ABSTRACT

MYERS, MARRIELLE. The Use of Learning Trajectory Based Instruction (LTBI) in Supporting Equitable Teaching Practices in Elementary Classrooms: A Multi-Case Study. (Under the direction of Dr. Paola Sztajn).

Learning trajectories (LTs) are gaining prominence in mathematics education as an empirically validated tool that details students’ mathematical growth and development over time. Research on LTs has occurred along three fronts: a) the development and validation of LTs across different mathematical domains; b) the ways in which LTs can be useful in assessment and curriculum development; and c) how LTs can be useful in instruction. While early research in each of these areas is promising, the academic field is just beginning to understand the ways in which teachers use LTs in instruction. Additionally, very little is known about how teachers’ uses of LTs in instruction support or hinder equitable instruction. Therefore, this study fills a critical gap and explores how teachers’ uses of LTs in their instruction supported four dimensions of equity: access, achievement, identity, and power (Gutierrez, 2007).

The four teachers that participated in this study were each a part of the Learning Trajectory Based Instruction (LTBI) Project, which explored the ways in which teachers use LTs in instruction. This multi-case study explored the potential benefits of LTs and the LTBI model in relation to equitable instruction. Specifically, a theoretical framework was articulated from the literature that posited the ways in which LTBI might support equitable instructional practices. This framework was then empirically tested using a series of interviews and classroom observations. Ongoing and retrospective data analysis led to a revised LTBI Equity framework. Results showed that the LTBI model can support many aspects of equity along the dominant axis and fewer along the critical axis. These results
suggest that while LTs and LTBI are useful constructs for enacting some elements of equitable instruction, they are insufficient alone. Additionally, for teachers with strong deficit perspectives, LTs and LTBI could be misused and create further inequities.
The Use of Learning Trajectory Based Instruction (LTBI) in Supporting Equitable Teaching Practices in Elementary Classrooms: A Multi-Case Study

by
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DEDICATION

For my parents, who sacrificed so much so that I would have the opportunity to pursue my dreams. I love you both dearly.
BIOGRAPHY

Marrielle Myers was born on December 24, 1981, in Durham, North Carolina. She is the proud daughter of Lewis and Cheryl Myers. In 1999, Marrielle graduated from Northern High School in Durham, North Carolina, and left home to attend Hampton University for her undergraduate studies. She graduated from Hampton University in Hampton, Virginia, with honors in May 2003. While at Hampton, she earned a B.S. in Mathematics and became a member of the Gamma Theta Chapter of Alpha Kappa Alpha Sorority, Incorporated. Upon graduation, Marrielle enrolled in graduate school at North Carolina State University (NCSU) where she studied Mathematics Education. Marrielle graduated from NCSU with her Master of Education degree in May 2007. While completing her Master’s degree, Marrielle applied to and was accepted into the doctoral program for Mathematics Education at NCSU.

During her tenure as a graduate student, Marrielle worked as a research assistant on various projects including Nurturing Mathematics Dreamkeepers (NMD), Diagnostic E-Learning Trajectories Approach (DELTA), and Learning Trajectories Based Instruction (LTBI). She also served as a teaching assistant for two undergraduate courses. During this time, she also taught high school mathematics at Word of God Christian Academy and Southeast Raleigh Magnet High School. Additionally, she taught academically gifted students through Duke University’s Talent Identification (TIP) Program as well as SAT Prep with Educational Services Center.
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CHAPTER ONE

Over the past six decades, national initiatives have been developed to address mathematics reform (Grouws, 1992; National Commission for Excellence in Education, 1983; NCTM, 2000). During this time period, scholars and policy makers have debated the best approaches to teaching mathematics. Tensions brewed throughout the 1960’s and 1970’s, leading to our nation’s abandoning the efforts of the New Math dialogues and implementing a Back-to-Basics movement. The 1980’s and 1990’s were laden with dialogue about the deterioration of mathematics education in the United States and its subsequent impact on the workforce. These scathing reports (such as The National Commission on Excellence in Education’s 1983 report “A Nation at Risk: The Imperative for Educational Reform”) were a pre-cursor to the 1989 NCTM Principles and Standards, which highlighted the need for student-centered approaches to learning and the role that research on student learning could play in mathematics education. The release of these standards also spawned numerous research-based curricula. During the last 20 years, the NCTM Standards have been revised and in doing so, the NCTM made a diligent effort to seek the advice of various stakeholders in these revisions (NCTM, 2003). More recently, the NCTM released the report Curriculum Focal Points (2006) in which they outlined critical mathematics topics by grade level as well as specific skills and procedures students should demonstrate.

In the more than 30 years through which NCTM and other organizations continued to work on developing and refining standards, scholars in mathematics education and other content areas were amassing a breadth of research-based knowledge on learning trajectories (Note: Although some scholars and fields use the term learning progressions, I use learning
trajectories and interpret these two constructs as identical). From their inception, learning trajectories (LTs) have shown great promise for several reasons: a) they simultaneously attend to specific skills as well as broader concepts in a domain; b) they focus on the gradual growth and refinement of student thinking over time (NRC, 2007); c) they are empirically developed based on what students have demonstrated (Smith et al., 2006); d) they offer probable pathways of student growth and development as well as misconceptions faced along these paths (Confrey, 2006); and e) they provide a coherent picture of learning as a continual pathway as opposed to isolated events (Heritage, 2008).

Current discourse in mathematics education has transitioned from the usefulness of LTs as a construct to their potential benefits in classrooms. To this end, LTs have gained prominence as an empirically validated tool that can be used to unpack the nuances of student learning over time (Battista, 2004; Clements, 2007; Clements and Sarama, 2004; Confrey et al., 2009; Corcoran, Mosher, & Rogat, 2009). Specifically, the development of the Common Core State Standards (CCSS) drew heavily upon the research on LTs with the goal of articulating standards with more clarity and specificity of student learning over time. With the adoption of the Common Core State Standards (CCSS), teachers are being required to increase their own understanding of mathematics, understand how student knowledge of mathematics develops over time, and use this as a basis for student-centered instruction.

**Learning Trajectories in Instruction**

Research regarding LTs has occurred along three primary fronts: a) developing and validating LTs (Battista, 2006; Clements & Sarama, 2007; Maloney & Confrey, 2010); b) using LTs to design curriculum and assessment (NRC, 2007); c) exploring the ways teachers
use LTs in instruction (Bardsley, 2006; Edgington, 2012; Mojica, 2010; Wilson, 2009).
Regarding the latter, previous research on LTs has shown promise. Using knowledge of
student thinking as represented by LTs, teachers have: a) set goals based on students’
developmental level (Clements, 2007); b) described student work with greater detail (Wilson,
2009); c) assessed students more effectively (McKool, 2009); and d) anticipated students’
strategies as well as misconceptions (Edgington, 2012).

Building from this work, the Learning Trajectories Based Instruction (LTBI) project
took a direct approach to develop a conceptual model for how LTs could be used in various
facets of instruction. Specifically, this research project addressed the ways in which LTs
inform lesson planning, instruction, and assessment. The research team posited what each of
these aspects of instruction looks like when informed by LTs and developed a 60-hour
professional development program.

While initial results of LTs and the LTBI model as a tool to support student-centered
mathematics practices are positive (Edgington, 2012; Wilson, 2009), this work has not
focused on whether or not these outcomes are equitable. Specifically, when using LTs in
instruction:

1) Do teachers set goals for all students in the same manner?

2) Do teachers describe students’ work in similar ways? Is what counts as evidence for
one student the same as evidence for another?

3) Are all students assessed in the same manner and how do teachers use LTs to
interpret the results of this assessment?
4) How do teachers anticipate for different groups of students and use these anticipations to design meaningful instruction?

5) Do teachers use LTs to provide all students with opportunities to grow and progress over time?

The aforementioned questions are not intended to indicate that LTs are inherently problematic; rather, they are meant to offer a rationale for the work conducted in this study and describe a critical gap that the results of this study fills. These questions highlight the need for the work conducted in this dissertation in conjunction with a framework for equity to understand the ways in which LTs can be used to support equitable instructional practices.

**Purpose and Significance of the Research Questions**

While many scholars praise the potential of LTs, the field has yet to move beyond teachers’ uses of LTs at the classroom level to understand if LTs are used equitably in instruction. To study this phenomenon, the first part of this research includes a detailed review of literature related to LTs and equity. Specifically, Gutierrez’ (2007) framework for equity was selected as a way to frame this study. The first part of this study was the development of a conceptual framework, which provides indicators of how LTs could support equitable instruction along the four dimensions Gutierrez’ (2007) outlines: access, achievement, identity, and power.

This framework under-girded a multi-case research study that investigated the ways in which teachers use LTs and LTBI to enact equitable instructional practices along four dimensions of equity: access, achievement, identity, and power. Specifically the following research questions were addressed in this study:
1. In what ways do teachers use LTs and LTBI to provide access to students in their classrooms?

2. In what ways do teachers use LTs and LTBI to promote achievement for students in their classrooms?

3. In what ways do teachers use LTs and LTBI to help students develop their identity?

4. In what ways do teachers use LTBI to empower students in their classrooms?

This study is significant in its attempt at generating theory about how LTs can support equitable instruction. One important outcome of this work is a refined LTBI-Equity framework. Further, the discussion of uses and tensions that arose from data analysis sheds light on critical factors that scholars must consider when designing and implementing content-only based professional development.

**Overview of Methodological Approach**

This multi-case study is a part of a larger design study focused on developing a conceptual model for the ways teachers use their knowledge of LTs in instructional practice. Because the goal of this work was to examine the conceptual framework in light of teachers’ practices, a case study approach was used (Eisenhardt, 1989; Merriam, 1998; Yin, 2009). This approach allows for in-depth analysis of an individual teacher as well as cross-case analysis to understand themes across teachers (Merriam, 1998; Yin, 2009). The study focused on the ways in which teachers used LTs and LTBI to teach equitably in their classrooms as well as the tensions that arose in doing so. Results from this study describe what teachers’ enactment of equitable LTBI instructional practices looked like and offer insight into why some aspects of equity are more clearly aligned with LTs than others.
Outline of the Dissertation

This dissertation is organized into five chapters. In this first chapter, I discussed the rationale for this work and situated it in respect to current trends in mathematics education. In Chapter Two, I discuss my theoretical framework and the ways in which it shapes this study. I also review and synthesize literature about learning trajectories as well as equity. I conclude the chapter with a presentation of my conceptual framework, which was under investigation in this study. Chapter Three begins with a presentation of the refined research questions and a description of the methodology, participants, and data analysis methods that were used to answer the questions. In Chapter Four, I provide detailed case profiles of each of the four participants as well as a cross-case analysis to answer the research questions. In Chapter Five, I frame the findings in light of the research questions, present a revised framework for LTs and equity, and discuss the contributions, implications and limitations of the work. I conclude Chapter Five by suggesting areas for future research related to this topic.
CHAPTER TWO

In this chapter, I present the theoretical framework that guides this study. Next, I review the literature that is relevant to this work. The literature review begins with a discussion of learning trajectories including their emergence over the years as well as the benefits espoused for teachers and students. I also include a discussion of the critiques of learning trajectories both from extant literature as well as from a Critical Race Theory (CRT) perspective. Next, I present various definitions of equity that exist in the literature and tensions that arise from a synthesis of these definitions. Afterwards, I discuss the potential for using frameworks of students’ thinking as a baseline for equitable instruction and cite promising examples from the literature, specifically two cases from the Cognitively Guided Instruction (CGI) Project. I conclude this chapter with a presentation of the conceptual framework that will be used to direct this study along with the research that was used to assist in reinterpreting Gutierrez’ (2007) equity framework in light of its application in LTBI.

Theoretical Perspective

Various theoretical frameworks have been used to study different aspects of mathematics education. Two theoretical perspectives informed this study: social constructivism and critical theory (specifically, critical race theory or CRT). Although they are not always discussed in tandem, I will give a brief overview of the theories followed by a discussion of why both theories are necessary for this study.

Social constructivism is grounded in constructivist theory. Constructivism, which draws heavily from the work of Piaget (1970), seeks to understand what makes “the human mind go from a state of less sufficient knowledge to a state of higher knowledge” (pp. 12-
Piaget believed that any time we encounter a situation; we try to make sense of it. Individuals first attempt to make sense of a new situation by assimilating it into previous situations they have experienced. If this does not work, a modification of a current scheme must occur providing an accommodation for the new experience (Piaget, 1975). We are constantly in a state of trying to make sense of the world; therefore, assimilation and accommodation are always active processes. While Piaget’s theory did consider the interaction between the learner and the object, it did not focus on to the social environment in which the learner and object are located.

Lev Vygotsky, a pioneer of social constructivism, posited that knowledge is constructed in social settings. Therefore, to understand knowledge construction we must seek to understand not only the learner→object interaction, but also the learner→object→setting interaction. Davis and Sumara (2002) stated that the fundamental shift social constructivism makes is “[the] departure from the Kantian/Piagetian assumption that the learner is the locus of cognition” and that human cognition is “more diffuse, distributed, and collective” (p. 414). This was a crucial distinction in theories of learning. Where Piaget (1970) argued that new situations or occurrences force us to seek equilibration, Vygotsky (1978) believed that a new experience coupled with a more experienced person (a teacher or peer) is actually the impetus for learning. This is the cornerstone of the zone of proximal development (ZPD), which he articulated as:

\[
[T]he \ distance \ between \ the \ actual \ developmental \ level \ as \ determined \ by \ independent \ problem \ solving \ and \ the \ level \ of \ potential \ development \ as \ determined \ through
\]
problem solving under adult guidance or in collaboration with more capable peers (p. 86).

The activity and setting in a social constructivist theory now become the locus of cognition. Specifically, “learning awakens a variety of internal developmental processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers” (Vygotsky, 1978, p. 90).

A key premise of social constructivism is that the culture we are in shapes how we think, and how we think, in turn, shapes our culture. Whereas the work of social constructivism was a step forward in acknowledging forces outside of the learner, it did not go far in identifying oppressive facets of the social environment, specifically race and culture. Where Vygotsky and other theorists of the social constructivism paradigm loosely defined culture as the classroom, the school, and the environment in which learning takes place, critical theorists would problematize such a loose definition and argue that failing to acknowledge race and culture, particularly of those marginalized in society, disregards how learning and cognition may differ for those students. In particular, critical theory suggests that it is not sufficient to simply acknowledge that social influences exist, especially when these influences cast some students to the periphery.

Thus, the second perspective used to guide this study is critical theory. Kincheloe, McLaren, and Steinberg (2011) listed the following assumptions of critical theory:

• All thought is fundamentally mediated by power relations that are social and historically constituted;
• Facts can never be isolated from the domain of values or removed from some form of ideological inscription;

• The relationship between concept and object and between signifier and signified is never stable or fixed and is often mediated by the social relations of capitalist production and consumption;

• Language is central to the formation of subjectivity (conscious and unconscious awareness);

• Certain groups in any society and particular societies are privileged over others and, although the reasons for this privileging may vary widely, the oppression that characterizes contemporary societies is most forcefully reproduced when subordinates accept their social status as natural, necessary, or inevitable;

• Oppression has many faces, and focusing on only one at the expense of others (e.g., class oppression versus racism) often elides the interconnections among them; and finally

• Mainstream research practices are generally, although most often unwittingly, implicated in the reproduction of systems of class, race, and gender oppression. (p. 164)

While critical theorists focus on different aspects of society (e.g., race, religion, gender, socio-economic status), these approaches in education are concerned with how the larger ills of society manifest in schools and how schools “sort, select, favor, disenfranchise, silence, or privilege particular groups of students or people” (Bogdan & Biklen, 2007, p. 23). Inherent in critical theory are issues of power, morality, and justice. The purpose of critical
research, then, is to expose injustice and “transform existing social inequalities” (McLaren, 1994, p. 168).

Newer to the field of mathematics education is CRT. This perspective of CRT emerged in the 1970’s with its initial articulation in the legal field (Tate, 1997). The scholars, lawyers, and social activists who defined themselves as critical race theorists set out to understand race and racism and the implications of these constructs in the post-civil rights society. The fundamental belief in CRT is that racism is not an event that occurs in isolation, nor is it anomalous, but that it is a part of American history and is predominant in this society (Ladson-Billings, 2003). Because of this, scholars from this line of thought see racism as inherent in educational contexts and seek to understand how “traditional interests and cultural artifacts serve as vehicles to limit and bind the educational opportunities of students of color” (Tate, 1997, p. 234).

In this study, I contend that to fully understand teachers’ use of LTs in their classroom and how the LTBI model may be used for equitable instruction, elements from both a social constructivist theory and a critical theory are necessary to guide both data collection and analysis. Thus, in this study, I attend to the school culture, the classroom culture, the culture of the students, and how each of those impact teacher/student interactions. Taking a step further, I use CRT to not only acknowledge the cultural environment in which this study is situated, but also acknowledge the influence that race and culture have on the teaching-learning environment. Thus, I situate issues of teaching and learning within larger frames of power and oppression.
In the LTBI research project, teachers participated in a weeklong professional development institute in the summer and three monthly meetings during the fall semester on the topic of an early number and counting learning trajectory. The focus of the professional development was to introduce the LT to teachers as well as the introduction of the LTBI model which situates the LT at the center of instructional decisions and calls for explicit attention to who the individual students are as learners. The social constructivist and CRT theoretical frameworks articulated above influences this research of the LTBI professional development in two ways. First, because the focus of this study is to understand how teachers use LTBI to teach equitably in the classrooms, a social constructivist orientation warrants classroom observations to determine how teachers use the LT when interacting with their students.

Second, justice and social transformation, which are both driving forces behind a critical framework, allow me to question teachers in post-observation interviews about the ways in which they are using this new knowledge with all students. Through these interviews, I aided teachers in examining their own uses of LT’s with individual students in their classrooms with a goal of understanding how their use of LTBI serves individual students. Thus, I examined teachers’ use of the trajectory in their classroom, and in doing so, I also encouraged them to consider and reflect upon issues of equity in the classrooms. From the viewpoint of knowledge construction, I believe that the development of LTs is a valuable tool for students and teachers. However, I do not accept that LTs are a perfect construct; therefore, I wish to examine how they are used with all students.
I start this review of the literature with a discussion of LTs and conclude the discussion by raising concerns about the use of LTs and the ways in which they promote or hinder equitable instruction. After this, I discuss equity and provide a synthesis on varying definitions of equity. I then offer examples of research that have considered attention to student thinking and implications that this work has had on equity for those students. Finally, I propose a framework that draws upon LTs and equity as a potential merger of these two constructs.

**Learning Trajectories**

Over the past two decades, a number of scholars have written about and argued for the use of learning trajectories in mathematics and science education (Battista, 2004; Brown, Clements & Sarama, 2007; Clements & Sarama, 2008; Clements, Wilson & Sarama, 2004; Clements & Sarama, 2008; Confrey et al, 2009; Duncan, Rogat & Yarden, 2009). While the terminology differs (e.g., learning progressions vs. learning trajectories), many of these scholars agree that learning trajectories have the potential to transform classroom practice. In this section, I provide a brief overview of the history of learning trajectories as well as their evolution. Although I present research conducted on learning trajectories and learning progressions, I treat these two definitions as synonymous, and for the purpose of this work, I use the words learning trajectories. First, I present research that highlights instructional uses of learning trajectories. I follow this discussion by sharing results of studies that have linked student achievement to instruction and curricula based on learning trajectories. Finally, I offer a brief critique learning trajectories using both extant literature from the field and a CRT perspective. I conclude by highlighting the potential benefits and challenges of LTs.
The History of Learning Trajectories

In 1990, Case and Griffin argued *big ideas* should guide classroom instruction. In 1996, Brown and Campione introduced the term *developmental corridor*. These scholars were some of the first to address learning across grades and place an emphasis on students’ knowledge increasing both in quantity (learning more concepts) and quality (refinement of those concepts). Catley, Lehrer and Reiser (2004) used the term *learning performances* and believed that instruction should begin by focusing on a small number of concepts and that student cognition should be explicitly aligned with objectives and standards. Brown, Sarama, and Clements (2004) built from Simon’s 1995 articulation of a *hypothetical learning trajectory* to indicate that learning trajectories are comprised of three pieces: a mathematical goal, a developmental path that many children progress along, and activities/tasks that are related to each level. Duncan, Rogat, and Yarden (2009) offered a *learning progression* for modern genetics and highlighted three key areas that learning progressions should address: big ideas, progression across grades, and the development of learning performances and assessment for progression. More recently, Battista (2011) defined an LT as “a detailed description of the sequence of thoughts, ways of reasoning, and strategies that a student employs while involved in learning a topic, including how the student deals with all instructional tasks and social interactions during this sequence” (p. 4).

In some of her earlier work, Confrey (2006) defined a *conceptual trajectory* and a *conceptual corridor*. Figure 1 is a representation of these ideas. The key components of this figure and of Confrey’s conception are:
1. Student learning is not a linear process (as evidenced by the squiggled, dashed line).

2. Each student’s trajectory may be unique (the dashed lines will vary with each student). This is not a trivial conclusion. Because each student’s prior knowledge varies, they may each begin at various places in the trajectory.

3. Students will predictably encounter certain landmarks and obstacles as they progress through the trajectory. Because these can be anticipated based on literature and previous research, teachers can be more deliberate in their planning and instruction.

4. Although the corridor is flexible and allows different students to pass through in different ways, the tasks and instruction serve as boundaries on their likely behaviors.
Confrey et al. (2009) later refined her initial definition of learning trajectories and articulated the following:

[A learning trajectory is a] researcher-conjectured, empirically-supported description of the ordered network of constructs a student encounters through instruction (i.e., activities, tasks, tools, forms of interaction and methods of evaluation), in order to move from informal ideas, through successive refinements of representation, articulation, and reflection, towards increasingly complex concepts over time (p. 347).

Not only has the topic of learning trajectories increased among research programs, it has also gained national attention in reform documents that challenge traditional approaches.
to instruction, most recently in the articulation of the Common Core State Standards. In 2007, the National Research Council (NRC) defined learning progressions as “successively more sophisticated ways of thinking about a topic that can follow one another as children learn about and investigate a topic over a broad span of time” (p. 214). The NRC posited that the concept of progressions can be a viable tool for providing a framework for a broader vision of science education, highlighting foundational topics that students must learn and synthesizing what we know about students and how they learn. They went on to argue that progressions can provide a common language for researchers, curriculum developers, policy makers, and assessment writers (NRC, 2007, p. 214). Corcoran, Mosher, and Rogat (2009) also articulated a number of uses for LTs specifically improved instruction, curriculum, standards and assessment. Specifically, the authors argue that to bring about change in education LTs is a more “deliberate approach, one that employs scientific principles and which engineers solutions that can be tested and refined until they work” (Corcoran, Mosher & Rogat, 2009, p. 11).

Over time, slight nuances continue to arise in defining LTs. A few ideas do seem to be central to this construct and are particularly relevant for this study. First, LTs are based on empirical work with students and challenge more traditional approaches to curriculum development and instruction. Second, there is a specific goal for students to meet. Learning may start with small or informal understandings, but it is expected that these understandings can become more sophisticated over time. The fluid nature of LTs can provide multiple opportunities for students to engage with a concept and coordinate ideas around various concepts to build a more robust understanding. Finally, LTs and student learning are not
independent of tasks. Student learning depends on task quality, selection, and sequencing. This is a critical component because students do not just progress on their own as a result of age or other experiences. Movement through a trajectory must have a deliberate component enacted by teachers.

**Learning Trajectories in Instruction**

Recently, researchers have shifted their attention from the development of learning trajectories in various content domains to thinking about how LTs can be used in instructional settings and the positive effects they can have on students’ achievement. A number of studies have been and are currently being conducted with in-service teachers. These studies range from small case studies of one or a few teachers (Bardsley, 2006; Brown, Sarama & Clements 2007; Edgington, 2012) to large-scale studies with multiple teachers across multiple schools (Clements, Sarama, Wolfe & Spitler, 2013; Wilson, 2009). In a study of one pre-K teacher, Brown, Sarama and Clements (2007) reported that using an LT assisted the teacher in several areas: setting clear goals for students based on their developmental level; selecting and modifying tasks; and ensuring that students were engaged in tasks that would help to solidify their current understandings while building related skills. The teacher stated, “Learning trajectories permitted me to be flexible, adaptable, and responsive to the children’s changing needs” (Brown, Sarama, & Clements, 2007, p. 181). In another study, Clements and Sarama (2008) found that teachers using LTs in their instruction were more likely to monitor and be actively involved in activities as well as develop and use formative assessments that are tied to their knowledge of developmental progressions.

Other findings indicate that LT’s can help teachers foster rich mathematical
conversations (Clements & Sarama, 2008); deepen their own mathematical knowledge (Mojica, 2010); use student thinking to influence instructional decisions (Bardsley, 2006; Mojica, 2010); describe student work with more detail (Wilson, 2009); make sense of students’ understanding (Confrey, 1990); select developmentally-appropriate activities (Brown, Sarama & Clements, 2007); provide a shared language for teachers to discuss student thinking (Wilson, 2009); and anticipate and address students’ misconceptions (Edgington, 2012).

**Learning Trajectories and Student Achievement**

As we continue to learn more about LTs and their usefulness in instruction, it has become equally important for the field to consider what potential impact LTs can have on student achievement. To date, there have been few studies that explicitly relate students’ achievement to learning trajectories. There are, however, a few seminal studies that show promise.

In a study of students working on the *Building Blocks* curriculum, Clements and Sarama (2008) reported that students in the experimental group out-performed students in the comparison group on post-test scores. After experiencing favorable results with a smaller sample size, Clements and Sarama (2013) decided to scale the study up to see if the same results would hold in a larger sample given that the curriculum was implemented with fidelity. In a randomized, cluster trial design with over 40 schools and over 1,000 students, they found that students who were in the experimental group significantly outperformed their counterparts on achievement measures.
Critiques of Learning Trajectories

Because CRT is one of the theoretical frameworks that undergird this study, it is important to consider LTs from a CRT perspective. Recall that CRT asserts that racism is an everyday facet in our society, and as such, racism undergirds the day-to-day activities in schools and educational systems. As a lens for critiquing research and practice, critical race theorists would question which students are used in the empirical research to develop LTs, what the goals of the researchers developing LTs are, and how LTs were chosen to be the basis of the Common Core standards.

While many authors do not provide demographic information detailing which students were used in the development or validation of their LTs, Szilagyi, Clements and Sarama (2013) do offer evidence that may address one concern of critical theorists. In the development of an LT for length, Szilagyi et al. worked with students from two different countries to investigate if the development of big ideas for measurement was dependent on culture. They found that students from various levels and countries followed similar paths, and thus they developed the trajectory based upon patterns that a majority of the students followed. While this work is still in the early stages, it does show promise.

A second critique of LTs is that they are too rigid. Because many scholars concerned with equity employ a post-structuralist theory in their work (Fardon & Schoeman, 2010; Gutierrez, 2010; O’Donovan, 2004;), it is worth acknowledging this viewpoint (although I do not use this framework in my work). Post-structuralists would critique learning trajectories, but from a different angle. While CRT is concerned with societal factors and implications, post-structuralists question the idea of a trajectory in and of itself. Post-structuralists often
reject the notion of certainty and finality, and they would argue that learning is much more unstable and cannot be bound by a trajectory. To this argument, I offer that LTs are hypothetical and provide suggested paths that students may follow. They are probabilistic (instead of deterministic), and if implemented with fidelity, they should not limit what students and teachers do; rather, LTs provide a framework for understanding.

Another critique of LTs is that a focus on LTs “may lead us to oversimplify or ignore critical drivers of learning associated with teaching” (Empson, 2011, p. 573). Empson went on to state that while we have a vast knowledge of students’ thinking related to LTs, we have less of an understanding on how teachers gather information related to student thinking and use this information to make decisions in the moment of instruction. She argued that since teaching is a “relational act,” it should not be regulated to a list of rules. To these points I offer the following argument in rebuttal. While research studying teachers’ use and implementation of LTs is in its infancy, initial results from this work show promise for teachers. Earlier in this section, I articulated a number of findings that indicate that LTs and curriculum built on LTs can support teachers and have positive results on their students. To Empson’s second point about diminishing the relational nature of teaching, none of the existing literature reviewed for this study has indicated that LTs have a negative impact on the relationships between teachers and students. Much of the work conducted in this dissertation will actually address that concern since a focal point of this work is to enhance our understanding of how teachers use LTs and LTBI to meet the needs of and support students in their classrooms.

I conclude this section by offering my own caution about LTs and situating the
necessity for this study. Over the last six decades, educational reform with a specific focus on mathematics has been a topic of national publications and mathematics education interest groups. From the New Math dialogue of the 1950’s and 1960’s to the Back-to-Basics movements of the 1970’s, American teachers and students have been required to learn more mathematics and demonstrate this knowledge on state and federally mandated assessments. Although the aforementioned programs were far-reaching and contributed to research in mathematics education, there was virtually no attention paid to issues of equity and culture in the early documents. As national calls for reform continued in the 1980’s with NCTM’s *Agenda for Action* and the release of *A Nation at Risk*, equity was still neglected or severely underdeveloped (Meyer, 1989). More recently, the *Principles and Standards for School Mathematics* (NCTM, 2000), *National Mathematical Advisory Panel Report*, and *Common Core State Standards for School Mathematics* have been released. Although these documents continue to challenge the field to provide all students with access to high quality mathematics and require teachers to provide better educational opportunities for their students, certain students continue to be underserved in schools (Martin, 2000; Howard & Terry, 2011). Not only are these students underserved, but also the very programs put in place to help bring this vision of reform in mathematics education to pass view these students with a deficit orientation. I argue that it is not enough to simply increase mathematics achievement among various groups of students if we do so at the risk of perpetuating stereotypes. Because LTs (as utilized in the CCSS) are the newest call for reform at the national level, it is critical that the field understand the ways in which LTs are used among various groups of students, particularly those who have been “cast aside” by previous reform movements.
As I transition into my discussion of equity, I use a quote from a widely-published research program to highlight the need for attending to equity in the context of work with LTs. In their work with LTs, Clements and Sarama (2004, 2007) have consistently observed that using an LT-based curriculum can have positive outcomes for students. In one of their most recent publications on a large-scale study, they offered the following finding: “Interventions to address these early differences may benefit low-resource and minority children more because they have fewer educational opportunities in their homes and communities” (Clements, Sarama, Wolfe & Spitler, 2013, p. 813). The authors acknowledged that “low-resource and minority children” out-performed their counterparts that were not in LT-based classrooms. While this statement highlights that LTs can be productive for increasing achievement (which should be an outcome in equitable classrooms), it includes a deficit perspective because it equates minority children with having fewer opportunities in their homes. Additionally, their study addresses issues of opportunity to learn and achievement, but it fails to address a more robust notion of equity including identity and power.

Equity

Throughout the years, equity has taken on different definitions. Scholars that study different content areas and employ different research paradigms interpret equity in varying ways. To understand how equity is defined in this study, I begin by presenting various definitions of equity found in the literature (see Table 1). A number of different issues arise from these varying conceptions of equity. First, there is a tension between equity and equality. As demonstrated in the table, some definitions allude to equity as sameness while
others do not. Second, some of the definitions presented focus on equitable outcomes among students, while others speak to equitable access and processes. Thus, there is tension between equity as a process vs. equity as an outcome. These varying definitions lend themselves to different approaches to equity, and I explore these variances in greater detail following the table of definitions.

Table 1

*Definitions of Equity in Mathematics Education Literature*

<table>
<thead>
<tr>
<th>Source</th>
<th>Definition/Concept</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fennema &amp; Meyer (1989)</td>
<td>Students must have equal opportunity to learn mathematics, equal educational treatment, and equal educational outcomes.</td>
</tr>
<tr>
<td>Allexsaht &amp; Hart (2001)</td>
<td>All students regardless of their race, ethnicity, class, gender, or language proficiency will learn and use mathematics. All interested parties must be aware of social, economic, and political contexts that hinder equity. Equity in mathematics education requires: (a) equitable distribution of resources to schools, students and teachers, (b) equitable quality of instruction, and (c) equitable outcomes for students. (p. 93)</td>
</tr>
<tr>
<td>Lipman (2004)</td>
<td>Equitable distribution of material and human resources, intellectually challenging curricula, educational experiences that build on students’ cultures, languages, home experiences, and identities; and pedagogies that prepare students to engage in critical thought and democratic participation in society. (p. 3)</td>
</tr>
<tr>
<td>Civil (2006)</td>
<td>All students have access to opportunities to engage in rich mathematics. (p. 31)</td>
</tr>
<tr>
<td>Gutierrez (2007)</td>
<td>Equity is being unable to predict mathematics achievement and participation of students based solely upon student characteristics such as race, class, ethnicity, sex, beliefs, and proficiency in the dominant language. (p. 41)</td>
</tr>
<tr>
<td>Goffney (2010)</td>
<td>Practices that provide equitable access to challenging and meaningful opportunities for learning mathematics through instruction that increases participation and academic success, especially for marginalized students. (p. 13)</td>
</tr>
</tbody>
</table>
Why Equity and Not Equality

One tension that exists in the equity discussion, and one that has been previously addressed (Goffney, 2010; Gutierrez, 2002; Secada, 1989), is the difference between equity and equality. Some posit that this confusion arose partly due to the civil rights movement when the phrase “separate but equal” was introduced (Tate, 1997). The thought was that if schools that served children of color had all of the same materials as those that served white students, then there would be no need to integrate schools. Thus, white students and black students could remain in separate schools as long as the schools were equal. While on the surface equality and equity may seem identical and mistakenly viewed as producing the same outcomes, consider the words of former President Lyndon B. Johnson. In his famous June 4, 1965, address to graduates at Howard University he stated, “You cannot take a man who has been in chains for 300 years, remove the chains, take him to the starting line and tell him to run the race, and think that you are being fair” (Retrieved from www.blackcommentator.com 49/49_cover.html on September 13, 2012). In effect, President Johnson addressed the absurdity that equality and equity are one in the same. Simply allowing marginalized students to compete in the same race and with the same resources as their white counterparts is not justice. It is this premise that I use as a lens to demystify the notion that equality and equity will produce the same outcomes. I argue that a focus on equality alone is not enough, and I illustrate this argument using the role of the classroom teacher as an example.

Teacher role. Focusing on the role of the teacher is one way to examine the equity versus equality debate. An equality orientation would suggest that teachers should spend the same amount of time with all students and interact with all students in the same way. The
literature refutes this notion. In her study of successful teachers of African-American students, Ladson-Billings (1995) identified characteristics of these teachers that she argues support African American students. She stated that these teachers (1) immersed themselves in the community of their students and saw themselves as members of the community, and (2) built deep relationships with these students. Teachers are members of the communities that they live in, and their experiences help them relate to the students with whom they are similar. Meeting the needs of those students and sharing common experiences can come easier if your students are members of a similar ethnic group and socioeconomic status. Because elementary schools teachers are primarily white, middle class women, their lived experiences are often times very different from the diverse classrooms in which they teach (National Center for Education Information, 2011). In order for these teachers to get to know their students, they will have to purposefully spend time getting to know about the cultures their students come from. When teachers teach students from communities that differ from their own, more time is needed to meaningfully understand the lived experiences of those students. Thus in this case, equity is not equality.

In their work with exploring small group interactions as a tool for equity, the authors of Complex Instruction (1999) stated that equity requires changes in the role of the teacher. The authors sought to determine what aspects of small group work were necessary to ensure equitable participation of students and equitable outcomes in learning. The impetus behind this work was that although many teachers use small group work as a tool for instruction and even differentiation, a number of unintended consequences can occur. First, students who do not read at grade level or who are English language learners may not be able to fully
participate in the group (Cohen, Lotan, Scarloss & Arellano, 1999). Another concern was that “students who are academically low achieving or who are social isolates are [often] excluded from the interactions” (Cohen et al., 1999, p. 80). Specifically, they cited the need for students working in groups to have equal status relationships. Without this, “higher status” students will often dominate the group, participate more, and therefore have the most opportunities to learn (Cohen et al., 1999).

To ameliorate this problem, the Complex Instruction team suggested that teachers must publicly assign competence to students and that this is “especially important and effective to focus attention on low-status students” (Cohen et al., 1999, p. 85). To do this, the teacher must first identify each student’s strengths and contributions to the group or class. By acknowledging these students publicly, the teacher changes the class’ expectations of that student which, in turn, allows that particular student more opportunities to participate in the group. An equality orientation requires that all students receive the same amount of praise and acknowledgement. An equity orientation rebuts this notion, as the argument is that to level the playing field, some students need more acknowledgement than others, and they need it publicly.

One primary difference that I see between an equity platform and an equality platform is that the goals differ significantly. An equality platform could be achieved by rewriting standards or other national documents that mandate that all students be treated the same and that we have high expectations for all students. An equality platform can be reduced to a simple prescription without attention to the underlying problems that have led to past inequalities. An equity agenda, however, requires a different approach. It is an ongoing
process that begins with looking back, analyzing previous inequities, and developing a long-term plan to repair past damage.

**Equity as a Process versus Equity as an Outcome**

Another tension that arises across definitions of equity is whether we should view equity as a process, whether we should view it as an outcome, or whether there is some combination of both. Equity as a process focuses on what teachers should do, what schools should do, what curriculum should do, etc. The process can be very descriptive, but what is missing from this view is an explicit link between what should be done during the teaching/learning process and how that relates to student outcomes. I argue that we must focus on equity both as a process and examine the related outcomes to determine whether we have been successful. Below, I use the “achievement gap” as an example of how focusing on equity as an outcome alone does little to move us forward.

Scholars that discuss equity as an outcome are concerned with the tangible results, most notably achievement as measured by standardized tests. When disparities in student achievement were first detected, the goal became to close the achievement gap. This “achievement gap” became the focus of research projects and continues to be the focus of research programs (Gutierrez, 2008). In fact, a large part of the No Child Left Behind Act (2001) was dedicated to acknowledging and making recommendations to close the gap. The authors included three sections devoted to addressing the needs of African-American students, Hispanic students, and American Indian students. In each of these documents, the authors noted that these groups of students have been left behind and under-educated for too long, and therefore the government was stepping in to make “closing the achievement gap a
national priority” (NCLB, 2005, p. 1). Moreover, the authors positioned themselves as the heroes, arguing that the achievement gap has been unaddressed for too long, but with government regulation and mandates these children would now succeed.

What the authors of this document have failed to fully consider, along with others who view equity solely as an outcome, are the historical events and social constructs that have caused and continue to cause so many students of color to be under-educated. In fact, Matthews (2005) and Strutchens et al. (2012) argued that before we can truly call for equity and equitable outcomes, we must first clearly articulate the causes of the inequity. In regards to African-American students, NCLB attributes their failure to the fact that our nation has two educational systems, and that they are “separate and unequal” (NCLB, 2005, p. 1). For Hispanic students, the document notes that cultural and language barriers have not been addressed and have therefore led to the failure of these students. No specific reasons are given for the failure of American Indian students; however, the documents state that prior to NCLB, American Indian students were not tracked and compared to their peers (NCLB, 2005).

Although trivial, NCLB does highlight that low expectations for African-American and Hispanic students must no longer be tolerated. While this appears to be an equity argument, a number of scholars have criticized this act, its immense focus on the achievement gap, and how only highlighting the gap actually further perpetuates inequities in the educational system (Gutierrez, 2008, 2011; Ladson-Billings, 2006).

