ABSTRACT

ASHOURI, MORTEZA. Modeling Microdamage Healing in Asphalt Pavements Using Continuum Damage Theory. (Under the direction of Dr. Y. Richard Kim).

Healing of microcracks has been proved to play an important role in the behavior of asphalt concrete mixtures especially by causing the strength and damage recovery in the material and increasing the fatigue life of pavements. This dissertation presents a method to quantify the amount of healing in asphalt mixtures using Simplified Viscoelastic Continuum Damage (S-VECD) theory and to utilize this model to predict the damage characteristic curve of the material under realistic loading of pavements. The ability of Simplified Viscoelastic Continuum Damage (S-VECD) theory to predict the fatigue behavior of asphalt concrete mixtures under various loading conditions has been demonstrated over the past decade.

Although, pavements are under a pulse-rest type of loading history which happens from axle to axle of passing vehicles, conducting pulse-rest healing tests in laboratory is a very time consuming task due to the inclusion of rest periods. In this study, group-rest tests (interrupted fatigue tests) that are much faster than the pulse-rest tests are utilized to calibrate the healing model. Group-rest tests were conducted at three different temperatures of 10°, 20° and 30°C, and four rest periods of 10, 30, 90 and 270 seconds. The rest periods were applied at different damage levels to investigate the effects of temperature, rest period, and damage state on healing potential of the material. Then it is shown that the effects of temperature and rest period can be combined in a joint parameter named reduced rest period. Using the results of group-rest healing tests and time-temperature superposition principle, a
method is proposed to generate healing mastercurves for different damage levels that relates the amount of healing in the material to the test temperature and rest period through the joint parameter, reduced rest period.

The final proposed healing model predicts the damage characteristic curve of pulse-rest healing test from the damage characteristic curve of continuous fatigue test by using the healing function obtained from the group-rest healing tests. Since damage characteristic curve is one of the most important material characteristics to describe the fatigue behavior and plays an important role in the response of the material under loading, the proposed method would be a great step in the prediction of asphalt concrete behavior under realistic loading conditions with rest periods without having to conduct the time consuming pulse-rest healing tests in the lab.
Modeling Microdamage Healing in Asphalt Pavements Using Continuum Damage Theory

by
Morteza Ashouri

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APPROVED BY:

Y. Richard Kim
Chair of Advisory Committee

Murthy N. Guddati

Christopher P. Bobko

Min Liu
DEDICATION

To my family, without their support, encouragement and most important love none of this would be possible.
BIOGRAPHY

Morteza Ashouri was born on March 1, 1983 in Ghazvin, Iran, a beautiful city 150 kilometer west side of the capital city of Iran, Tehran. He completed his elementary, middle and high school studies in his hometown. In August 2001, he was admitted to Sharif University of Technology (SUT) the most prestigious engineering university in Iran to earn his B.Sc. and M.Sc. in Civil Engineering. In August of 2008 he then moved to North Carolina State University to pursue his Ph.D. degree in Civil Engineering.
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CHAPTER 1. INTRODUCTION AND MOTIVATION

1.1. Introduction

One of the main distresses in asphalt concrete pavements is fatigue cracking. This type of distress happens in the pavements due to repeated loading of traffic in the roads. Small cracks (i.e., microcracks) start shortly after initiation of traffic loading. These micro-cracks densify and coalesce after many cycles to form larger cracks named macrocracks. It is inevitable to have a deep and thorough understanding of asphalt concrete constitutive behavior under realistic traffic loading conditions in order to better understand the processes of crack initiation and crack propagation under traffic loading.

Modeling of asphalt concrete constitutive behavior is not an easy task to carry. A true constitutive relationship has to reflect viscoelasticity of the matrix, fatigue damage growth and healing that happens due to the rest periods between the applications of loading cycles. It is a well-known fact that healing occurs in asphalt concrete mixtures and binders as different researchers have studied this phenomena in their researches through experimental tests. Due to the healing process, existing microcracks in the body of a pavement which caused from the previous cyclic loading can be cured and thus the mixture can regain its strength partially or completely.

Microdamage healing in an asphalt mixture is a function of several factors such as temperature, rest period duration, the physical and chemical properties of the binder used in the mixture, properties of the mixture itself, amount of damage preceding the rest period,
number of rest period applications, aging and pressure. There are also different expressions and definitions for healing such as reversal of micro-damage or micro-crack, increase in stiffness and integrity of the material, and increase in the fatigue life.

Researches by Kim and co-workers at North Carolina State University (NCSU) have shown that the simplified viscoelastic continuum damage (S-VECD) theory can be adopted for the characterization of fatigue damage growth in asphalt concrete materials independent of mode of loading. The S-VECD theory is based on work potential theory (Schapery 1990). In the S-VECD theory the overall integrity of mixture is represented by pseudo stiffness (C) of the material and the overall damage within the specimen is represented by an internal state variable (ISV), S. The relationship between C and S, the so-called damage characteristic curve, is found to be unique for a given mixture regardless of loading rate, stress/strain amplitude, and temperature. Since S-VECD can be used to characterize fatigue damage in asphalt concrete mixtures and healing is defined as partial or complete reversal of induced damage due to previous loading, S-VECD can then be adopted to model the microcrack healing. Intuitively, S value decreases and pseudo stiffness of material increases, as a rest period is applied to the material.

Research is needed to investigate how the S-VECD theory would apply for loading histories with rest periods and to develop a healing model that accounts for the effects of temperature, rest period, and damage state on the healing potential of a mixture. In addition, the characterization test protocol must be efficient enough for state highway agencies to
adopt the healing model in their specifications and design methods. The research presented in this dissertation is designed to answer these important questions.

1.2. Objectives

The primary objectives of the research are:

i. to develop a procedure to quantify the healing potential of asphalt concrete mixtures based on the reduction of damage parameter $S$ or possibly stiffness recovery during the rest period by using the S-VECD theory;

ii. to present a functional form which relates the healing potential of a mixture to rest period, temperature, and damage level;

iii. to bridge between pulse-rest and group-rest healing tests and prediction of damage characteristic curve $C(S)$ of realistic pulse-rest healing tests by using the results of group-rest tests which can be conducted in a much shorter time; and

iv. to study the effect of SBS modified binder on healing properties of asphalt mixtures.
CHAPTER 2.  BACKGROUND AND LITERATURE REVIEW

2.1. Healing: Definition and Its Significance?

One of the major difficulties in fatigue performance modeling of asphalt concrete is how to predict the fatigue performance of asphalt pavements under complex traffic loading from laboratory tests using simple loading histories. Previous studies have indicated that a shift factor is needed to correlate laboratory fatigue test results to field fatigue performance. For example, Prowell et al. (2010) reported from the NCHRP Project 9-38 that the shift factors determined from four test sections in the 2003 NCAT Test Track study range from 4.2 to 75.8. Harvey et al. (1997) also showed that the shift factor relating the fatigue life of laboratory tests to the fatigue life of real in-service pavements may range from 10 to 100. One of the major factors that cause the difference between fatigue performance of asphalt concrete measured from laboratory testing and field fatigue performance of asphalt pavements under traffic loading is rest period, which in turn causes healing in asphalt concrete. Many literatures including Prowell et al. (2010) and Harvey et al. (1997) suggest the significant role that rest periods have in the performance of asphalt concrete in actual pavements and the urgent need to investigate and model the healing phenomenon.

In the literatures related to asphalt concrete, healing is defined as a process in which the partial or total recovery of damage is induced in the material (increase in stiffness and strength). Although healing phenomena and crack closure has been a hot topic of investigation by different researchers during the last fifty years, an accurate method for the quantification and modeling of healing has not been generated yet and its effect on fatigue
life of asphalt mixtures is not clear. Some researchers think healing result in the recovery of modulus to some extent only temporarily while others think that this modulus recovery leads to an increase in the fatigue life of pavements too. Researches on healing effects in asphalt mixtures were conducted by many researchers through variety of test methods on different scales of mixtures from binder and mastic levels to fine aggregate mixture (FAM), mixture and slab scales in laboratories and field that will be presented in Section 2.4 of this chapter.

Most of the researches on this subject were focused on factors affecting the amount of healing in the mixtures such as temperature, rest period, loading mode, specimen damage level before healing period starts, effect of polymer modified binder on healing properties of the material, aging, properties of the binder used in the mix, etc. Primarily it was assumed that healing happens between load applications but nowadays it is well-known that healing may occur continuously as damage develops during the fatigue test and not only during the rest period applications between loading cycles. What is obvious about healing and most of the researchers agreed is that the phenomenon exists in asphaltic mixtures and it has a dramatic effect on the performance and fatigue life of asphalt concrete pavements which urges researchers to dig more into understanding the phenomenon.

It is important to have an insight about realistic pavement loading in the field to know what kind of stresses and strains the actual pavements feel in order to better simulate the field conditions in the testing program. The pavement loading in highways that comes from the tire pressure and weight of passing vehicles is neither controlled-strain nor controlled-stress and it is actually a combination of both types of loading. Also, loading cycles are the combined effect of both the axial and shear stresses. Of course, it is impossible to conduct a
testing sequence that exactly simulates the pavement condition in the field but by knowing the details of realistic loading on actual pavements it is desired to conduct a test which is as close to the field condition as it could be.

### 2.2. Healing Phases and Mechanisms

As soon as the external load is removed during the test time, two processes happen; viscoelastic recovery and microcracks healing. The difference between these two processes comes from the sources which drive them meaning that the viscoelastic recovery happens due to the rearrangement of molecules in the material while crack healing is a result of wetting (crack closure) and interdiffusion between the crack faces in order to reach to the original material properties such as stiffness and strength.

Wetting is directly dependent to the surface free energy of the binder used in the mixture which means that the higher the surface free energy the higher the wetting capability. Philips (1998) proposed a three step process in the healing of asphalt binders as follows:

1. Wetting (crack closure) of the two faces of the crack (adhesion of crack faces due to the surface free energy)
2. Diffusion of molecules between crack faces (binder flow)
3. Complete recovery of strength and other mechanical properties due to the randomization of the diffused molecules

It is believed that the first step is the fastest among others and causes the stiffness recovery only while the other two steps are slower and result in both stiffness and strength

2.3. Important Factors in Determining Healing Potential

A list of factors that affect the healing potential of asphalt mixtures and binders are as follows:

- Temperature: One of the most important factors. Increasing the temperature makes it easier for the binder to flow and increases the healing potential of the mixtures.
- Rest period: The time that the specimen is allowed to heal is another effective factor. The longer the rest period the more the mechanical properties recover to the undamaged state.
- Damage level: It is harder for the material to recover its original undamaged mechanical properties when the damage level is higher. In other words, when the damage level is low and the number of microcracks is limited it is possible for the material to get rid of a few microcracks.
- Physical and chemical properties of binder: The asphalt binder which has been used in the mixture is probably the most important component. The amount (percent) of the binder in the mix, its viscosity, aging, surface free energy and other properties of the binder would directly affect the healing properties of the mixture.
- Aging: It is believed that the more the mixture aged the less it can be healed from the damaged state. In other words, aging reduces the capability of the mixture to recover to the original undamaged state.
2.4. Previous Researches

In this section a quick review of the previous works done by other researchers during the last fifty years on healing subject will be presented. By conducting repeated flexure tests on beam specimens on elastic foundation Monismith et al. (1961) tried to investigate the effect of rest periods. The loading time was 1 second and three different rest period lengths of 1, 3 and 19 seconds were used in the study at a temperature of 77°F at 3, 15 and 30 cycles per minute of frequency. The research showed that increasing the rest period duration had nothing to do with fatigue life extension while later studies on the subject by other researchers revealed that applying rest periods causes fatigue life extension.

An example of this is a research done by Deacon (1965) who found out from the results of his tests that a reduction in rest period time has a significant effect on fatigue life of specimens.

Bazin and Saunier (1967) applied long rest periods on damaged specimens and noticed an increase in fatigue life of the samples. They used a frequency of 50 Hz at a temperature of 10°C in a dynamic two-point bending apparatus all the way to the failure of tested samples. Applying rest periods of several hours to 100 days, the same test was conducted on the specimens and a new fatigue life was reached which was as twice as the previous fatigue life without rest period applications.

To evaluate the effect of healing Raithby and Sterling (1970) applied varying ranges of storage periods in axial loading test on a square-section beam (75 mm × 75 mm × 225 mm). Sinusoidal tension compression load controlled mode test was conducted in the research at temperatures of 10°C and 25°C and frequencies of 2.5 Hz and 25 Hz. Since the
tests were done in load controlled mode, the load level was kept to zero during each rest period time. The researchers compared the fatigue performance (fatigue life) of specimens subjected to continuous fatigue test with the one from the healing tests with intermittent load cycles. By using the test results, they found out that applying storage periods of 40 ms to 800 ms resulted in an increase of fatigue life of specimens by five times on average. They also suggested that there could be an asymptote for this increase in fatigue life by increasing the rest time meaning that there might be an optimum rest period length beyond which no significant further increase in fatigue life would happen.

Van Dijk and Visser (1977) conducted healing tests in a three-point bending apparatus at a temperature of 20°C and frequency of 40 Hz in controlled strain mode. The results of their study showed that increasing rest periods would increase fatigue life which was consistent with previous results of other researchers. Likewise Raithby and Sterling (1970) suggested that there is a limit on increase of fatigue life by increasing healing time.

Bonnaure et al. (1982) also used pulse-rest healing test to investigate the fatigue life change due to rest period. They used three-point bending apparatus and beams with 230 mm × 30 mm × 20 mm dimensions. The domain of their testing conditions was much wider with respect to the previous researchers because they did the tests in both controlled stress and controlled strain modes, at three different temperatures of 5°C, 20°C, and 25°C, a single frequency of 40 Hz and various rest period lengths of 0, 3, 5, 10, and 25 times the length of the loading cycle. The main conclusion of their research was similar to previous studies meaning that they concluded that applying rest time increases the fatigue life of specimens.
According to their research, an optimum rest period length to have beneficial effect on fatigue life is equal to 25 times the loading cycle.

Lee and Kim (1998) proposed a viscoelastic continuum damage (VECD) based constitutive model for asphalt concrete that can be used for modeling of microdamage healing as well as rate dependent damage growth. Using work potential theory (Schapery 1990) a continuum damage model for elastic materials was presented and then by taking advantage of elastic-viscoelastic correspondence principle they extended the model to a viscoelastic continuum damage model. In order to represent the induced damage in a specimen under cyclic fatigue test they introduced an internal state variable (ISV) which can be calculated from a rate type damage evolution law. Using controlled-strain tensile uniaxial cyclic tests with different strain amplitudes they were able to derive their VECD model parameters. Then for the verification purposes, they successfully showed that their model was able to predict the damage growth and recovery in asphalt concrete in other types of loading.

Daniel et al. (2001) used third-point bending beam fatigue machine and impact resonance method together in order to evaluate the effect of mixtures made with different asphalt cements and healing temperature on microcrack healing. The increase in the modulus of specimen after healing period is only due to microcrack healing in the material and time-dependent relaxation does not have any contribution in the modulus recovery after rest period because impact resonance method is considered to measure the very short time (low temperature) response of the material. The test setup used in this research is shown in Figure 1.
Figure 1. Three-point bending beam with impact resonance setup (Daniel et al. 2001)

To quantify the healing potential of asphalt concrete mixture Daniel used some parameters as follows:

1) A damage indicator developed by Kim et al. (1998) which is defined as the ratio of number of cycles a specimen has endured before applying a rest period to the fatigue life for that specimen. Instead of using this definition for the damage level at the moment of rest period application, pseudo stiffness definition and a more rigorous internal state variable (ISV) from viscoelastic continuum damage (VECD) theory was used to indicate level of damage in the specimen at every moment during the test.

2) The increase in fatigue life of the specimen by applying healing period (as percentage of number of cycles to failure).

3) Horizontal shift in dynamic modulus vs. cycle curve meaning that how many cycles it would take for the material to get back to the same modulus value as it had before the introduction of rest period. In other words, how many cycles would be added to the fatigue life of the specimen by applying the rest period?
4) She expressed healing of the material as a percentage of the recovery in the modulus after a rest period with respect to the modulus loss of the specimen due to previous repetitive cycles.

Daniel realized that the phenomenon by which a rest period extends the fatigue life of a specimen is that it shifts the fatigue curve (modulus vs. number of cycles) to the right as can be observed from Figure 2.

Zhang et al. (2001) used Superpave Indirect Tensile Test (IDT) in their studies to inspect the effect of healing. First a repeated load (haversine load with frequency of 10 Hz followed by a 0.9 second rest period) was applied at 10°C temperature and then a 12-hour
rest at 30°C temperature was introduced after specific number of loading cycles. In this way, the testing sequence was a combination of both group-rest and pulse-rest healing tests. They presented a threshold concept based on dissipated creep strain energy limit and proved that healing exists during rest periods. They assumed that when the dissipated creep strain energy limit exceeds the proposed threshold, a macro-crack is formed which cannot be healed.

Grant (2001) in his thesis performed Superpave Indirect Tensile Test (IDT) on two superpave mixtures with fine blend and coarse blend at temperatures of 10°C and 15°C using hydraulic loading machine. Tests were conducted in the sequence of 0.1 second haversine loads followed by 0.9 second rest periods and normalized resilient modulus was used to measure damage accumulation in the specimen during the test. Using the model developed in University of Florida healing rate was determined in terms of the recovered dissipated creep strain energy. The results indicated that healing rate was much more at 15°C and coarse mixture with respect to 10°C and fine mixture, respectively. Another finding was that recovered strains are not a good measure of healing.

Si et al. (2002) considered pseudo-stiffness to be an appropriate parameter to calculate healing percentage due to rest periods. They used controlled strain repeated cyclic uniaxial tensile fatigue test with Superpave shear tester on cylindrical specimens of 100 mm diameter and 150 mm height to evaluate the effect of healing. Three LVDTs were attached to the specimens by Devcon 5-minute epoxy resin glue 120 degree apart from each other to measure the deformations. Four different rest periods of 2, 5, 10, and 30 minutes were applied at 1000 cycle intervals to investigate the effect of healing of micro-cracks. They
defined a healing index and used the jump in pseudo stiffness after each rest period directly to calculate the healing index and quantify the amount of healing. The research showed that there is a healing effect in pseudo stiffness due to the rest periods which causes a fatigue life extension. Observations also proved that more healing occurs by applying longer rest periods and so the extension in fatigue life would be more.

