ABSTRACT

MEHTA, VIRAJ KIRANKUMAR. Data fusion of multispectral remote sensing measurements using wavelet transform. (Under the direction of Dr. Hamid Krim.)

This thesis focuses on fusion of multispectral data available from remote sensing instruments. The aim is to develop fast and memory efficient algorithms that may be used for real-time implementation aboard satellites. Multiple channel data from the SSM/I instrument are used for experiments. Starting with a Bayesian estimation formulation of the data fusion problem, an attempt is made to take advantage of the sparseness resulting from wavelet transforms to optimize computational efficiency. After generating the necessary statistical models for the data to be estimated, a preconditioning whitening filter, which simplifies the choice of the required wavelet transform, is developed. The significant gains obtained by a compact representation in wavelet basis are shown. An input grid transformation leading to channel filters is then used to construct a real-time implementation of the optimal estimator. Simulated results of such a system are then used to demonstrate the achieved improvement in field resolution.

In conclusion, a direction for future work is laid out for improving the estimation optimality over non-stationarity by adaptive techniques and extension to future instruments.

DATA FUSION OF MULTISPECTRAL REMOTE SENSING MEASUREMENTS USING WAVELET TRANSFORM

by
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Biography

Viraj Mehta was born on March 22, 1980 in Ahmedabad, India. He completed his schooling at Prakash Higher Secondary School in May 1997. He graduated from Nirma Institute of Technology (Gujarat University), Ahmedabad in May 2001 with a Bachelor of Engineering degree in Electronics and Communications. His carried out his bachelor's final year project work at Physical Research Laboratory, Ahmedabad.

He then joined the Masters program in Electrical Engineering at North Carolina State University, Raleigh, NC. There he was a part of the Vision, Information and Statistical Signal Theories and Applications group (VISSTA), and worked on his thesis under the direction of Dr. Hamid Krim. While working towards the master's degree, he also spent summer and fall 2002 interning with Summus Inc., Raleigh, NC.

Acknowledgements

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Chapter 1

Introduction

1.1 Motivation

Remote sensing measurement instruments onboard satellites are increasingly complex and many multi-channel and multi-sensor issues arise as a result. With an insatiable demand for higher resolution, applications have led to an even greater number of channels, which in turn was reflected by a deployment of a large number of sensors, with a daunting quantity of data. From specifications [1] of most of these sensor instruments one can note the typical features that they exhibit. Very recent and current multispectral and hyperspectral instruments like the **M**ulti-**S**pectral **S**canner (MSS) & Thematic Mappers from the Landsat missions, **A**dvanced **S**paceborne **T**hermal **E**mission and **R**eflection Radiometer (ASTER) & **Mod**erate Resolution **I**maging **S**pectrometer (MODIS) onboard the Terra satellite, Hyperion and **A**dvanced **L**and **I**mager (ALI) instruments flown onboard the EO-1 satellite all indicate the trend in satellite-based remote sensing to a greater number of channels with obvious advantages. The information from these channels about vegetation, soil, rocks, oceans, air, is invaluable to applications like vegetation mapping, mineral exploration, military use and environmental conservation to name a few. Not only do the measurements at different frequencies provide different information about different geological, atmospheric or even man-made features, they also provide an opportunity to combine the wealth of data into more

useful enhanced resolution fields that represent all the information contained in the multiple bands in a more compact form.

To comprehensively analyze this data while preserving all the original information, one proceeds to achieve a coherent and composite image by way of clever sensor fusion. The sheer size of the data, non-linearity in the physical models that relates measured data, non-uniform spatial resolution, non-stationarity of the statistics of the data for global coverage and missing data with errors are some of the major impediments in the path towards the goal of efficient fusion. The myriad of applications of interest and constant emergence of new ones has pushed towards using reprogrammable hardware for onboard satellite data processing. This is to enhance flexibility and, most importantly, to reduce communication burdens by limiting the extent to which raw, unprocessed data are transmitted to the ground. It is thus essential to develop faster and more efficient data manipulation algorithms compatible with FPGA implementations: algorithms using only basic and efficient mathematical operations and very limited memory.

1.2 The SSM/I Instrument

The aim of this research work is to thus develop algorithms, which can be used to generate higher resolution fields, while preserving information contained in all the different sensor channels of an instrument. The **S**pecial **S**ensor **M**icrowave/**I**mager (SSM/I) is in that sense an adapted instrument that generates data consistent with our research goals, the results of which may be used to evaluate our algorithms by comparing to more highly performing and more recent instruments.

The SSM/I is a passive microwave radiometer flown aboard **D**efense **M**eteorological **S**atellite **P**rogram (DMSP) satellites. The SSM/I orbit is near circular, sun-synchronous, and near polar, with an altitude of 860 km and an inclination of 98.8°. The orbital period is 102 minutes. This orbit provides complete coverage of the Earth, except for two small circular sectors 2.4° centered on the North and South poles [2]. The SSM/I rotates continuously about an axis parallel to the local spacecraft vertical and measures the upwelling scene brightness temperatures. The absolute brightness temperature of the scene incident upon the antenna is received and spatially filtered to produce an effective input signal or antenna temperature at the input of a feedhorn antenna. The passive microwave radiometer output voltages are then transmitted to the ground base stations.

SSM/I consists of seven channels, each of which may be considered as separate total-power radiometers that simultaneously measure the microwave emission coming from the Earth and the intervening atmosphere. Dual-polarization measurements are taken at 19.35, 37.0, and 85.5 GHz, and only vertical polarization is observed at 22.235 GHz. Spatial resolutions vary with frequency. Table 1 lists the frequencies, polarizations and temporal and spatial resolutions of the seven channels.

Earth observations are taken during a 102.4° arc centered on the spacecraft sub track in the aft direction and corresponds to a 1394 km wide swath, as shown in the figure 1. During each scan, the 85 GHz channels are sampled 128 times over the 102.4° arc. Observations at the lower frequencies are only taken every other scan and are sampled 64 times over the arc. This sampling pattern results in a 12.5 km pixel spacing at 85 GHz and a 25 km pixel spacing at the lower frequencies. Scans during which the lower channels are sampled are called 'A-scans', and the other scans are called 'B-scans' [2].

The SSM/I data is well adapted to the goals of our research. It provides multiple microwave channels that offer an opportunity for fusion to generate an enhanced resolution field. The individual antenna gain patterns are different so that they can also be accounted for. The spatial and temporal resolutions and patterns are different for some channels so that they can be accounted for in the proposed models and help in development of real-time algorithms that are to match the data acquisition rates. In addition, the statistics of the data (as will be shown later) vary notably over global coverage. The use of SSM/I is an example for proposing a common algorithm that is sufficiently generic to be applicable for other for other measurement instruments.

SSM/I Channel		3 dB Footprint (km)		Approximate
Frequency (GHz)	Pol.	Track	Scan	Spacing (km)
19.35	V	69	43	25
19.35	H	69	43	25
22.235	V	60	40	25
37.0	V	37	28	25
37.0	H	37	28	25
85.5	V	15	13	12.5
85.5	H	15	13	12.5

TABLE I
SIZES OF THE 3-dB ANTENNA FOOTPRINTS AND THE APPROXIMATE SPACING OF THE MEASUREMENTS IN THE TRACK AND SCAN DIRECTIONS OF THE SSM/I CHANNELS (TAKEN FROM [3]).
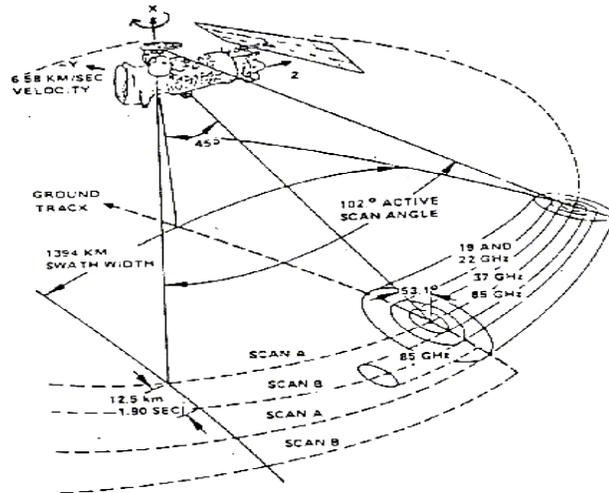
Fig. 1 SSM/I orbit and scan geometry from [4]

1.3 Thesis Overview

The work of this thesis has been organized into 7 chapters. The 1st chapter introduces the reader to the framework for data fusion in remote sensing and provides an understanding of the motivation behind this work and the reasons for selecting SSM/I data for experimental purposes.

In the 2nd chapter we deal with the mathematical formulation of the data fusion problem at hand. The rationale behind choosing a Bayesian estimation framework for fusion, and tools such as the wavelet transform for optimizing the formulation is explained. An overview of the size and statistical nature of the quantities we consider are also provided.

The 3rd chapter deals with statistical analysis of the captured SSM/I sample data to propose a covariance model for the underlying high-resolution field. The finer details of the assumptions, which led to the proposed covariance model, are explained in this, in addition to extending the procedure to more recent instruments. A simple covariance model for measurement errors is also proposed with emphasis on its use to customize weighting of various antenna channels.

In the 4th chapter we first describe the experiments carried out with the basic formulation. Then we carry out some mathematical manipulations to impose a pre-whitening condition on the data to be estimated. We then show, how when combined with a simple 2-D Haar wavelet transform it results in immense savings for memory and computational complexity.

A real-time implementation of the above formulation is described in the 5th chapter. We also explain our approach of using channel filters to represent the acquired data in a common basis where fusion is carried out by a manipulation of coefficients. A MATLAB simulation result for this implementation with a corresponding comparison to the previous matrix-based computations is given.

