<td>4 (17%)</td>
<td>6 (11%)</td>
</tr>
<tr>
<td>3</td>
<td>Procedures with connections but no justification</td>
<td>3 (13%)</td>
<td>12 (21%)</td>
</tr>
<tr>
<td>2</td>
<td>Procedures without connections</td>
<td>15 (65%)</td>
<td>37 (66%)</td>
</tr>
<tr>
<td>1</td>
<td>Memorization</td>
<td>1 (4%)</td>
<td>1 (2%)</td>
</tr>
</tbody>
</table>

Note: Percentages may not add to 100 due to rounding.
These descriptive statistics show that a higher percentage of the classwork tasks in Class A were low demand (rank 1-2). Sixty-nine percent of the tasks that students were assigned had the potential to engage them in using a specific procedure or reproducing previously learned facts. This trend was also evident in the ranking of the items as 68% of the classwork items were low demand as well.

**Assessments**

This section describes the cognitive demand of the assessment tasks that students encountered during the study. Students generally experienced two types of summative assessments, quizzes and tests. During this study, students received three quizzes and one unit test. Mr. Nimrick assigned 17 mathematical tasks on assessments during the study. The assessment tasks ranked in three of the four cognitive demand levels on the IQA task rubric (Table 4).

Table 4

*Task and item ranks for Nimrick's Class A Assessments*

<table>
<thead>
<tr>
<th>Rank</th>
<th>Description</th>
<th>Tasks (%) n=17</th>
<th>Items (%) n=38</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Doing Math or procedures with connections and justifications</td>
<td>1 (6%)</td>
<td>3 (8%)</td>
</tr>
<tr>
<td>3</td>
<td>Procedures with connections but no justification</td>
<td>2 (12%)</td>
<td>4 (11%)</td>
</tr>
<tr>
<td>2</td>
<td>Procedures without connections</td>
<td>13 (76%)</td>
<td>21 (55%)</td>
</tr>
<tr>
<td>1</td>
<td>Memorization</td>
<td>1 (6%)</td>
<td>10 (26%)</td>
</tr>
</tbody>
</table>

*Note: Percentages may not add to 100 due to rounding.*
These descriptive statistics show that a higher percentage of the assessment tasks in Class A were low demand. Eighty-two percent of the tasks that students engaged with had the potential to engage students in using a specific procedure or reproducing previously learned facts. This trend was also evident in the ranking of the items as 81% of the assessment items were low demand (rank of 1 or 2).

**Cognitive Demand of Tasks**

There were a total of 20 homework tasks, 23 classwork tasks, and 17 assessment tasks assigned in Class A during this study. Table 5 shows the number of high demand (ranks of 3-4) and low demand (ranks of 1-2) tasks and items for each type of coursework.

Table 5

*Cognitive demand of Nimrick's Class A Tasks*

<table>
<thead>
<tr>
<th>Type of Coursework</th>
<th>Tasks</th>
<th></th>
<th></th>
<th>Items</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low Demand</td>
<td>High Demand</td>
<td>Total</td>
<td>Low Demand</td>
<td>High Demand</td>
<td>Total</td>
<td></td>
</tr>
<tr>
<td>Homework</td>
<td>7 (35%)</td>
<td>13 (65%)</td>
<td>20</td>
<td>36 (40%)</td>
<td>55 (60%)</td>
<td>91</td>
<td></td>
</tr>
<tr>
<td>Classwork</td>
<td>16 (70%)</td>
<td>7 (30%)</td>
<td>23</td>
<td>38 (68%)</td>
<td>18 (32%)</td>
<td>56</td>
<td></td>
</tr>
<tr>
<td>Assessment</td>
<td>14 (82%)</td>
<td>3 (18%)</td>
<td>17</td>
<td>31 (82%)</td>
<td>7 (18%)</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>37 (62%)</td>
<td>23 (38%)</td>
<td>60</td>
<td>105 (57%)</td>
<td>80 (43%)</td>
<td>185</td>
<td></td>
</tr>
</tbody>
</table>

*Note:* Percentages are calculated across the row and may not sum to 100 due to rounding.

Students only received higher percentages of high demand tasks when doing homework, a trend that was also evident in the homework items that students solved. Sixty-five percent of the homework tasks were high demand compared to 30% of the classwork tasks and 18% of the assessment tasks. When looking at the items within task types, 60% of
the homework items were high demand compared to 32% of the classwork items and 19% of assessment items. In this case, there is a similar trend between the number of high demand tasks and the number of high demand items in each coursework type. Looking across all tasks, 38% of all tasks were high cognitive demand and 43% of all the items were high demand. Though the overall demand percentages are similar, the difference in the within-task percentages may indicate a lack of alignment among the task. To investigate whether there was alignment, Chi-squared tests were performed.

To describe the alignment of tasks and items, the researcher compared the proportion of high demand tasks to the number of low demand tasks in homework, classwork, and assessments using the Chi-squared test for homogeneity. This test compared the proportion of high-level (i.e. score of 3 or 4) and low-level (i.e. score of 0, 1, or 2) tasks in each type of coursework using all of the tasks used in Nimrick’s Class A for the five observation days to determine whether the potential of the tasks to support student justification was different across the three coursework types. Distributions that are not statistically different show alignment in that there is no evidence of different opportunities to engage with high demand tasks in each coursework type, while statistically different distributions show evidence of a difference in opportunities to engage with high demand tasks in each coursework type.

In Mr. Nimrick’s Class A, there is evidence that the number of high demand tasks used across the three types of coursework is significantly different ($\chi^2 \approx 9.7$, df=2, $p=0.008$). This suggests a lack of alignment among the cognitive demand levels of homework, classwork, and assessment tasks.
To investigate whether this trend continues when comparing cognitive demand levels of items, an identical Chi-squared test was used on the combined item rank data shown in Table 5. Results again indicate a difference in the number of items ranked high and low demand across coursework types ($\chi^2 \approx 23.3$, df=2, $p \approx .000009$). This supports the earlier finding that there is a lack of alignment among the items students were given in different types of coursework.

**Lesson Goals and Task Choices**

This section presents interview data on Mr. Nimrick’s lesson goals and how these goals may have influenced his classwork task choices. Though this is limited in the insight that it will provide for all of the task choices, it may shed light on why classwork tasks had the cognitive demand that they did.

For Teaching Set 1 Mr. Nimrick was asked “What are your goals/objectives for these lessons? What do you want your students to learn?”

Mr. Nimrick responded,

The primary goal is understand the major properties of a parallelogram. Understand and apply the major properties and I guess the goal is given a four-sided figure can you determine if it is a parallelogram using these properties. That’s the important part. A little less important, can you formally prove it using a two-column proof?

Can you at least get it started and get it most of the way there?

When asked “Can you tell me why you chose the problems you’re going to use in your lessons for the proofs? What are the characteristics of these problems that made you feel like they were good ones to choose?”
Mr. Nimrick responded,

There are a couple proofs I’m going to use for the project tomorrow and they are varying difficulty. There are some that are really straightforward and I’m going to divide those up, its not going to be random how I divide them up. Obviously I’ve got groups that are obviously going to be weaker than others and I want them to really-the goal I’m looking at for them is setting up the proof, coming up with mathematically correct statements. And I know I’ll probably spend most of my time working with those groups. But I’ve got some more difficult ones that maybe have more steps, they’ll have to prove things are parallel first, like the second one we did in the lesson today. I’ve got those, I’ve chosen those because I know that they’re more challenging, there are more steps, it takes a higher level of thinking. I’ll give those to my more, to the groups that have more accomplished proof writers in them.

In Teaching Set 1, Mr. Nimrick’s lesson goals were for students to “understand and apply the major properties of parallelograms … and to write proofs”. To understand and apply these properties students needed opportunities to engage with tasks that supported the learning of parallelogram properties, identification of parallelograms based on those properties, and justification or explanation of mathematical reasoning.

To meet these goals, Mr. Nimrick discussed using proofs that could support students’ understanding of parallelograms and their properties. In addition to the proofs that were discussed, students’ warm-up activities included items that required them to assess whether a figure was a parallelogram based on its visual characteristics. Further, homework and classwork tasks included items that required students to find angle and side lengths such that
the figure would be a parallelogram. Though tasks were not analyzed by teaching set, many of the tasks used in class during Teaching Set 1 were high cognitive demand. This may be a result of the learning goals.

For Teaching Set 2 Mr. Nimrick was asked, “What are your goals/objectives for these lessons? What do you want your students to learn?” Mr. Nimrick responded, “Students will be able to find the area of a sector and the measure/arclength of an arc.” When asked, “Why did you choose the mathematics problems that you are going to use during the lesson?” He responded, “The problems chosen were of increasing difficulty. In this way, I can isolate the cause of student difficulty.”

In Teaching Set 2, Mr. Nimrick’s lesson goal was for students to “find the area of a sector and the measure and arclength of an arc”. These goals stressed the procedures for finding the sector area and measure of arclength. To meet these goals, Mr. Nimrick discussed using problems with varying difficulty. Specifically, Mr. Nimrick used tasks where students were required to identify arcs and arc measures on different circles, calculate circumference and area of circles, and calculate areas of sectors on circles. The tasks used during this teaching set were procedural and therefore ranked low cognitive demand, which may have been a result of the learning goals.

Summary of the Case of Mr. Nimrick’s Class A Tasks

Overall, the tasks in Mr. Nimrick’s Class A were not aligned. Students were provided with more opportunities to engage with high demand tasks on homework assignments. This may have provided students opportunities and ample time to grapple with meaningful tasks. Conversely, assigning the majority of high cognitive demand tasks for
homework may have resulted in student frustration if students were not well-prepared to solve them (Dieker, Maccini, Strickland, & Hunt, 2009; Henningsen & Stein, 1997). The misalignment in the number of high demand tasks across the types of coursework was evident in the overall percentages of high demand task compared to low demand tasks, 38% versus 62%. However, tasks used during class and on assessments, the majority of which ranked at a low level, supported Mr. Nimrick’s learning goals. Goals aimed at procedural competence may have resulted in choosing tasks described as procedures without connections compared to goals aimed at application and understanding, which resulted in tasks described as procedures with connections and doing mathematics.

**Mr. Nimrick’s Class B**

Mr. Nimrick’s Class B is an Advanced Placement Statistics course with a total of 32 students. Students received unit overview sheets with all of the homework and classwork assignments and assessment dates for that unit. Figure 11 is an example of the unit sheet that students received. During the observations, Mr. Nimrick taught lessons on linear regression, correlation, causation, transforming for linearity, and comparison of population parameters. Mr. Nimrick used textbooks and other teachers as resources for tasks.
Figure 11: Nimrick’s Class B assignment schedule

Homework

Homework was generally assigned daily but was checked for completeness intermittently. The number of mathematical tasks in each homework assignment varied. There were eight mathematical tasks assigned for homework during the study. Table 6 shows the number of homework tasks and items ranked at each level.

Table 6

Task and item ranks for Nimrick’s Class B Homework

<table>
<thead>
<tr>
<th>Rank</th>
<th>Description</th>
<th>Tasks (%)</th>
<th>Items (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Doing Math or procedures with connections and justifications</td>
<td>7 (88%)</td>
<td>38 (83%)</td>
</tr>
<tr>
<td>3</td>
<td>Procedures with connections but no justification</td>
<td>1 (13%)</td>
<td>3 (7%)</td>
</tr>
<tr>
<td>2</td>
<td>Procedures without connections</td>
<td>0</td>
<td>5 (11%)</td>
</tr>
<tr>
<td>1</td>
<td>Memorization</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: Percentages may not add to 100 due to rounding.
These descriptive statistics show that all of the homework tasks in Class B were high cognitive demand. One hundred percent of the tasks that students were assigned had the potential to engage them in complex thinking and/or exploring and understanding mathematics. Though all of the tasks were ranked high demand, 83% of the individual items were of the highest demand while 11% of the items were procedural.

**Classwork**

This section describes the cognitive demand of the classwork tasks that students encountered based solely on task prompts. Students generally experienced two types of classwork tasks during class. Students completed warm-ups and mathematics tasks during the notes or lecture portion of class. There were 12 mathematical tasks assigned for classwork during the study. Table 7 shows the number of classwork tasks and items ranked at each level.

Table 7

Task and item ranks for Nimrick's Class B Classwork

<table>
<thead>
<tr>
<th>Rank</th>
<th>Description</th>
<th>Tasks (%) n=12</th>
<th>Items (%) n=24</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Doing Math or procedures with connections and justifications</td>
<td>7 (58%)</td>
<td>9 (38%)</td>
</tr>
<tr>
<td>3</td>
<td>Procedures with connections but no justification</td>
<td>2 (17%)</td>
<td>2 (8%)</td>
</tr>
<tr>
<td>2</td>
<td>Procedures without connections</td>
<td>3 (25%)</td>
<td>9 (38%)</td>
</tr>
<tr>
<td>1</td>
<td>Memorization</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>No Mathematical Activity</td>
<td>0</td>
<td>4 (16%)</td>
</tr>
</tbody>
</table>

*Note: Percentages may not add to 100 due to rounding.*
These descriptive statistics show that a high percentage of the classwork tasks in Class B were high demand. Seventy-five percent of the tasks assigned had the potential to engage students in complex thinking and/or exploring and understanding mathematics. A slightly opposite trend was evident in the ranking of the items as 54% of the classwork items were low demand.

**Assessments**

This section describes the cognitive demand of the assessment tasks that students encountered during the study. Students generally experienced two types of assessments, quizzes and tests. During this study, students received one quiz and three unit tests. There were 22 mathematical tasks assigned for assessments during the study, which ranked in each of the four cognitive demand levels (Table 8).

Table 8

*Task and item ranks for Nimrick's Class B Assessments*

<table>
<thead>
<tr>
<th>Rank</th>
<th>Description</th>
<th>Tasks (%)</th>
<th>Items (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Doing Math or procedures with connections and justifications</td>
<td>4 (18%)</td>
<td>7 (21%)</td>
</tr>
<tr>
<td>3</td>
<td>Procedures with connections but no justification</td>
<td>8 (36%)</td>
<td>8 (24%)</td>
</tr>
<tr>
<td>2</td>
<td>Procedures without connections</td>
<td>6 (27%)</td>
<td>13 (38%)</td>
</tr>
<tr>
<td>1</td>
<td>Memorization</td>
<td>4 (18%)</td>
<td>6 (18%)</td>
</tr>
</tbody>
</table>

*Note:* Percentages may not add to 100 due to rounding.

These descriptive statistics show that the tasks and items on assessments in Class B are nearly equally distributed between high demand and low demand for both tasks and
items. Fifty-four percent of the tasks that students were assigned had the potential to engage them in complex thinking and/or exploring and understanding mathematics and 45% of assessment items were high demand.

**Cognitive Demand of Tasks**

There were a total of 8 homework tasks, 11 classwork tasks, and 22 assessment tasks assigned in Class B during this study. Table 9 shows the number of high demand and low demand tasks and items for each task type.

Table 9

*Cognitive demand of Nimrick's Class B Tasks*

<table>
<thead>
<tr>
<th>Types of Coursework</th>
<th>Tasks</th>
<th>Items</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n (%)</td>
<td>n (%)</td>
</tr>
<tr>
<td></td>
<td>Low Demand</td>
<td>High Demand</td>
</tr>
<tr>
<td>Homework</td>
<td>0 (0%)</td>
<td>8 (100%)</td>
</tr>
<tr>
<td>Classwork</td>
<td>3 (25%)</td>
<td>9 (75%)</td>
</tr>
<tr>
<td>Assessment</td>
<td>10 (45%)</td>
<td>12 (54%)</td>
</tr>
<tr>
<td>Total</td>
<td>13 (31%)</td>
<td>29 (69%)</td>
</tr>
</tbody>
</table>

*Note:* Percentages are calculated across the row and may not sum to 100 due to rounding.

Students engaged with higher percentages of high demand tasks across all types of coursework, but this trend was not evident when examining the items within homework, classwork, and assessment tasks. One hundred percent of the homework tasks were high demand compared to 75% of the classwork tasks and 54% of the assessment tasks. When comparing task items, 89% of the homework items were high demand compared to 46% of
the classwork items and 45% of assessment items. This may indicate a moderate relationship between the number of high demand tasks and the number of high demand items. Looking collectively, 69% of all tasks and 64% of all items were high cognitive demand. Though the overall demand percentages are similar, the within-task percentages may indicate that there is misalignment among the tasks. To investigate whether there was true alignment, Chi-squared tests were performed.

To describe the alignment of tasks and items in each coursework type, the researcher compared the proportion of high demand task in each type of coursework. The results show that there is no significant difference in the proportion of high and low demand tasks used in Nimrick’s Class B ($\chi^2 \approx 6.0$, df=2, $p \approx .05$). This suggests alignment among the cognitive demand levels of homework, classwork, and assessment tasks. To investigate whether this trend continues when comparing cognitive demand levels of items, an identical Chi-squared test was used on the combined item rank data shown in Table 9. The test showed that the trend did not continue when the items data was compared ($\chi^2 \approx 22.0$, df=2, $p \approx .00002$), which suggests a significant difference between the cognitive demand of the items within homework, classwork, and assessment tasks. This suggests that students may not have received comparable opportunities to engage with high items within different types of coursework.

**Lesson Goals and Task Choices**

This section presents interview data on Mr. Nimrick’s lesson goals and how these goals may have influenced his classwork task choices. For Teaching Set 1, Mr. Nimrick was
asked “What are your goals/objectives for these lessons? What do you want your students to learn?”

Mr. Nimrick responded,

I want students to know when a linear regression model is appropriate. In other words when I’m taking 2 two quantitative variables and I want to know what their relationship is, I want them to take those 2 quantitative variables and be able to tell me is there a relationship, how strong is it, and what is the relationship. How can I sum up the relationship, i.e. the line of best fit, least square regression line. And with the other- I’m kind of combining two chapters in this unit. With the other one, chapter 4, finding ways to relate 2 categorical variables. So I guess the same goals there. Is there a relationship between these 2 categorical variables, what is it, how strong is it, that kind of stuff.

When asked, “Why did you choose the task/problems that you are going to use during the lesson? Why did you choose the tasks that you are going to use with categorical variables and to show what to do with nonlinear data?”

Mr. Nimrick responded,

I tried to use things in class that are simple that we can provide a very basic illustration of the concepts and we take them up to more difficult ones after that. Some of them I chose them to illustrate the Simpson’s paradox. I just used problems that I found in textbooks that illustrated that paradox and were of graduating difficulty.
In Teaching Set 1, Mr. Nimrick’s lesson goals were for students to “know when a linear regression model is appropriate, and to know how to relate quantitative and categorical variables.” To understand and apply these properties, students need to be given opportunities to engage with contextual situations that allow them to reason about how to describe relationships between categorical and quantitative variables statistically. To meet this goal, Mr. Nimrick discussed using basic items or tasks that illustrated the concepts and were of graduating difficulty. Specifically, Mr. Nimrick used tasks where students were required to evaluate scenarios and determine how to model them. The tasks used during Teaching Set 1 were conceptual because students were expected to complete the required procedures and explain their findings. Many of the tasks used in class during Teaching Set 1 were high cognitive demand. This may be a result of the learning goals.

For Teaching Set 2 Mr. Nimrick was asked, “What are your goals/objectives for these lessons? What do you want your students to learn?” Mr. Nimrick responded, “Students will learn how to perform a significance test for two parameters.” When asked, “Why did you choose the mathematics problems that you are going to use during the lesson?” He responded, “To provide a basic demonstration of the mechanics of the significance test.”

In Teaching Set 2, Mr. Nimrick’s lesson goal was for students to “learn how to perform a significance test.” These goals stressed the procedures for performing significance tests. To meet these goals, Mr. Nimrick discussed using problems that illustrated the mechanics of performing this type of test. The tasks used during this teaching set were scenario problems that required students to evaluate data and conduct significance tests.
Tasks used on this teaching set were both procedural and conceptual which may be a consequence of the learning goals.

**Summary of the Case of Mr. Nimrick’s Class B Tasks**

Overall, the tasks in Mr. Nimrick’s Class B were aligned. Students were provided with similar opportunities to engage with high demand tasks across all types of coursework. These opportunities may have resulted in students having time to grapple with high demand tasks independently on homework and assessments and collectively during class assignments. Alignment in the number of high demand tasks, however, did not result in alignment when examining individual items. This implies that students did not receive comparable opportunities to respond to high demand and low demand items within tasks. Still, overall, 69% of the tasks and 64% of all items were high cognitive demand. There is some evidence that tasks used during class supported Mr. Nimrick’s lesson goals. However, regardless to whether the learning goals were procedural or more geared towards explanation and justification, the majority of tasks that students interacted with could be described as procedures with connections or doing mathematics.

