ABSTRACT

HUNTER, ELIZABETH ELAINE. Improved Driver Yielding Model for Pedestrian Delay Estimation. (Under the direction of Dr. Nagui Rouphail.)

The interaction of pedestrians and motorized traffic at unsignalized intersections needs to be further explored to determine the effects on intersection operation and pedestrian safety. Pedestrian crossing delay at unsignalized intersections, specifically midblock crossings, is dependent on the availability of gaps between vehicles and driver yield events. Unsignalized midblock crossings are common in downtown areas and college campuses. The Highway Capacity Manual (HCM) 2010 has made an effort to combine pedestrian gap acceptance and driver yielding behavior to predict pedestrian delay, but the methodology has not been calibrated by field observations. Mean yield rates for several crossing treatments, for a few sites, are provided. Mixed-priority pedestrian delay models for single-lane roundabouts address the interaction where some drivers yield to pedestrians and some pedestrians accept gaps. But to date, no predictive model exists to estimate yield probability based on site-specific attributes.

Validation and improvement of the HCM 2010 and mixed-priority pedestrian delay estimation methods is the major objective of this research. Research consisted of using three widely used methods to determine critical gap, specifically, HCM calculation, maximum likelihood estimation (MLE), and the graphical method. A regression model to predict motorist yielding rate based on site characteristics was developed and results were then compared with observed rates. The two delay estimation methods, three critical gap methods, and observed and predicted yielding rates resulted in twelve values for estimated average pedestrian delay per site. These estimates were compared with field observed delay.

Data collected at 27 sites in North Carolina, Florida, and Alabama contributed to validation and improvement of the HCM and the mixed-priority pedestrian delay models. Both pedestrian delay estimation methods use pedestrian critical gap values and motorist yield rates. For maximum likelihood estimation and the graphical method, sites were grouped by crosswalk length. Longer critical gap values were typically seen at the sites with longer

crosswalks. Multiple linear regression was used to develop a model that could be implemented in the HCM to predict motorist yield rate from site characteristics. Model development consisted of both categorical and numerical data. The final motorist yielding model included on- or off-campus, whether the site was in Florida, one- or two-way, and crossing width as the independent variables. This model had an overall adjusted R^2 value of 0.7261. The model was found to be more accurate at predicting high and low motorist yielding rates than it was at mid-level motorist yielding rates. Over half of the sites were accurate within 15%. Results appeared more accurate when the sites were grouped by common characteristics, such as on- or off-campus and one- or two-way traffic. It is recommended that this model be used when field data is unavailable.

An effort was made to validate the HCM pedestrian delay method, but even after using three different critical gap estimation methods and two different yield values, the estimates never matched the observed values. Average pedestrian delay was underestimated using the HCM method. The mixed-priority model with the observed yield encounter probability and the critical gap from the graphical method showed the most accurate results. A trendline for the observed delay against the predicted delay from this method had an R^2 value of 0.894. This model was designed to apply to any pedestrian population, where perfect utilization is assumed for sighted pedestrians. It was determined that an additional conversion factor of 0.68 could be applied to more accurately predict the average pedestrian delay.

Further research in additional states should be performed in order to validate the findings of this research. It is recommended that the mixed-priority pedestrian delay model, with an adjustment factor, be used over the HCM method.

Improved Driver Yielding Model for Pedestrian Delay Estimation

by
Elizabeth Elaine Hunter

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APPROVED BY:

Bastian Schroeder

Billy Williams

Nagui Rouphail
Chair of Advisory Committee

BIOGRAPHY

Elizabeth Elaine Hunter was born in Kinston, North Carolina on September 2nd of 1990. She grew up in the small community of Back Swamp in Beulaville, North Carolina. Elizabeth graduated from Richlands High School in the spring of 2008 and began attending North Carolina State University in the fall. She completed her Bachelor of Science degree in Civil Engineering with a concentration in Transportation in the spring of 2012. After receiving her degree, she worked for the Institute of Transportation Research and Education as a graduate researcher while working towards her Master of Science degree. Once she completes her Master's degree, she plans to begin working for a local consulting firm.