Chapter 6 offers suggestions to develop methods to adapt the proposed algorithm with ranging statistical parameters. The resulting additional computational costs of an adaptive requirement are also explained. Finally, in chapter 7 we conclude with an outline of the directions in which further work may be carried out.

Chapter 2

Problem formulation for Data Fusion

2.1 The Problem

The first crucial assumption we make in formulating our data fusion problem is that of linearity. We, basically model the measurement signal of the brightness temperatures as a linear combination of the actual field and measurement errors. Consider the following equation:

$$\mathbf{Y} = \mathbf{GX} + \mathbf{E} \tag{1}$$

where, $\mathbf{X}$ denotes the underlying field being measured,

$\mathbf{G}$ denotes the Gain of the measurement antenna,

$\mathbf{E}$ denotes the measurement error, and

$\mathbf{Y}$ denotes the observed measurements.

Note that $\mathbf{Y}$ and $\mathbf{X}$ need to be clearly defined in terms of the actual data available and the field we are attempting to estimate. $\mathbf{G}$ is the gain operator which captures the integration process to yield the measurements. By taking $\mathbf{Y}$ to be all the data measurements available from various channels of the same field, the inverse problem of determining $\mathbf{X}$, then becomes the actual data fusion problem that we are after.

2.2 Bayesian Estimation

The data fusion problem, as described earlier, assumes data from multiple antenna channel measurements. Another way of stating it is to harness and combine all available information from all channel measurements to generate a higher resolution field. Several approaches have been proposed in this regard. The theory [5] is based on the fact that the density of the satellite measurements is higher than the resolution of the instrument, which means that it is possible to take advantage of over sampling to reconstruct a higher resolution image from low-resolution data. Several techniques have been proposed for image enhancement based on a matrix inversion method proposed by Backus and Gilbert [6]. These techniques are based on finding a linear combination of the surrounding measurements to yield images at higher spatial resolution than the original data. These algorithms require matrix inversion at each pixel, which means excessive time requirements. BGI methods are however adapted to noisy measurements by tuning parameters to trade off noise and resolution. Other techniques were based on the theory of image reconstruction and enhancement from noisy irregular samples using algebraic image reconstruction techniques (ART) [7]. Different algorithms based on ART have been adapted [8] to address the problem of resolution enhancement of the remotely sensed data. These adaptations include controlling noise and compensating for the attenuation introduced by the aperture function (Antenna gain function).

Our estimation approach here is Bayesian. Several assumptions are in order before formulating the estimation problem. We can safely assume that the measurement error $\mathbf{E}$ is gaussian distributed with zero mean and with the Covariance Matrix $\mathbf{R}$. In addition,

assuming that $\mathbf{Y}$ and $\mathbf{X}$ are Gaussian distributed i.e. by limiting our analysis to second order statistics and solving Eq. (1), we get the following estimation equation [9],

$$\hat{\mathbf{X}} = (\mathbf{P}^{-1} + \mathbf{G}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{G})^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{Y} \tag{2}$$

Where the only new term $\mathbf{P}$ is the *a priori* Covariance Matrix of $\mathbf{X}$. While this seems to yield a direct solution to the inversion problem, an accounting for the non-stationarity of the measured field and for the nature of the data makes it nontrivial. For example, the non-stationarity implies that Eq. (2) is valid for only local patches of data and that the inversions are required to be carried out at virtually every pixel of the data. This and several other problems are addressed as we focus on the specific formulation for SSM/I.

2.3 Sparseness by Wavelet Transformation

To gain an insight into how we plan to apply a wavelet transform for simplification in our data structure, we proceed as follows. We first effectively precondition the estimation problem using a wavelet transform W,

$$\overline{\mathbf{X}} = \mathbf{WX}, \tag{3}$$

leading to an estimated $\mathbf{X}$:

$$\hat{\overline{\mathbf{X}}} = ((\mathbf{WPW}^{\mathrm{T}})^{-1} + (\mathbf{GW}^{\mathrm{T}})^{\mathrm{T}}\mathbf{R}^{-1}(\mathbf{GW}^{\mathrm{T}}))^{-1}(\mathbf{GW}^{\mathrm{T}})^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{Y}, \tag{4}$$

With a judicious choice of a wavelet and application of a pre-whitening condition, as we later show, a great simplification of the above equations results which in turn impacts the computational costs of matrix calculations and inversions well adapted to FPGA implementations.

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2.4 Formulation for SSM/I

2.4.1 The Input Channels

To specialize the formulation of field estimations to SSM/I, we carefully format them in a vector $\mathbf{Y}$. In all we have seven available data channels namely: 19V, 19H, 22V, 37V, 37H, 85V, and 85H GHz channels. Since each channel is from a 2-dimensional grid, the first step is to vectorize this 2-D data into a 1-D column vector, i.e. $\mathbf{Y}_{85} = \text{Vec}[\mathbf{Y}_{85}]$. We carry out simple vectorization and append various channel data to the same column vector data as $\mathbf{Y} = [\mathbf{Y}_{85}, \mathbf{Y}_{37}, \mathbf{Y}_{22}, \mathbf{Y}_{19}]^{\text{T}}$ where $\mathbf{Y}_{85}$, $\mathbf{Y}_{37}$ … are the vectorized measurements of each channel. We determined that the mean of the data from different channels is different, and hence proceeded to mean removal prior to fusion. In our experiments we only use the mean removed vertical polarization channels at each frequency (four in all). Note that throughout, it is required that the data come from a finite local patch of the underlying field with stationary statistics. We also note that the size of the data coming from different channels is different for the same local patch. Our goal is to generate an underlying field grid that is 4 times finer in resolution in both dimensions than the highest resolution channel available to us (i.e. the 85GHz channels). Thus, for an NxN set of acquired data from 85V, we estimate 4Nx4N=16NxN pixels. The geometry of the measurements is such that the 19V, 22V and 37V channels are sampled at half the resolution (25km as opposed to 12.5 for 85V) and thus for every 4 pixels that the 85V contributes the other channels contribute one pixel each in the input data. This gives us a set of (NxN +3NxN/4)=7NxN/4 input data points. In Fig. 2 we outline the construction of $\mathbf{Y}$ for experiments.
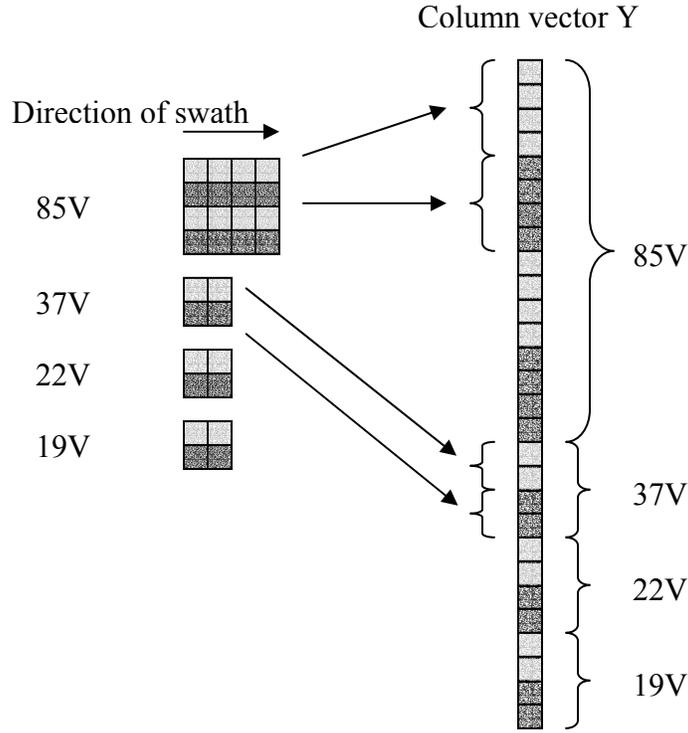
Fig. 2 Reformatting the input data

2.4.2 The antenna gain operator

To proceed with the formulation it is essential to model the antenna gain operator $\mathbf{G}$. According to Eq. (1), the $\mathbf{G}$ matrix operates on the underlying field $\mathbf{X}$ (size 16NxN column matrix) to produce the measured data $\mathbf{Y}$ (size 7NxN/4 column vector as shown before). To construct the required matrix $\mathbf{G}$ of size (7NxN/4) x (16NxN), we first model the individual channel gains. For each of the four channels we assume a jointly binomial gain pattern. For the sake of illustration, we consider the 85V channel. The geometry of the measurements indicates that the 3dB footprint of the channels is 15km in length along the track and 13km in length along the scan while there are measurements available 12.5 km in each of the two directions. Thus, there exists a clear overlap between various measurements. Since for each

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85V measurement, we have an assumed 16 pixels of the underlying field $\mathbf{X}$, the dimension of each pixel in $\mathbf{X}$ is (12.5/4 x 12.5/4) = (3.125 x 3.125) km^2. Now the actual footprint is 15km x 13km and thus approximately the gain pattern averages over a 6x4 patch of pixels of $\mathbf{X}$ (taking 6 instead of 5 for generating a symmetric gain pattern will help simplify the analysis later). Taking a jointly binomial gain pattern over this patch and after accounting for the approximations, we get the following pattern of Fig. 3.
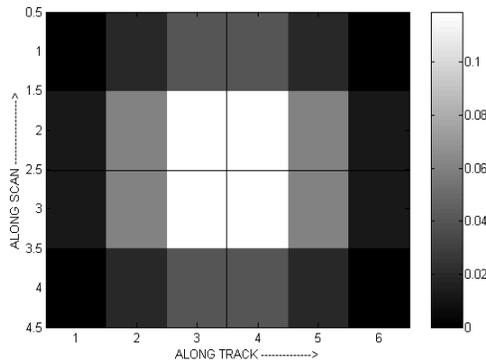
Fig. 3 The gain pattern for the 85V channel (Note: overall gain is normalized to unity)