**Cross-Case Analysis of Mr. Nimrick’s Class A and Class B Tasks**

To compare the overall distributions of Class A and Class B tasks, the Mann-Whitney U test was used (D. Jones & Tarr, 2007). Because a higher percentage of the tasks used in Mr. Nimrick’s class B were high demand (rank 3 or 4), the Mann-Whitney U test was used to see if the ranks of the tasks in Class B were different than those in Class A. The null hypothesis is that the task ranks of Nimrick’s Class A are the same as the task ranks of Nimrick’s Class B. Therefore, the alternative hypothesis is that the task ranks of Nimrick’s
Class A and Class B are different and specifically that the ranks of the tasks used in Nimrick’s Class B are higher overall that the ranks of the tasks used in his Class A, which implies higher cognitive demand of tasks used in Nimrick’s Class B. There is evidence that Mr. Nimrick’s classes, Class A and Class B, have task distributions that are significantly different (U=1982 Z-Score =-2.7334, p=0.00634). This implies that the cognitive demand of the tasks used in Class B, AP Statistics, is higher than those used in Class A, academic Geometry. This suggests that more of the tasks used in the accelerated course had the potential to provide students opportunities to explain and justify their thinking compared to those used in the academic course. This also suggests that students in the AP Statistics course received more opportunities to engage with tasks that connected conceptual and procedural knowledge.

Mrs. Moreland’s Class A

Mrs. Moreland’s Class A is a Geometry course with a total of 26 students. Unit overview sheets with all homework and classwork assignments and assessment dates for each unit were posted on Mrs. Moreland’s Blackboard site (Figure 12). During the observations, Mrs. Moreland taught lessons on transformations, parallelogram properties and proofs, and special segments and angles in circle. Mrs. Moreland used resources provided by her professional learning community as well as coursework from her teaching degree program as resources for tasks.
Homework

Homework was discussed each day but only collected assignments were checked for completeness. The number of mathematical tasks in each homework assignment varied. There were only four mathematical tasks assigned for homework during the study. Table 10 shows the number of homework tasks and items ranked at each level.

Table 10

<table>
<thead>
<tr>
<th>Task and item ranks for Moreland's Class A Homework</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

Note: Percentages may not add to 100 due to rounding.

These descriptive statistics show a high percentage of low demand homework tasks in Class A. This was one of the more difficult cases because there were so few homework assignments. Seventy-five percent of the tasks that students were assigned had the potential
to engage students in using previously learned procedures. Contrary to high demand tasks, a high percentage of low demand homework tasks indicates the same trends for task items because task ranks are based on the highest potential reasoning level apparent in the task. Therefore in Mrs. Moreland’s Class A, as expected, there were a higher percentage of low demand items as well with 90% of the items being ranked at level 2.

**Classwork**

Students had classwork tasks during three parts of class each day. Students completed warm-ups, mathematics tasks during the notes or lecture portion of class, and independent or group seatwork. There were 20 mathematical tasks assigned for classwork during the study, ranking in three of the four cognitive demand levels on the IQA task rubric (Table 11).

<table>
<thead>
<tr>
<th>Rank</th>
<th>Description</th>
<th>Tasks (%)</th>
<th>Items (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Doing Math or procedures with connections and justifications</td>
<td>2 (10%)</td>
<td>2 (6%)</td>
</tr>
<tr>
<td>3</td>
<td>Procedures with connections but no justification</td>
<td>1 (5%)</td>
<td>1 (3%)</td>
</tr>
<tr>
<td>2</td>
<td>Procedures without connections</td>
<td>17 (85%)</td>
<td>31 (91%)</td>
</tr>
<tr>
<td>1</td>
<td>Memorization</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

*Note: Percentages may not add to 100 due to rounding.*

A higher percentage of the classwork tasks in Class A were low demand. Eighty-five percent of the tasks that students were assigned had the potential to engage students in using
A specific procedure or reproducing previously learned facts. This trend was also evident in the ranking of the items as 91% of the classwork items were low demand.

**Assessments**

During this study, students received two tests. There were nine mathematical tasks assigned on assessments during the study. Table 12 shows the number of assessment tasks and items ranked at each level.

**Table 12**

*Task and item ranks for Moreland’s Class A Assessments*

<table>
<thead>
<tr>
<th>Rank</th>
<th>Description</th>
<th>Tasks (%) n=9</th>
<th>Items (%) n=41</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Doing Math or procedures with connections and justifications</td>
<td>1 (11%)</td>
<td>2 (5%)</td>
</tr>
<tr>
<td>3</td>
<td>Procedures with connections but no justification</td>
<td>5 (56%)</td>
<td>11 (27%)</td>
</tr>
<tr>
<td>2</td>
<td>Procedures without connections</td>
<td>2 (22%)</td>
<td>23 (56%)</td>
</tr>
<tr>
<td>1</td>
<td>Memorization</td>
<td>1 (11%)</td>
<td>5 (12%)</td>
</tr>
</tbody>
</table>

*Note:* Percentages may not add to 100 due to rounding.

A high percentage of the assessment tasks in Class A were high demand. Sixty-seven percent of the tasks that students were assigned had the potential to engage them in exploring and understanding the nature of mathematical concepts, procedures, and/or relationships. The opposite trend was evident in the ranking of the items as 68% of the assessment items were low demand.
Cognitive Demand of Tasks

There were a total of 4 homework tasks, 20 classwork tasks, and 9 assessment tasks assigned in Class A during this study. Table 13 shows the number of high demand and low demand tasks and items for each type of coursework.

Table 13

*Cognitive demand of Moreland’s Class A Tasks*

<table>
<thead>
<tr>
<th>Type of Coursework</th>
<th>Tasks</th>
<th>Low Demand</th>
<th>High Demand</th>
<th>Total</th>
<th>Items</th>
<th>Low Demand</th>
<th>High Demand</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homework</td>
<td>3 (75%)</td>
<td>1 (25%)</td>
<td>4</td>
<td>19 (90%)</td>
<td>2 (10%)</td>
<td>21</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Classwork</td>
<td>17 (85%)</td>
<td>3 (15%)</td>
<td>20</td>
<td>31 (91%)</td>
<td>3 (9%)</td>
<td>34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Assessment</td>
<td>3 (33%)</td>
<td>6 (66%)</td>
<td>9</td>
<td>28 (68%)</td>
<td>13 (32%)</td>
<td>41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>23 (70%)</td>
<td>10 (30%)</td>
<td>33</td>
<td>78 (81%)</td>
<td>18 (19%)</td>
<td>96</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Percentages are calculated across the row and may not sum to 100 due to rounding.

Within the types of coursework, students only received high percentages of high demand tasks during assessments. Sixty-six percent of the assessment tasks were high demand compared to 25% of the homework tasks and 15% of the classwork tasks. For individual items, there were a higher percentage of low demand items within each type of coursework. Ninety percent of homework items, 91% of classwork items, and 68% of the assessment items were low demand. In this case there is a similarity between the number of low demand tasks and the number of low demand items except for in the instance of assessment tasks. Looking across all tasks, 70% of all tasks and 81% of all items were low demand. There appears to be alignment among the tasks based on the percentages. To
investigate whether there was true alignment, a Chi-squared test was performed. The small number of tasks prohibits using the Chi-squared test for tasks.

To describe the alignment of items in each coursework type, the researcher compared the number of high and low demand items from tasks in each coursework type using the Chi-squared test for homogeneity. There is uncertainty about the accuracy of these results because only 66% of the expected counts were greater than five. However, there is evidence of a significant difference among the numbers of high and low demand items across the three types of coursework tasks ($\chi^2 \approx 7.9$, df=2, $p \approx .02$). This suggests a lack of alignment among the cognitive demand levels of homework, classwork, and assessment items because a high percentage of the items and tasks were low cognitive demand except for in the case of assessment tasks.

**Lesson Goals and Task Choices**

This section presents interview data on Mrs. Moreland’s lesson goals and how these goals may have influenced her classwork task choices. For Teaching Set 1, Mrs. Moreland was asked “What are your goals/objectives for these lessons? What do you want your students to learn?” Mrs. Moreland responded, “[I want them to be able to] translate and reflect 2D objects graphically and algebraically, identify that all of the vectors of the points of an object translated will be parallel and that perpendicular bisectors are formed with the reflection line when we reflect objects.” When asked “Can you tell me why you chose the problems you’re going to use in your lessons for the proofs? What are the characteristics of these problems that made you feel like they were good ones to choose?” Mrs. Moreland responded, “For reflections, a lot of the kids have seen reflections, the Miras and patty paper
are just new representations. The vector notation will just be a new representation of the translations we did the previous day.”

In Teaching Set 1, Mrs. Moreland’s lesson goals were for students to “translate and reflect figures and reason about the relationship between transformations and parallel and perpendicular lines.” To understand and apply these properties, students need opportunities to engage with transformations and justify and explain their thinking to show that they understand the process. To meet these goals, Mrs. Moreland discussed using hands-on activities so that students could experience and create the transformations. Mrs. Moreland also used tasks that required students to identify types of transformations and write transformation rules in coordinate and algebraic notation. The tasks used in Teaching Set 1 did support students’ reasoning about transformations, however more of the tasks were low demand.

For Teaching Set 2 Mrs. Moreland was asked, “What are your goals/objectives for these lessons? What do you want your students to learn?” She responded, “I want students to learn that tangents in a circle form 90 degree angles. Once we have a 90-degree triangle, all the old rules apply! I also want students to conceptually understand the angle a little better... that an angle really is part of a circle (or multiple circles!!!) that are all just similar.” When asked, “Why did you choose the mathematics problems that you are going to use during the lesson?” She responded, “Examples are to help them get ‘big ideas’. The stuff on the worksheet is to get them to go beyond big ideas.”

In Teaching Set 2, Mrs. Moreland’s lesson goal was for students to “gain a deeper understanding of angles” and learn that tangents and radii form right angles. These goals
stressed the conceptual knowledge for angle measure and tangency. To meet these goals, Mrs. Moreland discussed using examples that focused on big ideas and classwork that moved beyond those ideas. Specifically, Mrs. Moreland used tasks where students were required to use procedures to calculate central and inscribed angle measures. Classroom tasks also required students to use the Pythagorean theorem to find the lengths of triangle with radii for sides. The tasks used during this teaching set were procedural as students were given the rules to calculate the measures. Though tasks were not analyzed by teaching set, many of the tasks used in class during this teaching were low cognitive demand. This may have been a consequence of the learning goals.

**Summary of the Case of Mrs. Moreland’s Class A Tasks**

Overall, the items in Mrs. Moreland’s Class A were not aligned. Most of the tasks were low demand, which implies that tasks were misaligned as well. Based on the percentages, students were provided with more opportunities to engage with high demand tasks on assessments. This may indicate that students were assessed at a higher level than they practiced material on homework and classwork tasks. Overall, students did not receive comparable opportunities to respond to high demand and low demand tasks based on overall percentages, 30% high demand versus 70% low demand. There is some evidence that tasks used during class supported Mrs. Moreland’s lesson goals. Goals aimed at procedural competence may have resulted in tasks described as procedures without connections compared to goals aimed at application and understanding, which resulted in more tasks described as procedures with connections and doing mathematics.
Mrs. Moreland’s Class B

Mrs. Moreland’s Class B is an AP Statistics course with a total of 32 students. Unit overview sheets with all of the homework and classwork assignments and assessment dates for that unit were posted on Mrs. Moreland’s Blackboard site. Figure 13 is an example of the unit sheet that students could access. During the observations, Mrs. Moreland taught lessons on Simpson’s Paradox, simulations, and the $\chi^2$ test. Mrs. Moreland heavily relied on a colleague who also taught AP Statistics at her school as a resource for tasks and also used tasks from a teacher teaching AP Statistics in another school in the district.

![Figure 13: Moreland's Class B assignment schedule](image)

Homework

Students’ homework assignments were assigned at the beginning of a unit, but students were allowed to complete assignments at their own pace. Students’ graded homework assignments did not come from the textbook, homework was assigned through electronic templates such as Blackboard and Study Island. Also, homework was rarely discussed in class and homework solutions were posted online. The homework items analyzed in this study were items assigned during class in conjunction with a classwork
assignment. The only homework assignment was a test review that began in class and was completed at home. Table 14 shows the homework task and items ranked at each level.

Table 14

*Task and item rank for Moreland’s Class B Homework*

<table>
<thead>
<tr>
<th>Rank</th>
<th>Description</th>
<th>Tasks (%)</th>
<th>Items (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>n=1</td>
<td>n=15</td>
</tr>
<tr>
<td>4</td>
<td>Doing Math or procedures with connections and justifications</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Procedures with connections but no justification</td>
<td>1 (100%)</td>
<td>9 (60%)</td>
</tr>
<tr>
<td>2</td>
<td>Procedures without connections</td>
<td>0</td>
<td>2 (13%)</td>
</tr>
<tr>
<td>1</td>
<td>Memorization</td>
<td>0</td>
<td>4 (26%)</td>
</tr>
</tbody>
</table>

*Note*: Percentages may not add to 100 due to rounding.

The small number of homework tasks in this case does not provide enough evidence to draw conclusions about students’ opportunities to engage with high cognitive demand tasks; however, this task was high cognitive demand. It has the potential of supporting students’ reasoning about linear models of data. There also appears to be alignment between the number of high demand items and the rank of the task because 60% of the items were high demand as well.

**Classwork**

Students engaged with classwork tasks during two parts of class each day: the notes or lecture portion of class, and independent or group seatwork. There were nine mathematical tasks assigned for classwork during the study. Table 15 shows the number of classwork tasks and items ranked at each level.
Table 15

**Task and item rank for Moreland’s Class B Classwork**

<table>
<thead>
<tr>
<th>Rank</th>
<th>Description</th>
<th>Tasks (%) n=9</th>
<th>Items (%) n=43</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Doing Math or procedures with connections and justifications</td>
<td>6 (67%)</td>
<td>2 (5%)</td>
</tr>
<tr>
<td>3</td>
<td>Procedures with connections but no justification</td>
<td>1 (11%)</td>
<td>14 (33%)</td>
</tr>
<tr>
<td>2</td>
<td>Procedures without connections</td>
<td>2 (22%)</td>
<td>24 (56%)</td>
</tr>
<tr>
<td>1</td>
<td>Memorization</td>
<td>0</td>
<td>3 (7%)</td>
</tr>
</tbody>
</table>

*Note: Percentages may not add to 100 due to rounding.*

Seventy-eight percent of the tasks that students were assigned in Class B had the potential to engage students in creating meaning for, exploring and understanding mathematical concepts, procedures, and/or relationships. The opposite trend was evident in the ranks of the individual items as 63% of the classwork items were low demand.

**Assessments**

During this study, students received two tests. There were three mathematical tasks assigned on assessments during the study. The assessment tasks ranked in the two high-demand levels on the IQA task rubric (Table 16).
Table 16

**Task and item ranks for Moreland’s Class B Assessments**

<table>
<thead>
<tr>
<th>Rank</th>
<th>Description</th>
<th>Tasks (%)</th>
<th>Items (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>n=3</td>
<td>n=48</td>
</tr>
<tr>
<td>4</td>
<td>Doing Math or procedures with connections and justifications</td>
<td>2(66%)</td>
<td>9(19%)</td>
</tr>
<tr>
<td>3</td>
<td>Procedures with connections but no justification</td>
<td>1(33%)</td>
<td>22(46%)</td>
</tr>
<tr>
<td>2</td>
<td>Procedures without connections</td>
<td>0</td>
<td>10(21%)</td>
</tr>
<tr>
<td>1</td>
<td>Memorization</td>
<td>0</td>
<td>7(15%)</td>
</tr>
</tbody>
</table>

*Note: Percentages may not add to 100 due to rounding.*

A higher percentage of the assessment tasks in Class B were high demand. One hundred percent of the tasks that students were assigned had the potential to engage them in exploring and understanding the nature of mathematical concepts, procedures, and/or relationships. This trend was also evident in the ranking of the items as 65% of the assessment items were high demand.

**Cognitive Demand of Tasks**

There were a total of 1 homework task, 9 classwork tasks, and 3 assessment tasks assigned in Class A during this study. Table 17 shows the number of high demand and low demand tasks and items for each type of coursework.
Table 17

*Cognitive demand of Moreland's Class B tasks*

<table>
<thead>
<tr>
<th>Type of Coursework</th>
<th>Tasks</th>
<th></th>
<th>Items</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low Demand</td>
<td>High Demand</td>
<td>Total</td>
<td>Low Demand</td>
<td>High Demand</td>
</tr>
<tr>
<td>Homework</td>
<td>0</td>
<td>1 (100%)</td>
<td>1</td>
<td>6 (40%)</td>
<td>9 (60%)</td>
</tr>
<tr>
<td>Classwork</td>
<td>2 (22%)</td>
<td>7 (78%)</td>
<td>9</td>
<td>27 (63%)</td>
<td>16 (37%)</td>
</tr>
<tr>
<td>Assessment</td>
<td>0</td>
<td>3 (100%)</td>
<td>3</td>
<td>17 (35%)</td>
<td>31 (65%)</td>
</tr>
<tr>
<td>Total</td>
<td>2 (15%)</td>
<td>11 (85%)</td>
<td>13</td>
<td>50 (47%)</td>
<td>56 (53%)</td>
</tr>
</tbody>
</table>

Note: Percentages are calculated across the row and may not sum to 100 due to rounding.

These data were more difficult to analyze because of the small number of tasks used in Class B during the study. Nevertheless, within each type of coursework, students received higher percentages of high demand tasks. One hundred percent of the homework and assessment tasks were high demand compared to 78% of the classwork tasks. When looking at individual items, there were a higher percentage of high demand tasks in homework and assessment tasks only. Sixty percent of homework items and 65% of assessment items were high demand while 37% of the assessment items were high demand. In this case there may have been a relationship between the number of high demand tasks and the number of high demand items except for in the case of classwork tasks. Collectively, 85% of the all tasks and 53% of the individual items were high demand. There appears to be misalignment among the tasks based on the percentages. To investigate whether this misalignment was significant, Chi-squared tests were performed on the item data. The small number of tasks prohibiting using the Chi-squared test to test for difference in the distributions of tasks.
To describe the alignment of items in each coursework type, the researcher compared the number of high and low demand items for tasks in each type of coursework using the Chi-squared test for homogeneity. The null hypothesis for this test is that the proportion of high and low demand items are the same across the three coursework types. Therefore, the alternative hypothesis is that one of these proportions is not the same as the other two. There is evidence that supports the rejection of the null hypothesis, which implies a significant difference among the item distributions ($\chi^2 \approx 7.2, \text{df}=2, p\approx.03$). This suggests a lack of alignment among the cognitive demand levels of homework, classwork, and assessment items. This supports the appearance of misalignment of items based on the percentages but does not provide definitive evidence about the alignment of the tasks, as the majority of the tasks are ranked high demand. The small number of tasks renders this case inconclusive.

**Lesson Goals and Task Choices**

This section presents interview data on Mrs. Moreland’s lesson goals and how these goals may have influenced her classwork task choices. For Teaching Set 1 Mrs. Moreland was asked “What are your goals/objectives for these lessons? What do you want your students to learn?” Mrs. Moreland responded, “I want students to learn that they can use random digits to give them an idea of how a sample would work in real life, the usefulness of sampling, and how to detect bias and/or strata in sampling. If there is some bias/strata, I want them to also understand how to compensate for it when taking a sample.” When asked “Can you tell me why you chose the problems you’re going to use in your lessons for the proofs? What are the characteristics of these problems that made you feel like they were good ones to choose?” Mrs. Moreland responded, “We wanted to highlight common AP
questions and questions that tend to pose mental blocks/difficulties for students. Most of chapter five is easy for them to get (it's mostly vocabulary), which is why we made it an independent study. However, there are still topics that require some in class discussion. Hence, why I'm spending these couple of days to frontload!”

In Teaching Set 1, Mrs. Moreland’s lesson goals were for students to understand how to sample, the usefulness of sampling, and how to account for biases in sampling.” To understand and apply these properties, students need to be given opportunities to engage with data and strategies for sampling. To meet these goals, Mrs. Moreland discussed using common AP questions that may present as misconceptions for students. Specifically, Mrs. Moreland used tasks that required students to design simulations using the random digit table or the random number generator to design simulations given probabilities of success. Students were also given a task that required them to employ different sampling methods and discuss the benefits of each. The tasks used during this teaching set were both procedural and conceptual, because students were expected to complete the required procedures and explain their findings. Many of the tasks used in class during this teaching set were high cognitive demand. This may be a result of the learning goals.

For Teaching Set 2 Mrs. Moreland was asked, “What are your goals/objectives for these lessons? What do you want your students to learn?” She responded, “I want students to understand that they can test to see if a distribution is as we expect it to be (we have significance test for big/whole picture situations). I want them to understand when a GOF test is appropriate and when a homogeneity test is appropriate and to perform these tests with accuracy of course.” When asked, “Why did you choose the mathematics problems that you
are going to use during the lesson?” She responded, “My colleagues chose them, but I agreed that they would address the typical AP questions asked about Chi-Squared.”

In Teaching Set 2, Mrs. Moreland’s lesson goal was for students to understand when to use the chi-squared tests for homogeneity and goodness of fit. These goals stressed the conceptual knowledge for knowing when to apply this statistical test. To meet these goals, Mrs. Moreland discussed using typical AP questions. She used a hands-on activity and practice worksheet that had students compute Chi-squared statistics and interpret their findings. These tasks focused on linking procedural and conceptual knowledge.