Kim et al. (2003) used dynamic mechanical analysis (DMA) controlled-strain testing on two different sand asphalt mixtures. Controlled-strain torsional fatigue tests were performed at three different strain levels and at a temperature of 25°C with a loading frequency of 10 Hz. Change in three different damage indicators was used in this study in order to monitor the damage and healing in the specimens during the test: (a) pseudo stiffness; (b) dynamic modulus; and (c) dissipated strain energy and they were proven to be effective. As can be observed from Figure 3 another conclusion of this research was that rest periods lengthen the fatigue life of asphalt mixtures.
Castro and Sanchez (2006) carried out three-point flexural fatigue tests on beam specimens with dimensions of 300 mm × 50 mm × 50 mm which had an average air void of 3.7%. They used controlled-displacement sinusoidal loading pattern at a frequency of 10 Hz and temperature of 20°C and defined the failure point as the point at which the final force was equal to half of the initial force. Two different tests were conducted on the specimens; Continuous fatigue tests without rest periods and pulse-rest healing tests. Pulse-rest tests were cyclic tests with each cycle consisted of a sinusoidal loading wave with 0.1 second duration which was followed by a rest period of 1 second until specimen failure. They concluded that the rest periods and so the healing of the material resulted in an increase in mixture fatigue life by five to ten times. In order to run the tests and to avoid the testing
duration of healing tests to get very long, they also tried to find an optimum rest period
duration beyond which no significant increase in fatigue life would have happened. For this
purpose, previous research done by Raithby and Sterling (1972) showed that for rest periods
greater than ten times the loading time there would not be significant increase in fatigue life
of mixtures.

Little and Bhasin (2007) tried to explain the mechanism that causes healing in asphalt
mixtures and to quantify the healing amount in the mixtures. They suggested that healing is
the cumulative effect of two important steps; wetting of the crack faces and interdiffusion of
molecules from one face of a crack to the other. To quantify the amount of healing the
following healing index equation was used:

\[ H = \frac{R_A^R - R_B^R}{R_A^R} \]  \hspace{1cm} (1)

where \( R_A^R \text{ and } R_B^R \) are the pseudo-strain energy after and before a rest period, respectively.

One of the conclusions from the study was that the chemical and physical properties of
asphalt binders used in the mixture play an important role in the wetting and bonding of
 crack surfaces and so healing of the material. The authors believed that healing effect is
substantial in fatigue life increase of pavements so in order to incorporate healing effect in
the prediction of fatigue life of real pavements an empirical equation was suggested which
relates the measured fatigue life of mixture in the lab to the one in the field by using a shift
factor multiplier.
Seo and Kim (2008) successfully used acoustic emission (AE) method in order to monitor damage accumulation and healing in cyclic fatigue tests of asphalt concrete mixtures. Cylindrical specimens of 150 mm height and 75 mm diameter were made from a mixture with nominal maximum aggregate size (NMAS) of 19 mm and uniaxial tensile fatigue tests with and without rest periods were performed at 20°C temperature. The results of the research showed that acoustic emission parameters such as AE energy and AE counts which were obtained from two sensors attached to the middle of specimens may be used for identifying both the fatigue damage growth and quantification of healing effect in asphalt concrete mixtures.

Lee et al. (2010) conducted fatigue tests with rest periods on asphalt concrete slabs by using third scale model mobile loading simulator (MMLS3) and evaluated the fatigue damage growth and healing effect using nondestructive evaluation (NDE) techniques. The two NDE techniques used in this study were stress wave velocity test using impact resonance and ultra-sonic pulse velocity test. Both of these techniques indicated a reduction in phase velocities due to the wheel applications and an increase (recovery) in phase velocities because of healing effect during rest periods as can be observed from Figure 4. This figure confirms the healing effect on asphalt concrete slab as it shows a recovery in wave velocity (and so in the stiffness) of asphalt layer due to the rest period applications. They concluded that most of the healing happens at the first 30 minutes of rest period. It was also recommended by the researchers of this study to investigate the temperature dependence of the healing potential in asphalt concrete mixtures.
Liu et al. (2011) investigated the effect of induction heating in healing of asphalt mastic beams and found that it is possible to use induction heating in order to cure asphalt mastic beams many times even after they have been broken under fatigue testing. It was also recorded in their research that induction heating causes an increase in fatigue life as well as healing rate of asphalt mixtures. Asphalt mastic beams with dimensions of 125 mm × 25 mm × 15 mm were used in this study. The test procedure is explained in Figure 5. First the specimen underwent three-point beam bending test at a temperature of -20°C to get a brittle failure and the specimen was broken into two pieces. Then the two pieces of the broken beam specimen were put in a mold with their fractured surfaces facing together and induction heating was applied for duration of 2 minutes up to a temperature of 120°C. Three-point beam bending test was then conducted on healed specimen again to observe the strength

Figure 4. Stress wave phase velocity reduction and recovery due to MMLS3 loadings and healing effect (Lee et al. 2010)
recovery due to induction heating and the test procedure was followed until the two healed surfaces could not carry loads anymore. Authors believed that induction heating caused the flow of bitumen in asphalt mastic between the two cracked faces and so healing and strength recovery in the specimen achieved.

A group of researchers at Delft University worked on healing subject at different levels from binder and mastic to fine aggregate mixture (FAM) and mixture level and tried to investigate and quantify healing phenomenon in asphalt concrete materials. Qiu et al. (2008) in their first attempt investigated the healing potential in asphalt binder specimens. They conducted three different self-healing test procedures on three types of specimens:
All of these three tests were designed based on the ability of asphalt bitumen to flow between two separate surfaces of a binder sample and heal an existing crack. The crack was introduced in the body of the asphalt binder samples by using a knife (tests 1 and 3) or originally two samples of binder were faced and pushed together to join with each other due to the binder flow between their surfaces and then the formed specimen was tested (test 2). They proposed three equations to measure and quantify the cohesive healing in asphalt binder samples using the test results as follows:

(1) Ductility self-healing test:

\[
P_{\text{Ductility}} \% = \frac{L_{\text{Healed}}}{L_{\text{Original}}} \times 100\%
\]

where \( L_{\text{Healed}} \) is the length when the healed sample breaks; and \( L_{\text{Original}} \) is the length when the original sample (not healed) breaks.

(2) DSR self-healing test:

\[
P_{\text{DSR}} \% = \frac{M_{\text{Healed}}}{M_{\text{Original}}} \times 100\%
\]

where \( M_{\text{Healed}} \) is the measured stiffness (modulus) of the healed samples; and \( M_{\text{Original}} \) is the measured stiffness (modulus) of the original samples (not healed).

(3) Direct tensile self-healing test (DTT):
\[ P_{\text{DTT}} \% = \frac{S_{\text{Healed}}}{S_{\text{Original}}} \times 100\% \]  

where \( S_{\text{Healed}} \) is the measured maximum strength of the healed samples; and \( S_{\text{Original}} \) is the measured maximum strength of the original samples (not healed).

They concluded that the time needed for the recovery of samples strength (from DTT testing) was much longer than the healing time needed for their stiffness recovery (from DSR testing).

Same group of researchers at Delft University, Qiu et al. (2010), developed a test method to monitor self-healing capability of asphalt mastics. The test procedure they followed (DTT testing with healing and reloading) and the specimen shape is shown in Figure 6. They programmed the DTT machine to stop at predefined target elongations (TE) in the tested specimen in order to apply healing procedure and be able to investigate self-healing capability of the specimen at different crack phases.
They concluded that at small crack phases the SBS polymer modified asphalt mastic has better strength recovery with respect to the standard bituminous mastic but in large crack phases the strength healing capability of standard bituminous mastic is more. The results of this study also indicated that elastic recovery is more when the crack is small or non-visible but viscous recovery (due to the flow of binder) is much more important for healing of large cracks.

Following these researches Qiu et al. (2011) developed another test method to investigate healing in asphalt concrete mixtures. They used beam on elastic foundation
(BOEF) setup with beam dimensions of 400 mm × 70 mm × 35 mm. The test procedure consisted of loading-healing-reloading parts like their previous works. They used strength recovery and residual crack opening displacement (COD) recovery that happened during rest period in order to quantify the healing phenomena in asphalt mixes. The group continued their research on healing subject at material levels of binder, FAM and mixture with variety of test setups and specimen shapes till today.

Bhasin and his co-workers have been another group who put too much effort in investigation, quantification and modeling the healing behavior in asphalt concrete specimens from binder level to mixture. Bhasin et al. (2011) used dynamic shear rheometer (DSR) test method to investigate the effect of aging and temperature on healing of asphalt binders. As was expected they concluded that the more the specimen ages the more the healing potential of the material decreases and healing capability and rate is more at higher temperatures. In another research, Bhasin et al. (2011) used simulation of molecular dynamics to investigate healing process in asphalt binders. Through molecular modeling they were able to show that self-diffusivity of asphalt molecules across the crack interface which is one of the mechanisms of healing is very dependent on the chain length and degree of chain branching of the binder molecules.

In a research study that was conducted by Mamlouk et al. (2012) hot mix asphalt (HMA) fatigue and healing properties was under investigation using a four point bending beam fatigue test. Tests were conducted at deflection-controlled mode in both haversine and sinusoidal loading patterns with a loading frequency of 10 Hz, two strain levels of 400 and 800 microstrains and at three test temperatures of 4°C, 21°C and 38°C. A mixture with nominal
maximum aggregate size (NMAS) of 19 mm and a PG 64-22 binder was used to fabricate beam specimens. Pulse-rest healing tests with 5 and 10 second rest periods were performed on beam specimens as well as continuous four point bending beam fatigue tests. They concluded that sinusoidal load pattern is more consistent and accurate for testing because maintaining the haversine shape for stresses and strains during a test is not possible due to the permanent deformations which is a result of the viscoelastic nature of asphalt mixtures. The neutral axis of the beam under test would change after a few cycles of test beginning because of the permanent deformations. Comparing the stiffness vs. number of cycle curves proved that the curves of tests with rest period applications fall on top of those from continuous tests without rest periods.

Palvadi et al. (2012) presented a new method for healing quantification in asphalt concrete materials. They used work potential or viscoelastic continuum damage (VECD) theory which was previously developed and shown to be successful for modeling fatigue damage growth and healing processes by Lee and Kim (1998). Cylindrical FAM specimens of 20 mm diameter and 50 mm height were cored from larger superpave gyratory compactor specimens with 100 mm diameter and 75 mm height of four different mixes. The FAM specimens were subjected to controlled-stress sinusoidal cyclic torsional loading tests with rest intervals of 5, 10, 20 and 40 minutes at temperature of 25°C and frequency of 10 Hz using dynamic shear rheometer (DSR) test machine. Rest intervals were applied at predefined damage levels of 0.8, 0.7 and 0.6 of initial pseudo stiffness of tested specimen which means 20%, 30% and 40% damage to the specimen, respectively.
Figure 7 shows a typical fatigue test with a single rest period applied in the middle of the test. In Palvadi et al.’s method, a power law function was used to fit to the C vs. N test data before rest period which had the functional form of Equation (5).

\[ C^1 = A_1 - A_2 (N)^r \] \hspace{1cm} (5)

where \( A_1, A_2 \) and \( r \) are fitting parameters.

In order to define a corrected pseudo stiffness increase they used the fact that reloading curve is parallel to the original damage curve in region II. So using the fitting parameters obtained from Equation (5) and the measured data from region II along with Equation (6) the horizontal shift of the curve \( C^1 \), R, was determined.

\[ C^3 = A_1 - A_2 (N - R)^r \] \hspace{1cm} (6)
By using the value of N at which the rest period was applied, $C_i$ and $C'_i$ were calculated from Equations (5) and (6), respectively. $C'_i - C_i$ was considered to be the actual increase in pseudo stiffness due to the applied rest period. Then having the values of $C_i$ and $C'_i$, the values of $S_i$ and $S'_i$ were back calculated using $C(S)$ function of tested mixture. This requires that damage characteristic curve, $C(S)$ to be unique even by introducing the rest period and reloading which is considered to be the main assumption in their research. Internal state variable, S, which is an indicator of fatigue damage growth in the specimen and
was defined in VECD theory in Lee and Kim (1998) work, was used to quantify the amount of healing as shown in the following equation:

\[
\% \text{Healing} (C, t) \equiv \frac{(S_f - S_i)}{(S'_i)} * 100
\]  

(7)

where \( C \) represents the pseudo stiffness before a rest period which is a measure of damage level in the specimen, \( t \) is the rest period duration, and \( S_i \) and \( S_f \) are the internal state variables before and after applying the rest period, respectively.

Using the results of the performed tests in this study and curve fitting techniques, healing was modeled as a function of rest period duration and damage level before the rest period. Verification tests were conducted at modes of loading and sequence of rest period applications which were different than those used for the model development. From the verification tests results it was proved that the proposed healing function can predict the healing percentage correctly irrespective of the test sequence and mode of loading. The authors recommended more research not just on FAM specimens but also full asphalt mixtures and at other temperatures and aging levels to incorporate the effect of these factors in healing characteristic function too.

2.5. Summary of Healing Literature

According to the literature fatigue damage and healing phenomena in asphalt concrete materials has been under investigation by many researchers during the past fifty years. They used different testing programs and specimen geometries to quantify the healing and incorporate it into asphalt concrete modeling. As soon as fatigue loading starts, damage
grows in the specimen and microcracks form in the body of tested sample. It has always been a challenge for the researchers to find a true and valid correlation between the fatigue life of specimens tested in the labs and fatigue life of actual pavements in highways because in reality there is a rest period after each load pulse caused by the elapsed time between consecutive axis of passing vehicles and it is also impossible to maintain exactly the same testing conditions as there is in the field. Generating a test program which produces the same field stresses in the lab tested specimens is not an easy task to carry.

Researchers emphasized on four important aspects regarding healing modeling in asphalt concrete mixtures:

(1) Investigate the dependence of healing potential of mixtures on different factors such as temperature, rest period duration, damage level before the rest period, chemical and physical properties of asphalt binder used in the mixture, aging and other factors.

(2) Present a healing characteristic function which shows the dependency of healing to the factors that affect it.

(3) Propose a method to predict the fatigue life of asphalt mixtures considering the effect of healing without having to conduct time consuming pulse-rest healing tests.

(4) Find an optimum rest period duration beyond which no increase in fatigue life of asphalt mixture specimens would happen.

This research will focus on the investigation of these aspects.
CHAPTER 3. THEORETICAL BACKGROUND

The main goal in this chapter is to present theories that are used in the S-VECD model.

3.1. Linear Viscoelasticity (LVE)

Linear viscoelastic (LVE) materials exhibit time- and temperature-dependent behavior. That is to say, the current response is dependent on both the current input and all past input (i.e., input history). By contrast, the response of an elastic material is only dependent on the current input. Constitutive relationships for LVE materials are typically expressed in the convolution integral form:

\[ \sigma = \int_0^t E(t - \tau) \frac{d\varepsilon}{d\tau} d\tau \]  \hspace{1cm} (8)

\[ \varepsilon = \int_0^t D(t - \tau) \frac{d\sigma}{d\tau} d\tau \]  \hspace{1cm} (9)

where \( E(t) \) and \( D(t) \) are the relaxation modulus and creep compliance, respectively.

The time-dependent modulus values presented in Equations (8) and (9) are oftentimes difficult to obtain. The LVE properties can be measured in the frequency realm through the complex modulus. Theoretical considerations of the complex modulus can be found elsewhere (Chehab 2002). In short, the complex modulus provides the constitutive relationship between the stress and strain amplitudes of a material loaded in a steady-state
cyclic sinusoidal manner. It is then possible, through the theory of linear viscoelasticity, to convert this frequency-dependent property to E(t) and D(t).

Assessing the LVE properties via the complex modulus has the following advantages:

1. The long loading time required to assess the time-dependent material properties in the time domain may actually damage the specimen and lead to incorrect values. The cyclic complex modulus test, especially in the tension-compression mode with zero mean strain, can cover a wide range of conditions without inducing damage in the specimen.

2. It is difficult to obtain the relaxation modulus and creep compliance experimentally at very short times. However, through the use of complex modulus and linear viscoelastic theory, short time values of these properties can be calculated.

3. Small amounts of viscoelasticity can be measured more accurately using the complex modulus test than using creep or relaxation tests.

4. The modulus or compliance measured in the cyclic test can be used directly in the analysis of the dynamic response of structures to cyclic and transient loading.

Numerous methods of varying accuracy have been proposed to convert frequency-dependent properties to the corresponding time-dependent properties. Because the methods utilized in this research are slightly different than those found in previous efforts, a brief summary of the process is presented below.
3.1.1. Interconversion for the Relaxation Modulus

The interconversion method for obtaining the relaxation modulus utilizes an exact technique that incorporates the storage modulus in Equation (10). The storage modulus, when expressed in terms of reduced angular frequency ($\omega_R$), can be expressed using the Prony series representation given in Equation (11):

$$E'(\omega_R) = \left| E'(\omega_R) \right| \sin \left( \phi(\omega_R) \right)$$  \hspace{1cm} (10)

$$E'(\omega_R) = E_\infty + \sum_{i=1}^{m} \frac{\omega_R^2 \rho_i E_i}{\omega_R^2 \rho_i^2 + 1}$$ \hspace{1cm} (11)

where

$E_\infty$ = elastic modulus;

$\omega_R$ = angular frequency;

$E_i$ = modulus of $i^{th}$ Maxwell element (fitting coefficient); and

$\rho_i$ = relaxation time (fitting coefficient).

Similarly, it can be shown using the theory of viscoelasticity that the Prony representation of the relaxation modulus is given by Equation (12):

$$E(t) = E_\infty + \sum_{i=1}^{m} E_i e^{-t/\rho_i}$$ \hspace{1cm} (12)

In short, the storage modulus is calculated by Equation (10) and then fit to the Prony representation by using the collocation method (Park and Schapery 1999, Schapery 1961). The coefficients determined from this process are then used with Equation (12) to find the relaxation modulus directly.
3.1.2. Interconversion for the Creep Compliance

Using the theory of viscoelasticity, the exact relationship between the creep compliance and relaxation modulus is given by Equation (13):

\[
\int_{0}^{t} E(t-\tau) \frac{dD(\tau)}{d\tau} d\tau = 1
\]  

(13)

If creep compliance is written in terms of the prony representation (Equation (14)), substituted into Equation (13) along with Equation (12) and simplified, the result can be expressed as a linear algebraic equation, Equation (15). The coefficients, \{D\}, in this equation are solved by utilizing the least square function in Matlab\textsuperscript{TM}:

\[
D(t) = D_{g} + \sum_{j=1}^{n} D_{j}\left(1 - e^{-t/\tau_{j}}\right)
\]  

(14)

\[
[A]\{D\} = [B]
\]  

(15)

where

\[
[A] = \sum_{j=1}^{N} \left[ \sum_{m=1}^{N} P_{m} E_{m} \left( e^{-\frac{1}{\rho_{m} \tau_{j}}} - e^{-\frac{1}{\rho_{m}}} \right) + E_{\infty} \left(1 - e^{-\frac{1}{\tau_{j}}}\right) \right]
\]  

(16)

\[
\{D\} = D_{j}
\]  

(17)

\[
[B] = 1 - \frac{1}{E_{\infty} + \sum_{m=1}^{N} E_{m} e^{-\frac{1}{\rho_{m}}}} \left( E_{\infty} + \sum_{m=1}^{N} E_{m} e^{-\frac{1}{\rho_{m}}} \right)
\]

Once the coefficients, \(D_{j}\), are determined they are substituted directly into Equation (14) to find the creep compliance.
3.1.3. LVE Characterization Methodology

In this section a methodology for assessing and analyzing LVE properties of asphalt concrete mixtures through the dynamic modulus testing will be presented. However, it should be known that the test applies cyclic sinusoidal loading at several combinations of frequency and temperature. Load and axial deformations, measured at four locations separated by 90° intervals, are recorded for each combination of frequency and temperature. From these measurements stresses and strains are calculated based on the specimen area and gauge length of the deformation measurements, respectively.