The above figure shows the gain pattern for a 2D patch of data. The same pattern when made for a gain operator that converts a vectorized underlying field of 20x20 pixels into a vectorized 85V measurement of 5x5 pixels is shown below in Fig. 4.
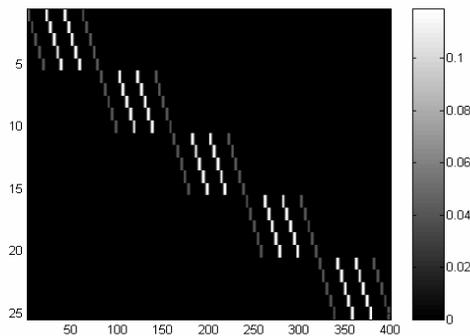
Fig. 4 A sample gain matrix for 85V
(Each set of 5 rows produces one row of 85V measurements in the 5x5 patch)

One can clearly note the overlap between 85V pixels by observing that some columns in the above diagram have more than one non-zero row elements. This overlap is much more evident for the lower frequency channels which have much larger footprints. It is thus clear that there is considerable information in the channels, which may be exploited for estimating a finer underlying field! The overall gain matrix $\mathbf{G}$ for all channels is again constructed by appropriately combining individual gain matrices for all channels in correspondence to the measured data $\mathbf{Y}$ on each individual channel.

Chapter 3

Generation of statistical models

3.1 Requirement

To estimate the underlying field using Equation (2), we need to empirically estimate the *a priori* covariance matrices $\mathbf{P}$ and $\mathbf{R}$ of the field $\mathbf{X}$ and the error in measurement $\mathbf{E}$ respectively. The non-stationarity (non-homogeneity) of the measured field makes the empirical estimation of $\mathbf{P}$ particularly challenging, and requires a careful evaluation of the statistics of the available data. In accounting for the specific data formats, we develop an analytical model which best reflects the empirical covariance statistics. In determining $\mathbf{R}$, some assumptions are made, and we in turn exploit that in weighting different measurement channels.

3.2 Estimating the covariance matrix P of X

3.2.1 Using statistics of 85V channel

For estimating the covariance matrix of the underlying field we use the measurements from the 85V channel. It is closest in terms of resolution to the underlying field than any of the other channels. More importantly, the 85V channel is the channel in which the measurement pixels have least overlap between adjoining pixels. Unlike the channel measurements, the pixels in the underlying field do not have any overlap between them.

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Moreover, if we look at the gain pattern (footprint) of the 85V channel along the scan then we observe that there is hardly any overlap in the scan direction. This is due to the width of the footprint along the scan being 13km and the sampling rate in the same direction being 12.5 km. This is tantamount to saying that if we use the covariance behavior of the rows along the scan direction, is such that its empirical estimate will be closest to the desired model of the underlying field.

3.2.2 Assumptions and procedures

As detailed above, the estimation of the covariance matrix **P** is entailed in the estimation of the covariance matrix for the 85V channel in the single dimension along the scan. Recall that our initial formulation was assumed for a local patch of pixels. In other words, the statistics may drastically vary for a different patch. The same problem is encountered in the course of its empirical estimation, which thus requires that certain procedures be adopted.

By way of experimentation on the 85V data sample we determined that if the global mean is kept zero, then the covariance behavior is highly non-stationary. (This is the single biggest challenge in obtaining the final model.) If we however, subtract the mean over local patches in the entire grid of available data, then the resulting data has a stationary covariance behavior. We develop an adaptive procedure for marching through the entire grid of data to carry out the local mean removal namely "windowing". For every single pixel in the entire grid, we compute the mean of its neighboring values in a window of size 8x8. This mean is then subtracted from the value of the pixel. This procedure achieves the "locally mean removed data", and yields homogeneity (stationarity) in the second order statistics.

The detrended data is subsequently used to estimate the covariance matrix of an 8-pixel row in the scan direction. The average over the entire field of available data is shown in Fig. 5.
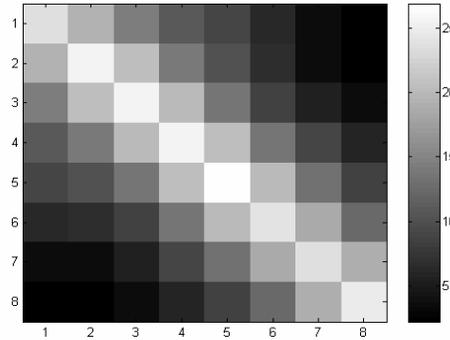
Fig. 5 Covariance Matrix for a row of 8 pixels along the scan direction for an 85V channel

This covariance matrix has a very specific structure. If we in fact consider the anti-diagonal of the above matrix it can then be shown to precisely fit an exponential model! Fig. 6 shows the fitting of the data points from an anti-diagonal and a parallel diagonal (for greater points) to the model

$$C(d) = A * \exp(B \mid d \mid) \tag{5}$$

with A = 26.5375 and B = -0.35398, where |d| is the distance between 2 pixels. We next handle generating the actual covariance matrix **P** using the above model.
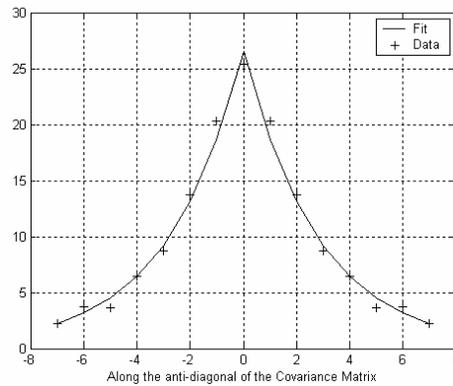
Fig. 6 Fitting of the covariance to an exponential model

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3.2.3 Mathematical model

Eq. (5) is generated for a row of data along the scan direction in the 85V channel. Unlike the 85V the pixels in the underlying field suffer no overlap and their statistics can be assumed isotropic. With this assumption it is straightforward to construct $\mathbf{P}$. We need to keep in mind the vectorization constraints imposed on the column vector for the formatted underlying field. For a swath width N in terms of the underlying pixels one may see that $\mathbf{P}$ has the following form.

$$P(x, y) = C(d) =$$

$$= C\left(\sqrt{(\lfloor x/N \rfloor - \lfloor y/N \rfloor)^2 + (x - \lfloor x/N \rfloor - y + \lfloor y/N \rfloor)^2}\right)$$

$$= A * \exp\left\{-B * \sqrt{(\lfloor x/N \rfloor - \lfloor y/N \rfloor)^2 + (x - \lfloor x/N \rfloor - y + \lfloor y/N \rfloor)^2}\right\} \tag{6}$$

Note that the formulation of the problem for a patch of NxN pixels leads to this special structure of the Covariance matrix. This basically means that since a pixel exhibits high correlation with its neighbors, the covariance structure is high between not only adjoining data points, but also between points at multiples of N because of the vectorization. For example, observe the covariance matrix generated for a patch of 20x20 pixels, (i.e. of size 400x400) as shown in figure 7.
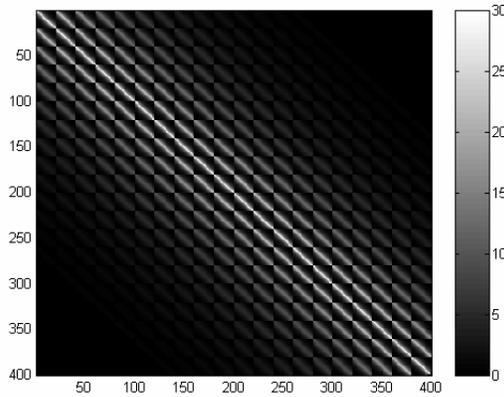
Fig. 7 The final covariance matrix

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3.2.4 Generalization

Even though, the method outlined here for determining **P** is specific to the SSM/I instrument, several key points may be generalized. For any given instrument, the corresponding process would involve the following steps:

- Observe characteristics of sample data to determine what input channel(s) provide statistical data that is closest to the underlying field and thus has minimum overlap.

- Apply statistical modifications (e.g. local mean removal) to selected data to guarantee the imposed assumptions of stationarity.

- Compute the covariance matrix for the relevant input data points.

- If possible, fit this covariance matrix with a mathematical model.

- From the generated model, develop the final form that is adapted to the vectorized field with a specified swath length.

3.3 The covariance matrix for measurement errors

The measurement error **E** in every channel is assumed to be zero mean additive white gaussian noise. Assuming non-correlation and equal variance for one channel, the error covariance matrix is just $\sigma^2 \mathbf{I}$ for each of the channels, where σ is the standard deviation for measurement errors in that channel.

We can have different error variances for different channels. Ideally, one would like to determine empirical estimates of the error variance for subsequent computations. In the absence of dependable empirical results we may however interpret the error variance as a

weighting factor. In other words, by selecting higher error variance for a particular channel we are estimating our underlying field to a lesser extent from it. This turns out to be very useful in the SSM/I setting. Since, the lower resolution channels give us less accurate measurements then the higher ones, we might want to derive our estimation more from the higher resolution channels than the lower ones. This leads us to selecting $\sigma_{19}^2 > \sigma_{22}^2 > \sigma_{37}^2 > \sigma_{85}^2$ where the terms are the measurement error variances of the respective channels.