Summary of the Case of Mrs. Moreland’s Class B Tasks

Overall, the tasks in Mrs. Moreland’s Class B appeared aligned. Students were provided with more opportunities to engage with high demand tasks on across all task types. Though there appeared to be alignment in the tasks, individual items were misaligned though students received comparable opportunities to respond to high demand and low demand items based on overall percentages, 53% versus 47%. There is some evidence that tasks used during class supported Mrs. Moreland’s lesson goals. Goals aimed at application and understanding resulted in tasks described as procedures with connections and doing math.

Cross-Case Analysis Mrs. Moreland’s Class A and Class B Cases

To compare the overall distributions of Class A and Class B tasks, the Mann-Whitney U test was used. Because a higher percentage of the tasks used in Mrs. Moreland’s Class B were high demand, the Mann Whitney U test was used to see if the cognitive demand of tasks in Class B tended to be higher than those in Class A. The null hypothesis for this test is that the rank of the tasks used in Mrs. Moreland’s Class A have the same distribution as the ranks
of the tasks used in Mrs. Moreland’s Class B. The null hypothesis is rejected which provides evidence that Mrs. Moreland’s Class A, Geometry, and Class B, AP Statistics, have task distributions that are significantly different (U=3804, Z-Score =3.4702, p=0.00052). This implies that the cognitive demand of Class B tasks is higher than the cognitive demand of Class A tasks. This also suggests that students in the accelerated class received more opportunities to explain and justify their thinking based on the potential of the mathematics tasks chosen for use in the course compared to the students in the academic course, and that students in the accelerated course received more opportunities to engage with tasks that connected conceptual and procedural knowledge.

Mrs. Kaiser’s Class A

Mrs. Kaiser’s Class A is a Geometry course with a total of 28 students. Homework packets were given to students at the beginning of each unit with all of the homework assignments. Figure 14 shows a portion of the homework assignment that students were assigned for Day 1 of the unit. Students were expected to complete homework assignments in the homework packets. During the observations, Mrs. Kaiser taught lessons on triangle proofs, radicals, and Pythagorean theorem. Mrs. Kaiser used her professional learning community as well as other teachers, an Internet search engine (i.e. Google), and old textbooks as resources for tasks.
Figure 14: Kaiser's Class A Homework Packet

Homework

Homework was generally assigned and checked for completeness daily. The number of mathematical tasks in each homework assignment varied. There were 10 mathematical tasks assigned for homework during the study. Table 18 shows the number of homework tasks and items ranked at each level.

Table 18

Task and item ranks for Kaiser's Class A Homework

<table>
<thead>
<tr>
<th>Rank</th>
<th>Description</th>
<th>Tasks (%)</th>
<th>Items (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Doing Math or procedures with connections and justifications</td>
<td>5 (50%)</td>
<td>29 (31%)</td>
</tr>
<tr>
<td>3</td>
<td>Procedures with connections but no justification</td>
<td>2 (20%)</td>
<td>27 (28%)</td>
</tr>
<tr>
<td>2</td>
<td>Procedures without connections</td>
<td>1 (10%)</td>
<td>20 (21%)</td>
</tr>
<tr>
<td>1</td>
<td>Memorization</td>
<td>2 (20%)</td>
<td>19 (20%)</td>
</tr>
</tbody>
</table>

Note: Percentages may not sum to 100 due to rounding.
These descriptive statistics show that a higher percentage of the homework tasks in Class A were high demand. Seventy percent of the tasks that students were assigned had the potential to engage students in complex thinking and/or exploring and understanding mathematics. As previously stated, a higher percentage of high demand homework tasks does not necessarily imply the same for task items because task ranks are based on the highest potential reasoning level apparent in the task. However, in Mrs. Kaiser’s Class A there were a higher percentage of high demand items as well with 59% of items being ranked at level 3 or 4.

Classwork

Students had classroom tasks during three parts of class: warm-ups, mathematics tasks during the notes or lecture portion of class, and independent or group seatwork. There were 14 mathematical tasks assigned for classwork during the study, with rankings shown in Table 19.

Table 19

<p>| Task and item ranks for Kaiser's Class A Classwork |
|---|---|---|---|</p>
<table>
<thead>
<tr>
<th>Rank</th>
<th>Description</th>
<th>Tasks (%)</th>
<th>Items (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Doing Math or procedures with connections and justifications</td>
<td>9 (64%)</td>
<td>23 (43%)</td>
</tr>
<tr>
<td>3</td>
<td>Procedures with connections but no justification</td>
<td>1 (7%)</td>
<td>2 (4%)</td>
</tr>
<tr>
<td>2</td>
<td>Procedures without connections</td>
<td>4 (29%)</td>
<td>28 (53%)</td>
</tr>
<tr>
<td>1</td>
<td>Memorization</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: Percentages may not sum to 100 due to rounding.
These descriptive statistics show that a higher percentage of the classwork tasks in Class A were high demand. Seventy-one percent of the tasks that students were assigned had the potential to support their understanding of the nature of mathematical relationships and processes. This trend was also somewhat evident in the ranks of individual items as 47% of the classwork items were high demand.

**Assessments**

This section describes the cognitive demand of the assessment tasks that students encountered during the study. Students generally experienced two types of assessments, quizzes and tests. During this study, students received two quizzes. There were only three mathematical tasks, comprised of 28 items, assigned on assessments during the study. Table 20 shows the number of assessment tasks and items ranked at each level.

Table 20

*Task and item ranks for Kaiser’s Class A Assessments*

<table>
<thead>
<tr>
<th>Rank</th>
<th>Description</th>
<th>Tasks (%) n=3</th>
<th>Items (%) n=28</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Doing Math or procedures with connections and justifications</td>
<td>1(33%)</td>
<td>1(4%)</td>
</tr>
<tr>
<td>3</td>
<td>Procedures with connections but no justification</td>
<td>1(33%)</td>
<td>7(25%)</td>
</tr>
<tr>
<td>2</td>
<td>Procedures without connections</td>
<td>1(33%)</td>
<td>20(71%)</td>
</tr>
<tr>
<td>1</td>
<td>Memorization</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

*Note: Percentages may not sum to 100 due to rounding.*
had the potential to engage students in complex thinking. However, the opposite trend was evident in the ranking of the items as 71% of the assessment items were low demand. This may have been a consequence of the common assessments used in this course that were developed collectively by all of the teachers teaching this material. Note, the small number of assessment tasks do not provide substantial evidence to support claims about assessments.

**Cognitive demand of tasks**

There were a total of 10 homework tasks, 14 classwork tasks, and 3 assessment tasks assigned in Class A during this study. Table 21 shows the number of high demand and low demand tasks and items for each type of coursework.

<table>
<thead>
<tr>
<th>Type of Coursework</th>
<th>Tasks</th>
<th></th>
<th></th>
<th>Items</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Low Demand</td>
<td>High Demand</td>
<td>Total</td>
<td>Low Demand</td>
<td>High Demand</td>
<td>Total</td>
</tr>
<tr>
<td>Homework</td>
<td>3 (30%)</td>
<td>7 (70%)</td>
<td>10</td>
<td>39 (41%)</td>
<td>56 (59%)</td>
<td>95</td>
</tr>
<tr>
<td>Classwork</td>
<td>4 (29%)</td>
<td>10 (71%)</td>
<td>14</td>
<td>28 (53%)</td>
<td>25 (47%)</td>
<td>53</td>
</tr>
<tr>
<td>Assessment</td>
<td>1 (33%)</td>
<td>2 (66%)</td>
<td>3</td>
<td>20 (71%)</td>
<td>8 (29%)</td>
<td>28</td>
</tr>
<tr>
<td>Total</td>
<td>8 (30%)</td>
<td>19 (70%)</td>
<td>27</td>
<td>87 (49%)</td>
<td>89 (51%)</td>
<td>176</td>
</tr>
</tbody>
</table>

Note: Percentages are calculated across the row and may not sum to 100 due to rounding.

Students received higher percentages of high demand tasks in each type of coursework. Seventy percent of the homework tasks, 71% of classwork tasks, and 66% of assessment tasks were high demand. For individual items, 59% of the homework items were high demand compared to 47% of the classwork items and 29% of assessment items. In this
case there does not appear to be a relationship between the number of high demand tasks and the number of high demand items. Looking across all tasks, 70% of all tasks were high cognitive demand, with 51% of the individual items ranked as high demand. In spite of the differences in high demand task percentages, the overall demand percentages may indicate alignment among the tasks. To investigate whether there was true alignment, Chi-squared tests were performed. The small number of tasks prohibits using the Chi-squared test for tasks.

To describe the alignment of items in each task type, the researcher compared the number of high demand items to the number of low demand items in each task type using the Chi-squared test for homogeneity. The null hypothesis is that the proportion of high and low demand items within the three coursework types are the same, and the alternative hypothesis is that at least one of these proportions is not the same. In Mrs. Kaiser’s Class A there is evidence of a significant difference among the item distributions \((\chi^2 \approx 8.3, \text{df}=2, p \approx .02)\). This supports the appearance of misalignment of items based on the percentages and but does not confirm misalignment among the cognitive demand levels of homework, classwork, and assessment tasks as tasks are ranked based on the highest ranked items within that task. However, these statistics suggest that students have the opportunity to engage in higher demand tasks during homework and classwork but not on assessments, based on the item distributions.

**Lesson Goals and Task Choices**

This section presents interview data on Mrs. Kaiser lesson goals and how these goals may have influenced her classwork task choices. For Teaching Set 1 Mrs. Kaiser was asked
“What are your goals/objectives for these lessons? What do you want your students to learn?”

Mrs. Kaiser responded,

The students by the end of the day should know how to construct a hypotenuse leg proof and a CPCTC proof. I’m doing hypotenuse leg first because I’m thinking that’s going to be the harder one and if I’m seeing some resistance, if I’m seeing they’re struggling, then I may not get to the CPCTC. Then I would just expand that over to Monday, so my goal is to get through those two today so that over the weekend, they get a chance to go through those problems and then Monday is just a mixed review of all the different things we have…

When asked “Can you tell me why you chose the problems you’re going to use in your lessons for the proofs? What are the characteristics of these problems that made you feel like they were good ones to choose?”

Mrs. Kaiser responded,

It’s the level of difficulty. From our perspective its not that difficult, but I think it’s the picture having different figures which I think when they see the same picture all the time, they first of all think its the same proof every single time, and they carry that over and so I wanted to find some that had overlapping pictures just because I need for them to have that exposure to all different types. Of course I don’t have every different shape up there, you can’t prepare them for that, but just to get them out of their comfort zone a little bit to say hey these are two overlapping triangles, where is the triangle that I’m proving? How do I handle that? I wanted them to model that
today, so I put one of those in my lesson and that’s also part of the classwork that they’re going to have to turn in today, so I have similar but not the same.

In Teaching Set 1, Mrs. Kaiser’s lesson goals were for students to engage with hypotenuse leg CPCTC (corresponding parts of congruent triangles are congruent) proofs. To understand and apply these properties, students need opportunities to engage with triangle proofs. To meet these goals, Mrs. Kaiser discussed using proofs that could support students’ understanding of how to prove different statements using similar figures. The tasks that Mrs. Kaiser used during this teach set were high demand. Students had multiple opportunities to complete proofs and justify their reasoning in response to tasks.

For Teaching Set 2 Mrs. Kaiser was asked, “What are your goals/objectives for these lessons? What do you want your students to learn?” Mrs. Kaiser responded,

Going into the lesson I was hoping that they would know how to identify radicals, know that radicals have indexes, and that they could do more than just take the square root, be able to simplify them, be able to do operations with them. Adding, subtracting, multiplying, dividing radicals was my goal for today.

When asked, “Generally, can you talk as a whole about the problems you picked, what were you trying to accomplish?” She responded,

I had a set of problems I was picking from...The ones I wanted to show them were the ones that simplified completely because it amazes me when it’s a perfect square they still want to leave the square root there. They don’t understand what to do and I’m thinking by definition, if it’s the perfect square, you just have that number. So I wanted them to have the perfect square.
In Teaching Set 2, Mrs. Kaiser’s lesson goal was for students to “simplify and operate with radicals.” These goals stressed the procedures for operations with radical expressions. To meet these goals, Mrs. Kaiser discussed using problems that focused on perfect squares and how to simplify radicals. All of the tasks used during this teaching set were ranked at level 2 because there focus was on improving students’ procedural facility with simplifying and operating with radicals.

**Summary of the Case of Mrs. Kaiser’s Class A Tasks**

Overall, the tasks in Mrs. Kaiser’s Class A appeared aligned. Students were provided with similar opportunities to engage with high demand tasks on homework, classwork, and assessment tasks and overall, 70% of the tasks were high demand. However this alignment was not present among the item distributions, because only 51% of the items across all types of coursework were high demand. The tasks chosen by Mrs. Kaiser seem to support lesson goals. Goals aimed at procedural competence may have resulted in tasks described as procedures without connections compared to goals aimed at application and understanding, which resulted in more tasks described as procedures with connections and doing math.

**Mrs. Kaiser’s Class B**

Mrs. Kaiser’s Class B is an Honors Algebra II class with a total of 31 students. Homework packets were given to students at the beginning of the each unit with all of the homework assignments listed on a schedule. Figure 15 is an example of the homework schedule that students received. During the observations, Mrs. Kaiser taught lessons on strategies for solving quadratics and translations of quadratics. Mrs. Kaiser used teacher’s
personal websites, a professional and social site for teachers and students (i.e. Edmodo), NCTM articles, textbooks, and her professional learning community as resources for tasks.

**Unit 4: Quadratic Functions & Equations**

NC Objectives to be covered:
1.03 Operate with algebraic expressions (polynomial, rational, complex fractions) to solve problems.
2.02 Use quadratic functions and inequalities to model and solve problems.
   a. Solve using graphs.
   b. Interpret the constants and coefficients in the context of the problem.

<table>
<thead>
<tr>
<th>Day</th>
<th>Date</th>
<th>Lesson</th>
<th>Assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Wed. Feb. 27</td>
<td>Complex Numbers</td>
<td>Packet pg. 1 #2-18 even</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Packet pg. 2 #2-18 even</td>
</tr>
<tr>
<td>2</td>
<td>Thurs. Feb. 28</td>
<td>Factoring &amp; Solving:</td>
<td>Packet pg. 3 #2-20 even</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Trinomials &amp; Difference of Squares</td>
<td>Packet pg. 13, section 3 only: #1-14</td>
</tr>
</tbody>
</table>

Figure 15: Kaiser's Class B Homework schedule

**Homework**

Homework was generally assigned daily and checked for completeness each day. The number of mathematical tasks in each homework assignment varied. There were nine mathematical tasks assigned for homework during the study. Table 22 illustrates the number of homework tasks and items ranked at each level.
Table 22

Task and item ranks for Kaiser's Class B Homework

<table>
<thead>
<tr>
<th>Rank</th>
<th>Description</th>
<th>Tasks (%) n=9</th>
<th>Items (%) n=98</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Doing Math or procedures with connections and justifications</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Procedures with connections but no justification</td>
<td>2 (22%)</td>
<td>14 (14%)</td>
</tr>
<tr>
<td>2</td>
<td>Procedures without connections</td>
<td>7 (78%)</td>
<td>84 (86%)</td>
</tr>
<tr>
<td>1</td>
<td>Memorization</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: Percentages may not sum to 100 due to rounding.

Seventy-eight percent of the tasks that students engaged with were lower demand and had the potential to support their use of a procedure that was either specifically called for or evident based on prior instruction, experience, or placement of the task. In Mrs. Kaiser’s Class B there were a higher percentage of low demand items as well, with 86% of the items being ranked at level 2.

Classwork

Students had classroom tasks during three parts of class: warm-ups, mathematics tasks during the notes or lecture portion of class, and independent or group seatwork. There were 15 mathematical tasks assigned for classwork during the study with rankings for tasks and items displayed in Table 23.
### Table 23

**Task and item ranks for Kaiser's Class B Classwork**

<table>
<thead>
<tr>
<th>Rank</th>
<th>Description</th>
<th>Tasks (%)</th>
<th>Items (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Doing Math or procedures with connections and justifications</td>
<td>1 (7%)</td>
<td>1 (2%)</td>
</tr>
<tr>
<td>3</td>
<td>Procedures with connections but no justification</td>
<td>5 (33%)</td>
<td>16 (36%)</td>
</tr>
<tr>
<td>2</td>
<td>Procedures without connections</td>
<td>9 (60%)</td>
<td>28 (62%)</td>
</tr>
<tr>
<td>1</td>
<td>Memorization</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

*Note: Percentages may not add to 100 due to rounding.*

Sixty percent of the tasks that students engaged with were limited to engaging students in using a procedure without connections. This trend was also evident in the ranking of the items as 62% of the classwork items were low demand.

**Assessments**

Students generally experienced three types of assessments, quizzes and tests. During this study, students received six quizzes, four of which were warm-up quizzes. There were seven mathematical tasks assigned on assessments during the study. The assessment tasks ranked in two of the four cognitive demand levels on the IQA task rubric, as illustrated in Table 24.
### Task and item ranks for Kaiser's Class B Assessments

<table>
<thead>
<tr>
<th>Rank</th>
<th>Description</th>
<th>Tasks (%) n=7</th>
<th>Items (%) n=26</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Doing Math or procedures with connections and justifications</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Procedures with connections but no justification</td>
<td>2 (29%)</td>
<td>2 (8%)</td>
</tr>
<tr>
<td>2</td>
<td>Procedures without connections</td>
<td>5 (71%)</td>
<td>24 (92%)</td>
</tr>
<tr>
<td>1</td>
<td>Memorization</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

*Note: Percentages may not sum to 100 due to rounding.*

These rankings show that a higher percentage of the assessment tasks in Class B were low demand. Seventy-one percent of the tasks that students engaged with were limited to engaging students in using a procedure. The same trend was evident in the ranking of the items as 92% of the assessment items were low demand.

**Cognitive Demand of Tasks**

There were a total of 9 homework tasks, 15 classwork tasks, and 7 assessment tasks assigned in Class B during this study. Table 25 shows the number of high demand and low demand tasks and items for each task type.
Table 25

*Cognitive demand of Kaiser's Class B tasks*

<table>
<thead>
<tr>
<th>Type of Coursework</th>
<th>Tasks</th>
<th>Low Demand</th>
<th>High Demand</th>
<th>Total</th>
<th>Items</th>
<th>Low Demand</th>
<th>High Demand</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Homework</td>
<td>7</td>
<td>(78%)</td>
<td>2</td>
<td>(22%)</td>
<td>9</td>
<td>84</td>
<td>(86%)</td>
<td>14</td>
</tr>
<tr>
<td>Classwork</td>
<td>9</td>
<td>(60%)</td>
<td>6</td>
<td>(40%)</td>
<td>15</td>
<td>28</td>
<td>(62%)</td>
<td>17</td>
</tr>
<tr>
<td>Assessment</td>
<td>5</td>
<td>(71%)</td>
<td>2</td>
<td>(29%)</td>
<td>7</td>
<td>24</td>
<td>(92%)</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>21</td>
<td>(68%)</td>
<td>10</td>
<td>(32%)</td>
<td>31</td>
<td>136</td>
<td>(80%)</td>
<td>33</td>
</tr>
</tbody>
</table>

Note: Percentages are calculated across the row and may not sum to 100 due to rounding.

Within types of coursework, students received higher percentages of low demand tasks. Seventy-eight percent of the homework tasks, 60% of classwork tasks, and 71% of assessment tasks were low demand. When looking at items within types of coursework, 86% of the homework items, 62% of the classwork items, and 92% of assessment items were low demand as well. Looking across all tasks, 68% of tasks were low cognitive demand and 80% of items were low demand. Because the within coursework type percentages are close in number, the overall demand percentages may indicate alignment among the tasks. To investigate whether there was alignment, Chi-squared tests were performed on items. The small number of tasks prohibits its use to compare the distributions of the tasks.

To describe the alignment of items for each type of coursework, the researcher compared the number of high demand items to the number of low demand items using the chi-squared test for homogeneity. The null hypothesis is that the proportion of high and low demand items within each tasks for each type of coursework is the same. There is evidence that suggests the rejection of the null hypothesis ($\chi^2 \approx 13.6, \text{df}=2, p \approx .001$). There are
significant differences among the item distributions in Mrs. Kaiser’s Class B. This suggests a lack of alignment among the cognitive demand levels of homework, classwork, and assessment items. This contradicts the appearance of alignment of items based on the percentages. This also indicates misalignment in the tasks as well, as more of the tasks were low cognitive demand.

Lesson goals and task choices

This section presents interview data on Mrs. Kaiser lesson goals and how these goals may have influenced her classwork task choices. For Teaching Set 1 Mrs. Kaiser was asked “What are your goals/objectives for these lessons? What do you want your students to learn?” Mrs. Kaiser responded, “Today’s lesson, the objective is to solve the quadratic using completing the square, and Monday’s lesson will be solving quadratic equations using the quadratic formula.” When asked “Can you tell me why you chose the problems you’re going to use in your lessons for the proofs? What are the characteristics of these problems that made you feel like they were good ones to choose?” Mrs. Kaiser responded,

The variety of the problems that I picked, I try to start pretty easy, they got bored pretty quickly, like oh this is easy, got it. Then try to ramp it up, try to show them all the different examples like when \( a \) is not equal to one, what happens, how do I handle it, how do I complete the square. I try to give them a little bit of all the different scenarios that we could have so that’s why I chose the problems that I chose.