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1. INTRODUCTION AND OBJECTIVES

With increasing focus on pedestrian facilities and mobility, questions about pedestrian safety, the interaction of pedestrians and motorized traffic, as well as their operational effects, need to be explored. Pedestrian crossing behavior has not been explored to the same degree that vehicle gap acceptance has been investigated. Pedestrian crossing delay is dependent on the availability of gaps between vehicles or driver yields at unsignalized crossings. Unsignalized crosswalks, specifically midblock crossings, are common in downtown areas and college campuses. Contrary to what is implied in the terminology, midblock crossings are not necessarily located in the middle of a block, but rather can be found anywhere along a roadway at locations away from an intersection crossing. A thorough understanding of pedestrian and driver interaction at unsignalized street crossings is lacking.

Safety of pedestrians, as well as drivers, is affected by the delay to the pedestrian and the driver's willingness to yield. A total of 4,743 pedestrian fatalities and 76,000 pedestrian injuries were identified by the National Highway Traffic Safety Administration, NHTSA, in traffic collisions nationwide in 2012 (NHTSA, 2014). The report further cites that pedestrian fatalities were highest in California (612), Texas (478), and Florida (476). When the states are sorted by "Pedestrian Fatalities per 100,000 Population," four of the top ten states were in the southeast (South Carolina, Louisiana, Florida, and North Carolina). These statistics show the importance of improving pedestrian safety in the southeast region and the states included in the STRIDE consortium. Improper crossing of a roadway or intersection, or failure to yield right-of-way, caused 45% of pedestrian fatalities in 2001, according to a study by NHTSA (2003). The accessibility of an unsignalized crossing for pedestrians is a function of the following: availability of crossing opportunities in the form of yields and crossable gaps, rate of utilization of these opportunities by pedestrians, delay incurred until opportunity is utilized, and level of risk experienced when attempting to cross (Schroeder, et al. 2011).

The Highway Capacity Manual (HCM) 2010 (TRB, 2010) has made an attempt at combining pedestrian gap acceptance and driver yielding behavior to predict pedestrian delay, but the documented methodology is not based on, nor calibrated by, empirical observations and has not been calibrated by field observations. The pedestrian mode is compared to minor street traffic operation at two-way stop controlled intersections, with adjustments for pedestrian critical gap and the percentage of yielding. Schroeder and Rouphail (2010) refer to the more complex interaction of the two modes, in which some drivers yield to pedestrians and some pedestrians accept gaps in traffic, as a mixed-priority crossing. In their research, mixed-priority pedestrian delay models were created for single-lane roundabouts using behavioral crossing data. NCHRP Report 674 includes models for two-lane roundabouts and channelized right-turn lanes (Schroeder, et al. 2011).

In the HCM pedestrian delay method, mean yield rates are given for several crossing treatments, such as overhead flashing beacons, high-visibility signs, and rectangular rapid-flashing beacons. However, only a few sites were studied for each of these treatments and no predictive model exists to estimate the yield probability based on site-specific attributes. For the mixed-priority pedestrian delay models (Schroeder and Rouphail, 2010), the probability of crossing is based on the driver yield occurrence/acceptance and critical gap occurrence/acceptance. Three single-lane roundabouts were studied. It is recommended that practitioners supplement mean yield rates with local knowledge and engineering judgment.

This thesis uses data recently collected in three states (North Carolina, Florida, and Alabama) to validate or improve the HCM pedestrian delay method and the mixed-priority pedestrian delay models. This effort will be based on observed driver yielding behavior at 27 unsignalized crosswalks, as well as observed delay to pedestrians. Statistical models will be used to predict motorist yield rate (M_y) based on several explanatory variables. The model will be validated by comparing the predicted yield rates with the observed rates. Critical gap will be determined using the HCM equation, maximum likelihood estimation, and the graphical method. For pedestrian delay, each critical gap value and the observed and predicted yielding rates will be entered into the HCM method and mixed-priority model. All pedestrian delay estimation values will then be compared to the field estimated delay.