Note that this still does not provide us with an exact method to determine the individual channel measurement error variances, but merely imposes constraints. We thus have to empirically and a bit qualitatively select them. The final Covariance Matrix R is thus diagonal with elements of the diagonal being the variance of the measurement error for the channel that the measurement corresponds to, i.e.,

$$
\mathbf{R} = \begin{bmatrix}
\sigma_{85}^2 I_{n^2 x n^2} & 0 & 0 & 0 \\
0 & \sigma_{37}^2 I_{n^2/4 x n^2/4} & 0 & 0 \\
0 & 0 & \sigma_{22}^2 I_{n^2/4 x n^2/4} & 0 \\
0 & 0 & 0 & \sigma_{19}^2 I_{n^2/4 x n^2/4}
\end{bmatrix}
\tag{7}
$$

Note that all the zeros in the above matrix indicate that there is no correlation between measurement errors of various channels. Each of the individual error covariance matrices for individual channels contributing to the above matrix is different in size. This is due to the measured data for the underlying field which depends on n which is the width of the local square patch under observation.

Chapter 4

Experiments and optimizations

4.1 Direct method

With the statistical models for the covariance matrices of the underlying field and of the measurement error in place, we proceed to generate fusion results using direct but costly matrix computations. We use Bayesian estimation Eq. (2), which, recall, is written as:

$$\hat{\mathbf{X}} = (\mathbf{P}^{-1} + \mathbf{G}^T\mathbf{R}^{-1}\mathbf{G})^{-1}\mathbf{G}^T\mathbf{R}^{-1}\mathbf{Y}$$

A sample experiment was carried out on a patch of data of size 10x10 pixels in the 85V channel and 5x5 in the other channels. The corresponding size of the underlying field is thus 16 times that of the 85V channel, i.e. 40x40 pixels. Note that in the adopted format the X and Y matrices are column vectors of size 1600 and 175 (100+25+25+25) respectively. The result is shown in Fig. 8.

A clear resolution improvement in the fusion based over the highest resolution measured field is evident. While the optimality of this estimation method under the stated assumptions is unquestionable, the computational cost is prohibitive. For a non-stationary field with a varying covariance matrix the inversion $(\mathbf{P}^{-1} + \mathbf{G}^T\mathbf{R}^{-1}\mathbf{G})^{-1}$, is required to be performed for every stationary local patch. In general, without an adaptive algorithm this would virtually mean inverting a 400x400 matrix twice at every pixel that needs to be estimated (assuming stationarity over a 10x10 patch in the 85V channel). The inversion of the error covariance matrix is not so much a costly affair as it is a diagonal matrix.
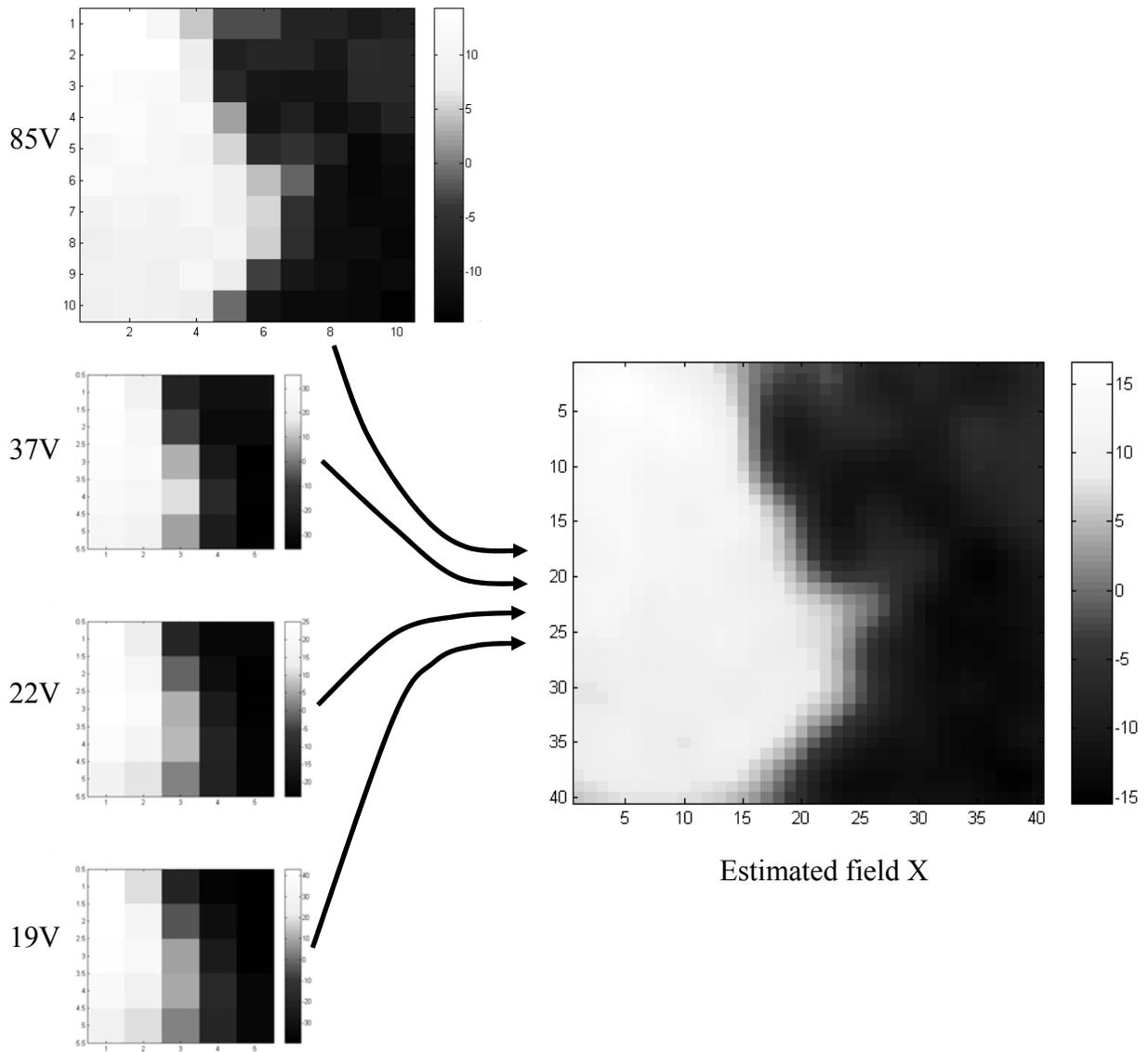
Fig. 8 The estimation result of underlying field X from 4 input SSM/I channels using direct matrix computation. The error variances are assumed to be 48, 36, 16, & 1 for the channels 19V, 22V, 37V & 85V respectively.

4.2 Modification by a pre-whitening filter

To simplify the analysis and improve the estimation technique, we proceed to transform the measurements and hence the development. Clearly, in the process of estimating the underlying field $\mathbf{X}$, we have two sources of information: one that is based on the prior knowledge of the statistical nature of the field and the other being the real time information available as the data is acquired. One way of estimating the field $\mathbf{X}$ could be to combine this information in two sequential data processing blocks as shown in Fig. 9.
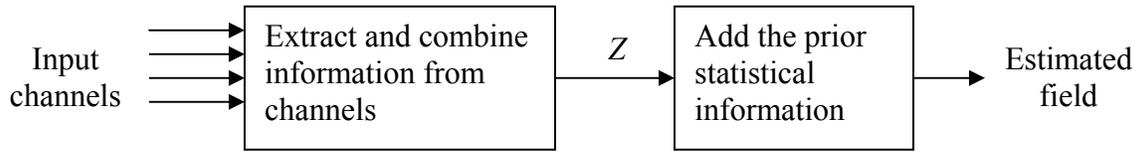

Fig. 9 Block representation of our estimation technique

From this perspective, the intermediate quantity Z in the estimation procedure is free of the prior covariance statistics $\mathbf{P}$ and is thus white.

The key to partitioning the estimation process into two such blocks thus lies in applying a preconditioning transform that makes the estimated field white. We obtain this preconditioning transform by taking the Cholesky Factorization of the *a priori* covariance matrix of $\mathbf{X}$, i.e. $\mathbf{P} = \mathbf{AA}^{\mathrm{T}}$ where $\mathbf{A}$ is a full rank Upper Triangular matrix. Now, letting $\mathbf{F}_{\mathrm{W}} = \mathbf{A}^{-1}$ and rewriting the problem statement yields:

$$\mathbf{Y} = \mathbf{GF}_{\mathrm{W}}^{-1}\mathbf{F}_{\mathrm{W}}\mathbf{X} + \mathbf{E} \tag{8}$$

Defining, $\mathbf{G}_{\mathrm{W}} = \mathbf{GF}_{\mathrm{W}}^{-1}$ we get the Bayesian estimate of the quantity $\mathbf{F}_{\mathrm{W}}\mathbf{X}$ as:

$$\hat{\mathbf{X}}_{\mathrm{W}} = \mathbf{F}_{\mathrm{W}}\hat{\mathbf{X}} = (\mathbf{I} + \mathbf{G}_{\mathrm{W}}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{G}_{\mathrm{W}})^{-1}\mathbf{G}_{\mathrm{W}}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{Y} \tag{9}$$

The above equation is exactly same as Eq. (2) except that the Gain matrix has now changed and the prior on the estimated field is now an identity matrix, making it statistically white! Note that this whole procedure requires the first block in Fig. 9 to restructure the covariance statistics of the input channels, which are subsequently fused. This suggests the possibility of further simplifying the first block by distinguishing the decorrelation and fusion steps as further detailed in the next chapter.