The procedure for analyzing the dynamic modulus utilizes a least squares regression technique that first assumes stress and strain are represented by the functional form presented in Equation (18).

\[ y(t) = A_0 + C_1 \cos(2\pi ft + \theta) \]  

If the addition law for cosines is applied to Equation (18), then the function may be rewritten as:

\[ y(t) = A_0 + A_1 \cos(2\pi ft) + B_1 \sin(2\pi ft) \]  

where \( A_1 \) and \( B_1 \) are given by;

\[ A_1 = C_1 \cos(\theta) \]  

\[ B_1 = -C_1 \sin(\theta) \]  

The angle, theta, can be calculated through Equations (20) and (21) as
\[ \theta = \tan^{-1}\left(\frac{-B_1}{A_1}\right) \]  

(22)

It should be noted that if \( \theta \) is larger than \( \pi \) then \( A_1 \) will be less than one, but from Equation (22), \( \theta \) will be calculated as less than \( \pi \). It is therefore more accurate to present Equation (22) in piecewise form.

\[
\theta = \begin{cases} 
\tan^{-1}\left(\frac{-B_1}{A_1}\right), & A_1 > 0 \\
\tan^{-1}\left(\frac{-B_1}{A_1}\right) + \pi, & A_1 < 0 
\end{cases} \]

(23)

The amplitude of the function, \( C_1 \), can similarly be calculated from Equations (20) and (21) where it is found that:

\[ C_1 = \sqrt{A_1^2 + B_1^2} \]  

(24)

Applying a least square model to Equation (19) it can be showed that the solution to coefficients \( A_0 \), \( A_1 \) and \( B_1 \) is given by:

\[
\begin{bmatrix}
\sum \cos(2\pi ft) \\
\sum \sin(2\pi ft)
\end{bmatrix}
\begin{bmatrix}
\sum \cos^2(2\pi ft) \\
\sum \cos(2\pi ft)\sin(2\pi ft)
\end{bmatrix}
\begin{bmatrix}
\sum \sin^2(2\pi ft) \\
\sum \sin(2\pi ft)\sin(2\pi ft)
\end{bmatrix}
\begin{bmatrix}
A_0 \\
A_1 \\
B_1
\end{bmatrix} =
\begin{bmatrix}
\sum y \\
\sum y \cos(2\pi ft) \\
\sum y \sin(2\pi ft)
\end{bmatrix}
\]

(25)

If number of data points, \( N \), is such that whole cycles are analyzed then Equation (25) may be written as:

34
\[
\begin{bmatrix}
N & 0 & 0 \\
0 & N/2 & 0 \\
0 & 0 & N/2
\end{bmatrix}
\begin{bmatrix}
A_0 \\
A_1 \\
B_1
\end{bmatrix} = \begin{bmatrix}
\sum_{i=1}^{N} y \\
\sum_{i=1}^{N} y \cos(2\pi ft_i) \\
\sum_{i=1}^{N} y \sin(2\pi ft_i)
\end{bmatrix}
\]

(26)

It is then easily shown that the coefficients \( A_0 \), \( A_1 \) and \( B_1 \) are given by:

\[
A_0 = \frac{\sum_{i=1}^{N} y}{N}
\]

(27)

\[
A_1 = \frac{2}{N} \sum_{i=1}^{N} y \cos(2\pi ft)
\]

(28)

\[
B_1 = \frac{2}{N} \sum_{i=1}^{N} y \sin(2\pi ft)
\]

(29)

Applying this methodology for stress, it is found that it is necessary to center the stress \( \sigma' \) so that the mean value is zero (Equation (30)). Equations (23), (24), (27), (28) and (29) are then applied to the centered stress with Equations (30) to (35).

\[
\sigma' = \sigma - \frac{\sum_{i=1}^{N} \sigma_i}{N}
\]

(30)

\[
A_{\sigma 0} = \frac{\sum_{i=1}^{N} \sigma'_i}{N}
\]

(31)

\[
A_{\sigma 1} = \frac{2}{N} \sum_{i=1}^{N} \sigma'_i \cos(2\pi ft_i)
\]

(32)

\[
B_{\sigma 1} = \frac{2}{N} \sum_{i=1}^{N} \sigma'_i \sin(2\pi ft_i)
\]

(33)
\[
\theta_{\sigma} = \begin{cases} 
\tan^{-1}\left(-\frac{B_{\sigma}}{A_{\sigma}}\right), & A_{\sigma} > 0 \\
\tan^{-1}\left(-\frac{B_{\sigma}}{A_{\sigma}}\right) + \pi, & A_{\sigma} < 0
\end{cases}
\] (34)

\[|\sigma^*| = \sqrt{A_{\sigma}^2 + B_{\sigma}^2} \] (35)

The results of these equations can then be used with Equation (19) to verify the fitting procedure.

The same methodology is used for strain measurements; however, to center the strain measurements it is necessary to remove the effects of drift (D). Drift is removed by finding the mean strain as a function of time, best fitting this drift to a linear function and then subtracting the drift from all measured values. In this study drift is removed by finding the moving average of the measurements at intervals equal to the number of samples per cycle. Therefore, for fitting the strain Equations (23), (24), (27), (28) and (29) are applied as:

\[
\varepsilon' = \varepsilon_i - D(t_i) - \frac{\sum_{i=1}^{N} \varepsilon_i}{N}
\] (36)

\[A_{\varepsilon_0} = \frac{\sum_{i=1}^{N} \varepsilon'_i}{N}\] (37)

\[A_{\varepsilon_1} = \frac{2}{N} \sum_{i=1}^{N} \varepsilon'_i \cos(2\pi ft_i)\] (38)

\[B_{\varepsilon_1} = \frac{2}{N} \sum_{i=1}^{N} \varepsilon'_i \sin(2\pi ft_i)\] (39)
\[ \theta_\varepsilon = \begin{cases} 
\tan^{-1}\left( \frac{-B_{\varepsilon_1}}{A_{\varepsilon_1}} \right), & A_{\varepsilon_1} > 0 \\
\tan^{-1}\left( \frac{-B_{\varepsilon_1}}{A_{\varepsilon_1}} \right) + \pi, & A_{\varepsilon_1} < 0 
\end{cases} \]  

(40)

\[ |\varepsilon^*| = \sqrt{A_{\varepsilon_1}^2 + B_{\varepsilon_1}^2} \]  

(41)

As with stress, the results of these equations can then be used with Equation (19) to verify the fitting procedure.

To calculate the dynamic modulus Equations (35) and (41) are used;

\[ |E^*| = \frac{|\sigma^*|}{|\varepsilon^*|} \]  

(42)

Phase angle (\(\phi\)) is finally calculated with Equation (43).

\[ \phi = \theta_\sigma - \theta_\varepsilon \]  

(43)

It is interesting to note from Equation (43) that even though it is known that stress leads strain the phase angle is given by subtracting the fitting angle of strain from the fitting angle of stress. This peculiarity results from the original assumption of the functional form (Equation (18)) where it is seen that a higher fitting angle corresponds to a larger shift in the negative time direction. The results of this process for several different temperature and frequency combinations are presented for a mixture in Figure 8.
Figure 8. Typical unshifted dynamic modulus in: (a) semi-log space; (b) log-log space
Asphalt concrete in the LVE range is known to be thermorheologically simple (TRS) and, as such, the effects of time and temperature can be combined into a joint parameter, reduced time/frequency, through the time-temperature shift factor (a_T) with Equation (44).

\[ f_R = f \times a_T \tag{44} \]

Figure 9. Typical dynamic modulus mastercurve in: (a) semi-log space; (b) log-log space
In practical terms, this behavior allows for the horizontal shifting of the data in Figure 8 to form a single curve, the mastercurve, for describing the constitutive behavior of asphalt concrete over a wide range of reduced frequencies. Figure 9 presents such mastercurves for the replicate tests shown in Figure 8. The time-temperature shift factor is the amount of horizontal shift, in log scale, required to create the continuous curve. The amount of shifting is dependent on the temperature chosen as the reference and, therefore, varies by temperature, as shown in Figure 10.

![Figure 10. Log shift factor function for a typical mixture](image)

The mastercurve is fit to a sigmoidal functional form, Equation (45), and the relationship between the shift factor and temperature is fit to a second order polynomial.
function, Equation (46). To determine the sigmoidal coefficients and the time-temperature shift factors, the Solver function in EXCEL is used by minimizing the error between Equation (45) and the logarithm of the measured data. Both the sigmoidal coefficients and the log shift factors are optimized to minimize this error. It is noted that one of the log shift factors is constrained to a value of zero. This temperature is subsequently referred to as the “reference temperature” and is typically taken as the median test temperature. The optimum log shift factors are finally fit with Equation (46). With the coefficients of Equations (45) and (46) it is then possible, through Equation (44), to find the modulus at any temperature and frequency combination.

\[
\log |E^*| = a + \frac{b}{1 + \frac{1}{e^{d + g \ast \log (f_e)}}}
\]

(45)

\[
\log (a_r) = \alpha_1 T^2 + \alpha_2 T + \alpha_3
\]

(46)

3.2. VECD/S-VECD Model and Formulations

3.2.1. Damage Model for Elastic Solids

The mechanical behavior of an elastic medium with constant material properties (i.e., without damage growth) can usually be described using an appropriate thermodynamic potential. These potentials are point functions of thermodynamic state variables. When a system is under loading, strain energy which is defined as the work done on the body by external loading will be stored in the system. However, when damage occurs due to the external loading, a portion of the work done on the body under loading would not be stored
as strain energy and is consumed to cause damage to the body. According to the concept of thermodynamics of irreversible processes the amount of work required to generate a given extent of damage can be expressed as a function of internal state variables (ISV’s).

Schapery (1990) applied the method of thermodynamics of irreversible processes to develop a theory applicable in describing the mechanical behavior of elastic media with growing damage and other structural changes.

The work potential theory is based on the following three elements (Schapery 1990):

1. Strain energy density function:

\[ W = W(\varepsilon_{ij}, S_m) \]  \hspace{1cm} (47)

2. Stress-strain relationship:

\[ \sigma_{ij} = \frac{\partial W}{\partial \varepsilon_{ij}} \] \hspace{1cm} (48)

3. Damage evolution law:

\[ -\frac{\partial W}{\partial S_m} = \frac{\partial W_s}{\partial S_m} \] \hspace{1cm} (49)

where \( \sigma_{ij} \) and \( \varepsilon_{ij} \) are stress and strain tensors, respectively. \( S_m \) are ISV’s (or damage parameters), and \( W_s = W_s(S_m) \) is the dissipated energy due to micro-structural changes. The left hand side of Equation (49) is the so-called available thermodynamic force for damage growth while the right hand side is the required force.
3.2.2. Correspondence Principle (CP)

An elastic-viscoelastic correspondence principle (CP) was proposed by Schapery (1984) that is applicable to both linear and nonlinear viscoelastic materials. He suggested that the same constitutive formulation forms used for the elastic materials could be used for viscoelastic body but physical stresses and strains are instead pseudo variables in the form of convolution integrals as follows:

\[
\varepsilon_{ij}^{R} = \frac{1}{E_{R}} \int_{0}^{t} E(t - \tau) \frac{\partial \varepsilon_{ij}}{\partial \tau} d\tau
\]

(50)

\[
\sigma_{ij}^{R} = E_{R} \int_{0}^{t} D(t - \tau) \frac{\partial \sigma_{ij}}{\partial \tau} d\tau
\]

(51)

where

\( \sigma_{ij}, \varepsilon_{ij} = \) physical stresses and physical strains,

\( \sigma_{ij}^{R}, \varepsilon_{ij}^{R} = \) pseudo stresses and pseudo strains,

\( E_{R} = \) reference modulus that is an arbitrary constant, and

\( E(t), D(t) = \) relaxation modulus and creep compliance, respectively.

Equation (52) shows the uniaxial stress-strain relationship for a linear viscoelastic (LVE) material:

\[
\sigma = \int_{0}^{t} E(t - \tau) \frac{d\varepsilon}{d\tau} d\tau
\]

(52)

Using the definition of pseudo strain in Equation (50), Equation (52) can be rewritten
as:

\[ \sigma = E_R \varepsilon^R \]  \hspace{1cm} (53)

where

\[ \varepsilon^R = \frac{1}{E_R} \int_0^t E(t-\tau) \frac{\partial \varepsilon}{\partial \tau} d\tau \]  \hspace{1cm} (54)

The similarity of Equation (53) to the linear elastic stress-strain relationship is obvious. The results on experimental verification of the correspondence principle (CP) were documented by Kim and Little (1990), Kim et al. (1995), and Lee and Kim (1998) using uniaxial monotonic and cyclic tests on asphaltic materials.

### 3.2.3. Damage Model for Viscoelastic Solids

According to the Section 3.2.2, it is possible to use the work potential theory to account for viscoelastic materials by using the CP (Schapery 1990). In this case, the strain energy density function \( W = W(\varepsilon_{ij}, S_m) \) would change to a pseudo strain energy density function \( W^R = W^R(\varepsilon^R_{ij}, S_m) \) meaning that the strains in the work potential theory would be replaced with pseudo strains.

Since both available force for growth of \( S_m \) and the resistance against the growth of \( S_m \) are rate-dependent for most viscoelastic materials it is not possible to directly transform the damage evolution law in Equation (49) into evolution law for viscoelastic media (Park et al. 1996).
Thus, the following form of evolution law that has been employed by Lee and Kim (1998) would be used in this research to describe the damage evolution process in asphalt concrete material:

\[
\dot{S}_m = \left( - \frac{\partial W}{\partial S_m} \right)^{\alpha_m}
\]  

(55)

where \( \dot{S}_m \) is damage evolution rate and \( \alpha_m \) are positive constants. The form of Equation (55) is similar to the power-law crack growth equations for viscoelastic materials.

3.2.4. Constitutive Modeling with Growing Damage and Healing

Modeling the mechanical behavior of asphalt concrete considering both time-dependent damage growth and microdamage healing is very complicated. In this study the stepwise approach that was developed by Lee and Kim (1998) is used. First, a uniaxial viscoelastic constitutive model that accounts for growing damage was established. The constitutive model was then adopted to analyze the tests with microdamage healing.

According to Lee and Kim (1998) the following three characteristics can be observed due to the damage induced in specimens under fatigue testing:

a. nonlinear behavior of the loading and unloading paths in each cycle,

b. change in the slope of each \( \sigma - \varepsilon^R \) cycle (i.e., reduction in the pseudo stiffness of the material) as cyclic loading continues, and

c. accumulation of permanent pseudo strain \( \varepsilon^R_S \) in the controlled-stress mode (i.e., shift of the \( \sigma - \varepsilon^R \) loop from the origin as cyclic loading continues).
Secant pseudo stiffness $S^R$ was defined to show the change in the slope of $\sigma - \varepsilon^R$ loops as follows:

$$S^R = \frac{\sigma_m}{\varepsilon_m^R}$$  \hspace{1cm} (56)

where $\varepsilon_m^R$ is the peak pseudo strain in each stress-pseudo strain cycle, and $\sigma_m$ is a stress corresponding to $\varepsilon_m^R$.

The following uniaxial constitutive equations were presented by Lee and Kim (1998) for linear elastic and linear viscoelastic bodies with and without damage based on the observations made in fatigue tests conducted on asphalt mixtures.

Elastic Body without Damage: \hspace{1cm} $\sigma = E^R \varepsilon$ \hspace{1cm} (57)

Elastic Body with Damage: \hspace{1cm} $\sigma = C(S_m) \varepsilon$ \hspace{1cm} (58)

Viscoelastic Body without Damage: \hspace{1cm} $\sigma = E^R \varepsilon^R$ \hspace{1cm} (59)

Viscoelastic Body with Damage: \hspace{1cm} $\sigma = C(S_m) \varepsilon^R$ \hspace{1cm} (60)

where $E_R$ is a constant, and $C(S_m)$ is a function of $S_m$. $E_R$ in Equation (57) is Young’s modulus. The function $C(S_m)$ represents the change in stiffness of the material due to changes in microstructure of the material, e.g., growing damage or healing (increase or reduction of internal state variable $S_m$). As was stated before and is obvious by comparing Equation (59) with Equation (57) and Equation (60) with Equation (58), the viscoelastic
constitutive equations can be described by the elastic equations when physical variables are replaced by corresponding pseudo variables.

Lee and Kim (1998) used three ISV’s to describe the inelastic stress-pseudo strain behavior in Equation (60). These variables are: (1) $S_p$, a time-dependent damage parameter (Schapery 1981); (2) $\varepsilon^R / \varepsilon^L$ which is the ratio of current pseudo strain to the largest absolute value of pseudo strain up to the current time; and (3) $\varepsilon^R_0$, pseudo strain amplitude. They employed $S_p$ to describe the change in pseudo stiffness due to the damage occurred in the material while $\varepsilon^R / \varepsilon^L$ and $\varepsilon^R_0$ were used to differentiate the loading and unloading paths in the stress-pseudo strain cycles.

Lee and Kim (1998) proposed the following uniaxial pseudo strain energy density function based on the experimental study of cyclic test data:

$$ W^R = \frac{I}{2} MC(S)(\varepsilon^R_p)^2 + \frac{I}{2} G \left( \frac{\varepsilon^R}{\varepsilon^L}, \varepsilon^R_0 \right) \left( \varepsilon^R_p \right)^2 \quad (61) $$

where $I = \text{initial pseudo stiffness}$,

$\varepsilon^R_p = \varepsilon^R - \varepsilon^R_S$,

$\varepsilon^R_S = \text{accumulated permanent pseudo strain in controlled-stress mode}$, and

$$ M = \frac{\varepsilon^R_m}{\left( \varepsilon^R_m - \varepsilon^R_S \right)} $$

is a mode factor used to show the slope of $\sigma - \varepsilon^R$ loop.
The parameter I was used to minimize the sample-to-sample variability. The function $C(S)$ represents the change in the slope of each $\sigma - \varepsilon^R$ loop due to growing damage, while the hysteresis function $G$ represents hysteretic behavior of the loading and unloading paths in each cycle.

Then, the stress-strain relationship becomes

$$\sigma = \frac{\partial W^R}{\partial \varepsilon^R_e} = I \left( MC(S) + G \left( \frac{\varepsilon^R_e}{\varepsilon^R_L}, \varepsilon^R_0 \right) \right) \varepsilon^R_e$$

(62)

$S$ in Equation (62) is the only undetermined ISV that can also be obtained from the time-dependent damage evolution law, Equation (55).