The major objectives of this research are as follows:

- Determine critical gap using three methods: HCM calculation, maximum likelihood estimation, and the graphical method
- Develop a regression model to predict motorist yielding rate based on site characteristics that could be implemented into a new edition of the HCM
- Compare model predicted motorist yielding rates with observed rates
- Compare 12 average pedestrian delay estimates for each site with the observed field delay for the site
- Validate and improve HCM 2010 and mixed-priority pedestrian delay estimation methods

The contribution of this thesis differs from the STRIDE final project report. The STRIDE research team developed a microscopic driver yielding model at the observation level, while a macroscopic model of driver yielding at the site level using site characteristics was developed as a part of this thesis. Schroeder (2008) developed models for yield probability based on event level variables for two unsignalized midblock crossings. Microscopic characteristics for these models included vehicle speed, presence of an adjacent yield, which lane the vehicle was in, whether the driver was part of a low-speed platoon, presence of multiple pedestrians, presence of an aggressive pedestrian, necessary deceleration rate, presence of female pedestrians, and whether the site was located on-campus and in which state. Most of these variables change from one event, pedestrian-vehicle interaction, to the other. A microscopic model with behavioral characteristics would not be reasonable for implementation in the HCM.

This document is divided into six chapters. This chapter presented an introduction to the research and explains the objective. Chapter 2 discusses driver yielding behavior in the literature, the HCM pedestrian delay method, and the mixed-priority pedestrian delay models. Chapter 3 lays out a data collection and analysis methodology used for the overall research project and other input data required for the HCM method and mixed-priority model. Regression modeling techniques and explanatory variables are also discussed for driver yielding rate. Chapter 4 covers the development of the motorist yield rate model through linear regression and model validation. Chapter 5 discusses data exploration, specifically critical gap values, yield rates observed and predicted, and comparing field delay to the HCM method and mixed-priority model predictions. Chapter 6 summarizes the conclusions and recommendations.

2. LITERATURE REVIEW

2.1 Background

Crossing the road while walking is one of many potentially risky pedestrian maneuvers, where it is necessary to make an accurate assessment of the gap between vehicles (Hunt, et al. 2011). Pedestrians must determine if it is possible to complete the crossing maneuver before an oncoming vehicle arrives. Crossing is increasingly difficult on busy arterials with high pedestrian volume, such as Central Business Districts (CBD) of urban areas (Sun, et al. 2003). The decision to cross in a particular gap is based on the pedestrian's perception of the arrival time of the next vehicle, as well as how long it would take them to cross the road at a comfortable walking speed. This decision is further complicated by the possibility of a driver deciding to yield.

Driver yielding and pedestrian gap acceptance affect the capacity and delay of the system. Pedestrian delay is essentially a function of the critical gap (for a single pedestrian or a group) and the probability that a driver will make the decision to yield for a given crossing event. The term critical gap is often used to describe the threshold a particular pedestrian uses to determine acceptability of a gap (Cambridge Systematics, Inc. 2004). Critical gap can also refer to the time between consecutive vehicles when a pedestrian is waiting at the crosswalk, where the pedestrian is equally likely to accept or reject a gap (Schroeder and Rouphail 2011a). For the pedestrian population and analysis method in the Highway Capacity Manual (HCM), critical gap is defined as the minimum average gap length accepted by half of all pedestrians to safely cross the street.

2.2 Pedestrian Behavior

Pedestrian walking speed affects gap acceptance. The variability of walking speed is important when discussing crossing behavior, because it is directly proportional to the time required to cross a given distance (Fitzpatrick, et al. 2006). Bennett, et al. (2001) investigated pedestrian walking speeds at signalized intersections and mid-block crossings and found slower average speeds at mid-block locations. One would expect walking speeds to be lower for older pedestrians and groups of pedestrians. Fitzpatrick, et al. (2006) recommended in a National Cooperative Highway Research Program (NCHRP) report to lower the pedestrian walking speed used by the Manual on Uniform Traffic Control Devices (MUTCD) from 4.0 ft/s to 3.5 ft/s, but further acknowledged that even lower speeds may be appropriate in some cases, such as where pedestrians typically walk more slowly or use wheelchairs in the crosswalk. The research found that as many as one third of pedestrians travel at a slower pace, specifically children, older pedestrians, and persons with disabilities. Tarawneh (2001) suggested a walking speed of 1.11 m/s (3.6 ft/s), with small groups and those aged 21-30 walking faster than groups of three or more and other age groups. The mean start-up time (from the start of the Walk signal to the moment the pedestrian steps off the curb and starts to cross) was 2.5 s for older pedestrians, compared with 1.9 s for younger ones (Fitzpatrick, 2006).