Because of hundreds of years of inequity and oppression, it is not surprising that students of color have not achieved at the same level of their white counterparts. In fact, this
is the heart of the equity as a process versus equity as an outcome debate. It is this same argument that dispels the myth that equity and equality are one in the same. Consider the following words from Robinson (2000) as cited in Ladson-Billings (2006):

No nation can enslave a race of people for hundreds of years, set them free bedraggled and penniless, pit them, without assistance in a hostile environment, against privileged victimizers, and then reasonably expect the gap between the heirs of the two groups to narrow. Lines, begun parallel and left alone, can never touch. (p. 74)

If we truly expect to see changes in the achievement of students of color and students in poor and rural communities, we must be equitable in the process of their education (e.g., funding, resources, rigorous curriculum, qualified/culturally relevant teachers, additional supports for learning after school, and parent/community involvement). In many cases, equity requires that we do more for students who have traditionally been underserved to compensate for what was lacking. If we focus on equity in the process and refine it as we move along, equitable outcomes should be a natural occurrence.

**Attention to Student Thinking as a Vehicle to Equitable Instruction**

Although many mathematics education researchers agree that teacher content knowledge and attention to student thinking are critical for high quality mathematics instruction (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996; Kazemi & Franke, 2004), there is less consensus on the role of equity in mathematics education. As a result, some scholars argue that equity is a separate construct and is not related to mathematics (Eisenberg, 2014). Still others argue that equity and the environment that learning takes place
in are inseparable from the content being learned, and that the best teachers are those with strong content knowledge, precise knowledge of student thinking, and a strong sense of their students’ cultural capital (Gay, 2000; Ladson-Billings, 1994, 1995).

In the context of this study, I argue that teachers’ attention to student thinking can be used as a bridge to equitable instructional practices. In this section of the literature review, I discuss Cognitively Guided Instruction (CGI) and how researchers in two different projects have studied teachers’ implementation of CGI as a tool for equitable instruction. I also use the results from one study that used an LT-based curriculum and reported that an outcome of their work was equitable achievement outcomes.

**Cognitively Guided Instruction**

One promising body of research that has been seminal in addressing teachers’ knowledge of student thinking and the ways in which teachers can build from students’ thinking is CGI (Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996; Franke, Carpenter, Levi, & Fennema, 2001). CGI was built from researchers’ desire to understand how students solved different addition and subtraction problems. Researchers found that while a group of problems may appear to be simply addition or subtraction, there are nuances within these problem types that make them easier or more difficult for students. Using results from student interviews, the researchers developed a hierarchy of problem types along with strategies students use within each problem type. After the team fleshed out the problem types, this knowledge base was used to provide professional development for teachers with a goal of understanding how teacher knowledge of the CGI framework impacted students. Results indicated that when provided with a lens (CGI), teachers could use it to understand
individual students’ thinking process, and that teachers could build from students’ informal understandings when making instructional decisions. Researchers also found that attention to students’ thinking helped teachers to reorganize their own mathematical understandings, and that this shift in pedagogy has a positive impact on student learning (Fennema et al., 1993). Despite the promise this framework of student thinking has demonstrated, only recently have scholars begun to think about how it impacts equitable classroom instruction.

Battey and Chan (2010) reported on their efforts to use CGI as a vehicle to challenge teachers’ beliefs about their students. They noted that teachers often described students by metanarratives, and that these stories positioned African-American students and other populations as inferior to their White and Asian counterparts, casting them as “other.” In this multi-year project, they worked to counteract these metanarratives by drawing teachers’ attention to what students can do as opposed to what they cannot do. The researchers then challenged teachers to use this new way of talking to re-structure the discourse in their professional development program. Their findings indicated that focusing on student thinking did change teachers’ discourse about students, and subsequently, teachers rooted their claims about students in evidence instead of assumptions. Additionally, they found that CGI helped teachers focus on individual students’ thinking and caused them to shift from larger notions about the group to which students belong to focus on their individual needs. The authors posited that these behaviors can help dispel the deficit orientation teachers have towards students (Battey & Chan, 2010).

In a study of two first grade classrooms also using CGI, Marshall, Musanti and Celedon-Pattichis (2007) sought to understand how Latino teachers could support the
mathematical thinking and learning of their students by aligning problems to these students’ cultural and linguistic backgrounds. In a similar study, Turner, Celedon-Pattichis, and Marshall (2008) also studied three kindergarten classrooms to specifically examine the following:

How young Hispanic students, including those who are ELLs learn to solve problems and communicate their mathematical thinking, and more specifically, how teachers draw upon other [emphasis original] knowledge bases, such as students’ language, culture, or home experiences, as resources to support students understanding (p. 22).

Because reform-oriented mathematics programs call for an increased attention to solving open-ended tasks, writing about mathematics, and sharing mathematical thinking, students whose home culture and language are different from that of the mainstream population are often denied opportunities to participate in class. Both of these studies used CGI as a tool to think about how a focus on student thinking supported the needs of culturally and linguistically diverse students. I argue that this approach is different from professional development that specifically addresses culture in that the researchers used students’ ways of thinking as a basis for teachers’ enacting equitable instruction.

Findings from these studies supported the conjecture that by focusing on student thinking and embedding problem solving in contexts that are familiar to students, teachers can engage students successfully. For example, Turner, Celedon-Pattichis, and Marshall (2008) found that the use of story problems invites students into mathematical problems, and that the structure of the story problem can be used to help students scaffold their mathematical explanations. They argued that knowledge of student thinking assists teachers
in (a) viewing students as “competent problem solver[s] who had something important to explain to members of the class” (Turner, Celedon-Pattichis & Marshall, 2008, p.31), and (b) asking both open- and closed- questions which assisted students in communicating their thinking. Finally they posited that “strategic use of questioning and explicit positioning . . . work together to support students’ participation in mathematical discourse” (Turner, Celedon-Pattichis & Marshall, p.32).

**Summary**

These examples indicate that by focusing on student thinking, we can have equitable outcomes for students. These examples also highlight that by drawing teachers’ attention to students’ thinking, metanarratives can be challenged and teachers can begin to counteract deficit-oriented dialogue. In the following section, I present an existing framework for equity. I provide a brief overview of literature related to each of the four constructs in the existing framework, and then I propose potential ways in which LTs and this framework can support one another. I conclude by presenting the hypothetical framework that was created in light of literature about both equity and LTs.

**Conceptual Framework**

To examine the potential for LTs to assist with equitable teaching practices in the classroom, I draw on the literature presented and use the work of Gutierrez (2007) as a way to frame this body of work. In contrast to themes of equality in education that arose during the civil rights movement, Gutierrez stated that “equity means fairness, not sameness” and went on to define equity as “the inability to predict mathematics achievement and participation solely on student characteristics such as race, class, ethnicity, sex, beliefs, and
proficiency in the dominant language” (p. 4). Gutierrez described four dimensions of equity—access, achievement, identity and power—and discusses how each of these constructs is critical to developing a robust understanding of equity. Access and achievement comprise what Gutierrez terms the “dominant axis” while identity and power comprise the “critical axis” (2007, p. 4). By naming the first dominant, the intent was not to say that one is more important, but rather to distinguish between what students will need to know to show mastery of the content as it is valued in this society (access and achievement) compared with what students will need to know in order to maintain a sense of self and their ability to use mathematics to critically view the world (identity and power).

Gutierrez (2007) not only framed equity, she also presented findings from different research studies she previously conducted and explicated how those findings informed her definition of equity. Gutierrez’s framework is a powerful tool for defining equity, and inherent in it are elements of both equity as a process and equity as an outcome. In what follows, I present constructs within her framework along with related research within each piece that influenced my definition of equity. I conclude with my re-interpretation of this framework based on the potential of learning trajectories as a tool to promote equity.

Access

For a number of years, scholars in mathematics education conceived access as opportunity-to-learn (Elmore & Furhman, 1993; Tate, 1995). While opportunity-to-learn remains a central concept, Gutierrez argued that it alone is insufficient in defining equity. She stated that access is what students physically have or do not have and includes:

- Quality mathematics teachers;
• Adequate technology and supplies in the classroom;
• A rigorous curriculum;
• A classroom environment that invites participation;
• Reasonable class sizes; and
• Supports for learning outside of class hours (Gutierrez, 2007, p. 2).

There are a number of studies where researchers have demonstrated that these factors do, in fact, contribute to students’ gaining more access in mathematics classrooms (Campbell, 1996; Flores, 2007; Silver & Stein, 1996). For example, the “Quantitative Understanding: Amplifying Student Achievement and Reasoning Project” (QUASAR) was developed to combat the notion that lack of participation and low achievement of urban students is due to their lack of ability. Instead, researchers argued, these students are not provided with exposure to high-quality problems; in addition, the typical classroom environment impedes their participation (Silver & Stein, 1996). Although a number of themes were present in QUASAR classrooms, researchers reported that teachers consistently utilized tasks that “encouraged students to use rather sophisticated mathematical thinking and reasoning—either connecting problems to underlying concepts and meaning or tackling complex mathematical problems in novel ways” (Silver & Stein, 1996, p. 483). Another salient feature of these classrooms was the focus on student discourse and collaborative problem solving. Findings from this study indicated that as a result of access to rigorous problems and collaborative learning situations, students in QUASAR classrooms performed comparably to, and in some cases outperformed, similar students on NAEP exams (Silver & Stein, 1996). On other measures, these students showed a significant increase in their ability
to fully articulate and justify their solution strategies to open-ended assessment items (from 18% to 40% accuracy over three years). Thus, QUASAR began to show the importance of access to high-quality instruction and how providing that access was related to equitable outcomes in mathematics education.

Similarly, Project IMPACT (Increasing the Mathematical Power of All Children and Teachers) was based on the premise that in order to meet the needs of all students, particularly those who have not been successful in traditional mathematics classrooms, reform must occur on a school-wide level. The primary aim was that teachers must increase their own content knowledge and revamp their traditional instruction to include solving meaningful problems and requiring that students “communicate their reasoning, document their approaches, and interpret the strategies of others” (Campbell, 1996, p. 454). This type of approach provided students in these schools with access in that their teachers had stronger content knowledge, and the structure of the classroom instruction now provided space for students to share and engage in mathematical reasoning as a group. Overall, students in IMPACT schools outperformed students in comparison schools, and although significant changes were not apparent immediately, over time, students showed significant gains on problem solving and reasoning tasks (Campbell, 1996).

In spite of successes experienced in these mathematics classrooms and the best practices and recommendations resulting from these studies regarding access, there are still examples that highlight the need for continual study of the importance of this construct and inequitable learning experiences that ensue when the goal of access is not explicitly addressed in instruction. In a study of one elementary classroom and one high school
classroom, Civil and Planas (2004) found that student participation in mathematics discussions was largely based on social and organizational structures in the classroom. They went on to state that not only do students recognize the “location” of students in the classroom (e.g., gifted, special education), but that these labels influence how students participate in the classroom and that “high-status students may have easier access to the mathematical discourse, whereas students placed in the low-status system are still supposed to prove their value” (Civil & Planas, p. 13). Because participation in classroom discussions is a critical component of reform-oriented mathematics programs, students must have access to classroom discussions.

**Access with LTs.** In an equitable mathematics classroom, it is critical that all students be able to share in mathematics discourse. Because language and discourse are critical factors in learning and knowledge construction (Vygotsky, 1986), students should be provided with as many opportunities as possible to engage in math talk. Strutchens (2000) said that students should not only have to find and justify their own solutions, but they should also be given the opportunity to question other students about their work. Thus, the definition of access for the purpose of this study includes both physical and cognitive access to rich-tasks as well as access to mathematical discussions about ideas generated in the context of these tasks. The specific construct under evaluation in this study is how teachers use LTs and LTBI to design instruction and select meaningful mathematics tasks such that all students have an entry point and are able to engage in the classroom. Further, I investigate how teachers use LTs to ensure that tasks are rigorous for students.
Achievement

As Gutierrez (2007) continued to frame her definition of equity, she stated that “beyond opportunities to learn, we must also care about student outcomes” (p. 3). This is a critical issue and refers back to the inadequacy of equity as a process as an isolated construct. If we were to provide students with all of the resources mentioned above (e.g., quality teachers and a rigorous curriculum) and still see the same types of results we have traditionally seen, we will still continue to underserve students. Because our educational system places a high emphasis on student outcomes, it is important that we think of them in terms of defining equity. Gutierrez offers the following as a part of the achievement dimension:

- Participation in a Given Class
- Course Taking Patterns
- Standardized Test Scores
- Participation in the Mathematics Pipeline (p. 3)

In both of the studies cited for the access dimension (QUASAR and Project IMPACT), students experienced higher assessment scores than their counterparts who did not receive the same curriculum and instructional practices. Because of these results as well as others, Gutierrez stated that, “access is a precursor to achievement” (p. 4).

**Achievement with LTs.** Traditionally, achievement is measured in terms of academic standards. States and school districts create broad standards that students must meet. Teachers are then required to teach content related to those standards and assess their students. One benefit that has been articulated for using LT’s is that teachers are able to set
individual learning goals for students based on their current location in the trajectory. Mosher (2011) states:

If we expect substantially all children to meet standards, schools and teachers have to take responsibility for monitoring students’ progress and intervening on a timely basis when needed…In order for students to get back on track, they also need feedback and supporting experiences that are responsive to the particular difficulties they are having (Mosher, 2011, p. 1).

What we don’t know is how teachers use LT’s to set goals for individual students. An equity approach requires that teachers set high expectations for all students. One goal of this study is to examine how teachers use LTs to set short-term and long-term goals for students, monitor their progress, diagnose their misconceptions, and provide relevant intervention when needed. Heritage (2008) stated that LTs help teachers,

See connections between what comes before and after a specific learning goal, both in the short and long term…this means that teachers have the opportunity to build explicit connections between ideas for students that thread the development of increasingly complex forms of a concept or skill together (p.5).

For students to achieve, it is necessary that teachers know precisely what their students do and do not understand and build on it in meaningful ways towards positive outcomes.

Identity

In the late 1980’s, Fordham and Ogbu (1986) began to discuss a phenomenon called “acting white.” What they found was that many African American students who were able to do well in school did not exert the necessary effort because they had become disillusioned
with success. When presented with what they viewed as a choice between maintaining friendships and popularity, through giving up what they viewed as their cultural identity and succeeding in school, some African American students chose to disengage. This is because doing well in school and speaking “proper” English are seen as something outside of their culture. To maintain their “sense of culture” and establish a clear boundary between themselves and their white classmates, African American students participate in athletics, act out in class, or fly under the radar (Fordham & Ogbu, 1986). This behavior, which still exists among students today, is an important part of an equity framework. Gutierrez (2007) stated that,

Because there is a danger of students having to downplay some of their personal, cultural, or linguistic capacities in order to participate in the classroom or the math pipeline…issues of identity have started to play a larger role in equity research in mathematics education (p. 3)

She went on to identify the following as indicators of what it looks like for students to maintain and develop their identity in a mathematics classroom. She offers the following:

- Maintain and draw upon cultural and linguistic capacity
- A balance between self and others
- Students see themselves in the curriculum (mirror)
- Students use the curriculum as a tool to view and analyze the world (window)
- Students find mathematics meaningful to their lives
- Students sense that they have become a better person (Gutierrez, 2007, p. 3)

In the early 1990’s, Claude Steele (1995) introduced a concept of “stereotype threat”
to education literature. This theory asserts that when an individual is placed in a situation where a known stereotype exists about them, their potential to perform is hindered for fear of confirming the negative stereotype. This was particularly important in his studies of African American students. Because race and culture are arguably the most salient portions of the identity each student brings to school, stereotype threat can arise in the classroom. Numerous studies have confirmed the validity of stereotype threat among women in mathematics (Spencer, Steele & Quinn, 1999) and African Americans on standardized achievement tests (Steele & Aronson, 1995). Steele and his colleagues also sought to understand differences in stereotype threat and why some members of a population are affected while others are not. Because everyone belongs to at least one group that is subject to being stereotyped (e.g., race, culture, religion, sexual orientation), it was important for them to understand how these played out in various settings. What they discovered was that:

A person has to care about a domain in order to be disturbed by the prospect of being stereotyped in it. That is the whole idea of disidentification—protecting against stereotype threat by ceasing to care about the domain in which the stereotype applies. (Steele, 2003, p. 255)

Scholars that attribute student failure to a deficit orientation argue that some portion of a student’s identity is deficient; therefore, they should change it to adapt to mainstream traditions. Steele (2003) posited that we can reduce this phenomenon by developing an atmosphere of trust such that students do not feel like their performance is going to be judged in accordance to a set of stereotypes, but rather their performance will only be used to identify their knowledge.
Identity with LTs. This line of work has major implications for instruction in classrooms. Students should not have to “dis-identify” with whom they are or where they come from to compete on a level playing field in schools. Therefore, teachers should work to understand who their students are and value what they bring to the classroom. Teachers must also work to build instruction around the lived experiences of their students and allow them to work on and solve tasks in ways that are meaningful to them. This does not negate their responsibility to teach the curriculum and introduce “formal” mathematics, but it does challenge the one-size-fits-all approach to mathematics teaching and learning. Because LTs often highlight a variety of strategies and approaches to solving problems, one focal point here is whether teachers are able to harness this awareness to support and encourage a variety of algorithms that might be observed among different students. Another point of investigation is whether teachers can extend the LTBI definition of open tasks to include tasks that are relevant to students’ out-of-school experiences and use those experiences to draw connections between school mathematics and the broader society.

Power

An underlying theme of many arguments for equity is social justice (Gonzalez, 2009; Gutstein, 2006). As I have mentioned earlier in this review, a study of equity must consider the original source of the inequity. After these inequities are identified, scholars argue that we only achieve equity or justice by changing schooling and society on a larger level. Because equity is not a static quality, one that you either have or do not have, it is important that we think not only about ourselves, but also the effect our actions have on the field of mathematics education. Because of this, Gutierrez (2007) states that it is not enough to
provide students with access, achieve at high levels, and maintain their identities if
“mathematics as a field and/or our relationships on this planet do not change” (p. 3). She
defines identifies different ways this social transformation may occur including:

- Who gets to talk in the classroom (voice)
- Who decides on curriculum
- Alternative notions of knowledge
- Rethinking the field of mathematics as a humanistic enterprise
- Opportunities for students to use math as an analytical tool to critique society

(Gutierrez, 2007, p. 3-4).

Many scholars that argue for social justice as an outcome of school instruction have
rooted their arguments in the work of Paulo Freire. Most notably recognized for his book
Pedagogy of the Oppressed, Freire (1970) argued that the oppressed are prime candidates for
a liberatory pedagogy because they suffer the effects of their oppressors and an oppressive society. Much of the research on teaching for social justice or critical pedagogy examines the role of the teacher in the classroom. In many traditional classrooms, the teacher is viewed as the keeper of knowledge, and his/her primary function is to take what they know and pour it into the less knowledgeable students. Gutstein (2003) further expounded on this by noting that at the start of his study of teaching mathematics for social justice with Mexican American students, “Mathematics [was viewed] as a rote-learned, decontextualized series of rules and procedures to memorize, regurgitate, and not understand” (p. 46). This approach to teaching, which Freire (1970) calls a “banking” approach, impedes students’ ability to become critical thinkers and problem solvers. In fact, as long as students are viewed as
receptacles of the teacher’s knowledge, they will never learn to see society critically as that would undermine the goal of the oppressor, which is to maintain the status quo. Instead of a “banking” approach to teaching, Freire (1970) and Gutstein (2003) argue for a model of instruction that liberates students by teaching them to read the world by reading the word.

**Power with LTs.** Where traditional curricula provide a specified set of lessons, activities, or tasks and sometimes even offer prompts for teachers to say, the focus of a learning trajectory is to outline a path of learning from less sophisticated ideas to more sophisticated ideas. Traditional instruction usually follows the banking approach. The LTBI model of instruction reorganizes the current instructional paradigm by using students’ current knowledge as a way to build and refine additional mathematics knowledge. LTs also provide teachers with a frame to understand where individual students are. Because of this, teachers should be able to see each student as a possessor of mathematical knowledge.

Through the process of selecting and sequencing student work, all students should also have the opportunity to share their strategies and justifications, thus giving them a voice in the classroom. Teaching mathematics for social justice or social transformation (sometimes referred to as critical mathematics pedagogy) has two goals: the first is that students develop competency in mathematics, and the second is that students develop critical consciousness (Gutstein, 2007). While I am not suggesting that LT’s or LTBI has the capability to develop critical consciousness or help students critique society, I do think that it can be used to develop student voice and mathematics competency. A prerequisite to students’ using their mathematical knowledge to critique society is forming critical thinking skills and being able to use those skills to pose and defend mathematical arguments.
A Framework for LTBI and Equity

Gutierrez (2002) stated, “because equity is a value-laden term and requires human judgment, we have had fewer examples of what equity might mean empirically. That is, how might we know it if we saw it?” (p.148). To conclude this chapter, I present the following framework as an initial conjecture of what indicators for equity may look like in the context of a learning trajectories-based lesson (See Table 2). Looking for these indicators and the role the LT and LTBI instructional model can play in supporting them can help us to identify (if observed) when and how teachers are using LTs to support equitable instruction.
### Table 2

**Conceptual Framework for LTBI and Equity**

<table>
<thead>
<tr>
<th>Access</th>
<th>Teachers use their knowledge of LTs and LTBI to:</th>
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</thead>
<tbody>
<tr>
<td>Design instruction and instructional tasks such that they are accessible for all students.</td>
<td></td>
</tr>
<tr>
<td>Identify and use up-to-date research based materials and technology</td>
<td></td>
</tr>
<tr>
<td>Be accessible to and attend to all students in the class.</td>
<td></td>
</tr>
<tr>
<td>Foster classroom discussions such that all students can participate and engage.</td>
<td></td>
</tr>
<tr>
<td>Provide all students with opportunities to engage in rigorous mathematics.</td>
<td></td>
</tr>
<tr>
<td>Achievement</td>
<td>Set high, yet appropriate, academic standards for all students.</td>
</tr>
<tr>
<td>Unpack and build upon their students’ prior mathematical knowledge and use it as a basis for understanding more meaningful and complex mathematics.</td>
<td></td>
</tr>
<tr>
<td>Select and use a variety of forms of assessment (e.g., formative, summative, projects, class discussions) to gauge student achievement.</td>
<td></td>
</tr>
<tr>
<td>Identity</td>
<td>Support the development of a robust mathematical identity</td>
</tr>
<tr>
<td>Listen to and consider students out-of-school experiences and design instructional activities that incorporate elements from their homes and communities.</td>
<td></td>
</tr>
<tr>
<td>Validate the use of students’ own algorithms and strategies to solve problems.</td>
<td></td>
</tr>
<tr>
<td>Assist students to build connections between the mathematics they learn and the broader world/society.</td>
<td></td>
</tr>
<tr>
<td>Encourage students to engage in mathematical tasks according to their preferences and participate in mathematical discourse in ways that are comfortable for them.</td>
<td></td>
</tr>
<tr>
<td>Power</td>
<td>Ensure that students have voice in the classroom.</td>
</tr>
<tr>
<td>Position students as experts in the classroom (this includes things they know in school and things they know from outside of school).</td>
<td></td>
</tr>
<tr>
<td>Allow students to solve problems that are relevant to them (these problems can exist inside or outside of school).</td>
<td></td>
</tr>
<tr>
<td>Encourage all students to present, justify, and defend their mathematical ideas/arguments.</td>
<td></td>
</tr>
<tr>
<td>Help students to see themselves as sources of mathematical knowledge.</td>
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</table>
This is a hypothetical framework for what equitable instruction *may* look like at the classroom level. In this study, I am investigating the role LTs and LTBI in these areas. Through working with a small group of case teachers, I sought to refine these existing categories, present new ones that may emerge, and offer examples of how these categories are manifested in the classroom. In the next chapter, I present the study design and the methodology that led to this research.
CHAPTER THREE

The purpose of this study was to design and empirically examine a framework for the ways in which LTs and LTBI can be used to support teachers in providing equitable instruction. Although previous research has examined how pre-service and in-service teachers implemented learning trajectories in instruction (Bardsley 2006; Edgington, 2012; Mojica, 2010; Wilson, 2009); how teachers developed an equitable mathematics pedagogy (Wager, 2008; Yoder, 2008); and how equitable mathematics instruction could be identified and measured (Goffney, 2010), this study is unique because it combines these two research foci to specifically examine how teachers’ implementation of LTBI promotes equitable instruction in their classrooms and what this phenomenon looks like. In the next few paragraphs, I discuss features of qualitative research methods and case study methodology. I conclude this section with a description of Eisenhardt’s (1989) process for building theory from case study research and discuss the rationale for why this process was appropriate for the present study.

Study Design

Because I wanted to know how teachers used LTs and LTBI to promote equitable instruction, a qualitative case-study design was warranted (Yin, 2009). Bogdan and Biklen (2007) defined five features of qualitative research. First, qualitative research is naturalistic in that data is collected in the setting in which it normally occurs because context is important. Second, qualitative research is descriptive and includes “interview transcripts, field notes, photographs, videotapes, personal documents, memos, and other official records” (Bogdan & Biklen, 2007, p. 5). Third, this methodology focuses on the process and is
concerned with how things happen, not just what happens. Fourth, qualitative research is inductive and qualitative researchers “do not search out data or evidence to prove or disprove hypotheses they hold before entering the study; rather, the abstractions are built as the particulars that have been gathered are grouped together” (Bogdan & Biklen, 2007, p. 6). Finally, qualitative researchers are concerned with accurately capturing and portraying the lived experiences of the participants. In this study, as I will discuss later in this section, the fourth feature was not applicable. Because I had a specific goal of generating theory, I brought a framework to the study.

Case studies as defined by Merriam (1998) are also particularistic, descriptive, and heuristic. They are particularistic because they focus on a “particular situation, event, program, or phenomenon” (Merriam, 1998, p. 29). This study is particularistic in that the focus was on four participants in the LTBI project and their interpretation of equitable teaching practices in the context of LTs and the LTBI model. Case studies are descriptive because the end result is a detailed description of the phenomenon that is built from multiple data sources over time (Merriam, 1998). This study is descriptive because an outcome of this study is a revised framework for describing and understanding the ways in which LTBI can support equitable instructional practices. As I will detail in the upcoming sections of this chapter, the revised framework and the data supporting these revisions were captured via interviews, classroom observations, field notes, and student portraits. Finally, case studies are heuristic because they “illuminate the reader’s understanding of the phenomenon. . .bring about the discovery of new meaning. . .or confirm what is known” (Merriam, 1998, p. 30). This study is heuristic because findings indicate that although some aspects of the initial
framework were present in teachers’ instructional practices, additional practices were also implemented in their efforts to enact equitable instruction. Additionally, some constructs included in the initial conceptual framework were not supported by the data and thus were removed from the LTBI-Equity framework. Thus, by building understanding based on detailed analysis of a particular phenomenon, this study provides confirmation as well as revisions for various aspects of what constitutes equitable instruction using LTs.

This study utilized a multi-case study design to look within and across cases to generate theory about teachers’ uses of LTs and LTBI to promote equitable instruction. Eisenhardt (1989) built from the work of Yin (1981, 1984) to articulate the process of building theory from case study research using within-case and cross-case analysis. This process included the following steps:

• Getting started;
• Selecting cases;
• Crafting instruments and protocols;
• Entering the field;
• Analyzing the data;
• Shaping hypotheses;
• Enfolding literature; and
• Reaching closure (Eisenhardt, 1989, p. 533).

In the upcoming sections, I discuss each step of the process and how it unfolded in this dissertation study including the following steps: the refinement of the research questions,
a rationale for the selection of cases, data collection instruments and timeline, and data analysis. Because the final step of Eisenhardt’s process (reaching closure) addresses how researchers situate their findings in relation to extant literature in the field, it is not addressed in the methods section. I discuss this step in Chapter 5 of the dissertation. I conclude Chapter 3 with a detailed discussion of how I worked to enhance the validity and reliability of the findings from this research as well as a subjectivity statement.

**Refined Research Questions**

In the “getting started” phase of Eisenhardt’s (1989) process, she posits that the researcher must select and narrow the research questions to tighten the focus of the work. She also states that in this phase “a priori specification of constructs can. . .help to shape the initial design of theory building research” (p. 536) and that these constructs allow the researcher to measure the phenomenon more accurately. These constructs (also referred to as propositions) come from “the literature, personal/professional experiences, theories and/or generalizations based on empirical data” (Baxter & Jack, 2008, p. 551). In light of these recommendations, the research questions initially presented were refined to reflect each of the constructs in the conceptual framework.

1. In what ways do teachers use LTs and LTBI to provide access to students in their classrooms?

   a. How do teachers use LTBI to design instruction and instructional tasks such that they are accessible for all students?

   b. How do teachers use LTs and LTBI to identify and use up-to-date research-based materials and technology?
c. How do LTs and LTBI help teachers think about being accessible to all students during instruction?

d. How do teachers use LTs and LTBI support students such that all students can engage in classroom discussions?

e. How do LTs and LTBI assist teachers in providing all students with opportunities to engage in rigorous mathematics?

2. In what ways do teachers use LTs and LTBI to promote achievement for students in their classrooms?

a. How do teachers use LT’s to set high academic standards for all students?

b. How do teachers use LT’s to unpack and build upon students’ prior mathematical knowledge and use it as a basis for building more complex mathematics?

c. How do teachers use LTBI to implement various types of assessment to gauge students’ achievement?

3. In what ways do teachers use LTs and LTBI to help students develop their identity?

a. How does using the LTBI model assist teachers in developing a robust mathematical identity in students?

b. How do teachers use LTBI as a means to listen to and consider students out-of-school experiences and design instructional activities that incorporate elements from their homes and communities?

c. How do teachers use LTs and LTBI to solicit and validate various algorithms and solutions to tasks?
d. How do teachers use the LTBI model to provide opportunities for students to build connections between the mathematics they learn and the broader world/society?

e. How do teachers use LTBI to encourage students to engage in mathematical tasks according to their preferences and participate in mathematical discourse in ways that are comfortable for them?

4. In what ways do teachers use LTBI to empower students in their classrooms?

a. How do teachers use LTs and LTBI to give all students voice in the classroom?

b. How do teachers use LTs and LTBI to position students as experts in the classroom?

c. How do teachers use LTs and LTBI to provide students with opportunities to solve problems that are relevant to them?

d. How do teachers use LTBI to provide opportunities for all students to present, justify, and defend their mathematical ideas?

e. How do teachers use LTs to see all students as sources of mathematical knowledge?

Additionally, Yin (2009) argued that study propositions “direct your attention to something that should be examined within the scope of the study” (p. 28). In essence, study constructs or propositions developed at the onset of the study serve to

- Direct the researcher as to what to study (Yin, 2009);

- Direct the researcher about where to look for relevant evidence (Yin, 2009); and
• Help the researcher develop interview protocols and other data collection measures (Baxter & Jack, 2008, Eisenhardt, 1989).

Baxter and Jack (2008) argue that propositions are similar to hypotheses in quantitative studies in that they are educated guesses about what may occur. Using this theory, Table 3 was developed. It includes the original conceptual framework articulated from the literature along with my propositions about how teachers may use LTs and LTBI to teach mathematics equitably within each proposed construct.
### Table 3

**Conceptual Framework Constructs/Propositions**

<table>
<thead>
<tr>
<th>Constructs</th>
<th>Constructs with LTs and LTBI</th>
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<tbody>
<tr>
<td>Design instruction and instructional tasks such that they are accessible for all students.</td>
<td>Teachers use their knowledge of the student profiles and associated LT topics (e.g., producer to five, solving join problems using direct modeling) to ensure that students at various levels have entry points to the task.</td>
</tr>
<tr>
<td>Identify and use up-to-date research based materials and technology</td>
<td>Teachers use their knowledge of the content in each LT to identify relevant and research based materials and technology that support the development of skills listed in the LTs (e.g., recognize what curricular materials are aligned with LT and thus supportive of their learning goal).</td>
</tr>
<tr>
<td>Be accessible to and attend to all students in the class.</td>
<td>Teachers use their knowledge of students’ current mathematical understandings (as indicated by the LTs) to determine the level of support they need to provide for students to ensure they are able to access the task and engage in the mathematical content of the task. This knowledge influences teachers as they monitor their students’ work.</td>
</tr>
<tr>
<td>Foster classroom discussions such that all students can participate and engage.</td>
<td>Teachers use their knowledge of the content in the LTs to scaffold the conversation so that all students have a place in the conversation and to use students’ current mathematical understanding (as evidenced by the LT) to build connections between various mathematical ideas that arise.</td>
</tr>
<tr>
<td>Provide all students with opportunities to engage in rigorous mathematics.</td>
<td>Teachers use their knowledge of where students are currently working in the LT as well as their knowledge of the upcoming concepts in the LT to ensure that the work students are engaged in is rigorous and has the potential to help students progress toward more advanced mathematics.</td>
</tr>
<tr>
<td>Encourage students to engage in mathematical tasks according to their preferences and participate in mathematical discourse in ways that are comfortable for them.</td>
<td>Teachers use their knowledge of the LT and the various strategies in the LT to allow students to work in ways that are comfortable to them and represent their work (written work and verbal descriptions) in ways that align with the students’ understanding of mathematics.</td>
</tr>
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### Table 3 cont’d

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<thead>
<tr>
<th>Achievement</th>
<th>Identity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set high, yet appropriate, academic standards for all students.</td>
<td>Support the development of a robust mathematical identity</td>
</tr>
<tr>
<td>Teachers use their knowledge of the topics of the LT to set goals for students that are appropriate based on students’ current understandings.</td>
<td></td>
</tr>
<tr>
<td>Unpack and build upon their students’ prior mathematical knowledge and use it as a basis for understanding more meaningful and complex mathematics.</td>
<td>Teachers use their knowledge of the LT and how students progress through the LT to support students’ efforts and encourage them to keep moving. The LT serves as a tool for teachers to acknowledge students’ current understandings as well as the knowledge that all students can progress.</td>
</tr>
<tr>
<td>Select and use a variety of forms of assessment (e.g., formative, summative, projects, class discussions) to gauge student achievement.</td>
<td>Teachers use listen to their students and use their knowledge of open tasks to create tasks that are relevant to their homes and communities.</td>
</tr>
<tr>
<td>Select and use a variety of forms of assessment (e.g., formative, summative, projects, class discussions) to gauge student achievement.</td>
<td>Validate the use of students’ own algorithms and strategies to solve problems.</td>
</tr>
<tr>
<td>Teachers use their knowledge of formative assessment along with the LT to think of a variety of ways to solicit evidence about students’ understanding.</td>
<td></td>
</tr>
<tr>
<td>Identify Listen to and consider students out-of-school experiences and design instructional activities that incorporate elements from their homes and communities.</td>
<td>Assist students to build connections between the mathematics they learn and the broader world/society.</td>
</tr>
<tr>
<td>Teachers use their knowledge of the LTs to recognize and validate (encourage) a variety of strategies, algorithms, and tools to solve problems.</td>
<td>Teachers use the connecting portion of the LTBI instructional model to assist students in making not only mathematical connections, but also real world connections.</td>
</tr>
</tbody>
</table>
**Table 3 cont’d**

<table>
<thead>
<tr>
<th>Power</th>
<th>Ensure that students have voice in the classroom.</th>
<th>Teachers allow all students to have voice, or take ownership of the ideas and activities that are a part of the mathematics lesson and draw upon topics from the LT as a means to include all students (e.g., voting on a classroom decision and using mathematics from the LT to analyze the results and make a decision).</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Position students as experts in the classroom (this includes things they know in school and things they know from outside of school).</td>
<td>Teachers use their knowledge of the LTs and various skills listed and position students as experts based on their usage of certain skills or strategies.</td>
</tr>
<tr>
<td></td>
<td>Allow students to solve problems that are relevant to them (these problems can exist inside or outside of school).</td>
<td>Teachers select or create open tasks that are relevant to their students’ lives and impact their communities.</td>
</tr>
<tr>
<td></td>
<td>Encourage all students to present, justify, and defend their mathematical ideas/arguments.</td>
<td>Teachers use their knowledge of the LT to recognize various mathematical ideas present in the classroom and encourage all students to present, justify, and defend their ideas. Teachers’ knowledge of the LT will be useful as they select and sequence students and encourage students to make mathematical connections.</td>
</tr>
<tr>
<td></td>
<td>Help students to see themselves as sources of mathematical knowledge.</td>
<td>Teachers’ knowledge of the LT will be a tool that will assist them to frame every student as a possessor of mathematical knowledge. Therefore, teachers recognize what students already know and empower students by helping them see themselves as possessors of mathematical skills.</td>
</tr>
</tbody>
</table>
Participants and Study Context

The next phase of Eisenthaldt’s (1989) process is case selection. The participants in this study were a subset of teachers from the LTBI project. The project as a whole included seven elementary teachers from one partner school in a professional development program focused on LTs and LTBI. Because the content of the LTs was focused on early numbers, counting, and addition and subtraction, the work was particularly relevant for K-2 teachers. Using a purposeful sampling technique, all of the K-2 teachers in the project were invited to participate. The four teachers who participated were three kindergarten and one first grade teacher in the project; there were no second grade teachers in the professional development as a whole, and the three upper-grade teachers in the projects were not invited to be part of this study. Each of the participating teachers agreed to participate and thus served as the cases for this research. I chose the pseudonyms of Elsie, Elizabeth, Carolina and Anna for the four cases I will present.

At the onset of this study, all seven LTBI participants had completed a mathematics content assessment as well as a beliefs assessment. Differences in teachers’ content knowledge and beliefs as well as their participation in project meetings indicated that these four cases would provide variability in implementation of LTBI. Table 4 provides average information from all seven PD participants and situates participating teachers in relation to the average of the group. Although the beliefs instrument (see Appendix A) did not specifically address issues of equity in pedagogy, it did address teachers’ views of mathematics, the teachers’ role in the classroom, as well as the student’s role in the classroom which do relate to constructs of the equity framework.
Table 4

Summary of Mathematical Knowledge for Teaching Pre/Post Test Scores

<table>
<thead>
<tr>
<th>Assessment</th>
<th>Mean (n=7)</th>
<th>St. dev</th>
<th>Elsie</th>
<th>Elizabeth</th>
<th>Caroline</th>
<th>Anna</th>
</tr>
</thead>
<tbody>
<tr>
<td>LMT (Pre)</td>
<td>17.14</td>
<td>4.14</td>
<td>13</td>
<td>17</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>LMT (Post)</td>
<td>18.28</td>
<td>5.50</td>
<td>20</td>
<td>14</td>
<td>8</td>
<td>19</td>
</tr>
<tr>
<td>Beliefs Instrument</td>
<td>137.71</td>
<td>4.96</td>
<td>137</td>
<td>134</td>
<td>132</td>
<td>140</td>
</tr>
</tbody>
</table>

This table presents a few important ideas. Elizabeth had the highest content score among the four participants on the LMT pre-assessment. Her score was aligned with that of the three upper grades teachers in the LTBI Project. Although Elizabeth’s mathematics content score was initially one of the highest, she articulated numerous times that she did not feel comfortable with math and did not teach math daily in her classroom as a result. (Note: This is discussed in greater detail in her case portrait, which I provide later in this chapter.) Another important point is that both Elsie and Anna made large gains in their LMT post-assessments and were above the group mean on the post LMT assessment. Caroline did not complete all of the items on her post assessment, which explains the decrease in her LMT score. The scores on the beliefs instrument did not have a large variance, but Elizabeth’s and Caroline’s scores both indicate that their classrooms were more teacher-centered than those of other teachers in the study.

Context

All participants for this study taught at the one partner school that was part of the LTBI larger project. The school was located in a suburban area in the southeastern United
States. The partner school was built in the late 1920’s with continual additions and improvements being made through the 1990’s. The school sits on a five-acre campus in the heart of a historic area and has an enrollment of 370 students. There are 21 teachers in the school. Four of them are kindergarten teachers and four teach first grade. Average class sizes are smaller than both the district and state averages. Demographic data indicates that 36.2% of students are considered economically disadvantaged and 27.1% of students have limited English proficiency. The teacher turnover rate is lower than the district average but higher than the state average.