It would be of great help to reduce Equations (61) and (62) to simpler forms. When $\varepsilon^R$ becomes $\varepsilon^m$, $G \rightarrow 0$ because $\varepsilon^R / \varepsilon^R_L \rightarrow 1$. Observing that $M(\varepsilon^m - \varepsilon^S) = \varepsilon^m$, they could obtain the following forms of reduced constitutive equations from Equations (61) and (62):

$$W^R_m = \frac{I}{2} C(S) \varepsilon^R_m \varepsilon^R_{me}$$

(63)

$$\sigma_m = IC(S) \varepsilon^R_m$$

(64)

where $\varepsilon^R_{me} = (\varepsilon^R_m - \varepsilon^S)$. Comparing Equations (64) and (56) reveals that the function $C(S)$ represents pseudo stiffness $(S^R)$ and so the changes in the slope of $\sigma - \varepsilon^R$ loops during fatigue testing. $W^R$ in the evolution law, Equation (55), is replaced with $W^R_m$ for $S$ as follows:
\[ \dot{S} = \left( -\frac{\partial W_m^R}{\partial S} \right)^\alpha \]  

(65)

### 3.2.5. Characterization Procedures for C

C values can be computed by combining Equation (64) with the measured stresses and pseudo strains. Then, we need to obtain the values of \( S \) corresponding to the strain input by solving the evolution law and then by cross-plotting the corresponding C and S values the functional form of \( C(S) \) can be determined. However, it is not easy to find \( S \) using the evolution law, Equation (65), because according to the equation a prior knowledge of \( C(S) \) is required before it can be solved for \( S \). For notational simplicity, \( \sigma_m, \varepsilon_m^R, \) and \( W_m^R \) will be denoted as \( \sigma, \varepsilon^R, \) and \( W^R, \) respectively.

Lee and Kim (1998) used the following chain rule in order to eliminate \( S \) in the right-hand side of the damage evolution law (Equation (65)):

\[ \frac{dC}{dS} = \frac{dC}{dt} \frac{dt}{dS} \]  

(66)

From Equations (63) and (65),

\[ \frac{dS}{dt} = \left[ -\frac{I}{2} \frac{dC}{dS} (\varepsilon^R)^2 \right]^\alpha \]  

(67)

Substituting Equation (66) into Equation (67) and rearranging yields

\[ \frac{dS}{dt} = \left[ -\frac{I}{2} \frac{dC}{dt} (\varepsilon^R)^2 \right]^\frac{\alpha}{(1+\alpha)} \]  

(68)
Now, \( S \) in Equation (68) may be obtained from the following numerical scheme:

\[
S \cong \sum_{i=1}^{N} \left[ \frac{I}{2} \left( \epsilon_{i}^{R} \right)^{2} \left( C_{i-1} - C_{i} \right) \right]^{\frac{\alpha}{1+\alpha}} \frac{1}{\left( t_{i} - t_{i-1} \right)}
\]  

(69)

In many viscoelastic crack growth problems, the \( \alpha^{th} \) power in pseudo energy release rate governs the crack growth speed and \( \alpha \) itself is related to the material’s creep or relaxation properties (Schapery 1975a-c).

Lee and Kim (1998) investigated the validity of the two expressions for \( \alpha \) suggested by Schapery under different modes of loading and found that \( \alpha = 1 + 1/m \) works better for the controlled-strain mode while \( \alpha = 1/m \) is more appropriate for the controlled-stress mode where \( m \) is the slope of the \( \log D(t) - \log(t) \) relationship. Thus, \( \alpha = 1/m \) is used in this study as the initial value for the \( \alpha \) since the healing tests in the present research have been conducted in the controlled-stress mode.

It is a well-known fact that a continuous fatigue test on asphalt concrete specimen reaches the steady state condition after the application of the very first cycles. In such a condition, it is possible to make some beneficial simplification in the calculation of pseudo strain (FHWA method of analysis). Equation (50) represents the rigorous formula for the calculation of pseudo strain. In the steady state condition the integral in Equation (50) would be simplified as follows:

\[
\epsilon_{0}^{R} = \frac{1}{E_{R}} \int_{0}^{t} E(t - \tau) \frac{\partial \epsilon_{\mu}}{\partial \tau} d\tau \rightarrow \epsilon_{0}^{R} = \epsilon_{0} \times \left| E^{*} \right|_{LVE}
\]  

(70)
This simplification that is referred to as the “steady-state assumption” saves a significant amount of computational time without causing large error in the calculation. According to the simplification in the calculation of pseudo strain Underwood et al. (2010) and Lee and Kim (1998) considered two steps in the analysis of test data. First step which is related to the very first cycle of the test in which the condition of steady state has not been reached yet (transient) and the second step which starts after the first cycle and material is considered to be at the steady state condition (cyclic). As a result of the piecewise pseudo strain equation, the pseudo stiffness and damage parameter equations are also piecewise.

These equations that were also used in this study are presented in Equations (71) to (73).

\[ \varepsilon^R = \begin{cases} \varepsilon^R_j = \frac{1}{E_R} \int_0^{\xi} E(\xi - \tau) \frac{d\varepsilon}{d\tau} d\tau & \xi \leq \xi_p / 2 \\ \left( \varepsilon_{0,sa} \right)_{cycle i} = \frac{1}{E_R} \left( \frac{1}{2} \left( \varepsilon_{0,pp} \right) \right)^* |E_{LVE}^*| & \xi > \xi_p \end{cases} \]  \hspace{1cm} (71)

\[ C = \begin{cases} \frac{\sigma}{\varepsilon_{0,sa}^* I} & \xi \leq \xi_p \\ \frac{\sigma_{0,sa}}{\varepsilon_{0,sa}^* I} & \xi > \xi_p \end{cases} \]  \hspace{1cm} (72)

\[ dS = \begin{cases} (dS_{transient})_{timestep} = \left( -\frac{1}{2} \left( \varepsilon^R_j \right)^2 \Delta C_j \right)^{\alpha' + \alpha} \left( \Delta \varepsilon \right)^{\alpha' + \alpha} & \xi \leq \xi_p \\ (dS_{cyclic})_{cycle i} = \left( -\frac{1}{2} \left( \varepsilon_{0,sa}^2 \Delta C_i \right) \right)^{\alpha' + \alpha} \left( \Delta \varepsilon_p \right)^{\alpha' + \alpha} (R_D) & \xi \geq \xi_p \end{cases} \]  \hspace{1cm} (73)

where \( R_D = \left( \frac{1}{\xi_f - \xi_i} \int_{\xi_i}^{\xi_f} \left( f(\xi) \right)^{2\alpha} d\xi \right)^{\alpha'.} \)
CHAPTER 4. MATERIALS AND TEST METHODS

4.1. Study Materials

The first mixture used in this healing study was a prepared RS9.5B loose mixture that had a nominal maximum aggregate size (NMAS) of 9.5 mm and 30% of Recycled Asphalt Pavement (RAP) material and the binder grade of the mixture was PG 64-22.

Sometimes running fatigue tests on specimens made from a specific mixture mostly result in failures outside the LVDTs measurement range that is called end failures. One of the main reasons for this end failure is the air void gradient along the height of samples meaning that air void percent is higher in regions close to the ends which makes it easier for the samples to break near the end plates. Reducing the height of tested specimens is a convenient way to avoid this issue and to have approximately constant air void along specimen height.

To investigate the effect of this height reduction on the measurements of specimen properties, fatigue and dynamic modulus tests have been conducted by other researchers on specimens with 150 mm and 130 mm heights. Damage characteristic curves from fatigue tests on specimens with different combinations of specimen heights and gauge lengths are shown in Figure 11. Phase angle, dynamic modulus, and shift factor graphs of the mixture are also shown in Figure 12, Figure 13, and Figure 14 respectively. It is clear from these figures that results from these two geometries match very well. So one could test specimens with 150 mm height using 100 mm LVDTs or in order to get more middle failures reduce the height to 130 mm and use 75 mm LVDTs mounted on tested samples. Figure 15 shows that the gradation curve of the mixture lies within the control points.
Figure 11. Damage characteristic curves of RS9.5B mixture for different geometries

Figure 12. Phase angle curves of RS9.5B mixture for different geometries
Figure 13. Dynamic modulus curves of RS9.5B mixture for different geometries: (a) log-log scale, (b) semi-log scale
Figure 14. Shift factor curves of RS9.5B mixture for different geometries

Figure 15. Gradation chart of RS9.5B mixture
4.2. Laboratory Specimen Fabrication

All specimens were compacted by the Superpave gyratory compactor, to a height of 178 mm and a diameter of 150 mm. To obtain specimens of uniform quality, the fabricated samples were cored and cut to a diameter of 75 mm and a height of 150 mm for testing. Air void measurements were done using the core-lock method and specimens were stored in sealed bags until testing in order to avoid aging of the material. The target air void was 4.0% and the air voids for all samples in this study were between 3.5% to 4.5%. All test specimens were being tested within two weeks after fabrication for keeping the consistency among tests.

4.3. Selection of Test Temperatures and Rest Periods

In order to study the effect of different factors on healing properties of asphalt mixtures and investigate the functional form of healing which includes all different effective factors, control load group loading-rest healing tests have been conducted at a frequency of 10 Hz, four different rest periods of 10, 30, 90 and 270 seconds and three different temperatures of 10°, 20° and 30°C for the total of 12 different testing conditions. The four rest periods in this study were not chosen randomly. According to Prowell et al. (2010), for a pavement design life of 40 years and a total of 20 million Equivalent Single Axle Load (ESAL) applications, the average rest period between load applications in real pavements would be about 60 seconds (i.e., 40 years divided by 20 million ESALs). So a decision was made to use four different rest periods two of which to be less than 60 seconds (10 and 30 seconds) and the other two to be more than 60 seconds (90 and 270 seconds).
4.4. Selection of Mode of Loading and Loading History

Fatigue tests can be categorized in two different modes, controlled-stress and controlled-strain. In controlled-stress mode, a constant stress amplitude is applied during the test and the strain increases with load cycles while in controlled-strain mode, a constant strain amplitude is applied and the stress decreases with the application of load cycles. At the moment when a rest period is planned to start, the on specimen load must be zero in order to let the healing process initiate. If the MTS machine used to conduct the tests in this study is programmed in a controlled-strain loading mode in the healing tests, it is possible to have nonzero on specimen load at the time when applying a group of controlled-strain load cycles is finished and rest period is planned to start. Because of this limitation in the testing machine a controlled-stress mode of loading has been adopted in this study to be able to zero the load level when each rest period starts. Two types of controlled-stress test can be conducted: (1) zero minimum controlled-stress test and (2) zero mean controlled stress test.

In order to figure out which of the two controlled-stress types of test must be chosen for the healing study one should investigate the loading history that occurs in actual pavements. This task has been done by simulation using the LVECD program which is a pavement analysis program developed at NCSU. Figure 16 shows the typical stress history that occurs at the bottom of an asphalt layer as a result of passing vehicles and rebounding of the base layer. As can be observed from this figure, there is not just tension stress but also compression in the loading history. This means that zero mean controlled-stress test that has both tension and compression parts simulates the actual pavement stress history at the bottom.
of asphalt layer better than the zero minimum controlled-stress test that only has the tension (positive) stresses.

Another observation is that if zero minimum loading history is used in the tests it will produce residual stresses in the material that would be involved in crack closure when the loading is removed and healing is supposed to start. So in that case it would be difficult to differentiate between the damage recovery caused by healing and the damage recovery caused by the residual stresses. Because of the above mentioned reasons the researcher decided to use zero mean controlled-stress history to conduct the healing tests in this study.

![Typical loading history at the bottom of asphalt layer (using LVECD program)](image)

Figure 16. Typical loading history at the bottom of asphalt layer (using LVECD program)
As a final check to decide between zero mean and zero minimum controlled-stress modes of loading the researcher decided to conduct some group-rest tests in both modes and compare the consistency of the test results considering the fatigue life of tested specimens. Table 1 presents the results of these tests. As the table suggests the general trend of $N_f$ in zero mean controlled-stress tests are reasonable regarding the fact that one could expect a longer fatigue life when the rest period time increases while on the other hand looking at the test results for zero minimum controlled-stress tests reveals that this expected trend is not observed.

<table>
<thead>
<tr>
<th>Loading mode</th>
<th>Temperature (°C)</th>
<th>Rest Period (Second)</th>
<th>10</th>
<th>30</th>
<th>90</th>
<th>270</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero mean</td>
<td></td>
<td></td>
<td>43000</td>
<td>48000</td>
<td>63500</td>
<td>115000</td>
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<td></td>
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<td></td>
<td>7500</td>
<td>27500</td>
<td>41000</td>
<td>29000</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td></td>
<td>13000</td>
<td>14500</td>
<td>18500</td>
<td>18000</td>
</tr>
<tr>
<td>Zero minimum</td>
<td>20</td>
<td></td>
<td>34000</td>
<td>28000</td>
<td>27500</td>
<td>48000</td>
</tr>
</tbody>
</table>

Table 1. Comparison of $N_f$ for zero mean and zero minimum controlled-stress tests
Depending on different ways that rest periods are applied during a test, two types of healing tests can be considered and carried out:

1) Interrupted loading test (group-rest healing test): This test is a “short healing test” in which rest periods are applied in the middle of continuous fatigue test at predefined cycles. Different lengths of rest periods are inserted at different damage levels to determine a modulus or energy recovery as a function of rest period and damage level.

2) Test with intermittent loads (pulse-rest healing test): In this type of healing test, there is a single rest period after each loading cycle. This “long healing test” reflects a more realistic traffic loading history because in real pavements there is a rest period between load applications of successive axles of passing vehicles.

Considering the fact that the group-rest test is much faster than the pulse-rest test it would be great if a link can be found between these two types of healing test so that one can predict and model the material behavior in pulse-rest test just by conducting the group-rest test. As an example, performing a group-rest test with 60 seconds rest period would take a testing time of 3 hours including the temperature conditioning but a pulse-rest test with the same rest period may last for several days.

The load level at each temperature was determined in such a way that the testing sequence result in a reasonable number of cycles to failure of the specimen to avoid long testing time. As was mentioned in the definition of group-rest healing test, rest periods were introduced a few times at different damage levels in each test in order to be able to capture
the effect of the amount of damage prior to the application of rest period on the healing potential of the material as well.

Damage level before applying the rest period \( (C) = \frac{E'}{E_0} \)

where

\( E_0 = \) Initial modulus of specimen from finger print test and

\( E' = \) Modulus of the specimen before applying rest period

In Figure 17 a schematic of interrupted loading sequence (group-rest healing test) which was used in the evaluation of functional form of healing is shown. As can be seen from the figure the stress controlled tension-compression cyclic test was interrupted by rest periods at different percentage reduction in specimen’s initial stiffness. By employing this type of test it would be possible to monitor the healing potential change with respect to the damage level before applying rest period and since the tests are conducted at different temperatures and rest periods, the effect of these two factors on the healing potential of the material can be captured as well. Finally by combining the results of these tests at different temperatures and rest periods applied at various damage levels of tested specimens a functional form for the healing potential of the material can be obtained.
Figure 17. A typical group-rest healing test with RP = 30 seconds
4.5. Testing Machine and Measurements

After coring and sawing of fabricated specimens and before testing them, a few steps need to be performed. First of all, both ends of specimens were glued to steel end plates with DEVCON steel putty. End plates and both ends of specimens were carefully cleaned before gluing because failure in doing a nice job in cleaning them would cause a premature failure of the test. In order to minimize the eccentricity and to make sure of the proper alignment of specimen a special gluing jig was used to hold the end plates parallel with respect to the both ends of specimen.

Each 100 mm LVDT was mounted on the specimen vertically by using two brackets at the two ends of the LVDT. The brackets were held in place through steel targets which were glued to the specimen using 5-minute DEVCON glue and a specific gluing jig.

Measurements of axial deformations were done in 90° intervals over the middle 100 mm of specimen height by using loose-core LVDTs from IPC Global during the test time. Crosshead movement, load (from load cell on the testing machine) and deformation data (from LVDTs) were acquired and stored in the computer using a data acquisition program.

A MTS (Material Testing System) closed-loop servo-hydraulic loading machine with a 25 kN load cell capacity was used to conduct all the tests. The device had an environmental chamber, equipped with liquid nitrogen to control and maintain the test temperature with 0.5°C accuracy of the target temperature.
CHAPTER 5. DETERMINATION OF PERCENT HEALING

5.1. S-VECD Calculation of Group-Rest Healing Tests

As was explained in Section 4.3, primary tests have been done on specimens at three different temperatures (10°, 20°, and 30°C), four different rest periods (10, 30, 90, and 270 seconds) with rest periods applied at different stages of each test (i.e., different damage levels of specimens). Looking at the modulus curves of different specimens shows a bigger jump for longer rest periods as is expected. Figure 18 is an example of this observation.

Figure 18. Visual comparison for the jump in modulus between: (a) 90 seconds and (b) 270 seconds rest periods at 20°C
Another reasonable observation is that for two different temperatures with the same rest period length, the amount of jump in the modulus is bigger for the higher temperature with respect to initial modulus of tested specimen. Figure 19 presents a good example.

Figure 19. Visual comparison for the jump in modulus between: (a) 20°C and (b) 40°C for the same rest period (RP = 270 seconds)
The results of S-VECD analysis of group-rest tests for RS9.5B mixture are presented in APPENDIX A. Here the results for one of the tests (T = 30°C and RP = 30 seconds) are presented in Figure 20 to Figure 22 as an example. An increase in the modulus of specimen after each rest period can be observed as is expected. This means that during each healing period, the material regains its strength partially. Another observation comes from a comparison between the original damage characteristic curve and the curve after each rest period. It can be seen that damage growth in the material is faster after each rest period and during the reloading part with respect to the original damage characteristic curve $C(S)$ of intact material. This means that the increase of modulus due to the crack closure during the healing does not necessarily guarantee that the material has actually gained its strength that much and in fact some fraction of modulus recovery comes from weak joints between the crack faces during the resting time. In other words some of the microcracks in the body of tested specimen just heal partially during rest period and these weak bonds will be reopened fast during the reloading part which results in the faster rate of damage growth in the specimen.

Figure 20, Figure 21, and Figure 22 show modulus, phase angle, and damage characteristic curves of a group-rest healing test on 150 mm tall, 75 mm diameter specimen of the first mixture at 30°C with 30 seconds rest periods, respectively. As was explained in Section 4.5 four LVDTs of 100 mm length were used in 90° intervals to measure the deformation in the axial direction of cylindrical specimens. In some cases, one of the LVDTs does not work properly so it would not read the deformations correctly. In such a test,
readings from the bad LVDT and the LVDT in the opposite direction (180° from the bad LVDT) would be removed during the analysis of raw data files of the test. That is why in some cases calculations have been made just for two LVDTs. If two LVDTs have this problem and they are placed in 90° from each other, the test would be considered as an incorrect test and the test data can not be used at all.
Figure 20. Modulus curves of group-rest healing test at 30°C with 30 seconds rest period:
(a) individual LVDTs (b) average of 4 LVDTs
Figure 21. Phase angle curves of group-rest healing test at 30°C with 30 seconds rest period:
(a) individual LVDTs (b) average of 4 LVDTs
Figure 22. Damage characteristic curve of group-rest healing test at 30°C with 30 seconds rest period

5.2. Percent Healing Calculation

The next step after obtaining $C(S)$ curve of group-rest healing tests is to find the functional form of the healing potential of the material with respect to temperature, damage level before applying rest period, and rest period duration. From the first set of tests the results of which are presented in APPENDIX A, the healing potential dependency on mentioned factors has been captured.