Pedestrians may lose patience due to longer delays when trying to find gaps and may decide to take a risk on a shorter gap. Sun, et al. (2003) and Wang, et al. (2010) examined the relationship between waiting time and gap acceptance. Their results showed a marginal increase in minimum gap acceptance as waiting time increased. This is likely due to the fact that some pedestrians who wait longer are risk-free and only accept very safe gaps. The waiting time is increased for them, simply because they are waiting for a larger gap. On the other hand, Dunn and Petty (1984) found that pedestrians at midblock crossings who have been waiting for 30 or more seconds showed more risky behavior. One of the characteristics of risky behavior is accepting shorter gaps in traffic. Accordingly, the HCM predicts an increasing likelihood of non-compliance as pedestrian delay increases (TRB, 2010). Therefore, non-compliance can be translated into adjusting the critical gap to a lower value as the pedestrian delay or waiting time increases. Tarawneh (2001) found through a study in Jordan that pedestrians tended to walk faster after longer wait times.

Pedestrians can accept the same gap simultaneously, unlike queued vehicles on a single-lane approach (Schroeder and Rouphail, 2011a) (Schroeder, 2008). Sun, et al. (2003) found that the mean minimum gap accepted by groups increase with group size and the probability of a group of 4 pedestrians accepting shorter gaps is less than the probability for an individual. One reason for this could be that pedestrians in a group are unwilling to take risks because of the presence of other people, and are thus forced to wait for a gap that is acceptable for the whole group. Yagil also found that a group of pedestrians behaves differently than individuals. Pedestrians were more likely to wait at an intersection if encountering a group of pedestrians already waiting at the crosswalk (Yagil, 2000).

2.3 Methods for Determining Critical Gap

The critical gap of an individual driver or pedestrian is the minimum gap that is found to be acceptable, and is thus smaller than the accepted gap and greater than the largest rejected gap (Troutbeck, 1992). The mean critical gap for the population is the minimum average gap length accepted by half of all pedestrians to safely cross the street. More difficult crossing maneuvers, such as having to cross multiple lanes of traffic, results in increased critical gaps. Two-stage crossings, where a median of some sort provides refuge to the pedestrian, may reduce the critical gap for the near lane. Several methods are available for determining the critical gap. The method in the HCM 2010 is based on the crosswalk length, pedestrian walking speed, and start-up and end clearance time.

Other methods include the maximum likelihood method and graphical method. Log-normal distribution of critical gaps is assumed for maximum likelihood estimation, as well as the pedestrian population being consistent and homogeneous. Troutbeck (1992) reviewed ten methods for determining critical gap and concluded that the maximum likelihood method gave the best results. Troutbeck created an excel spreadsheet coded with a visual basic macro that can be used to calculate the mean critical gap. Critical gap measurements based on paired observations of the maximum rejected gap and the accepted gap require user entry in the maximum likelihood method (Tian, et al. 2000). The resulting critical gap value can be taken to be the average value of all pedestrians observed. The method also provides a standard deviation of the critical gap estimate. If all pedestrians accept the first gap offered without rejecting any gap, other methods should be considered or more data should be collected. Similarly, if the difference between the accepted and largest rejected gap is very large, the method sometimes cannot converge at an estimate.

The graphical method for pedestrian gap acceptance uses all of the rejected gaps and accepted gaps, rather than only being able to use paired observations (Chae, 2005). Rejected gaps and accepted gaps are plotted by gap size and cumulative distribution of the gaps. The critical gap is the intersection of these two probability plots, where the probability of accepting a gap is the same as rejecting a gap. Results are easily biased and provide approximate results.

Methods for vehicular gap acceptance typically include the concept of follow-up time. Follow-up time is not considered for pedestrians since they are not required to wait in a single queue as vehicles do, but can instead cross together in an acceptable gap (Schroeder, 2008). The HCM (2010) includes calculations for the spatial distribution of pedestrians and the group critical headway.

2.4 Yield Behavior

The decision of a driver to yield or of the pedestrian to GO is generally dependent on the following classes of variables: vehicle dynamics, driver characteristics, pedestrian characteristics, and confounding factors (Schroeder and Rouphail 2011b). Vehicle dynamics include maximum deceleration rate, travel speeds, and distance from the crosswalk. Driver characteristics include the willingness to yield and driver courtesy. Pedestrian characteristics include assertiveness, the presence of multiple pedestrians, or the willingness to accept risk. In addition to the personal attributes above, confounding factors describing the circumstances surrounding the interaction may impact the decision-making process. Examples include the presence of a downstream queue/congestion after the crosswalk or a yield event in the opposing direction or adjacent travel lane. Yielding behavior is also intuitively related to whether or not a vehicle is traveling in a platoon of vehicles. Other factors may include the cross-section of the road, the type of crossing treatment or the general level of congestion at the crossing location.