In Teaching Set 1, Mrs. Kaiser’s lesson goals were for students to solve quadratics using complete the square and the quadratic formula. To understand and apply these
properties, students need opportunities to engage with different types of quadratics. To meet these goals, Mrs. Kaiser discussed using graduating difficulty problems that engage students with quadratics with different features. The tasks that Mrs. Kaiser used during this teach set were low demand. However, students had multiple opportunities to use different strategies to factor quadratics.

For Teaching Set 2 Mrs. Kaiser was asked, “What are your goals/objectives for these lessons? What do you want your students to learn?” Mrs. Kaiser responded, “Their goals were to and are to develop the rules or apply the rules to the parent functions to get the transformation that’s desired.” When asked, “Can you tell me why you picked that math problem and what were the features of that math problem that you really feel helped your students understand?” Mrs. Kaiser responded,

It is to come up with a generic function. Instead of me using \( |x| \), use a picture or use an object to say this represents our function. Now, if I put this into the function, how do you expect that to change the output? Or if I change my function on the outside of it, if I apply this to the outside of it, how does that change the output? So try to keep it generic, a little more abstract so it takes away from a one particular function at a high level.

In Teaching Set 2, Mrs. Kaiser’s lesson goal was for students to engage abstractly with function transformations. These goals stressed opportunities for understanding the nature of mathematical relationships between parent and child functions because students could not solve to find solutions. They had to reason. To meet these goals, Mrs. Kaiser discussed using problems that focused on transformation of abstract functions and figures
that were not clearly linked to an equation. Many of the tasks used during this teaching set were ranked at level 4 because they focused on linking procedural and conceptual understanding.

**Summary of the Case of Mrs. Kaiser’s Class B Tasks**

Overall, the tasks in Mrs. Kaiser’s Class B appear to be aligned. Though the distribution of items was statistically different, at least 60% of the tasks in each coursework type were low demand. The trend was also evident in the overall percentages with 68% percent of all tasks being low demand, which means that students generally received more opportunities to engage with tasks that required the use of prescribed or single procedures. The tasks used during the instruction seemed to support lesson goals. Goals aimed at procedural competence may have resulted in tasks described as procedures without connections compared to goals aimed at application and understanding, which resulted in more tasks described as procedures with connections and doing math.

**Cross-Case Analysis of Mrs. Kaiser’s Class A and Class B Cases**

To compare the overall distributions of Class A and Class B tasks, the Mann-Whitney U test was used. Because a higher percentage of the tasks used in Mrs. Kaiser’s Class A were high demand, the Mann Whitney U test was used to test whether the distribution of task ranks in Class A and B were the same or if there was a significant difference in the ranks of the tasks between the two courses. There is evidence that Mrs. Kaiser’s Class A and Class B have task distributions that are significantly different (U=214.5, Z-Score = -3.1722, p=0.00152), and based on the percentages of high demand tasks computed for each course, the cognitive demand of tasks in Class A, Geometry, is higher than the cognitive demand of
tasks in Class B, Honors Algebra II. This suggests that students in the academic class received more opportunities to explain and justify their thinking based on the potential of the task than did the students in the accelerated class. This also suggests that the students in the academic class received more opportunities to engage with tasks that connected conceptual and procedural knowledge.

**Cross-Case Analysis 1**

The remainder of this chapter relies on cross case analysis to characterize and compare the six cases of collections of tasks within a teacher’s course by collectively considering the cognitive demand of tasks, the alignment of the tasks across coursework, and the alignment of teachers’ learning goals and classroom tasks.

Based on the IQA Task Rubric, ranks of 1 and 2 for cognitive demand do not provide students opportunities to justify or explain solution strategies beyond description of procedural steps. Ranks of 3 or 4 support students’ opportunities to justify their thinking, use multiple strategies to arrive at solutions, and connect procedures with the underlying concepts. Thus, task collections that seem to succeed in providing students’ opportunities to learn should have equal or greater amounts of high cognitive demand tasks compared to low demand tasks across coursework categories (i.e. homework, classwork, and assessment).

Another feature to consider is alignment among the tasks in the coursework categories. Students who are expected to connect concepts and procedures on homework tasks or classroom tasks opportunities to make these connections on assessments as well (Hiebert et al., 1997). To measure alignment, Chi-squared tests for homogeneity were used to detect differences in the distribution of low demand and high demand tasks for homework,
classwork, and assessment tasks. Aligned tasks support students’ opportunities to learn because they ensure that students encounter the same types of tasks in different settings: in class, outside of class, and on assessments. However, alignment coupled with high demand tasks provide students opportunities to encounter similar tasks that require them to reason about mathematics.

The final feature used to locate the collections of tasks on the continuum is based on teachers’ interview data. Hiebert and Grouws (2007) define teaching as “classroom interactions among teachers and students around content directed towards facilitating [all] students’ achievement of learning goals” (p. 377). Therefore, setting appropriate learning goals shapes students’ learning opportunities. Hence, the final criteria to consider is the alignment of teachers’ learning goals with tasks they chose to use in class during observations. While the cognitive demand of classwork tasks influence students’ opportunities to learn mathematics through exposure to worthwhile tasks, an equally important feature of students’ opportunity to learn is exposure to tasks that align with the teacher’s learning goals. Appropriately set learning goals also support students’ opportunities to solve mathematics problems using prior knowledge. Therefore, each of the six cases was ranked by considering the alignment between learning goals and tasks used.

Based on the analysis of tasks and alignment across coursework and to learning goals, Mr. Nimrick’s Class A, Mrs. Moreland’s Class A, and Mrs. Kaiser’s Class B seem to represent struggles to support students’ opportunities to learn. In each of these three collections of tasks, there was evidence that students in those classes had some opportunities to engage in higher cognitive demand tasks. In Mrs. Kaiser’s Class B, the majority of the
tasks in different coursework types ranked low, the learning goals were procedural, and learning goals and tasks were all well aligned at lower levels of demand. Mr. Nimrick’s Class A had two coursework types that were majority low demand and one that was typically high demand. Nimrick’s Class A also had mixed learning goals, but the tasks that students engaged with overall tended to be of low demand. Ms. Moreland’s Class A tasks also shared a similar profile. Two of the three coursework types had more tasks ranked at a low demand, coursework types were not aligned, but learning goals seemed aligned with the lower cognitive demand of most tasks used. While Mrs. Kaiser’s Class B tasks seem to represent very limited opportunities for students to engage in tasks that provide higher cognitive demand, there were more moderate opportunities in Mr. Nimrick’s Class A and Mrs. Moreland’s Class A.

Conversely, Mr. Nimrick’s Class B, Mrs. Moreland’s Class B, and Mrs. Kaiser’s Class A seem to represent examples of successes to support students’ opportunities to learn. In all three cases, the presence of higher demand tasks was more persistent. All of the coursework types in Mr. Nimrick’s Class B had more tasks ranked high demand and were well aligned across coursework, though this was not true for the analysis at the item level. Mr. Nimrick’s Class B also had mixed learning goals that aligned with classroom tasks. The tasks used in Mrs. Moreland’s Class B and Mrs. Kaiser’s Class A have similar profiles.

**Continuum and the Conceptual Framework**

In order to describe how collections of tasks are placed on the opportunity to learn continuum for tasks, the conceptual framework must be revisited. Recall teachers’ mathematical task choice includes identifying learning goal and choosing tasks that support
learning goals and offer opportunities for both procedural and conceptual knowledge building. Students’ opportunities to learn in this study are defined as opportunities to engage with high demand tasks that support (a) connecting procedural and conceptual knowledge, (b) justifying and explaining mathematical thinking, and (c) using prior knowledge. These factors of OTL are included as necessary features of collections of tasks that support students’ OTL. These opportunities are used to classify collections of tasks, as those collections that provide students more opportunities to learn are considered more equitable than those that provide fewer opportunities. Before describing the placement of the collections of tasks, two points should be addressed. First, prior knowledge is addressed in the next chapter when task implementation is discussed. Second, the classification of collections of tasks as a struggle to support students’ learning opportunities may be a consequence of the lessons observed, as well as, the researcher’s access to all student work. These classifications only serve to illustrate what students’ opportunities to learn might be if a particular collection of tasks embodies a teacher’s task choices over the course of an academic year.

**Placement of Collection of Tasks on a Continuum**

Each collection of tasks is a unit of analysis and there are two combinations of unit features that represent teacher success with supporting students’ opportunities to learn and two that represent teacher struggles with supporting students’ opportunities to learn. The combinations of task collection features that represent teacher struggles with supporting students’ opportunities to learn are labeled limited and moderate. The term limited refers to a lower tendency for students to have opportunities to justify and explain thinking and engage
with high cognitive demand tasks; the classification of limited does not refer to student achievement or teachers’ ability. The combinations of features that represent teacher success with supporting students’ opportunities to learn are labeled moderately persistent and persistent. The term persistent refers to a higher tendency for students to have opportunities to justify and explain thinking and engage with high cognitive demand tasks. Figure 16 shows the continuum of student’s OTL based on the collection of tasks. Finally, Mrs. Kaiser’s Class A seemed to have the most persistent opportunities for students to engage in higher cognitive demand tasks. All coursework types in Class A had a majority of tasks which ranked high, the learning goals were mixed, and learning goals and tasks were well aligned.

Each teacher had one course that seemed to represent a struggle in providing students’ opportunities to engage with high cognitive demand tasks and one class that appeared to be successfully providing students’ opportunities to learn. A characterization of collections of tasks as a struggle to support students’ learning opportunities may be a consequence of the particular lessons observed. Thus a characterization only serves to illustrate what students’ opportunities to learn might be if a particular collection of tasks embodies a teacher’s task choices over the course of an academic year.

The cases characterized as limited (Mrs. Kaiser’s Class B) and moderate (Mr. Nimrick’s Class A and Ms. Moreland’s Class A) provided students opportunities to engage with high cognitive demand mathematics at an emergent level. Students experienced opportunities to engage with demanding tasks less frequently than they engaged with more procedural tasks, but still had opportunities to engage with demanding tasks in at least one
coursework category. The cases considered as successful in providing student’s opportunities to learn provided students opportunities to engage with demanding tasks more often than they engaged with more procedural ones. Each case represents a persistent (Mrs. Kaiser Class A and Mrs. Moreland’s Class B) or moderately persistent (Mr. Nimrick’s Class B) opportunity for students to learn because students were given many opportunities in each case to respond to tasks that required them to think and reason about mathematical concepts. The issue here is frequency of opportunities.

**Development of a Continuum Framework**

Recall teachers’ mathematical task choice includes identifying learning goal and choosing tasks that support learning goals and offer opportunities for both procedural and conceptual knowledge building. Students’ opportunities to learn in this study are defined as opportunities to engage with high demand tasks that support (a) connecting procedural and conceptual knowledge, (b) justifying and explaining mathematical thinking, and (c) using prior knowledge. These factors of OTL are necessary features of collections of tasks that support students’ OTL. As shown from the cross-case analysis, all of the collection of tasks had these necessary features to varying degrees. None of the cases represented the complete absence of these features. The degree to which each collection of tasks fulfilled these features was characterized as ranging from limited to persistent.

While only based on analysis of six cases, the evidence suggests that a continuum of opportunities to learn can be used to characterize collections of tasks within a teacher’s course. On a continuum, collections of tasks that provide students more opportunities to learn, and are more aligned across different types of coursework and to a teacher’s learning
goals for students, are considered more equitable than those that provide fewer opportunities. A proposed continuum is shown in Figure 16.

The combinations of task collection features that represent teacher struggles with supporting students’ opportunities to learn are labeled limited and moderate. The term limited refers to a lower tendency for students to have opportunities to justify and explain thinking and engage with high cognitive demand tasks; the classification of limited does not refer to student achievement or teachers’ ability. The combinations of features that represent teacher success with supporting students’ opportunities to learn are labeled moderately persistent and persistent. The term persistent refers to a higher tendency for students to have opportunities to justify and explain thinking and engage with high cognitive demand tasks.
Figure 16: Opportunity to Learn Continuum based on Tasks
**Limited OTL**

A case of tasks may be considered as limited if (1) students engage with a high percentage of low cognitive demand tasks, (2) tasks are aligned, and (3) learning goals and classroom tasks are misaligned or learning goals and tasks are procedural and aligned. A high percentage of low demand tasks indicates that students’ opportunities are limited to using procedures and recalling previously learned facts. Though these tasks may support students’ facility with procedures or definitions, low demand tasks limit students’ opportunities to explain or justify mathematical work. Low demand tasks also convey mathematics as a combination of memorizing definitions and using procedures. For limited collection of tasks, alignment indicates that students engage with more low demand tasks across all types of coursework. This may indicate a lack of opportunities to explain and justify thinking on homework, classwork, and assessment tasks. Misalignment in the learning goals expressed by a teacher and tasks that students engage with, or alignment between procedural learning goals and tasks, result in a collection of tasks being characterized as limited. Students’ ability to learn a particular topic is heavily dependent upon the tasks that are assigned and tasks should be assigned based on teachers’ learning goals. Therefore misalignment between learning goals and classroom tasks or alignment between lesson goals and procedural tasks may provide limited opportunities for students to justify and explain their thinking or to engage with tasks that support complex thinking.

**Moderate OTL**

A case of tasks may be classified as moderate if (1) students engage with a high percentage of low cognitive demand tasks on two of the coursework types, (2) tasks are not
aligned, and (3) learning goals and classroom tasks are procedural and aligned. This combination of task features is considered as moderate instead of limited because students have more opportunities for explanation or justification on at least one of the coursework types. Opportunities to engage with tasks that have the potential to engage students in complex thinking can change how students view mathematics and its purpose. In the moderate case, students engage with tasks that may support facility with procedures or important definitions and at least one task type that allows them to think critically or share explanations or justifications for their work. For moderate cases, misalignment of tasks indicates that there may be noticeable differences in the cognitive demand of the tasks that students engage with for homework, classwork, and assessment. This is desirable because the cognitive demand of one of the coursework types is high and the other two are low. Finally, if a teacher’s learning goals align with the assigned tasks and the goals were procedural, the opportunities to learn are considered moderate.

**Moderately Persistent OTL**

A collection of tasks may be considered as moderately persistent if (1) students engage with a high percentage of high cognitive demand tasks on two of the types of coursework, (2) tasks are not aligned, and (3) learning goals are both conceptual and procedural and learning goals and classroom tasks aligned. This combination of task features is classified as moderately persistent instead of persistent because students have more opportunities for explanation or justification on two of the coursework categories. Opportunities to engage with tasks that have the potential to engage students in complex thinking can change how students view mathematics and its purpose. In this case, students
engage with tasks that may support critical thinking and justification of mathematical reasoning in two coursework categories and with tasks that support students’ facility with procedures or important definitions in one coursework category. In moderately persistent task cases, misalignment of tasks indicates a noticeable difference in the cognitive demand of tasks that students engage with across coursework categories. This is desirable because only one of the task cognitive demand level is low compared to the other two. If the teacher’s learning goals are aligned with the tasks, and the goals are both conceptual and procedural, the collection of tasks is characterized as providing moderately persistent opportunities to learn.

**Persistent OTL**

The final characterization of opportunities to learn based on tasks is that of persistent. A collection of tasks may be considered as persistent if (1) students engage with a high percentage of high cognitive demand tasks, (2) tasks across coursework are aligned, and (3) learning goals are both procedural and conceptual and classroom tasks and learning goals are aligned. This combination of task features are classified as persistent because having a high percentage of high demand tasks indicates that students interact with tasks that have the potential for supporting complex thinking and justification of mathematical reasoning. Being engaged with high demand tasks may convey that mathematics is about reasoning, justification of strategies, complex thinking, and non-algorithmic approaches to problem solving. In this combination, alignment of tasks indicates that students engage with high demand task regardless of the coursework category. This means that students receive many opportunities to explain and justify their thinking on homework, classwork, or assessment
tasks. If the teachers’ learning goals align with the assigned tasks and the goals were procedural and conceptual, the collection of tasks is ranked as persistent.

**Chapter Summary**

The focus of this chapter was the first research question of this study:

1. How do the tasks that ECMTs choose support students’ opportunities to engage with high cognitive demand tasks in class, outside of class, and on assessments?

   a. How do the task choices of an ECMT differ when teaching academic versus accelerated courses?

In an effort to address this question, analyses of the six cases were presented. The cases were defined at the level of a teacher’s class. The source of data for these analyses was the homework, classwork, and assessment tasks assigned to the students in each class during the study. After individual analyses, two levels of cross case analysis was conducted, one at the teacher level and one at the case level. The cross case analysis at the teacher level investigated differences in the distribution of high cognitive demand and low cognitive demand tasks. The emphasis of the cross case analysis at the case level was the three features of tasks that support students’ opportunities to learn: percentage of high cognitive demand tasks, alignment of tasks, and alignment of learning goals and task choices. The characterizations of the six cases resulted in an emergent framework to describe a continuum of opportunities to learn based on a teacher’s task choice (Figure 16).

The mathematical tasks used in this study provided students opportunities to engage with mathematical content that aligned with the course curriculum. Within each case there were similarities and differences in the learning goals of the teachers and how they used tasks
to meet their learning goals. The learning goals expressed by teachers were not clear indicators of the type of tasks that were employed in each class. Each participant’s goal was for students to understand new content, however some used more conceptual tasks to support this goal while others used more procedural tasks. Overall, each teacher desired conceptual understanding and procedural fluency for his/her students but some participants more successful than others in consistently using tasks that could aid them in meeting both of these goals.

The connections between course level and use of high cognitive demand tasks were fairly uniform. Of the three accelerated courses, one was considered as moderately persistent and the other as persistent. These represent successes of choosing tasks that support student’s opportunities to engage more often with high cognitive demand tasks. Likewise, of the three academic Geometry courses, two were ranked as moderate. Moderate is a ranking that represents a struggle in choosing tasks that support students’ opportunities to engage with high cognitive demand tasks. Only one collection of tasks presented the opposite trend. This resulted in one of the academic Geometry classes being ranked as persistent in providing students more opportunities to engage with high cognitive demand tasks and one of the accelerated classes being characterized as limited.

The next chapter seeks to address the second research question relative to task implementation in the 15 academic Geometry lessons and the 15 accelerated class lessons.
CHAPTER 5: ANALYSIS OF TASK IMPLEMENTATIONS AND FINDINGS

The second research question for this study was the following:

2. How do ECMTs implementations of mathematical tasks support equitable classroom spaces?

   a. How do the task implementations of academic and accelerated courses differ?

To examine this question, each teacher was observed during classroom instruction for 10 lessons. To ensure a focus on instruction and not particular teachers, each course level, academic or accelerated, defined a case and each lesson was the unit of analysis for each case. This chapter describes the implementation of tasks across all participants. For these analyses, the cases are the two levels of courses, academic Geometry and accelerated, which include AP Statistics and Honors Algebra II. Each case includes 15 lessons, 5 by each teacher. Lessons were delineated by observation day. Each lesson was analyzed for features of implementations that supported students’ opportunities to learn. These features are: (1) using high demand tasks, (2) giving clear expectations for engagement with tasks, (3) providing students with necessary resources to fully participate in lessons, (4) providing students opportunities to share written work publicly and (5) to justify and explain thinking, (6) modeling high-level thinking, and (7) answering questions with questions to encourage peer dependency. Taken together, these features are used to place implementations on a continuum of equitable classroom spaces. After discussing each feature as it pertains to students’ OTL, three portraits of task implementations are presented to provide a contextual picture of emergent, moderate, and prominent equitable classroom spaces. Because each teacher taught Geometry, each portrait will be based on a geometry task. Each portrait is a
combination of the challenges and successes observed across all three teachers. Portraits include fictitious classroom dialogue to show the three levels of equitable classroom spaces.

This chapter is structured in two sections. The first section describes the three teachers’ practice collectively by evaluating the instruction observed during individual lessons. The second section answers research question 2 by describing how these lessons \((n=30)\) are placed on a continuum of equitable classroom spaces. The second section also presents portraits of mathematics teachers’ instruction to provide a more contextual view of the different levels of equitable classroom spaces.

**Opportunity to Learn and Features of Task Implementation**

The analysis of class observation data resulted in the observation of instances of teacher practice in mathematics classrooms that support students’ opportunities to learn, specifically students’ opportunities to engage with mathematical tasks using prior knowledge, connect procedural and conceptual knowledge, and justify and explain thinking. It is possible that the teachers observed in the study made instructional decisions based on a few students in the class or only a limited number of students shared justifications and explanations of their mathematical thinking. This study is not focused on individual students’ opportunities and therefore does not report on individual students. Therefore, the instances of instruction described here collectively give examples of how teachers actively supported these opportunities.