To represent the results in a way that one can see the general trend of healing with respect to different factors, calculation of some parameters is required. These parameters are the damage level prior to the application of the rest period and the amount of healing during each rest period. In order to calculate these parameters, stiffness of the intact material before
running the test is needed. That is why before conducting each healing test, a so called finger-print test was done on the specimen with a low level of load amplitude which resulted in strain amplitude between 50-75 microstrain in the material to measure the stiffness of the intact material before the test starts. This initial stiffness value is called $E_0$.

Following the three steps below, it is possible to calculate the damage level ($C$) before and percent healing ($%H$) during each rest period.

Step 1. Damage level calculation:

$$C = \frac{E}{E_0}$$  \hspace{1cm} (74)

where

$C =$ Damage level before applying rest period,

$E =$ Modulus of specimen before applying rest period, and

$E_0 =$ Initial modulus of intact specimen (from finger print test).

Step 2. Percent healing calculation:

According to Figure 23 there are three different ways to interpret healing percent in asphalt concrete mixture using various parameters obtained from group-rest healing tests data analysis. These three different methods (three different sets of parameters) to calculate the amount of healing are as follows:

Method A. Equation (75) is the calculation formula of healing percent in this method.
\[ \%H_C = \frac{C' - C}{1 - C} \times 100 = \frac{\Delta C}{1 - C} \times 100 \]  

(75)

where

\( \%H_C \) = Percent healing of the material due to an applied rest period, 

\( C \) = Pseudo stiffness of specimen before applying the rest period (damage level), and 

\( C' \) = Recovered pseudo stiffness of specimen at the end of the rest period.

Figure 23. Parameters for the calculation of damage level and percent healing

Method B. In this method percent healing is calculated using Equation (76).
\% H_s = \frac{\Delta S}{S_0} \times 100 \quad (76)

where

\% H_s = \text{Percent healing of the material due to an applied rest period,}

\Delta S = \text{Horizontal shift of } C(S) \text{ curve due to the healing caused by the application of the rest period, and}

S_0 = \text{Damage parameter value at the moment of the rest period application.}

Note that after the application of each healing period the \( C(S) \) and \( C(N) \) curves shift to the right by an amount of \( \Delta S \) and \( \Delta N \), respectively which means that with the same integrity (same \( C \) value) the material can undergo more damage \( (S_0 + \Delta S) \) in the first case and in the second case it means that after the rest period and when reloading starts, \( \Delta N \) cycles must be applied to the specimen to get to the same \( C \) value as it had before the initiation of the rest period or in other words the application of that specific rest period caused an increase in the fatigue life \( N_i \) of the specimen by \( \Delta N \) cycles. Shifting \( C(S) \) and \( C(N) \) curves to the left by \( \Delta S \) and \( \Delta N \), respectively and ignoring the jumps in pseudo stiffness value caused by each rest period application, two smooth curves may be obtained which are believed to be the \( C(S) \) and \( C(N) \) curves of the specimen if no rest period was applied during the test (continuous fatigue test).

Method C. In this method, the increase in fatigue life due to a rest period application is being used to calculate the healing percent of the material.
\[
\% H_N = \frac{\Delta N}{N_0} \times 100
\]  

(77)

where

\( \% H_N \) = Percent healing of the material due to an applied rest period,

\( \Delta N \) = Horizontal shift of \( C(N) \) curve due to the healing caused by the application of the rest period (increase in \( N_i \)), and

\( N_0 \) = Number of load cycles applied to the specimen before the rest period starts.

Figure 24 to Figure 26 show that the general trend of healing curves would be the same no matter which of the above methods being used in the calculation of healing potential of material. The only difference between these three methods would then be the amount of healing calculated from each method.
Figure 24. Pseudo stiffness increase, ($\Delta C_1$), calculated from method A

Figure 25. Shift of C(S) curve, ($\Delta S$), calculated from method B
Step 3. Curves at specific damage levels:

Since the rest periods have been applied randomly during each test, most probably the damage levels at which these rest periods have been applied are not rounded values. To be able to draw healing curves at specific rounded damage levels linear interpolation has been done between the curves and healing curves with damage levels of 0.9, 0.8, 0.7, 0.6 and so on were obtained.

Using the experimental and analytical procedure explained in method B, the percent healing was calculated for the mixture after each rest period. The percent healing is a function of the damage level at the moment of applying the rest period, rest period length and temperature. A function of the form presented in Equation (78) was used to fit to the
computed healing percent values and to relate healing potential of the material to the three mentioned factors.

\[
\% \text{Healing} = m_1 \times (t_R)^{m_2}
\]  

(78)

where

\[ t_R = \text{Rest period length (second), and} \]

\[ m_1 \text{ and } m_2 = \text{Fitting parameters.} \]

The results clearly show that when rest periods are introduced at a similar level of damage, longer rest periods result in a greater amount of healing as is expected. Furthermore, greater amount of healing is achieved when rest periods are introduced at lower damage levels (or higher values of pseudo stiffness \( C \)) of specimen. Fitting Equation (78) to the test data results in a fitted healing curve that reaches an asymptotic value and this asymptotic value becomes smaller as the level of damage before the introduction of rest periods becomes higher (smaller \( C \) values).

Figure 27 to Figure 29 show the calculated actual healing of the material using method B (explained schematically in Figure 23) as a function of rest period and damage level preceding the rest period at three testing temperatures. As it can be observed from these figures the percent healing follows a power law form at each temperature. There is also an excellent compatibility between fitted healing function in Equation (78) and calculated healing percentages from the test results as shown in Figure 30 to Figure 32.
Figure 27. Percent healing as a function of damage level and rest period at 10°C

Figure 28. Percent healing as a function of damage level and rest period at 20°C
Figure 29. Percent healing as a function of damage level and rest period at 30°C

Figure 30. Fitted healing curves and percent healing points from tests at 10°C
Figure 31. Fitted healing curves and percent healing points from tests at 20°C

Figure 32. Fitted healing curves and percent healing points from tests at 30°C
After the fitting of power form function in Equation (78) to the percent healing values from the tests is done it is possible to find functional forms relating the fitting parameters \( m_1 \) and \( m_2 \) to the two other factors; test temperature and damage level.

Once this is done Equation (78) would represent a function through which it is possible to calculate percent healings at different temperatures, damage levels and rest period lengths for the tested mixture but instead of doing this the researcher decided to use another method to express this dependency.

A similar healing study was done by Si et al. (2002) with different rest periods. Figure 33 and Figure 34 show the results of their research. As these two plots suggest the smallest rest period applied by Si and his co-workers was about 2 minutes. The results from the current healing study with the smallest rest period of 10 seconds shows that even with this small rest period the initial sudden healing (crack wetting) happens very fast and as soon as the load is removed (Figure 27 to Figure 29). So the smallest rest period should be much smaller than 2 minutes that has been used by Si and co-workers.
The fitting exercise in Figure 30 to Figure 32 makes it very clear that percent healing changes with respect to the rest period length at any temperature and damage level follow a
power form behavior. This fact reminds us of the relationship between dynamic modulus and frequency at any temperature (Figure 8). Asphalt concrete material has been proven to be thermorheologically simple (TRS) in the viscoelastic range as was shown in CHAPTER 3.

5.3. Percent Healing Mastercurves

It has never been tried to combine time-temperature effect in healing range. If it can be proven that asphalt concrete mixtures are thermorheologically simple (TRS) even when the damaged material is allowed to heal one can combine the effect of rest period time with temperature using time-temperature shift factors obtained from dynamic modulus test and form a mastercurve for percent healing. Table 2 represents the procedure to combine the effect of rest period with temperature into a joint parameter called reduced rest period, using the time-temperature shift factor \( a_{T} \) along with Equation (79):

\[
RP_{(Reduced)} = \frac{RP}{a_{T}}
\]

(79)

Figure 35, Figure 36, and Figure 37 show the percent healings with respect to the reduced rest period at different damage levels in arithmetic, semi-log and log-log space, respectively. It is obvious that there is a single healing mastercurve for each damage level. After finding the mastercurve functional form, fitting the function to percent healing data would reflect the dependency of the healing to both rest period and temperature (through reduced rest period parameter). Once done with the fitting for all damage levels one can express the fitting parameters of the mastercurve in terms of damage level. This way the final
mastercurve equation would show the dependency of healing to reduced rest period and damage level.

Table 2. Combining rest period and temperature effect in reduced rest period parameter

<table>
<thead>
<tr>
<th>Actual T(°C)</th>
<th>LOG (SF)</th>
<th>SF</th>
<th>Target T(°C)</th>
<th>Rest Period (RP)</th>
<th>Reduced RP</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.9</td>
<td>1.49E+00</td>
<td>3.06E+01</td>
<td>10</td>
<td>10</td>
<td>0.3</td>
</tr>
<tr>
<td>9.9</td>
<td>1.49E+00</td>
<td>3.06E+01</td>
<td>10</td>
<td>30</td>
<td>1.0</td>
</tr>
<tr>
<td>9.725</td>
<td>1.51E+00</td>
<td>3.25E+01</td>
<td>10</td>
<td>90</td>
<td>2.8</td>
</tr>
<tr>
<td>9.75</td>
<td>1.51E+00</td>
<td>3.23E+01</td>
<td>10</td>
<td>270</td>
<td>8.4</td>
</tr>
<tr>
<td>19.95</td>
<td>7.00E-03</td>
<td>1.02E+00</td>
<td>20</td>
<td>10</td>
<td>9.8</td>
</tr>
<tr>
<td>19.65</td>
<td>4.91E-02</td>
<td>1.12E+00</td>
<td>20</td>
<td>30</td>
<td>26.8</td>
</tr>
<tr>
<td>19.425</td>
<td>8.07E-02</td>
<td>1.20E+00</td>
<td>20</td>
<td>90</td>
<td>74.7</td>
</tr>
<tr>
<td>19.4</td>
<td>8.43E-02</td>
<td>1.21E+00</td>
<td>20</td>
<td>270</td>
<td>222.4</td>
</tr>
<tr>
<td>29.85</td>
<td>-1.31E+00</td>
<td>4.88E-02</td>
<td>30</td>
<td>10</td>
<td>204.8</td>
</tr>
<tr>
<td>29.75</td>
<td>-1.30E+00</td>
<td>5.03E-02</td>
<td>30</td>
<td>30</td>
<td>596.7</td>
</tr>
<tr>
<td>30</td>
<td>-1.33E+00</td>
<td>4.68E-02</td>
<td>30</td>
<td>90</td>
<td>1925.1</td>
</tr>
<tr>
<td>29.75</td>
<td>-1.30E+00</td>
<td>5.03E-02</td>
<td>30</td>
<td>270</td>
<td>5370.7</td>
</tr>
</tbody>
</table>
Figure 35. Percent healing vs. reduced rest period in arithmetic space

Figure 36. Percent healing vs. reduced rest period in semi-log space
Figure 37. Percent healing vs. reduced rest period in log-log space

The healing mastercurves at different damage levels are fit to sigmoidal functional form, Equation (80), and the relationship between the shift factor and temperature is the second order polynomial obtained from dynamic modulus test on this mixture. To determine the sigmoidal coefficients, the solver function in EXCEL is used by minimizing the error between Equation (80) and the logarithm of the measured data. The reference temperature is taken as the median test temperature (T_{ref} = 20°C). With the coefficients of Equations (80) and (81), it is then possible, through Equation (79), to find the percent healing at any temperature and rest period combination.
\[
\log |\%H| = a + \frac{b}{1 + \frac{1}{e^{d+e \log(kp_{\mu,1})}}} 
\]  
(80)

\[
\log(a_T) = \alpha_1 T^2 + \alpha_2 T + \alpha_3 
\]  
(81)

Table 3 shows coefficients of the shift factor (SF) function in Equation (81) when the reference temperature is set to T = 20°C.

<table>
<thead>
<tr>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.99E-04</td>
<td>-1.68E-01</td>
<td>3.08E+00</td>
</tr>
</tbody>
</table>

Figure 38, Figure 39, and Figure 40 show the measured percent healing at different temperatures (10°, 20°, and 30°C), rest periods (10, 30, 90, and 270 seconds) and damage levels along with the fitted mastercurves in arithmetic, semi-log and log-log space, respectively. Percent healing values are not available for damage level of 0.5 and below at 10°C because specimens were failed at a C value larger than 0.5 at this testing temperature.
Figure 38. Percent healing mastercurves at various damage levels in arithmetic space

Figure 39. Percent healing mastercurves at various damage levels in semi-log space
Figure 40. Percent healing mastercurves at various damage levels in log-log space.

Table 4 and Figure 41 show the fitting coefficients of percent healing mastercurves for different damage levels. As can be observed the values of parameters $b$ and $d$ are constant.
Table 4. Fitting parameters for percent healing mastercurves

<table>
<thead>
<tr>
<th>Damage Level (C)</th>
<th>0.9</th>
<th>0.8</th>
<th>0.7</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>-0.011</td>
<td>-0.086</td>
<td>-0.242</td>
<td>-0.420</td>
</tr>
<tr>
<td>b</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>d</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>g</td>
<td>0.644</td>
<td>0.525</td>
<td>0.609</td>
<td>0.696</td>
</tr>
</tbody>
</table>

Figure 41. Fitting parameters for percent healing mastercurves vs. damage level
According to Table 4 and Figure 41 the parameters of percent healing mastercurves are either constant or could be represented as a function of damage level. Also the effect of rest period and temperature were already combined in reduced rest period parameter at healing mastercurve formula, Equation (80). So the finalized healing function is:

\[
\log |\% H | = a + \frac{2}{1 + e^{g \log(RP_{red})}}
\]

where

\[
RP_{red} = \frac{RP}{a_T},
\]

\[
\log(a_T) = 6.99 \times 10^{-4}T^2 - 0.168T + 3.08,
\]

\[
a = 1.3839C - 1.2277,
\]

\[
g = 33.529C^3 - 70.305C^2 + 47.949C - 10.006, \text{ and}
\]

\[
C = \text{Pseudo stiffness indicating damage level.}
\]
CHAPTER 6. DEVELOPMENT OF MODEL
CHARACTERIZATION PROCEDURE

In CHAPTER 5 it was proven that asphalt material is thermorheologically simple (TRS) even in the healing region which means that by doing time-temperature shifting based on an arbitrary reference temperature percent healing data would fall on sigmoidal curves called healing mastercurves if they are plotted in reduced rest period domain instead of rest period. A direct conclusion is that applying different rest periods at arbitrary different temperatures same percent healing could be obtained if the resulting reduced rest time is equal for those combinations of temperature and rest period.

Now it is time to take advantage of this property of healing phenomena and reduce the number of necessary tests in order to develop the healing function and form the healing mastercurves at different damage levels for a mixture.

From a practical point of view doing all 12 healing tests (three temperatures at four rest periods) would need four days of testing time which is not appropriate. In the following few pages the fitting procedure is done on healing data but instead of using all healing data from 12 tests in the fitting process, only the results of three tests were used meaning that solver option in Microsoft EXCEL was adopted to minimize the error between measured percent healings and the fitted sigmoidal mastercurve at three points (three tests) only. If it turns out to be a good enough fit the testing time to develop the healing mastercurves would be reduced to only one day which is great.
These three tests must be chosen such that they cover a large range in reduced rest time scale (or equivalently cover a large range along the percent healing axis from a test with very low healing percentage to a test with very high healing percentage). So the three chosen tests to do the curve fitting procedure are as shown in Table 5:

<table>
<thead>
<tr>
<th>Position</th>
<th>Temperature (°C)</th>
<th>Rest Period (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>small reduced RP</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>medium reduced RP</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>large reduced RP</td>
<td>30</td>
<td>270</td>
</tr>
</tbody>
</table>

Figure 42 and Figure 43 show the healing mastercurves following the curve fitting to the three data points shown in Table 5 in semi-log and log-log space, respectively.
Figure 42. Healing mastercurves fit using three selective tests only (semi-log space)

Figure 43. Healing mastercurves fit using three selective tests only (log-log space)
The actual measured healing percentages from all tests are plotted in these two figures as well as mastercurves for different damage levels. As expected tests at the lower temperature (10°C) failed at larger C values (less damage levels) and as a result mastercurves for C = 0.5 and under does not exist. Although the fitting has been performed using three points only Figure 42 and Figure 43 show a very good agreement between mastercurves and actual percent healings.

Table 6, Table 7, and Table 8 represent a numerical comparison between percent healings predicted by the summarized fitting procedure using three tests, i.e., %Healing (2), and those obtained from curve fitting using all 12 tests, i.e., %Healing (1) for temperatures of 10°, 20°, and 30°C, respectively.
Table 6. Comparison of two curve fitting procedures at T=10°C

<table>
<thead>
<tr>
<th>C</th>
<th>RP (Second)</th>
<th>Reduced RP</th>
<th>%Healing (1)</th>
<th>%Healing (2)</th>
<th>Percent difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>10</td>
<td>0.33</td>
<td>6.8</td>
<td>7.8</td>
<td>-13.9</td>
</tr>
<tr>
<td>0.9</td>
<td>30</td>
<td>0.98</td>
<td>9.7</td>
<td>10.6</td>
<td>-9.2</td>
</tr>
<tr>
<td>0.9</td>
<td>90</td>
<td>2.77</td>
<td>13.5</td>
<td>14.2</td>
<td>-4.9</td>
</tr>
<tr>
<td>0.9</td>
<td>270</td>
<td>8.37</td>
<td>19.0</td>
<td>19.1</td>
<td>-0.9</td>
</tr>
<tr>
<td>0.8</td>
<td>10</td>
<td>0.33</td>
<td>6.1</td>
<td>6.2</td>
<td>-0.6</td>
</tr>
<tr>
<td>0.8</td>
<td>30</td>
<td>0.98</td>
<td>8.2</td>
<td>8.2</td>
<td>-0.3</td>
</tr>
<tr>
<td>0.8</td>
<td>90</td>
<td>2.77</td>
<td>10.7</td>
<td>10.7</td>
<td>0.0</td>
</tr>
<tr>
<td>0.8</td>
<td>270</td>
<td>8.37</td>
<td>14.2</td>
<td>14.1</td>
<td>0.2</td>
</tr>
<tr>
<td>0.7</td>
<td>10</td>
<td>0.33</td>
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Table 7. Comparison of two curve fitting procedures at T=20°C

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<th>C</th>
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<th>Reduced RP</th>
<th>%Healing (1)</th>
<th>%Healing (2)</th>
<th>Percent difference</th>
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<td>23.9</td>
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Table 8. Comparison of two curve fitting procedures at T=30°C

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<th>Reduced RP</th>
<th>%Healing (1)</th>
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<th>Percent difference</th>
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</table>
From the percent difference column in these tables it is obvious that the fitted healing mastercurves are about the same using these two different fitting methods. This fact is shown graphically in Figure 44 and Figure 45.