In the past few years, the school has not made significant growth according to standardized testing models. The school improvement plan suggests that they are spending considerable time identifying students that need additional assistance, using common assessments to group students, and providing teachers with additional professional development in various subject matters.

The research team gained access to the school by approaching district personnel as well as the principal of the school. At the end of the 2011-2012 school year, the research team gave a presentation to the entire staff detailing the goals of the project, requirements for participation, and intended benefits for teachers. Twelve teachers initially expressed interest in the project and participated in the summer professional development. Seven teachers completed the full year of the project.

**LTBI Professional Development**

This study is a part of a larger designed study entitled Learning Trajectory Based Instruction. Learning trajectory-based instruction is defined as instruction that situates LTs
at the center of the teaching practice. The goals of the larger LTBI project were to (1) explore the impact of LTs on elementary teachers mathematics instruction; (2) build a conceptual model of instruction that is centered on LTs; and (3) confront teachers’ stereotypes of students (Harré & van Langenhove, 1999) by focusing on what students can do (as seen in LTs) and building upon it through supportive classroom practices (instead of focusing on what they cannot do).

The professional development began with a 30-hour summer institute in which participants learned about Clements and Sarama’s (2009) LTs for early number and counting, addition and subtraction problem types. Because the content of the LT was dense, the LTBI research team organized multiple LTs from Clements and Sarama’s work into what we called “Learner Profiles.” These profiles—named perceptual child, direct modeling child, counting on child, place value child and multi digit child—provided a way for teachers to “chunk” information about students and created a broader perspective. The hypothesis was that these profiles would provide a more manageable grain size of information for teachers as opposed to each individual level of each individual LT (across the four LTs presented to teachers in the project, there were a total of 36 levels). Over the summer, teachers learned the characteristics of each student profile (see the rows in Table 5) and began to examine the individual LTs (see columns in Table 5).

During the summer PD, we also discussed Confrey’s (2006) conceptual corridor (see Figure 1). Specifically, we discussed the idea that students may enter at different places and move through concepts at different times. Although movement through LTs is probabilistic, we do know that there are certain landmarks and obstacles that students encounter as they...
move from less sophisticated understandings to more sophistication. Using the conceptual corridor as a backdrop, we discussed that the LT profile labels are not intended to be “fixed” labels for students, rather they are to be used to help teachers “locate” students in the LTs and help them move within the corridor.
Table 5  Clements & Sarama’s (2009) LT organized into Profiles

<table>
<thead>
<tr>
<th>LT Level</th>
<th>QUANTITY</th>
<th>COUNT</th>
<th>ADDITION/SUBTRACTION</th>
<th>PLACE VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perceptual</td>
<td>Perceptual Subtizing</td>
<td>Perceptual Subtizer to 4 then 5</td>
<td>Perceptual Subtizer to 4 then 5</td>
<td>Perceptual Subtizer to 4 then 5</td>
</tr>
<tr>
<td>Child</td>
<td></td>
<td>Recter (10)</td>
<td>Corresponder</td>
<td>Small Number</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Counter Small Number</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Counter to 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Producer to 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct</td>
<td>Conceptual Subtizing</td>
<td>Counter and Producer to 10+</td>
<td>Find result (joining, part-part-whole with direct modeling, counting all; take away using objects)</td>
<td>Composer to 4, 5, 7 then 10 (knows number combinations, doubles to 10)</td>
</tr>
<tr>
<td>Modeling</td>
<td></td>
<td>Counter backward from 10</td>
<td>Make it N (adds on to make another number)</td>
<td></td>
</tr>
<tr>
<td>Child</td>
<td></td>
<td>Counter from N (N+1, N-1)</td>
<td>Find change (finds missing addend using objects)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Skip Counter by 10s to 100</td>
<td>Join-to—count all, Separate-to—count all, and Match—count rest</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Counter to 100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Counting</td>
<td>Extended Subtizing</td>
<td>Counter on using patterns (keeps track using numerical patterns)</td>
<td>Counting strategies (join and part-part-whole)</td>
<td></td>
</tr>
<tr>
<td>on Child</td>
<td></td>
<td>Counter on by fives and twos with understanding</td>
<td>Using finger patterns, Counting on, Counting up to</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Counter of imagined items (counts mental images of objects)</td>
<td>Part-whole (flexibly solve all previous problem types, sometimes start unknown)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Counter on keeping track</td>
<td>Number-in-number (keeps part and whole in mind simultaneously; uses counting strategies for start unknown)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Counter of quantitative units and place value (understands base 10 system, decomposes a ten into ones when useful)</td>
<td>Derive (flexibly uses strategies and derived combinations)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Counter to 200 (recognizes patterns)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Place Value</td>
<td>Place Value Subtizing</td>
<td>Number Conserver</td>
<td>Problem solver (flexibly uses strategies and known combinations)</td>
<td>Composer with 10s and 15 (understands 2 digit number as tens and ones)</td>
</tr>
<tr>
<td>Child</td>
<td></td>
<td>Counter Forward and Back</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multi Digit</td>
<td>Multiplication Subtizing</td>
<td>Multiplication Subtizer with Place Value and Multiplication</td>
<td>Multidigit solved by incrementing or using tens and ones</td>
<td></td>
</tr>
<tr>
<td>Child</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In addition to learning about the LT in the summer, teachers also learned about Stein et al.’s (2008) framework for five practices that support mathematical discourse as well as formative assessment. Figure 2 is a visual display of what was presented to teachers in the project about how LTs can inform each of these important facets of teaching and learning.

Figure 2. Placing LTs at the Center of the Five Practices (LTBI Model for Instruction)

Figure 2 is the basis for the instructional model on which the LTBI PD focused. In 2012, Sztajn, Confrey, Wilson, and Edgington defined LTBI as “teaching that uses students’ LTs as the basis for instructional decisions” (p. 147). Additionally they stated that teachers could use their knowledge of strategies, misconceptions, and how student thinking develops
over time to accomplish several things: anticipate students’ approaches in relation to the learning goal; monitor and probe students as they worked in accordance to their anticipations; select and sequence a variety of strategies to put important mathematical ideas on the floor for discussion; and use their knowledge of to build important mathematical connections (Sztajn et al., 2012).

Another important component of the LTBI instructional model is the role that LTs play in formative instruction (evidence and feedback). Specifically, the PD included discussions of how the LT can be used to solicit evidence of student thinking. Teachers were also asked to use the LT to make sense of this evidence as opposed to a using a disciplinary-approach to mathematics. Additionally, teachers used the content of the LT to provide specific feedback to students. An assumption of the larger LTBI project was that LTs can support teachers in this model of instruction by assisting them to focus on the individual student and designing instruction that meets their needs based on their current conceptions.

Following the initial 30 hours of professional development on the LT and LTBI instructional model, teachers met with the project team for three sessions after school during the fall semester and four additional sessions during the spring semester. Each of those sessions lasted three hours, and the agenda for each was developed after reviewing video and audio from previous sessions (which is a critical component of the design research process). These sessions provided teachers with additional exposure to the LT, LTBI, and allowed teachers to share results of LTBI tasks and assessments they were asked to implement in their classrooms in between meetings. Having introduced each of the profiles and individual LTs to teachers in the summer, we revisited them throughout the year as we discussed how these
individual pieces came together during instruction. Teachers examined how one task could draw on multiple LTs simultaneously as well as how students could be working at one profile level for one LT and at a different level for another LT. The LTBI team also asked teachers to engage in clinical interviews with their students, assessment, and lesson planning that helped foster these connections and reinforced the initial discussion in the summer of LTs being a “fluid” construct. Table 6 provides a summary of what was covered during each session during the year (30 hours in the summer over four days, plus follow-up meetings and tasks implemented between the meetings).

Table 6

*Summary of LTBI PD Sessions*

| Summer Institute | Day #1 | · Defined LTs (discussed them as being fluid and probabilistic)  
| | · Discussed idea of LTs’ conceptual corridor (Confrey, 2006)  
| | · Introduced the perceptual child profile  
| | · Introduced the early counting child profile  
| | Day #2 | · Introduced the five practices  
| | · Introduced the counting on child profile  
| | · Introduced the place value child profile  
| | Day #3 | · Introduced the multi-digit child profile  
| | · Discussed Formative assessment and the multi-digit child  
| | · Introduced the counting LT  
| | Day #4 | · Discussed using the counting LT in whole class instruction  
| | · Connected the LTs to the CCSS  
| | · Used the counting LT to plan lessons (in grade level groups)  

Table 6 cont’d

Summary of LTBI PD Sessions

| Monthly Sessions | Session #5 | · Reviewed the counting LT  
|                 |           | · Introduced the addition and subtraction LT  
|                 |           | · Discussed assessment using the addition and subtraction LT  
|                 | Task: Teachers were asked to use an addition and subtraction task to assess all of their students  
|                 | Session #6 | · Reviewed addition and subtraction LT  
|                 |           | · Discussed assessing students using the addition and subtraction LT  
|                 | Task: Teachers were asked to select two students to conduct clinical interviews with. They were asked to select students that they thought were working at “different” places in the LTs based on their responses to the whole class assessment.  
|                 | Session #7 | · Debriefed from student interviews  
|                 |           | · Presented an episode of “Experiencing LTBI in the classroom”  
|                 |           | · Introduced the place value LT  
|                 | Session #8 | · Introduced the quantity LT  
|                 |           | · Co-planned a lesson using the addition and subtraction LT  
|                 | Task: Teachers were asked to teach the lesson that we co-planned during this PD session.  
|                 | Session #9 | · Debriefed from the addition and subtraction LT lessons  
|                 |           | · Discussed how to use LTs in whole class instruction  
|                 |           | · Teachers planned an LTBI lesson individually  
|                 | Task: Teachers were asked to teach the lesson that they planned during this PD session. An LTBI team member came to observe.  
|                 | Session #10 | · Discussed how to use LTs and LTBI to make instructional decisions  

At the onset of this dissertation study, the participants had completed the 30 hours of professional development during the summer institute as well as an additional 9 hours during the fall. This previous engagement with LT and LTBI indicated that when this study began,
teachers were already familiar with the overall ideas of LT and LTBI, allowing this research to examine connections between these ideas and equity. This study was conducted concurrently with the remaining 12 hours of professional development in the spring of 2013.

**Data Sources and Data Collection**

The third phase of Eisenhardt's (1989) framework is the development of research instruments and protocols. Equity and teaching equitably are not compartmentalized concepts; therefore, it was important to examine various facets of teachers’ practice to see where and in what ways they might be using LTBI to teach equitably. One strength of the case study method is “its ability to deal with a full variety of evidence—documents, artifacts, interviews, and observation” (Yin, 2003, p. 8). Thus, the primary data sources for this study were transcripts of pre-observation and post-observation interviews using researcher-designed protocols; pre- and post- teacher-developed student portraits; videos of classroom observations; and field notes. Secondary data sources included pre- and post- LMT scores; beliefs instrument scores; Instructional Quality Assessment (IQA) scores; teacher lesson planning documents; and samples of student work. These secondary items were not directly collected by the researcher and not used in cross-case analysis; rather, the LMT, beliefs, and IQA scores were used solely for providing descriptive information about the participants and to help the reader better understand the participants’ mathematics instruction. The lesson planning documents and samples of student work were used as reference points during pre- and post- observation interviews when participants chose to refer to them (Note: I did not require participants to submit these items for the dissertation study).
See Figure 3 for the data collection sequence and Table 5 for a list of data by participant. It is important to note that Caroline chose to not complete her final round of data collection. Additionally, Anna was unable to complete her final observation due to repeated scheduling conflicts. I also discuss how I dealt with missing data in the upcoming paragraphs.

*Figure 3. Data Collection Sequence*

Data collection occurred from February 2013 through May 2013. The following paragraphs describe each of the data sources. For some data sources, I list the research questions that were answered as a result of evidence from those sources. Because each of the interview protocols are listed in the appendices and directly relate to the research questions, I will not include a discussion of how the research questions were linked to the interview protocols.
Table 7

List of Data Sources by Participant

<table>
<thead>
<tr>
<th></th>
<th>Primary Data Sources</th>
<th>Secondary Data Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Interview #1</td>
<td>Interview #2</td>
</tr>
<tr>
<td>Initial Mathematics Portraits</td>
<td>Interview #3</td>
<td>Interview #4</td>
</tr>
<tr>
<td>Elsie</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Elizabeth</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Caroline</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Anna</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

Mathematical Student Portraits

The participants were asked to create a student portrait for each of their students that had consent to participate in the study (See Appendix B). In the portrait, I asked teachers to write about their students’ knowledge of mathematics, evidence to support teachers’ knowledge claims about the student, instructional goals for the student, and the opportunities needed for the student to progress. For each of these four areas, I asked the teachers to discuss the role of the LT in responding to each question. These mathematical portraits were used to gather evidence to answer the following research questions:
1a. How do teachers use LTs and LTBI to design instruction and instructional tasks such that they are accessible for all students?

1e. How do LTs and LTBI assist teachers in providing all students with opportunities to engage in rigorous mathematics?

2a. How do teachers use LTs to set high academic standards for all students?

2b. How do teachers use LTs to unpack and build upon students’ prior mathematical knowledge and use it as a basis for building more complex mathematics?

2c. How do teachers use LTs and LTBI to implement various types of assessment to gauge students’ achievement?

Not only were the student portraits designed to allow for the examination of how teachers discussed individual students, they were also used to compare and contrast how teachers talked about groups of students. Additionally, the student portraits allowed me to examine how individual students and groups of students made progress or did not make progress during the study and what factors teachers credited towards that movement/growth.

Three of the four participants completed the initial round of mathematical portraits and used them during our first interview. Because one teacher did not complete the portraits, I used the first interview as an opportunity for her to discuss her students in relation to the questions in the portraits. Each of the participants revisited her initial portraits during the second interview and discussed her students in relation to the LT. Only Anna completed and submitted mathematical portraits at the end of the study. I asked Elsie and Elizabeth to discuss their students in relation to the mathematics profiles during their final interviews.
Interview Protocol #1

In the first interview, I asked teachers a series of questions related to each of the four dimensions of equity. While the questions did relate to the constructs in the LTI-Equity framework, I did not use the language of access, achievement, identity, and power during this interview (see Appendix C). I also used this time to discuss the initial student mathematical portraits with each teacher. Teachers noted during this interview that as they completed their portraits, they found it very repetitive as groups of their students had similar characteristics and learning needs in relation to mathematics. Therefore, teachers found it helpful to group their students for ease of discussion and thus “sorted” their portraits into groups according to LT profile level (e.g., perceptual child, counting on child, etc.). An external transcriber transcribed each of the interviews in its entirety.

Interview Protocol #2

During the second interview, I asked each teacher to reflect on the lesson they had taught generally and then in relation to the four equity dimensions. During this portion of the interview, I still did not use the specific language from the four dimensions, but I introduced the ideas that each dimension represented. After teachers had reflected on the lesson during the interview, I presented to them the LTBI-Equity framework and discussed each of the constructs with them. I asked teachers to share their thoughts on each of the constructs as well as any examples from their instruction that they thought fit into those constructs. I concluded this interview by asking teachers to think of one construct they would work on incorporating into their next observation. An external transcriber transcribed all of the interviews in their entirety.
**Interview Protocol #3**

During the third interview, I started by asking teachers to reflect on the LTBI-Equity framework since I had introduced it to them. I then asked them to reflect on the lesson I had observed in relation to each construct and each dimension in the framework. I concluded this part of the interview by asking them to talk to me about the next lesson I would observe and how they would work to promote access, achievement, identity, and power during that lesson (See Appendix F). I also used this interview as an opportunity to explore emerging themes that resulted from on-going data analysis (e.g., the use of the word confidence). Question 8 from the interview protocol was used as a way to frame the discussion about the mathematics portraits and students’ movement during the semester. Question 9 from the protocol was included to unpack themes and language that had arisen from previous interviews. An external transcriber transcribed all of the interviews in their entirety.

**Interview Protocol #4**

During the final interview, I started by asking teachers to reflect on the most recent class lesson I had observed. I also wanted to collect more direct evidence of the role of the Five Practices in the LTBI-Equity framework. Therefore, I added an additional question about the practices to elicit this information from the participants (See Appendix G). I also used this interview to probe two emerging themes that had arisen from my work with the teachers in this study as well as during the LTBI project. I asked teachers again about their meaning for the word “confidence” and about how they interpreted students being at various levels of a profile (e.g., moving out of the perceptual level, moving into the place value level). An external transcriber transcribed all of the interviews in their entirety.
Videotaped Classroom Observations

All classroom observations were videotaped and occurred over one to two days. I manned the video camera during each of the observations and zoomed in when teachers were working with individual students. All teachers wore a Bluetooth microphone to capture whole class dialogue as well as their conversations with small groups or individual students. During each of these observations, I collected field notes (see Appendix D) and acted as a silent observer. While observing the teachers and reviewing the videotaped observations, I looked for evidence to answer the following research questions:

1c. How do LTs and LTBI help teachers think about being accessible to all students during instruction?

1d. How do LTs and LTBI support students such that all students can engage in classroom discussions?

3b. How do teachers use LTBI as a means to listen to and consider students out-of-school experiences and design instructional activities that incorporate elements from their homes and communities.

3c. How do teachers use LTs and LTBI to solicit and validate various algorithms and solutions to tasks?

3d. How do teachers use the LTBI model to provide opportunities for students to build connections between the mathematics they learn and the broader world/society?
3e. How do teachers use LTBI to encourage students to engage in mathematical tasks according to their preferences and participate in mathematical discourse in ways that are comfortable for them?

4a. How do teachers use LTs and LTBI to give all students voice in the classroom?

4b. How do teachers use LTs and LTBI to position students as experts in the classroom?

4c. How do teachers use LTs and LTBI to provide students with opportunities to solve problems that are relevant to them?

4d. How do teachers use LTBI to provide opportunities for all students to present, justify, and defend their mathematical ideas?

4e. How do teachers use LTs to see all students as sources of mathematical knowledge?

**Instructional quality assessment (IQA)**

The Instructional Quality Assessment (IQA) Rubrics can be used to assess teachers’ instruction as well as their students’ work. According to the authors, these rubrics were designed to serve the following purposes:

- Supplement school assessment systems based on student achievement,
- Evaluate equity in students learning opportunities,
- Research and evaluation of reform oriented instructional practices as well as effectiveness of professional development initiatives, and
- Serve as a learning *tool* for improving instruction (Junker et al., 2004, p. 2)
Teachers’ lessons were evaluated on four rubrics: a) potential of the task (AR1); b) implementation of the task (AR2); c) student discussion following the task (AR3); and d) teacher questioning (ARQ). Scores on these rubrics can range from zero to four and each of these four rubrics reflects important aspects of the LTBI-Equity framework. This assessment was selected for use in the LTBI project because it nicely aligned with the Smith and Stein (2011) instructional framework presented to teachers.

Two raters were hired externally and trained by an IQA expert trainer who reported that raters participated in a two-day training where they reached at least 80% exact-point agreement on training videos before rating lessons individually. Twenty-four percent of lessons in the LTBI Project sample (13 of which were from my dissertation study) were double coded, with ninety-one percent exact-point agreement.

The IQA data was used in this study for three reasons. First, the IQA measure was theoretically aligned with the LTBI as proposed during the PD. Second, it was necessary to establish that the lessons from the participants were of high quality, which is a pre-cursor to looking for other instances of equity. Finally, I was able to compare my results from classroom observations and field notes to the results of the scoring from external coders. Since I created descriptive portraits of each teacher in relation to their overall mathematics instruction as well as task selection and implementation, the results from the IQA coding served to triangulate my descriptions of the observed instruction. The IQA scoring results were completed after I completed the participants’ mathematics teaching portraits, and allowed me to examine my findings in relation to those from the external coders, which increased the validity of my findings.
The other reason this scoring was necessary was to provide an external evaluation of the rigor of teachers’ mathematics instruction. I considered that if all of teachers’ lessons were rated poorly, further discussions of issues of equity would be null because sound mathematics instruction would not have been in place. The results from the IQA scoring (presented with each teacher portrait) shed light on some of the tensions teachers faced as they attempted to use the LT to promote equitable instruction.

**Data Analysis**

According to Eisenhardt’s (1989) framework, a hallmark of the case study methodology is “the frequent overlap of data analysis with data collection” (p. 538). Other scholars who conduct case study research also support ongoing data collection and data analysis as a way to manage/organize data, reveal emerging themes, and ensure that your data is focused on your research questions (Glaser & Strauss, 1967; Merriam, 1998; Yin, 2009). In the upcoming paragraphs, I discuss my data analysis for this study, which occurred in two phases: ongoing and retrospective (Cobb, Confrey, DiSessa, Lehrer, & Schauble, 2003). These two phases of data analysis address steps four, five, and six of Eisenhardt’s process, namely entering the field, analyzing the data, and shaping hypotheses.

**Ongoing Analysis**

The goal of the ongoing data analysis was to understand emergent themes as they related to the LTBI-Equity framework. Each interview and notes from the interview were reviewed immediately following the interview. Notes were made about the language teachers used during their interviews and other concepts from the teachers that were unclear to the researcher. I also made notes of trends that occurred across teachers as preliminary themes
(e.g., confidence, high vs. low). Coding was done using Atlas.ti qualitative analysis software. Videos and interview transcripts were coded directly using the software.

During the on-going analysis, both open coding and pre-determined codes were used. The pre-determined codes consisted of the four dimensions of the equity framework as well as the practices that comprise the LTBI model of instruction. These codes all resulted from the research questions as well as the LTBI model of instruction. They were applied to interview transcripts, student mathematics portraits, and classroom videos. Some data were double or triple coded when appropriate. Table 7 lists each of the pre-determined codes as well as examples from the data that received the associated code during the initial round of coding. Although I did not code the data during the initial round according to each of the individual constructs/propositions in the LTBI-Equity framework (e.g., 1a, 1b, 1c…), the examples presented in the following table are meant to be exemplars of different constructs within a dimension.
Table 8

**Examples of pre-determined codes**

| Access | “Common Core is beginning to look at place value, and so tens and ones and how that goes together, and we really haven’t done a lot with just seeing how tens come together in that tens frame, or what that grouping of ten is to then work into having a ten and then your ones. So my intent right now is to get the kids using the tens frame, being flexible with it, and then moving into thinking about place value with tens and ones.”

“For the number to choose, I wanted to choose something that even my lowest could access and I think eight was kind of in the middle. I didn’t want to pick something higher where it would be kind of...beyond ten. I didn’t want to pick ten because that’s kind of known facts for a lot of them. I didn’t want to pick past ten because that’s too much for them to access ‘cause they’re really working with their numbers up to ten...”

| Achievement | “I guess I haven’t really come up with a succinct goal, but in my mind I was trying to see if I could work through how they’re looking at breaking ten in half and counting on from a higher number.”

“Some of my groups have worked with ten already so that’s why I hope that they can do combinations of twelve.”

| Identity | “Monkey soccer is what they were playing at recess that day and the others, they like to play tag too. So I just chose two things that most of them are interested in playing outside so they can understand that things in their everyday lives will help them think about math concepts.”

| Power | The teacher offered this response when asked how she affirms students as they are sharing their work:

“Oh. Well that’s an interesting idea...you know. Tell me more...why you’re thinking that way....or...Wow! This is different that so and so thought about it, but do they both work?”

| Task & Learning Goal | “So the intended learning goal is for them to be able to list different combinations of eight.”

| Anticipate | “I anticipated that a lot more students would have tried something and [I] ended up having a lot of students not try something or tried something completely inappropriate...I anticipated that some of my kids would have been doing more than what they were doing cause I know they were choosing to not do things they’ve done in the past”

| Monitor | “I think with that group, like I said, I wanted to leave it open-ended when I present it and then kind of as I’m conferencing with them see what they do.”


And so… and seeing the videos of kids sharing this summer was one of the things, because you want to have kids who maybe aren’t doing as high level of math, so sharing theirs, the talking about it I think helps solidify it in their mind, but also there are other kids who aren’t doing as high a level of math so it’s good for them to see that model as well.”

“Well, I guess I try and make sure everyone is able to share, but I try to be strategic with who I share or just keep note as I go of who shared on what days so I can make sure that I’ve pulled everyone within at least a week’s time.”

| Select & Sequence | “And so…and seeing the videos of kids sharing this summer was one of the things, because you want to have kids who maybe aren’t doing as high level of math, so sharing theirs, the talking about it I think helps solidify it in their mind, but also there are other kids who aren’t doing as high a level of math so it’s good for them to see that model as well.”

| Connect | No data was coded as connecting in the initial round.

As an example of multiple coding, the following piece of data was triple coded in the initial round of data analysis as achievement, anticipating, and monitoring:

Before the lesson, I had kind of gone through and done the pre-entry on each of them so I kind of in my head had a mental note of what I know they can do, what I want them to be doing, or what I’m pushing them to do or what they’re working on so I kind of had that mental in my head as I was going around to each of them. So I had that in my head, like the pre-entry that I had done on them and I was walking around looking for those specific things or pushing them towards those specific things and I think for a lot of them like I said what I had anticipated or what they knew they could do they weren’t doing or some of them, some things that I thought they would be doing they weren’t doing. So it kind of made me say, okay, well…I really have to backtrack here.

This piece of data received all three codes because the teacher noted that she anticipated what students could do when she wrote her pre-entry (mathematical student portraits). She goes on...
to discuss that as she was walking around the classroom she was looking for evidence of her
anticipations. Finally, this data was coded as achievement because the teacher indicated that
she tried to move kids forward or “push them” towards a goal she set for them based on their
current level of understanding.

During on-going analysis, open coding was also used. The goal of the open coding
process was to allow for themes not originally included the framework to emerge. The
following open codes were developed during on-going data analysis: curriculum, high versus
low students, comparing, literacy connection, usefulness of LT, using “high” students as
examples, confidence, deficit orientation, and motivation. As data analysis continued, these
codes were refined. For example, as I examined data that was coded “high” versus “low,”
and using “high” students as examples, it became clear that both of these instances reflected
the teachers’ deficit orientation. Thus both of these codes were merged with “deficit
orientation” as a larger emerging theme in the study.

Another example of an important code that resulted from open coding was
“confidence.” This was a word repeatedly and consistently used by all four case teachers.
Because I was initially unsure of how teachers were using this term, I coded all pieces of data
that had the word confidence. I then added additional questions to future interview protocols
to probe teachers’ use of this word. Through this process, the following ideas emerged from
what teachers were saying:

i. Confidence is an important skill that needs to be developed in kindergarten so
   students do not always resort to relying on the teacher.
ii. Students need to be confident of their mathematical ideas so they can take ownership of them.

iii. Students’ confidence with mathematical skills at a certain LT level gives teachers an idea of whether or not they are ready to progress.

iv. If students are not confident, they will be reluctant to share their ideas.

v. If students are not confident they will be less likely to attempt a problem or try a new strategy that the teacher suggests.

These new meanings that emerged for confidence allowed me to re-examine each piece of data that received this code and determine its relevance to the study. For example, the idea present in statement (a) was discarded because it was related to a general skill that kindergarten teachers work on throughout the year. The second idea was recoded as identity as the teacher indicated that she needed to validate students as a means to help them own their thinking. The third idea was recoded as achievement because the statement related to teachers’ making sense of what students are demonstrating, how they demonstrate this knowledge, and how goals should be set for them as a result. The fourth idea was recoded as power because it related to sharing mathematical ideas. The final idea was recoded as access because it related to the teacher providing a more rigorous/sophisticated strategy for a student and whether or not the student took that strategy up.

At the conclusion of the on-going analysis, all data sources had been coded using codes related to the equity dimensions, the LTBI models of instruction, and large emerging themes such as the deficit perspective. In the next phase of data analysis, each of these
examples was examined again, within and across cases, looking specifically for presence of the LT or LTBI model and the role of LT and LTBI in equitable instruction.

**Retrospective Analysis**

**Within-Case.** Merriam (1998) states that for multiple cases studies there are two phases of analysis necessary, within-case and cross-case. During within-case analysis, each case is treated as an individual study, and the goal of this analysis is for the researcher to learn as much about the case as possible. Eisenhardt (1989) and Patton (1990) both state the need for a “case record” or a detailed description of each case. While these write-ups are descriptive in nature, “they are central to the generation of insight” (Gersick, 1988; Pettigrew, 1988). Therefore, the first phase of my retrospective analysis was to complete a detailed case portrait of each teacher. In these portraits, I reviewed all data collected for each individual teacher and described each case teacher according to demographic background, mathematics instruction, participation in the LTBI project, classroom organization/structure, task selection and implementation, lesson quality, how they talked about their students, how they used the LT in their instruction, and what the dimensions of equity looked like in their classrooms. The first part of chapter four includes the case portraits and is intended to provide the reader with insight into who each teacher is and what their classrooms consisted of. Because the last two items in the original case portraits (use of LT in instruction and equity during instruction) directly related to the research questions, they were later removed from the individual teacher portraits presented in the first part of the following chapter and were used in the cross-case analysis that constitutes the second part of that chapter.
Cross-Case. After the within-case analysis was completed, I began the cross-case analysis. For this stage of the data analysis, I reviewed all data that had been coded across teachers to look for evidence of the LT and/or LTBI. I then connected these data to the dimensions of the equity framework. After this step, I read through all data that was coded as “Identity & LT” or “Identity & LTBI” to examine where they fit within the various constructs of the identity dimension. The final step of the analysis was to review all data coded as, for example, as “Identity/LT/3c” as well as “Identity/3c” to look for themes of how the LT was used as well as examples of non-use for each construct, within each dimension of the framework. One important code that resulted from this round of data analysis was tensions. Tensions are defined as examples when the teacher a) recognized the importance of the construct, but had difficulty enacting it; b) enacted a construct, but did not draw upon the LT in doing so; c) failed to implement a construct due to deficit orientations; or d) implemented instruction that was not aligned with the LTBI instructional model.

Eisenhardt (1989) argued that looking for cross-case patterns is important because it forces the researcher to examine data in diverse ways (including similarities and differences). Yin (2009) suggests creating “word tables” in which all of the data related to a single construct are located in one table. This format assists the researcher in the “examination of cross-case patterns” and helps the researcher “draw cross-case conclusions” (Yin, p. 160). The result of analyzing these word tables is the construction and refinement of themes. It is critical in this phase that the researcher verifies each theme across all evidence to confirm, revise, disconfirm, or throw out the theme (Eisenhardt, 1989). Because tensions were an
important theme that had arisen from the data analysis, I kept conflicting themes in my data corpus as both the use of and tensions related to a construct had important impacts on equity.

Table 8 is an abbreviated example of a word table I created to reflect one construct in the identity theme. In this table, I present a portion of the data that was coded as “Identity and 3c” as well as “Identity/LT/3c.” Some of these examples indicate presence of the LT or LTBI model and others do not. Note that all examples are not included in this word table because they are too numerous. The goal of this table is only to illustrate the process used in the cross-case analysis as I sought to answer my research questions.
<table>
<thead>
<tr>
<th>Teacher</th>
<th>Construct: Validating the use of students’ own algorithms and strategies to solve problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elsie</td>
<td>“I mean, I think a lot of times when you ask open-ended questions like you have an idea of what they’re going to say. It wasn’t the first time we had done tens…used tens frames, but we hadn’t used them tons. You do kind of have to go with what the kids are saying and then try to move it back to the point where you’re trying to get to.”&lt;br&gt;“Like balancing letting the kids really be creative versus saying “No, this is the way we have to do it”…Like I had one little girl Mira…she wasn’t even thinking about the number I don’t think. She was thinking about how many different ways she could make it look on the [tens frame]…so that was something I struggled with because, yes she thought about doing it a different way or something, but it wasn’t a way that I think was helping her or helping her friends. Although mathematically in her brain it was helping her and I just couldn’t see it.”&lt;br&gt;“I’m definitely not a ‘it’s my way and my way is the only way and this is how you need to do it’ kind of person…I think we all have different ways that we think about things and I never have a problem with letting kids try to do things a little bit differently.”</td>
</tr>
<tr>
<td>Elizabeth</td>
<td>“I think similar to how I handled the ladybugs, the basic subtraction, concept of subtraction. If we’re working as a whole group to have a lesson on subtraction, we’ll stick with the strategy that I’m trying to get across and make sure everyone across the board gets this general strategy. And if you have something interesting or complicated or you’ve noticed multiplication or something like that, let’s write id down, let’s talk about it later…”</td>
</tr>
<tr>
<td>Caroline</td>
<td>“I gave them numbers, so some of the numbers were like up into the 30s and 40s and some were just 0-10, and so they each had to flip over a card they had to decide…they were working with partners and they had to decide which number was greater and then I told them…they had to record it on their paper in a way that other people would be able to understand what they were writing. So they came up with their own…we didn’t introduce the sign yet. This is like the first day.”&lt;br&gt;“Well I guess the trajectory helped me do that with the greater than and less than. I was like “Is it okay for me to just let them come up with their own solutions?” So I probably would not have done that before.”</td>
</tr>
</tbody>
</table>
Table 9 cont’d

*Word Table Example*

<table>
<thead>
<tr>
<th>Anna</th>
<th>“When I’m watching them solve, when I’m contemplating them to solve, I’m looking for you know…I’m not asking them to solve it one…I mean if they’re coming up with a different way to solve it. I’m thinking of that as, oh great, this is a different way that I could share that might be helpful for another student. So I’m pushing them to try something that I know that they need to be trying, but at the same time if they’re coming up with something that I know you know the strategy could be useful to a student you know I would like them to share as many different strategies and as many different you know ideas that they have as possible.”</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>In the beginning I was trying to leave it completely open. If you want counters, there’s counters up here. If you want your number line, all those things are there. But as we’re getting…trying to push for different strategies and try to push for the next strategy, I’ve been being more limited on it…So I try in the beginning to leave it open-ended and then I guess as we get closer to the end of the year I try to scale down and maybe work away. I think in first grade the go-to for everybody would be direct modeling it and I’m trying to steer a lot of my kids that I know don’t have to draw 25. And we’re getting to 60 take away 40. Okay, well that’s probably not a great time to do that.”</td>
</tr>
</tbody>
</table>

From analysis of this table, the following themes emerged for the ways in which teachers used LTs and LTBI in this construct: a) supporting a variety of strategies; b) allowing students to modify the task; and c) allowing students to invent representations. The following tensions were present in this construct: a) open versus moving towards sophistication; and b) No LT/LTBI (teacher focused). The data supports that teachers’ knowledge of the LT allowed them to recognize a variety of strategies and recognize their appropriateness in instruction. The data also supports an important tension that arose in this data: teachers’ responses to this question indicated that they had difficulty striking a balance between allowing students to be open and using their own strategies/representations while
also feeling compelled to push them towards sophistication. This tension is one example of an important issue not only for equity, but also related to the LTBI model. Word tables were created for each construct in each dimension to answer each research question. The analysis resulting from these word tables is presented in the second part of Chapter 4.

**Validity and Reliability**

Although validity and reliability in qualitative research are not conducted in the same manner as they are in quantitative research, they are equally important constructs to address. For this work to make a valuable contribution to the field and impact professional development, it was important that the data collection and subsequent analysis be conducted rigorously. Because reliability is a pre-cursor to validity (Guba, 1981), I discuss it first in this section.

**Reliability**

In quantitative research, reliability is a measure of sameness. Given the same experiment is conducted under the same conditions researchers should expect to see the same results. In qualitative research, reliability does not and should not hold the same meaning, particularly in education. Because human behavior, schools, students and many of the related entities are not static, it is unrealistic to think that the same study or repeated observations would produce identical outcomes (Merriam, 1998). Instead, Lincoln and Guba (1985) offer an alternative notion of reliability. Instead of the traditional definition, they suggest that in qualitative research, we really want the findings of the data to be consistent with the data collected (Guba, 1981; Merriam, 1998). In essence, reliability addresses the trustworthiness
of the analysis process and warrants that the results of the analysis accurately reflect the data collected.

Three strategies were used to enhance the reliability of this dissertation study. First I used triangulation of data sources. Using multiple data sources to uncover evidence provides support that the findings are aligned with the data as opposed to being aligned with the researcher’s opinions. Additionally, Guba (1981) suggests that overlapping methods are another measure of reliability. In this study, the IQA scoring conducted by independent coders that were hired externally strengthens the stability of my qualitative analysis of the teachers’ mathematics instruction. Finally, the “investigator’s position” should be explained as well as the “assumptions and theory behind the study. . .the basis for selecting informants and a description of them, and the social context from which data were collected” (Merriam, 1998, pp. 206-207). My theoretical framework provided in Chapter 2 of this study serves as a way to frame my assumptions and guiding theory as I approached this study and the subsequent analysis. Additionally, my description of the school context and the selection of participants for this study in this chapter also provide evidence that can be used to assess the dependability of the findings.

**Internal Validity**

Merriam (1998) offers a discussion of both internal and external validity of qualitative studies. She suggested six practices to address issues of internal validity: triangulation; member checks; long-term observation; peer examination; participatory or collaborative modes of research; and researcher bias disclosure. Krefting (1990) summarizes Guba’s (1981) strategies for internal validity (also referred to as credibility) and provides the
following strategies: prolonged and varied field experience, time sampling, field journal, triangulation, peer examination, interview technique, establishing authority of the researcher, and structural coherence (p. 217). To enhance the internal validity of this study, I looked across these two frameworks and used triangulation, long-term observations/prolonged and varied field experience, peer examination/participatory or collaborative modes of research, and a researcher bias disclosure. Each of these four strategies is explained in detail in the following paragraphs.

As mentioned above, I used triangulation of data sources as one strategy to enhance internal validity. The use of multiple data sources and multiple cases provides a more “holistic understanding” of the data as well as “plausible explanations about the phenomena being studied” (Mathison, 1988, p. 17). Additionally, triangulation of data reduces the likelihood that the researcher focuses too intently on one source of data and forms a biased opinion from a single data source. In this dissertation study, data sources were compared with one another to “cross-check data and interpretation” (Krefting, 1990, p. 219).

Krefting (1990) argues that sufficient time in the field is useful to “identify reappearing patterns, allow the informants to be accustomed to the researcher (increasing rapport), and enhanced research findings through intimate familiarity and discovery” (p. 217). I met the standard of long-term observations by doing repeated visits to teachers’ classrooms over the course of the whole spring semester (February 2013-May 2013). In addition to being at the partner school for data collection for this study, I was also present for all LTBI PD sessions from August 2012-June 2013 as well as other school-wide events (e.g., school spelling bee). My presence for these sessions and other activities constituted an
additional 70 hours of face-to-face time with the participants and 10 hours of time spent involved in school-related activities.

Participatory or collaborative modes of research involve multiple researchers being involved in various phases of the study. Peer examination is a part of this process and involves the primary researcher discussing the research process and emerging themes with colleagues. I met this strategy of internal validity in four ways. First, I shared all elements of the proposed research project with my committee members during the dissertation proposal defense. At this time, my committee members made recommendations about the study and interview protocols. Second, I piloted my interview protocols with a 2nd grade teacher from a prior LTBI partner school. During this session, I asked her the interview questions to assess the following elements: the clarity of the questions, whether or not the questions would lead to the responses needed to answer the research questions, and the length of the interview.

As a third step, I had a four hour face-to-face meeting with Rochelle Gutierrez, whose work I drew on to develop the conceptual framework for this study. During this meeting we discussed each of the constructs developed along the four dimensions of equity, the design of the dissertation study, additional research questions that may be of importance in future work related to this study, and indicators of what to look for during data analysis. My meeting with Dr. Gutierrez was audiotaped. Another step in this strategy was holding bi-weekly meetings with my dissertation advisor. During these meetings, I shared emerging themes and ideas that arose from my observations, field notes, and research journal. These discussions allowed me to “present working hypotheses for reactions and discuss the evolving design of the study”
(Krefting, 1990). I maintained a detailed log of our discussions including questions, responses, and feedback on emerging themes.