The matrix $\mathbf{F}_W$ is in effect a whitening filter. Fig. 10 shows the structure of the inverse filter matrix with the corresponding impulse response of the whitening filter on the right. Closer observation of Fig. 10(b) reveals the short and long term dependency of a pixel on the neighboring pixels in the same scan and the adjoining scans respectively. Note that in practice we never use the whitening filter, and hence the inverse of the cholesky factorization never needs to be computed. We only use $\mathbf{F}_W^{-1}$ (i.e. $\mathbf{A}$) to "recolor" the estimated quantity $\hat{\mathbf{X}}_W$ as the final step in the estimation process.


(a)   (b)
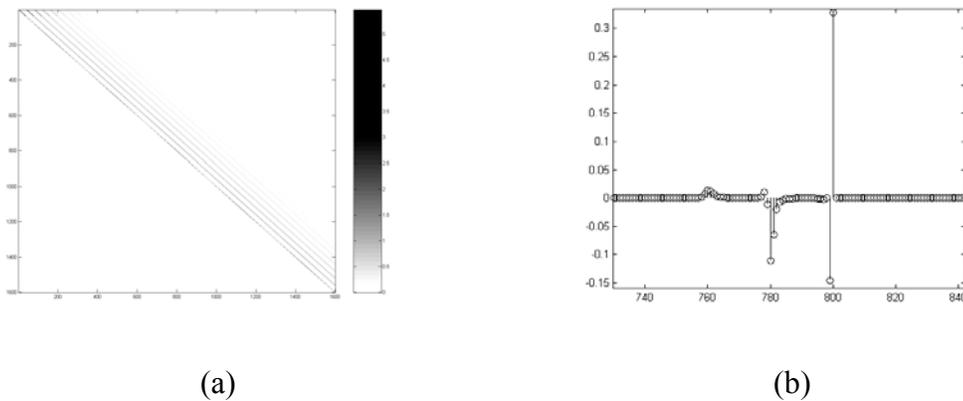
Fig. 10 (a) the inverse filter matrix, (b) the impulse response of the whitening filter

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4.3 Wavelet preconditioning

As indicated earlier, we plan on conquering the daunting computational burden by taking advantage of sparseness resulting from wavelet transform. With the pre-whitening condition imposed, the choice of a suitable wavelet for doing this becomes easier. In our experiments, we carried out wavelet decomposition of the estimated quantity $\hat{\mathbf{X}}_W$ (reformatted as 2-D image) using a 2-D Haar wavelet. We have experimentally established that the only significant coefficients are the low frequency averages at the first level. The remaining coefficients are all close to zero and may hence be thresholded.

$$\text{(a)} \qquad\qquad\qquad\qquad\qquad\qquad \text{(b)}$$
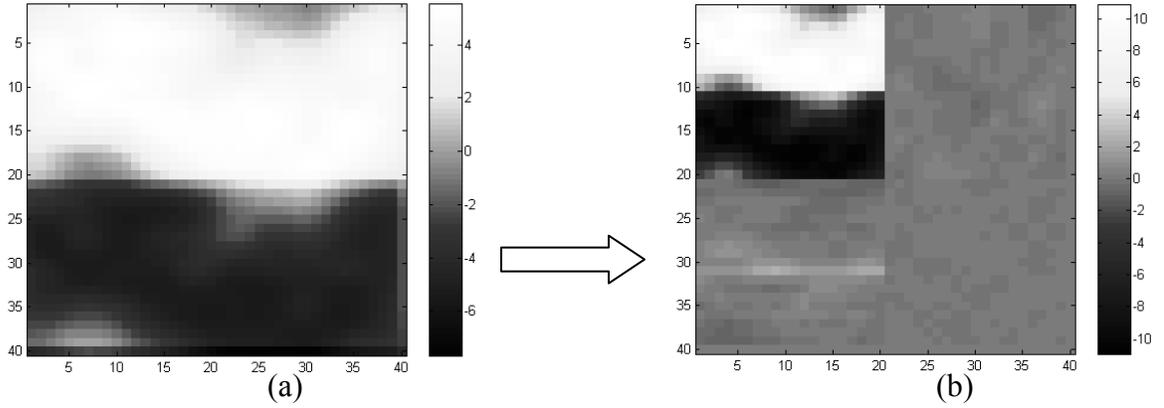
Fig. 11 (a) The reformatted matrix $\hat{\mathbf{X}}_W$ and (b) its level-1 2D Haar wavelet decomposition. This amounts to saying that an application of a wavelet transform $\mathbf{W}_1$ to the vectorized white data $\hat{\mathbf{X}}_W$ will extract the low frequency portion (normalized averages of 4 adjoining pixels) and preserve most of the information while reducing the size of data by a factor of 4. Mathematically, defining $\mathbf{X}_1 = \mathbf{W}_1\mathbf{X}_W$, and $\mathbf{G}_1 = \mathbf{G}_W\mathbf{W}_1^{\mathrm{T}}$ we can write the modified estimation equation as:

$$\hat{\mathbf{X}}_1 = \mathbf{W}_1\mathbf{F}_W\hat{\mathbf{X}} = (\mathbf{I} + \mathbf{G}_1^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{G}_1)^{-1}\mathbf{G}_1^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{Y} \tag{10}$$

$\hat{\mathbf{X}}_W$ $\mathbf{W}_1\hat{\mathbf{X}}_W$
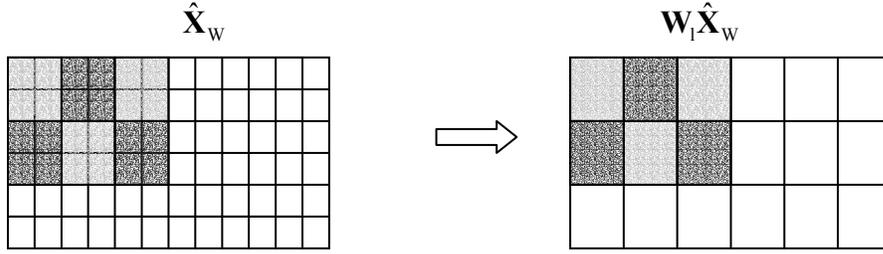
Fig. 12 Pictorial description of the effect of the wavelet transform in 2-D

With these definitions in place, the estimation of the underlying field is a two-step process as originally envisioned in Fig. 9, the first step entailing the estimation of the quantity $\hat{\mathbf{X}}_1$ by means of fusion of the input data channels using Eq. (10), and the second effecting the reconditioning of the estimated quantity by applying an inverse wavelet transform followed by an inverse whitening filter. The appeal of this technique lies in the fact that the intermediate quantity $\hat{\mathbf{X}}_1$ is represented in a compact wavelet basis. This has multiple advantages. From a real-time FPGA implementation viewpoint, the size reduction of the matrices $\mathbf{X}_1$ and $\mathbf{G}_1$ by a factor of 4 in one dimension and smaller size of (a factor of 4 in both dimensions) the inversion $(\mathbf{I}+\mathbf{G}_1^T\mathbf{R}^{-1}\mathbf{G}_1)^{-1}$ provides a significant gain. This leads to great savings in memory and computational time. Moreover, considering the case of an onboard satellite implementation, it makes sense to decrease the communication burdens by representing the estimated data in a compact form. This is again achieved by the wavelet transformation if we carry out the necessary reconditioning filtering at ground base stations.

4.4 Comparison

For the same input data as described in Section 4.1, we now show estimation results with the pre-conditioning simplifications. The field to be estimated before reconditioning is

now of size 400x1(20x20) and the equivalent antenna gain matrix $\mathbf{G}_1$ is 175x400 instead of the earlier 1600x1 and 175x1600 respectively. We have eliminated the necessity for inverting the *a priori* covariance matrix P and the only significant inversion is that of the matrix $(\mathbf{I} + \mathbf{G}_1^T \mathbf{R}^{-1} \mathbf{G}_1)$ which is of size 400x400 instead of the earlier 1600x1600. This is at a cost of computing $\mathbf{G}_1$, which may be taken care of offline and hence at a very minimal additional cost.

(a)   (b)
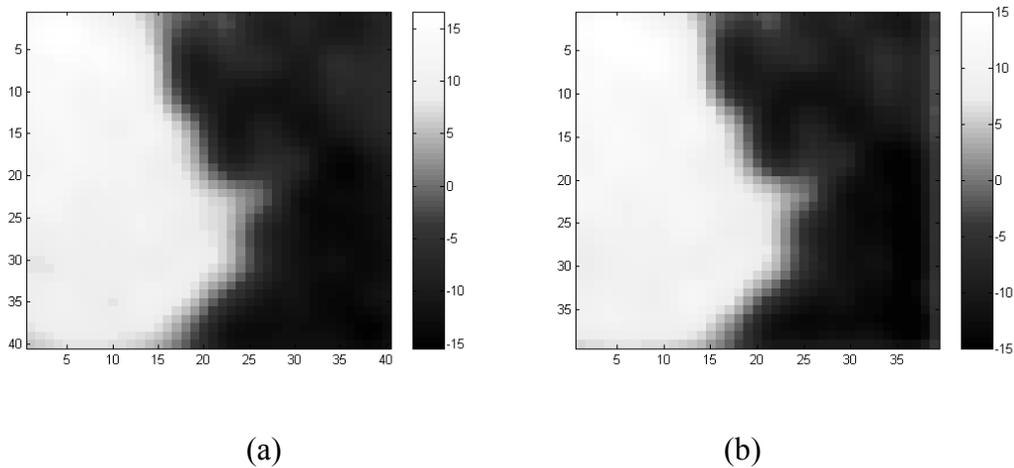
Fig. 13 Results of (a) direct method (b) with preconditioning

The above figure shows the results of the two methods side by side for comparison. Except for some border effect pixels that are introduced because of the nature of the inverse whitening filter, the two methods yield near exact estimation results. The achieved simplifications are also justified from a frequency domain viewpoint. A field typically contains high low-resolution information and low high-resolution information. By imposing the pre-whitening condition our modified field is white meaning that the high frequency components are emphasized. The reduction of coefficients by application of the Haar wavelet transform is then just a de-emphasis of the high frequency components which does not cause loss of information since it is counteracted upon by the pre-whitening preprocessing.