**Focus on Teaching, Not Teachers**

The classroom observations are presented collectively so that the illustrations and claims represent instances of instruction across teachers. Classes will be described as
Geometry and Accelerated to further blind the results. Each of the lessons by course level, Geometry and Accelerated, were analyzed separately to show the course-specific characteristics of teacher practice. Once lessons were grouped based on common characteristics within course level, the differences between course levels are discussed. Finally, groups of lessons with similar characteristics across courses will be used to describe a continuum of equitable classroom spaces.

Features of Task Implementations

The first four features of the task implementation that were assessed describe physical features of the implementation that support students’ opportunities to learn. They are:

1. use of high cognitive demand tasks
2. expression of clear expectations for engagement in the task,
3. availability of resources, and
4. students’ opportunities to display written work publicly.

The ranks of the classwork tasks assigned in the previous chapter were used to categorize the task used during lessons taught on each observation day. The use of any high cognitive demand task (rank 3-4) during the task implementation resulted in the observations being described as using high cognitive demand tasks. Task implementations were described as having clear expectations for engagement if the teacher gave explicit directions for how students were to interact during different parts of the lesson and during work with tasks. This feature attended to the ease with which teachers managed transitions between parts of the lesson and between modes of engagement while students were working on a task. The third feature refers to availability of resources, which describes students’ access to all necessary
materials. The final characteristic that described a physical aspect of the implementation was students’ opportunities to share written work publicly. This feature was used to describe task implementations during which students were asked to present their work on the board or on a poster to share with their peers.

The remaining three features of task implementation describe how teachers verbally support students’ opportunities to learn. They are:

5. providing students’ opportunities to justify and explain their thinking,

6. providing models of high-level reasoning, and

7. answering questions with questions.

Task implementations where teachers asked questions that encouraged students to justify their solution strategies or explain their solutions were described as implementations that encouraged explanation or justification of mathematical thinking. Implementations in which teachers provided justifications for solution strategies and connected tasks to students’ prior knowledge were described as implementations where teachers modeled high-level reasoning. Finally, implementations in which teachers responded to students’ questions with a return question or questions to the class were described as implementations where teachers answered questions with questions. This feature does not describe all instances where students answer their peers’ questions, but instead instances where the teacher structures such an exchange. Observations were assigned these features if there was one instance during a lesson. The assumption being made here is that features observed at least once are in the repertoire of teachers’ instructional practices and are likely to surface again.
Having high cognitive demand tasks, providing clear expectations for engagement with tasks, and providing resources for engagement with tasks directly affect students’ OTL. These features of task implementation describe the basic level of access in a classroom. That is, students who are not presented with high cognitive demand tasks, are not provided clear expectations for engagement with high demand tasks, and do not have the proper resources to engage with high cognitive demand tasks cannot fully engage in mathematic classrooms and may have a limited view of what mathematics is and what doing mathematics means (Hiebert et al., 1997; M. S. Smith & Stein, 1998; Yackel & Cobb, 1996). In order for students to have the opportunity to connect conceptual and procedural knowledge, they must engage with tasks that explicitly prompt for or have the potential to engage students in mathematical reasoning.

Providing clear expectations for engagement with the task is also an important feature of task implementation that supports students’ opportunities to learn (Goffney, 2010; Henningsen & Stein, 1997). In order for students to meet their teachers’ learning goals they must be informed about those goals and given clear directives on what is required to meet them. Likewise, when teachers properly manage classroom transitions, students are able to prepare to fully engage in each part of the class.

Finally, to engage with these high demand tasks, students must have access to all necessary resources and materials. These resources include any materials that directly influence students’ ability to complete the task (e.g. random number tables, graphic calculators, patty paper) as well as those that support students’ ability to engage in the class.
(e.g. paper and pencil) (Henningsen & Stein, 1997). Students should not be restricted from participation due to a lack of basic materials.

Two strategies that support students’ opportunities to learn are allowing students to share their written work publicly and providing students opportunities to justify and explain their thinking. Using student work as the basis for whole class discussion allows students to see their peers as knowledgeable and as authorities in the classroom (Yackel & Cobb, 1996).

Finally, the practice of answering questions with questions supports students’ opportunities to learn because this practice encourages students to depend on themselves, their peers, and other resources for solutions instead of on the teacher. In this sense, students’ prior knowledge is a personal resource. Teachers should consider their students’ prior knowledge in order to help them build connections between previously learned and new material (Confrey, 2006; National Research Council, 2001a). Further, teachers should consider what features of their lessons support this self-dependence. Task implementations were assessed to learn about the combination of these features present in each lesson as a way to describe equitable classroom spaces because these features represent instructional practices that support students’ opportunities to learn.

**Features of Task Implementations in the Geometry Classes**

Across the 15 academic Geometry lessons, only availability of necessary resources was a feature of all lessons. Teachers provided paper and pencil for students who did not have them during lessons and had the necessary number of manipulatives during activities that required their use. Also, most lessons (13 out of 15) were described as having clear expectations for engagement with tasks and engagement in the class during instruction.
Teachers commonly provided explicit cues for transitions during class. Teachers used explicit prompts (e.g. “Get out paper and pencil to record notes” or “Once you have completed your proof, compare your work with a partner”) and timers to help students transition between activities.

Exactly two-thirds of the lessons (10 out of 15) used high cognitive demand tasks and provided students opportunities to justify and explain their thinking. It should be noted that there were lessons (2 out of 15) where students engaged with high cognitive demand tasks but were not given opportunities to justify and explain their thinking. During these lessons, students contributed answers to questions and presented solution strategies but were not expected to justify or explain their solutions or solution strategies. There were also lessons (2 out of 15) that did not use high cognitive demand tasks during which students had opportunities to justify and explain their thinking. During these lessons, students were asked to justify their solutions strategies for tasks that were initially ranked as procedures without connections.

Using high cognitive demand tasks included using tasks that built on students’ prior knowledge. Teachers provided evidence of how tasks build in students’ prior knowledge. There were two ways on which teachers considered prior knowledge, separately and collectively. Teachers discussed evidence of students’ prior knowledge separately by sighting instances of student dialogue and ease of response to warm-up/bellwork tasks as evidence that the lessons built on recently obtained prior knowledge. For example, teachers stated that if students used vocabulary or properties of figures previously presented in the course (e.g. using properties of triangles to explain properties of parallelograms), then
students were accessing prior knowledge. However, one teacher struggled with this concept of prior knowledge based on a view of students’ prior knowledge. This teacher stated that without a pretest, changes in students’ knowledge could not be solely attributed to her instruction because of what students had learned in previous classes.

Teachers also discussed evidence of students’ prior knowledge collectively by describing students’ ability to perform tasks that were not a part of the current curriculum. This notion of prior knowledge includes skills or knowledge that students may have learned in previous courses. Teachers sighted evidence of students using mnemonic devices learned in previous classes to aid in finding solutions to current tasks. Though some students were able to use their collective prior knowledge to correctly approach tasks, this form of prior knowledge was more problematic, because students sometimes struggled to enact skills assumed to have been learned in previous courses (e.g. graphing on a coordinate plane and operating with radicals). Again teachers discussed student dialogue and questioning to provide evidence of how students might have used their prior knowledge.

Opportunities for students to share written work, observe modeling of high-level reasoning, and answer their peers’ questions were more infrequently observed features of lessons during the study (nine, eight, and four lessons respectively). During lessons were students were invited to share their written work with their peers, they generally recorded solutions to the bellwork or warm up tasks, tasks that were usually review of previously learned material. They were also provided opportunities to present their solution strategies for tasks during lessons that did not introduce brand new information. For example students
were asked to go to the board during the second or third day of a lesson sequence on highly connected material.

Opportunities to observe high-level reasoning were also a feature of instruction observed more infrequently. Teachers often told students how to find correct answers but did not always help students to think about how features of problems and prior knowledge could inform the problem solving process. Teachers that provided opportunities to observe high-level thinking provided students with structured pre-work activities. Pre-work or sidebar activities were activities during which students were able to engage in the thinking that must take place to solve a task before the actual task is attempted. Teachers used these pre-work activities to help students create definitions of important terms, consider what pathways for solutions were fruitful and which were not, and how prior knowledge interacts with new tasks. The final and least observed feature across the lessons was the opportunity for students to answer their peers’ questions. During these lessons teachers generally answered students’ questions. When students raised their hands, teachers circulated and discussed with the students whether they were on the right track. During the classes where students were expected to answer their own questions and the questions of their peers, teachers suggested that students consult notes, foldables, prior work, and group partners in response to a raised hand or questions. Teachers also asked student originated questions to the class to see if other students could answer the questions that was posed when this feature of instruction was observed.

Table 26 shows which of the seven features of task implementation were present during each Geometry observation. Overall, 10 of the 15 lessons provided students
opportunities to engage with some high cognitive demand tasks, clear expectations for engagement with tasks and during lessons, and access to necessary and appropriate resources. Of those lessons, nine offered opportunities beyond the first three features, use of high cognitive demand tasks, clear expectations and access to necessary resources, with four lessons offering at least one or two features beyond the three and five lessons offering six or all seven of the features of task implementation described in the table.
Table 26

*Features of Task Implementations in the Geometry Classes*

<table>
<thead>
<tr>
<th>Task Implementation Features</th>
<th>Lessons</th>
<th>Geometry (n=15)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3 4 5 6 7 8 9 10 11 12 13 14 15</td>
<td></td>
</tr>
<tr>
<td>1. Using high cognitive demand tasks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Task that link to prior knowledge</td>
<td>x x x x</td>
<td></td>
</tr>
<tr>
<td>b. Tasks that require justification</td>
<td></td>
<td>x x x x x x x x</td>
</tr>
<tr>
<td>2. Giving clear expectations for task engagement</td>
<td>x x x x x</td>
<td>x x x x x x x</td>
</tr>
<tr>
<td>3. Providing students with resources</td>
<td>x x x x x x x x x x x</td>
<td></td>
</tr>
<tr>
<td>a. Providing students with materials to engage with tasks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>i.e. calculators, patty paper</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. Providing students with materials to engage in class</td>
<td></td>
<td></td>
</tr>
<tr>
<td>i.e. paper, pencils, calculators</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Opportunities to share work publicly</td>
<td>x x x x</td>
<td></td>
</tr>
<tr>
<td>a. Allowing students to present their work on the board and posting student work on the</td>
<td></td>
<td></td>
</tr>
<tr>
<td>walls (posters, completed assignments)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Providing opportunities to justify and explain</td>
<td>x x x x x x x x x x x</td>
<td></td>
</tr>
<tr>
<td>a. Asking questions that require explanation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. Expecting students to provide justification for solutions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Modeling high-level reasoning</td>
<td>x x x x x x x x x x x</td>
<td></td>
</tr>
<tr>
<td>a. Providing explanations for solutions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. Connecting new instruction to prior knowledge</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Answering questions with questions</td>
<td>x x x</td>
<td></td>
</tr>
</tbody>
</table>

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Features of Task Implementations in the Accelerated Classes

Across the 15 lessons observed in the accelerated courses, only availability of necessary resources was a feature of all lessons. Teachers provided paper and pencil for students who did not have them during lessons and had the necessary number of manipulatives during activities that required their use. Also, most lessons (12 out of 15) were described as using high cognitive demand tasks. Many of the tasks that students engaged with during these lessons explicitly prompted for justification and explanation of mathematical reasoning and interpretation of solutions. Nine of the lessons were described as having clear expectations for engagement with tasks and engagement in the class during instruction. This was a consequence of classroom structure. During lessons where students listened to lectures, sometimes there were no explicit directions about what to do during implementation. This lead to lessons where some students interjected relevant thoughts and ideas, others recorded notes, still others listened without any note taking, and some seemed disengaged. In the classes where there were clear expectations, students were commonly provided explicit cues for transitions during class.

Building on prior knowledge and explicit prompts to justify and explain thinking are features of high cognitive demand tasks. Teachers provided evidence of how tasks built in students’ prior knowledge. For the accelerated classes, teachers mostly discussed prior knowledge collectively. Teachers discussed evidence of students’ prior knowledge collectively by describing students’ intuitions of previously learned concepts and applications of prior skills to new materials. For example, one participant described students’ ability to apply linear relationships to statistical scenarios as evidence of students’ using prior
knowledge, while another described how students’ conception of probability informed students’ learning of random sampling. In the accelerated classes, the idea of students’ transcripts as a résumé surfaced from one participant. This implies that students are expected to understand previously taught courses because have been added to their transcripts. Teachers sighted student dialogue and questioning to provide evidence of how students might have used their prior knowledge.

Only nine of the lessons were described as providing students opportunities to justify and explain their thinking. Even though the tasks that were used in these classes were high cognitive demand, students were not always given the opportunity to share task solutions or their reasoning related to their solutions publicly. This was also true when describing students’ opportunities to share written work. Only six of the lessons observed in the accelerated classes provided students opportunities to share their written work. However, when students in these classes did share their work it was usually during the portion of the class that involved instruction and not bellwork.

Exactly two-thirds of the classes (10 out of 15) contained opportunities for students to observe a teacher modeling high-level reasoning. Teachers often included clear explanations and interpretations of the mathematics content in their lectures. They also regularly described common misconceptions that students might face during lectures as well. The final and least observed feature (1 out of 15) across the lessons was the opportunity for students to answer their peers’ questions. During these lessons teachers generally answered students’ questions during lessons. When students raised their hands, teachers circulated and discussed with the students whether they were on the right track.
Table 27 shows which of the seven features of task implementation were present during each Accelerated lesson observation. Overall, seven of the lessons provided students opportunities to engage with high cognitive demand tasks, clear expectations for engagement with tasks and during lessons, and access to necessary and appropriate resources and all seven offered opportunities beyond those three with four lessons offering at least one or two features beyond the three and three lessons offering six or all seven of the features of task implementation described in the table.
Table 27

Features of Task Implementations in the Accelerated Classes

<table>
<thead>
<tr>
<th>Task implementation Features</th>
<th>Lessons</th>
<th>Accelerated (n=15)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 2 3 4</td>
<td></td>
</tr>
<tr>
<td>1. Using high cognitive demand tasks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Task that link to prior knowledge</td>
<td>x x x x</td>
<td></td>
</tr>
<tr>
<td>b. Tasks that require justification</td>
<td>x x x x</td>
<td></td>
</tr>
<tr>
<td>2. Giving clear expectations for task engagement</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>x x x x</td>
<td></td>
</tr>
<tr>
<td>3. Providing students with resources</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Providing students with materials to engage with tasks (i.e.</td>
<td>x x x x</td>
<td></td>
</tr>
<tr>
<td>calculators, patty paper)</td>
<td>x x x x</td>
<td></td>
</tr>
<tr>
<td>b. Providing students with materials to engage in class (i.e.</td>
<td>x x x x</td>
<td></td>
</tr>
<tr>
<td>paper, pencils, calculators)</td>
<td>x x x x</td>
<td></td>
</tr>
<tr>
<td>4. Opportunities to share work publicly</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Allowing students to present their work on the board and</td>
<td>x x x x</td>
<td></td>
</tr>
<tr>
<td>posting student work on the walls (posters, completed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>assignments)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Providing opportunities to justify and explain</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Asking questions that require explanation</td>
<td>x x x x</td>
<td></td>
</tr>
<tr>
<td>b. Expecting students to provide justification for solutions</td>
<td>x x</td>
<td></td>
</tr>
<tr>
<td>6. Modeling high-level reasoning</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Providing explanations for solutions</td>
<td>x x x x</td>
<td></td>
</tr>
<tr>
<td>b. Connecting new instruction to prior knowledge</td>
<td>x x</td>
<td></td>
</tr>
<tr>
<td>7. Answering questions with questions</td>
<td></td>
<td>x x x x x x x x x</td>
</tr>
</tbody>
</table>
**Cross-Case Analysis 2**

The remainder of this chapter relies on cross case analysis to characterize and compare the cases of academic Geometry lessons and accelerated lessons by collectively considering the seven features of instruction that have been used in this study as components of students’ learning opportunities during lessons: (1) using high demand tasks, (2) giving clear expectations for engagement with tasks, (3) providing students with necessary resources to fully participate in lessons, (4) providing students opportunities to share written work publicly and (5) to justify and explain thinking, (6) modeling high-level thinking, and (7) answering questions with questions to encourage peer dependency.

**Equitable Classroom Spaces**

Students’ opportunities to learn during task implementations depends on their ability to fully engage with high demand worthwhile tasks, justify and explain their mathematical thinking, observe models of high-level thinking, and use their resources to complete tasks. This section first provides backing from literature of the relative importance of the seven features of classroom instruction that were analyzed in this study. The relative importance of certain features was used in the cross-case analysis of the 15 academic classes and the 15 accelerated classes. By grouping lessons together that contain similar aspects of the seven features, three types of classroom equitable spaces emerged that can be used to characterize opportunity to learn on a continuum for task implementations. The lessons described within each of the three types of equitable classroom spaces are presented in a table in each of the subsequent sections.
Prioritizing Features of Instruction

As stated in the *Professional Standards for teaching Mathematics* (NCTM, 1991), opportunities for student learning are not created by simply putting students into groups, by placing manipulatives in front of them, or by handing them a calculator. Rather it is the level and kind of thinking in which students engage that determines what students learn (Stein et al., 2000, p. 11).

The sentiment of this statement is that the resources and classroom organizational structures are important but more important is how the tasks that students engage with support mathematical reasoning. In addition, the expectations for engaging with the task must be clear to support students’ opportunities to learn. Therefore, the absence of high cognitive demand tasks, clear expectations for engaging with tasks or in class, or access to necessary resources to engage with tasks during a lesson or a lesson that only offered these features was described as providing students limited opportunities to learn. This implies that lessons that provide students opportunities to engage with high cognitive demand tasks that have clear expectations and to use all necessary resources in working with tasks seem to succeed in providing students more opportunities to learn compared to those that are lacking these features.

In the presence of the basic features necessary to support students’ learning opportunities, lessons that afford students more learning opportunities including opportunities to share their work with others support students’ opportunities to justify and explain their thinking. Opportunities to justify and explain one’s mathematical thinking can support the development of coherent mathematical concepts and position students as active
members in the community of learners (Franke et al., 2007; Hiebert et al., 1997; Yackel & Cobb, 1996). Thus, lessons that provide students opportunities to engage with high demand tasks that are clearly defined using necessary resources and share their written work and explanations and justifications of this work appear to provide students more opportunities to learn compared to lessons that do not support students’ opportunities to share their written work or explain and justify their mathematical thinking.

The final two features of opportunity to learn based on task implementation, modeling high-level reasoning and answering questions with questions, are related to particular classroom norms observed during instruction. When students are provided models of high-level reasoning, they can engage with tasks and their peers as doers of mathematics (Stein et al., 2000; Yackel & Cobb, 1996). Clear models of high-level reasoning assist students in knowing how to justify and explain their mathematical thinking and help them evaluate what counts as valid mathematical arguments. Answering questions with questions supports students’ dependence on themselves and peer for mathematical solutions (Reinhart, 2000). The more that students understand about how to engage as an active participant in the mathematics class, the more they are able to experience intellectual autonomy in this setting (Yackel & Cobb, 1996). Hence, when students experience these features as norms in their classrooms coupled with the presence of high demand tasks with clear expectations, access to all necessary resources, and opportunities to share their written work and justifications and explanations, they have the most opportunities to learn.

Both academic Geometry and accelerated lessons represented struggles, partial success, and success in providing students’ opportunities to learn based on the features of
implementation. Given that these characterizations may be a consequence of the lessons observed, these categories only serve to illustrate students’ opportunities to learn if a particular combination of features embodies a teacher’s task implementations over the course of an academic year.

**Characterizing similar lessons**

Based on the analysis of the 30 lessons with a focus in the seven features that support students’ opportunities to learn, 14 lessons struggled to provide the basic opportunities to learn, seven lessons offered a mixture of different opportunities beyond basic, and nine offered most or all of the seven features. The next sections describe the features of academic Geometry and accelerated lessons at each level of equitable classroom spaces.