The rate of driver yielding to pedestrians at unsignalized crosswalks varies across locations (Rodegerdts, 2007), but in nearly all cases is less than 100%. Findings from NCHRP report 572 (Rodegerdts, 2007) show that 43% of the drivers at two-lane approaches of the roundabout do not yield to pedestrians. The lack of yielding is only 17% for single-lane roundabouts. It can be said that the number of lanes affect driver yielding, at least for roundabouts. Fitzpatrick (2006) noted that certain roadway design elements, such as number of through lanes and posted speed limit, have a significant impact on driver yielding at unsignalized midblock crossings. Motorists are also more likely to yield on narrow, low-volume roadways.

Pedestrians will experience less delay if motorists yield to create crossing opportunities. Motorist yield rates vary and can be influenced by factors such as roadway geometry, travel speeds, pedestrian crossing treatments, local culture, and law enforcement practices. A range of treatments exist that are intended to increase the rate of driver yielding (Fitzpatrick, 2006). Figure 1 is provided in the HCM 2010 as exhibit 19-17 and provides yielding rates for various crossing treatments from data collected for staged (controlled) and unstaged pedestrians. These values are based on a limited number of sites and also do not cover the full range of available crossing treatments. It is suggested that local knowledge and engineering judgment be used to supplement these values (TRB, 2010).

Crossing Treatment	Staged Pedestrians		Unstaged Pedestrians	
	Number of Sites	Mean Yield Rate, %	Number of Sites	Mean Yield Rate, %
Overhead flashing beacon (push button activation)	3	47	4	49
Overhead flashing beacon (passive activation)	3	31	3	67
Pedestrian crossing flags	6	65	4	74
In-street crossing signs (25–30 mi/h)	3	87	3	90
High-visibility signs and markings (35 mi/h)	2	17	2	20
High-visibility signs and markings (25 mi/h)	1	61	1	91
Rectangular rapid-flash beacon	N/A	N/A	17	81

Source: Fitzpatrick et al. (*11*) and Shurbutt et al. (*12*).

Figure 1: Yielding Rates for Various Crossing Treatments (TRB, 2010)

In previous research, Sun, et al. (2002) collected data on driver yielding and pedestrian gap acceptance at an unsignalized midblock pedestrian crossing. They found that drivers are more likely to yield to a group of pedestrians and that older drivers were more likely to yield than younger drivers. The authors looked at only one crosswalk and did not analyze any pedestrian treatment effects. The authors collected 1.5 hours of each AM and PM peak data over 5 days, for a total of 15 hours of data. The resulting samples included 687 accepted gap, 938 rejected gap and 1254 motorist yield data points.

A study at two two-lane roundabouts in Maryland shows that the speed of the vehicle significantly influences the driver yielding rate (Geruschat and Hassan, 2005). The authors found that 65% of the variability of driver yielding rate could be explained by speed. Based on their study, they estimated that the driver yielding rate with speeds lower than 15 miles per hour is about 75% as opposed to 50% with speeds higher than 20 miles per hour. The authors also observed that pedestrian behavior influenced the driver yielding rate. Drivers were almost twice as likely to yield to a pedestrian standing one foot in the crosswalk as one foot from the curb. Their study also showed that drivers had significantly higher yielding rates to pedestrians carrying a white cane.

A study by Salamati, et al. (2012) showed that the drivers tend to yield to pedestrians with white canes more often than sighted pedestrians. Drivers traveling in the far lane, relative to pedestrian location, have lower probability of yielding to a pedestrian than drivers in the near lane. As the speed increases the probability of driver yielding decreases. The results show that factors such as vehicle platooning, downstream conflict and pedestrian waiting position do not have a significant impact on the probability of a driver yielding to a pedestrian.