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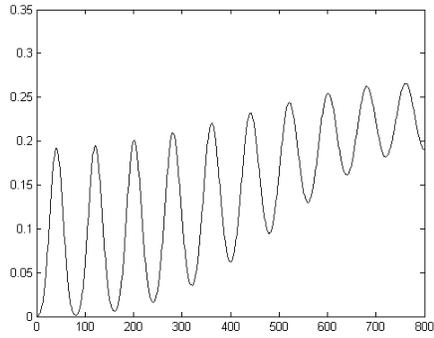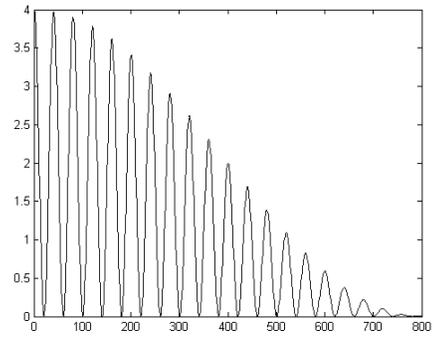
(a)   (b)

Fig. 14 Frequency domain views of the 2-D versions of the (a) whitening filter and (b) the low frequency averaging Haar wavelet. Note how the high frequency de-emphasis of the Haar wavelet transform is counteracted by emphasis of the same by the whitening filter.

Chapter 5

Real-time implementation

5.1 Input grid modification

We now proceed to develop a real-time implementation of the optimized estimation formulation. Looking back at Fig. 9, it is the clear that the focus of the work here is to develop a real-time implementation for the first block, since the second is already in the form of a cascade of the inverse wavelet and inverse whitening filters. As was briefly discussed in Chapter 4, there exists the possibility of simplification of the first block into de-correlation and combination sub-blocks. The need for de-correlation of input channels arises from the fact that the intermediate quantity to be estimated is white, and the combination is just a restatement of fusion.

To visualize the intuition behind the following manipulations aimed towards achieving a real-time implementation, consider the matrix representation of the first block, i.e. $\mathbf{M} = (\mathbf{I} + \mathbf{G}_1^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{G}_1)^{-1}\mathbf{G}_1^{\mathrm{T}}\mathbf{R}^{-1}$. The estimation of the intermediate quantity is then just a multiplication of the input with this matrix, i.e. $\hat{\mathbf{X}}_1 = \mathbf{M}\mathbf{Y}$ and the final estimation is achieved by reconditioning this intermediate quantity, i.e. $\hat{\mathbf{X}} = \mathbf{F}_{\mathrm{W}}^{-1}\mathbf{W}_1^{\mathrm{T}}\hat{\mathbf{X}}_1$. Fig. 15 shows the $\mathbf{M}$ matrix for input restricted to the 85V and 37V channels. (Note: Throughout this chapter we use the 85V and 37V channels only for experimental purposes. Extension to other channels is straightforward.)
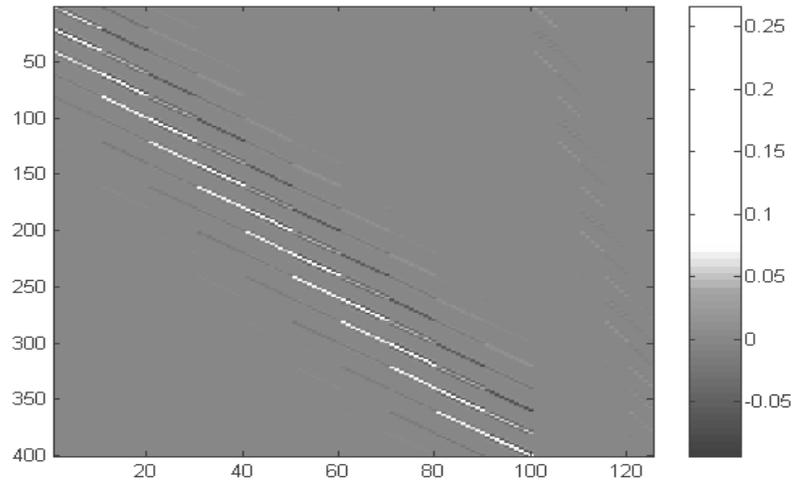
Fig. 15 The M matrix

The dimensions of the **M** matrix shown above are 400x125 for 400 pixels to be estimated for a final field of 1600(40x40) pixels. The first 100 columns indicate operation on the 100 input pixels from a 10x10 field in the 85V channel while the remaining 25 indicate operation on the 25 input pixels from a corresponding 5x5 field in the 37V channel. Thus, the first important conclusion is that we can separate the contributions to a pixel from various input channel as follows:

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_{85} & \mathbf{M}_{37} & ..... \end{bmatrix} \tag{11}$$

where $\mathbf{M}_{85}, \mathbf{M}_{37},...$ correspond to different channels.

The estimation statement follows as:

$$\hat{\mathbf{X}}_l = \mathbf{MY}$$

$$= \begin{bmatrix} \mathbf{M}_{85} & \mathbf{M}_{37} & ... \end{bmatrix} \begin{bmatrix} \mathbf{Y}_{85} \\ \mathbf{Y}_{37} \\ ... \end{bmatrix} = \mathbf{M}_{85}\mathbf{Y}_{85} + \mathbf{M}_{37}\mathbf{Y}_{37} + ... \tag{12}$$

This implies that the operators $\mathbf{M}_{85}, \mathbf{M}_{37}, ...$ can individually de-correlate the input channels, and the estimation of the intermediate preconditioned quantity is then just the addition of the corresponding coefficients for each pixel in the adopted wavelet basis. Let's now consider the portion for the 85V channel first, i.e. the first 100 columns or $\mathbf{M}_{85}$. We notice a certain pattern here. Looking at sets of 10 columns, each column in a set is a vertical shift of its adjoining column by 2 pixels and each set is a vertical shift of the adjoining column by 20 (2x(Swath width)) pixels (one swath is 10 pixels here, since we are estimating from a 10x10 field of the 85V channel). It is now easy to see that our aim of developing a real-time implementation is realized if we carry out some input reformatting that makes every column a vertical shift of the adjoining column by one pixel, i.e. construct a matrix filter to achieve such shifts. This is easily most efficiently accomplished by effectively increasing the 85V channel input sampling rater (to a higher resolution grid corresponding to the quantity to be estimated) by a systematic insertion of zeros. This is pictorially shown in Fig. 16.
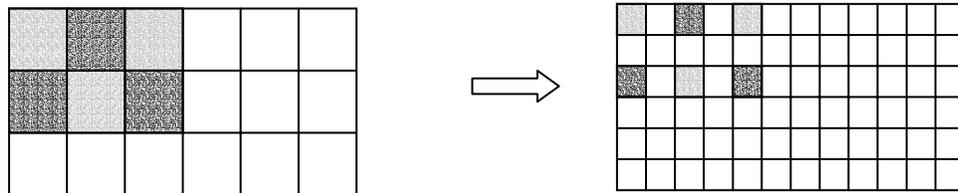
Fig. 16 Pictorial representation of the input modification that leads to a representation of the matrix operator on the input as a filter.

A similar grid modification is required for the 37V channel. Given its original lower sampling rate, more zeros are required for it as the 37V channel. With each $\mathbf{M}$ matrix as a separate filter for each channel, we have a bank of input filters. Let the input data be

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modified from $\mathbf{Y}_{85}, \mathbf{Y}_{37},...$ to $\mathbf{Z}_{85}, \mathbf{Z}_{37},...$ by the aforementioned technique. Now the matrices

$\mathbf{M}_{85}, \mathbf{M}_{37},...$ become $\mathbf{F}_{85}, \mathbf{F}_{37},...$

$$\therefore \mathbf{X}_1 = \mathbf{F}_{85}\mathbf{Z}_{85} + \mathbf{F}_{37}\mathbf{Z}_{37} + ...$$

$$\therefore \mathbf{X}_1 = \mathbf{f}_{85} * \mathbf{Z}_{85} + \mathbf{f}_{37} * \mathbf{Z}_{37} + ... \quad \text{where * are convolution} \tag{13}$$

Thus, $\mathbf{X}_1$ is obtained by filtering $\mathbf{Z}_{85}, \mathbf{Z}_{37},...$ by the filters $\mathbf{F}_{85}, \mathbf{F}_{37},...$ and adding up the output

coefficients. The simplification of the $\mathbf{M}$ matrix into a bank of filters thus completes the real-

time implementation of the optimal estimator.

5.2 MATLAB simulation experiment

We now proceed to show the exact construction of the real-time implementation. Fig.

17 shows the block diagram representation of such a system. It can easily be extended to

include additional channels using a similar approach. On the left hand side is what might be

called a bank of filters for the channels. An "input reformatting" block, which carries out the

input grid modification as previously explained, precedes every channel filter. The outputs of

the various channel filters are added after accounting for synchronization delays. The sum is

then the conditional estimate of the field of interest. The inverse wavelet and "re-coloring"

filters applied to this sum yield the final estimate as shown on the right hand side of the block

diagram. We now explain in detail the construction of each individual block.