Figure 44. Fitted healing mastercurves using three and twelve tests (semi-log)
At the end of this chapter more results of group-rest healing tests are presented from different points of view by the three methods explained in CHAPTER 5. Parameters of these three methods are extracted using the procedure showed graphically in Figure 23 and are displayed in different spaces to show their relationship with healing factors (temperature, rest period, and damage level before applying rest period). It is obvious that all these parameters ($\Delta S$, $\Delta N$ and $\Delta C_1$) generally follow a clear trend with respect to healing factors so that it may also be possible to find a functional form for their behavior too. This subject is beyond the domain of the presented study and the researcher recommends it as a future task.
Although healing function can be defined based on any of these methods and parameters - as is obvious from changes of these parameters with respect to healing factors - method B was the basis of percent healing calculations in this research because this method uses the internal state variable \(S\) (damage parameter) which is directly connected to the damage and healing concept of the material (increase and reduction of \(S\) is related to the damage and healing respectively).

The whole calculations and presented results for the amount of healing and the proposed healing function in this research were done based on the second method explained in Section 5.2 (method B).

In Figure 46 to Figure 66 changes of \(\Delta C_1\), \(\Delta S\) and \(\Delta N\) with respect to rest period duration, damage level and temperature are shown. Just by looking at these figures one can see the obvious trend of these parameters with respect to the healing factors (damage, temperature and rest period length).

Finding the functional dependency of these parameters to healing factors could be beneficial in the prediction of asphalt concrete material behavior. For example knowing the function which relates \(\Delta N\) to damage level, rest period length and test temperature, it may be possible to predict the increase in fatigue life \((\Delta N_t)\) of asphalt mixtures due to the healing phenomena during a pulse-rest healing test with respect to a continuous fatigue test at the same temperature.
Figure 46. $\Delta S$ vs. rest period for different damage levels ($T = 10^\circ C$)

Figure 47. $\Delta S$ vs. rest period for different damage levels ($T = 20^\circ C$)
Figure 48. ΔS vs. rest period for different damage levels (T = 30°C)

Figure 49. ΔN vs. rest period for different damage levels (T = 10°C)
Figure 50. ΔN vs. rest period for different damage levels (T = 20°C)

Figure 51. ΔN vs. rest period for different damage levels (T = 30°C)
Figure 52. $\Delta C_1$ vs. rest period for different damage levels ($T = 10^\circ C$)

Figure 53. $\Delta C_1$ vs. rest period for different damage levels ($T = 20^\circ C$)
Figure 54. $\Delta C_1$ vs. rest period for different damage levels ($T = 30^\circ C$)

Figure 55. $\Delta S$ vs. $C$ (damage level) for different rest periods ($T = 10^\circ C$)
Figure 56. ΔS vs. C (damage level) for different rest periods (T = 20°C)

Figure 57. ΔS vs. C (damage level) for different rest periods (T = 30°C)
Figure 58. $\Delta C_1$ vs. $C$ (damage level) for different rest periods ($T = 10^\circ C$)

Figure 59. $\Delta C_1$ vs. $C$ (damage level) for different rest periods ($T = 20^\circ C$)
Figure 60. ΔC1 vs. C (damage level) for different rest periods (T = 30°C)

Figure 61. ΔN vs. C (damage level) for different rest periods (T = 10°C)
Figure 62. $\Delta N$ vs. $C$ (damage level) for different rest periods ($T = 20^\circ C$)

Figure 63. $\Delta N$ vs. $C$ (damage level) for different rest periods ($T = 30^\circ C$)
Figure 64. ΔS vs. ΔN for different rest periods (T = 10°C)

Figure 65. ΔS vs. ΔN for different rest periods (T = 20°C)
Figure 66. $\Delta S$ vs. $\Delta N$ for different rest periods ($T = 30^\circ C$)
CHAPTER 7. PREDICTION OF DAMAGE CHARACTERISTIC CURVE FOR PULSE-REST HEALING TESTS

One of the main goals of this research as was discussed and explained from the beginning of this thesis is to run both group-rest and pulse-rest healing tests and try to find a link between them so that just by conducting short group-rest healing tests and using damage characteristic curve of these tests one could find the characteristic curve for long pulse-rest healing tests.

7.1. Steady State Check

A few steps need to be taken before heading to this main goal. Because of the noises associated with the experimental testing and the measurements, VECD formulation and analysis could only be applied to the very first few cycles of a test. In other words, analyzing raw data files of a test using the time-wise VECD formulation would not result in a clear \( C(S) \) curve for the reason of measurements noises are too high to do the analysis in a time step manner. So the first step is to confirm that S-VECD model formulation (cycle-wise analysis method) which was explained in detail in CHAPTER 3 and Equations (71) to (73), is an exact enough approximation of the VECD time-wise method in this kind of tests. Not only because of the noise in experimental measurements S-VECD formulation must be used instead of VECD but using S-VECD formulation also reduces computational time a lot.
For this purpose and for the simplification of pseudo strain integral in Equation (71)a and obtaining Equation (71)b, it must be proven that strain growth is gradual in the domain of the integral (i.e., the steady state condition). According to observations from continuous fatigue test, the sudden increase in strain amplitude happens at the very beginning of the test where the steady state condition has not been reached yet. That is why for the analysis of the very first cycle the exact time-wise VECD formulation is being used to calculate the substantial damage growth in the material caused by this cycle. After the first cycle though, the strain amplitude increases gradually with time until the failure of tested specimen. During this period (from second cycle to failure) because of the steady state condition and gradual increase in strain amplitude, the S-VECD formulation can be employed for the analysis of data files. The same observation can be made for the increase of phase angle during a test.

Figure 67 and Figure 68 show the strain amplitude and phase angle history during a pulse-rest healing test with rest period of 10 seconds, respectively. It is obvious that strain amplitude and phase angle are increasing gradually after a few first cycles and during each test until the failure. This observation suggests that the S-VECD cycle-wise formulation would be sufficient approximation for the VECD formulation and can be used in order to analyze the data from the pulse-rest healing tests.
Figure 67. Strain amplitude for a pulse-rest healing test with RP = 10 seconds and T = 30°C

Figure 68. Phase angle for a pulse-rest healing test with RP = 10 seconds and T = 30°C
7.2. Damage Characteristic Curves of Pulse-Rest Healing Tests

Up to now, it has been shown that the steady state assumption is valid not only for the group-rest and continuous fatigue tests but also for pulse-rest (intermittent) healing tests. Now that this is a proven fact, it would be possible to implement S-VECD formulation into a code and make use of it in order to do the analysis of pulse-rest healing tests. Similar to the previous analysis done on raw data files of continuous and group-rest healing tests, this task was done using the MATLAB software. Figure 69 to Figure 72 are the results of such analysis on the continuous fatigue test and pulse-rest healing tests with 10, 30, and 90 seconds rest periods at the temperature of 30°C.

Figure 69. C(S) curve for pulse-rest test with RP = 10 seconds at T = 30°C
Figure 70. $C(S)$ curve for pulse-rest test with $RP = 30$ seconds at $T = 30^\circ C$

Figure 71. $C(S)$ curve for pulse-rest test with $RP = 90$ seconds at $T = 30^\circ C$
Figure 72 shows that the rate of damage growth in pulse-rest healing tests is much less than that of the continuous test and also more damage can be absorbed by (accumulated in) the specimens in pulse-rest healing tests at the same integrity of the material as expected (higher $S$ value at the same pseudo stiffness $(C)$ with respect to the continuous test).

Of course, the damage growth rate would be less and the specimen could reach higher $S$ values (accumulate more damage without losing the material integrity) when the healing time is longer which means that as was expected the $C(S)$ curve for pulse-rest healing tests with longer rest periods (i.e., 90 seconds) lies on top of those with shorter rest periods (i.e., 10 seconds).
7.3. Prediction of Damage Characteristic Curve of Pulse-Rest Tests Using Palvadi et al.’s Method

Palvadi et al. (2012) proposed a method to quantify the amount of healing in asphalt mixtures. Point A in Figure 73 is the status of specimen before the rest period. Since the damage growth rate in healed specimens during reloading after rest period is much faster than the damage growth in original specimen (curve $C^2$ versus $C^1$ in Figure 73) they concluded that the increase in pseudo stiffness which happens as a result of rest period application (jump from point A to point B) does not reflect the actual amount of healing and some kind of correction must be applied.

A power law function was used to fit to the $C$ vs. $N$ test data before rest period which had the functional form of Equation (83).

$$C^1 = A_1 - A_2 (N)^r$$  \hspace{1cm} (83)

where $A_1$, $A_2$ and $r$ are fitting parameters.

In order to define a corrected pseudo stiffness increase they used the fact that reloading curve is parallel to the original damage curve after point C (region II). So using the fitting parameters obtained from Equation (83) and the measured data from region II along with Equation (84) the horizontal shift of the curve $C^1$, $R$, was determined.

$$C^3 = A_1 - A_2 (N - R)^r$$  \hspace{1cm} (84)
By using the value of \( N \) at which the rest period was applied, \( C_i \) and \( C'_i \) were calculated from Equations (83) and (84), respectively. \( C'_i - C_i \) was considered to be the actual increase in pseudo stiffness due to the applied rest period at point A. Then having the values of \( C_i \) and \( C'_i \), the values of \( S_i \) and \( S'_i \) were back calculated using \( C(S) \) function of tested mixture. This requires that damage characteristic curve, \( C(S) \) to be unique even by introducing the rest period and reloading which is considered to be the main assumption in their research. The healing amount was then quantified based on the following equation:
\[ \%\text{Healing}(C,t) = \frac{S_i - S_f}{S_i} \times 100 \] (85)

In order to predict the damage characteristic curve of pulse-rest healing tests first the researcher attempted to use this approach to check the reasonableness of results. The assumption of uniqueness of \( C(S) \) even when rest period and reloading involve can be considered by adding a new term in damage evolution law. So the new damage evolution law would take the form of Equation (86).

\[ \frac{\partial S}{\partial t} = \left( -\frac{1}{2} \frac{\partial C}{\partial S} \varepsilon^2 \right)^{\alpha} - \frac{\partial W_H}{\partial S} \] (86)

Since the first term on the right hand side of Equation (86) is zero during a rest period, the changes in damage parameter, \( S \), would be all because of the second term. So during rest period Equation (86) will be simplified to the form of Equation (87).

\[ S_f - S_i = \int_0^{t_r} -\frac{\partial W_H}{\partial S} \, dt \] (87)

where \( t_r \), \( S_i \) and \( S_f \) are the rest period length and the values of damage parameter before and after the rest period, respectively. Having \( \%H \) from Palvadi’s work in Equation (85) it is possible to find the value of \( S_f \).

\[ S_f = S_i \left( 1 - \frac{H}{100} \right) \] (88)
Then using $C(S)$ of the material it is possible to find $C_i$ and $C_i$ that correspond to $S_i$ and $S_i$, respectively. So the new point $(S_i, C_i)$ is obtained which would be followed by next load cycle during a pulse-rest healing test. The pseudo stiffness of specimen in the next load cycle is calculated using S-VECD and surprisingly it was observed that reduction of pseudo stiffness caused by this load cycle is less than the increase caused by healing phenomena during the applied rest period.

The result of this analysis is shown in Figure 74 in which instead of having a gradually descending $C(N)$ curve, the method proposed by Palvadi et al. (2012) and their assumption of unique $C(S)$ would result in an ascending $C(N)$ curve that is not acceptable.

Figure 74. Ascending $C(N)$ curve as a result of Palvadi et al.’s approach
7.4. Proposed Method

Looking at the damage characteristic curves of interrupted (group-rest) healing tests and pulse-rest healing tests together in different spaces would be helpful in finding a probable link between these curves and hopefully obtain the curve for pulse-rest healing tests without having to conduct the actual test and just by using the curves from continuous and group-rest healing tests. For this purpose Figure 75 represents continuous test along with the group-rest and pulse-rest tests for different rest periods of 10, 30, and 90 seconds at 30°C.

Finding a link between these curves just by using this representation of $C(S)$ would be hard. Plotting the characteristic curves in other spaces may then be helpful. For this purpose, one should study the best functional forms that can be fitted to these curves. There are two functional forms that fit to the damage characteristic curves of continuous fatigue tests. Equations (89) and (90) show these two functions with their fitting parameters.
Figure 75. Damage characteristic curve for all the tests at T = 30°C

\[ C = e^{(-aS^b)} \] (89)

\[ C = 1 - C_{11}S^{C_{12}} \] (90)

where

\[ a, b, C_{11}, C_{12} = \text{Fitting parameters and } a, b > 0 \]

Equations (89) and (90) can be rearranged and shown in the forms of Equations (91) and (92), respectively:

\[ \log(-\log(C)) = \log(\log(e)) + \log(a) + b \times \log(S) \] (91)
\[ \log(1-C) = \log(C_{11}) + C_{12} \times \log(S) \]  

This basically means that damage characteristic curves would be approximately linear in \(\log(-\log(C))\) vs. \(\log(S)\) and \(\log(1-C)\) vs. \(\log(S)\) spaces. Figure 76 and Figure 77 show the damage characteristic curves of Figure 72 in these two new coordinate systems. Lines are fitted to these curves using Microsoft EXCEL. It is obvious from these two figures that the linear approximation in \(\log(1-C)\) vs. \(\log(S)\) space is more accurate. Now, it seems to be much easier to find a correlation between these curves since they are approximately parallel.

Figure 76. Damage characteristic curves of continuous fatigue test and pulse-rest healing tests in \(\text{LOG} (-\log(C))\) - \(\text{LOG} (S)\) space at \(T = 30^\circ C\)
Since the lines in Figure 77 are approximately parallel, the first idea in order to generate the pulse-rest curves from the continuous curve would be by shifting them along the horizontal axis which is $\log(S)$ and try to fit them into a single line. This idea comes from the fact that asphalt concrete material in the linear viscoelastic range is thermorheologically simple (TRS) which was explained thoroughly at CHAPTER 3.

Figure 78 is the result of this shifting of pulse-rest healing tests along $\log(S)$ axis. Since the objective is to develop the damage characteristic curves of the pulse-rest healing
tests using the one for the continuous fatigue test, the later is considered to be the reference line and so the continuous test curve remains unchanged.

Figure 78. $C(S)$ of continuous test and pulse-rest healing tests in LOG (1 - C) - LOG (S) space after shifting at T = 30°C.

Shifting the curves of pulse-rest tests in Figure 78 was done manually. It is obvious from this figure that a good collapse of these curves is not possible by conducting a horizontal shifting only. It seems that a kind of rotation of the curves is needed as well.
A trial and error method was followed in order to make the collapse of damage curves happen. According to damage growth rate in Equation (67) from CHAPTER 3:

\[
\frac{dS}{dt} = \left[ -\frac{I}{2} \frac{dC}{dS} \varepsilon_R^2 \right]^\alpha
\]

Equation (93) is obtained by applying a new parameter called \( h(C, \text{RP}_R) \) which is assumed to be a function of damage level and reduced rest period in Equation (93).

\[
\frac{dS}{dt} = h(C, \text{RP}_R) \left( -\frac{1}{2} \frac{dC}{dS} \right)^\alpha \varepsilon_R^{2\alpha}
\]

Equation (94) is obtained by applying a new parameter called \( h(C, \text{RP}_R) \) which is assumed to be a function of damage level and reduced rest period in Equation (93).

\[
S \cong \left( h(C, \text{RP}_R) \right)^{\frac{1}{1+\alpha}} \sum_{i=1}^{N} \left[ \frac{I}{2} \left( \varepsilon_i^R \right)^2 \left( C_{i-1} - C_i \right) \right]^{\frac{\gamma_i}{1+\alpha}} \left( t_i - t_{i-1} \right)^{\frac{1}{1+\alpha}}
\]

Integrating both sides of Equation (94) and using the chain rule of \( \frac{dC}{dS} = \frac{dC}{dt} \frac{dt}{dS} \) along with some mathematical simplifications the following numerical scheme is drawn:

\[
S \cong \left( h(C, \text{RP}_R) \right)^{\frac{1}{1+\alpha}} \sum_{i=1}^{N} \left[ \frac{I}{2} \left( \varepsilon_i^R \right)^2 \left( C_{i-1} - C_i \right) \right]^{\frac{\gamma_i}{1+\alpha}} \left( t_i - t_{i-1} \right)^{\frac{1}{1+\alpha}}
\]

This is basically the same as Equation (69) including the new parameter \( h(C, \text{RP}_R) \).

Using trial and error the dependency of this new introduced parameter to damage level and reduced rest period was found to be in the form of Equation (96):

\[
\left( h(C, \text{RP}_R) \right)^{\frac{1}{1+\alpha}} = \frac{1}{a_{RP}}
\]

and

\[
a_{RP} = S^{H(C, \text{RP}_R)}
\]
where

\[ S = S(C) = \text{Damage parameter at different C values in continuous test, and} \]

\[ H(C, RP_R) = \text{Healing index as a function of C and } RP_R. \]

Figure 79 shows the dependency of \( h(C, RP_R) \) to damage level and reduced rest period. In order to show the dependency in a clearer graph \( \frac{1}{h(C, RP_R)} \) is plotted instead of \( h(C, RP_R) \).

![Graph showing dependency](image)

Figure 79. \( \frac{1}{h(C, RP_R)} \) as a function of damage level and reduced rest period
Substituting Equation (97) in Equation (96) the following equation can be derived for

\[ h(C, RP_R) \]

\[ h(C, RP_R) = S^{-\frac{(H(C, RP_R))(1+\alpha)}{}} \]  \hspace{1cm} (98)

If Equation (95) is used for the computation of damage parameter in all of the tests instead of Equation (69), the damage characteristic curves of continuous test and the three pulse-rest tests in Figure 72 will collapse as shown in Figure 80. To be clear S value computed from the new numerical scheme in Equation (95) is called \( S_{\text{Reduced}} \) or \( S_{\text{New}} \).

Figure 80 clearly shows that

\[ S_{\text{New}}(C) = S_{\text{Continuous}}(C) \]  \hspace{1cm} (99)

There is no need to remind that in the case of continuous test (RP = 0) no healing occurs (%H = 0) and Equation (98) results in \( h(C, RP_R) = 1 \). This means that in the case of continuous fatigue test Equations (95) and (69) are identical.
The following relationship is obtained by combining Equations (95) and (96)

\[
S \approx \frac{1}{a_{RP}} \sum_{i=1}^{N} \left[ \frac{I}{2} (e_i^R)^2 (C_{i-1} - C_i) \right]^\alpha (t_i - t_{i-1})^{\nu_{i+\alpha}} 
\]

which means

\[
S_{Reduced} = S_{Continuous} = \frac{S_{Pulse-Rest}}{a_{RP}} \quad (101)
\]

And from Equations (101) and (97)

\[
S_{Pulse-Rest} = S_{Continuous} \times a_{RP} = S_{Continuous}^{[H(C,RP)_{i+1}]} \quad (102)
\]
Once damage characteristic curve of continuous test for a mixture is established it is possible to find corresponding S values at the same damage level (C) for pulse-rest healing tests (at any reduced rest period) using Equation (102) and so predict the damage characteristic curve for any pulse-rest healing test conducted on the same mixture.