2.5 Highway Capacity Manual 2010 Pedestrian Delay Estimation

In chapter 19 of HCM 2010, pedestrian level of service (LOS) at two-way stop-controlled intersections (TWSC) is defined for pedestrians crossing the traffic stream not controlled by a STOP sign (TRB, 2010). This method can be applied to midblock pedestrian crossings. LOS is determined by computed or measured control delay, where extremely long delays result in LOS F. The pedestrian methodologies were developed separately from the vehicle methodology and have different limitations. These pedestrian methodologies apply to TWSC intersections and midblock crossings where up to four through lanes can be crossed, or eight through lanes if a center median is provided for refuge. It is assumed that the crosswalk is isolated, away from signals.

The TWSC intersection methodology for the pedestrian mode is applied through a series of steps requiring input data related to vehicle and pedestrian volumes, geometric conditions, and motorist yield rates to pedestrians to calculate the average pedestrian delay. The following six steps are required:

(1) Identify two-stage crossings,

(2) Determine critical headway,

(3) Estimate probability of a delayed crossing,

(4) Calculate average delay to wait for adequate gap,

(5) Estimate delay reduction due to yielding vehicles, and

(6) Calculate average pedestrian delay and determine LOS

Pedestrians typically cross in two stages when a refuge is provided. Pedestrian delay should be estimated for each stage of crossing and summed to establish average delay for entire crossing (TRB, 2010).

Critical gap, or headway, is then estimated, which is the minimum average gap length accepted by half of all pedestrians. The critical headway for a single pedestrian (t_c, seconds) depends on the crosswalk length (L, feet), average pedestrian walking speed (S_p, ft/s), and pedestrian start-up time and end clearance time (t_s, seconds) (TRB, 2010). The equation is shown below. Rouphail, et al. (2005) described pedestrian gap acceptance as the sum of latency and actual crossing times, an approach similar to the HCM 2010 method discussed above. Field estimates of latency time could be used in place of the HCM 2010 start-up time.

$$t_c = \frac{L}{S_p} + t_s \tag{1}$$

The group critical headway ($t_{c,G}$, seconds) calculation requires that the spatial distribution of pedestrians (N_p, pedestrians) be calculated from the total number of pedestrians in the crossing platoon (N_c, pedestrians). Equations for group critical headway, spatial distribution of pedestrians, and total number of pedestrians in the crossing platoon are provided below (TRB, 2010):

$$t_{c,G} = t_c + 2(N_p - 1) \tag{2}$$

$$N_p = Int\left[\frac{8.0(N_c - 1)}{W_c}\right] + 1 \tag{3}$$

$$N_c = \frac{v_p e^{v_p t_c} + v e^{-v t_c}}{(v_p + v) e^{(v_p - v) t_c}} \tag{4}$$

Where: 8.0 = Default clear effective width used by a single pedestrian to avoid interference when passing other pedestrians (ft)

W_c = Crosswalk Width (ft)

v_p = Pedestrian Flow Rate (ped/s)

v = Vehicular Flow Rate (veh/s)

Probability of a pedestrian not incurring a delayed crossing is equal to the likelihood that a pedestrian will encounter a gap greater than or equal to the critical headway immediately upon arriving at the intersection. This probability can be estimated using a Poisson distribution and assuming random arrivals of vehicles. The probability of a blocked lane (P_b), gap not exceeding critical headway, is the complement. Probability of a delayed crossing (P_d), assuming independent distribution of traffic in each through lane, is a function of the blocked lane probability and the number of through lanes crossed (L) (TRB, 2010).

$$P_b = 1 - e^{\frac{-t_{c,G}v}{L}} \tag{5}$$

$$P_d = 1 - (1 - P_b)^L \tag{6}$$

The average delay to a pedestrian waiting for an adequate gap at an unsignalized crossing, assuming no driver yielding, depends on the critical headway and vehicular flow rate. Average gap delay for pedestrians who incur nonzero delay (d_{gd}, seconds) is a function of the probability of a delayed crossing and the average pedestrian gap delay (d_g, seconds). Both equations are given below (TRB, 2010):