5.2.1 Input grid modifier

Let's take the input grid modifier of the 85V channel as an example. Fig. 18 (a)

shows how this can be implemented in a real-time fashion. The input is first up sampled by
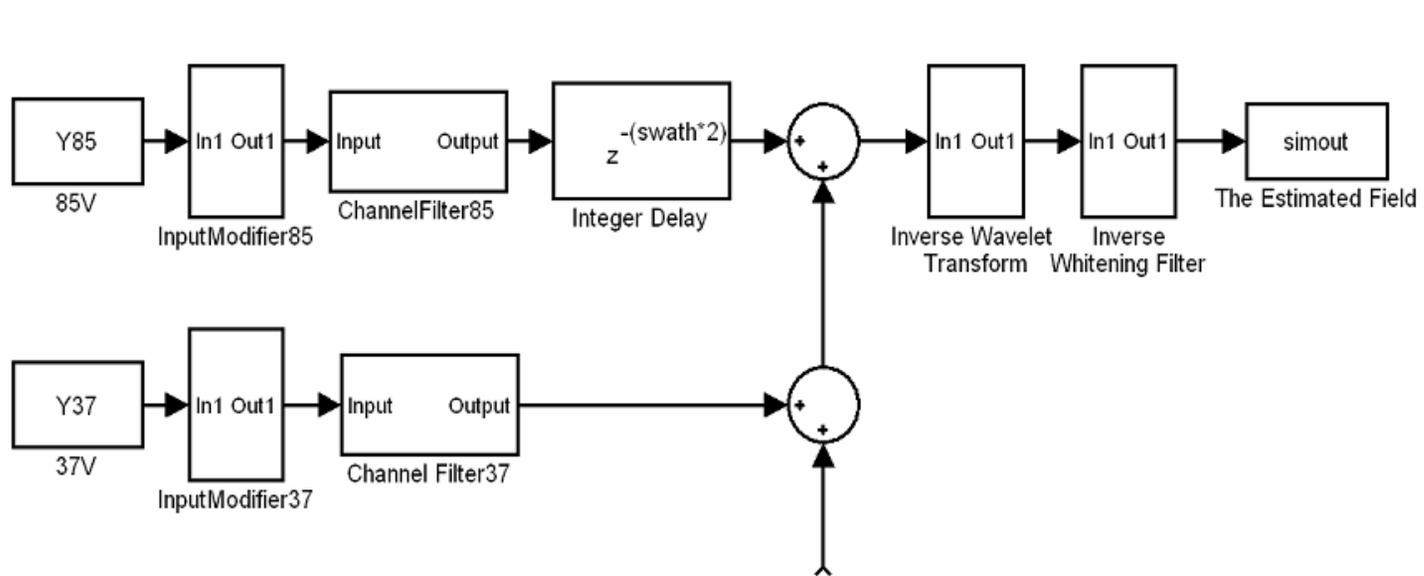
Fig. 17 Block Diagram representation of the real-time implementation of the optimal estimator.

insertion of one zero between every two pixels. The pixels corresponding to one swath are then buffered and appended by an equal number of pixels of value zero that form an entire zero swath. This is now unbuffered to produce the grid-modified input. For the 37V channel, the process is similar and is shown in Fig 18 (b).
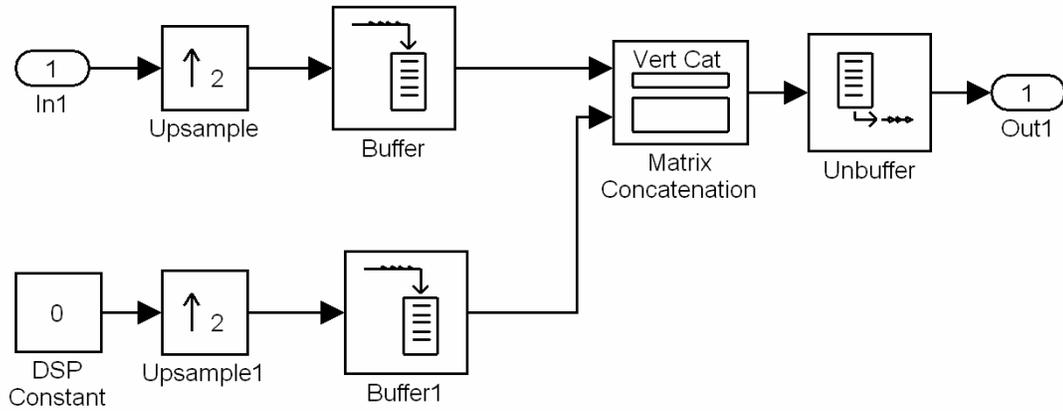
(a)
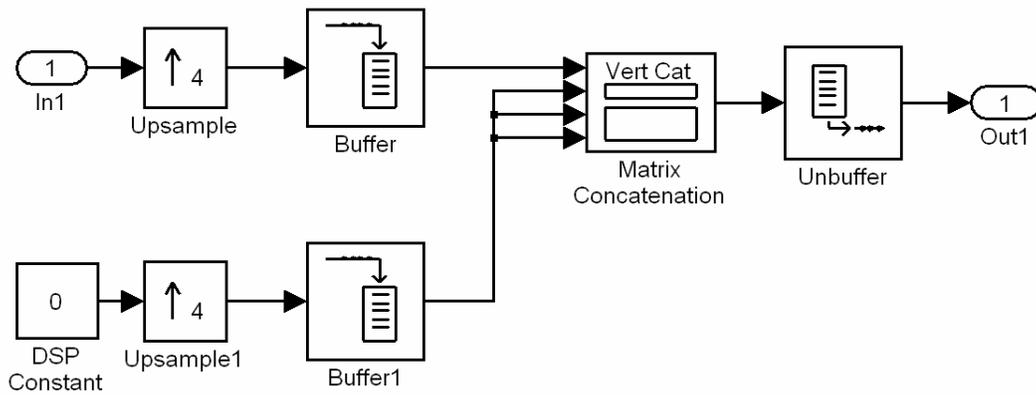
(b)

Fig. 18 The input grid modifiers for the (a) 85V channel and (b) 37V channel.

5.2.2 Channel filter

We again take as an example the construction of the 85V channel filter. The first step is to obtain a column of the $\mathbf{M}_{85}$ matrix, which defines the impulse response of the required filter.
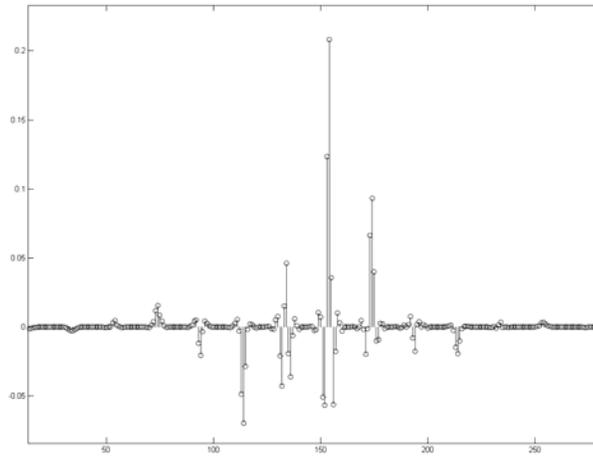
Fig. 19 Impulse response of the 85V channel filter.

We can easily construct a Direct Form I FIR filter with the given coefficients of the impulse response. Even though the column length is 400 for our example, the number of significant values in the impulse response is much smaller, and the resulting filter hence has a compact form. To make the filter independent of the swath width, a crucial observation is in order, namely that the impulse response has significant values in sets of points equivalent to a swath width. The zero values coincide with the middle of any two consecutive sets. This is basically just indicative of the short and long-range dependence of the acquired data on the pixels in the underlying field. For our filter design to be valid for any swath width, we just need to insert zero points between the sets such that the width of the swath still corresponds to the size of the sets. This translates to adding extra delays dependent on swath width

between parallel forward paths of the FIR filter. The construction of filters for other channels is similar.
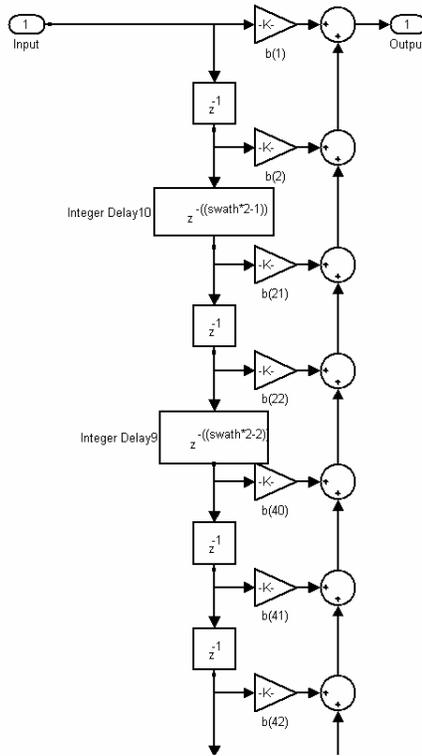
Fig. 20 A section of the Direct Form 1 implementation of the channel filter for 85V channel.
(Note the intermediate integer delays dependent on swath width.)

5.2.3 Inverse wavelet transform

The construction of the re-coloring filter is exactly the same as that of the channel filter, as it also needs care in accommodating the swath width in its structure. The only remaining block is the inverse Haar wavelet transform and is shown in Fig. 21. The multiplicative factor 0.5 occurs because of power normalization. The rest is a process of generating the entire 2D image from the available low frequency average. This just requires a four-field replication of the pixel value, first two times in a row and then a replication of an entire row corresponding to a swath. The buffering and unbuffering shown, is required to achieve filtering for vectorized data.