7.5. Prediction of C(S) for Pulse-Rest Healing Tests

According to the final conclusion of Section 7.4 it is possible to develop the damage characteristic curve of pulse-rest healing tests of a mixture (at any temperature and rest period) just by having the damage characteristic curve of continuous test and the healing mastercurves data from group-rest healing tests (%H) conducted on the same mixture.

Performing each pulse-rest healing test may take from hours to many days, depending on the test temperature and the length of rest period. If the above explained method turns out to be working well it will dramatically reduce the testing time because in that case there is no need to conduct time consuming pulse-rest tests and it is possible to get the damage characteristic curve of any pulse-rest healing test just by having the results of a continuous fatigue test and three group-rest healing tests that will take only one day to conduct.

Figure 81 shows the result of applying percent healing values obtained from group-rest healing tests to the damage characteristic curve of the continuous test at different damage levels from the beginning of the test to the failure as was explained in Equation (102).

The test data from actual pulse-rest tests are also plotted. As this plot suggests there is a very good agreement between predicted points and actual test data.
Figure 81. Comparison of $C(S)$ curves from actual pulse-rest healing tests and shifting procedure in LOG (1 - C) - LOG (S) space at $T = 30^\circ C$.
Figure 82. Comparison of $C(S)$ curves from actual pulse-rest healing tests and shifting procedure in C - S space at $T = 30^\circ C$

Since this comparison has been made in log-log space, it needs to be proven in the arithmetic space too because sometimes small differences in log-log scale would be tremendous in arithmetic scale. This comparison is shown in Figure 82 for different rest periods of 10, 30, and 90 seconds. It is obvious that the proposed method for the prediction of damage characteristic curves of pulse-rest healing tests works very well in arithmetic space as well as logarithmic space.
CHAPTER 8. VERIFICATION OF PROPOSED HEALING MODEL USING SBS MIXTURE

In CHAPTER 7 a method to develop damage characteristic curve of pulse-rest healing tests was proposed. The inputs of the proposed model are damage characteristic curve of continuous fatigue test and percent healing data obtained from group-rest healing tests and the output is damage characteristic curve of pulse-rest healing test at any desired reduced rest period (i.e., any temperature and any rest period length). Finally the reliability of the model was proven through comparing damage characteristic curves of actual laboratory conducted pulse-rest tests and predicted points using Equation (102) along with the data from continuous and group-rest tests.

In order to fully examine the trustiness of the model there is a need to follow the proposed protocol and see if it also works in a new mixture. This chapter will focus on the application and verification of the model in a mixture that is totally different from RS9.5B mixture which was used up to now.

8.1. SBS Mixture

Unlike the previous mixture which was a loose mixture containing 30% of RAP (Recycled Asphalt Pavement), the specimens that are going to be used in this research study from now on are made from component materials and a polymer modified SBS binder that is considered to heal way better than other frequently used binders and the mixture is called SBS mixture. This new mixture was chosen to be extremely different from the previous one.
regarding the healing properties and age of the mixture to prove the capability of the proposed model in a wide range of material properties.

Figure 83 represents the gradation chart of SBS mixture with Nominal Maximum Aggregate Size (NMAS) of 12.5 mm.

![Figure 83. Gradation chart of SBS mixture](image)

### 8.2. Dynamic Modulus, Phase Angle and Shift Factor Curves

Linear viscoelastic test was conducted on three different specimens of SBS mixture and dynamic modulus, phase angle and shift factor curves were extracted from the analysis of these tests. Figure 84 to Figure 86 show these curves in order.
Figure 84. Dynamic modulus mastercurves of SBS mixture in: (a) semi-log (b) log-log scales
Figure 85. Phase angle curves of SBS mixture

Figure 86. Shift factor curves of SBS mixture


8.3. Comparison of Healing Properties of SBS and RS9.5B Mixtures

According to summarized test protocol three characterization group-rest healing tests were performed on specimens of SBS mixture and percent healing at different damage levels was calculated using the definition of healing index presented in CHAPTER 5.

The temperatures and rest periods of these three tests were chosen in such a way that they form two points at the two ends and one point in the middle of healing mastercurves. Table 9 shows the conditions in which these three tests were conducted.

<table>
<thead>
<tr>
<th>Test ID</th>
<th>Temperature (°C)</th>
<th>Rest Period (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>90</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>270</td>
</tr>
</tbody>
</table>

In order to show the extreme difference in healing properties of SBS and RS9.5B mixtures a comparison was made between percent healings obtained from these three characterization tests and the corresponding group-rest tests from RS9.5B mixture. The result of this comparison is shown in Figure 87, Figure 88, and Figure 89 for the first, second and third characterization test respectively.
It is obvious from these bar charts that as expected SBS mixture in which SBS polymer modified binder was used heals much better than RS9.5B mixture which contains 30% of RAP material.

Figure 87. Comparison of healing properties (T = 10°C, RP = 10 seconds)
Figure 88. Comparison of healing properties (T = 20°C, RP = 90 seconds)

Figure 89. Comparison of healing properties (T = 30°C, RP = 270 seconds)
8.4. Healing Mastercurves for the SBS Mixture

Using solver option in Microsoft EXCEL it is possible to fit a sigmoidal function to percent healing results from the three mentioned characterization group-rest tests at each damage level and get the healing mastercurve at that specific damage level.

The following three figures show healing mastercurves obtained for SBS mixture at all damage levels in arithmetic, semi-log and log-log spaces respectively. As expected percent healing is higher at larger reduced rest periods and any healing mastercurve with larger C value (less damage) lies on top of the other ones that have smaller C values (more damage).

Figure 90. Healing mastercurves of SBS mixture in arithmetic space
Figure 91. Healing mastercurves of SBS mixture in semi-log space

Figure 92. Healing mastercurves of SBS mixture in log-log space
Figure 93 shows characteristic curve of continuous fatigue test for SBS mixture.

![Figure 93. Damage characteristic curve of continuous fatigue test](image)

8.5. Verification Tests

Three pulse-rest healing tests were performed to verify accuracy and reasonableness of the proposed model in CHAPTER 7. Table 10 shows the test conditions in which these three tests were conducted. Comparing Table 9 and Table 10 one could see that the test conditions in which the verification tests were done do not match with those at which the characterization tests were conducted. This was actually done on purpose in order to put the model to test in a more challenging way.
Table 10. Verification tests of SBS mixture

<table>
<thead>
<tr>
<th>Test ID</th>
<th>Temperature (°C)</th>
<th>Rest Period (Second)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>90</td>
</tr>
</tbody>
</table>

Having damage characteristic curve of continuous fatigue test and the healing mastercurves for SBS mixture from Section 8.4 it is possible to predict damage characteristic curves of pulse-rest healing tests listed in Table 10. Rewriting Equation (102):

\[ S_{\text{Pulse-Rest}} = S_{\text{Continuous}}^{[H(RP_k)^{-1}]} \]  \hspace{1cm} (103)

S on the right side of this equation comes from the damage characteristic curve of continuous test and H in the power comes from the healing mastercurves both of which are already obtained in Section 8.4. So every means is available for the prediction purpose.

Then a comparison between the predicted and actual curves shows the reliability of the proposed method in the prediction of damage characteristic curve of pulse-rest tests. Figure 94 and Figure 95 show the result of the explained prediction in comparison with actual tests in arithmetic and semi-log spaces respectively.
These two plots show a very good agreement between actual damage characteristic curves of pulse-rest tests and predicted points. Using these predicted damage characteristic curves it is possible to back calculate the strains in the material and so the displacements in real testing specimens.
Figure 95. Comparison of $C(S)$ curves of actual pulse-rest healing tests and prediction in semi-log space

8.6. Prediction of Strains in the Material

Damage parameter $S$ does not have a very clear physical meaning. So it is desirable to compare a predicted quantity which has a sensible physical meaning with the same quantity in actual test. This task has been accomplished. Since the predicted damage characteristic curve is available through the explained procedure it is possible to back calculate strains using the predicted curve and compare the results with the actual strains in the body of the
material during a pulse-rest test. Figure 96 and Figure 97 show the result of such analysis for a pulse-rest test on RS9.5B mixture.

Figure 96. Strain history in the material using predicted $C(S)$ and back calculation method.
Figure 97. Actual (measured) strain history in the material

The only difference between these two plots is that in actual test there is an accumulative permanent strain in the material during the testing time. That is why the mean strain (centerline in Figure 97) increases with test progress. What matters here is the strain amplitude because in the S-VECD the amplitudes are being used.
Figure 98. Comparison between measured and predicted strain amplitudes

In Figure 98 measured strain amplitudes during the pulse-rest healing test is compared to predicted strain amplitudes and the match is very desirable.

Table 11 shows an example to demonstrate the amount of time that can be saved by following the proposed characterization protocol. As the table represents the testing time needed to conduct the characterization tests (including the replicate testing and analysis) is four days while 60 days of testing is needed to conduct the pulse-rest tests. Three group-rest healing tests and one continuous fatigue test are needed for the proposed protocol which would be a total of four tests. In this example it was assumed that the damage characteristic curve of only 5 different pulse-rest tests is desired. Considering the fact that following the
proposed protocol it is possible to predict the damage characteristic curve of any pulse-rest test (at any temperature and any rest period) it is obvious that tremendous amount of time can be saved using this protocol.

Table 11. An example showing the amount of time saved using the proposed protocol

<table>
<thead>
<tr>
<th>Item</th>
<th>Proposed protocol</th>
<th>Empirical protocol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of tests</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Number of replicates</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Total number of tests</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>Required time (days)</td>
<td>4</td>
<td>60</td>
</tr>
</tbody>
</table>

Figure 99 presents a flow chart that summarizes the proposed test protocol and analysis procedure to obtain damage characteristic curve of pulse-rest tests using the damage characteristic curve of the continuous test and %H data from the group-rest tests.
Figure 99. Flowchart of the proposed protocol

1. Conduct continuous fatigue test and obtain C(S)
2. Obtain the S value that corresponds to different damage levels from C(S)
3. Measure the %H values and construct %H mastercurves
4. Calculate $a_{	ext{HFF}} = S^{|H(RP_{n}, C)}|
5. Shift the damage characteristic curve from the continuous test using $a_{	ext{HFF}}$ factor to obtain the C(S) curve of desired pulse-rest tests
CHAPTER 9. CONCLUSIONS AND FUTURE WORK

Conclusions from this research are as follows:

- Time-temperature superposition (t-TS) concept holds for asphalt concrete material not just in viscoelastic range but also during rest periods when the material heals.

- A summarized test protocol was presented and explained in detail that consists of three group-rest healing tests at appropriate temperatures and rest periods. It was shown that if the percent healing data obtained from these tests are plotted in reduced rest time domain, a series of sigmoidal form healing functions can be fitted nicely to the data for different damage levels. These sigmoidal functions were named healing mastercurves.

- Using the proposed mastercurve healing functions, percent healing of the material at any damage level, rest period and temperature can be obtained without conducting the group-rest healing tests.

- A steady state check was done to make sure that S-VECD formulation can be used for the analysis of pulse-rest healing tests. Results of this study showed that steady state condition is reached in pulse-rest tests after a few first cycles (2-3 cycles) no matter how long the rest period is.

- Three different methods for the calculation of percent healing in the material were presented. The changes of percent healing with respect to damage level, rest period and temperature looks the same using these methods.
• Using the healing mastercurves obtained from the three characterization tests (summarized test protocol) it is possible to predict the amount of healing at different temperatures and damage levels for any rest period with an acceptable accuracy (CHAPTER 6).

• From the results of group-rest healing tests it is obvious that the rate of induced damage in the healed material after rest period (slope of reloading curve) is much more than the slope of the original damage curve for the tested specimen.

• Generally speaking, percent healing in the material has a direct dependency to test temperature and rest period length meaning that by raising the test temperature or increasing the rest time the healing increases as well. Healing has a reverse correlation with damage level. It means that increasing the damage level in the specimen reduces the ability of the material to heal back to its original undamaged state.

• A method was suggested which uses percent healing data (obtained from healing mastercurves) to predict damage characteristic curves of pulse-rest healing tests at any desired rest period and temperature.

• Using the predicted damage characteristic curve of pulse-rest healing test and back calculation method, strain history in the material was predicted. Then it was shown that the predicted strain match very well with the measured strain.

The following subjects are suggested for future research:
• Development of a relationship between damage rate of the healed material during reloading and damage rate of the virgin material.

• Prediction of the C(N) curve for pulse-rest healing tests using a model similar to the model proposed in this dissertation (i.e., using C(N) curve of continuous and group-rest healing tests) and try to link the fatigue life of specimens under pulse-rest healing tests to the fatigue life of specimens in group-rest and continuous tests.

• Investigation of the healing in large cracks (macrocracks) and its strength and importance.

• Investigation of healing at high temperatures where the VECD theory is not valid due to additional viscoplastic strains.

• Dependency of percent healing to other factors such as volumetric percentage of binder in a mixture, aging, and confining pressure.

• Mechanistic Investigation of the $a_{RP}$ parameter and its dependency to the healing potential and damage level of the material.
REFERENCES


APPENDIX A. RESULTS OF S-VECD ANALYSIS OF GROUP-REST HEALING TESTS FOR RS9.5B MIXTURE

Figure 100. Average modulus of LVDTs 1 and 3 with T = 10°C and RP = 10 seconds

Figure 101. Average phase angle of LVDTs 1 and 3 with T = 10°C and RP = 10 seconds
Figure 102. Damage characteristic curve with T = 10°C and RP = 10 seconds

Figure 103. Average modulus of LVDTs 2 and 4 with T = 10°C and RP = 30 seconds
Figure 104. Average phase angle of LVDTs 2 and 4 with $T = 10^\circ C$ and $RP = 30$ seconds

Figure 105. Damage characteristic curve with $T = 10^\circ C$ and $RP = 30$ seconds
Figure 106. Average modulus of LVDTs 1 and 3 with $T = 10^\circ$C and $RP = 90$ seconds

Figure 107. Average phase angle of LVDTs 1 and 3 with $T = 10^\circ$C and $RP = 90$ seconds
Figure 108. Damage characteristic curve with $T = 10^\circ C$ and RP = 90 seconds

Figure 109. Average modulus of 4 LVDTs with $T = 10^\circ C$ and RP = 270 seconds
Figure 110. Average phase angle of 4 LVDTs with $T = 10^\circ C$ and $RP = 270$ seconds

Figure 111. Damage characteristic curve with $T = 10^\circ C$ and $RP = 270$ seconds
Figure 112. Average modulus of 4 LVDTs with $T = 20^\circ C$ and $RP = 10$ seconds

Figure 113. Average phase angle of 4 LVDTs with $T = 20^\circ C$ and $RP = 10$ seconds
Figure 114. Damage characteristic curve with $T = 20^\circ C$ and $RP = 10$ seconds

Figure 115. Average modulus of 4 LVDTs with $T = 20^\circ C$ and $RP = 30$ seconds
Figure 116. Average phase angle of 4 LVDTs with $T = 20^\circ C$ and $RP = 30$ seconds

Figure 117. Damage characteristic curve with $T = 20^\circ C$ and $RP = 30$ seconds
Figure 118. Average modulus of 4 LVDTs with $T = 20^\circ C$ and $RP = 90$ seconds

Figure 119. Average phase angle of 4 LVDTs with $T = 20^\circ C$ and $RP = 90$ seconds
Figure 120. Damage characteristic curve with $T = 20^\circ C$ and $RP = 90$ seconds

Figure 121. Average modulus of 4 LVDTs with $T = 20^\circ C$ and $RP = 270$ seconds
Figure 122. Average phase angle of 4 LVDTs with $T = 20^\circ C$ and $RP = 270$ seconds

Figure 123. Damage characteristic curve with $T = 20^\circ C$ and $RP = 270$ seconds
Figure 124. Average modulus of 4 LVDTs with $T = 30^\circ C$ and $RP = 10$ seconds

Figure 125. Average phase angle of 4 LVDTs with $T = 30^\circ C$ and $RP = 10$ seconds
Figure 126. Damage characteristic curve with $T = 30^\circ C$ and $RP = 10$ seconds

Figure 127. Average modulus of 4 LVDTs with $T = 30^\circ C$ and $RP = 30$ seconds
Figure 128. Average phase angle of 4 LVDTs with $T = 30^\circ C$ and $RP = 30$ seconds

Figure 129. Damage characteristic curve with $T = 30^\circ C$ and $RP = 30$ seconds
Figure 130. Average modulus of 4 LVDTs with $T = 30^\circ C$ and $RP = 90$ seconds

Figure 131. Average phase angle of 4 LVDTs with $T = 30^\circ C$ and $RP = 90$ seconds
Figure 132. Damage characteristic curve with T = 30°C and RP = 90 seconds

Figure 133. Average modulus of LVDTs 2 and 4 with T = 30°C and RP = 270 seconds
Figure 134. Average phase angle of LVDTs 2 and 4 with $T = 30^\circ$C and $RP = 270$ seconds

Figure 135. Damage characteristic curve with $T = 30^\circ$C and $RP = 270$ seconds
APPENDIX B. MATLAB CODES USED IN THE ANALYSIS

The MATLAB codes used for the analysis of raw data files of tests in this research are presented in this appendix. Outputs of each code are inputs for the next code, so the codes must be run consecutively.