$$d_g = \frac{1}{v}\left(e^{vt_{c,G}} - vt_{c,G} - 1\right) \tag{7}$$

$$d_{gd} = \frac{d_g}{P_d} \tag{8}$$

For any yielding event, each through lane is either clear or blocked. If the lane is clear or the driver in the blocked lane chooses to yield, the pedestrian may cross. Otherwise, the pedestrian must wait for the next potential yielding event. This process will repeat until the wait exceeds the expected delay for an adequate gap in traffic, at which point the average pedestrian will receive an adequate gap in traffic and will be able to cross without having to depend on yielding motorists. The equation below shows the calculation for average pedestrian delay (d_p, seconds), where the first term represents the delay in waiting for a yield, the second term represents the delay in waiting for an adequate gap, and the third term represents the adjustment to delay in the event of a driver yielding (TRB, 2010):

$$d_p = \left(\sum_{i=1}^{n} h(i - 0.5)P(Y_i)\right) + d_g - d_{gd} \sum_{i=1}^{n} P(Y_i) \tag{9}$$

Where: i = Crossing Event (i = 1 to n)

h = Average Headway for each through lane

$P(Y_i)$ = Probability that motorists yield to pedestrian on crossing event i

n = Int(d_{gd}/h), average number of events before an adequate gap is available

The probability that a driver will yield for a given crossing event are provided with models for each of one, two, three, and four lane crossings. The equation is simplest when the pedestrian is crossing one through lane and is provided below for any crossing event:

$$P(Y_i) = P_d M_y \left(1 - M_y\right)^{i-1} \tag{10}$$

Where: M_y = Motorist Yield Rate (decimal)

The motorist yield rate in the HCM can be obtained from Exhibit 19-17, which is replicated above as Figure 1. However, that exhibit only shows average yield rates for some treatments, and is not sensitive to crosswalk geometry or behavioral attributes. No method or equation exists in the HCM to estimate M_y, short of field measurements.

For multi-lane crossings, the probability that motorists yield requires motorists in both/all lanes to yield simultaneously if both/all lanes are blocked or a single motorist to yield if only one lane is blocked. The equations for two-, three-, and four-lane crossings are provided below, respectively (TRB, 2010):

$$P(Y_i) = \left[P_d - \sum_{j=0}^{i-1} P(Y_j)\right]\left[\frac{\left(2P_b(1-P_b)M_y\right)+\left(P_b^2M_y^2\right)}{P_d}\right] \tag{11}$$

$$P(Y_i) = \left[P_d - \sum_{j=0}^{i-1} P(Y_j)\right]\left[\frac{P_b^3M_y^3+3P_b^2(1-P_b)M_y^2+3P_b(1-P_b)^2M_y}{P_d}\right] \tag{12}$$

$$P(Y_i) = \left[P_d - \sum_{j=0}^{i-1} P(Y_j)\right]\left[\frac{P_b^4M_y^4+4P_b^3(1-P_b)M_y^3+6P_b^2(1-P_b)^2M_y^2+4P_b(1-P_b^3)M_y}{P_d}\right] \tag{13}$$

The final step in the HCM method is to calculate the average pedestrian delay, the service measure, and determine the LOS. For two-stage crossings, the pedestrian delay is equal to the sum of the delay for each stage of the crossing (TRB, 2010).

2.6 Mixed-Priority Pedestrian Delay Models

Schroeder and Rouphail (2010) developed mixed-priority pedestrian delay models at single-lane roundabouts by using behavioral crossing data. “Mixed-priority” refers to crosswalk operations where drivers at times yield to create crossing opportunities, but sometimes pedestrians need to rely on their own judgment of gaps in traffic to safely cross the street. Probabilistic behavioral parameters were measured for controlled crossings by blind pedestrians. The delay model is designed to apply to any pedestrian population, where perfect utilization can be assumed for sighted pedestrians. Delay is predicted as a function of the following probabilities: encountering a crossing opportunity in the form of a yield or crossable gap and utilization of said yield or gap, with these factors being combined to produce an overall probability of crossing. A multi-linear, log-transformed regression approach was used to predict the average pedestrian delay. The variables were transformed by applying the natural logarithm because the delay distribution suggested a log-normal distribution by being skewed to the left. The resulting model was statistically significant and produced good estimates of pedestrian delay that matched the observed field data.