**Struggling Equitable Classroom Spaces.** Based on the analysis of classroom observations, six academic lessons and eight accelerated lessons offered or struggled to offer the basic features of students’ opportunities to learn. More accelerated lessons received this description than geometry lessons. These lessons either lacked the combination of the three basic features of task implementations to support students’ opportunity to learn or only had those features. Table 28 shows how the lessons for the geometry and the accelerated courses compare. The equitable classroom spaces in the accelerated lessons provided students more opportunities to engage with high demand tasks, justify and explain their thinking and observe models of high-level thinking compared to the emerging geometry lessons. None of these lessons provided students opportunities to answer the questions of their peers and fewer of the accelerated lessons provided students clear expectations for engagement with tasks compared to the accelerated lessons.
Table 28

*Features of Struggling Equitable Classroom Spaces by Course Type*

<table>
<thead>
<tr>
<th>Task implementation Features</th>
<th>Accelerated (n=8)</th>
<th>Geometry (n=6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lessons</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1    2    3  4  5  7  11  12</td>
<td>3  4  5  6  8  9</td>
</tr>
<tr>
<td>1. Using high cognitive demand tasks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Task that link to prior knowledge</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. Tasks that require justification</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>x    x    x    x    x    x    x</td>
<td>x</td>
</tr>
<tr>
<td>2. Giving clear expectations for task engagement</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>x    x    x    x    x    x</td>
</tr>
<tr>
<td>3. Providing students with resources</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Providing students with materials to engage with tasks (i.e.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>calculators, patty paper)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. Providing students with materials to engage in class (i.e.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>paper, pencils, calculators)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>x    x    x    x    x    x    x    x</td>
<td>x    x    x    x</td>
</tr>
<tr>
<td>4. Opportunities to share work publicly</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Allowing students to present their work on the board</td>
<td></td>
<td></td>
</tr>
<tr>
<td>and posting student work on the walls (posters, completed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>assignments)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>x    x    x    x</td>
</tr>
<tr>
<td>5. Providing opportunities to justify and explain</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Asking questions that require explanation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. Expecting students to provide justification for solutions</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>x    x    x    x</td>
<td>x    x</td>
</tr>
<tr>
<td>6. Modeling high-level reasoning</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Providing explanations for solutions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. Connecting new instruction to prior knowledge</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>x    x    x    x    x</td>
<td></td>
</tr>
<tr>
<td>7. Answering questions with questions</td>
<td></td>
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</tr>
</tbody>
</table>
Partially successful equitable classroom spaces. Based on the analysis of classroom observations, three academic lessons and four accelerated lessons seem to represent partial success in supporting students’ opportunities to learn. One more lesson taught in the accelerated courses was described as partially successful compared to the geometry courses. These lessons exhibited the three basic features of task implementations to support students’ opportunity to learn and provided students opportunities beyond that. The majority of these lessons provided students opportunities to share written work and justify and explain their thinking, one accelerated lesson did not offer students any opportunities to share their written work or explain and justify their thinking. This lesson provided students opportunities to observe high-level thinking and answer their peer’s questions. Table 29 shows how the lessons for the geometry and the accelerated courses compare. The equitable classroom spaces in the accelerated classes provided students the same number of opportunities to share written work, justify and explain their thinking, and observe models of high-level thinking. In the technical sense, students in the geometry classes received more opportunities to share written work, justify and explain their thinking, and observe models of high-level thinking because two-thirds of the geometry lessons offered these opportunities compared to one-half of the accelerated courses. The only feature observed during accelerated lessons that were not observed during geometry lessons was the opportunity for students to answer their peers’ questions.
Table 29
Features of Partially Successful Equitable Classroom Spaces by Course Type

<table>
<thead>
<tr>
<th>Task implementation Features</th>
<th>Accelerated (n=4)</th>
<th>Geometry (n=3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lessons 6 9 10 13</td>
<td>7 10 13</td>
</tr>
<tr>
<td>1. Using high cognitive demand tasks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Task that link to prior knowledge</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>b. Tasks that require justification</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>2. Giving clear expectations for task engagement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Providing students with resources</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Providing students with materials to engage with tasks</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>(i.e. calculators, patty paper)</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>b. Providing students with materials to engage in class</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>(i.e. paper, pencils, calculators)</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>4. Opportunities to share work publicly</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>a. Allowing students to present their work on the board and</td>
<td></td>
<td></td>
</tr>
<tr>
<td>posting student work on the walls (posters, completed assignments)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Providing opportunities to justify and explain</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Asking questions that require explanation</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>b. Expecting students to provide justification for solutions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6. Modeling high-level reasoning</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Providing explanations for solutions</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>b. Connecting new instruction to prior knowledge</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. Answering questions with questions</td>
<td></td>
<td>x</td>
</tr>
</tbody>
</table>
Consistently successful equitable classroom spaces. Based on the analysis of classroom observations, six academic classes and three accelerated classes can be characterized as consistently successful in supporting students’ opportunities to learn. These lessons exhibited the three basic features of task implementations to support students’ opportunity to learn and provided students opportunities to share written work and justify and explain their thinking, observe models of high-level thinking and answer their peers’ questions. Table 30 shows how the lessons for the geometry and accelerated courses compare. The equitable classroom spaces in the accelerated classes provided students similar opportunities to share written work, justify and explain their thinking, and observe models of high-level thinking. However, only the geometry lessons provided students opportunities to answer their peers’ questions.
Table 30

*Features of Consistently Successful Equitable Classroom Spaces by Course Type*

<table>
<thead>
<tr>
<th>Task Implementation Features</th>
<th>Lessons</th>
<th>Accelerated (n=3)</th>
<th>Geometry (n=6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>1. Using high cognitive demand tasks</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Task that link to prior knowledge</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>b. Tasks that require justification</td>
<td>x</td>
<td>x</td>
<td>x</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Giving clear expectations for task engagement</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Providing students with resources</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. Providing students with materials to engage with tasks (i.e.</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>calculators, patty paper)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. Providing students with materials to engage in class (i.e.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>paper, pencils, calculators)</td>
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<td></td>
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<td></td>
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</tr>
<tr>
<td>4. Opportunities to share work publicly</td>
<td></td>
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</tr>
<tr>
<td>a. Allowing students to present their work on the board and posting student work on the</td>
<td>x</td>
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<tr>
<td>walls (posters,</td>
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<tr>
<td>5. Providing opportunities to justify and explain</td>
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<tr>
<td>a. Asking questions that require explanation</td>
<td>x</td>
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<tr>
<td>b. Expecting students to provide justification for solutions</td>
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<tr>
<td>6. Modeling high-level reasoning</td>
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</tr>
<tr>
<td>a. Providing explanations for solutions</td>
<td>x</td>
<td>x</td>
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<tr>
<td>b. Connecting new instruction to prior knowledge</td>
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<td></td>
</tr>
<tr>
<td>7. Answering questions with questions</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Development of a Continuum Framework

In order to describe how task implementations are placed on a continuum of equitable classroom spaces, the conceptual framework used in this study must be revisited. Recall teachers’ task implementations attend to the how teachers support students’ engagement with demanding mathematical tasks and students’ public participation. Students’ opportunities to learn in this study are defined as opportunities to engage with high demand tasks using prior knowledge, connect procedural and conceptual knowledge, and justify and explain mathematical thinking. These opportunities are used to classify task implementations, as those implementations that provide students more opportunities to learn are considered more equitable than those that provide fewer opportunities. Note that the presence or absence of features of task implementation that support students’ opportunity to learn may be a consequence of the lessons observed. These classifications of task implementation represent different levels of equitable classroom spaces based on the collective presence of features that support students’ opportunities to learn. These classifications may appropriately describe students’ opportunities to learn during task implementation if the collection of features described embodies a teacher’s lessons over the course of an academic year.

Based on analysis of two cases containing 15 lessons each, the evidence suggests that a continuum of opportunities to learn can be used to characterize the ways lessons may provide equitable classroom spaces for students. On a continuum, lessons that have clear expectations and provide students opportunities to engage with high demand tasks, access to necessary materials, opportunities to share work and observe and produce high-level
reasoning are considered more equitable than those that provide fewer of these opportunities. Figure 17 shows the proposed continuum of equitable classroom spaces.

The combination of lesson features that represents teacher struggles with supporting students’ opportunities to learn is labeled emergent. The combination of lesson features that represents partial success with supporting students’ opportunities to learn is labeled moderate, while prominent is the label given to the combination of features that represent consistent success with supporting students’ opportunities to learn. The term emergent depicts the presence of basic features of equitable classroom spaces. These features; high cognitive demand tasks, clear expectation for engagement with tasks and in class, and access to necessary resources, represent the most ideal foundation for all the other features of higher level equitable classroom spaces. Moderate spaces contain the three basic features and include opportunities for students to share their work or opportunities for students to observe high-level thinking and serve as resources for their peers. Prominent refers to the presence of the basic features that support students’ opportunities to learn and opportunities for students to share their work, observe high-level thinking, and serve as resources for their peers.
Figure 17: Continuum of equitable classroom spaces

- **Emergent**
  - Inconsistency in the use of appropriate resources to engage with high demand mathematical tasks with clear expectations
  - OR
  - Only the use of appropriate resources to engage with high demand mathematical tasks with clear expectations

- **Moderate**
  - Use of appropriate resources to engage with high demand mathematical tasks with clear expectations
  - Consistent opportunities to justify and explain thinking and/or share written work
  - Inconsistent opportunities to observe models of high-level thinking
  - Inconsistent expectations to use personal resources to answer mathematical questions

- **Prominent**
  - Use of appropriate resources to engage with high demand mathematical tasks with clear expectations
  - Consistent opportunities to justify and explain thinking and/or share written work
  - Consistent opportunities to observe models of high-level thinking
  - Consistent expectations to use personal resources to answer mathematical questions
Emergent Equitable Classroom Spaces

Equitable classroom spaces are classified as emerging when students either do not have opportunities to engage with high cognitive demand tasks with clear expectations and the appropriate resources during a lesson or only have these opportunities and no more. Engaging with high cognitive demand task with clear expectations and having access to any necessary resources encompasses the basic needs of students as learners (Hiebert et al., 1997; NCTM, 2000; Stein et al., 1996). Therefore lessons that fail to meet or just meet these basic needs are considered emergent. Lessons labeled as emergent may have been coded for features beyond using high cognitive demand tasks, providing clear expectations of engagement with task and provide necessary materials to engage with the task, but those features were not used to place a lesson beyond this category. That is, the presence of student justification or the sharing or students’ work during emerging lessons did not result in a higher classification of the lesson. Students’ opportunities to learn are highly connected to the demands of the tasks that they are assigned in addition to their ability to fully engage with demanding tasks (Stein et al., 2000). Therefore, an inability to engage affects students’ opportunities to learn and results in a label of emerging. Note that emergent is still a description of an equitable classroom space. That is, students are in environments that have some features that support their opportunities to learn. These emergent equitable spaces provide students opportunities to engage with demanding tasks that encourage them to make connections between procedural and conceptual knowledge.
Moderate Equitable Classroom Spaces

Equitable classroom spaces are described as moderate if they provide students opportunities beyond those described at the emergent level. Ideally at this level, students are given opportunities to share written work and/or justify and explain their thinking when engaged with high cognitive demand tasks that have clear expectations using appropriate materials. Opportunities for students to share written work and justify and explain their mathematical thinking directly affects their opportunities to learn. These moderate equitable classroom spaces provide students opportunities to justify and explain their thinking, which may result in opportunities to display their prior knowledge when engaged with demanding tasks that encourage them to make connections between procedural and conceptual knowledge. Also, these spaces give students opportunities to engage in the mathematics community more completely because they are able to observe models of high level reasoning and able to be a resource for themselves and their peers.

Prominent Equitable Classroom Spaces

The highest description of equitable classroom spaces is prominent. Prominent equitable classroom spaces include all of the features described in moderate spaces but include students’ opportunities to observed teachers modeling high-level thinking and/or answer their peers’ questions. This is described as the most successful of the equitable classroom spaces because students are provided opportunities to justify and explain their mathematical thinking while engaging with demanding tasks and are able to observe the sociomathematical norms of the class. These norms include what constitutes high-level reasoning and how student might use their resources to answer mathematical questions. The
prominent equitable classroom spaces not only provide students opportunities to learn but also provide them opportunities to become more accomplished doers of mathematics (Yackel & Cobb, 1996).

**Portraits of Equitable Classroom Spaces**

This section reports more results from the task implementations through portraits of task implementation. These portraits allow the reader to experience the classroom as an observer. The researcher chose to represent these data in portraits because portraits provide realistic examples of task implementations at each level while protecting the anonymity of the study participants. Sharing classroom episodes of specific participants may encourage the description of particular teachers’ practices as inferior, which is not favorable because each teacher had at least one lesson can be described as an emergent, moderate, and prominent equitable classroom space. Because each participant taught a course in Geometry and the geometry lessons were placed each level on the continuum of equitable classroom spaces, each portrait will use the same geometry task. Three fictional teachers will teach the same task at different levels of implementation to show equitable classroom spaces in action. Each of the teachers will also have the same gender to prevent matching of implementation with study participants. A discussion of how each portrait exemplifies the characteristics of equitable spaces at each level of the continuum follow the three portraits.

**Introduction and Outline**

The following portraits are accounts of three teachers’ task implementation. Sample dialogue is included in each portrait to show what an exchange between students and a teacher may look like at each level of equitable classroom spaces. This sample dialogue is
based on the data collected from 17 task implementations, six pre-lesson interviews, and six post lesson interviews. Seventeen implementations are included instead of the entire 30 because 13 of the lessons observed at the level of emerging did not represent the basic three elements of equitable classroom spaces, and these elements are present in each portrait. Mr. Smith, Mr. Jones, and Mr. Brown were created as fictitious teachers, but the experiences described here are representative of all study participants. Additional characters are included in the story for the purpose of illustrating experiences of students in implementations that exemplify major points in the continuum. Elements of the portraits directly related to characteristics outlined in Figure 17 are denoted with an *. Each portrait will use the same task to reinforce the focus on students’ opportunities as a result of teacher instruction.

The Task. In its presentation, the task (Figure 18) used for each portrait is high demand (edHelper.com, 2013). Students are not given specific procedures to solve the proof and are not given specific pathways to find the solution. However, providing a specific format for expressing the solution does not necessarily influence the pathway. Each portrait provides a picture of task implementation at each level.
Emergent Equitable Classroom Space. Mr. Smith, an early career high school mathematics teacher, is preparing a lesson on triangle proofs. He has been teaching for three years and feels confident in his capabilities to teach this material to his students. Students have been working with simple triangle proof for a few days and are going to be engaging in practice of triangle proofs during these portraits.

In this lesson, the bellwork and homework review has been completed. Now Mr. Smith is ready to do a triangle proof example with his students.

Mr. Smith: Alright guys, take out a piece of paper because today we are going to do more proof practice. If you need paper you know where to find it*. Now I know that we have been working on proofs for a few days now. When we started we did algebraic proofs, and now we are on triangle proofs. I just want to make sure that you are all comfortable with proofs before we move one. So before we get started can anyone tell me how we set up this proof*?
Student A: Yes. First we make the chart and write down statement and reason. Then we number the chart, then we write the Givens under statements and Given as the reason.

Mr. Smith: Good so here is the proof that we are going to do (Figure 18). Let’s everybody set it up. [Mr. Smith walks around the room to see if students are on track.] Ok. Now that we all have it set up, who can tell me what our first step should be? Remember when they give us a picture we should use it.

Student B: Well after you write the givens in, you can mark the picture.

Mr. Smith: Great so what do I mark?

Student C: The right angles and the congruent sides.

Mr. Smith: Good, why?

Student D: Well because congruent is given and perpendicular means right triangles.

Mr. Smith: Now that we have that what can we do?

Student E: Put the angles up there.

[Mr. Smith writes $\angle ABC$ and $\angle ABD$ is a right angle in the two column proof chart]

Mr. Smith: What’s the reason? It’s definition of perpendicular lines like Student D just said. Now what?

Student B: We can say that we have right triangles, triangle $ABC$ and triangle $ABD$.

Mr. Smith: Good. We are using the definitions of right triangles. What next?

Student F: Well are they congruent triangles?

Mr. Smith: No we can’t say that yet because we don’t have enough evidence. So what can get us that evidence?
Student E: Segment AB is congruent to segment AB.

Mr. Smith: Right by the reflexive property and now we can say that the triangles are congruent. What is the reason that we use for right triangles?

Students in unison: HL

Mr. Smith: So are we done? What are we trying to prove?

Student G: We want segment BD congruent to segment BC.

Mr. Smith: And how do we say that parts of a triangle are congruent?

Students in unison: CPCTC [corresponding parts of congruent triangles are congruent]

Mr. Smith: Right. Corresponding parts of congruent triangles are congruent, and remember we need to check to make sure that the parts are corresponding. How do you tell? You would match up the picture. If triangle ABC is congruent to triangle ABD, then AB equals AB check, AC equals AD check, and BC equals BD check. Now we are sure. Now get with your small group and work on these next two problems. I will come around to see how you are doing.
Figure 19: Board Work for an emerging equitable classroom space

**Moderate Equitable Classroom Space.** Mr. Jones, an early career high school mathematics teacher, is preparing a lesson on triangle proofs. He has been teaching for three years and feels confident in his capabilities to teach this material to his students. Students have been working with simple triangle proof for a few days and are going to be engaging in practice of triangle proofs during these portraits.

In this lesson, the bellwork and homework review has been completed. Now Mr. Jones is ready to do a triangle proof example with his students.

Mr. Jones: Alright guys, take out a piece of paper because today we are going to do more proof practice. If you need paper you know where to find it*. Now I know that
we have been working on proofs for a few days now. When we started we did algebraic proofs, and now we are on triangle proofs. I just want to make sure that you are all comfortable with proofs before we move one. So before we get started can anyone tell me how we set up this proof?

Student: Yes. First we make the chart and write down statement and reason. Then we number the chart.

Mr. Jones: Good so here is the proof that we are going to do. I want everybody to try and come up with what they think it will look like. I’ll give you three minutes to get started. Talk to your neighbors if you get stuck. [Mr. Jones starts a timer and walks around the room to see if students are on track. As he walks, if students raise their hands, he answers their questions.] Ok. Now can I have someone come to the board and get us started? I want you to write down your first three statements and reasons and then we are going to talk about them. Don’t forget to use the picture.

Student A: [Figure 20 shows the student’s work]. The first thing I did is mark the right angles because AB and CD are perpendicular because it’s given. Then I named the right triangles because just saying B wouldn’t tell you which angles were right. Then I said since we have right angles in triangles that we have right triangles.
Mr. Jones: Ok. Did everyone follow that? Student A used the given and then recorded what she knew on the picture and in the table. Now is this what you had to start with? Could you have started another way? Yes, but let’s follow this line of thinking all the way out. Now, Student A, why did you want triangles?

Student A: Well because if I have triangles, I think that it will be easier to prove that side CB is congruent to side BD.

Mr. Jones: Do you see how Student A kept her end goal in mind? She wants to prove sides congruent but knows she needs congruent triangles before she can move on. Good. Ok now I want someone else to come up and do the next two steps.

Student B: (Figure 21) Ok. So I added the next given. Segment AC is congruent to segment AD. Then I said that segment AB was equal to itself-
Mr. Jones: Equal?

Student B: Congruent to itself and I marked the picture.

Mr. Jones: Now why is it congruent?

Student C: Reflexive.

Mr. Jones: Right, now why would this help us get to the statement we are trying to prove?

Student C: Well because if we have two sides and an angle, we have SSA, isn’t that one of them?
Mr. Jones: Well since we don’t curse in math class, we use the HL theorem. But why is that helpful?

Student D: Well because if we have congruent triangles, then we can use CPCTC to get to the prove statement.

Mr. Jones: Right: So add the final steps onto your paper. Then you can get started on your independent proofs. You can discuss with your peers but you have to turn in your own work.

**Prominent Equitable Classroom Space.** Mr. Brown, an early career high school mathematics teacher, is preparing a lesson on triangle proofs. He has been teaching for three years and feels confident in his capabilities to teach this material to his students. Students have been working with simple triangle proof for a few days and are going to be engaging in practice of triangle proofs during these portraits.

In this lesson, the bellwork and homework review has been completed. The bellwork and homework review has been completed. Now Mr. Brown is ready to do the triangle proof example with his students.

Mr. Brown: Alright guys, take out a piece of paper because today we are going to do more proof practice. If you need paper you know where to find it*. Now I know that we have been working on proofs for a few days now. When we started we did algebraic proofs, and now we are on triangle proofs. I just want to make sure that you are all comfortable with proofs before we move one. So before we get I want you to look at the givens and the picture and discuss in your groups what conclusions you can draw about the figure* [after a brief discussion, students share out].
Student A: Our group said that the perpendicular lines give us right angles*.

Student B: Ok so that means we have right triangles right?

Mr. Brown: Does that mean we have right triangles?

Student C: Yes because whenever we have right angles in a triangle we have right triangles. That’s just the rule. The definition.

Mr. Brown: Ok what else?

Student D: We have an isosceles triangle too. Segment AC is congruent to segment AD.

Mr. Brown: Ok. What else?

Student E: Since the triangle is isosceles, the base angles, C and D are the same measure, congruent.

Mr. Brown: Ok that is a good start. I want you to use the information that you have gathered and come up with a proof. You can work in groups or independently. When you are done, share it with your neighbor to get feedback. Remember ask three then me. You have flipbooks, notecards, notes, books, and your peers. You should consult all of those things before you raise your hand.

[Students have worked for approximately seven minutes and two different proofs have surfaced.]