Finally, a bias disclosure is included at the end of this chapter. In this disclosure, I offer information about my background, my motivation to attend graduate school, and my overall orientation towards research. This disclosure provides insight into my “assumptions, worldview, and theoretical orientation” (Merriam, 1998, p. 205) and is included after my discussion of external validity.

**External Validity**

External validity in qualitative research is a measure of generalizability or whether or not the findings from one study can be extended to other situations (Guba, 1981; Merriam, 1998). While there is disagreement about whether or not generalizability is appropriate for qualitative research, there is agreement on how to meet the burden of external validity. First, cross-case analysis in a multi-case study is one way to ensure generalizability as the themes in one case can be checked against themes in another case. Another way to ensure generalizability of findings is to use thick, rich descriptions so that the reader can determine “the extent to which a study’s findings apply to other situations” (Merriam, 1998, p. 211). Specifically the reader should be provided with enough information such that they can determine how applicable the data is to themselves and “how closely their situations match the research situation, and hence, whether findings can be transferred” (Merriam, 1998, p. 211). Thus in this dissertation study, I utilized a multi-case study, cross-case analysis, and provided thick rich descriptions of each case via a case portrait.
Bias Disclosure

Background of the Researcher

I always loved math, and as a young child I made a decision to be a teacher. I decided to major in mathematics as an undergraduate, but as I approached my senior year I felt that something was lacking. While in college, I took a class with a professor that I thought was brilliant. Despite the vast knowledge he had of mathematics, there seemed to be a disconnect in his ability to reach us (the students). I wondered why it was so difficult to take something you know and teach it to someone else. Because of this, I decided to apply to graduate school to seek a master’s degree in mathematics education. While I was comfortable with the content knowledge that I had gained, I wanted to gain the skills that I would need to teach and interact with my future students in a meaningful way. Before graduate school, I saw mathematics as a sophisticated system of rules and algorithms, some of which had applications in other fields, and some that you just needed to memorize. I no longer think this way.

As a mathematics educator, I believe that each person must construct his/her own understanding of mathematics and that this construction is deeply tied to the informal experiences that students bring to school with them. I believe that all students have knowledge of mathematics, and that each student’s individual knowledge should be valued and viewed as starting point instead of a roadblock. I do believe that a lot of children are being underserved by our traditional educational model, but instead of continuously discussing all of the problems we face, I have the opportunity to help teachers learn how to
reach each of their students and help them develop the skills to see the beauty and practicality of mathematics.

Although we did have a “job” to accomplish during the course of this research project, this project meant a lot more. I had the opportunity to gain tremendous knowledge about the impact this professional development model had on our teachers, and hopefully this knowledge will be used to help additional teachers in the future. Teachers have a huge task on their hands, and I am honored to have been able to work with the teachers in this study and to have learned as much from them as possible. Just as all students have a starting point that teachers must build on, each teacher has a starting point; the purpose of this study was to understand their journey as they learned and grew. Through my work with these four teachers, I was able to understand how LTs and LTBI could be useful for them. I was also able to witness their successes and struggles with trying to implement this in their classrooms.

**Role of the Researcher**

Classroom observations comprised a major part of this research study. As a critical part of observational research, Merriam (1998) states that the role of the researcher in the observational setting is critical. I incorporated a reflection prompt into my classroom observation protocol to make notes about my role during each observation. While in classrooms, my role was to observe the teachers’ instruction as well as their interactions with their students. At the conclusion of each lesson, I met with each teacher for a post-observation conference. Again, my role was to engage in discussion with the teachers and prompt them to reflect upon their practice. While I did ask questions to clarify some issues
that arose during instruction, it was not my intent to coach teachers through what they “should have done.” I wanted to genuinely understand how their participation in the LTBI professional development helped them or did not help them attend to issues of access, achievement, identity, and power in their classroom. After the introduction of the equity framework, I did ask teachers to refer back to the framework when they asked “what they should have done or could have done.” By neither answering their questions nor providing my opinion, I was able to observe teachers’ sensemaking with constructs in the framework.

In previous research projects, I have always developed a great rapport with the teachers I have worked with, and the same was the case during this study. Because I had worked with all four teachers during the fall semester (during LTBI PD sessions, collecting data for the LTBI project, etc.), they were very comfortable with me, and our conversations were open and genuine. Additionally, I observed teachers in their classrooms prior to data collection for my study, so their students were familiar with me as well as the idea of having a video camera in the classroom during their instruction.

**Ethical Issues**

Prior to the start of this study, the LTBI project went through the Institutional Review Board (IRB) process at NC State University. All potential participants for the study were invited to attend a meeting held at their school to gain information about the research project. From there, teachers were offered the opportunity to participate, and those teachers that were interested signed consent forms. Each teacher was given consent forms (in English and Spanish) for the students in their class, and only the students who returned consent forms were videotaped. The other students were seated outside of the range of the video camera.
There were no foreseen risks to any of the participants in this study. All teachers who volunteered to participate in the LTBI grant did receive stipends for their participation. All data was kept on the researcher’s personal computer and back-ups were stored on an external hard drive. All data was coded, and pseudonyms will be used in all reports generated from this study.

**Limitations of the Study**

A common limitation articulated in case study research is that because there are only a few cases, the results may not be generalizable to the larger population that the case was selected from. I argue that generalizability as defined in quantitative research was not the purpose of this study. I used a small group of teachers to understand how participating in a design study on a specific set of topics related to LTBI affected their ability to implement equitable teaching practices in their classrooms. Therefore, I do not see the design of this study as a limitation, but rather a strength of the study.

One limitation of this study is that I am a novice researcher. Although I have six years of experience as a research assistant on various projects during my master’s and doctoral programs, this study represents my first attempt at designing and conducting all phases of the research process. Additionally, this study was completed independently. In order to further increase the reliability of the results, all data could have been reviewed and coded by another researcher to establish inter-rater reliability.

**Chapter Summary**

In this chapter, I articulated the design of this research study including the context, content, and methods as well as the necessary steps for generating theory from case study
research. Four K-1 teachers participated in a semester-long, multi-case research project that consisted of classroom observations and a series of interviews. The following chapter contains the findings from this research study. The findings are presented in two sections. First, I present a detailed case portrait of each teacher that is intended provide the reader with a rich description of each case. These portraits are organized into the following sections: mathematics instruction profile, participation in the LTBI Project, classroom organization, task selection and implementation, and how the teacher talked about their students. In the second part of the chapter, I present the results of the cross-case analysis and offer evidence to answer the research questions. I begin each section by presenting the overall research question. I then present each construct along with themes that were present in the construct. Each of the themes is supported with examples from the data, and I conclude each section with a summary of the ways in which LTs and LTBI were present (or not present) in that dimension of equity.
CHAPTER FOUR

In this chapter, I first present individual portraits of each case teacher. Each teacher profile includes information about their professional background, their mode of participation in the LTBI PD, a description of their mathematics instruction, information about their classroom organization, how they selected and implemented tasks, a discussion of their lesson quality as measured by IQA, and how each teacher talked about their students. The purpose of these profiles is to provide an overview of each case and to give the reader insight into the participants. In the second part of the chapter, I present a cross-case analysis to answer each research question. I begin by presenting the overarching research question for each dimension (access, achievement, identity, and power). I then present the constructs within each dimension followed by themes that arose from the data analysis. Next, I present examples from the data to illuminate each theme. While it is not possible to present all examples observed during the course of this study, the examples selected in both parts of this chapter are meant to be exemplars of the themes that emerged during data analysis.

Individual Case Portraits

Elsie

At the time of the study, Elsie had 27 years of teaching experience. She had been teaching at the partner school for 17 years and had six years of experience teaching kindergarten. She previously taught students with autism for 12 years and spent nine years as a literacy specialist. Elsie, a white female, earned her undergraduate degree in therapeutic recreation. She always had an interest in children and wanted to be involved in community settings. Because most jobs available in her field at the time she graduated college were
hospital settings, she returned to school to earn her Master’s degree in special education. Although she was never certified as a regular education teacher, her background and previous experiences were credited to her and she was awarded an elementary license. In addition to her degree in special education, she is certified in ESL, literacy, and has obtained her National Board Certification.

When asked why she chose to teach at the partner school, she indicated a number of reasons. First, she lived in the community surrounding the school and believed that she should not take her child across town to another school when one existed in her neighborhood. Second, she noted that she really liked the teachers and staff at the partner school. She concluded this discussion by stating the kids at her former school “already had everything,” and that she “wanted to make an impact on the kids who needed extra.” She also wanted to be a part of a school community where parents advocated for all children as opposed to just their children.

**Mathematics instruction profile.** Elsie described the typical components of a mathematics lesson in her room including a whole group lesson (where she modeled the task with manipulatives) followed by small group/individual practice of the new concept. She stated that she concluded her lessons with whole class sharing to solidify what was learned. The use of manipulatives was very important in her instruction, and Elsie wanted to ensure that her students could model mathematical situations. She also believed strongly in allowing students to contribute to the mathematical discourse in the classroom.

Initially when asked about teaching mathematical ideas, Elsie indicated that students need lots of opportunities to explore concepts. Later she articulated that although exploration
is important, it is also important for teachers to cultivate opportunities for students to explore and that these opportunities should be related to their LT profile level. She also mentioned frequently that her focus is not for students to just “rattle off addition” problems, but to be aware of the mathematics involved. She wanted her students to have a solid understanding of numbers.

**Participation in the LTBI project.** Elsie played an active role throughout the professional development. She was eager to learn new concepts and share her ideas with the group. Her math content knowledge as measured by the Learning Mathematics for Teaching (LMT) assessment was lower than that of other participants in the PD. Generally, Elsie participated more in discussions related to pedagogy or content related specifically to kindergarten students. In discussions that were heavily-focused on mathematics at the upper levels, Elsie tended to withdraw or discuss secondary topics as opposed to the focus of the discussion.

**Classroom organization.** Elsie’s classroom was very inviting. Of the 16 students in her classroom, six were African-American, six were Caucasian, and four were Hispanic. Elsie’s class was evenly split with eight girls and eight boys. During each of my visits there, she had all students’ work represented both inside and outside the classroom. Students had assigned seats, and each table name was represented by a color. During the course of the semester, the students’ seats changed periodically. Student seating appeared to be random, and each table represented a mix of gender and ethnicities. When students were called to work on the carpet, they were allowed to choose their own seat.
It was customary for Elsie to open up her math discussions (as well as other content areas) by allowing students to share their thoughts on the topic for the day. During the course of her whole class discussions, Elsie would often call on every student that had their hand raised and allow students to share more than once if they liked. She actually articulated that this was one of her “downfalls.” She credited her poor “time management” to her desire to allow every student to speak.

Elsie’s students were highly engaged in her lessons (e.g., attentive, participatory, volunteering to answer questions) and my presence as an observer did not detract from their participation. As students transitioned from one task to the other, they would sometimes come and speak to me. They were very affectionate towards Elsie. During individual work time, Elsie would often have multiple students competing for her attention simultaneously. Elsie’s TA interacted with the students in a similar manner.

**Task selection and implementation.** For each of the lessons I observed, Elsie articulated clear learning goals for her students and created tasks that aligned with those goals. When asked about her lesson goals and tasks, Elsie drew upon elements from her pacing, the Common Core State Standards, and the LTs.

Elsie’s goal in her first lesson was for “students to recognize that numbers can be represented in different ways, see numbers as a complete set, a group of smaller sets coming together, 10s & 1s.” During the observation for this lesson, Elsie began by showing students a tens frame and asking them what it looked like to them. She then held up dot cards and quickly flashed them to students. Students had to subitize the number on the dot card. Students were given a dry erase marker, a laminated tens frame, and a tissue. For the
remainder of this whole group lesson, the teacher flashed a dot card (she used the numbers 5, 8, 4, 7, 3, and 10) and then asked the students to represent the number they saw on their tens frame. A student was called on to share their response each time. After the initial student presented, the teacher solicited another volunteer to share who had represented the number differently.

On the second day of this observation, Elsie extended her work with using ten frames by using an activity titled, “I can make ___. ” She modeled the activity whole class doing “I can make 4.” The objective of the lesson was to use a tens frame and two colored counters to represent the number as many different ways as possible. Each student was given their own materials including a recording sheet, two color counters, and two crayons to match the counters. Some students worked on “I can make 5” while other students worked on “I can make 8.” Students worked individually for the remainder of the lesson, and those that finished early completed their morning work. There was no whole group discussion or conclusion for this lesson.

Elsie’s next observation was a few days after the class had returned from a trip to a local creek. Prior to the field trip, the students had been working on solving story problems. The goal for this lesson was for students to be “flexible with different combinations of 10.” The respective task for this lesson was the following problem: “On a recent trip to Titus Creek, George helped us roll a log. Under the log were ten critters. Some were caterpillars and some were worms. How many caterpillars and worms could we have seen?” Although her students had previously been successful with join story problems, many students expressed difficulty with this task. After clarifying the task for students and modeling one
solution with them, they were allowed to work individually. When the teacher called the
students up to share at the conclusion of the lesson, they were very playful and were unable
to focus. Elsie said that they would go ahead and take recess early and share their solutions
later. The next time I saw Elsie, she told me that she felt like this lesson bombed. When
discussing it with her, she was surprised that students had so much difficulty with the
problem as she thought it was a simple addition story problem. I indicated that this problem
was actually a part-part-whole addition problem and that the difficulty may have been
because there was no “action” in the problem for students to model and that there were two
missing addends. Elsie indicated that she had not thought about the lack of “action” in the
story problem and that now she understood why it was difficult for some of her students

Elsie’s final observation started with a whole class opening followed by individual
work. The goal of the lesson was for students to be flexible solving a variety of story
problems. Students solved the following problems during whole class instruction:

1) Elias has 3 rings. Siwa has some more rings. They have 9 rings all together. How
   many rings does Siwa have?

2) DeLisa has 2 flowers. Ms. L has 3 flowers. How many flowers do they have all
together?

Each student was then given a worksheet with additional story problems to solve
individually. Some students finished their individual work early. Elsie told students to
challenge themselves by writing doubles facts or number facts on the back of their paper.
After students completed their work individually, the class reassembled on the carpet for
whole-class sharing.
**Lesson quality.** In this section, I discuss the analysis of Elsie’s lesson quality across each of the four IQA rubrics. All of her lessons were scored, and this discussion centers on the average scores across the observations. It is meant to provide additional details about the rigor of Elsie’s instruction as well as the opportunities for high-quality mathematics instruction present in her classroom.

Table 10

_IQA Results for Elsie’s Lesson Quality_

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Elsie’s mean score of three on the task potential rubric (AR1) indicate that, in general, her lessons offered the opportunity for complex thinking about mathematical procedures and relationships. Because these lessons did not extend these ideas beyond engaging in complex thinking to generating theory based on patterns and evidence, Elsie was not awarded the highest score of four on this rubric. Her task implementation (AR2) score suggests that while
the task had the potential for complex thinking, evidence of such was not always apparent in the lesson, and that connections were not consistently made between concepts and procedures. A score of a two on the student discussion rubric (AR3) means that while students did have opportunities to share their work, these discussions were limited to “show and tell” presentations. Elsie’s questioning (ARQ) demonstrates the highest variance among the four rubrics. Her score here indicates that Elsie’s questions ranged from simple questions about procedures/facts, to questions that required one-word answers, to questions that required students to clarify their thinking and build connections between mathematical ideas. Because Elsie consistently indicated that she had difficulty with whole class discussions, it is not surprising that these two rubrics (AR3 and ARQ) had the “lowest” mean scores and “highest” variance.

**How Elsie talked about her students.** Two themes emerged in relation to Elsie’s dialogue about her students. First, Elsie did not use fixed labels when discussing where her students were working in the LTs. In each grouping, Elsie indicated that these students were in transition. For example, she described them as “perceptual moving towards early counting,” “in-between,” or “moving towards counting on.”

Second, Elsie spoke positively of her students and at each “level,” as she articulated her ideas about students based on what students had accomplished and areas for potential growth. For example, Elsie wrote the following in an initial student mathematics portrait about a student that she labeled as perceptual and moving towards early counting:

Feliz is able to recite count to 10 and count sets to 10, but he loses 1:1 correspondence and numeral knowledge past 10. He is unable to subitize simple sets
of 3 & 4. He understands concept of same, but reverses > and <. Feliz needs to go back and recount when doing simple addition and subtraction problems.

When setting a goal for this student, Elsie wrote that Feliz “needs practice recognizing numerals 0-10 and chances to work with small numbers (counting & producing sets) to solidify understanding of set values and number relations <, >, = with sets 0-10.” Compare these statements to Elsie’s statements about one of her students that she indicated was moving into counting on (which is a more sophisticated profile level):

Eliot knows that numbers represent a value. He easily counts, produces sets to 20, and can count to 100. He subitizes to 3 but needs to count some sets to 4. He is able to keep track of the original number when small units are removed or added back. . . . He is able to do join addition ≤ 10.

When setting a goal for Eliot, Elsie wrote “Eliot will need more opportunities to practice a variety of add/subt problems to develop knowledge of number combos. Also use of 10’s frame to solidify subitizing sets to 10 & move towards place value of 10’s & 1’s.”

Although these two students were identified as working at different levels in the LTs, the examples highlight how Elsie sought to identify what her students could do and equally identified their strengths as well as areas for growth. Her examination of students was not focused on labeling them in relation to higher or lower levels of the trajectory. Instead, she sought to understand each student and support learning at all levels.

Elizabeth

At the time of the study Elizabeth had 15 years of teaching experience. She was in her fourth year of teaching kindergarten and had been teaching at the partner school for nine
years. She entered the school system as a teacher assistant and then an ESL assistant before becoming a full time teacher. She previously taught ESL for 1st through 5th grade students in Japan and then in the United States when she returned from overseas. She became infatuated with how mathematics was taught in other cultures/countries when a friend from Japan told her that math in the U.S. is taught “all wrong.” She stated that this friend worked with a small group of students, taught them math in a “different” way, and that her students were successful. Elizabeth began to read and learn about how math was taught in other countries and tried to incorporate various aspects of what she learned into her instruction.

Elizabeth earned her undergraduate degree in East Asian languages and literature. Her particular focus was Mandarin. Her passion is language, and she chose to work with kindergarten students so that she could share her love of language with all students. Although she attended graduate school, she indicated that she never earned her degree because she did not complete her final exam. Elizabeth is licensed to teach ESL as well as K-6.

When asked what was special about the partner school, her initial response was that she could walk to the school. She went on to say that when she first joined the school nine years ago, the school was 98% free and reduced lunch and that violent acts had occurred at the school. She said that the school used to be considered a school that you would not want your children to attend. She stated that over time, the school became more balanced according to socio-economic status and that now the school is a place where students can come to have their needs met in a safe environment.

Mathematics instruction profile. From the start of the study, Elizabeth was very open about the fact that she was not a strong mathematics student herself, that she did not
have positive experiences as a learner of mathematics, and that she previously did not teach mathematics daily in her classroom. Although she was not happy as a mathematics student, she stated that she did not want to pass her negative feelings of mathematics on to her students. When asked how she would organize a mathematics lesson and what her role was during instruction, she indicated that the focus of mathematics in kindergarten should be on number sense and that it is important to understand how numbers tell a story. Over the course of this project, she continued to reiterate how important it was for students to use numbers to answer questions and tell stories about the world around them.

**Participation in the LTBI project.** Elizabeth did not play an active role in the PD. Although she would respond and contribute when asked, she did not readily initiate ideas or contribute to the group discussion. She was often late to meetings and only completed three of nine homework assignments. Elizabeth’s math content knowledge as measured by the LMT at the beginning of the PD was the highest of the four teachers in this study.

**Classroom organization.** Elizabeth’s classroom was very organized, and she always had students’ work displayed both inside and outside of the classroom. Of the 17 students in Elizabeth’s classroom, two were African-American, seven were Caucasian, and eight were Hispanic students. Elizabeth had twelve girls and five boys in her class. Students had assigned seats and table names were represented by both a color and geometric shape (e.g., red circle). Student seating appeared to be by random assignment, and each table was representative of students at all “four levels,” as Elizabeth initially labeled them. (Note: I will explain Elizabeth’s groupings of her students later in this profile in the section titled “How Elizabeth talked about her students”). When students were called to the carpet for whole
class instruction, they were allowed to select their own seats. If students were disruptive, Elizabeth would tell them why their behavior was wrong and reassign their seat elsewhere on the carpet if necessary.

Typically, Elizabeth’s students were engaged for the opening of the lesson. This engagement did not continue throughout the lesson, and by the end of the lesson, only two or three students remained engaged. Because many of the lessons involved one student participating at a time (e.g., one person picking hearts for their box, one student moving beads on the rek-en-rek, and one student modeling subtraction on the number line), other students played with each other and got up out of their seats.

Elizabeth opened up her math lessons by presenting the tasks to the students. She would then ask students how they might approach the task or their thoughts on a solution. Although Elizabeth allowed various students to respond to this prompt, she quickly went to her “top” student that she knew would provide the “correct” response. For example, Elizabeth asked the class how they could take apart the number ten. Diana, a student that Elizabeth had ranked in the next to lowest group, raised her hand to answer the question. Although Elizabeth called on her, she only allotted three seconds of wait time before redirecting the question to Paul, who she stated is her top student in the class. In another instance, Elizabeth asked what number you would have to add to zero to make ten to fill a tens frame. Although a few students called out answers (e.g., zero and five), she selected Joey’s response of ten. She then asked Joey how he knew the answer was ten. He responded because $0+10=10$. Elizabeth did not address the other answers that were provided, nor did
she explain Joey’s solution to the class. (Note: Elizabeth stated that Joey was her second highest student in the class).

**Task selection and implementation.** Elizabeth did not articulate clear learning goals for her students and had difficulty connecting the learning goals to the task. For example, Elizabeth stated that her goal for her first observation was for students to explore composing and decomposing the number 10. She stated that she wanted to understand how students looked at breaking ten in half, and that the goal was to understand that five was half of ten. Elizabeth went on to state that she wanted her students to fill in a tens frame and understand that it is easier to add a smaller number to a larger number. Because Elizabeth articulated so many goals for this lesson and some were unclear, I asked her to clarify her goal for the lesson. She was unable to do so. This lack of clarity with the goal translated into the lesson implementation.

Elizabeth started her first lesson by asking students about building things and how you can put cubes together and take them apart. Elizabeth spent the first eight minutes of a fifty-two minute lesson talking about building things and showing the students how she made the paper number cubes that she used as dice in the lesson. Students did not actually begin the task until twenty-four minutes into the lesson. For this lesson, each pair of students was provided with a red number cube, a tens frame, red/yellow counters, and a recording sheet. Students were asked to roll the red number cube and place that number of red counters on their tens frame. They were to fill in the remaining spaces on their tens frame with yellow counters. In addition to this task, Elizabeth asked the students to remember and think about how they filled in their yellow chips on the tens frame. She wanted the students to keep track
of whether they filled in the row first (to make a five) or if they filled in columns first. After modeling this activity for the whole class, students were partnered up and asked to complete the activity with a partner. Student initially seemed confused about the task. During individual work time, Elizabeth worked with one group of students and her TA worked with another. The other two tables were not assisted. The class was called back to the carpet at the end of the individual work time (which was ten minutes of a fifty-two minute lesson). During this time, students were very distracted, and only a few students were called on to share their answers. During the first portion of this discussion, Elizabeth asked the students how they felt about this game compared to “Shake and Spill” and if they “liked this.” The only three students that were allowed to share their mathematical thinking at the end of this lesson were three of the four students in her “highest” group.

On the 2nd day of this observation, the teacher indicated that she thought the previous lesson was “above the students’ heads,” and so she decided to modify her plans. She mentioned that one student in particular could not recognize the numbers on the number cube, so she needed to make the lesson easier. Because of Valentine’s Day, Elizabeth decided to use a tens frame with pink and red hearts to review the previous lesson. In this lesson, students were called to the carpet individually and each rolled a red number cube. They were asked if they wanted that number of red hearts. From there they had to choose a combination of red hearts and pink hearts such that they would have a total of ten hearts to decorate their Valentine’s Day box.

Again, the goal of this lesson was unclear. It was not clear why students rolled the number cube if eventually they would get to choose the number of red and pink hearts they
wanted. Because students came up one-by-one to select the hearts, most of the class was not engaged and played with one another. The observation concluded with students gluing the hearts to their mailbox.

The same lack of clarity about the goals for the lesson was present in the second observation as well. When I asked Elizabeth to articulate the goal for her next lesson she stated:

Maybe I guess since I . . . I know I’ve in each case said it’s hard to get to all students, but if I could really try to make sure that I build in time in a lesson to make sure that some of the kids who normally don’t attend would be able to show their work and I could try and figure out some way to do that. They might not share or they wouldn’t even do the work and I usually . . . It has to be with a lot of support and one-on-one. Maybe I could make it so that they really have an opportunity to get excited because they can accomplish something. I will try . . . Like I said, I was thinking of doing the rek-en-rek and that’s what I had written out, but I’ve been toying with possibly switching it to Cuisenaire Rods, but they love beads so we might try the rek-en-rek and then I could do something like that and gear it more towards the kids who are having a hard time and talk about sharing at the beginning of the day.

She started this lesson by introducing the “rek-en-rek” to the class (see Figure 4). She told the students that she did not know what the word rek-en-rek meant or where it was from, but that they would be using it for the lesson. She started by asking students what they thought it meant. One student suggested that maybe it means “ten and ten.” Elizabeth began the lesson by selecting various numbers and asking students to represent that number on the
rek-en-rek. Elizabeth stated multiple times that students could represent numbers different ways. She then had stack of cards with numbers on them. Students were randomly given a card and told that they could only move that number of beads on the rek-en-rek. Elizabeth stated that the goal of the activity was to make twenty. She also said she wanted to find out how many people it would take to get it to twenty, and so she was going to call on someone with a high number. She started by asking for someone who had a number larger than five. A student with seven came up to the board. Elizabeth told this student that she could count seven beads or that she could slide over the five red beads and then add two white ones onto it. Two students that had seven cards came up and two students that had threes came up. Elizabeth pointed out that it took four students to make twenty. She then posed a question to the students: “What if I wanted more than four people to make twenty. What would I need?” Someone in the class yelled out “lower numbers.” Elizabeth affirmed that students’ answer and continued the lesson.

![Figure 4. Rek-en-rek](image)

When I arrived for my final observation, the students were very excited and had just returned from recess. Elizabeth asked all of the students to come and sit at the carpet. The
topic of this lesson was subtraction using the number line. Elizabeth stated the following goal for this lesson:

I want them to be solid on the idea that when you subtract, you end up with a smaller number. We touched on the idea today. They rolled the dice and the first number we wrote it down and then we rolled it again. We wrote that number down and we talked about which one was greater and that one is the first number in a number sentence or in the equation. And then the smaller number you subtract. I said to the kids who are higher level, I said, “You’ve probably heard of negative numbers and there are…when you get older you’ll be taking larger numbers away from smaller numbers but right now we are learning about subtraction where we take a smaller number or the same sized number from a number.” And they talked about 4 minus 4. “What do you have?” “Zero.” But that after the equal sign, that number will always be smaller than, unless. . . . Well, if it’s zero it’s still smaller. That’s a big difference from addition. So maybe with my number line story and working on numbers I’ll have them all confirm that, “Yes we go back and we end up with a smaller number.”

Elizabeth had a number line and paper fish (to be used as manipulatives) for the class. After she laid out a given number of fish, individual students were called up to roll a die and whatever number they got, they had to remove that number of fish. This lesson was whole class and was very teacher-centered. It was more of a hands-on activity or game than a mathematics lesson, as students did not formalize the concept of subtraction.

**Lesson quality.** In this section, I discuss an analysis of Elizabeth’s lesson quality across each of the four IQA rubrics. This discussion centers on her average scores across
each of the observations and is meant to provide additional detail about the rigor of
Elizabeth’s instruction as well as the opportunities for high-quality mathematics instruction
present in her classroom.

Table 11

*IQIA Results for Elizabeth’s Lesson Quality*

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<th>AR1 Task Potential</th>
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In relation to the potential of the task (AR1), Elizabeth varied from selecting tasks
that had the potential for high cognitive demand and the use of multiple representations to
selecting tasks that required students to memorize or reproduce facts. Even when the task had
potential for high cognitive demand, this was not reflected in the implementation, as the
implementation score was a 1.75. This score denotes that the implementation was a cross
between rote memorization and reproduction of ideas and focused on producing the correct
answer rather than building connections. The student discussion score (AR3) highlights
discussions that were either non-mathematical or “show and tell” approaches with little questioning from the teacher. Finally the teacher questioning score (ARQ) indicated that Elizabeth’s questions typically required a simple recall of facts or procedures and lacked academic relevance.

How Elizabeth talked about her students. Two themes were present in Elizabeth’s dialogue about where her students were working in relation to the trajectory. First, Elizabeth did not consistently use the LT profiles to label where students were working in the LT. She used a combination of high/low language, describing what her students did and did not do, as well as the LT profile names when asked to describe where students were working. Second, Elizabeth discussed her “higher” students more favorably than she did her “lower” students. In the following paragraphs, I provide examples to illustrate each of these themes.

Elizabeth initially grouped her students into four groups: lower group, next up from low, not quite counting on, and higher group. Halfway through the study, Elizabeth shifted her language and described students using the LT profiles (e.g., Counting on and Place Value). At the end of the study, I asked Elizabeth to tell me about where her students finished the year. She asked me if she should start with her “high performing or the lowest,” showing that she was not using the LT language to examine what her students knew. Elizabeth proceeded to discuss the students by stating that her two lowest students were her only two students that couldn’t count by 10s to 100, and that this “was consistent with their beginning of the year check.” In relation to her high students, she said, “Those I expected to do well. . . . All did a great job.” Elizabeth provided detailed descriptions of the types of problems these students solved correctly on their end-of-year assessments (e.g., “She was
able to count up to twenty objects, but not in groups”). Although she described what students did, she did not use the LT profiles to examine what the students were doing until I pushed her to think about the profiles. Once prompted, Elizabeth used the LT profile language to label where all of her students were (e.g., perceptual, early counting, counting on, and place value).

Elizabeth’s language when describing students from the higher group and the lower group was quite different. For example, when asked about how she knows where her students are, Elizabeth stated:

For example, this morning. . . I always do this with the short day (2-11-13). I say, “Can you make a connection? What can you do with the dashes to make it a number sentence or math sentence? And Nikki said, “You can make it a plus and equals thirteen.” And she did that without writing anything down, and so there were four other kids that got it right away when [Nikki] said that. They knew what a plus sign was, they knew what an equal sign was, so they’re definitely the higher group.

When asked to anticipate how the students in her “higher” group would engage in her first lesson, she stated, “I’m anticipating that they’ll get this relatively quickly, but that the other kids might not.” When I asked how she knew those students were her “higher” students, she stated that they are:

Absolutely confident in cardinality, subitizing, knowing. . . comparing numbers, knowing that if you take a certain amount away, you need to add a certain amount to get to ten. . . . Whenever I’ve done assessments, they’ll explain how they got their answer sometimes using addition. . . . They just know it like that, so most of them
show they’re counting on, if not more. They’re definitely confident in counting on.

But I wouldn’t put them as far as confident in place value yet.

Although Elizabeth initially described these students as being in the “higher group,” when asked to discuss her label in greater detail, she used language from the LT to do so. When asked to describe the students in the remaining groups, Elizabeth also used language from the LT and indicated that these students “need a lot of work still on counting and cardinality” and that for some of the students “it’s a question of language,” and that one student, although not ESL, needs a better “command of English.”

Elizabeth proceeded to describe her ESL students as “illiterate” and stated that she uses this term in literacy to indicate that a student had “no words for the concept.” For example, Elizabeth stated they “can’t express things like saying less, [and] more clearly.” Because both Elizabeth and her TA were able to speak Spanish, I asked if these students were able to articulate these concepts to them in Spanish. She said that these students “don’t talk about these things with their parents.” Later when asked to discuss these students in relation to the LT, Elizabeth included a number of non-mathematical factors in her discussions of students including age, ESL status, lack of motivation, lack of maturity because she has not been able to “get to them” during a lesson, and lack of attentiveness.

In another conversation with Elizabeth, I asked her about her efforts to engage all of her students in classroom discussions. She indicated that some students usually don’t raise their hands and that typically it’s the “second language learners [and] kids on the lower end of the trajectory.” She stated:
Sometimes they’ll just put their hand up because they just want to talk, and I’ll let them try, and then we’ll work to make sure that they’re not embarrassed or don’t feel bad about giving the incorrect answer, and then we’ll talk about how do we get the right answer. And what’s been helpful is a lot of the kids who are, you know, zooming way ahead and are way much above the others on the trajectory, they teach the other kids how they got it, how they got the answer.

While the examples presented in the last few paragraphs are not exhaustive, they were selected to highlight some of the differences both in language used to describe students and in Elizabeth’s interactions with the students. Specifically students in the “higher” group were described primarily by the mathematics they demonstrated, their strengths, and were afforded high expectations, while students in the “lower” groups were often described using deficit language, as being needy, or being time-consuming.

**Caroline**

Caroline taught kindergarten during this study. She did not complete her demographic information, and after additional attempts to collect this information, she did not respond. Therefore, I do not have data about Caroline’s years of experience, grade-level experience, certifications, or why she chose to teach at the partner school.

**Mathematics instruction profile.** Early in the professional development, Caroline stated that she did not use whole group lessons as her primary mode of mathematics instruction. She typically opened her lessons with a brief mathematics activity and proceeded by breaking students into ability groups to participate in math centers/stations. One station was always a math game/activity on the computer. One station was a small group of students
working with her; this group focused on a skill that they had not yet mastered. The other stations consisted of other games or hands-on activities that were based on skills in the kindergarten mathematics curriculum. The students were typically in groups of four and would rotate through one station per day such that by the end of the week, they had completed all five stations.

Caroline reported that an important component of her mathematics instruction was exploration. At the start of the summer institute, Caroline wrote that children should be given the opportunity to explore tasks and work with manipulatives rather than being shown or told specifically what to do. At the end of the summer institute, she wrote that mathematics tasks should be open-ended and that teachers could use LTs to group students. She went on to say that students should have time to work individually and then the teacher should allow students to share and make connections between the ideas shared. At the conclusion of the PD, Caroline stated that her mathematics instruction had totally changed from the beginning of the year, and that next year she was going to focus more on having students share their work and mathematical thinking. Specifically, she said:

The way I teach now has changed since starting this workshop. Now, I find a task and think about how the students might solve the task. If it is a new idea, I will do a mini-lesson and then give students time to solve the problem on their own or in pairs. I know that I need to do better with having students share their responses. As we start a new school year, I will try my best to put that in place for next year. I think the major problem this year is that I didn't stick to a solid sharing time because it is so
hard in the beginning. Having that expectation from the beginning will make it go
smoother eventually and in the long run will only benefit the students.

Although exploration and working in small groups became an integral part in Caroline’s
mathematics instruction, her classroom was still run very tightly, and she had very clear
expectations regarding how students should work while exploring a concept.

**Participation in the LTBI project.** Of the four cases in this study, Caroline’s
participation in the PD changed the most during the course of the project. During the initial
30-hour summer PD session, Caroline would often leave the room during a 15-minute break
and not return for long periods of time. During these extended breaks, she would be seen
around the school working on other tasks related to the opening of the school year. It became
clear early on that Caroline was a leader in the school and that other teachers in the PD and
the staff in general looked to her for answers. Initially, the members of the research team
speculated that these behaviors indicated a lack of interest in the project. As time passed,
Caroline continued to play an up-front role in the school, but a back seat role in the project.
We later speculated that content and ideas of the project may have been very difficult for her
to grasp, and thus it was easier for her to drift into the background instead of confronting her
discomfort. At times in the project, she did articulate her confusion with some of the LTs and
indicated that all of the new material presented was overwhelming and confusing.

During one of our mid-year sessions, Caroline surprised the research team. Teachers
were given a homework assignment in which we asked them to teach a task in their
classroom, collect samples of students’ work, and be prepared to discuss their students in
relation to the trajectory. Caroline had very detailed information about each of her students and was able to use the language from the LT very fluently.

**Classroom organization.** Caroline’s classroom was highly structured. Once she gave instructions or directions, she expected students to comply immediately. Not doing so meant that students would quickly be removed from the larger group and asked to sit alone or with the TA. During all of my visits, she had students’ work represented throughout the classroom and in the hallway. Students had assigned seats and table names were represented by colors. During each of my visits, the seating assignments remained the same. Because five of Caroline’s students did not have consent to participate in the project, the seating assignments were often rearranged so that only participating students would be in view of the camera. The other students worked at the back table with the TA along with the students whom the teacher deemed to be disruptive. Thirteen of eighteen students in Caroline’s class had consent to participate in the study. Of the 13 participating students, four were African American, four were Caucasian, and five were Hispanic. There were six boys and seven girls. The five students that did not obtain consent were all African American and Hispanic.

**Task selection and implementation.** From the beginning of the study, Caroline was able to articulate clear learning goals for her students and select tasks that aligned with those goals. For her first lesson, she stated that her goal was number combinations and that students would be solving word problems. Caroline identified the following problem from a K-5 math teaching website that we provided at the summer institute: “You are at the grocery store and you need to get eight apples. Some are red and some are green. How many different
combinations can you make?” On the second day of this first observation, students worked on combinations of ten.

The focus of Caroline’s second lesson observation was subtraction. The lesson started whole class and the teacher read the book entitled *The Action of Subtraction*. After reading the book, Caroline modeled the activity with the students. The whole class activity consisted of monkeys falling out of a tree. The teacher had a tree and monkeys and placed some monkeys in the tree. She then pulled some down and asked the students how many monkeys were left. Various students were called on to participate in the whole class lesson. Students were asked to identify how many monkeys were originally in the tree, how many fell down, and how many were left. Some students were also asked if they could create a number sentence that represented the story. After the whole class activity, each student was given a pond and a set of frogs. The individual activity consisted of starting with a given number of frogs in the pond, allowing some frogs to “jump out” and counting the remaining number of frogs in the pond. Students were to keep track of their subtraction number sentences. After working on the task individually, students were called back to the carpet for whole class sharing. Because the students were so distracted when she called them back to the carpet, Caroline asked me to cut the video and said she was going to stop the lesson there.

**Lesson quality.** In this section, I discuss an analysis of Caroline’s lesson quality across each of the four IQA rubrics. Again, this discussion centers on her average scores across each of the observations and is meant to provide additional detail about the rigor of her instruction as well as the opportunities for high-quality mathematics instruction present in her classroom.
Table 12

IQA Results for Caroline’s Lesson Quality

<table>
<thead>
<tr>
<th></th>
<th>Lesson 1 Part 1</th>
<th>Lesson 1 Part 2</th>
<th>Lesson 2</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR1 Task Potential</td>
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<td>3</td>
<td>2</td>
<td>2.3</td>
<td>.47</td>
</tr>
<tr>
<td>AR2 Task Implementation</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2.3</td>
<td>.47</td>
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<tr>
<td>AR3 Student Discussion</td>
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<td>.47</td>
</tr>
<tr>
<td>ARQ Teacher Questioning</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1.67</td>
<td>.94</td>
</tr>
</tbody>
</table>

Caroline’s task potential score of 2.3 indicated that while some of her tasks had the potential for high level mathematics in which students could identify patterns and make connections among ideas, the other two only required students to engage in a procedure without connecting the procedure to the underlying mathematics. Caroline’s score on the implementation rubric is interesting in that it is identical to the potential rubric. This suggests that when Caroline planned a task with the potential for complex thinking, she maintained that level of rigor throughout the task. Additionally, for tasks with lower potential, Caroline’s implementation reflected the same standards. The final two rubric scores (AR3 and ARQ) both attest to Caroline’s uncertainty with orchestrating classroom discussion and her own feelings of inadequacy in this area. In both of these areas, students were not prompted or required to explain the “why” and “how” of their mathematical work. These scores indicated that when students were provided with opportunities to share, they often described about
“what” they did and the teacher’s questions during this process were either non-mathematical or procedural.