1. Reading Raw Data Files and Calculation of Stress and Strain

```matlab
clc;
clear all;
close all;
format long e
%% Inputs
dt=0.001;
ER=1;
Area=pi/4*(75/25.4)^2; % inch2
L=100; % mm
f=10; % Frequency
np_cycle = floor(1/(f*dt)); % number of sample point in each cycle
Omega=2*pi*f; % Angular Frequency
Eps=5*dt; % Range for computing Strain Drift
NDriftUpdate=5;
ApproximateN=4000000;
Psi2KPa=6.894757293168361;
NFilterCycle=10;
TruncLoad=50;
Prony_Folder='C:\Users\Morteza Ashouri\Desktop\C-S (1st Set, T=30, Rp = 10, 30, 90, 270)\T=30, Rp=30';
Prony_File='EmCoefficients75-150-2-20.00C reference.pcf';
Test_Folder='C:\Users\Morteza Ashouri\Desktop\C-S (1st Set, T=30, Rp = 10, 30, 90, 270)\T=30, Rp=30';
Test_File='Healing-';
nTest_File=57;
```
%% Read File
% read Prony File
idProny=fopen([Prony_Folder '\ Prony_File]);
Data = fscanf(idProny,'%s',[1 1]);
Einf = fscanf(idProny,'%f',[1 1]);
Data = (fscanf(idProny,'%f %f',[2 inf])).';
ShiftFactor=5.03 * 10^-2;
Rhoi=Data(:,1);
Rhoi=Rhoi*ShiftFactor;
Ei=Data(:,2);
% read Test File
InitialTime=zeros(nTest_File+1,1);
index=1;
for i=0:nTest_File
idTest=fopen([Test_Folder '\ Test_File num2str(i)]);
Data = fscanf(idTest,'%s',[1 1]);
j=strfind(Data,:');
Data=Data(j+1:end);
t0 = str2num(Data);
tempvar=fscanf(idTest,'%s',[1 4]);
Data=(fscanf(idTest,'%f %f %f %f %f %f %f %f %f %f %f %f',[11 inf])).';
fclose(idTest);
read_data(i+1).LVDT = Data(:,3:6);
n = size(Data,1);
read_data(i+1).time = (t0+dt:dt:t0+n*dt).';
read_data(i+1).stress = Data(:,2)/Area*Psi2KPa;
end
%% cutting begining and ending part of the files
for i = 0:nTest_File
for icomp = 1:5
if icomp < 5
value = read_data(i+1).LVDT(:,icomp);
else
value = read_data(i+1).stress;
end
```matlab
% cutting beginning part
if i == 0
    % do nothing
else
    dv = diff(value);
    if dv(1)>0
        ind = find(dv<=0,1,'first');
        value(1:ind-1) = [];
        time(1:ind-1) = [];
    elseif dv(1)<0
        ind = find(dv<=0 & value(2:end)>value(1), 1, 'first');
        value(1:ind-1) = [];
        time(1:ind-1) = [];
    else
        % do nothing
    end
end

% cutting ending part
value = rot90(rot90(value));
time = rot90(rot90(time));
dv = diff(value);
if dv(1)>0
    ind = find(dv<=0,1,'first');
    value(1:ind-1) = [];
    time(1:ind-1) = [];
elseif dv(1)<0
    ind = find(dv<=0 & value(2:end)>value(1), 1, 'first');
    value(1:ind-1) = [];
    time(1:ind-1) = [];
else
    % do nothing
end
value = rot90(rot90(value));
time = rot90(rot90(time));
```
% saving data
modified_data(i+1,icomp).time = time;
modified_data(i+1,icomp).value = value;
end
end

%% filling gaps
for icomp = 1:5
final_data(icomp).value = [];
final_data(icomp).time = [];
end
% merging condition
% zero for copying from left - 1 for copying from right
merg_cond = zeros(nTest_File+1,1);
% merg_cond(4) = 1;
i = 0;
for i = 0:nTest_File-1
if merg_cond(i+1) == 1 % copy from right
for icomp = 1:5
gapb = modified_data(i+1,icomp).time(end)+dt;
gape = modified_data(i+2,icomp).time(1)-dt;
ngap = round((gape-gapb)/dt + 1);
num_cycle = floor(ngap/np_cycle);
fill_time = (gapb:dt:gape).';
num_fill = length(fill_time);
% adding complete cycles
fill_data = zeros(ngap,1);
fill_data(1:ngap,1) = modified_data(i+2,icomp).value(1:ngap); % copy data
final_data(icomp).value = [final_data(icomp).value ;
modified_data(i+1,icomp).value];
final_data(icomp).value = [final_data(icomp).value ; fill_data];
final_data(icomp).time = [final_data(icomp).time ;
modified_data(i+1,icomp).time];
final_data(icomp).time = [final_data(icomp).time ; fill_time];
end
else % copy from left
for icomp = 1:5
    gapb = modified_data(i+1,icomp).time(end)+dt;
    gape = modified_data(i+2,icomp).time(1)-dt;
    ngap = round((gape-gapb)/dt + 1);
    num_cycle = floor(ngap/np_cycle);
    fill_time = (gapb:dt:gape).';
    num_fill = length(fill_time);
    fill_data = zeros(num_fill,1);
    eind = length(modified_data(i+1,icomp).value);
    % adding complete cycles
    fill_data(1:num_cycle*np_cycle) = modified_data(i+1,icomp).value(eind-
num_cycle*np_cycle+1:eind); % copy data
    % adding remaining data points
    fill_data(num_cycle*np_cycle+1:end) = modified_data(i+1,icomp).value(eind-
np_cycle+1:eind-num_cycle*num_fill-num_cycle*np_cycle,1);
    final_data(icomp).value = [final_data(icomp).value ;
modified_data(i+1,icomp).value];
    final_data(icomp).value = [final_data(icomp).value ; fill_data];
    final_data(icomp).time = [final_data(icomp).time ;
modified_data(i+1,icomp).time];
    final_data(icomp).time = [final_data(icomp).time ; fill_time];
end
end
end

% Analysis Inputs
nt = zeros(5,1);
for icomp = 1:5
    nt(icomp) = length(final_data(icomp).time);
end
nt = min(nt);
Time = (0:dt:(nt-1)*dt).';
Stress = final_data(5).value(1:nt);
LVDT(:,1) = final_data(1).value(1:nt);
LVDT(:,2) = final_data(2).value(1:nt);
LVDT(:,3) = final_data(3).value(1:nt);
LVDT(:,4) = final_data(4).value(l:nt);

%% plotting
figure;
plot(Time,Stress,'b.');
figure;
plot(Time,LVDT(:,1),'b.');
figure;
plot(Time,LVDT(:,2),'b.');
figure;
plot(Time,LVDT(:,3),'b.');
figure;
plot(Time,LVDT(:,4),'b.');

2. Calculation of Modulus and Phase Angle

StartLoad=[14.547 45.557 80.559 125.563 195.566 310.568 450.572 680.575 1010.575 1490.576]; % Start Loading Cycle
EndLoad=[15.547 50.557 95.560 165.563 280.566 420.569 650.571 980.573 1460.574 1760.994]; % End Loading Cycle
InitialTime=StartLoad(1);
InitialIndex=find(Time>=InitialTime,1);
Time=Time(InitialIndex:end);
Stress=Stress(InitialIndex:end);
InitialDisp=mean(LVDT(1:InitialIndex-1,:),1);
LVDT=LVDT(InitialIndex:end,:);
NumLVDT=length(InitialDisp);
Strain=zeros(size(Stress,1),NumLVDT);
for i=1:NumLVDT
Strain(:,i)=(LVDT(:,i)-InitialDisp(i))/L;
end

% total Number of Data points
nData=length(Time);
% Number of points in cycle
N=round(1/(f*dt));
NCycles=floor(nData/N)+1;
CenterStress=zeros(nData,1);
CenterStrain=zeros(nData,NumLVDT);
AmpStress=zeros(nData,1);
AmpStrain=zeros(nData,NumLVDT);
ThetaStress=zeros(nData,1); % Stress Phase angle
ThetaStrain=zeros(nData,NumLVDT); % Strain phase angles
Estar=zeros(nData,NumLVDT);
PAngle=zeros(nData,NumLVDT);
iDriftUpdate=NDriftUpdate;
PlotIndex=zeros(nData,1);
for jLoad=1:length(StartLoad)
iBegin=find(Time>=StartLoad(jLoad),1);
 iEnd=find(Time>=EndLoad(jLoad),1);
PlotIndex(iBegin:iEnd)=1;
for i=iBegin:N:iEnd
  % i start of each cycle
  j=i+N-1;
  if j>iEnd
    j=iEnd;
  end
  CycleStress=Stress(i:j);
  CycleStrain=Strain(i:j,:);
  % average responses
  AveStress=mean(CycleStress);
  AveStrain=mean(CycleStrain,1);
  % center Stress
  CenterStress(i:j)=CycleStress-AveStress;
  % Stress Coeffs
  AStress0=mean(CenterStress(i:j));
  AStress1=2*mean(CenterStress(i:j).*cos(Omega*Time(i:j)));
  BStress1=2*mean(CenterStress(i:j).*sin(Omega*Time(i:j)));
  % Stress Amplitude and Phase Angle
  AmpStress(i:j)=sqrt(AStress1^2+BStress1^2);
  if AStress1>=0
    ThetaStress(i:j)=atan(-BStress1/AStress1);
  else
    ThetaStress(i:j)=atan(-BStress1/AStress1)+pi;
end

end
% Strain Amplitude
for iLVDT=1:NumLVDT
if iDriftUpdate==NDriftUpdate
if i+NDriftUpdate*N-1<size(Time,1)
x=Time(i:i+NDriftUpdate*N-1)-Time(i);
y=Strain(i:i+NDriftUpdate*N-1,iLVDT);
else
x=Time(i:end)-Time(i);
y=Strain(i:end,iLVDT);
end
p=polyfit(x,y,1);
D=p(1);
end
CenterStrain(i:j,iLVDT)=CycleStrain(:,iLVDT)-D*(Time(i:j)-Time(i))-AveStrain(iLVDT);
% Strain Coeffs
AStrain0=mean(CenterStrain(i:j,iLVDT));
AStrain1=2*mean(CenterStrain(i:j,iLVDT).*cos(Omega*Time(i:j)));
BStrain1=2*mean(CenterStrain(i:j,iLVDT).*sin(Omega*Time(i:j)));
% Strain Amplitude and Phase Angle
AmpStrain(i:j,iLVDT)=sqrt(AStrain1^2+BStrain1^2);
if AStrain1>=0
ThetaStrain(i:j,iLVDT)=atan(-BStrain1/AStrain1);
else
ThetaStrain(i:j,iLVDT)=atan(-BStrain1/AStrain1)+pi;
end
end
if iDriftUpdate==NDriftUpdate
iDriftUpdate=0;
end
iDriftUpdate=iDriftUpdate+1;
end
end
% Calculating E* and Phase Angle
for iLVDT=1:NumLVDT
    Estar(:,iLVDT)=AmpStress./AmpStrain(:,iLVDT); % E* - Modulus
    theta0 = -(ThetaStrain(:,iLVDT)-ThetaStress(:,1));
    theta0 = mod(theta0,pi);
    PAngle(:,iLVDT)=((theta0)*180/pi); % Phase Angle
end
figure;
plot(Time(PlotIndex==1),CenterStress(PlotIndex==1),'x',Time(PlotIndex==1),
AmpStress(PlotIndex==1),'x');
ylabel('CenterStress');
xlabel('Time');
legend('CenterStress','Amplitude');
figure;
plot(Time(PlotIndex==1),CenterStrain((PlotIndex==1),1),'x',Time(PlotIndex==
1),AmpStrain((PlotIndex==1),1),'x');
ylabel('CenterStrain1');
xlabel('Time');
legend('CenterStrain1','Amplitude');
figure;
plot(Time(PlotIndex==1),CenterStrain((PlotIndex==1),2),'x',Time(PlotIndex==
1),AmpStrain((PlotIndex==1),2),'x');
ylabel('CenterStrain2');
xlabel('Time');
legend('CenterStrain2','Amplitude');
figure;
plot(Time(PlotIndex==1),CenterStrain((PlotIndex==1),3),'x',Time(PlotIndex==
1),AmpStrain((PlotIndex==1),3),'x');
ylabel('CenterStrain3');
xlabel('Time');
legend('CenterStrain3','Amplitude');
figure;
plot(Time(PlotIndex==1),CenterStrain((PlotIndex==1),4),'x',Time(PlotIndex==
1),AmpStrain((PlotIndex==1),4),'x');
ylabel('CenterStrain4');
xlabel('Time');
legend('CenterStrain4', 'Amplitude');
figure;
plot(Time(PlotIndex==1), Strain(PlotIndex==1,:), 'x');
ylabel('Strain');
xlabel('Time');
legend('1', '2', '3', '4');
figure;
plot(Time(PlotIndex==1), PAngle(PlotIndex==1,:), 'x');
ylabel('Phase Angle');
xlabel('Time');
legend('1', '2', '3', '4');

% E* - Modulus
figure;
plot(Time(PlotIndex==1), AmpStress(PlotIndex==1)./AmpStrain((PlotIndex==1), 1), 'bx');
hold on
plot(Time(PlotIndex==1), AmpStress(PlotIndex==1)./AmpStrain((PlotIndex==1), 2), 'rx');
hold on
plot(Time(PlotIndex==1), AmpStress(PlotIndex==1)./AmpStrain((PlotIndex==1), 3), 'gx');
hold on
plot(Time(PlotIndex==1), AmpStress(PlotIndex==1)./AmpStrain((PlotIndex==1), 4), 'kx');
hold on
ylabel('Modulus');
xlabel('Time');
legend('1', '2', '3', '4');
hold off

% Averaging of Good LVDTs
ilVDT=[1 2 3 4];

% Strain Averaging
AveStrain=mean(Strain(:, ilVDT), 2);
figure;
plot(Time(PlotIndex==1), AveStrain(PlotIndex==1), 'x');
ylabel('Ave Strain');
xlabel('Time');
% E* Averaging
AveEstar=mean(Estar(:,iLVDT),2);
figure;
plot(Time(PlotIndex==1),AveEstar(PlotIndex==1),'
');
ylabel('Ave E*');
xlabel('Time');
% Phase Angle Averaging
AvePAngle=mean(PAngle(:,iLVDT),2);
figure;
plot(Time(PlotIndex==1),AvePAngle(PlotIndex==1),'
');
ylabel('Ave Phase Angle');
xlabel('Time');

3. Calculation of Pseudo Strain

% Analysis
iLVDT=[1 2 3 4];
% Strain=CenterStrain;
Strain=mean(Strain(:,iLVDT),2);
nTime=length(CenterStress);
Time=Time(1:nTime);
PStrain=zeros(nTime,1);
Eta0=zeros(length(Ei)+1,1);
for n=1:nTime-1
EtaNext=zeros(length(Ei)+1,1); % +1 for Einf
EtaNext(1)=Einf*Strain(n+1); % Inf Term
EtaNext(2:end)=exp(-dt./Rhoi).*Eta0(2:end)+Ei.*exp(-
dt./(2*Rhoi))*(Strain(n+1)-Strain(n)); % Porny Terms
PStrain(n+1)=1/ER*sum(EtaNext);
Eta0=EtaNext;
end
PStrain=PStrain; % KPa
figure;
plot(Time,PStrain,Time,CenterStress);
ylabel('PseudoStrain (CenterStress)');
xlabel('Time');
figure;
plot(PStrain,CenterStress);
ylabel('CenterStress');
xlabel('PseudoStrain');

% total Number of Data points
nData=length(Time);
% Number of points in cycle
N=round(1/(f*dt));
nCycle=floor(nData/N);

CenterPStrain=zeros(nCycle*N,1);
AmpPStrain=zeros(nCycle*N,1);
ThetaPStrain=zeros(nCycle*N,1);

for i=1:N:nCycle*N
    % i start of each cycle
    j=i+N-1;
    if j>nData
        j = nData;
    end
    CyclePStrain=PStrain(i:j);
    % average responses
    AvePStrain=mean(CyclePStrain);
    % center Stress
    CenterPStrain(i:j)=CyclePStrain-AvePStrain;
    % PStrain Coeffs
    APStrain0=mean(CenterPStrain(i:j));
    APStrain1=2*mean(CenterPStrain(i:j).*cos(Omega*Time(i:j)));
    BPStrain1=2*mean(CenterPStrain(i:j).*sin(Omega*Time(i:j)));

    % PStrain Amplitude and Phase Angle
    AmpPStrain(i:j)=sqrt(APStrain1^2+BPStrain1^2);
    ThetaPStrain(i:j)=atan(-BPStrain1/APStrain1);
end

figure;
plot(Time(1:nCycle*N),CenterPStrain,Time(1:nCycle*N),CenterStress(1:nCycle*N));
ylabel('CenterPseudoStrain (CenterStress)');
xlabel('Time');
figure;
plot(CenterPStrain,CenterStress(1:nCycle*N));
ylabel('CenterStress');
xlabel('CenterPseudoStrain');
figure;
plot(PStrain,Stress(1:nTime));
ylabel('Stress');
xlabel('PseudoStrain');

% Initial Curve
figure;
iStress=Stress(1:20);
iPStrain=PStrain(1:20);
plot(iPStrain,iStress);
ylabel('Stress');
xlabel('PseudoStrain');

4. S-VECD Implementation and Calculation of C(S)

% LVE dynamic modulus
w = f*2*pi;
E_LVE = sqrt( (Einf + sum( (Ei.*w^2.*Rhoi.^2)./(w^2*Rhoi.^2+1)) )^2 + (sum((Ei.*w.*Rhoi)./(w^2*Rhoi.^2+1)) )^2 );

% calculating damage Cycle
I = 1.09711;
alpha = 3.05;
beta = 0;
dXi = dt/ShiftFactor; % Reduced time
in_ind = 48;
C_in = Stress(1:in_ind)./(PStrain(1:in_ind)*I); % damage factor
C_in(1) = 1;
dC_in = diff(C_in);
dS_in = (-I/2 * (PStrain(1:in_ind-1)).^2 .* dC_in).^(alpha/(1+alpha)) * (dXi)^(1/(1+alpha));
S_in = cumsum([0 ;dS_in]);
Filter = 5;
S_cycle = zeros(nCycle/Filter,1);
C_cycle = zeros(nCycle/Filter,1);
jcycle = 0;
for icycle = 1:Filter:nCycle
  jcycle = jcycle + 1;
  begin_ind = in_ind + (icycle-1) * N;
  end_ind = begin_ind + Filter*N -1;
  if end_ind > length(Strain)
    end_ind = length(Strain);
  end
  ind = (begin_ind:end_ind).';
  E = Strain(ind);
  E0pp = max(E)-min(E);
  E_0_ta = 1/ER * (beta+1)/2 * (E0pp * E_LVE);
  S_0_ta = (max(Stress(ind)) - min(Stress(ind)))/2;
  C_cycle(jcycle) = S_0_ta/(E_0_ta*I);
  if icycle == 1
    dC = C_cycle(jcycle) - C_in(end);
  else
    dC = C_cycle(jcycle) - C_cycle(jcycle-1);
  end
  if dC > 0
    dC = 0;
  end
  f_xi = (PStrain(ind))/E_0_ta;
  f_xi_tension = f_xi .* (f_xi>0); % Tension Part of f(Xi)
  dXi_cycle = sum(dXi* (f_xi>0)); % Xif - XiI
  RD = ( 1/dXi_cycle * sum((f_xi_tension).^((2*alpha) * dXi))^(1/(1+alpha)));
  dS_cycle = Filter*(-I/2*E_0_ta^2*dC)^(alpha/(1+alpha)) * ((N-1)*dXi)^(1/(1+alpha)) * RD;
  if icycle == 1
    S_cycle(jcycle) = S_in(end) + dS_cycle;
  else
    S_cycle(jcycle) = S_cycle(jcycle-1) + dS_cycle;
  end
end
C = [C_in ; C_cycle];
S = [S_in ; S_cycle];
figure;
plot(S,C,'b.');
xlabel('S');
ylabel('C');
ylim([0 1]);