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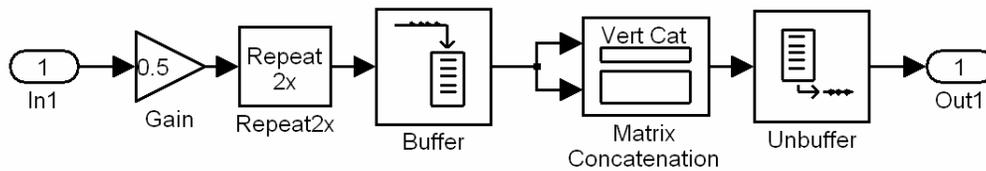
Fig. 21 Implementation of the inverse Haar wavelet transform for vectorized 2D data.

5.2.4 Simulation result

We now show the MATLAB simulation result of the real-time implementation with an 85V channel data size of 256x128 pixels resulting for an estimated data size of 1024x512 $= 2^{19}$ pixels! This is shown in Fig. 22 along with an inset of a smaller size of data to note the improvement in resolution. While the estimation of the inset data seems flawless one can notice some boundary value problems in the estimated image. This is mainly because of the dependence of the rightmost pixels on the leftmost ones as a result of the vectorization. This may be taken care of by traditional methods such as increasing the data points by replication of data at boundaries so as to push the erroneous values out of the required estimation grid. More careful examination reveals unexpected values of estimated pixels around some regions. These are mainly because of the assumption of stationary statistics throughout the image. Adaptive methods to incorporate non-stationarity that improve overall estimation accuracy are discussed in the next chapter.
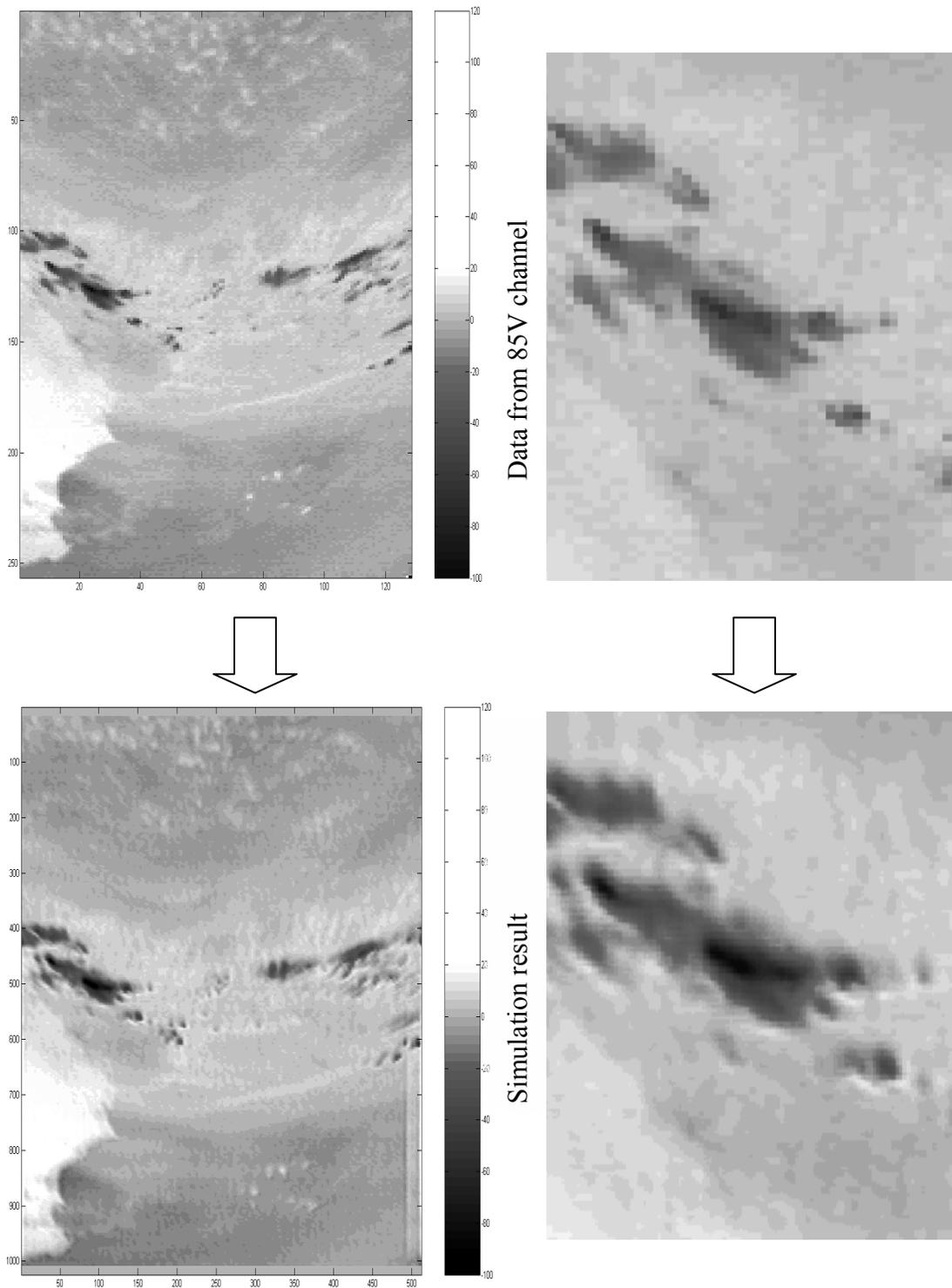
Fig. 22 Comparison of the simulation result with the finest available input data, i.e. the 85V channel. The two diagrams on the right side show a smaller inset of the larger images to indicate the improvement in resolution.

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Chapter 6

Adapting to non-stationarity and errors

6.1 Requirement

Recall, we initially made assumptions of stationarity over fixed 10x10 local patches of data in the 85V channel. Although our empirical estimation of the covariance matrix was based on averaging the covariance statistics over a larger set of data, the mathematical assumptions only guarantee optimal estimation over the local stationary patches. This is evident from the simulation experiments carried out on large data fields. This is especially true for areas where the change in statistics is drastic, like for brightness measurements from borders between sea and ice regions. To optimize the estimator for all such non-stationarity, including instrumental errors in measurement, we need an adaptive method.

6.2 Method

While there may be more than one practical implementation to achieve the required adaptation, we outline the general direction of work here along with the associated costs. Assuming that an exponential covariance model is still valid, we basically need to adapt the two parameters A and B in the model $C(d) = A * \exp(B \, | \, d \, |)$. This amounts to changing the *a priori* covariance matrix $\mathbf{P}$ in the initial Bayesian estimation Eq. (2). In other words we carry out real-time empirical estimation of the two parameters in the covariance model and compute the optimal estimate of the underlying field using the modified $\mathbf{P}$ matrix. The

empirical estimation here may be carried out from the input data from 85V channel based on the assumptions of least overlap direction and isotropy shown earlier. It may also be carried out in a predictive feedback manner from the computed estimation of the underlying field.

The same real-time implementation that we developed for the non-adaptive model may be extended by adaptively modifying the filter coefficients. The inverse whitening filter, which is the last block in the real-time implementation, is directly dependent on the prior covariance matrix (it being the cholesky factorization of the prior). The channel filters, which depend on the modified antenna gain matrix, also depend on the prior. Thus, an adaptive implementation requires a real-time update of the filter coefficients of these blocks. These measures leave the modular structure of the implementation largely unchanged with an additional cost for the improved estimation depending on the complexity of the computing mechanism of the filter coefficients.

Although such an exact computing mechanism is yet to be realized, we expect it to be less or equally time intensive as the forward estimation procedure (even by choosing memory intensive methods if required). In that case, the overall real-time performance of the system would be unaffected.

The other challenge arising with the non-stationarity environment emanates from instrumental measurement errors. While, it is possible to track erroneous measurements by observing statistical discontinuities, often in satellite antenna measurements an instrument already accounts for it. The problem thus reduces to incorporating the information of the erroneous measurement in the estimation model. This is easily done by controlling the variance of error measurement for every channel in the error covariance matrix. For instance, in an adaptive mechanism, an erroneous pixel in the 85V channel would cause an increase of

the error variance for that channel and hence lead to an estimation for the field values that is closer in quality to those of the lower resolution 37V channel. The dependence on expected statistics would ensure that the error be reduced if neighboring pixel values are correct.

Chapter 7

Conclusion

The challenges of efficient data fusion and resolution enhancement in satellite imaging have been the driving motivation for this work. Exploiting the representation gain offered by the wavelet transform, we have proposed a real-time Bayesian estimation algorithm consistent with our goals. The evaluations and validations have been carried out on SSM/I data for which we have also developed empirical statistical models.

In furthering the wavelet transform induced sparseness, we proposed a convenient pre-whitening filtering process. This, in combination with a simple averaging low frequency Haar wavelet transform, led to a modular implementation resulting in tremendous savings in memory and computational time. A comparison of results from the direct method with the modified method showed that the fidelity of the estimation was preserved while keeping the cost low.

Manipulations of the input grid yielded a real-time implementation. The result was a bank of input transformations and channel filters that effectively decorrelated and combined input information and represented them in a compact basis. The inverse wavelet and "recoloring" transforms carried out on this compact information led to the final optimal estimation of the field long sought. The simulation environment of the proposed system was all in MATLAB and showed the enhanced resolution field generated.

In the future work section we have provided insights on how an adaptive implementation may be approached to further improvement of estimation over non-stationarity and measurement errors. In all, we have successfully shown the general approach of using compact wavelet representation to gain simplifications in re-configurable hardware for satellite based image processing. While, some of the assumptions specifically pertained to the SSM/I instrument, the proposed approach and methods are sufficiently generic to apply to multispectral remote sensing instruments and thus should prove to be useful for further research.

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