**How Caroline talked about her students.** Three themes were present in Caroline’s discussion of her students. First, when Caroline talked about her students in relation to the LT, she typically talked about them in a fluid nature (as opposed to a fixed position) or as being in transition. She believed that in kindergarten most kids start at the same place but that “kids who might have had more access to different things at home kind of zoom through the profiles or levels quicker than others.” For example, in discussing her plan for the first lesson, she stated that her students that are “just transitioning into the early counting child, I think they’re sort of . . . . They seem to be past the perceptual child, but they’re not . . . they don’t have a . . . like, a full understanding of [the next level].”

Second, when Caroline discussed her students at various levels of the LT, she talked about how the LT provided her with agency to meet her students’ needs. Even in instances when deficit thinking was present in Caroline’s discussion about her students, she coupled deficit-orientates statements with how LTBI assisted her in moving students forward. Consider the following quote from Caroline below, coupled with an activity she created to try to address a concern she identified. When discussing her perceptual students, Caroline stated:

I think so many things that we take for granted like that, you know, we might do with our children or our friends’ children. These kids haven’t gotten anything at home. In literacy they’ve never really been read to on a consistent basis. And so I think that’s kind of the same way as far as math. They weren’t introduced to an environment of
like learning different things and communicating with one another. I think that’s really low in a lot of our families here.

This idea was very important for Caroline because she found it difficult to have students share in class. She thought that because some of her students did not have opportunities to talk and share at home, it was difficult for her to foster productive whole-class sharing. To address this and help her students build their communication skills, Caroline designed a lesson to help kids think about sharing in math. She stated, “They need to understand like talking to each other about math, so I thought that if I gave them the sentence to use and then they could start trying to explain and be able to explain their thinking to others, sort of.” For this lesson, students were given a sentence frame that said, “I have _____. I need _____ more to get to 10.” Working in pairs, students rolled a die. That number went in the first blank. They then had to find out how many they needed to make ten and communicate that to their partner. Caroline stated that she though this lesson was horrible and that it did not go well because of classroom management issues.

As we continued the discussion of her perceptual students, I asked Caroline to elaborate on how the trajectory helped her think about the types of activities she would do with them and about how it influenced the time she spent with them to move them forward. She responded:

Mainly it was looking at the objectives before and seeing what they weren’t able to understand and then re-teaching and doing small groups with them. And so now it’s like, what this does help me do is think about what they can do, and whereas before I might say, “Well, he needs to work on this. He needs to work on this. He needs to
work on this.” And now it’s like, “Well this is what you can do and then let’s go back and see what he’s sort of missing and then what he needs help on.” So that’s a different way that I . . . . This has helped me look at my kids to like sort of see what their strengths are in math and then start from there. Because it’s always good for them to say, “Oh look. You can, you can count to five. Now you can count to ten.” And sort of like there’s a graph that we use when I’m doing an assessment like that so they can see their graph get colored more and more. So I think that’s helped me use this to help the kids understand that, “Oh, I can count to five now. I can count to ten now. I’m going to work on 15 or whatever.”

The final theme that came up when Caroline talked about her students was the use of a number of non-mathematical factors to characterize students’ difficulties as they worked on tasks. As Caroline talked about how her students solved tasks, she used language from the LTs to describe what students did mathematically. However, when reflecting on why some students experienced difficulty with a task, Caroline used non-mathematical statements as justifications for what some of her students did. For example, Caroline made statements such as:

- Tyson is just lazy,
- I think he just didn’t want to do it,
- Brandon differs from day-to-day,
- One day I think she’s getting it and then the next day it’s like…I think she has some sort of processing disorder…I don’t know if it’s just her being lazy.
While she was very adept at using the trajectory to discuss where her students were, what “gaps” they had, what opportunities they needed, and how they might approach a task, the LT was not consistently used as a tool to help explain students’ difficulties.

**Anna**

At the time of the study, Anna had seven years of teaching experience and all of her teaching experience was at the partner school. While her student teaching occurred at various grade levels and she even gained some experience in a class for exceptional students, all of her full-time teaching experience has been at 1st grade. Anna’s current certifications include Birth-6th grade (Regular Educational), Birth-12th grade (Exceptional Children), and Instructional Technology.

When asked why she chose to teach at the partner school, Anna indicated that it was because the school had a “small, community feel” and that the teaching staff had a “family feel.” She went on to say that one of the things that made the school special to her is that the teachers “stick together and look out for each other.” She also liked the level of parent and community involvement that was present in the school.

**Mathematics instruction profile.** Of the four teachers in the study, Anna demonstrated the most change in her mathematics instruction. Early in the project, Anna acknowledged that she did not think her mathematics instruction was very successful, and in the first few days of the PD, she indicated that she wanted to change her instructional model. Initially, Anna articulated that a formal introduction to topics in mathematics was important. Although her initial instructional approach was very teacher-centered and focused on direct
instruction, Anna began to make changes during the project, and during the second half of the project, she was consistently implementing elements of the LTBI model.

The changes in Anna’s instructional model reflected some of her general changes in beliefs about mathematics pedagogy. This shift included statements such as:

[Mathematics teaching] involves a greater focus on finding out where students are and discovering how to meet their needs and move them forward in the process.
Teaching students math is not subjecting them to the same math idea over and over (if it is not developmentally appropriate for them) and just hoping they will eventually get it. I also have a better understanding that teaching a student does not just come from the teacher. Students can and need to learn from the experiences of other students!

At the conclusion of the LTBI project, Anna wrote the following reflection:

After completing the work for this training I believe my ideas and practice have changed. In my previous teaching years I spend a majority of time giving my students direct instruction and then time to practice. I thought providing them with the correct most direct approach to solving a problem would be the best way for the most students to understand the math. I thought what I was doing by having students share answers was the same as peer sharing. During the past year I have seen that I need to give my students time to think, try, and make mistakes while learning the math before I go ahead a provide them with instruction. I also learned how powerful peer sharing can be. My students may not understand the approach that I think is best but when listening to their peers share their ideas they listen more and can find ways that they
understand better. I learned that I was not only teaching by standing up in the front of the room speaking but that I could teach my students through the sharing of their peers.

**Participation in the LTBI project.** From the start of the professional development, Anna made it very clear to the research team that she was eager to learn an instructional model for her mathematics instruction. While she found the content of the LTs to be very helpful and informative, she pushed the research team early on to provide her with a model for classroom instruction. For example, in the first week of our PD, she asked the team if we were going to tell her how to teach “this.” She was very actively engaged during the PD and frequently made contributions to discussions related to both mathematics and pedagogy.

**Classroom organization.** The first thing that caught my eye upon entering Anna’s classroom was a letter that she had written to her students on large chart paper and posted on the wall. This letter read:

Dear Students,

1) I believe in you.

2) I trust in you.

3) You are listened to.

4) You are cared for.

5) You are important.

6) You will succeed.

Love, Ms. Anna
Although Anna’s classroom was inviting and provided opportunities for students to engage and explore, it was very clear that she had specific rules in place, and her students were very aware of how they should conduct themselves in various settings. In fact, one fear that Anna articulated early in the study was the uncertainty of allowing students to engage in mathematics talk freely and what this would mean for classroom management. To address this, Anna developed a set of mathematics expectations for her classroom shown in Table 11:

Table 12

*Anna’s Mathematics Expectations*

<table>
<thead>
<tr>
<th>When working alone</th>
<th>When working with a partner</th>
<th>When Sharing Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>-Thinking</td>
<td>-Helping</td>
<td>-Level 2 Voice (Teacher Voice)</td>
</tr>
<tr>
<td>-Use fingers, number line,</td>
<td>-Ask for help</td>
<td>-Listen with a Level 0 voice (Silent)</td>
</tr>
<tr>
<td>etc…</td>
<td>-Math talk</td>
<td></td>
</tr>
<tr>
<td>-Draw a picture</td>
<td>-Level 1 Voice (Whisper)</td>
<td>-Ask questions</td>
</tr>
<tr>
<td>-Try something</td>
<td>-Teaching</td>
<td></td>
</tr>
</tbody>
</table>

Anna opened up her classroom discussions with a review of her expectations for students. They would then work on a task that was similar to the task they would engage in during the lesson. While launching the story problem for the day, Anna would ask students to read the problem aloud and then solicit volunteers to share how they planned to approach the problem. Anna’s students were consistently engaged in the classroom activity, and she encouraged input from all students. Anna even made sure to include/assist students that did not have consent so that they were not neglected during the lesson.
**Task selection and implementation.** Although not required for this study, Anna provided me with a lesson plan for her first lesson observation. In it, she demonstrated a clear understanding of the task, the ways in which she planned to support students in engaging in the task, and her anticipations for students’ strategies. Because none of her students earned “full credit” on a part-part-whole story problem during her mid-year testing, Anna decided to choose that problem type for this lesson. Specifically, the goal of her first lesson was for students to work on different number combinations of eight and to present them in equation form. The task was, “There are 8 students getting ready for recess. Some of the students will have indoor recess and some will go outside for recess. How many of each could you have inside and outside? Write your equations.” Each student was given a recording sheet that listed the problem at the top and had ten spaces for students to write their responses. Before allowing the students to work individually, Anna asked the students how they planned to approach the problem and if they had any ideas for how to solve it. One student said, “There can be eight inside and eight outside.” Anna realized that this student did not understand the task and so she called eight students up to the front. She told the class that these eight students represented the set of eight kids, and so they needed to think about how they could break these students up. Students were then allowed to work individually. As they worked, Anna circulated the room and engaged with students. Anna was very deliberate about observing how her students solved the problem and was able to speak about their strategies in great detail. She also had a pre-determined list of questions she wanted to ask students including:
1. How is this solution similar to this one?

2. How do you know that you have all of the solutions?

3. How do you know where to start?

4. Why did you pull eight blocks?

At the end of the lesson, Anna paired the students up with someone sitting next to them for “pair share” time. For the whole group sharing, Anna had pre-determined the strategies that she wanted the students to share and selected a few students to share at the end of the lesson.

When reflecting on this lesson, Anna indicated that she did not think the lesson went well because it did not go as she anticipated. When I asked her to explain more, she stated:

I anticipated that a lot more students would have tried something and they ended up having a lot of students not try something or tried something completely inappropriate just to be doing something, but not really thinking about the problem while they were trying something. I anticipated that some of my kids would have been doing more than what they were doing ‘cause I know they were choosing to not do things that I know that they could have been doing that they’ve done in the past. And my sharing didn’t go as anticipated, I had picked students to share because I saw them doing great things and then when we went to kind of express it or share it some of them said completely crazy things, some of them weren’t articulating what they were doing right or changed what they were doing so it was just overall. . . . And yeah, it was just overall not. . . . Didn’t go as well as I thought.

When I arrived for Anna’s next observation, the students were working on a math problem. Anna stated that they had worked on a series of math problems throughout the day
and that my observation would only cover a few of the problems they were working on. The problem that students were working on was “Our class went to Ben & Jerry’s and got some ice cream cones. 8 kids got chocolate cones, 7 kids got vanilla cones, and 2 kids got strawberry cones. How many cones did Ben & Jerry’s give to our class?”

After students completed this “warm-up” problem, Anna had all of the students come back to the carpet to start the new task. The problem was, “At the museum, our class used Legos to build Robots. Pink group used 15 Legos, blue group used 10 Legos, and green group used 20 Legos. How many Legos did they use to build the robots?” Anna selected a student to read the problem to the class. The whole class then read the problem together. Anna concluded her launch of the problem by telling the students:

This sounds easy, but I made the numbers a little bit bigger, so using counters may not be your best bet. You may have to think of a different strategy to use. You may have to think about some other ways that we’ve practiced counting numbers: in your head, base ten blocks. Think of some other ways that we’ve been practicing doing our adding to make it quicker.

Students were then released to go to their seats and work. Each student was given a recording sheet (see Appendix H) to record their “blank equation,” their “final equation,” and a space to show their work. As Anna circulated the room and monitored her students, I observed the following behaviors:

1) She asked some students how they were going to start the problem and what number they were going to start with.

2) She suggested how to start the problem for some students.
3) She asked students to explain their strategies to her.

4) She asked a student who had completed the problem to share their strategy with a student that was having difficulty getting started.

5) She asked two students who had used different strategies to share with one another.

6) She encouraged students who had done the work in their head to think about to write their work or draw what they did.

7) She directed the Aqua group to do a share at their table.

8) She pushed students to think about a “quick way” to do their addition and asked them to think back about how they add in their morning meeting.

9) She helped students to think of more efficient ways of grouping the numbers in the problems (e.g., adding the 10 and 20 first and then decomposing the 15 into a ten and fives).

During the monitoring phase, Anna got around to all students in her classroom and was able to work with them in some capacity.

Anna concluded this lesson by asking students who wanted to share what they saw their partner doing. She took volunteers and called on certain students. The strategies that were shared represented a variety of levels of the LT (e.g., addition facts, direct modeling, base-10 blocks). With each strategy that was presented, Anna recorded the student’s work on the board, talked about the strength of that strategy, and asked other students who used the same strategy to contribute to the discussion of that strategy.

**Lesson quality.** In this section, I discuss an analysis of Anna’s lesson quality across each of the four IQA rubrics. Again, this discussion centers on her average scores across each
of the observations and is meant to provide additional detail about the rigor of her instruction as well as the opportunities for high-quality mathematics instruction present in her classroom.

Table 14

*Table 14*

**IQA Results for Anna’s Lesson Quality**

<table>
<thead>
<tr>
<th></th>
<th>Lesson 1</th>
<th>Lesson 2</th>
<th>Mean</th>
<th>SD</th>
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<tr>
<td>AR1 Task Potential</td>
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<td>3</td>
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<tr>
<td>AR2 Task Implementation</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>AR3 Student Discussion</td>
<td>2</td>
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<td>ARQ Teacher Questioning</td>
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</tbody>
</table>

Anna consistently selected tasks that had potential to engage students in complex thinking. Her score on AR1 indicated that her tasks required students to use multiple representations and identify patterns. Anna maintained the rigor of each of her tasks during implementation indicating that students engaged in the task according to her plan during task selection. Anna also maintained rigor during student discussion, and in general, students discussed important mathematical ideas, provided explanations for their work, and presented multiple strategies. Finally, the score on the ARQ rubric indicates that Anna consistently asked probing questions and used her questioning strategies to generate discussion among the entire class.
How Anna talked about her students. Three themes were present in Anna’s discussion of her students. First, she used the LT profiles labels to discuss where students were working and indicated that these labels were not fixed. Second, she used language and skills from the LT to identify what all students could do. Finally, she used ideas from the LT to think about how to support students during instruction.

In Anna’s initial grouping of her students, she grouped her students into five groups: perceptual, beginning early counting, early counting, moving into counting on, and counting on. Anna’s initial groupings of students indicated not only that she had adopted language from the LT and was able to use the profile names to place her students, but also that she was able to recognize distinctions in students to indicate whether they were “early on” at a profile level, “at” a profile level, or “moving out” of a profile level.

When discussing her students at the perceptual level and those beginning the early counting level, Anna indicated which skills her students were able to demonstrate on their own, which skills they could successfully demonstrate with scaffolding, and which skills they struggled with. As Anna discussed her students that were early counters, she also described what students could do and what they struggled with, but she went into greater detail. For example, she provided information on how these students could “find 1-2 solutions for part-part whole problems,” or that these students could “use a variety of strategies to solve problems.” When discussing what these students needed to progress, the focus was split between the role of the teacher and the role of the LT. For example, Anna indicated that these students would need to work on “strategies to find all possible solutions for part part whole
problems” or understanding that “part part whole problems are asking for combinations of a number and that they have multiple solutions.”

Additionally Anna provided statements such as “I know that in order for this student to begin moving into the counting on stage he has to master finding the change in problems.” When Anna did reference her role, instead of a focus solely on direct instruction, she indicated that these students needed “direct instruction from her as well as work shared from other students.” She also indicated that these students needed “multiple opportunities to practice these types of problems.”

When discussing her students that were moving into counting on and beginning counting on, Anna again referred to the types of problems that they were able to solve; she also knew that they were consistently able to use certain strategies and that they could find multiple solutions to a problem. In reflecting on the type of work these students did in class, Anna offered information regarding the ease with which students solved a problem and that she would like these students to begin using known facts to “make problem solving quicker.”

Finally, when discussing her early counting students, Anna discussed how students at this level attended to precision, were “solid with all benchmarks from the Early Counting Child,” and could use known number facts. Although she did not use the word “flexible” or “flexibility,” this was a hallmark of how Anna described these students. Anna set goals for these students such as “building a mental library of doubles +1, doubles +2 and doubles -1, doubles -2.”

To conclude this section, I will share some similarities I observed in the ways in which Anna talked about her students across the five groupings. First, Anna talked
specifically about the skills students needed to progress and the ways in which she would assist in that process. Whether it was direct instruction, small group work, a one-on-one interview, or additional opportunities to practice, Anna offered specific examples of the next task or activity she would like each student to engage in to further understand where s/he was working in the LT. Second, when Anna discussed non-mathematical factors (e.g., exceptional students, English language learner, Attention Deficit Hyperactivity Disorder, lack of confidence, enthusiastic math learner), she coupled her descriptions of these outside factors with topics from the LT again to discuss how she would support students. Third, Anna indicated that direct instruction was necessary across all five of her groupings. It seemed that when Anna wanted to introduce a new strategy to students or address a misconception students had with a problem or strategy that she planned to use direct instruction to do so. Finally, Anna observed and was able to articulate growth in students at all five groupings.

The following example highlights the similarities between Anna’s discussion of Jamir (one of her exceptional students working at the perceptual level) and Destin. Anna stated the following about Jamir:

He struggles with number sense [and] he can count and compose numbers to 10 but struggles past that. He is not able to decipher on his own what a story problem is asking. With guidance, he is able to solve result unknown problems. . . . When looking at Jamir’s district assessment, I can see that he only solved one result unknown addition problem. For the result unknown subtraction problem, he tried to solve it with addition. He attempted to solve the problem direct modeling using a picture.
As Anna talked about Destin, whom she labeled as being a counting on student, she stated that he:

- Can currently use counting on mentally and is beginning to use known facts in his solutions. He can use a variety of strategies when solving problems. He has an understanding of part-part whole, but we are working on strategies for finding all possible solutions. He can find many solutions, but does not have a strategy to check if he has all solutions.

While these two students were at very different places in the LT, Anna was able to talk about both their strengths and their areas for growth as well as the necessary experiences to help them progress.

**Summary**

In the first part of this chapter, I presented a detailed portrait of each case. Recall that the purpose of these portraits was two-fold. First, the portraits provide the reader with an understanding of each teacher’s professional background, classroom structure, beliefs about mathematics, and views of their students. Second, the portraits help the reader frame how some teachers successfully used LTs and LTBI to promote equity and others did not. From these portraits, it is clear that the four teachers in this study represent varying degrees of professional experience and level of involvement in the LTBI PD. While some teachers looked at students and focused on what they did know, other teachers focused on what they did not know.

As you read the cross-case analysis, the variance of themes present in each construct as well as tensions observed are not surprising. Teachers who were more involved in the PD
and more intentional about implementing LTBI demonstrated more instances of equitable instruction in their classrooms. Teachers with strong deficit orientations, however, typically did not enact equitable instruction and oftentimes used the LT as a mathematical justification of why some students were “low” and others were “high.” In Chapter 5, I summarize the findings from each dimension of the cross-case analysis and then offer conjectures about how variance in teachers’ beliefs and instructional practices shaped the findings of from this study.

Cross-Case Analysis

In this section, I present the results from the cross-case analysis to answer the research questions. Within each research question, I present themes that emerged from the data analysis along with examples to support those themes. Examples were taken from classroom observations, interviews, and student portraits. It is important to note that all teachers were not present in all themes. Additionally, some teachers were much stronger in their use of the LT related to some dimensions of equity and not others. The examples chosen throughout this section reflect varying degrees of use of the LT and LTBI and are intended to be exemplars.

Along with presenting instances of how the LT and LTBI supported teachers in implementing equitable instruction, I also offer examples of tensions that arose as teachers engaged in this process. Recall from my discussion of retrospective analysis in Chapter Three that tensions were defined as instances when the teacher a) recognized the importance of the construct, but had difficulty enacting it; b) enacted a construct, but did not draw upon the LT
in doing so, c) failed to implement a construct due to deficit orientations; or d) implemented instruction that was not aligned with the LTBI instructional model.

**Question 1: Access**

This section addresses finding related to access. Specifically, I offer evidence to answer the first research question: *In what ways do teachers use LTs and LTBI to provide access to students in their classrooms?* In this dimension, multiple ideas of access were addressed. Specifically, this section offers details on how teachers used LTs and LTBI in planning tasks, selecting materials, being available to students during instruction, facilitating mathematical discussions that all students can participate in, and providing opportunities for all students to engage in rigorous mathematics.

**Question 1a: How do teachers use LTBI to design instruction and instructional tasks such that they are accessible for all students?** Themes that emerged related to designing tasks that are accessible for students were a) selecting open tasks, b) differentiating tasks, c) unpacking standards, and d) selecting “relevant” tasks. Teachers were able to utilize their knowledge of the LT to select or modify tasks such that students at various profile levels could engage in the task. Teachers also harnessed their knowledge of the LT to think about various ways to differentiate the task (by keeping the same task and using different numbers for students working at various LT levels) and thinking about the ways in which they could support students as they worked. Teachers also realized the importance of modeling the action in story problems and thus tried to select tasks that students had previous experience with.
**Selecting open tasks.** Caroline stated that it is important to have open-ended tasks to provide access for students. Anna also recognized the need for selecting an open task so that, “all students will be able to try or begin or work on regardless of if they’re perceptual, early counting, or counting on, they can all try it. . . . They just might try it different ways or be working on different strategies to access it.” For Anna, this involved creating tasks where her students could do the problem the following way:

Solve it however they wanted, so some students might need to direct model it and draw pictures…some of my higher students [can] start thinking of the numbers like place value, thinking of the numbers in tens and ones. So, I thought that it was a problem that my lowest could access, my middle students could do counting on, or my higher students could do some place value work with.

**Differentiating the task.** Teachers used their knowledge of the LT to differentiate tasks. Instead of using a disciplinary approach to mathematics (e.g., my “higher” students should work with larger numbers and my “lower” students should work with smaller numbers), teachers drew upon ranges of numbers from the LTs when determining which task students would work on. During her first observation, Elsie’s students were working with number combinations of either five or eight. When asked how she determined who would get an “I can make 5” or who would get “I can make 8” recording sheet, she stated:

I looked at assessments that we had done for end of second quarter and looked at where the kids were on the trajectory. So the children who were still more perceptual moving to early counting kids, then I had them with the fives because I felt like those were numbers that they were really comfortable with. The kids that were solid early
counting moving into counting on, I wanted them to really...I had them use the bigger number because there are more number combos that come together and so I felt like they’d be able to handle that.

Caroline used the LT in the same manner. In her first lesson, she selected an activity where students were going to the grocery store and making bags with red apples and green apples. She modified the original task so that students were working with a total of either five, ten, or twelve. She articulated that her counting on students would work on combinations of twelve, her students at the early counting level would work on combinations of ten, and that students moving from the perceptual level to the early counting stage would work with five. She articulated why she differentiated numbers for two groups of her students. First, she stated, “Some of my groups have worked with combinations of ten already so that’s why I hope that they can do combinations of twelve. I don’t want it to be, like, too repetitive for them so I’m hoping that works out.” Additionally, she stated the following for her students moving into the early counting stage: “

So I think that’s why they’re going to stay with five, so I can see how well they do. And then if they do okay, then it all depends on how they are during the sample one. If I think they can handle it, then I might give them a different number; maybe six or...you know. I’m not thinking that they’d be able to do eight.

Caroline drew on her knowledge of the LT profiles to think about which numbers would be accessible for students.

Elizabeth also stated that the LT could be useful for differentiation. She said that she can look at the accomplishments for each profile to think about how to make tasks fit
different learners. Although Elizabeth stated this as a use of the LT, I did not observe this use of the LT during my visits.

**Unpacking standards.** Teachers also provided access by unpacking state standards. The LT was used to identify smaller goals that were precursors to standards. For example, when asked about her goal for her first observation, Elsie indicated that the goals for Common Core were to look at place value and how tens and ones come together. She went on to state that she did not feel comfortable jumping right into that standard because she had not previously looked at how tens were made up. Therefore, the goal for this lesson was for students to “see number patterns on the tens frame and . . .subitizing of numbers bigger than four or five, but really . . .starting to make those grouping of numbers.” Elsie used the LT to identify where the CCSS standard was and to identify smaller goals that students should master while working towards the overarching standard.

**Selecting “relevant” tasks.** Another way in which teachers provided access to mathematical tasks was by considering how relevant the tasks they selected would be to their students. This particular theme is interesting because on the surface it relates to the identity dimension. While the focus of the selecting relevant tasks in the identity construct is to value elements of students’ cultural backgrounds and lived experiences, the focus in access was selecting tasks that had “relevant” action for students (e.g., counting, joining, separating).

Elsie articulated this as selecting “familiar activities that they could connect back to.” For example, she selected a counting activity that she assigned to students to complete for homework that she thought they would find relevant. She had students count how many eyes they had, count the number of eyes in their family, count the number of steps it takes to go
from the kitchen to the bedroom, and find items in their house that matched the geometric shapes they learned about. In another lesson, her class had recently returned from a field trip to the local creek, so she created a story problem related to looking for bugs under a log that they rolled over. This was an activity the students did while at the creek. Caroline designed a task that was related to monkey soccer and tag as those were both games that her students had played in PE, and students understood the concept of splitting up into teams in different ways. She also had students “make” bags of apples with red and green apples to model combinations problems. Anna used the LT to think about the different ways in which she would need to structure the task to support students. During our first interview, Anna had not yet decided on the story problem she was going to use in her lesson. She wanted to choose a context that the students would be familiar with and stated that the “relevance of a problem can make direct modeling easier.”

In each of these examples, the LT was present for teachers. They first decided on their topic or learning goal. Next, they considered that some of their students were working at the direct modeling profile and would need to “act out” the task. To provide the students access to the action in the task, teachers drew upon familiar experiences students had. Teachers simultaneously considered the mathematical task as well as the types of strategies students would use to solve it when determining how they could make the task accessible.

**Tensions.** In contrast the other teachers articulating the need to select open tasks that students at various levels can engage in, Elizabeth struggled with this concept. Her interpretation of access was checking up on students for each step of a problem/task. She stated that:
I think in planning ahead you plan for all contingencies of say a child doesn’t get the idea of – like with our subtraction ladybugs – that you take it step by step and for each step you plan for them not understanding it possibly, and for the ones it’s not accessible because they just don’t comprehend it, so making sure through the activity progressively each step is accomplished and comprehended and then move on to the next step. Check for comprehension at the lowest level, and once that’s clear move to the next step. . . . Basically just check for comprehension and review it and then move to the next.

I have labeled this particular instance as a tension because although Elizabeth is attempting to make the task accessible for students, check for understanding, and acknowledges that students are at different levels, this statement is not indicative of selecting an open task. She did not speak of selecting a task that students at various LT levels can work on, and it seemed procedural because the focus was on specific steps. Instead of using the LT as a tool to think about entry points for and how to structure the task so that all students could engage, Elizabeth chose to select tasks with strict instructions about how to solve them. This is problematic for equity because students’ opportunity to engage in rigorous, open-ended mathematics are greatly reduced when tasks are limited to procedures and drills.

**Question 1b: How do teachers use LTs and LTBI to identify and use up-to-date research based materials and technology?** A critical component of access is students having access to the most current research-based tasks and materials. Each of the teachers used their knowledge of the LT to create or modify existing tasks. In the case portraits, I discussed each teacher’s tasks/lessons. As teachers created tasks, they generated story
problems similar to those presented in the LTBI PD and used problems from websites provided in the LTBI PD.

Initially, I conjectured that teachers would use their knowledge of the LT to evaluate which curricular materials and technology aligned to their learning goals. Although I did not observe teachers evaluating curricular materials, they did generate their own. One explanation for the lack of evidence for the initial conjecture is that this study was conducted during the first year of Common Core implementation. Subsequently, the teachers only had district pacing guides and did not have books, workbooks or other curricular materials. Additionally, none of the teachers found the LT useful for incorporating technology in their lessons.

**Question 1c: How do LTs and LTBI help teachers think about being accessible to all students during instruction?** Equitable instruction requires that students have access to the teacher. This construct was one of the most difficult constructs for teachers. While each teacher recognized the importance of working with all students and understood that some students needed more one-on-one time than others, making this a reality during instruction was challenging. Subsequently, I only observed tensions for this construct.

**Tensions.** Teachers expressed the following struggles they faced in being accessible to all students: a) classroom management issues, b) spending too much time with some students, and c) grouping students. In the following paragraphs, I offer examples that support each of these themes. I conclude this section with why these tensions are problematic for equity.
Elsie struggled with being accessible to all of her students during instruction, and stated:

I always feel like I have a big ‘S’ on my forehead so they know that if they come up to me that I’m going to turn and listen. Like they just know that. And setting those boundaries of, “No. Right now I’m with this person so you need to wait.” But then also, sometimes there are quick little I can give like a nod to, “Yeah, that’s. . .you’re thinking the right way,” and get back to that person. So trying to balance between that. In conjunction with that, some of the kids, building that confidence that, “No I don’t have to have Ms. Elsie right then. I can try it on my own.” And so trying to figure out how to help them with that.

As the study went on, she indicated that working on her time management and trying to spend time with every child was a constant struggle for her, but one that she continuously addressed. In our final interview, she stated that another approach she tried to be accessible to all of her students:

I try to be aware of. . . Like, if I’m sitting at a table, that I’m sitting next to someone that I know is going to need me pretty consistently but I’m close to people that I can look over, students that I think they just need, “Oh, have you tried this?” and they’ll be able to do it. And then just kind of checking in visually with kids I think are doing it on their own.

Similarly, in Anna’s whole class instruction, she found that grouping students by skill level caused tension due to the classroom management aspects of it. Although she recognized the potential benefit of heterogeneous grouping, she was concerned that it made it too
difficult to help all students. Her goal was to maximize her chances of meeting everyone’s needs.

*Spending too much time with some students.* Caroline shared that it was difficult to think about being accessible to all students during the lesson. Specifically, she indicated that it is difficult for her to meet the needs of her students that are moving out of the perceptual stage for the following reason:

They aren’t really able to work independently. . . . They need it broken down in steps so I try and have my assistant work with them on some days so I can make sure I’m getting to the other kids and giving them enough time. . . . Since they can’t work so well independently, they need a lot of checking up on, so when I can’t get to them or when I want to get to someone else, then she sort of steps in and makes sure that they’re still understanding what to do since they need so many.

While this language is deficit-oriented, Caroline’s statements indicate an internal struggle she faced between wanting to spend more time with those students that need it versus spending time with all students in the class. Because she felt like some of her perceptual students lacked opportunities to learn mathematics and were in the perceptual stage for a long time, she really wanted to spend a lot of one-on-one time with them so they could move forward. For Caroline, access to time with the teacher was the most important thing she could provide for these students, but she struggled to do this if it meant never getting around to the rest of her class.

Anna indicated that she also groups her students and allows them to work in centers at times during her math instruction. When I asked her to describe how she used centers, she
indicated that sometimes she groups students based on how independently they are able to work and that the topics are:

- Kind of like that so like activities that fall on the same topic and then kind of more guided instruction for kids, but it’s hard to do it that way cause then I feel like I’m always with that lower group who can’t participate and I’m not always having an attempt to get to my higher kids.

Because Anna’s centers were intended to be short rotations 10-15 minutes per center and students rotate through the centers in one day as opposed to a week (which is how Caroline designed her centers), she experienced difficulty not spending too much time with her “lower” students. In our later conversations, Anna indicated that allocating time with students continued to be a challenge for her and that she continues to group for different purposes.

When asked how she allocates her time with students, Elizabeth indicated that she wished she could get to all of her students and that her TA works with students who need a “double dose.” Elizabeth also stated, “If I have kids that are on the ball and moving ahead, they can work individually. Or sometimes I’ll ask them to work with another student and explain it and I talk about how, “If you can explain it, you really know it.” She went on to say that she will have students work together and tells them, “You go ahead and help this person who’s maybe just about to get it,’ and then I’ll work with the kids and Mr. Rojas will also work with the kids who are really lost and need a lot of review.”

This idea of teacher time is a very important tension to address for two reasons. First, spending more time with the students who need it or those that have had fewer opportunities to engage in mathematics is an equitable instructional practice. For teachers to recognize that
some students need more opportunities to engage in mathematics and therefore need more access to the concepts is in line with an equitable pedagogy. In this regard, the tension that the teachers felt is because they knew what students needed, but the struggle between equity versus equality was a factor. In these examples, language from the LT was used in a deficit perspective to indicate that it was problematic to spend so much time with the “low” or perceptual students.

The second reason that this tension is important is because oftentimes, access to more teacher time equated to access to more direct instruction or drill work. Teachers were able to use the LT to discuss which skills students needed to work on, but because teachers viewed these students as “lacking” or in need of a “double dose,” they saw themselves as the primary provider of knowledge. For example, when I asked Caroline about how her students progressed from the perceptual level to the beginning of the early counting level and what she credits for this growth, she stated, “I think it was just like they would just be with me all the time, just like sort of drilling.”

**Question 1d: How do teachers use LTs or LTBI to support students such that all students can engage in classroom discussions?** Mathematical discussions are a central aspect of reform mathematics, thereby making it an important construct of the access dimension. Of the four teachers in the study, I only observed mathematical discussions in Elsie and Anna’s classrooms. Most of Elizabeth’s lessons were very teacher-centered. Caroline attempted to add discussions into her lesson structure but gave up quickly if she felt it did not go as planned. One theme (building connections between strategies) and two tensions (time management, unproductive discussions) were present in this construct.
Building connections between strategies. Elsie and Anna worked to ensure that the mathematical discussion in the classroom was accessible for all students by building connections between mathematical ideas or strategies that were presented during the launch of the lesson or during sharing. Elsie allowed a student to present and then called on another student to bring them into the conversation and help them see specifically how their work related to the student that just presented. In the following example, Elsie was working on subitizing seven. After showing a dot card with seven dots on it, she asked two different students to share how they represented it on their tens frame. She first called up Mario (a student she labeled as perceptual) and then Siwa (labeled as moving towards counting on). Consider the following exchange in Elsie’s classroom:

Siwa: I did the four like, one, two, three, (as she pointed to the four dots on the top row of the tens frame) something like this. But I did it in a neat way, but I do it with them not touching a lot.
Figure 5. Siwa’s Tens Frame Representation

Elsie: Mario, Mario. I noticed when you did it, you filled in all the way across and went to the bottom. That was a really neat way to do it, Mario, but look. Siwa did, two and then two more to make four, and then five, six, seven. She kind of spread hers out on the tens frame.

Since the goal of this lesson was to have students represent numbers on a tens frame and record “different ways to make a number,” Elsie provided access for Mario by specifically drawing his attention to a different strategy and helping him contrast it with his.

While Elsie worked to achieve accessibility to mathematical discussions during the launch of the task, Anna worked to achieve it at the close of the lesson. During her final observation, Anna selected students and asked them to share what they saw their partner do. As students told her what their partners did, she wrote the strategies on the board and re-voiced the students’ strategy to the class. As students shared their strategies, she also called
upon other students who used the same tool. She concluded this discussion by drawing students’ attention to the fact that while they used different strategies, they all got the same answer. Each of Anna’s behaviors in this example was in line with building connections between strategies according to the LTBI model.

**Tensions.** Two tensions were present in this construct. Some teachers had difficulty fostering classroom discussions due to time management and perceived lack of productivity. If teachers did not monitor time during their lesson, there was not adequate time to foster classroom discussions. Additionally, if teachers did not think students were capable of engaging in discussions, they were reluctant to reserve time for them.

*Time management.* Elsie stated that she had difficulty closing a lesson with discussion/connections due to lack of time management. So while she consistently allowed students to share their thinking during the opening of the lesson, she stated:

I’m not good at doing it at the end of the lesson. It tends to be more at the beginning, and mainly because I’m bad about time management and we run out of time. So that’s why my sharing isn’t consistently at the end.

In the LTBI model, teachers use their knowledge of the LT to make connections between various strategies, to help students understand the efficiency of strategies, and to understand how certain approaches can be used to solve a variety of problems. Because these types of connections typically occur at the end of a lesson, this tension was noteworthy.

*Making connections productive for learning.* Another tension in fostering discussion for all students is that some teachers indicated that they did not know how to make these productive for learning. Because Caroline believed that her lower students “do not have
control and they’re not like even actively listening to what is being said,” she didn’t think whole-class discussions were a useful learning tool for her students. She did also indicate that this was “her issue” and “not an LT issue.” This tension is noteworthy because the LTBI team spent a lot of time during the project talking with teachers about how to structure whole class discussions and the powerful mathematics content that can come when teachers work to select and sequence students’ responses in a meaningful way. Because Caroline’s deficit perspectives prevailed in this instance, she did not find the LT useful in incorporating mathematical dialogue in her classroom.

**Question 1e: How do LTs and LTBI assist teachers in providing all students with opportunities to engage in rigorous mathematics?** This construct addresses two important issues related to equity. At the basic level, this construct addresses opportunity to learn in that students have opportunities to learn mathematics (both during formal instruction times and throughout the day). The second component of this construct is that the mathematics students were exposed to was rigorous given their current level of understanding. Themes that emerged in relation to this construct included: a) a fluid view of the LT for progression, b) varying problem types, and c) increased opportunity to learn.

**A fluid view of the LT for progression.** This particular theme was a use of the LT for some teachers and a tension for others. Both Elsie and Anna were able to look across each of the LTs simultaneously and think of the entire LT table fluidly, meaning their views of where students were in the LT was not fixed. They believed that it was okay for students to be working at various places in the LT simultaneously. Elsie indicated that she could look at students’ work to think about where they are working in each LT. Throughout the duration of
the study, Elsie consistently looked across each of the LTs when discussing what students knew and where she thought they were working. When I asked Elsie about students being fluid in the LT, she stated:

My thoughts are that kids definitely can be flexible and so there may be something that clicks for them on one hand but doesn’t in another area. And so they might seem lower in one area and higher in another. But I don’t think that means that you leave out those other areas because then I think that’s when you end up having problems. You might take that string to help support and build up those areas that are more difficult but just because. . . . So like I said, you know, those guys really get the tens and ones but they’re still working down in here as well. And so, just because they get tens and ones doesn’t mean that, like, they need to have more experiences with why would 8+7 then turn into 15 or some…Or why would 3+3 turn into 6? Like, they’ve got to have that flexibility and fluidity with those lower numbers too.

Throughout the study, Elsie consistently used the language from various LTs to describe which skills her students demonstrated, which skills they experienced difficulty with, and which specific skills students need to work on to progress. Her student mathematics portraits were not limited to one LT.

In my final interview with Anna, she explained how her use of the LT had changed over the course of the year, specifically in relation to how she provides opportunities for students. She stated:

Before, I was kind of set on, like, a student is an early counting child and that’s what they are. And now as I’ve kind of gone through the process, I could see that
sometimes they could be an early counting child and work on these skills, but they’re also at the same time ready to start. . . . They may be ready to start or successfully work on a strategy inside of another trajectory or the next trajectory out. Like, it doesn’t make them a counting on child. They’re still an early counting child but they might just be really good at counting those numbers beyond 100 and recognizing them and understanding them. In the beginning I thought it was more like, “This is where you are and this is where you stay,” and I’m kind of using it more now to think about strategies for my students, to think about how. . . before I was just kind of looking at it, “Okay, this is where you are.” Now it’s like, “Okay, I can group them like this and I can push this group to use this strategy to move them forward, and this group I need to push for this.” So kind of using it more now to move them forward rather than just looking at where they are and, “Okay, they need this.”