Mr. Brown: Ok so I have been walking around looking at your work and I have seen some different proofs show up. I would like to see one that used a triangle congruency theorem or postulate and one that used isosceles triangle. Come on and put your proofs up on the board. Then we are going talk about them.
Figure 22: Group A’s Triangle congruency proof for persistent implementation

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\overline{AB} \perp \overline{CD}$, $\overline{AC} \cong \overline{AD}$</td>
<td>Given</td>
</tr>
<tr>
<td>$\overline{AC} \cong \overline{AD}$</td>
<td></td>
</tr>
<tr>
<td>2. $\angle ABC$ and $\angle ABD$ are right angles.</td>
<td>Definition of perpendicular lines</td>
</tr>
<tr>
<td>3. $\triangle ABC$ and $\triangle ABD$ are right triangles.</td>
<td>Definition of right triangles</td>
</tr>
<tr>
<td>4. $AB \cong AB$</td>
<td>Reflexive property</td>
</tr>
<tr>
<td>5. $\triangle ABC \cong \triangle ABD$</td>
<td>HL</td>
</tr>
<tr>
<td>6. $BC \cong BD$</td>
<td>CPCTC</td>
</tr>
</tbody>
</table>

Figure 23: Group B’s Triangle congruency Proof Persistent Implementation

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\overline{AB} \perp \overline{CD}$, $\overline{AC} \cong \overline{AD}$</td>
<td></td>
</tr>
<tr>
<td>$\overline{BC} \cong \overline{BD}$</td>
<td></td>
</tr>
<tr>
<td>2. $\triangle CAD$ is an isosceles triangle</td>
<td>Definition of isosceles triangle</td>
</tr>
<tr>
<td>3. $AB \perp CD$</td>
<td>Given</td>
</tr>
<tr>
<td>4. $AB$ is an altitude</td>
<td>Definition of an altitude</td>
</tr>
<tr>
<td>5. $BC \cong BD$</td>
<td>Altitudes in isosceles triangles are perpendicular bisectors of the base.</td>
</tr>
</tbody>
</table>
Mr. Brown: Are both of these proofs right? How do you know?

Student A: I agree with the first one. That is what I got. Once you use HL, you can go straight to CPCTC. I don’t know how they knew that AB was an altitude.

Mr. Brown: Would someone like to respond to that? How did they know AB was an altitude?

Student E: Hey look at the flipbook I think that one is in there. [Students consult their flipbooks].

Student F: Well, in the flipbooks it says that an isosceles triangle has an altitude that is a perpendicular bisector of the base. That means it bisects angle A, makes two right angles, angle ABC and angle ABD and it bisects CD at point B,

Student A: Yeah I know but how did you automatically know that AB was an altitude?

Student G: Well since we were given that AB was perpendicular to CD and we knew it was an isosceles, and then we used our knowledge of the properties of isosceles triangles to define AB as an altitude. Make sense?

Student A: yeah. I see it now.

Mr. Brown: So do they both work?

Students in unison: Yeah

Mr. Brown: Ok. Let’s wrap it up. The first proof used our knowledge of right triangles and treated the figure as two right triangles. We could have drawn the figure to express that. We would have had triangle ACB and triangle ADB. From there we
could think about the different ways that we learned to prove right triangles congruent. Remember HL is one that is specifically used for right triangles, but all of the other ones could have been applied to this proof as well, they may not have been the most efficient strategies though. For example, we could have used SSS to prove the triangles congruent but that would have required the use of HL and CPCTC, which would have results in us solving the proof. SAS could also have been used to complete this proof if we considered the isosceles altitude method that was presented today as well but again, this would have required more work than necessary. If we used the fact that segment AB was a perpendicular bisector, to prove that angles CAB and BAD we would also have proved that segment CB was congruent to segment BD. You see multiple strategies to solve proof problems are important but so is efficiency. The prework that we did at the beginning of class was to help us think about the ways that we could approach the problem to help us progress in an efficient manner. Any questions? [brief silence] No, then your assignment is to record the two proofs and write in words how each version gets you to the proof statement. I know that you all did this in a two-column format but this is going to get you closer to a paragraph proof. When you are done, you can begin on the proofs on this worksheet I am handing out.

Discussion

These three portraits of task implementations exhibit the features described in the three levels of equitable classroom spaces. The portrait of an emerging equitable space contains the three basic elements of equitable classroom spaces. First, the task that students
are engaging with is described as a high cognitive demand task. Geometric proof problems can allow students to use a general procedure that is closely connected to the underlying concepts. That is, students may be given a structure to format their proofs but cannot blindly use that format. Students must think about and apply mathematical definitions, rules, and concepts to make sense of geometric proof problems. Students are also provided the opportunity to show evidence of their reasoning in their work. Next, students in this portrait are clear about expectations for engagement with the task and during the lesson. There are specific prompts for students to indicate that they should work on the task. The teacher set up the task by telling students how it related to their previous work and that they would be engaged in practice of triangle proofs during this lesson. Students are also confident about the structure of the proofs that they were expected to complete as evidenced by their description of the structure of proofs. Finally, students are reminded of the availability of any necessary materials for participation, such as paper and pencils.

This portrait does not exemplify a moderate or prominent equitable classroom space because during engagement with the task, the norm for implementation appears to be that students provide specific solutions to the questions asked by the teacher. Students answered the teacher’s questions and did not offer justification for their solutions or ask conceptual questions of their own. Further, the teacher wrote student responses on the board, which prohibited students from sharing their written work, and he provided the reasons for the proof statements.

The portrait of moderate equitable classroom space also includes the three basic elements of equitable classroom spaces: a high cognitive demand task with clear expectations
for engagement and access to necessary materials. However students in this portrait are also allowed to share their written work with the class and justify and explain their mathematical thinking. Students hear the reasoning of their peers and the teacher pushes them to justify their choices. In this portrait, students are the originators of the knowledge and the teacher facilitates the knowledge exchange.

The final portrait shows prominent equitable classroom space also includes the three basic elements of equitable classroom spaces: a high cognitive demand task with clear expectations for engagement and access to necessary materials. It also provided opportunities for students to share their written work and explanations and justification for their mathematical thinking. Above these features of the implementations, the prominent equitable classroom space provides students opportunities to observe models of high level thinking and reasoning and to depend on their peers and other resources to answer their own questions. When the lesson began, the teacher asked students to work in groups to decide before they begin working what the possible conclusions could be drawn based on the givens and the figure that was provided. This is an important practice because having students consider fruitful pathways for solving a problem before fleshing out full solutions is necessary for students to make sense of and persevere through problems (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010b). Students were also given the opportunity to compare two viable solution strategies for completing the proof, which also supports perseverance. After using these strategies, the teacher explains the benefits of the pre-work and explains to students how other solutions strategies may have been less fruitful. This exchange helped students to see how the current
work related to earlier lessons and may have cemented for students the ideas provided by their peers during the discussion. Finally, students are also allowed to depend on themselves and their peers to answer questions that arose during the lesson.

The use of these three portraits shows that the same task can be presented to students and provide very different opportunities. Taken together each group of students experienced opportunities to learn that results in equitable classroom spaces, but the student in the moderate and prominent portraits were able to engage in the class as mathematical authorities and engaged with the material in a deeper way. The differences in the opportunities afforded the students in the three portraits have been described in the work of Stein et al. (2000). Their work differentiates between the task as written, the task as setup, and the task as implemented. They posit that mathematical instructional tasks are tasks that “are situated squarely in the interactions of teaching and learning” (Stein et al., 2000, p. 25) and state that teachers’ actions in support of students’ engagement with tasks can either expand or limit students’ opportunities to engage mathematical thinking and reasoning.

Likewise, the differences between the portraits are in the norms for communication during discussion and in the role of the teacher during these discussions. These portraits show that minor shifts in instructional practice can provide students more opportunities to learn. The difference in opportunities to learn for students between the emerging and moderate portraits may seem vast, in that students greatly benefit from opportunities to justify and explain their thinking and sharing their work, but small changes to the norms of classroom discussion can help teachers provide students these opportunities. Likewise, the difference between the opportunities to learn provided during the moderate and the
prominent portraits are vast in that students become more aware of how they are to engage with mathematical tasks, but again changes in how students are involved in lessons can provide these opportunities. These portraits are not presented to suggest that all lessons follow these specific instructional strategies (e.g. peer learning); they only serve to show one way that these opportunities can be provided. Other ways to provide these opportunities were described in previous sections.

**Chapter Summary**

This chapter addressed how teachers’ task implementations support equitable classroom spaces and how these spaces may differ in academic and accelerated courses. The primary sources of data were video and or audio recording of classroom instruction and field notes. The secondary sources of data for analysis were teachers’ post lesson interviews. Teachers’ implementations of mathematical tasks support equitable classroom spaces by using high cognitive demand tasks; expressing clear expectations for engagement with high cognitive demand tasks; providing necessary resources for engagement with tasks and in class; and by fostering classroom norms that support the sharing of students’ work and mathematical ideas, the modeling of high level thinking and self reliance for information. The more integral each of these features is during instruction in a classroom, the more evidence there is that a teacher’s instructional practices may be aimed at reducing barriers for student participation. The next and final chapter will discuss the findings as they relate to the overarching research question: *How do early career high school mathematics teachers’ classroom practices support students’ opportunities to learn?* make connections to the
literature, note the limitations of this study; and discuss implications and areas for future research.
CHAPTER 6: DISCUSSION AND CONCLUSIONS

Introduction

The purpose of this study was to examine how early career high school mathematics teachers’ classroom practices around task choice and implementation support students’ opportunities to learn. The participants of this study are three early career high school mathematics teachers teaching in a large high-need school district. Each participant was observed teaching two courses: one academic Geometry course and one accelerated course-either AP Statistics or Honors Algebra II. The overarching question that guided this study was “How do early career high school mathematics teachers’ classroom practices support students’ opportunities to learn?” Based on this question, two research questions were developed.

1. How do the tasks that ECMTs choose support students’ opportunities to engage with high cognitive demand tasks in class, outside of class, and on assessments?
   a. How do the task choices of an ECMT differ when teaching academic versus accelerated courses?

2. How do ECMTs implementations of mathematical tasks support equitable classroom spaces?
   a. How do the task implementations of academic and accelerated courses differ?

These two research questions were addressed through separate case studies. Six cases, representing the individual courses taught by each teacher, were examined for the first research question. Collections of tasks were analyzed for each case using the IQA rubric for the Potential of the Task (see Figure 8). The conceptual framework served to guide the
classification of teachers' task choices by how the collections of tasks used in Class A or Class B supported students' opportunities to learn. The opportunity to learn continuum based on the potential of tasks used in a course (see Figure 16) emerged as a way to characterize collections of tasks based on three features: cognitive demand of tasks, the alignment of tasks with learning goals, and the similarity of the cognitive demand of homework, classwork, and assessment tasks.

There were two cases for the second research question of this study. The cases consisted of the two types of courses: academic and accelerated. Each case consisted of five academic lessons (Geometry) and five accelerated lessons (AP Statistics or Honors Algebra II) for a total of 15 lessons in each case. The overarching conceptual framework of a mathematics teaching cycle with opportunities to learn (see Figure 6) served to guide the analysis. Again, a characterization of teachers’ task implementations based on seven features of classroom instruction was used to create a continuum for equitable classroom spaces (see Figure 17).

In this chapter, the findings with respect to two research questions will be discussed and linked to existing research. In addition, this chapter discusses limitations and implication for teacher education and the early career support for practicing teachers. Finally, possible areas for future research to extend this work will be discussed.

Summary of Research Questions and Findings

The overarching question that guided this study was “How do early career high school mathematics teachers’ classroom practices support students’ opportunities to learn?”
To answer this question, the task choices and task implementations of three early career mathematics teachers were analyzed.

Research Question 1

Analysis and findings for the first research question were discussed in Chapter 4. Research Question 1: How do the tasks that ECMTs choose support students’ opportunities to engage with high cognitive demand tasks in class, outside of class, and on assessments?

To describe how ECMTs’ task choices support students’ opportunities to engage with high cognitive demand tasks on homework, classwork, and assessment tasks, a continuum of opportunity to learn based on task choice emerged (see Figure 16). This continuum describes features of collections of tasks (i.e. homework, classwork, and assessment tasks) that support students’ learning opportunities: (1) percentage of high cognitive demand tasks within homework, classwork, and assessment task collections, (2) comparable distributions of high cognitive demand tasks across coursework types, and (3) learning goals that describe both procedural and conceptual learning. Three task collections represented successes in supporting students’ opportunities to learn while three represented struggles. The successes, labeled as moderately persistent and persistent on the continuum, were characterized by higher percentages of high cognitive demand tasks across all tasks and learning goals that attend to both procedural and conceptual knowledge building and align with tasks. The struggles, limited and moderate, were characterized by higher percentages of low cognitive demand tasks across all tasks and learning goals that were procedural in nature, or the goals were conceptual but did not align with the task collections.
Based on the analysis of task choices from three early career mathematics teachers, there is evidence that relatively novice teachers are able to support students’ opportunities to learn, but that struggles still occur.

*Research Question 1a: How do the task choices of an ECMT differ when teaching academic versus accelerated courses?*

Each teacher’s learning goals were aimed at students’ understanding of particular content but some learning goals were aimed at both procedural and conceptual understanding while others were aimed more at procedural understanding. The participants in this study had significant differences in the cognitive demand of tasks that were used in their academic and accelerated classes. However, each teacher’s task choices did not follow the same trend. Two of the teachers’ accelerated classes had task collections that were described as moderately persistent or persistent and academic classes that were described as moderate. One teacher had an accelerated class that was described as limited and an academic class that was described as persistent. Thus, these teachers are all capable of choosing tasks that provide opportunities to learn on at least a moderately persistent basis, but these opportunities are not consistently applied in the different levels of courses they teach. However, the more successful opportunities to learn are not necessarily only appearing in accelerated courses.

*Research Question 2*

Analysis and findings for the second research question were discussed in Chapter 5.

*Research Question 2: How do ECMTs’ implementations of mathematical tasks support equitable classroom spaces?*
To describe how ECMTs’ implementations of mathematical tasks support equitable classroom spaces, 30 lessons (15 academic Geometry and 15 accelerated) were analyzed and a continuum of opportunity to learn based on implementations was developed (Figure 24).
Figure 24: Continuum of equitable classroom spaces
This continuum (Figure 24) describes features of task implementations that support students’ opportunities to learn. The features are (1) use of high cognitive demand tasks, (2) clear expectations for engagement with tasks and in class, (3) access to appropriate resources; and opportunities to (4) share written work publicly, (5) justify and explain thinking, (6) observe models of high level reasoning, (7) and answer their peers’ questions. Taken together, these features characterize equitable classroom spaces.

Three characterizations—emergent, moderate, and prominent—were developed to describe how the features work together in task implementation to support equitable classroom spaces and students’ opportunities to learn. Emergent equitable classroom spaces either do not offer engagement with high demand tasks with clear expectations and access to appropriate resources, or that is all that is offered. Eight accelerated and six academic Geometry lessons were classified as emergent. Moderate equitable spaces offer the features of emerging equitable spaces and also provide students opportunities to share both written work and explanations and justification of mathematical thinking. Four accelerated and three academic Geometry lessons were classified as moderate. Finally, lessons that included all of the features of moderate equitable classroom spaces and provided students opportunities to observe teachers model high level reasoning and answer their peers’ questions were characterized as prominent equitable spaces. Three accelerated and six academic Geometry lessons were classified as prominent equitable classroom spaces.

Based on the analysis of task implementation from three early career mathematics teachers, there is evidence that relatively novice teachers are able to support equitable classroom spaces by the instructional practices that they employ. However, the use of
Instructional practices that support equitable classroom spaces can be uneven within the same course and across courses taught by the same teacher.

**Research Question 2a: How do the task implementations of academic and accelerated courses differ?**

Among the lessons that were characterized at each level there were differences. At the level of emergent equitable classroom spaces, the academic Geometry classes were more likely to use low cognitive classroom tasks (5 out of 6 lessons) than the accelerated classes (3 out of eight lessons). However, the academic geometry lessons were more likely to have clear expectations for engagement during tasks and in class (4 out of 6 lessons) compared to the accelerated classes (2 out of 8 lessons). At the level of moderate equitable classroom spaces, the only difference between accelerated and academic task implementations was the opportunity for students to answer their peers’ questions. Only one accelerated lesson provided students this opportunity while no academic Geometry lessons provided students this opportunity. Otherwise, there were similarities on the frequency of the other opportunities to learn. In moderate equitable spaces, there were similar opportunities for students to share written work publicly, justify and explain thinking, and observe the teacher model high level thinking; 2 out of 4 accelerated lessons and 2 out of 3 academic Geometry lessons. The highest level, prominent equitable classroom spaces had fewer differences in implementation. Most features were present in each of the implementations, however; only the academic geometry lessons provided students opportunities to answer their peers’ questions.
Thus, task implementations in both academic and accelerated courses are capable of supporting prominent equitable classroom spaces. At each level of equitable classroom spaces (emergent, moderate, and prominent), the combination of features that support students’ opportunities to learn differ by course, but there are learning opportunities at each level. The more successful opportunities to learn are not necessarily only appearing in accelerated courses.

Overarching Question: How do early career high school mathematics teachers’ classroom practices support students’ opportunities to learn?

The first major contribution of this study is the continuum of students’ opportunities to learn based on task collections. It aims to connect the literature on students’ OTL to practical descriptions of teachers’ classroom practice and describe how early career mathematics teachers use tasks to support students’ OTL. This use of OTL expands on previous use because it aims to describe the quality of students’ instructional experiences based on teachers’ classroom practices instead of the quantity of such experiences (Tate, 1995, 2001). OTL has been described in literature as “the circumstances that allow student to engage in and spend time on academic tasks such as working on problems, exploring situations and gathering data, listening to explanations, reading texts or conjecturing and justifying” (National Research Council, 2001b). Equity also shares an associated interpretation because it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students (Alder, 2001; Gutiérrez, 2007; NCTM, 2000). Considering these conceptions of opportunities to learn and equity, this study
aimed to link practical instructional practices with these instructional ideals to remove some of the vagueness around teaching for equity.

The continuum of students’ OTL based on collections of tasks is a framework for analyzing the tasks that students are presented in classroom settings. This continuum expands on the work of Smith, Stein, and Boston (Boston, 2012c; Boston & Smith, 2009; Boston & Wolf, 2006; M. S. Smith & Stein, 1998; Stein & Smith, 1998). The literature on choosing mathematical tasks is clear, in that high level mathematical tasks should provide reasoning opportunities for students and require students to justify and explain their thinking and engage with mathematical concepts above the level of memorization and use of prescribed procedures. Mathematical tasks that meet these requirements are ranked high demand. Boston extended this thinking and ranked tasks at a high level that had the potential to encourage or support complex thinking even if there were no explicit prompts for student explanation.

This study is in accordance with the literature on mathematical tasks and ranked tasks as high-level that encouraged complex thinking and explicitly prompted for explanation, justification, or interpretation of solutions. In addition to the task ranking, this study also ranked the individual items that comprised each task. The ranking of individual items showed contrasting trends in a few cases when compared to the ranks of the tasks. Though the number of items within a task that ranked high or low demand had no bearing on the rank that the task received, this does raise an interesting point. Tasks that require students to complete a high number of low demand items before being asked to make connections to the mathematical concepts may not always have the desired effect. Students may still believe
that mathematics is mostly characterized by using prescribed procedures if the majority of
their work is completed at that level (Hiebert et al., 1997).

Also, the continuum for opportunities to learn based on tasks includes teachers’
learning goals in the task selection process. Stein and Smith (1998) describe the task analysis
guide as a tool for helping teachers reflect on the ways that tasks support student reasoning.
Boston (2012a) describes the IQA Potential of the Task Rubric as a tool for assessing the
level of cognitive demand of a task. Neither of these tools for assessment of tasks considers
the learning goals that teachers have set for students. Learning goals influence task choices
because learning goals express teachers’ short term and long-term goals for student learning
(Hiebert et al., 1997; Hiebert et al., 2007). Hiebert and colleagues suggest that task choice
should be a result of teachers’ goals for student learning and teachers’ idea of how those
goals can be met (Hiebert et al., 1997).

The final piece of this continuum that is based on literature but also adds to it is the
idea of alignment in the cognitive demand of tasks that students engage with for homework,
classwork, and assessment tasks. Students’ academic activities both inside and outside of the
classroom affects students’ OTL, and engaging with tasks that are assigned for homework
influences students’ intellectual resources (Trautwein, 2007). Also, the formal assessments
that students engage with should reflect the opportunities that students experience on tasks
presented in the classroom and on homework (Husen, 1967; NCTM, 2000). Considering
tasks across different coursework (in class, homework, assessment) as collections can
support coherence across teachers’ tasks and can support students’ OTL.
The second major contribution of this study is the continuum of equitable classroom spaces based on task implementations. It aims to connect the literature on students’ OTL to practical descriptions of teachers’ classroom practice and describe how early career mathematics teachers implement tasks to support students’ OTL. This continuum adds to the literature that describes effective classroom practices and the literature that described classroom practices that support students’ OTL (Ladson-Billings, 1995a; Reynolds, 1992). This study supports the findings that engaging with tasks with clear expectations and appropriate resources supports students’ OTL and that these features of implementation that support students’ OTL also describe effective teaching. However, this continuum considers these the basic needs of equitable classroom spaces. Considering Gutiérrez’s (2012) features of equity access to physical resources support the use of measurable outcomes of student learning, inclusion of students’ identities in curricula, and experiences of social transformation as doers of mathematics. In this way access to the physical resources is necessary to support the types of interactions that are necessary to engage all students.