Both Elsie’s and Anna’s statements suggest that students are not fixed at one area of the LT. Their views support equitable instruction in two critical ways. First, it allows opportunities for rigorous mathematics because these teachers recognize that student growth can and should occur across multiple domains simultaneously. This view supports developing more robust mathematical understanding where the focus is on building connections as opposed to the development on specific skills. Second, these teachers did not indicate that it was necessary to hold students back or make them work in one area of the LT. Instead of “retaining” them at one place in the LT, they maintained rigor by offering development at different LT levels across different LT strands.
Varying problem types. Elsie used her knowledge of the trajectory to vary the problem types she exposed her students to as a way to challenge them. Previously, she had only introduced one problem type per lesson. After students had been exposed to a variety of problems, Elsie decided to increase the rigor of her activity. When I asked Elsie about her upcoming lesson goals and whether she was going to continue working with students on addition problems where the change is unknown, she stated:

Actually, I think what I’m going to do, and you can tell me what you think…I’d like your opinion. I could either do more change unknown or I was thinking about throwing some where it’s the final answer unknown to see if they can tell the difference.

In this example, she drew upon her knowledge of the different types of story problems and decided to increase rigor for students by exposing them to various problem types in one lesson.

Increased opportunity to learn. Another way teachers provided access to rigorous mathematics was by increasing the amount of time they spent on mathematics during the instructional day. This occurred in different ways and was particularly relevant for Elizabeth, as she reported that she did not teach math daily prior to participating in the project.

In addition to teaching math daily, Elizabeth also incorporated specific ideas from the LT during her morning meeting time, and she incorporated general math topics into snack time and when students were working on class projects. After looking at her quarterly assessments, Elizabeth realized that a number of her students had difficulty counting and making the change from “9 to 10, 19 [to] 20, and 29 [to] 30.” To address this, she stated:
I was able to incorporate something that would allow them to practice that switch that getting to 9 and the next level of 10. Every morning we count to 100 and the person who’s counting on the abacus will get to say 19 and then pick out a person and I will guide that morning meeting helper to pick on someone who I know needs practice and we’ll review and we’ll count up and give them all help. Every morning there’s a chance to count with us and then make the transition from 9 to 10.

Another example of this is Elizabeth’s incorporating students’ birthdays into her calendar time. She notes that one particular week there were a lot of birthdays. She posed to the students, “If we have five days and three people have birthdays how many days in the week did we not have birthdays?” She concluded by saying that “every morning message now has been a subtraction problem.”

These examples support equitable instruction in two ways. First, Elizabeth used her knowledge of the LT to identify specific skills (e.g., counting, finding result problems) that students should develop. Next, she found opportunities during the instructional day to incorporate these activities. By doing this, Elizabeth increased the number of opportunities students had to engage in mathematics. Second, she stated that she picks students “who I know need practice” to give them additional opportunities.

**Tensions.** Tensions arose in this construct in four ways including a) not using to the LT to modify tasks, b) mastery before movement, c) using the LT to increase expectations for some students, and d) classroom management.

*Not using the LT to modify tasks.* One non-use of the LT I observed for this particular construct is when Elizabeth did not use the LT to modify a task that she thought was too
difficult for students. On day one of her first observation, the students had a lot of difficulty with the task (rolling a number cube and placing the associated number of counters on a tens frame). Earlier in this chapter (in her profile), I conjectured that this was due to the lack of clarity of the learning goal and the setup of the task. While Elizabeth did use her knowledge of the LT to state difficulty that one student had, this difficulty did not translate into how the second part of the lesson was revised. Elizabeth stated that some of her students did not have “number recognition” and went on to say:

Well the first day the task I think I was overreaching and trying to put every part of a lesson that could have maybe stretched for a few days into a forty-minute lesson and I was expecting they would catch on and it didn’t work that way. So I thought it would be better to have a whole group lesson and a couple kids modeling it and then they each had a chance one by one to try out breaking up the number ten and also having a sort of a relationship to it. Because the first day it was just random and they were rolling the die and they were kind of confused and there were too many options. I thought oh we’ll do this and then we’ll move on to twenties and that was just asking too much. So I realized we had to simplify it, make a more direct shorter task and connect it to something that they were really happy about which was their valentine box.

When asked if the trajectory helped her think about how to modify the task or helped her think about what was difficult for the students, Elizabeth said:

I think well mostly that I noticed I had to work with them one on one. There was a group that I had that I was sort of exclusive to the rest of the class, and I knew they
were capable, like, if I had them profiled and they were more capable than others. If I had to help them that much, then I knew the ones that were lower that they were on the trajectory were probably floundering or just looking at other things. So in order to make sure that they were confident, I realized from placing them on that trajectory or giving them a profile I needed to work out a simpler task.

Elizabeth modified the task by having students roll a die and then grabbing that corresponding number of pink and red hearts to glue on their Valentine’s Day box. After students selected their hearts, the remainder of the math lesson was used for decorating their boxes. Not only was the second part of lesson not aligned to the original learning goal, it did not provide students with access to rigorous mathematics.

When reflecting on the second day of the lesson, Elizabeth stated that it was better because the class “came together,” the students were able to “share and know each other’s feelings,” and that the students “were all having fun with each other.” This is really a non-example of using the LT to modify a task to meet the needs of different learners as the focus was on changing the task completely and changing her role during the task.

Mastery before movement. While some teachers did not think of the levels of the LT as static and believed that students should be able to work and develop in different areas of the LT simultaneously, one tension that I observed during this study was the opposing viewpoint. When Elizabeth discussed how she grouped her students and the potential to use the LT to move students forward, I pushed her to think about whether or not the LT or framework had changed the ways in which she viewed students, specifically her belief from early on in the study that students must master one level of the LT before moving to the next.
Elizabeth stated that students should stay at one profile level and master those skills before moving to the next so that they don’t get discouraged and feel poorly about themselves. She went on to say, “I think my greatest concern is from learning from the trajectory that if you’re not confident at the level, to move on is not beneficial.” She also made the following observation:

There’s an order in which you can proceed that makes it easier for the child learning these things, so stick with that, make sure that they are confident, and then…and by having the trajectory map out the steps you can really see why it’s important you can’t go on to counting on until you’re really solid on cardinality and that type of thing. So it just seems logical.

While the LT does discuss progression of ideas in terms of refinement and sophistication, the idea of not allowing students to progress until they have completely mastered an idea is problematic for equity. Rather than using the LT as a tool to understand students’ current conceptions and difficulties and then use the LT as a tool to think about how to move students forward, Elizabeth used it as a way to justify limiting students’ exposure to varied content.

*Increased expectations for some students.* Another tension observed was that rigor was only appropriate for some students. This was a tension because the LT and LTBI model supported teachers in providing *some* students to engage in rigorous mathematics. Consider Elizabeth’s comments in relation to this construct:

I remember there were some things that would not really relate to the age range that I’m working with. Provide all students with opportunities to engage in rigorous
mathematics seemed a bit tough for kindergarten. I remember...or we do set appropriate academic standards, and I think since our last meeting when I mentioned that we thought that the idea of them counting to 100 and counting to 100 by tens would be pretty much asking too much but we did it every day and now a good majority of them can. Not successful for I’d say 50% of the class but we work it into our...I work things into my daily routines so they got daily practice, and those that could achieve it at least have the opportunity to try and the opportunity to practice.

Elizabeth stated that this rigorous goal was appropriate for “those that could achieve,” indicating that it wasn’t appropriate for everyone. Another reason this comment was problematic for equity is because it occurred early in the third quarter. Elizabeth had three more months of instructional time, yet she did not use the LT to consider additional opportunities for students to engage to meet this goal.

Classroom management. Another tension that emerged when observing Caroline teach her lesson on combinations of five, ten, and twelve is that some students were not exposed to the most rigorous level of the task because in the moment, Caroline thought the management of the task would be too difficult. Although Caroline had previously planned to have three different numbers in the task and offered justifications of why those numbers were appropriate for students, in the moment she changed her mind. She stated:

Because since it was their first time doing it, I felt like they needed to have a number that they were a little more comfortable with. I didn’t want to throw something higher. I decided at the last minute that that probably wasn’t...And it was sort of confusing already because some were doing 5 and some were doing 10, and if I threw
in another number that would be more confusing. The example was 10 so the kids who were doing 5, they really, they were thinking they were doing 10. Then I had to go around and explain to each of them, “Okay, you guys are doing 5.” That didn’t work well. I’ve never actually done that before. . . . That’s a little confusing.

I see this example as a tension in two primary ways. First, although the LT was useful in helping Caroline differentiating her task for students and providing them with an appropriate level of rigor, she was unable to translate her plans into her instruction. This suggests that she needed more support in how to structure her lesson so that she could effectively carry out her goals and challenge all of her students. Her statement in the early part of the quote about wanting students to work with a number that they were “more comfortable with” indicates that she experienced tension between wanting students to be challenged and wanting them to be comfortable. Again, while the LT made the idea of rigor, challenge, and what’s appropriate for students available to her for planning purposes, it was not enough to help her support her goals during instruction.

Summary of Access

Teachers used LTs and LTBI to think about access in a number of ways. They attended to how to make tasks appropriate for students on a number of levels including designing tasks, ensuring tasks were open, differentiating tasks for students at various LT levels, and matching the action of the task to students’ previous experiences. A hallmark of this dimension was that in order to make mathematics accessible for their students, teachers had to anticipate the needs of students and plan to meet those needs as they launched the task,
monitored students, and engaged in whole class discussions. Therefore, teachers used LTs and thought about LTBI as they planned their instruction.

The tensions occurred in this dimension when teachers did not use the LT/LTBI, had deficit orientations to students, experienced classroom management issues, struggled with equity vs. equality, and limited students opportunities to engage across concepts from multiple LTs simultaneously. Some tensions occurred during teachers’ planning while others were apparent as they attempted to enact LTBI in the classroom.

**Question 2: Achievement**

The second dimension under investigation in this study was achievement. This dimension addressed the ways in which teachers set high standards for students, designed instruction to meet those standards, and assessed students to see if they had achieved. In this section, I present findings related to teachers’ use of the LT as they completed their student portraits and discussed their academic goals for students. I also present findings related to the ways in which teachers worked to understand where their students were working in the LT. This section addresses my second research question: *In what ways do teachers use LTs and LTBI to promote achievement for students in their classrooms?*

**Question 2a: How do teachers use LTs to set high, yet appropriate, academic standards for all students?** An important component for equitable instruction is that teachers set high standards for all students in their classrooms. Although county and school districts set goals for students, my conjecture was that teachers would use their knowledge of the LT as a tool to set goals for students. The word “appropriate” in this construct indicates that goals should be appropriate for students based on their current mathematical
understandings as demonstrated in the LT. The only theme that emerged for this construct is that teachers used their knowledge of the LT to set short and long term goals for students. No tensions were observed for this construct.

**Using the LT to set short- and long-term goals.** Not only were teachers able to think about long-term goals and standards for students, they were also able to use the LT to generate intermediate short-term goals. Three of the four teachers used their knowledge of the LT to both describe what students were doing and to set a goal to help students progress to more sophisticated understanding. Elsie thought that it was important to note what students did and did not know when setting appropriate goals. For example, she wrote the following in her student portrait that one of her students: “She knows that numbers represent values of items…is able to count sets up to 15 successfully. She knows how to compare numbers < 10. She is not subitizing numbers (3 or 4) yet and she doesn’t skip count. She is beginning to be a counter from n, n+1.” Based on this knowledge, Elsie’s academic standard for this student was that she “develop subitizing skills of #’s 3,4 & then 5-10, count from n (n+1, n-1) consistently, and understand 10’s frame to support development of find result, make it N, find change, composing to 10.”

Caroline was also able to use the LT to set goals for students, and she provided very clear mathematical descriptions of what these students may do. For example, Caroline stated that her students who just came out of perceptual are “solid with numbers to five and they can count higher than ten. But…well…I guess that’s reciting.” For her students in the counting on stage, Caroline stated:
Right now we’ve done some word problems for adjoining and things like that and right now I know that they can do it with direct modeling, but that’s like the only thing they know how to do…I’m going to be looking for them to use different strategies besides just direct modeling.

Additionally, she stated that these students can “count by twos at least to twenty” and that some of them can “count by five.” She really wanted to push these students to use some strategies other than direct modeling to solve this problem.

In addition to setting goals in the context of a particular lesson or domain, Caroline was able to think about long-term goals for her students. In the following example, the LT served as a tool to somewhat disrupt the belief that kindergarten students should only master numbers 11-19. She stated:

I would hope that they would get a solid. . . they would be solid with the early counting and then. . . but I would hope everyone would be on the counting on child stage, which would probably be I guess more advanced. I don’t know. With our standards, it’s basically numbers 1-10 and we do this 11-19, but not really just knowing that 10 and whatever is 13. You know? So, they’re not working a lot with 11-19, but I don’t know. I guess I want everyone to keep on moving up as…but I would hopefully try to get everyone to have the solid grasp on the early counting.

In this example, Caroline acknowledged what the district-wide goal was for kindergarten (numbers 1-10), but she used her knowledge of the LT profiles to set an additional long-term goal for students.
Anna used the LT to set goals for students, and oftentimes her objective was to work on specific skills or standards. For example, Anna had two students in her classroom that she indicated were exceptional students. Because one student had difficulty “counting and composing numbers” and “his difficulties with attention make it hard for him to stay on task,” Anna felt that this student needed small group instruction or one-on-one instruction. Another one of her exceptional students had fine-motor skill issues. Anna was able to separate his physical impairment from what he was able to do mathematically. Specifically, she wrote that “[he] has always had a solid concept of how to solve addition and subtraction problems” and that “when he is presented with addition and subtraction problems he knows what he needs to do to solve them.” Anna went on to state that, “his fine motor skills. . . impact his counting and the solutions he comes up with” and that “when he used manipulatives to direct model his calculations were always off.” To address this, Anna worked with the student on matching his counting with numbers/objects, and after improving some of his motor skills, Anna stated that he was “no longer at that level and has moved on within the LT.” In addition to setting a skill-specific goal for this student, Anna focused on working with him to move him forward in the LT.

In each of these examples, the LT served as a reference point for teachers to identify students’ current understandings and misconceptions. They used that knowledge to set short- and long-term learning goals for students and oftentimes described how they would work to support students in achieving those goals. When teachers use their knowledge of the LT to identify specific short and long term goals, these goals serve as checkpoints for them to monitor students’ progress. When teachers have more information about students’ strengths
and areas for growth, they can continue to better meet their needs, thus increasing their potential for high academic achievement which is an important outcome of equitable classrooms.

**Question 2b: How do teachers use LT’s to unpack and build upon students’ prior mathematical knowledge and use it as a basis for building more complex mathematics?** This construct is very closely aligned with the previous construct. It is important for teachers to have goals and expectations for their students to ensure they are progressing. In order to meet these goals, teachers must know what their students know, and in an equitable classroom, teachers build from what students *do* know instead of focusing on what students have not yet demonstrated. The LT provided teachers with agency to make instructional decisions grounded in their students’ understanding. The two themes that emerged in this construct (using the LT as a checklist and customizing instruction) represent different examples of agency. One tension was observed in this construct.

**Using the LT as a checklist.** The bullet points in the LT table served as a list of skills that students need to learn over time; all teachers indicated that it was useful as a checklist to see what their students did or did not know. Elsie stated, “I notice that the students are doing and say ‘Oh well, they’re not doing this yet because right now this is their understanding.’ I feel like I’ve used the trajectory to really help me think about what my kids do and do not understand.” Similarly, Anna stated:

> It helps me you know kind of have a checklist kind of to go through, like, you know, does the student know how to do this really, does the student you know can they do this should they be moving forward into counting on or is there something I need to
back up and work on before I start, you know are they missing a piece somewhere else so it kinda. . . I use it kind of like as a checklist of what they should know before they’re trying something else, you know. I don’t want to have any gaps in there in the process, in the learning process for it.

All of the teachers were also able to use the LT to help them understand their students’ work, quarterly assessments, and district benchmarks. The LT helped teachers move beyond solely determining whether students got the answer right or wrong and helped them focus on the content of the questions and provide a description of where students were working in the LT based on checking off what they observed from the assessments. Caroline articulated this use of the LT in our first interview. She stated:

I’ve always wondered. . . why you can’t count these and say the number and remember that is three. I was always confused on why they don’t get it. . . . Like this year, this is all new to me, but it fits into the Common Core. I use this chart to see kind of what they can do and look at sort of like the assessments that I have done. . . and see where they fit in. For instance, I have a group that is just starting the counting on child stage. . . . I know they can do it [adjoining problems] with direct modeling, but that’s like the only think they know how to do.

In each of these examples, the teachers noted which skills students had mastered and made note of related skills that they needed to learn. Using the LT as a checklist was helpful for teachers as it provided them with a framework to situate students’ understanding. Thus, the LT helped teachers unpack what students currently knew. This theme was a precursor for the second theme in this construct. Once teachers understood where their students were
working in the LT, they had pedagogical agency to design appropriate instruction for them. The next theme highlights how teachers used the information they gained from the checklist to build customize instruction for students.

**Customizing instruction.** This theme highlights how teachers combined their knowledge of students’ current conceptions with an instructional plan. This theme differs from the checklist theme because the purpose of a checklist was evaluative of students or used to report their current level of “achievement.” This theme builds upon the checklist and was directed at helping students progress and learn more sophisticated mathematics.

Elsie articulated how she could use the trajectory to think about setting students’ current level of understanding as well as to explore how to build upon their current understandings to move to a more sophisticated concept. Specifically, she stated:

> I think the trajectory gave me a grounding so I was able to go back and look at that and say, “Oh, so if this is what they’re doing right now maybe these are the kind of activities that I should be doing and then this would be my next step.” So it kind of gave me a guiding point I guess.

Additionally, Elsie stated that:

> The breakdown under each LT strand into skills that are representative of a perceptual child, an early counting child, a counting on child etc. helps me to get a view of what Julia knows and where she should go next. When I look at the math work Julia produces, I can compare what she demonstrates she knows to where she is working on each LT. . . . The LT helps me see Julia where she is and what her next steps will be.
In one of Elsie’s lessons, students worked on number combinations problems using numbers smaller than ten. During this lesson, she wanted to extend students understanding of decomposing numbers with an ultimate goal of “fast, flexible understanding of numbers 0-10 and the combinations and how they work together, to then start working on more complex addition problems and things like that. So that was my goal, is to try to solidify their base ten understanding.” Based on how her students engaged in this task, she devised the following plan to extend their understanding:

My learning goal was I think still… I was wanting them to become flexible with the different combinations that are involved with the number 10. For some of my kids, I was hoping…when I pulled the chips out and they were all like, “I want those,” there were some kids that I felt like already had a significant number of 5+5=10, 9+1=10, that they already knew some of those combinations that I wanted them to think, “Oh I know this so I can kind of just go ahead and do this without needing manipulatives so much.” But I think it was just like, “No. I think I really would like that manipulative,” and then it kind of got in their way. And then for some of the kids it was more…so, for the kids that I felt like already have a sense of, “Yes I’m putting these two numbers together and I’m getting 10, and I can put two different numbers together and get 10,” and, “Oh, wow I can really see all these different number combinations.” For some of the kids, it was actually recognizing, “Oh, so if I have 3 ants here and I have 7 worms here, that’s what’s making it 10.” And the same thing, “If I had 7 ants and 3 worms, it’s different because it’s more ants and less worms but it’s still 10.” So kind of making that connection.
Elsie had a variety of “sub-goals” in mind (e.g., writing number sentences without using manipulatives and recognizing the connection between 3+7 and 7+3), and she structured this task such that she could build instruction on her students’ current understandings.

Similarly, Elizabeth talked about the importance of monitoring student’s responses and how this was a critical part of unpacking what they know. She stated that:

The key word “listening” – listening to their answer, and if I have the time, if it’s a specific math task like a time. . . not just morning meeting or throughout the day but a specific lesson, listening to how they derived their answer. And then from. . . I guess I should go in order. But just, to me, monitoring is the big part. The listening is most important. And from that, you can consider strategies. I would think, “What can I do next to help move on from where they are?” I guess the final outcome determines your future task selection.

Caroline echoed this sentiment and stated “I have a better understanding of what the children can do and I focus on that, and then that helps think about what they sort of need to be able to do next.” She also discussed how the LT has changed how she understands students’ background knowledge, particularly what students are “missing” and where they are “coming from.” She used her understanding of this in her lesson planning by recognizing students’ current understandings (including misconceptions) and planned instruction that built upon where students were. For example, Caroline stated that a sub-goal of her combinations activity (where students found combinations of 5, 10, or 12 red/green apples) was:
I just really wanted them to understand that like they all for some reason know that 5+5=10 but they think that’s the only thing that makes 10. So I wanted to kind of get them practice to understand that different combinations will get you to a certain number that they were working with.

Anna set goals by assessing students, determining their profile level, looking at what they had already achieved in a particular profile and setting a goal based on what they should work on next. She indicated that prior to being introduced to the LT, if she knew students were weak in math, she just pushed more math on them. After working with the LT, Anna stated that the LT has provided her with a way to specify goals for students. She stated when unpacking students current ideas she thinks,

Okay, this child can’t produce numbers past 5 so this is where he is and these are the skills he needs to master. So kind of, I guess, differentiating a little bit better and pushing them to work on skills that are appropriate for them rather than, “This is what you have to do and just keep doing it until you get it” kind of really didn’t know that there were all these…there was a stage of it, different levels that they had to do to get from here to here.

For Anna, the LT was useful to see where students were coming from and how she needed to help them progress. Her statement about “differentiating” instruction indicated that the LT was helpful as she customized activities to help individual students progress.

**Tensions.** While all teachers were able to unpack students’ current understandings and think about how to help these students progress, this was not initially the case. When I
initially asked Elizabeth to think about what skills her students currently had and how she could build upon them, she responded:

They have learned to count to ten, which is great because they didn’t do that before. I’m trying to think. They don’t have a number sense and…I guess it’s hard to say what they do have, honestly, I guess because I worry so much about what they don’t have. But they have gained since the very beginning of the year and just being able to count to ten, and even to twenty. Not completely accurately but they know if they put one number after the other, they’re seeing things in order. I guess I should be more positive but really they’re. . . . I don’t know how to express it in a positive way. I worry that they don’t grasp just the whole idea of numbers and amount, and why it’s important.

I chose to mention this example here because it highlights a few important ideas. For Elizabeth, being introduced to the trajectory was not enough for her to shift her views of students’ knowledge from a deficit perspective to thinking about how to use what students have as a basis for more sophisticated mathematics. I also used this example to contrast with Elizabeth’s statements in the previous theme. While she did indicate that it was important to listen to students, and that monitoring can help teachers think about where to move students, Elizabeth did not practice this with all of her students. The deficit orientation that she displayed for her “lower” students hindered her from developing roadmaps for all students in her classroom. This is troublesome for equity because in order to build more sophisticated mathematics for students, you must first acknowledge their foundation. Although Elizabeth was able to note the growth some of her “lower” students made, it was difficult for her to
customise instruction for them since her overarching view was that these students “don’t grasp the whole idea of numbers.”

**Question 2c: How do teachers use LTs and LTBI to implement various types of assessment to gauge students’ achievement?** This construct is important for equity in two ways. First, students should be given multiple opportunities to demonstrate their understandings. As students are given more opportunities to demonstrate their knowledge, teachers have more opportunities to customise instruction and provide them with meaningful opportunities to learn. Second, in equitable classrooms, students are given opportunities to demonstrate their knowledge in multiple formats. No teachers indicated that LTs or LTBI could specifically help them think of various forms of assessment. All teachers indicated that the new standards, common assessments, and quarterly benchmark assessments were their primary way of assessing students. Therefore, I did not observe any themes for this construct. I provide examples to support the tension of not using various forms of assessment in the next paragraph.

**Tensions.** Elsie did not articulate an explicit connection between LTBI and assessments. She stated that “formative and summative is more of what the district is giving us,” and that during class, she looks at their recording sheets and listens to their conversations to assess her students’ understanding. Similarly, Elizabeth did not see concrete connections for LTs and LTBI in this construct. She did indicate that trajectory can be helpful for assessment by looking at particular profiles and determining when students have “passed that level,” which is similar to the idea of a checklist as mentioned earlier in the achievement section. Caroline stated that she primarily uses the county and district objectives
to help guide her assessment practices. She also stated that she found the LT useful for focusing on what students can do. These statements reflect themes that came out of the previous two constructs in this dimension. While it is clear from those examples that the LT helped teachers think about and interpret the results of their district assessments, it did not influence the ways in which teachers thought about assessing their students.

**Summary of Achievement**

The first two constructs in this dimension were very fruitful and teachers could easily see the role of the LT and instructional model. The uses observed in the first two constructs were similar to those observed in other work conducted with LTs (e.g., goal setting and assessing students). The tensions observed were twofold. First, deficit orientations interfered with teachers’ ability to use what students do know to customize instruction. Because the LT alone was not enough to disrupt deficit orientations, all teachers did not fully utilize the potential of the LT. (Note: I discuss deficit orientations and how it impacted findings from this study more in Chapter 5). Additionally, the overpowering presence of state mandated test rang out across all teachers and the LTBI instructional model did not help teachers think about assessment.

**Question 3: Identity**

Research questions three and four relate to the critical axis and begin a shift in some of the responses I got from the teachers. One theme that came out in my discussion with Anna of this dimension is that this is not something she thinks about regularly, and that she doesn’t keep many of these ideas in the “forefront.” She also did not see as many direct connections here between the LT and this construct. Specifically she said, “It is not
necessarily that the LT can’t be used for it. I think a lot of it is, I just haven’t been using it for that piece.” Overall, there were fewer instances of the LT being useful to support these two dimensions than the dominant dimensions. In the following sections, I offer evidence to answer the third research question: *In what ways do teachers use LTs and LTBI to help students develop their identity?*

**Question 3a: How does using the LTBI model assist teachers in developing a robust mathematical identity in students?** The hallmark of this construct is that if students believe in themselves, they will persist in mathematics despite difficulty and obstacles. The concept of a mathematical identity was new for teachers. When I introduced this construct to during our second interview, Anna stated “I don’t think it comes up as much in math ‘cause I never even thought about it until you just brought it up.” Teachers used their knowledge of the LTs and LTBI to help students develop positive mathematical identities in two ways: a) building confidence and b) encouraging students to try the task.

**Building confidence.** In general, an important part of developing mathematical identity for students was for them to enjoy mathematics and be comfortable with mathematics. Elsie stated, “Like, my goal as a kindergarten teacher is I want the kids to be excited about school and feel like school is a fun place.” This idea was very important to Elsie, and I think that it undergirded her idea of students being “mathematicians.” So for students to think of themselves as mathematicians or scientists, a prerequisite is a love for school and schooling. Elsie constantly worked to include all of her students in the classroom and reassure them that they were “number experts” and “mathematicians.” These are phrases that she often used with her class, and the idea was that students could see mathematics and
mathematical connections throughout the day. For example, a student noticed the following arrangement of students in line and said:

Student: Ms. Elsie, listen to this pattern. Boy, Boy, Girl, Girl, Boy, Boy…

Elsie: You’re using your mathematical brain. Yes! Wow, you’re being a mathematician right there. Yeah we can use math to figure this part out.

The LT served as a tool for Elsie to recognize different aspects of mathematics throughout the school day and then validate students’ knowledge of mathematics as a way to boost their confidence.

Caroline indicated that to build students’ confidence, it is helpful to know where they are in the trajectory and what they have a “solid base” with. From there she stated, “You can move them up and since they already have a solid grasp on it, then if you’re giving them the…if you’re moving them into the next stage, they feel comfortable with doing that work since they’ve…they’re all….you know what they’re comfortable with.” Her statements indicated that developing confidence is about balancing support with appropriate challenge for students.

Anna indicated that although she had not thought about identity in relation to math, this is something she would like to start working on. Specifically, she wanted students to know that although they are in different places and may work with different numbers, everyone is doing the same problem. She saw this idea as an important step to building students’ identity so that even though students were working on “different problems,” they were all still doers of mathematics. She also saw building confidence as a precursor to
students’ defending their mathematical ideas. To support this goal of building students’ identity, Anna stated that she uses her monitoring time to reassure them, ask them to share their strategy and talk them through their strategy. She stated:

I mean, because even some of my shier students will share math if they’re proud of their work and they feel good about it they’ll share. But I have some of those reluctant ones that even if they’re right. . . . Like, I have one who’s really reluctant, but she’s usually always right but she just never wants to share. So I kind of just try to build her confidence and encourage her maybe when I’m, “Oh, you did really good. You solved this right. You had a great solution, a great strategy,” and then I’ll try to call on them. And sometimes I could say, “Come on, you know, share what we talked about. You did really good,” and pull back to our private conversation and sometime it works and sometimes…yeah, they look at me and look at me and look at me.

Teachers used LTs and LTBI to build students’ confidence in different ways. The LT served as a tool to help teachers recognize different skills in mathematics and acknowledge students as “mathematicians.” Additionally, teachers built on students’ current understandings so that they felt confident to move on to more challenging topics. Teachers also used their monitoring time to identify students’ strategies from the LT and talk them through with students. Finally, teachers assured students that they were all “doers” of mathematics even if they approached problems differently.

**Encouraging students to try the task.** Another theme in this construct was that a part of developing a mathematical identity is taking risks. Specifically, teachers wanted students
to feel safe to try a strategy and know that learning can come even in mathematical mistakes. Anna worked to support the development of her students’ identities by telling them to “try something.” She further explained what she meant using an example of when a student told her that they couldn’t do a problem. Her response to that student was:

Well try it. What could you do? and then once they are doing something, I’ll be like, “Oh, that’s exactly the right thing that you need to do to solve it. You didn’t even know you could do it, and look, you’re using...you’re counting up on a number line or...look you’re drawing the picture it’s like, And then once they’re trying something, like, “Look. You tried it and it’s getting you into the right direction.”...So just verbally doing it and when they’re... When we do the share time, just highlighting like, “Oh...” You know, one day I had a student, Jasper, “I can’t do this, I stink at math,” and he sat down and he tried the problem and he did it and I highlighted him during share time, “Jasper thought he couldn’t do it and when he tried something this is what he tried and as he worked it out and talked us through it, it was the correct answer.”

In this example, Anna’s knowledge of the LT served as a framework for her to recognize students’ strategies and affirm their efforts. Additionally, her use of the LTBI model provided her with an opportunity to highlight Jasper’s strategy and use it as the basis for her class discussion. In this way, Anna’s efforts worked to build Jasper’s identity as a doer of mathematics.

**Question 3b: How do teachers use LTBI as a means to listen to and consider students out-of-school experiences and design instructional activities that incorporate**
elements from their homes and communities? This construct is critical to identity because it builds from Gutierrez’ (2007) idea of the curriculum being a mirror. When she articulated this dimension, one important component of identity was that students saw elements of their homes and cultural backgrounds in the curriculum. I did not observe any instances of teachers designing instruction that built upon their students’ out-of-school experiences. This construct was particularly challenging for teachers to enact because they focused heavily on selecting tasks that they thought all students would understand. While the LTBI instructional model did support the use of open tasks in the classroom, it was insufficient to push teachers to think about students’ individual out-of-school and cultural experiences as a basis for instruction. Therefore, I only report tensions for this construct.

Tensions. Caroline did not find the LT or LTBI useful when thinking about students’ out-of-school experiences or building connections between school mathematics and the broader world. Instead, she stated that any time students can relate to what they’re doing, “they’ll understand it more,” and that teachers should “just know to try and relate what you’re doing with what the kids have experienced.” For example, she found a part-part-whole task using an online resource we provided for PD participants. She modified the task to involve a grocery store and choosing apples because “I’m sure everyone’s been to the grocery store in my class. [and] sometimes you’re choosing fruit and so I would think that they could relate to: choosing some green apples and choosing some red apples.” She stated that beyond this example relating to the grocery store, she found it hard to think about what all kids do outside of school. She stated that it was much easier to select and create tasks based on what students do in school (e.g., lunch choices, games in PE).
Elsie made similar statements in this construct. While she felt comfortable allowing students to talk about their home experiences and bring things in from home, using their out-of-school experiences as the basis of mathematical tasks was much more difficult. As she began to incorporate more story problems into her instruction, she used students’ names in the problem as well as school-related concepts (e.g., their class field trip). When Elizabeth was asked about incorporating students’ out-of-school experiences, she also offered examples that were related to school day experiences including: “How many people are absent today?” or “How many people want Beef-a-roni for lunch?” and “How many days left until Delta’s birthday?”

Even though Caroline offered the grocery store example as evidence of drawing on students’ out-of-school experiences, I listed it as a tension because it does not meet the criteria for identity. In fact, as we discussed this example further, her underlying goal was access because the students would understand the action of putting apples together in a bag. Each of these tensions highlights an important discussion from the literature review, equity versus equality. Although teachers did try to build on students’ out-of-school experiences, they selected common or school-based experiences so all students would understand. They opted to select common experiences in favor of equality as opposed to incorporating cultural elements from students who are typically marginalized in curricula. In equitable classrooms, teachers purposefully learn about and include the elements from students’ homes and communities that are not represented in mainstream curriculum.

**Question 3c: How do teachers use LTs and LTBI to solicit and validate various algorithms and solutions to tasks?** The teachers’ uses of the LT and LTBI in relation to this
construct were closely aligned to their overall lesson goals and classroom structure. Because Elsie, Caroline, and Anna attempted to incorporate the LT as well as the five practices into their instruction, they were much more open to accepting and supporting a variety of strategies during their instruction. Elizabeth, on the other hand, was very teacher-focused during her instruction, which diminished these types of opportunities in her classroom. Three themes emerged in relation to this construct: a) supporting a variety of strategies, b) allowing students to modify the task, and c) allowing students to create representations. Two tensions arose in this construct: a) when to be open versus when to move, and b) no LT or LTBI.

**Supporting a variety of strategies.** When asked about validating students’ algorithms/strategies and encouraging them to engage in mathematical tasks according to their preferences, Elsie indicated that she is “supportive” of both of these ideas. She stated, “I’m definitely not an ‘It’s my way and my way is the only way and this is how you need to do it’ kind of person.” She went on to say that, “I think we all have different ways that we think about things” and “I never have a problem with letting kids try to do things a little bit differently.” For Elsie a key component of allowing students to approach tasks from a variety of ways was that she must first, “set up the task open-ended enough.”

Caroline also indicated that the change in her instructional model (from direct instruction in centers to the use of more open tasks) also changed her views in how students worked. She stated that since learning about the trajectory, “It is okay for me to just let them come up with their own solutions.” She further stated that she allowed students to share the strategies they created on their own and then she allowed students to use strategies that their peers had come up with.
In Anna’s final observation, she read the following problem out loud with the class:

“At the museum, our class used Legos to build robots. Pink group used 15 Legos. Blue group used 10 Legos and green group used 20 Legos. How many Legos did they use to build the robots?” After presenting students with the problem, she told students:

Now…it sounds easy…but I made these numbers a little bit bigger. So using counters may not be your best bet. You may have to think of a different strategy to use. You may have to think of some other ways that we’ve practiced counting numbers….in your head…base ten blocks. Think of some other ways that we’ve been practicing our adding to make it quicker. That’s all I’m going to say…I want you to go to your seat, turn over your paper, and get started.

As she monitored students while they were working, she observed the strategy or representation they chose and questioned them to understand how they were thinking about the problem. The purpose of Anna’s questions was to understand the strategy the student chose to use and ensure that they were using it correctly.

**Allowing students to modify the task.** Elsie presented a story problem to her class which read “On our trip to Rover Creek, George helped us roll a log. Under that log we saw ten animals. Some were worms and some were ants. How many of each could we have seen with George?” The following exchange happened after Elsie presented the problem:

Mary: I need one more animal…I already know what to do.

Elsie: You need one more animal? We’re just going to use two right now, but if you think of a different way and you can show me how you use that, I’d like to see it.
Mary: I would use 5, 4, and 1. Because I need one more animal to use for the one.

Elsie: So what I want you to do is to think about what that might be and if you can show me how you did it with math then I will definitely…because we definitely saw rollie pollies.

Elsie then modeled the task for the entire class using counters. She told students that the red counters could represent the ants and that the yellow side could represent the worms. After using one and nine as an example and writing \( 1 + 9 = 10 \) on the board, she called on Mary to revisit her approach to the problem. The next part of the conversation went as follows:

Elsie: Maya was thinking if she had an extra animal what would that look like, and so Mary said, Mary how many did you have, five ants or five worms?

Mary: It would be like five worms, and maybe four ants, and then you need one more animal to make ten.

(While Mary is talking, Elsie writes the following number sentence on the board \( 5 + 4 + \_ = 10 \))

Elsie: So Mary was saying if she had…pretend like…there’s my worms and there’s my ants (she pointed to the five and the four in the number sentence written above) and then we had one rollie pollie. Would that be right? Let’s see.

Mary: So 4+5 is 9 and then we needed one more animal to make ten.

Elsie: What do you think? Would that work? So that is what you guys are going to do. If you want to just use the ants and worms, you can. If you think that there
might be some more critters under there you can think about that to, but how many does it have to be total?

Class: Ten

Not only did Elsie allow Mary to modify the task to support her own mathematical needs, she clarified Mary’s thinking for the class so that this “modified task” was available for other students to use as well.

Allowing students to create representations. When teaching a lesson on greater than and less than, Caroline gave the students numbers and told them:

They had to decide which number was greater and then I told them they had to... record it on their paper in a way that other people would be able to understand what they were writing. So they came up with their own. We didn’t introduce the sign yet. This is like the first day. And so, they just... Each group came up with a different way so some kids did like 54... They would write 54 and then the next number might be 61. So like one group drew a 54 and then an arrow and then 61 or whatever the number. There was one group that did do the greater than sign and then one group did like 54 and then they’d put a down arrow, and then 61 and they’d put an up arrow next to it.

Caroline went on to say that, “I guess the trajectory helped me do that with the greater than, less than. I was like, ‘Is it okay for me to just let them come up with their own solutions?’ So I probably wouldn’t have done that before.”

Tensions. The two tensions observed for this construct are very different from each other. In the first example, the strength of the LT was also a tension caused by the LT and
LTBI model. Anna struggled with balancing validating students’ own algorithms with grade level mandates and pacing. In the second example, the LT and LTBI model did not aid Elizabeth in clarifying her instructional goals or disrupting her skill-specific views of mathematics. Therefore she required students to use the strategy she taught. This tension represents a non-use of the LT or LTBI.

*When to be open versus when to move.* An interesting tension that came up related to this construct was Anna’s desire to be completely open and allow students to solve problems with their own strategies and those that they created versus her need as a teacher to push them on to more sophisticated strategies. Anna indicated that their first choice of a strategy is not always the most sophisticated they can do. Anna stated:

In the beginning, I was trying to leave it completely open. “If you want counters, there’s counters up here. If you want your number line, all those things are there.” But as we’re getting…trying to push for different strategies and try to push for the next strategy, I’ve been being more limited on it. Like I had a student, I think during the lesson on camera, who was arguing with me because he wanted to use place value blocks, and I was like, “No, I want you to do it ‘cause on the test there are no place value blocks. I want you to do it without it.” So I try in the beginning to leave it open-ended and then I guess as we get closer to the end of the year I try to scale down and maybe work away. I think in 1st grade the go-to for everybody would be direct modeling it and I’m trying to steer a lot of my kids that I know don’t have to draw 25 and we’re getting 60 takeaway 40. Okay, well that’s probably not a great time to do
that. So, I think I was more open with it in the beginning but I’m trying to… Push away, ‘cause I feel like if I do give them that first choice, it will be to do that.

In the “supporting a variety of strategies” theme for this research question, I shared an example from Anna’s class related to the Lego problem. This same example highlights some of the tension Anna felt. As she was launching the problem, she indicated that counters might not be the best approach for this problem. Although she did allow students to use counters to solve the task, she really wanted them to move towards a more sophisticated strategy. This tension was two-fold for Anna. First, this lesson occurred at the end of the school year, and she wanted students to be prepared for 2nd grade. An additional part of this challenge was preparing students for quarterly testing under timed conditions.

*No LT or LTBI.* This particular example is a tension and reflects a matter of preference for the teacher. When I asked Elizabeth about how she validates students’ algorithms and solution strategies in our second interview, she replied:

For example, drawing a vertical line, if you start at the top and go to the bottom, each time you’re not wondering should I go up today? Should I go down? Your mental energy is focused and you don’t have to be confused. You can just go right into calculation, for example, we always do the adding on, the counting on, you punch in your number three and then four, five, six, seven, eight, nine, ten. How many fingers do you have that you had to add to three to get ten? And I know I have five here and two here so that’s seven. So and you know we don’t say okay you’re going to punch in and put your hands, put your fingers down, it’s just always the same way so that
they can branch out later and ad lib and be creative but I want them to have a solid base.

Her response was intended to illustrate that if students just learn to do things one way and use that representation each time, they will have less confusion. Later in the study I asked Elizabeth to talk again about validating students’ algorithms. Elizabeth indicated that during whole class instruction, “we’ll stick with the strategy I’m trying to get across and make sure that everyone across the board gets this general strategy. And if you have something interesting or complicated, let’s write it down and talk about it later.”

The first example really highlights Elizabeth’s difficulty specifically addressing this particular piece of the framework. While her underlying point was that young students need a prescribed way of counting and that “consistency of numbers is sometimes more comforting” than allowing students to get “crazy” with mathematics, it was difficult to tease this apart with the response provided. The second example highlights the teacher-centered approach that was often evident during Elizabeth’s lessons. Both of these instances are problematic for equity because students’ contributions can be dismissed in favor of Elizabeth’s preferences.

**Question 3d: How do teachers use the LTBI model to provide opportunities for students to build connections between the mathematics they learn and the broader world/society?** The focus of this construct is to help students use the mathematics they learn in school to understand aspects of society. Moving beyond school-related topics and activities to thinking of how mathematics relates to the broader world/society was difficult for teachers. While this construct is not as closely tied to the LT itself as some other constructs, I did conjecture that teachers may recognize skills and topics from the LT and
then think about how what students are learning is useful outside of school once presented with the construct. While this connection was not made, the tension was not with the LT itself. The lack of evidence for this construct and teachers’ difficulty responding to it indicates that this construct needs to be addressed specifically for teachers to reach this goal.

**Tensions.** This particular component did cause tension for teachers, as there was the concern that making connections to the broader world is either not appropriate for young children or very difficult to do in mathematics because of the mathematical content appropriate for this age group. When asked about this construct, Elsie discussed a project that she participated in with her students. The class worked with the Pediatric Bone Marrow Unit at a local hospital to make pillows for incoming patients. She described how this project connected to this construct of the identity dimension by stating:

> And so talking about how many pillows would they need and how many are there of us and so would we each just make one pillow or do we need to make more than one pillow? So I don’t know if specifically that I said “Oh. Well they have 36 kids and we have 18, so we all make two.”

The second part of this statement reflects the fact that Elsie was unsure “if this ties in or not” as she was trying to make sense about this category in relation to kindergarten mathematics. Because she thought, “It’s kindergarten, it’s a little bit more limited in regards to what I maybe would do with kids who had more ability to work with higher numbers.” I think Elsie found it difficult to make a strong claim for this construct.
Similarly, when Elizabeth was asked about building connections between school mathematics and the broader world, Elizabeth provided two classroom examples (weather and number of books in the library), but went on to say:

I don’t think they have the ability to really grasp say, say I think maybe fourth, fifth grade demography and geography. That might be something a little bit more of a mathematical connection but for their level and I’ll introduce grander ideas or bigger numbers and they kind of get it but I think my kids who are challenged with second language and also just the beginning of number sense, they can’t get it.

Later in the study, Elizabeth revisited her earlier belief that numbers tell a story and that she wants students to be “comfortable about talking about the numbers in their life.” Elizabeth offered two examples to illustrate including how the students’ shoe sizes changed during the year and how frogs can jump four times the length of their body. Each of these examples offered by Elizabeth relate to topics they discuss in school. Although she mentioned across the duration of the study that “numbers can be used to tell a story,” these stories were related to school-based topics and not broader societal connections.

In my final interview with Anna, she immediately went back to this construct and stated:

I remember thinking a lot about trying to make the connection for them like to real world and I know that I really don’t ever do that and I’m thinking more of the math numbers and that part of it and not really thinking of how to make it real world for them. So I haven’t been able to extend it a ton to like outside world but I’ve tried to at least make some of the problems relate to things that we’re doing or that we’re
working on or that are going on in science. I’ve kind of tried to make them relate.

Today we were doing balancing and we talked about roller coasters ‘cause we’ve been writing about roller coasters, so how we need measurement and balance and force of motion in roller coasters. So, kind of trying to touch on that piece more, which I usually don’t really ever do, to try to make it more relevant for them.

This idea of relevance to students was very important to Anna as she mentioned it a few times during the initial interview about the framework and again during our final interview. She also speaks to her “progression” on this from not making math relevant at all, to relating it to ideas they are doing in school. She stated that she hasn’t been very successful with extending it to connections outside of school.

**Question 3e: How do teachers use LTBI to encourage students to engage in mathematical tasks according to their preferences and participate in mathematical discourse in ways that are comfortable for them?** The primary idea in this construct is that for teachers to support the development of students’ identities, it is important to allow students some agency in how they participate in mathematics classrooms. To this effect, I observed two themes related to this construct: a) allowing students to choose their own manipulative or tool, and b) allowing students to share in their own ways.

*Allowing students to choose their own manipulative or tool.* In general, one way teachers allowed students to engage was to choose whether or not they wanted to use a manipulative, what type of manipulative to use, if they wanted to draw a picture, or use number facts. During a reflection of Elsie’s second observation, she stated that she allowed her students to use counters, their fingers, or drawings to solve a task. She saw her role as a
teacher to provide multiple options for students, but she ultimately allowed students to engage according to their choice. This theme closely related to the construct about validating students’ algorithms and strategies, and after analyzing the data in this study, it seems that allowing students to have their choice of tool or manipulative was a pre-cursor to validating their work.

**Allowing students to share in their own ways.** Anna stated that she doesn’t force students to use certain vocabulary as they are sharing their work. Her primary goal is to “just encourage some type of sharing; maybe you could lean over to a friend and say I have no idea how to do this can you help me do this? So I’m hoping that they’re all comfortable with sharing and in any way that’s easy for them, a picture, words, or asking a friend.” In Anna’s classroom, this meant that she did not “correct” students as they were sharing. Rather, she allowed them to share and asked students question to help them clarify their thinking when necessary. This example is important for equity because the focus is on validating the students’ mathematical contributions as opposed to focusing on the language or dialect used when sharing.

**Summary of Identity**

It was apparent in this dimension that while the LT and LTBI can be used in supporting equitable instructional practices, it lends itself more to certain constructs (3a, 3c, and 3e) than others (3b and 3d). When teachers attended to the thinking of individual students in relation to the bigger picture of the LT, they were able to validate their identity as a doer of mathematics, support the strategies they used when problem solving, and allow them to share mathematics using their own language. The two constructs that were not
supported by evidence were less directly related to the LT and would need to be addressed specifically for teachers to incorporate students’ cultural experiences into their tasks and use mathematics to make sense of the world. Although teachers found these two constructs important and they indicated that these ideas were important, the LT and LTBI model did not directly support their efforts to incorporate this into instruction.

**Question 4: Power**

For the final research question, I looked for evidence of the ways in which teachers worked to empower their students. This particular dimension had the least evidence of the four dimensions. As with the identity dimension, the constructs that were more tightly aligned to ideas from the LT had the most evidence. The following paragraphs offer evidence to address the fourth research question: *In what ways do teachers use LTs and LTBI to empower students in their classrooms?*

**Question 4a: How do teachers use LTs and LTBI to give all students voice in the classroom?** I did not observe any uses or tensions related to this construct. The hallmark of this construct is not that teachers allow all students to speak during the lesson, but that students have a voice to make decisions related to classroom activities. Although Elsie did indicate that she has previously allowed students to vote on whether or not they would go outside for recess or have indoor recess, this was mentioned in relation to the “making math relevant” construct, and there was not clear evidence about how she used the LT with this task.

**Question 4b: How do teachers use LTs and LTBI to position students as experts in the classroom?** This construct relates to teachers’ publicly assigning students competence
during instruction. By positioning students as experts, teachers share their “power” in the classroom subsequently empowering students. Four themes emerged for this construct a) assigning students’ mathematical competence, b) referring students, c) building connections, and d) taking ownership of mathematical thinking.

Assigning mathematical competence. Anna used her knowledge of the LT to position students as experts by publicly acknowledging what they did and encouraging them to share their work with a classmate or the entire class. She said:

When I’m looking at the LT, I’m picking out, like, ‘There’s a set of things that I want you to be able to do,’ and I think that I’m always looking for, like, ‘Okay, you know, Omar did something really cool with the number line; I want Omar to share this and then, you know, somebody did a really cool thing with counting on and I wanna share that so I think giving them that, like, ‘I saw you do this job really well, I saw you.’ So kind of assigning them based on what I know I’m looking for in the trajectory so that I’m kind of highlighting it, like, ‘Okay look at this idea, so here’s an idea that you might not know about but I want you to know about it,’ or ‘Here’s an idea that you haven’t tried yet but I want you to possibly think about trying it.’ So I do that most of the time during like that share time is the time to really highlight like a job that somebody was doing really well.

Additionally, she told her students that sharing was a big deal and that it gave them an opportunity to “be the teacher.” To ensure that this was a reciprocal relationship, Anna stated that it was important for the students to teach each other and that since “not everybody is going to get a chance to share at the end, this is your time where everybody gets to share
and have an important job.” This last statement is very important for equitable instruction in that Anna realized that for all students to get to be “the teacher,” it was important to include sharing time while students were working as well as at the end of the lesson. This structure maximized the number of opportunities students had to share.

**Referring students.** Another way Anna positioned students as experts was by using them as a “reference point” for mathematical ideas and strategies. For example, Omar was the “number line expert” in her class. When we discussed the idea of positioning students as experts, Anna stated:

> In math, I think Omar was our first one to kind of introduce some of our number line strategies, so he’s our number line expert. And when we’re talking about number lines, a lot of the time we’re, like, if nobody else has used the number line or nobody else had a strategy with it, we can go, “Oh, Omar’s always using his number line.” So he’s kind of our number line expert that we go to when we’re using the number line as a strategy, and kind of use it now like, “All right. Remember how Oscar was using it? I want everybody to try using a number line, see if that will help you.”

Anna used her knowledge of the LT and strategies to recognize that Omar had created a valid strategy and was skilled at using it. She then assigned him the title of “number line expert” and directed other students to him when they had number line-related questions. This example highlights an equitable practice because despite Anna’s labeling Omar as a perceptual student, she still recognized his mathematical competence and empowered him based on what he demonstrated.
**Building connections.** Another way students were positioned as experts was when mathematical connections were made. When one student mentioned something similar to what another student shared, Elsie encouraged the students to make connections between the similarities. She stated, “If somebody does something that you did, that means you gave them a good idea. They’re standing on your shoulders to help themselves learn. And so that’s a good thing.” Not only did this idea position students as experts, it also built a community in the classroom so that students did not think of someone as “stealing their idea;” rather, they used their thinking to build their own understanding.

Another example of this is when Elsie asked Isa, a student working at the perceptual level, to share how she filled in seven on the tens frame. Isa was a very quiet student and did not frequently volunteer to share. After assisting Isa with sharing her solution with the class, Elsie asked the class, “If you did like Isa, show me with a connection.” She then stated, “There were a lot of people that did it like you did, Isa.” These statements are an indication that Elsie validated Isa’s strategy and publicly assigned her mathematical competence.

**Taking ownership of mathematical thinking.** Caroline’s idea of positioning students as experts tied to the idea of them taking ownership of their ideas and being content if they solved a problem using a different strategy. She spoke to this element of the dimension using an example from a lesson on greater than and less than. Although this lesson is not one I observed, Caroline used this example when discussing the framework. She stated that she gave students two numbers and that she wanted them to create a symbol or sign to let someone know which number was the larger number. She did not formally introduce the less than or greater than symbol to students. One group, however, knew about the symbol and
used it. As other groups presented, they discarded their original ideas in favor of using the symbol. Caroline told those students:

Well, that’s not what you did on your paper. Your idea is good too, so share how you. . . .” I was like, “You can leave that up on the board but share also what you guys were working on because it’s a good idea as well.” I guess I want everyone to understand that Tyson had this idea but your idea is good too, so you can focus on your way too.

**Tensions.** Both of the tensions I observed in relation to this construct came during a discussion with Elizabeth. Because she did not generally have positive views about her “lower” students, she was unable to position them as experts. Both of these themes are listed as tensions; although she recognized a counting strategy from one of her “lower” students, she did not use her knowledge of the LT to think about how that counting strategy may be accessible for other students in her class. In the second example, the LT was not enough to disrupt Elizabeth’s belief that only the “higher” students can be positioned as experts.

**Deficit orientation.** When I began my first discussion with Elizabeth about power, I started by re-voicing an example that Elizabeth had mentioned earlier in our dialogue. She was talking about some students in her class using a counting method where they tap their chin as they count. So as I explained to her what it meant to position a student as an expert, I used this as an example of the ways in which students could contribute to the classroom dialogue. Elizabeth indicated that she would allow students to share such a method, but that “usually they can’t express it because they don’t have. . . . They don’t know what they’re doing. They’ve just seen adults do it and they think that’s counting.” I asked Elizabeth to
clarify if they were counting and if their counting was associated with the gestures they were doing. Her response was:

No, it was just sort of like the touching their chin with their finger and then their knuckle to their chin, you know it was. . . I assume some type of counting method that is used in that country, but I usually would ask them about it and they could continue doing that if it helped them. . . . But I wouldn’t encourage the rest of the class to start doing it too. I just said this is the way we’re going to do it here and if that’s the way you do it, you can do that, that’s fine.

While Elizabeth’s responses here acknowledge that this student does have ownership of this idea, they suggest that she doesn’t see this as something that could be useful for all students. Her final statement serves as a way to cast these students as “outsiders.”

Only the “higher” students are experts. Later in the study, I asked Elizabeth to think again about positioning students as experts and how she works to do that. Although Elizabeth did position students as experts during the course of the study, these students were all members of her “higher” group. Elizabeth said:

We just had a center – a subtraction center – that was manned by Paul, Natalie and Jared. Natalie had her first grade workbooks and she asked me if she could have a center during math time that if people were done they would be able to visit the math center and practice subtraction because she knew how to do it and she wanted to teach them how to do it.

The three students placed in charge of this center were all part of her “higher group.” After this comment, I asked Elizabeth:
Interviewer: Okay. Can you think of any other examples? And it could be related to... ’cause positioning as experts can be related to something in school or even something outside of school that the students may have some type of expertise in.

Elizabeth: I think when we were studying weather and the degrees, we all had our thermometers and checking the temperature. Knowing that freezing was below 30 or... and different ways of measuring centigrade and Fahrenheit, Paul was right on it and he checked the weather on his mother’s iPhone and every morning he would come in and tell us whether it was above 65, and so, whether we needed a jacket for recess. We had a snowman on the door and it had a speech bubble that we would change every now and then, and so he said, “I need to change it so that it’s saying the temperature is too warm.” So he wrote...we erased it and he wrote, “The temperature is too warm for me. It is 65 degrees” or something like that. His keeping track of things, using a thermometer or a scale, he just... We all counted on him.

Even when pushed to think about other examples, Elizabeth could only think of examples related to her “top students.”

**Question 4c: How do teachers use LTs and LTBI to provide students with opportunities to solve problems that are relevant to them?** This construct is similar to two of the constructs in the identity dimension of this framework. In the identity dimension, the
goal is for teachers is two-fold. First, teachers should learn about their students and design tasks related to this personal knowledge of their students. Additionally, teachers should help students make connections between topics they do in school and society as a way to help students identify why school mathematics is useful for them. In this dimension, the idea is that teachers are knowledgeable about topics and issues related to their students’ lives, and they could use mathematics as a tool to address these issues/concerns (whether at the school level or on a larger scale). The purpose of this construct is that mathematics can be used as a tool to negotiate real-world issues and cause change. While students’ age range and mathematical skill level for the early grades does not lend itself to the level of depth you can achieve with this construct at the high school level, this is an important component for equity, and thus I looked for evidence of it. There was no evidence of any uses or tensions for this construct.

**Question 4d: How do teachers use LTBI to provide opportunities for all students to present, justify, and defend their mathematical ideas?** In general, this construct was something very new for teachers to consider, especially since they work with such young students. The language of “present, justify, and defend” mathematical ideas seemed strong for teachers, and when they referred to it, they talked about whole-class sharing. One theme emerged in relation to this construct: using the LT to determine who shares. This theme manifested in slightly different ways across the different teachers. Both of the tensions present for this construct resulted from deficit orientations.

**Using the LT to determine who shares.** The LT supported teachers in relation to this construct in two ways. First, it was useful for teachers to identify different strategies that they
wanted to highlight during their lesson. Second, it was useful as a tool to help teachers ensure that students at different profile levels all had opportunities to share.

*Different strategies.* Anna used the learning trajectory to anticipate the ways in which students would approach problems in her classroom and how these anticipations relate to how she structures whole-class sharing. Specifically, during her first lesson, Anna stated:

I really wanted to share the different strategies that could have been used like I wanted to share a student who direct models with manipulatives. I wanted to share a student who used the number line to count on. I wanted to share the student who mentally held the number and then counted on.

Anna was able to anticipate students’ strategies as well as misconceptions and think about what mathematical content she wanted to make available during her lesson.

Caroline stated that sharing in math is something new that she is trying to do. She stated that she keeps notes to ensure that everyone has shared in a week’s time, but that she will allow someone to share more than once if “they’ve done something that the class needs to see.” For Caroline, it was a balancing act to make sure all students are able to share given a week’s time period and that she selected students to share certain strategies she wanted to introduce to the class.

*Selecting a variety of students.* When first introduced to the framework, Elsie indicated that it was important for students at various levels to share not only for themselves, but also to see their similar peers share. Elsie identified how the profiles could be useful for students. First, when a student on the “lower end” makes a connection, particularly if they don’t typically volunteer to share, Elsie stated that this student needs to “verbalize it and
share it.” In contrast, Elsie had a number of students who always wanted to share and tried to dominate the conversation. Elsie attempted to balance that and stated, “There are lots of kids that are in their level that I would like to see be able to share too.” Because Elsie was able to use the profiles to group students, it helped her think about how to balance who shared in the classroom, and she found that the profiles were useful as a tool to include all students in classroom discussions. Specifically, she stated, “I try to be aware of different levels where the kids are so it’s not always just the kids who are the counting on kids who are sharing, but that it’s someone from each of the different levels that are sharing.” She summarized her ideas by saying, “I’d like for you to share and you to share and you to share because we’re going to get some different points of view, different places on the trajectory.” A key component of this construct for Elsie was that all students have contributions to make and while they may be different, they are all worthwhile. She stated, “Like, perceptual child versus counting on child. . . . What they’re able to present is important. And so it might look different…but they both have important thinking that they’re doing.” She went on to say that she encouraged all students to do what they know how to do and share it with others.

Tensions. Both of the tensions related to this construct reflect a deficit-orientation towards students. Although Elizabeth did encourage some of her students to present, there was an element of concern about her students with “less confidence” or the “group that is not quite as confident in their counting.” Elizabeth stated:

We will generally have a basic concept presentation, whether it be cardinality, counting in sequence, combinations to 10. Anything like that we’ll go over a basic idea concept and then an operation to show that concept and I will invite them to
show how they would approach showing the concept. For example, on the tens frame
I wanted to see how they looked at that tens frame and thought. . . did they see a row
of 5 and a row of 5 or did they count by twos and then could they explain why they
put their markers there? And I said, “Why are you doing it that way?” and they would
say, “Because it’s a pattern,” or “I like the way it looks.” I said, “Is it easier to count
that way to see the numbers or what do you think?” and some of them would just…it
was just because it was pretty or something like that…I allowed them to present their
idea, something like…or the way that they would count out that way, but if I feel that
it’s confusing to a group that is not quite as confident in their counting that they don’t
have to use the shape…the physical frame to divide into groups of five, I would say,
“That’s great. That helps you. Let’s see if someone has an easier way of doing it or
someone feels more comfortable putting all the frogs on one line, all the raindrops on
another,” and invite students who are not quite as confident to come up and do it their
way and see if it helps to use a different way of presenting things or using
manipulatives.

I find this tension to be interesting because the LT could have played a role in
resolving it. When sharing, some students will have more in-depth, intricate, or
mathematically sophisticated solution strategies. In our work over the course of the year, we
discussed this idea with teachers and talked about how teachers can leverage strategies from
all areas of the LT as a basis for providing powerful instruction for all students in their
classrooms. The tension Elizabeth experienced seems to be partly related to not drawing
upon the LT as a way to think about building connections between students’ strategies and
representations. Another tension is not wanting the students who are “less confident” to struggle.

Caroline also experienced tension in relation to encouraging all students to share their ideas. While her tension was not identical to Elizabeth’s, they both had elements of a deficit orientation to some students in their class. As I mentioned in Caroline’s profile and earlier in this cross-case analysis, sharing in mathematics was new for Caroline and something she expressed difficulty with throughout the study. When asked about sharing in her classroom, she indicated that:

That’s so difficult. I’m still. . . .You know, it’s hard to. . . . I have a group of kids who just. . . .they’re self-controlled, and they’re able to sit and listen to the others. Like, I still feel like they’re not really getting anything from the other kids. That’s just something I just need to work better on. Even when I thought it went okay with the kids sharing with the greater than and less than, I just don’t think that they were getting anything out of it, just because they do not have control and they’re not like even actively listening to what is being said.

**Question 4e: How do teachers use LTs to see all students as sources of mathematical knowledge?** The idea of this final construct is that teachers can first recognize what their students know mathematically and then help their students see themselves as doers of mathematics. While I did not observe specific evidence or tensions related to this theme, it did become apparent that this theme was closely related to other constructs in the framework. Specifically, when teachers assigned mathematical competence to their students and encouraged them to share, this was a way to help students see themselves as mathematical
beings. Additionally, when teachers recognize what students do mathematically and encourage them to share that with the class, this is also an example of helping students see themselves as sources of mathematics.

**Summary of Power**

The findings from this dimension are similar to those in the identity dimension in that some constructs were much more aligned to LT and LTBI than others. Three of the five constructs in this dimension were not observed across all teachers. When teachers used the content of the LT to determine what evidence students showed and where they were working in relation to the trajectory, they used this knowledge to position students as experts and share their expertise with the class. The tensions present in this dimension all shared a common theme: deficit orientation. This suggests that although the LT has the potential to empower students, it did not disrupt teachers’ deficit orientations held towards some of their students.

**Chapter Summary**

In this chapter, I presented profiles of each individual case to provide insight into who they were, how they taught mathematics, and how they structured their classrooms. I then looked across each of the four cases to answer the research questions. The purpose of the detailed case write ups was to provide a backdrop of who each teacher was such that the reader could situate some of the responses to in the cross-case analysis in relation to the teacher. Findings across all cases indicated that while the LT can be a powerful tool for providing access, achievement, identity, and power in elementary classrooms, it can also be dangerous when not situated within specific constructs of equity. In the next chapter, I reflect
across the findings of both the dominant and critical axis and discuss important implications these reflections have for mathematics education.
CHAPTER FIVE

The purpose of this study was to fill a critical gap in the field’s knowledge of how LTs might support equitable instruction. Toward that goal, I developed, empirically tested, and refined a framework outlining the ways in which LTs were used to teach equitably. In the previous chapter, I provided a description of each case and answered each of my research questions using cross-case analysis. In this chapter, I return to the LTBI-Equity Framework.

I begin this chapter by summarizing my findings in relation to each dimension in the initial framework. I include a discussion of the uses and tensions in each dimension and frame the uses and tensions in light of other research. Based on the findings from the analysis, I present a revised LTBI-Equity framework that captures evidence from this study and describes how teachers used LTs and LTBI in each dimension. Next, I present a set of conjectures for why some teachers in this study were able to use LTs and LTBI to teach equitably and why others were not. I conclude this chapter by discussing implications of this work, presenting the limitations of the study, and framing an agenda for future research related to equity and LTs.

Summary of Findings

The purpose of this study was to develop and empirically test a framework that examined the potential of LTs to support equitable instructional practices. To this end, an initial framework was designed based on the literature from equity and mathematics education; propositions/constructs were articulated regarding how teachers may use LTs and LTBI to implement these practices. While empirically testing the framework, it became apparent that it was necessary to not only understand the ways in which teachers used LTs to
teach equitably, but also to comprehend the tensions they faced as well as instances of inequity. Moreover, because Critical Race Theory was used in the theoretical perspective that guided this study, it was important to note instances when tensions occurred to examine if these tensions affected particular groups of students. In this section, I summarize the findings related to each dimension of the framework and discuss the uses and tensions that occurred in each. I situate my findings in relation to existing literature from equity, culturally relevant/responsive pedagogy, and cultural competence. Each of these bodies of work is closely related to the others, and they often represent similar core principles; therefore, I used research from these domains interchangeably. I also discuss instances where no evidence was present for a construct as well as cases when a new construct manifested and was added to the framework.

Access

Each of the five constructs in this dimension was supported by the data. An important trend in this dimension was that teachers used the LT and LTBI to make decisions that had the potential to lead to equitable instruction when selecting their tasks, anticipating their students’ needs and likely responses, and planning for classroom discussion. Thus, in relation to access, teachers primarily used LTs and LTBI to implement equitable instruction during their lesson planning. Teachers used their knowledge of LTs when creating tasks, ensuring that they were open, and providing multiple access points for students at different LT levels. Teachers also selected tasks that modeled real-life situations such that students could draw on their experiences to model the action/operation (e.g., splitting up into teams to play monkey soccer). The LT was also used during the planning phase as teachers considered how to build
a more robust understanding of mathematics by drawing upon multiple trajectories in a task and connecting mathematical ideas during classroom discussions. Finally, teachers utilized their knowledge of the LT to increase and maintain mathematical rigor during their instruction.

My findings of teachers’ uses of LT and LTBI to teach equitably are supported by other studies of equity in classrooms. Previous research has articulated many critical components for equitable instruction. Here I present results from a few studies that focused specifically on equitable instruction in mathematics classrooms. Specifically, these recommendations are necessary pre-cursors to equitable instruction and can be addressed during teachers’ planning. The following recommendations were reported:

- Tasks must be cognitively demanding and high quality (McKenzie, 2009; Perry, 2013);
- Tasks must incorporate multiple representations and require complex mathematical analysis (TEACH MATH, 2012);
- Tasks should be differentiated to meet the needs of individual learners and build upon students’ unique strengths (McKenzie, 2009); and
- Teachers must have both a “breadth” and “depth” approach to teaching mathematics such that they know a range of topics and strategies available for students that can be used to build a robust understanding (Lee, 2007, p. 112).

The findings from this study add specificity to some of this previous work that was conducted in other reform-oriented classrooms. First, the individual LTs (see the columns in Table 5) helped teachers develop a depth of knowledge about students’ mathematics in one
content strand. The LT table as a whole helped teachers develop a breadth of understanding about a range of topics in elementary mathematics. Teachers used this knowledge to differentiate based on mathematical evidence their students demonstrated. Additionally, teachers used their knowledge of the LT and developing open tasks from the LTBI model to create high quality problems.

The tensions that arose in this dimension fell into three categories: a) non-use of the LT, b) classroom management concerns, and c) deficit perspectives. In the case of non-use of the LT, teachers were unable to unpack the specific difficulties the students were having with the task, to use the LT as a referent to understand why students had difficulty, and to use that knowledge to modify the task. The classroom management tensions could be resolved with an explicit focus on developing norms to support LTBI classrooms. Although norms are not directly linked to the LT, they are critical for classrooms that utilize open tasks, group work, and whole class sharing which are elements of the LTBI model. The tensions that resulted from teachers’ deficit orientations highlighted that while teachers found LTs useful to provide access to rigorous mathematics, rigor may not be appropriate for all students.

A final reflection on this theme is that the empirical evidence from this study warranted a revision in the second construct: teachers use LTs and LTBI to identify and use up-to-date research based materials and technology. Initially, I conjectured that teachers would use the LT to examine existing curricula. Since this study occurred during the first year of implementation of the Common Core Standards, teachers did not have a curriculum, making it difficult to observe this construct. Although I did not observe teachers using the LT as a tool to examine existing curricula, they did use it as a tool to create their own. In most of
my observations, teachers created their own story problems and tasks by visiting websites and using other resources provided in the LTBI PD. Therefore, I revised this construct to include teachers identifying or creating research-based materials as well as modifying their existing materials to align with the content of LTs. There was no evidence to indicate that the LT or LTBI encouraged the use of technology during instruction; therefore, this portion of the construct was removed.

**Achievement**

As with access, much of the evidence from this dimension occurred as teachers discussed how they selected tasks and planned instruction. Even in cases when teachers used LTs to understand the results of their assessment, they used these results during planning to set the next “round” of goals for students and individualize instruction for them. These findings are supported in the literature as other scholars who have conducted work with LTs in classrooms found that LTs can be used to help teachers set specific goals and describe students’ work in relation to the LT (Edgington, 2012; Wilson, 2009).

The tensions in this construct were twofold: district mandates and, again, deficit perspectives. Although formative assessment was a part of the LTBI PD, teachers primarily used their district-wide assessments as evidence of what their students did and did not know. Even when teachers indicated that they gained evidence by listening to students and collecting samples of their student work, it was not clear whether the LT or formative assessment model was used to determine how to assess students. This is an important tension for equity for two reasons. First, if teachers limit the number of times they collect evidence about their students, they limit the number of opportunities they can use the LT to customize
instruction for these students to promote growth. This tension is also important because
students should be able to display their knowledge in a variety of ways and not be limited to
formal assessments.

One construct not initially included in this dimension that should be added is:

*Teachers use LTs and LTBI to produce equitable outcomes in student growth and
achievement.* While the first two constructs in this dimension are pre-cursors to growth, and
the assessment construct provides a construct to measure growth, I did not initially include a
construct directly related to measurable outcomes. In particular, the first construct in this
dimension “teachers use LTs and LTBI to set high, yet appropriate, academic standards for
all students” refers to teachers setting short-term goals for individual students. These goals
may relate to a specific lesson or unit, and relate to whether teachers use LTs and LTBI to
enact achievement benchmarks for students as instruction unfolds. The new construct, on the
other hand, refers to the long-term growth of collective groups of students. This construct
was necessary to capture measurable outcomes of students’ growth over time and to examine
patterns in how short-term goals for individual students may or may not connect to long-term
growth across groups of students.

As I analyzed student portraits and asked teachers to think about their students’
growth throughout the study, it was apparent that some students progressed and others did
not. Even in cases when all students moved “up a level” in the LT, students generally
remained in the “same groups” meaning that the “lowest students” remained in the “lower
levels” throughout the study. In Elizabeth’s classroom, the students who she ranked the
“lowest” at the start of the study did not progress and remained in the “lowest” group at the
conclusion of the study. Her “higher” students remained the higher students throughout the study, and some of her “higher” students progressed two levels in the LT. This finding is not surprising, as the data in chapter four indicated that Elizabeth provided more opportunities for her “higher” students to participate in mathematics and take leadership roles during mathematical discussions than she did her other students. In essence, even though all students progressed in the LT as the semester unfolded, the “gap” between the students in this classroom widened over the course of the semester.

There was one exception to this finding. At the start of the study, Elsie grouped her students according to four profile levels: perceptual child (three students), early counting child (seven students), emerging counting on child (one student), and early counting moving to counting on (five students). In our next discussion, 15 out of 16 students had moved up a level and some students moved two levels. When I asked Elsie about one of her students whom she initially listed as a perceptual child who had progressed to the counting on level (a gain of two levels), she stated:

I met with mom and talked about this is what I still keep consistently seeing, and I sent home flash cards with numbers and gave her activities for counting and things like that. So mom did a lot of stuff with him at home too and I think both of us working together. . . . Yeah, he’s probably come one of the longest ways.

This type of response is a hallmark of equitable instruction in that Elsie recognized that for this student to grow, he needed additional opportunities to engage in mathematics. She used her knowledge of the LT to determine which skills this student should work on, and then she
partnered with his mother to maximize the number of opportunities this student had to be successful.

In Gutierrez’ original framework, she stated that access is a precursor to achievement, and that when students are provided with equitable access/opportunities, equitable outcomes are necessary (Gutierrez, 2007). Additionally, equitable instruction should level the playing field over time such that there are no longer trends or patterns in students’ achievement based on their race, socio-economic status, or fluency with the dominant language (Gutierrez, 2007). Examining whether the LT helps level the playing field is an important facet of equitable LTBI. Therefore, the original achievement dimension needed to be modified to capture these types of trends and provide a clear way to link issues of access to tangible student outcomes.

Identity

In contrast to the first two dimensions, much of the evidence supporting identity was apparent during instruction, particularly as teachers launched tasks and monitored students. While LTs and LTBI did not support teachers in all of the constructs initially proposed in this dimension, it was present in three of the five. Recall that the impetus behind this construct is to develop students that will persist in mathematics in spite of challenges. Having a strong mathematical identity indicates that students see themselves as doers of mathematics. Teachers embraced the idea of students developing a mathematical identity, and in doing so worked to build the confidence of their students and encouraged them to take risks. The LT served as a tool to mathematically determine “appropriate” challenges for students, and teachers worked to encourage students to draw on what they knew to be successful. This idea
of “appropriate challenges” ties to Vygotsky’s (1978) ZPD as students should be provided with *appropriate* levels of challenge and that when teachers support them in those efforts students feel *safe* to try other skills.

The tensions in this dimension were non-use of the LT and “inherent” LT issues. The non-use of the LT resulted from focusing too much on the development of a single skill and being resistant to students’ using their own approaches to tasks. This is problematic for equity because when students are allowed to explore and create their own strategies, they make fewer mistakes, gain more robust understandings of mathematics, and experience increased confidence as a result of their “invention” (Van de Walle, Karp & Bay-Williams, 2013). This tension was surprising, as it is one that is directly addressed throughout the PD; the first step of the LTBI model is selecting open tasks that allow the use of multiple strategies and representations.

The second tension here is particularly challenging when considering equitable instruction. On one hand, the LTBI PD encouraged teachers to allow students to select and use their own strategies/algorithms—an important aspect of identity. As teachers learned about LTs, they learned that students’ understandings and strategies will become more sophisticated over time. Whereas teachers embraced this idea early on, the end of the school year brought additional challenges and frustrations, and teachers felt pressure to force students to use more sophisticated strategies, especially in light of standardized testing and expectations for the next grade level. I labeled this an “inherent” tension because teachers grappled with the duality of affirming and supporting students where they are as well as meeting grade level mandates and expectations.
Two of the constructs here (3b and 3d) were not supported by evidence in this study. Teachers’ difficulties designing tasks that are relevant to students’ cultural backgrounds as well as tasks that make connections between in-school and out-of-school mathematics is documented in the literature (Gay, 2000; McCulloch & Marshall, 2011). In 2011, McCulloch and Marshall reported on a three-year study in which they focused on helping teachers understand the importance of drawing on students’ out-of-school experiences when teaching mathematics. Findings from their work indicated that although teachers did make out-of-school connections, they were infrequent, shallow, and did “not effectively capitalize on the children’s out-of-school prior knowledge and experiences in mathematically meaningful ways” (p. 60). Their subsequent review of teachers’ reflections indicated that although teachers were aware that they should make these types of connections, they struggled to do so.

This same tension that McCulloch and Marshall (2011) reported was present in this study. My findings also indicated that teachers had difficulty developing tasks that drew upon students’ out-of-school experiences. Furthermore, my findings suggest that as teachers attempted to generate these kinds of tasks, they followed a “progression” of 1) using generic tasks, 2) adding students’ names to existing story problems, to 3) using “out-of-classroom” activities (e.g., games from PE class, field trips). The fourth level of this progression may be using students’ out-of-school experiences, but this was not observed in the study. Additionally, in instances when teachers did try to use students’ out-of-school experiences, those with strong deficit perspectives only used the experiences of their “high” students.
One explanation that McCulloch and Marshall (2011) offered for teachers’ difficulty with task development was that in previous studies of teachers who successfully made these connections, they engaged in curriculum development activities that explicitly addressed how to design these types of tasks. This explanation is also applicable to this work. In our sessions, we asked teachers to select, modify, adapt, and eventually create tasks that related to the LT. A part of our instructional model included a discussion of open tasks and making mathematical connections, but the LTBI PD did not extend this to selecting tasks that draw on the lived experiences of students or making connections that extend outside of the classroom. Because teachers were not asked to explicitly think about students’ culture in the PD, it is not surprising that they did not incorporate it into their tasks. Even when presented with the LTBI Equity framework, teachers’ knowledge of the LT and LTBI did not facilitate these types of connections.

Because this framework is intended to be a direct reflection of the ways in which LTs can be used to support equity, these two constructs were removed from the framework. This does not mean that they are not important; rather, it means they are not directly addressed by LTs or LTBI. In the “progression” I noted earlier in this section, teachers were working towards creating more inclusive tasks. If attention to students’ culture was a part of the LTBI PD, I do think teachers could draw on their knowledge of open tasks, topics from the LT, and students’ culture to create tasks that support students’ identity.

**Power**

Most of the evidence for this dimension was apparent during whole-class discussions, particularly when teachers selected students to share. Only two of the five
constructs in this dimension were supported with evidence. While the LT was useful for teachers’ positioning students as experts in relation to mathematics content, it was not useful in thinking about positioning students in out-of-school contexts. Additionally, the LT was useful for helping teachers recognize that students at different levels of the LT have mathematical knowledge to share and thus should be encouraged to participate in class discussions.

These findings do support equitable instruction and are aligned with findings from similar work. For example, in the Complex Instruction program, researchers found that in order to achieve equity in classrooms, teachers had to disrupt notions of who was “smart” or who was “dumb” to ensure that all students could participate actively and equally (Cohen, 1998). To do so, teachers often positioned students who were not viewed as academically strong as experts to shift their “group status” and help their classmates recognize that all students had something to contribute. In Chapter Four, I presented an example in which Anna positioned Omar as an expert in relation to the number line. Although Anna initially indicated that Omar was working at the perceptual level, she recognized that he used the number line flexibly with various problem types, and by announcing this to the class, she affirmed that Omar was proficient.

Each of the tensions in this dimension resulted from deficit orientations. Teachers with deficit orientations did not believe that all students had expertise; therefore, they focused on what students did not know or did not have, making it unlikely to see leadership potential in that student. In fact, in these cases, the LT served as a tool to reify teachers’ beliefs about who the “low” and “high” students were, and the LT served as a mathematical
justification to set lower expectations for their students who were still developing mathematical ideas at the perceptual level.

As with the identity dimension, some constructs within power were not present in the data. Specifically, student voice and relevancy of problems were not directly supported by the LT or LTBI. Both of these issues are important for equity but were not influenced by teachers’ knowledge of the LT. Thus, the first (4a) and third (4c) constructs for this dimension were removed from the framework.