The next two features of the equitable spaces continuum describe students’ opportunities to participate publicly in the classroom. Public participation in this study is defined as students having opportunities to share written work and their justification and explanations of mathematical concepts and solution strategies. Opportunities for students to share their thinking in both written and verbal formats have been described in literature as an effective teaching practice (e.g. Stein et al., 2008). Using students’ work to support discussions of student learning can help students become a more integral part of the classroom community (Stein et al., 2008; Yackel & Cobb, 1996) and support the use of
student work to advance learning goals (Stein et al., 2008). Consideration of tasks linked to
students’ opportunities to share work is a part of the practices described by Stein and
colleagues (2008) for orchestrating productive mathematical discussions and does attend to
the opportunities for students to experience authority in their classes, however, the focus of
that framework is not equitable classroom practices.

The final features of task implementations described are more closely related to the
classroom norms and sociomathematical norms that should characterize equitable classroom
spaces. Yackel and Cobb (1996) describe sociomathematical norms as “what counts as
mathematically different, mathematically sophisticated, mathematically efficient, and
mathematically elegant… and what counts as an acceptable mathematical explanation and
justification” (Yackel & Cobb, 1996, p. 461). Students’ opportunities to observe teachers’
models of high-level thinking support their understanding of sociomathematical norms.
Including the development of sociomathematical norms in the continuum of mathematical
task implementations supports teachers’ considerations of how students can engage in the
mathematical classroom community.

The final feature of an equitable classroom space is a classroom norm that affects the
ways that students interact with the teacher and each other. Classroom norms that encourage
students to depend on themselves and their peers for information support the teacher in
assisting students’ learning without telling them everything (Reinhart, 2000). Reinforcing
classroom norms that require students to ask questions of their peers is a way to explore the
knowledge of their peers and supports students’ opportunities to learn and their becoming a
part of a classroom community as well.
Overall, these continuums show how teachers’ classroom practices can be considered collectively as a way to support students’ learning opportunities. Literature describes each of these practices in a disjoint manner and has not previously connected these particular features of instruction as a way to support students’ OTL. This joint presentation is a major contribution of this study.

Limitations

While this study was designed to reduce limitations, they are unavoidable in any research study. The first limitation was that a novice researcher conducted the study. This presented as a limitation because of the amount of data that had to be managed and analyzed. The other limitations of this study were related to participants, time frame, and data collection. Though each of these represented limitations, the study was designed to mitigate the effect of some of these factors.

The participants in this study were the result of a convenience sample. Six early career high school mathematics teachers were invited to participate in this study and only three agreed. Though the participation of three teachers resulted in less data than expected, the use of rich, thick descriptions of these three teachers’ practice supported external reliability and the triangulation of different data sources supported internal validity (Creswell, 2007; Merriam, 2002).

Time frame was also a limitation of this study. The local education agency put constraints on the time frame for data collection. Data collection had to be completed by April 30 in order not to interfere with exam preparations. Further, because teachers’ practice changes over times, this study did not account for long-term change. However, the
researcher did observe classroom instruction at the beginning of and closer to the end of the semester to account for some of the change in teachers’ practice that may occur over a course with a group of students.

Data collection presented the third and largest limitation. Because the study was conducted in teachers’ classrooms, the researcher needed permission to video and audio-record teachers’ instruction. Only one participant permitted the video recording of instruction while the other two participants only permitted audio recording. To mitigate this limitation, the researcher collected field notes using a pen that recorded sound and linked field notes to the audio recording. Also, the researcher positioned multiple audio recorders around the classroom, in order to ensure that all data was collected and the teacher was given a blue-tooth microphone that recorded their discourse. In the classroom where video recording was permitted, the camera was stable and aimed at the board to reduce the probability that students’ faces would be captured. This also presented as a limitation but again, the teacher had the Bluetooth microphone so even though the teacher was outside of the camera’s view, classroom discourse was still recorded.

**Implications**

**Implications for Teacher education**

This study has shed light on features of task choice and task implementations that can support students’ OTL. Teacher education programs need to invest a great amount of time in developing teachers that are aware of and sensitive to issues of equity (Strutchens et al., 2011) to complement their current focus on the identification and selection of high cognitive demand tasks (Osana, Lacroix, Tucker, & Desrosiers, 2006; Zelkowksi, 2009). The
continuums proposed as a result of this study describe task choice and task implementations as a way to support students’ OTL. Thus, they may be used to support beginning teachers’ conceptions of instruction and their awareness of how instruction can influence students’ learning opportunities.

Another implication for teacher education is related to the ways in which some research described high leverage practices (TeachingWorks, 2012). Similar to the issues discussed with regards to the non-prescriptive nature of theories of equitable instruction, non-descriptive teaching practices can also prove too vague to influence teachers’ practices. This study aims to describe specific teacher practices and provides fictional portraits of task implementations to help prospective teachers think about the feasibility of using these practices in their classrooms. Also, connecting learning goals with the cognitive demand of tasks and linking homework, classwork, and assessment tasks in a framework can support more cohesive thinking around all of the mathematical tasks that are used in classrooms.

Implications for Early Career Support for Practicing Teachers

In addition to identifying features of collections of tasks and task implementation that support students’ OTL, this study also describes the teaching practice of three early career high school mathematics teachers in academic and accelerated courses and provides portraits of task implementation at different levels of equitable classroom spaces. These descriptions of beginning teachers’ task choices can be used to support teachers’ pedagogical development. Research suggests that descriptions of teacher practice in the form of cases or portraits (Henningsen & Stein, 1997; Stein et al., 2000) used in a professional development setting can support teachers in being reflective about their practice and using research-based
frameworks as a tool for their reflection (Sowder, 2007). These descriptions of teacher practice can also be used to help provide a practical, concrete aspect to teachers’ conceptions of equity (Bartell & Meyer, 2008).

The continuums are also useful tools in supporting beginning teachers’ classroom practice. Mentor teachers that observe early teachers’ lessons can use these continuums to provide feedback on how a teacher supports students’ OTL based on common teaching practices (Jensen, 2013). Use of these continuums as an observation tool could support teacher’s holistic reflection on their practice. These tools should not be used to tally the number of times that a specific instructional practice is used because teaching practice is not just a combination of repetitive behaviors but a professional practice (Simon et al., 2000).

**Areas for Future Research**

“Lurking behind the framing of any study is the question of what is valued by the investigators, and what is privileged in the inquiry” (Schoenfeld, 2007, p. 70). The overarching question that guided this study was “How do early career high school mathematics teachers’ classroom practices support students’ opportunities to learn?” This question foregrounds two constructs: classroom practices and students’ opportunities to learn. These constructs are most important because the work of researchers is to support the improvement of student learning (Sowder, 2007). Based on this assertion, the first area for future research involves students’ input. Using this study design and adding student interviews or surveys where students can discuss what features of the classroom support their learning may provide data that can be used to help teachers learn about the features of equitable classroom spaces that are most important or most readily described by students.
This type of study may also allow disparate experiences of students within the same class to surface and provide the teacher and researchers with viable research avenues in differences within classrooms as well.

Another viable avenue for future research was suggested by Franke et al. (2007). It is the investigation of the connection of three features of classroom practice: discourse, norms, and building relationships. This study describes discourse and classroom norms from the perspective of teachers’ intent (learning goals) and actions (task choice and task implementation). Learning about how teachers build relationships with their students and how these relationships influence students’ perceptions of equitable classroom spaces would also be fruitful research.

The second area for future research is aimed at the description of classroom practice. Franke et al. (2007) suggest that frameworks, such as the Orchestrating Productive Mathematical Discourse framework (Stein et al., 2008), may need to look different when it is applied at different levels of mathematics (accelerated versus academic) and in different mathematics courses (Algebra I versus Discrete Mathematics). To respond to this notion of necessary differences, a complementary study using the continuums constructed by this study can be used to evaluate the task choices and classroom practices of a high school mathematics department over the course of a semester to see whether the findings differ by course. An extension of this work would also be to apply these continuums to a middle school mathematics department to see if similar findings emerge.

Finally, reform practice has been used as a unifying concept to assess the teaching of science and mathematics (Sawada et al., 2002): equity has the potential to be another.
Because teaching equitably is a focus of science teachers and science teacher educators, these continuums can be applied in science classroom settings. This could be a fruitful area of future research because though the descriptions of task choices and task implementations developed are based on mathematics teacher’s practices, application in any setting where students are presented with tasks and explanation and justification of concepts are valued may serve to describe equitable classroom spaces.
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APPENDICES
### Appendix A: Overview of Data Collection Plan

Table A1

Mapping of research questions to data collected and data analysis tools

<table>
<thead>
<tr>
<th>Research Question</th>
<th>Data Used</th>
<th>Data Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>How do early career high school mathematics teachers classroom practices support students’ opportunities to learn?</td>
<td>Homework Tasks, Classwork Tasks, Assessments, Field notes</td>
<td>IQA Academic Rigor: Mathematics Rubric for Potential of the Task (Boston, 2012c)</td>
</tr>
<tr>
<td>How do the mathematical tasks that ECMTs choose support students’ opportunities to engage with high cognitive demand tasks in class, outside of class, and on assessments?</td>
<td>Audio/Video from each Teaching Set, Pre-lesson/Post-lesson Interviews, Field notes, Analysis from Research Question 1</td>
<td>Adaptation of factors that maintain cognitive demand (Henningsen &amp; Stein, 1997)</td>
</tr>
<tr>
<td>How do ECMTs’ implementations of mathematical tasks support equitable classroom spaces?</td>
<td></td>
<td>IQA Academic Rigor Mathematics Rubric for Task implementations and Mathematics Lesson Checklist (Boston, 2012c)</td>
</tr>
<tr>
<td>How do the implementations of mathematical tasks differ in academic and accelerated courses?</td>
<td></td>
<td>Open Coding</td>
</tr>
</tbody>
</table>
Appendix B: Interview Protocols for Pre/Post Lessons Conference

Pre-lesson Interview Protocol
Say: Thanks for letting me observe your classes. I have some questions related to your upcoming lessons. I would like to tape this interview. Is that alright? It will allow me to focus on our conversation and not worry about missing your responses.

1. How confident are you in teaching the content in these lessons? Do you have any concerns?
2. Can you briefly describe how this topic relates to the math that you have been doing so far?
3. How does this lesson build on what your students already know?
4. Consider the difficulties you may be anticipating students will have with the mathematics in the lessons. What have you planned to try to address these difficulties?
5. What are your goals/objectives for these lessons? What do you want your students to learn?
6. How will you know that your students have met the goals/objectives for these lessons?
7. What do you plan on doing during class and what instructional materials will you be using? [Probe: Are students familiar with these materials? Can you briefly describe the progression of this class?]
8. Were there any resources that you used to develop the lessons? What were they?
9. Why did you choose the task/problems that you are going to use during the lesson? Is this new material or a new representation?
10. Is there anything in particular I should know about the students I will be observing? [Probe: mathematical ability, ELL, special needs, behavioral/emotional issues, absenteeism, parent support] What supports are you providing for these learners?

Post-Lesson Protocol
Say: Now that I have observed your lesson, I would like to get your reaction to it. I would like to tape this interview. Is that alright? It will allow me to focus on our conversation and not worry about missing your responses.

1. How did you feel about teaching [insert class here] topic?
2. Did you meet your lesson’s goal/objective in this class?
3. How well prepared were you to guide student learning of this content?
4. In hindsight, are there any content related changes that you wish you had made when teaching this topic?
5. What do you think your students learned today?
   a. How do you know?
6. What evidence do you have that these lessons built in what your students already knew?
7. How did the mathematical task/problems you planned help you reach your lesson’s goal?
8. What features of the task were most helpful in helping you to meet your lesson goal?
9. Was the lesson you taught today different from the lesson that you planned to teach? If so, what prompted the changes to the lesson plan?
10. Did any student’s questions surprise you or make you change your lesson plan?
11. Have you taught this lesson before? If yes, how different was today from previous lessons?
12. Can you talk about how your students reacted to the lesson? Were most students engaged? Why or why not?

Assessment Protocol

Questions to be used if these lesson topics were assessed on a quiz or test:
1. How did your students perform on the items that assessed [insert observed topics here]?
2. Are there any topics that I observed you teach that were not assessed? Why or why not?
3. Did their performance on this item(s) surprise you?
4. Based on these assessment scores, are there any changes that you intend to make when you teach this material again?

Extension Questions

Questions to be used after all Teaching Sets are completed.

1. How do you generally engage your students during lessons?
   a. For instance, do you choose particular problems or activities thinking about how students will engage with them, or is there another focus?
2. What would you say encompasses students’ opportunities to learn?
3. How do you provide those opportunities to your students?
4. What do you use to inform your decisions on how you group students?
   a. Are there different methods or reasons for grouping students?
   b. Does the activity that you plan to use affect how you group them?
Appendix C: Consent Forms

North Carolina State University

INFORMED CONSENT FORM for RESEARCH

Title: Examining Early Career High School Mathematics Teachers’ Planning and Implementation of Lessons
Principle Investigators: Karen Hollebrands and Hollylynne Lee

What are some general things you should know about research studies?
You are being asked to take part in a research study. Your participation in this study is voluntary. You have the right to be a part of this study, to choose not to participate or to stop participating at any time without penalty. The purpose of research studies is to gain a better understanding of a certain topic or issue. You are not guaranteed any personal benefits from being in a study. Research studies also may pose risks to those that participate. In this consent form you will find specific details about the research in which you are being asked to participate. If you do not understand something in this form it is your right to ask the researcher for clarification or more information. A copy of this consent form will be provided to you. If at any time you have questions about your participation, do not hesitate to contact the researcher named above.

What is the purpose of this study?
The goals of this study are to better understand the instructional strategies and technologies used by mathematics teachers in their early career years (student teaching through year 5).

What will happen if you take part in the study?
If you agree to participate in this study, you will be asked to participate in several activities:
- Initial Interview. 60-75 minutes. The purpose of this interview is to discuss your courses and overall planning strategies.
- Pre-lesson interviews. They should take no more than 30 minutes to complete and will be conducted at a mutually agreed upon location.
- About 10 field observations of you implementing several lessons throughout the semester. We will collect artifacts such as any written plans, files and handouts used in the lessons. These field observations may be videotaped, pending approval from the Local Education Agency (LEA). Videorecording of field observations will occur from the back of the room and every effort will be made to not capture children’s faces.
- Post-lesson interviews. They should take no more than 45 minutes to complete and will be conducted at a mutually agreed upon location. These interviews will primarily be used to focus or clarify points of interest in the observation.
- Final Interview. 60-75 minutes. The purpose of this interview is to follow-up on observations made throughout the entire semester.

Risks
There are no physical or emotional risks associated with participation in this study.

Benefits
The teachers that participate in this study will potentially gain a better understanding about using instructional strategies and various technologies for learning mathematics. Each participant will have the opportunity to participate in a feedback session with a Noyce staff member to discuss teaching strategies for their particular classroom context. In addition, this study is important because it may inform the field about the professional development of early career mathematics teachers and help teacher educators design experiences that can foster understanding of instructional strategies and use of technology.
Confidentiality
The information in the study records will be kept confidential to the full extent allowed by law. Data will be stored securely by the principle investigator. Pseudonyms will be used in oral or written reports to avoid linking you to the study. You will NOT be asked to write your name on any study materials so that no one can match your identity to the answers/materials that you provide.

Compensation
Upon completion of the study, your school will be offered a $50 gift certificate to the National Council of Teachers of Mathematics. If you withdraw from the study prior to its completion, your data will be destroyed and your school will not receive any compensation.

What if you are a NCSU student?
Participation in this study is not a course requirement and your participation or lack thereof, will not affect your class standing or grades at NC State.

What if you have questions about this study?
If you have questions at any time about the study or the procedures, you may contact Hollylynne Lee at Poe 502 Campus Box 7801 NCSU Raleigh, NC 27695, or (919) 513-3544.

What if you have questions about your rights as a research participant?
If you feel you have not been treated according to the descriptions in this form, or your rights as a participant in research have been violated during the course of this project, you may contact Deb Paxton, Regulatory Compliance Administrator, Box 7514, NCSU Campus (919/515-4514).

Consent To Participate
“I have read and understand the above information. I have received a copy of this form. I agree to participate in this study with the understanding that I may choose not to participate or to stop participating at any time without penalty or loss of benefits to which I am otherwise entitled.”

Please check the appropriate box
☐ I agree to participate in all aspects of this study including audio and video recording of interviews and field observations.
☐ I agree to participate in this study including the audio and video recording of interviews, but no video recording of field observations
☐ I agree to participate in this study including the audio recording of interviews and field observations. However, the interviews and field observations may not be video recorded.

Participant’s name ___________________________ Date ____________
Participant’s signature_________________________________________________________ Date ____________
Investigator’s signature__________________________________________________________ Date ____________
Investigator's signature___________________________________________________________ Date ____________
Appendix D: Student Permission forms

Permission to participate in video-recording
Dear Parents/Guardians of __________ High School Students:

Your student is currently in a mathematics course that is being taught by [teacher]. S/he is participating in a research study. This study focused on teachers' decision making during instruction. Classroom observations will be digitally video-recorded to capture what the teacher is saying and doing. The video camera will be stationary and will be positioned to capture the teacher and the board or the screen most used for instruction. The camera will be placed to reduce the probability of capturing students' faces. Classroom artifacts such as blank worksheets may also be collected. No student names or student work will be collected. These videos will be uploaded to a private, password-protected server. The videos will be used as part of the study analysis. Once the study is completed, the videos will be destroyed.

Consent To Participate
“I have read and understand the above information.”

Students and Parents/ Guardians,
Please check the appropriate box below and then sign.

☐ I understand that [teacher] is the focus of this research study. I agree to be video-recorded during class instruction.

☐ I understand that [teacher] is the focus of this research study. I do not agree to be video-recorded during class instruction.

Students who do not agree to be recorded will be positioned outside of the camera’s view on observation days.

If you have questions or concerns, please email me at hollylynne@ncsu.edu or call at 919-513-3544

__________________________________ Hollylynne Stohl Lee, Ph.D.

Print Student Name: ____________________________
Sign Student Name ____________________________

Print Parent/Guardian Name: __________________
Sign Parent/Guardian Name: __________________
Permission to participate in audio-recording

Dear Parents/Guardians of _________ High School Students:

Your student is currently in a mathematics course that is being taught by [teacher]. S/he is participating in a research study. This study focused on teachers’ decision making during instruction. Classroom observations will be audio-recorded to capture what the teacher is saying. Classroom artifacts such as blank worksheets may also be collected. No student names or student work will be collected. These audio files will be uploaded to a private, password-protected server. The audio files will be used as part of the study analysis. Once the study is completed, the audio files will be destroyed.

Consent To Participate
“ I have read and understand the above information.”

Students and Parents/ Guardians,
Please check the appropriate box below and then sign.

☐ I understand that [teacher] is the focus of this research study. I agree to have my voice recorded during class instruction.

☐ I understand that [teacher] is the focus of this research study. I do not agree to have my voice recorded during class instruction.

Students who do not agree to be recorded will be positioned as far away from the recording devices as possible on observation days.

If you have questions or concerns, please email me at hollylynne@ncsu.edu or call at 919-513-3544

______________________________________ Hollylynne Stohl Lee, Ph.D.

Print Student Name:_____________________

Sign Student Name ________________________

Print Parent/Guardian Name:________________________

Sign Parent/Guardian Name: _____________________
Appendix E: Sample Mathematical tasks with task and item ranks

Figure E1: Sample geometry assignment 1
Arccs and Angles in a Circle

1. \( \overparen{WX} \)
2. \( \angle XOY \)
3. \( \overparen{YY} \)
4. \( \angle XV \)
5. \( \angle WZY \)

6. Find the \( m \angle MRN \)
7. Find the \( m \angle PG \).
8. Find the \( m \angle JKL \).

9. Find the \( m \angle KIL \).
10. Find the value of \( x \).
11. Find the value of \( x \).

12. Find the value of \( x \).
13. Find the value of \( x \).
14. Find the value of \( x \).

Figure E2: Sample geometry assignment 2
Table E1

Ranks of Sample Mathematics Assignments 1 and 2

<table>
<thead>
<tr>
<th>Task</th>
<th>Task Rank</th>
<th>Item</th>
<th>Item Rank</th>
<th>Justification</th>
</tr>
</thead>
<tbody>
<tr>
<td>HW1</td>
<td>3</td>
<td>17</td>
<td>3</td>
<td>Justification</td>
</tr>
<tr>
<td></td>
<td></td>
<td>18</td>
<td>3</td>
<td>The item has the potential to engage students in creating meaning for the concept of parallelograms. Students may need to identify patterns but are not pressed to form or justify generalizations.</td>
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<td>1</td>
<td>The potential of the task is limited to engaging students in memorizing or reproducing facts, rules, formulae, or definitions about parallelograms.</td>
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<tr>
<td>HW2</td>
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<td>3</td>
<td>The item has the potential to engage students in creating meaning for the concept of parallelograms and linking these concepts to algebraic procedures, but the task does not explicitly prompt for evidence of students’ reasoning and understanding.</td>
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<td>The item potential is limited to engaging students in using a procedure(s) for finding arc measures in circles that is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. There is little ambiguity about what needs to be done and how to do it.</td>
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<td>HW4</td>
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<td>2</td>
<td>The item potential is limited to engaging students in using a procedure(s) for finding angle measures in circles that is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. There is little ambiguity about what needs to be done and how to do it.</td>
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