The final construct in this dimension was highly-related to the second construct making it repetitive. When teachers positioned students as experts, they assigned them mathematical competence, which, in turn, informed the student that s/he did possess mathematical knowledge. Many of the examples that fit in the “positioning students as experts” construct also fit in the “help students see themselves as sources of mathematical knowledge” construct. In the example above, Anna affirmed Omar’s mathematical knowledge (e.g., his use of the number line) and positioned him as an expert for the class. Therefore, the final construct 4e was collapsed with 4b.

Revised LTBI-Equity framework

Based on the evidence collected from this study, a review of the uses and tensions from the findings, as well as how these findings relate to previous research, I revised the LTBI-Equity framework. This revised framework still attends to the four dimensions of equity but it only includes those constructs which were directly supported by LTs/LTBI or have the potential to be supported by LTs/LTBI as presented in the PD. Although some constructs were removed from the table, this does not indicate that they are not important.
These constructs were removed because they did not directly relate to LTs and LTBI as presented in the PD. In order to observe some of the constructs that I removed from the framework, it would be necessary to significantly change the PD such that ideas of identity and power are more purposefully addressed. In what follows, I first present the original framework with notes about constructs that were added or deleted (Table 15). Next, I present the revised framework and include revised propositions about how the teachers used LTs and LTBI in each construct (Table 16).
Table 15

*Original LTBI-Equity Framework with Notations*

<table>
<thead>
<tr>
<th>Access</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teachers use their knowledge of LTs and LTBI to:</td>
<td></td>
</tr>
<tr>
<td>Design instruction and instructional tasks such that they are accessible for all students.</td>
<td></td>
</tr>
<tr>
<td>Identify and use up-to-date research based materials and technology</td>
<td>Revised</td>
</tr>
<tr>
<td>Be accessible to and attend to all students in the class.</td>
<td></td>
</tr>
<tr>
<td>Foster classroom discussions such that all students can participate and engage.</td>
<td></td>
</tr>
<tr>
<td>Provide all students with opportunities to engage in rigorous mathematics.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Achievement</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Access</td>
<td></td>
</tr>
<tr>
<td>Set high, yet appropriate, academic standards for all students.</td>
<td></td>
</tr>
<tr>
<td>Unpack and build upon their students’ prior mathematical knowledge and use it as a basis for understanding more meaningful and complex mathematics.</td>
<td></td>
</tr>
<tr>
<td>Select and use a variety of forms of assessment (e.g., formative, summative, projects, class discussions) to gauge student achievement.</td>
<td></td>
</tr>
<tr>
<td>Produce equitable outcomes in student growth and achievement.</td>
<td>Added</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Identity</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Support the development of a robust mathematical identity</td>
<td></td>
</tr>
<tr>
<td>Listen to and consider students out of school experiences and design instructional activities that incorporate elements from their homes and communities.</td>
<td>Removed</td>
</tr>
<tr>
<td>Validate the use of students’ own algorithms and strategies to solve problems.</td>
<td></td>
</tr>
<tr>
<td>Assist students to build connections between the mathematics they learn and the broader world/society.</td>
<td>Removed</td>
</tr>
<tr>
<td>Encourage students to engage in mathematical tasks according to their preferences and participate in mathematical discourse in ways that are comfortable for them.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Power</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Ensure that students have voice in the classroom.</td>
<td>Removed</td>
</tr>
<tr>
<td>Position students as experts in the classroom (this includes things they know in school and things they know from outside of school).</td>
<td>Revised</td>
</tr>
<tr>
<td>Allow students to solve problems that are relevant to them (these problems can exist inside or outside of school).</td>
<td>Removed</td>
</tr>
<tr>
<td>Encourage all students to present, justify, and defend their mathematical ideas/arguments.</td>
<td></td>
</tr>
<tr>
<td>Help students to see themselves as sources of mathematical knowledge.</td>
<td>Merged with positioning students as experts</td>
</tr>
</tbody>
</table>
Table 16

The Revised LTBI-Equity Framework

<table>
<thead>
<tr>
<th>Access</th>
<th>Teachers use their knowledge of LTs and LTBI to:</th>
<th>Revised Propositions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Design instruction and instructional tasks such that they are accessible for all students.</td>
<td>Teachers use their knowledge of the student profiles and associated LT topics (e.g., producer to five, solving join problems using direct modeling) to design tasks with multiple access points and differentiate tasks.</td>
</tr>
<tr>
<td></td>
<td>Identify, create, modify, or use up-to-date research based materials or other curricula.</td>
<td>Teachers use their knowledge of the content in each LT to identify/create/modify instructional materials such that they are aligned with their learning goal.</td>
</tr>
<tr>
<td></td>
<td>Be accessible to and attend to all students in the class.</td>
<td>Teachers use their knowledge of students’ current understandings as evidence in the LT to determine the level of support needed for each student to access the task and engage in the mathematical content of the task. This includes direct access to the teacher as well as grouping students in various ways using the LT.</td>
</tr>
<tr>
<td></td>
<td>Foster classroom discussions such that all students can participate and engage.</td>
<td>Teachers use their knowledge of the content in the LTs to scaffold mathematics discussions such that all students can engage and connections are built between various mathematical concepts.</td>
</tr>
<tr>
<td></td>
<td>Provide all students with opportunities to engage in rigorous mathematics.</td>
<td>Teachers use their knowledge of where students are currently working in the LT as well as their knowledge of upcoming concepts to provide opportunities for rigorous tasks as well as movement across LTs.</td>
</tr>
</tbody>
</table>
Table 16 cont’d

<table>
<thead>
<tr>
<th>Achievement</th>
<th>Identity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set high, yet appropriate, academic standards for all students.</td>
<td>Support the development of a robust mathematical identity</td>
</tr>
<tr>
<td>Unpack and build upon their students’ prior mathematical knowledge and use it as a basis for understanding more meaningful and complex mathematics.</td>
<td>Validate the use of students’ own algorithms and strategies to solve problems.</td>
</tr>
<tr>
<td>Select and use a variety of forms of assessment (e.g., formative, summative, projects, class discussions) to gauge student achievement.</td>
<td>Encourage students to engage in mathematical tasks according to their preferences and participate in mathematical discourse in ways that are comfortable for them.</td>
</tr>
<tr>
<td>Teachers use their knowledge of the LT to set goals for students that are appropriate based on students’ current understandings as evidence in the LT.</td>
<td>Teachers use their knowledge of the LT to elicit and validate a students’ strategies, invented representations, and modifications to the task.</td>
</tr>
<tr>
<td>Teachers use the content of each LT as a checklist to determine what students have currently mastered, what they are currently working on, and what skills they should build upon when designing future instruction. Teachers also use their content knowledge from LTs to make sense of district and quarterly assessments.</td>
<td>Teachers use their knowledge of the LT and various strategies in the LT to allow students to work in ways that they are comfortable and represent their work (both written and oral) in ways that align with students’ understandings.</td>
</tr>
<tr>
<td>Teachers use their knowledge of LTs in planning and instruction to ensure equitable outcomes in students’ achievement.</td>
<td>Teachers use their knowledge of LTs and progression through LTs to support/encourage students. The LT serves as a tool for teachers to acknowledge students’ current understandings as well as a referent for creating “safe” mathematical risks.</td>
</tr>
<tr>
<td>Power</td>
<td>Position students as experts in relation to mathematics in the classroom.</td>
</tr>
<tr>
<td>-------</td>
<td>------------------------------------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>Encourage all students to present, justify, and defend their mathematical ideas/arguments.</td>
</tr>
</tbody>
</table>
It is not surprising that the revised framework has more constructs present on the dominant axis than the critical axis. The dominant axis is what many would see as the focus of reform-oriented mathematics education and instruction otherwise known as “good teaching.” The critical axis, however, challenges the status quo (Gutierrez, 2013). Although LTs challenge some aspects of mathematics education (e.g., building on the natural development of children as opposed to disciplinary guidelines), this study shows that LTs alone do not challenge issues of race, power, and social justice. While the remaining constructs in the critical axis are important for equity, they are only a small piece of a larger equity framework, and alone they are insufficient in developing a robust understanding of equity.

In my initial literature review, I attempted to revise Gutierrez’ (2007) framework in light of LTs and LTBI. In doing so, I maintained the necessary components of each dimension of equity and hypothesized the ways in which LTs and LTBI might support teachers in equitable instruction. After completing this study and analyzing the results, it is clear that LTs alone cannot address every construct in the original framework. It is also apparent that although all teachers participated in the same PD sessions, some teachers’ classrooms had more elements of equitable instruction than others’.

**Discussion of Findings**

Findings from this study indicated great variance in how teachers used LTs and LTBI to teach equitably in their classrooms. While some teachers changed their pedagogy and the culture of their classrooms, others did not. This variance prompted me to investigate possible
explanations for why some teachers successfully used LTs and LTBI equitably and others did not. I also examined why some parts of the LTBI model were taken up more successfully than others (e.g., monitoring vs. connecting) and the implications this has for equity. In this section, I use Guskey’s (1986, 2002) model for teacher change as one possible explanation. I conclude this section with a discussion of deficit perspectives as another conjecture for variances observed.

**Teacher Change**

One possible explanation for teachers’ uses and tensions of LTs and LTBI to teach equitably can be understood using Guskey’s (1986, 2002) model for teacher change. Guskey (2002) argued that PD leaders assume that they can change teachers’ beliefs which will lead to a change in their practice, which will in turn change student learning outcomes (p. 382). Guskey, however, argued that teacher change is experiential, and he offers an alternative model.

![Figure 6. Guskey’s (2002) model of teacher change (p. 383)](image)
In this model, he argued that PD doesn’t directly change teachers’ beliefs. Rather, PD changes teachers’ practices, which changes students’ outcomes, and the change in student outcomes is what changes teachers’ beliefs. He goes on to state:

Practices that are found to work—that is, those that teachers find useful in helping students attain desired learning outcomes—are retained and repeated. Those that do not work or yield no tangible evidence of success are generally abandoned…Attitudes and beliefs about teaching in general are also largely derived from classroom experience. Teachers who have been consistently unsuccessful in helping students from educationally disadvantaged backgrounds to attain a high standard of learning, for example, are likely to believe these students are incapable of academic excellence. If, however, those teachers try a new instructional strategy and succeed in helping such students learn, their beliefs are likely to change. (Guskey, 2002, p. 384)

Guskey’s findings are relevant to this study in two ways. First, recall from Anna’s case portrait that early on in the summer institute, she asked if the LTBI program was going to provide her with a new instructional model. She was eager to learn a new way to teach mathematics. Therefore, she changed the types of tasks she used, how she organized her instruction, and developed new mathematical norms for her classroom. Based on the success of her students and how they progressed in the LT, Anna continued to use the LTBI model. Drawing on the LTBI model, Anna provided opportunities for all of her students to engage in rigorous mathematics, positioned students as experts, and encouraged all students to defend
their mathematical ideas. She looked for and saw positive outcomes of her new practices in her students and continued to use LTs and LTBI in her classroom.

If I contrast Anna’s case with Caroline’s and Elizabeth’s, we can see examples of how a negative change in student outcomes caused teachers to abandon or struggle to implement new practices. In Caroline’s case, she stated that her classroom sharing time was not effective, and she stated that her students did not get anything out of it. While part of this was related to her underlying beliefs that all students did not know how to share, she found this practice challenging. Therefore, in my observations of Caroline, she often abandoned this portion of the lesson. Many instances of identity and power were present during discussions, and since Caroline did not implement this practice, she did not fully use LTs and LTBI for equitable instruction. In Elizabeth’s first lesson, she created a task and implemented it with her students. Recall that students had difficulty with this task; therefore, Elizabeth diminished the academic rigor in favor of a task that the students would find more “enjoyable.” Because her first attempt at providing students with what she thought was a rigorous task was not successful, Elizabeth believed that the students could not do this type of task; thus, her beliefs remained unchanged.

There is a subtle link that I argue is present in Guskey’s (2002) model that leads into the next portion of this discussion about deficit perspectives. When teachers experienced success in moving from “change in practices” to “change in student outcomes,” the credit is often assigned to their efforts to change the practice. But when difficulties arise, teachers’ underlying beliefs and deficit perspectives became apparent. In the examples I presented for
Caroline and Elizabeth, they both articulated deficit orientations as a reason for why their students were “unsuccessful” and why they did not follow through with elements of the LTBI model. This conjecture leads into my next discussion of deficit perspectives. Because LTs describe probable paths that students should take, I argue that deficit perspectives can manifest as an explanation for students who “deviate” from probable path.

**Deficit Perspectives**

Instances of deficit perspectives and tensions related to them were present throughout the study. Therefore, I returned to this phenomenon to understand why the LT did not disrupt some teachers’ beliefs. All teachers in the study thought the LT was a useful tool and even articulated that all teachers at the partner school should have participated in the PD. This shared enthusiasm for the LT combined with my observation of how the LT was used by some teachers to help some students and hinder others concerned me and led me to further consider the relation between LTs and equity. I offer the following discussion to clarify some findings for my study, which leads into a discussion of implications from the research.

Although only one teacher in this study demonstrated strong and consistent deficit perspectives, there were a variety of examples of deficit perspectives that derailed the potential of LTs and LTBI to promote equity. To situate this discussion, I draw on previous work related to defining deficit perspectives, understanding the ways in which they manifest in practice, and how to dispel them. This discussion is critical to this work as findings from this study indicate that when teachers had strong deficit perspectives, they did not use the LT or LTBI to enact equitable instruction.
The term deficit thinking/perspective/orientation has been present in the literature for over thirty years (Delpit, 1995; Padilla, 1981; Valencia, 1997). Those that hold these beliefs view those other than themselves or those who are different from the majority as “genetically inferior or socially depraved” (Nelson & Guerra, 2014, p. 71). Since beliefs are “deeply personal, individual truths one holds about physical and social reality” and “shape how an individual sees the world, sees other people, and sees oneself,” it is clear that the beliefs of an individual will shape all of their personal and professional interactions (Nelson & Guerra, p. 70). In fact, Nelson and Guerra go on to say that “beliefs have such a strong effect that even in professional practice, personal beliefs are a greater predictor of a persons’ behavior than professional knowledge” (p. 70). These types of beliefs are particularly problematic for teachers that hold deficit orientations towards their students.

In a study of educational reform efforts, Harris (2012) sought to understand how teachers’ expectations interacted with their efforts to implement a standards-based curriculum. Her findings revealed that expectations of students varied based on teachers’ beliefs about who could obtain proficiency. Additionally, she stated that “the standards in and of themselves did not remedy the challenges that schools and teachers confronted with students of varying academic skills” (Harris, p. 143).

In this dissertation study, one proposition was that teachers would use their knowledge of the LT to locate all students on a trajectory (recognizing their mathematical understandings) and to use that knowledge as a basis for building more sophisticated knowledge over time. Additionally, I conjectured that this “location” of students along a
trajectory had the potential to disrupt teachers’ notions of who could do mathematics because they would see that all students possessed knowledge. While this was present in many cases, it was not present across the board, indicating that equitable instruction was not always present. Harris (2012) stated, “Although most teachers were committed to standards exposure, less were committed to whether all students would attain similar proficiency or mastery of standards” (p. 144). Although the teachers in this study committed to the power and usefulness of the LT and LTBI, they did not believe that LTs and LTBI were appropriate for all students.

**Implications**

This study was a part of a larger project that focused on unpacking the ways in which teachers use their knowledge of LTs in instruction. This work extended the goals of the larger project by considering how teachers can harness their knowledge of LTs and LTBI to teach equitably. The findings from this work have implications for scholars and policy makers.

**Scholars**

This work draws on the research of scholars studying LTs as well as equity. Results indicated that LTs alone do not guarantee equity. Therefore, scholars with expertise on equity and LTs should partner to discuss teachers can capitalize on the strengths of this framework and how to alleviate the tensions observed.

Now that mathematics teacher educators have a clearer picture of the ways in which teachers use LTs in instruction, more work needs to be done to determine how to structure PD such that LTs and LTBI can be used equitably. Nelson and Guerra (2014) argue that
“addressing deficit thinking is an essential aspect of school improvement” (p. 89). To disrupt deficit thinking, Nelson and Guerra state that researchers must create cognitive dissonance and:

- Confront and problematize occurrences of deficit thinking and offer alternative notions;
- Increase teachers’ awareness of their own and their students’ "culturally based behaviors’’;
- Increase teachers awareness and unpack the assumptions they hold about those different from themselves; and
- Challenge teachers to maintain academic rigor and high expectations for all students.

It is important that mathematics teacher educators and those that design professional development around LTs consider the implications this work has for pre-service teachers as well as in-service teachers. They should consider pre-service classroom and field-based experiences as well as professional development models that have proven to be successful in enacting teacher change. Specifically, it is critical that scholars not only attend to the four constructs of Guskey’s (2002) teacher change model, but also the “in-between” spaces. This means attending to the content of the PD, monitoring the implementation fidelity of teachers’ classroom practices, helping teachers understand that changes in student outcomes are multifaceted and require time, and then unpacking how teachers’ beliefs change as a result.

Additionally, scholars must support teachers as they continue through this cycle and help
them make sense of their transitions. This detailed support can help teachers link student outcomes to classroom practices instead of deficit perspectives.

Scholars who primarily focus on issues of equity and social justice should also consider the duality of expertise needed to enact this framework. Because it can be uncomfortable to bring up issues of race and class, scholars who have successfully challenged teachers to unpack their own stereotypes and biases should collaborate with mathematics educators focused on LTs. This dual focus can be very powerful for teachers. If implemented with care, the LTBI-Equity framework along with other issues related to equity can be used to simultaneously disrupt teachers’ beliefs about what mathematics should look like and what doers of math should look like.

Policy Makers

Because LTs undergird the Common Core State Standards, and these standards can be found in many elementary school classrooms across the country, it is critical that teachers receive adequate professional development in relation to LTs. This professional development must address both the mathematical content of the trajectory as well as the how to use LTs equitably. Teachers need adequate time and support to grapple with LTs and equity simultaneously. Teachers will require support to help them realize when deficit perspectives arise in their planning, discussion, and instruction such that they can be re-directed to think about the ways in which LTs can be used as a tool help re-write low expectations. Because issues of equity can be personal and uncomfortable, this work must occur in environments that are safe and non-evaluative.
Limitations

As with any study, there are limitations. First, this study took place during a very challenging year for participating teachers. This was the first year that teachers were required to teach and implement the CCSS in their classrooms and use new quarterly/benchmark assessments. In addition to this work, these teachers also elected to participate in the LTBI project, which involved 60 hours of professional development on LTs and their use in instruction. Finally, these teachers agreed to participate in my study, which required additional work with lesson planning, interviews after school, and the completion of student portraits. Although teachers were very eager to learn new material and participate in this study, it is reasonable to assume that the swarm of new experiences occurring simultaneously could have been overwhelming for teachers. While rich information was gained from this study, there could have been slight differences (particularly related to assessment) if teachers were more comfortable with all of the new requirements.

Second, this work was conducted with a small number of teachers in the context of one professional development project. These findings are based on the observations of four teachers. I chose to work with a smaller number of teachers for two reasons. First, the four teachers selected for this study were those most likely to use the content of the LT in their lessons as it was most closely aligned with their grade band. Second, working with a smaller number of teachers allowed me to gain a depth as opposed to a breadth of understanding about their uses and struggles in thinking about LTs and equity. Further work with more teachers is necessary to fully unpack the LTBI-Equity framework. Additionally, conducting
this work in classrooms with varying student demographics will add to the LTBI-Equity knowledge base.

Another potential limitation is the timing of the study in relation to the PD. This study was conducted after teachers had already completed the first half of the PD. Consequently, this study occurred during the second half of the school year, and classroom norms had already been established. In an ideal setting, this study would have been conducted the year following the completion of the PD, which would have given some teachers more time to implement changes in their instruction.

Finally, this work was completed as a part of my dissertation process where I was the sole researcher. While I did have outside lesson quality analysis (IQA) completed by an outside expert and had a colleague code my work for purposes of reliability, this is my first attempt at designing and implementing all phases of the research process.

Next Steps

This study was an initial attempt at combining two different frameworks to consider the ways in which they could support each other. Throughout the data analysis process, there were a number of challenging decisions to make regarding whether or not certain examples from the data fit into this framework. For example, many teachers did the following things during their lessons over the course of the study:

- Reading problems aloud to students at the start of the lesson (teachers are making the language and vocabulary in the task accessible);
• Telling students “great job” when they did something correctly (building their confidence and identity); and

• Asking students what a “tens frame” reminds them of (allowing students to make connections).

These particular instances were challenging because although they do illustrate important components of equitable classrooms, they are general best practices that do not necessarily draw upon elements of the LT or LTBI. Future research should address the viability of one single framework that can fully address the both the dominant and critical axes. While the framework developed in this study does acknowledge the role of the LT in equitable instruction, other “best practices” are not included in the framework.

Similar work currently being conducted by the TEACH-MATH group articulated a framework with six categories. The first three categories (cognitive demand, depth of knowledge and student understanding, and mathematical discourse) focus solely on issues that would be important in any reform-oriented mathematics classroom (Aguirre & Zavala, 2013). The remaining three categories (power and participation, academic language support for ELLs, and cultural/community-based funds of knowledge) address issues of equity that arise in a mathematics lesson (Aguirre & Zavala, 2013). Because the goal of this work had a dual focus on children’s thinking and equity, all components of the dominant and critical axes are addressed specifically.

Building on this work, future studies should involve professional development that explores LTs and equity simultaneously and use this revised framework as a central part of
the PD. An integral part of this research should require teachers to consider issues of equity during each phase of lesson planning, instruction, and assessment to more fully expound upon *uses* and *tensions*.

Another important component of equity that was not fully captured in this study is student outcomes. While there was evidence to support the fact that teachers could set high standards for students and design instruction to support those standards, these results varied across teachers. Therefore, future research should follow teachers over time with the goal of connecting the following ideas: a) how teachers use LTs to set goals for students, b) how teachers use LTs to plan meaningful instruction, and c) how these efforts affects student achievement outcomes. Since quarterly testing and benchmark testing are a focus of many schools, results from such tests can serve as a measure to determine if there is equitable growth across student groups as well as achievement outcomes. Issues related to identity and power should still be addressed during such studies.

**Closing Remarks**

Equity is more than best practices; it must be an intentional act. While LTs and LTBI did perturb teachers by bringing up sensitive issues in relation to equity (e.g., access, rigor, high standards, instructional models), the LT alone was not enough to help teachers fully address the four dimensions of equity. Deficit perspectives are very difficult to disrupt and some of the tensions observed in this study indicate that while the LT was useful for planning instruction and anticipating students’ needs mathematically, in the moment teachers retreated
into wanting students to feel comfortable, thereby reducing the challenge or demand of the task for students.

The variations observed in this study indicate great promise and great responsibility. The prominence of LTs in national standards, curriculum, and assessment makes professional development focused LTs welcomed by many schools and districts. Although the climate is receptive to this work, mathematics educators must prepare teachers to use this new knowledge in responsible ways. The uses of LTs and LTBI demonstrate that by focusing on student thinking, we can work towards equitable instruction. The tensions indicate that if we provide LTs and the LTBI model to teachers without challenging their views about themselves, their role in the classroom, how they view their students, their deficit orientations, and their beliefs about who can learn, LTs can actually be a dangerous construct. As the field of mathematics education continues to conduct work in this area, we must focus on preparing teachers to use LTs and LTBI proficiently and responsibly.
REFERENCES


TEACH MATH. (2012). *Culturally responsive mathematics teaching lesson analysis tool.*


APPENDICES
Appendix A: Beliefs Instrument

Identification Number: _______________________________________________
Date: ______________________________________________________________

This scale presents a listing of sentences. You are to indicate the degree to which you agree or disagree with the opinion or belief expressed in each of the sentences.

If you strongly disagree with the opinion or belief expressed in a sentence, circle the letters SD to the right of that sentence.

If you disagree with the opinion or belief expressed in a sentence, but not so strongly, circle the letter D to the right of that sentence.

If you are not sure how you feel about the opinion or belief expressed in a sentence, that is you cannot decide or you do not really have an opinion, circle the letter N to the right of that sentence.

If you agree with the opinion or belief expressed in a sentence, circle the letter A to the right of that sentence.

If you strongly agree with the opinion or belief expressed in a sentence, circle the letters SA to the right of that sentence.

There are no “right” or “wrong” answers. The only correct responses are those that reflect what you believe to be true. Be sure to respond to each item in a way that reflects your personal beliefs. Do not spend too much time pondering each sentence. Read a sentence carefully and then indicate your opinion.

Be sure to respond to every statement.

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University of Maryland
College Park, MD 20742-1175

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<table>
<thead>
<tr>
<th></th>
<th>The best way to teach students to solve mathematics problems is to model how to solve one kind of problem at a time until the students achieve mastery and then to foster frequent practice.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.</td>
<td>I learn about my students’ perceptions of their mathematical ability through explicitly asking them (e.g., students write about it, one-on-one discussions, group discussions).</td>
</tr>
<tr>
<td>3.</td>
<td>When teaching multi-step word problems, I allow students who are struggling with basic computation to use a calculator.</td>
</tr>
<tr>
<td>4.</td>
<td>I like to use mathematics problems that can be solved in many different ways.</td>
</tr>
<tr>
<td>5.</td>
<td>When planning mathematics lessons, teachers need to focus explicitly on rules and procedures.</td>
</tr>
<tr>
<td>6.</td>
<td>For the majority of my students, I have a good sense of whether or not they see how the mathematics we do in class connects to their everyday lives.</td>
</tr>
<tr>
<td>7.</td>
<td>My primary role as a mathematics teacher is to identify and teach the mathematics content that is on the state’s mathematics assessment.</td>
</tr>
<tr>
<td>8.</td>
<td>I often learn from my students during mathematics class because my students come up with ingenious ways of solving problems that I never thought of.</td>
</tr>
<tr>
<td>9.</td>
<td>In my class it is just as important for students to learn data analysis and probability as it is to learn a long division algorithm.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Strongly Disagree</th>
<th>Disagree</th>
<th>Not Sure</th>
<th>No Opinion</th>
<th>Agree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>SD</td>
<td>D</td>
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<td>2.</td>
<td>SD</td>
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<td>3.</td>
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<td>4.</td>
<td>SD</td>
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<td>5.</td>
<td>SD</td>
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<td>6.</td>
<td>SD</td>
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<td>7.</td>
<td>SD</td>
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<td>8.</td>
<td>SD</td>
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<td>9.</td>
<td>SD</td>
<td>D</td>
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<td>A</td>
<td>SA</td>
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</tbody>
</table>
11. During mathematics class, a teacher should limit the questions he or she asks of a student to those questions that the teacher knows the student can answer correctly.

<table>
<thead>
<tr>
<th>SD</th>
<th>D</th>
<th>N</th>
<th>A</th>
<th>SA</th>
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</table>

12. Mathematics skills are mastered incrementally, so instruction should only focus on one skill at a time, ordered by difficulty, and not move on until most students have mastered that skill.

<table>
<thead>
<tr>
<th>SD</th>
<th>D</th>
<th>N</th>
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</table>

13. Students in a mathematics class should be expected to question other students, to question the teacher, and to answer other students’ questions.

<table>
<thead>
<tr>
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<th>D</th>
<th>N</th>
<th>A</th>
<th>SA</th>
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</thead>
</table>

14. In order to prepare students for assessments, when students are working on a problem in mathematics, I highlight more than one approach to solving that problem.

<table>
<thead>
<tr>
<th>SD</th>
<th>D</th>
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</table>

15. A lot of things in mathematics must simply be accepted as true and remembered.

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<th>SD</th>
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<th>A</th>
<th>SA</th>
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</table>

17. Students who perceive themselves as ‘bad’ at mathematics need different mathematical experiences than students who see themselves as ‘good’ at mathematics.

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18. If students use calculators, they will not master the basic skills they need to know.

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19. Students learn mathematics best by paying attention when their teacher demonstrates what to do, by asking questions if they do not understand, and then by practicing.

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<td>20.</td>
<td>If students can use manipulatives correctly, then they understand the underlying mathematics.</td>
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<td>21.</td>
<td>I learn about my students’ perceptions of connections between mathematics and their everyday lives through explicitly asking them (e.g., students write about it, one-on-one discussions, group discussions).</td>
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<td>22.</td>
<td>Learning mathematics requires a good memory because you must remember how to carry out procedures and, when solving an application problem, you have to remember which procedure to use.</td>
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<td>23.</td>
<td>Grouping students for cooperative work during instruction is not efficient because teachers do not know what students have learned or who did the work.</td>
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<td>24.</td>
<td>Students learn mathematics best by working to solve accessible problems that entail a solution process that has not been demonstrated to them.</td>
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<td>25.</td>
<td>When two students solve the same mathematics problem correctly using two different strategies, they should share the steps they went through with each other.</td>
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<td>26.</td>
<td>During mathematics class, I do not necessarily answer students’ questions immediately but rather let them struggle and puzzle things out for themselves.</td>
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<td>27.</td>
<td>For the majority of my students, I have a good sense of their motivations for wanting to succeed in mathematics.</td>
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<td>28.</td>
<td>I like my students to master basic mathematical operations before they tackle complex problems.</td>
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<td>29.</td>
<td>During mathematics class, students should be asked to solve problems and complete activities by relying on their own thinking without teachers modeling an approach.</td>
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<td>30.</td>
<td>When working in groups, if students are not progressing, the teacher should help students who are stuck by demonstrating the intended solution.</td>
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<td>31.</td>
<td>When teaching an objective in the curriculum, a teacher should use knowledge of students’ prior mathematical performance to vary problem difficulty across groups of students.</td>
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<td>32.</td>
<td>During mathematics class, discussion should focus on students’ ideas and approaches, no matter whether their answers are correct or incorrect.</td>
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<td>33.</td>
<td>Students can figure out how to solve many mathematics problems without being told what to do.</td>
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<td>34.</td>
<td>For the majority of my students, I have a good sense of whether they see themselves as ‘good’ or ‘bad’ at mathematics.</td>
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<td>35.</td>
<td>The majority of my students think ‘doing mathematics’ is mainly about figuring out what rule or procedure to apply and then doing so.</td>
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<td>36.</td>
<td>To teach mathematics, first model the activity, then provide some practice and immediate feedback, and, finally, clarify what the assignment is and how it is to be completed.</td>
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<td>37.</td>
<td>I learn about my students’ perceptions of what ‘doing mathematics’ means through explicitly asking them (e.g., students write about it, one-on-one discussions, group discussions).</td>
<td>SD D N A SA</td>
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<td>38.</td>
<td>Students should be homogeneously grouped for instruction and assigned to a curriculum on the basis of their prior mathematical performance.</td>
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<td>39.</td>
<td>Students who hold the perception that mathematics is important and connected to their future need to engage in mathematical activities that are different than those needed by students who do not think mathematics is important and connected to their future.</td>
<td>SD D N A SA</td>
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<td>40.</td>
<td>A teacher’s role is to ask students questions and to question their answers so that students will make sense of the mathematics.</td>
<td>SD D N A SA</td>
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Appendix B: Student Portrait

1. Based on what we have learned in the professional development and what you have seen in your classroom, what do you know about this student’s knowledge of mathematics?
   a. How does the LT help you think about this?

2. What evidence do you have about what this student knows?
   a. How does the LT help you think about this?

3. What goals can you set for this student based on what you know about them and what you know about the LT?

4. What opportunities are needed for this student to progress?
   a. How does the LT help you think about this?
Appendix C: Pre-Observation Interview Protocol

Access

1. How will you use the LT in your lesson planning to help you think about reaching all of your students?
   a. Can you share some specific examples?
2. Where did you get the idea or motivation for this unit?

Achievement

1. What is the intended learning goal/outcome of this set of lessons?
   a. How will you use the LT to set academic goals for your students?
   b. Will you have the same goals for all of your students? Why or Why not?
2. How will you use what your students already know about mathematics to build upon it in these lessons?
   a. Can you share some specific examples of how you will build upon their prior knowledge?
3. How will you know if the students met the intended learning goals?
   a. What forms of assessment will you use to determine what the students learned? Why did you select these forms of assessments?
   b. How does using the LT help you when thinking about assessment?
   c. Will you assess all of your students the same way? Why or Why Not? Can you provide an example?

Identity

1. How do you know where each of your students is prior to the start of this lesson?
2. Did using the LT help you to think about your students’ out-of-school experiences and how you could use them in your instruction? Why or Why not?

Power

1. How do you plan to ensure that all students participate in this lesson?
2. How will you provide opportunities for students to present, justify, and defend their mathematical ideas?
Appendix D: Classroom Observation Protocol

Adapted From Merriam (1998)

Date: __________________
Time of Arrival: _________________
Time of Departure: _________________
Grade Level: __________________
Teacher Name: __________________

The Classroom Environment

1. What is the mood/tone of the classroom today?
2. Describe the allocation of space in the classroom (including seating arrangements).
3. What is written on the board?
4. What things are displayed on the walls in the classroom? In the hallway?
5. Is student work displayed in the classroom? If so, what students are represented?
   What students are not represented?
6. What are the classroom rules? Norms?

Who is in the classroom?

1. Who is present?
2. Who is absent?
3. What is the ethnic/racial breakdown in the classroom?

Lesson & Activities

1. What is the topic of the lesson?
2. Provide a general description of the lesson.
3. What activities are going on today?
4. Did the teacher provide entry points or access points for all students to participate in the lesson?
5. What time did the lesson start? What time did it end?
6. What was the structure of the lesson?
7. Did the teacher build upon students’ prior knowledge or informal experiences during the lesson?
8. Did the teacher miss any opportunities to build on students’ prior knowledge or informal experiences during the lesson?

**Interactions & Conversation**

1. What was the content of the classroom conversation?
2. Who spoke during the lesson? What did they speak about?
3. Who listened during the lesson?
4. Describe the types of questioning the teacher used during the lesson.
5. Describe the nature of the interaction in the classroom.
6. Did the teacher encourage all students to participate/speak?

**Subtle Factors**

1. Did any informal or unplanned activities occur?
2. What non-verbal communication was present during the visit?
3. Did anything happen that “should not” have happened?

**Notes About My Role**

1. What was my role during today’s observation?
2. How does my presence affect the environment?
3. What did I say or do during the visit?

**Notes**

**Questions That Arise During The Observation**
Appendix E: Post-Observation Interview Protocol

Access

1. Talk to me about this unit:
   a. Did you adapt or modify any of the tasks or activities? If so, how?
   b. Why did you (or didn’t you) modify any of the tasks or activities?
   c. Did you come up with any new examples?

2. How did you foster the discussion and conversation during these lessons?
   a. Were all students able to participate in the class discussions? Why or Why Not?
   b. In what ways did different students participate?
      a. Why do you think there were differences in participation?

Achievement

1. How did you use what your students already know about mathematics to build upon it during this unit?
   a. Are there any examples of student knowledge that emerged during the lesson?
   b. Can you share some specific examples of how you built upon their prior knowledge?

2. How do you know if the students met the intended learning goals?
   a. How did you assess students during this unit?
   b. How did you use the LT when thinking about assessment?
   c. Did you assess all of your students the same way?
   d. Is there anything you would change about the way you assessed your students in these lessons?

Identity

1. How did you use the LT to incorporate various strategies or algorithms in your lessons?
   a. How did you build upon those strategies to support learning for all students?
2. What connections can you identify between the set of lessons you taught and the real world? How did you assist students with building those connections?  
   a. Did the LT play a part in this process at all? Why or Why Not?

**Power**

1. Talk to me about the type of conversations that occurred during these lessons?  
   a. How did you talk to the students?  
   b. How did the students talk to you?  
   c. How did they talk to each other?

2. What does it mean for your students to have a “voice” in the classroom?  
   a. How do you help them develop this “voice”?  
   b. Does the LT help you at all in assisting/supporting the development of students’ voice?

3. Talk to me about who participated in this unit and who did not?  
   a. Why did some students participate and others did not?  
   b. Does using the LT help you ensure all students are able to:  
      i. Participate in the lesson?  
      ii. Share their strategies?  
      iii. Defend their strategies?  
   c. In what ways did you support all students in participating, sharing, and defending their strategies? Did the LT help you with this in any ways?
Appendix F: Interview Protocol for Interview #3

1. During our last talk, I introduced this equity framework to you.
   a. How has this changed anything you have done since then?
   b. Have you thought about the framework?
   c. Have you thought about ways to incorporate elements of it?
   d. Have you found it useful?

2. Talk to me about how you provided access for students during this lesson.
   a. How did you ensure that the task was accessible for all students?
   b. How did you think about allocating your time with students?
   c. How did you provide access to the conversation and classroom discussions?
   d. Did all students engage in rigorous mathematics?

3. Talk to me about how this lesson related to students’ achievement?
   a. What goals did you have set for your students?
   b. How did you unpack and build upon students’ understanding during this lesson or in the lessons that followed?
   c. Talk to me about the types of assessment you are using to understand what your students know?

4. Talk to me about how you work to support the development of students’ identity.
   a. How do you help students develop a positive mathematical identity?
   b. How do you listen to and incorporate students’ out of school experiences to include elements from their homes and communities?
   c. How do you validate the use of students’ strategies and algorithms?
   d. How do you help students build connections between the mathematics they learn in school and the broader world/society?
   e. How did you encourage students to engage according to their preferences?

5. Talk to me about how kids are empowered in the classroom.
   a. How do you ensure that all students have a voice?
   b. How do you position students as experts?
   c. How do your provide opportunities for students to solve problems that are relevant to them?
   d. How are students encouraged to present, justify, and defend their ideas?
   e. How do you help students view themselves as sources of mathematical knowledge?
6. Talk to me about your next lesson.
   a. What is the intended learning goal?
      i. Are the goals the same for all students?
   b. What are the activities?
   c. How does this lesson relate to the LT?

7. Talk to me about how you plan to promote access, achievement, identity and power in your next lesson.

8. Let’s look at your student profiles and where they were when we initially started working with each other. Can you talk to me about where your students are now?
   a. Which students have moved?
      i. Have all students moved the same “amount”?  
      ii. Can you talk to me about what things/experiences/opportunities have helped them progress?
      iii. What evidence do you have that these students have moved?
   b. Do you have any students that have not moved?
      i. Can you talk to me about why these students have not moved?
      ii. What evidence do you have that these students have not moved?

9. Other Questions of Interest:
   a. Has the way you used the LT changed at all over the course of the year? Are you thinking about it differently now that we are approaching the end of the year?

   b. Do you believe that students need to completely master one level of the LT before moving on to the next?
      i. Talk to me about that?
      ii. What are the implications of your thoughts on this in relation to the equity framework?

   c. Can you talk to me about the meaning of the word “confidence”? I have heard it mentioned a few times in a few different ways.
      i. One way comes out during classroom observations and it seems to relate to building confidence in students?
         1. How does that relate to the equity framework?
      ii. Another way it comes out is in relation to confidence at a level of the LT or confidence in relation to the content.
d. Talk to me about the way the LT or LTBI has changed your views of students?
   i. Do you think differently about students?
   ii. Has it affected any stereotypes you may have previously held about what students can and cannot do?
   iii. We have heard teachers talk about being surprised when “low” students can do something or when “high” students have difficulty with something. How does your knowledge of the LT interplay with this surprise?
Appendix G: Final Interview Protocol Additions

1. Ask about each student, what level were they at the end of the year? Ask about some students that moved a lot and other students that did not move as much.

2. Ask about how teachers interpret achievement. It seems like one indicator of achievement is how “confident” a student is at a level and how much assistance they need from the teacher.

3. Show the teachers the copy of the slide with the 5 practices on it and ask them what they think they do in each practice that promotes equity and anything from each practice that they think may hinder equity.

4. What does it mean for a student to be early in a level, at a level, or moving to the next level? How do you differentiate that?
Appendix H: Anna’s Recording Sheet

There are 8 students getting ready for recess. Some of the students will have indoor recess and some will go outside for recess. How many of each could you have inside and outside? Write your equations.

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