The mixed-priority pedestrian delay model allows engineers to make predictions about operational performance, both current and future (Schroeder and Rouphail, 2010). It is applicable where pedestrian delay is determined by a mix of pedestrian gap acceptance and driver yielding behavior. The following variables were calculated and considered for model development: probability a vehicle would yield to the pedestrian, probability a yield event is encountered, probability of yield utilization, probability of a gap being crossable, probability of encountering a crossable gap, observed delay per leg, minimum delay, probability of crossing in a yield, probability of crossing in a crossable gap, probability of crossing. The final model was determined by significant parameter estimates, a high adjusted R^2 value, and relatively simple and practical model form. Pedestrian delay was predicted as a function of *P*(Cross), the overall probability of crossing calculated from the four individual probability parameters, for the recommended model, shown below (Schroeder and Rouphail 2010):

$$d_p = -0.78 - 14.99\ln(P_{\text{cross}}) \tag{14}$$

Where: d_p = average pedestrian delay (seconds)

ln = natural logarithm

P_{cross} = probability of crossing = *P*(Y_and_GO) + *P*(CG_and_GO)

= *P*(Y_ENC)**P*(GO | Y) + *P*(CG_ENC)**P*(GO | CG)

P(Y_and_GO) = probability of crossing in a yield

P(CG_and_GO) = probability of crossing in a crossable gap (CG)

P(Y_ENC) = probability of encountering a yield event

P(GO | Y) = probability of yield utilization

P(CG_ENC) = probability of encountering a CG event

P(GO | CG) = probability of crossing in a critical gap

The overall model and the *P*(Cross) parameter were significant at $p<0.0001$ and the adjusted R^2 value suggests that 63.3% of the variability of the data was explained by the model. The minimum delay collected from the field represented the crossing experience of sighted pedestrians. P(Y_ENC), P(GO | Y), P(CG_ENC), and P(GO | CG) need to be field-measured or derived from literature, previous studies, and traffic theory. Average pedestrian delay of sighted pedestrians can be determined by assuming perfect opportunity utilization. Utilization rates for blind pedestrians and other special populations, such as children and the elderly, will be lower and requires judgment by the analyst. Rate of driver yielding, P(Yield), can be measured in the field using manual tally and stop-watch methods. In the absence of field data, results from past studies can be used as guidance. The probability of encountering a yielding vehicle (P(Y_ENC)) better represents the flow rate that the pedestrian is likely to experience, since it is calculated from all encountered vehicles rather than those that could have yielded. The equation for estimating P(Y_ENC) is provided below (Schroeder, et al. 2011):

$$P(\text{Y\_ENC}) = P(\text{Yield}) * \left(100\% - P(\text{CG\_ENC})\right) \tag{15}$$

Availability of crossable gaps can be field-measured or estimated using traffic flow theory concepts on the basis of traffic volume and an assumed headway distribution. May (1990) used a simple negative exponential distribution to determine the probability of observing a headway greater than the critical headway (t_c, seconds) calculated using HCM methods, which is the probability of encountering a crossable gap (P(CG_ENC)). The equation is provided below (Schroeder and Rouphail, 2010):

$$P(\text{CG\_ENC}) = P(headway \geq t_c) = e^{\frac{t_c}{t_{avg}}} \tag{16}$$

Where: t_{avg} = average headway, (3600 s/h)/(vehicles/h)

2.7 Limitations of Current Pedestrian Delay Models

Both the HCM 2010 method and the mixed-priority pedestrian delay model have some limitations. The method provided in the HCM 2010 has not been empirically calibrated or validated with field observations (TRB, 2010). The mixed-priority pedestrian delay model was developed using field data, but the data was only collected for controlled blind pedestrian crossings at single-lane roundabouts (Schroeder and Rouphail, 2010). Driver yielding rates for various crossing treatments from data collected for staged (controlled) and unstaged pedestrians are provided in the HCM 2010. These values were found from a limited number of sites and all available treatments are not covered. Local knowledge and engineering should be used to supplement these values.

Several assumptions are made for the HCM 2010 method. The analysis assumes random arrivals and equal directional and lane distribution by drivers, as well as consistent and homogeneous drivers and pedestrians. Interaction effects of upstream signalized intersections are not accounted for in the HCM 2010 method. When traffic signals are located less than a quarter mile from the crossing, flows will likely have some platoon structure and will not be random. Steady-state conditions are assumed and pedestrian cross-flows are ignored. It is assumed that pedestrians will reach the crossing without delay from other pedestrians, which may not be true under high pedestrian volumes. Most importantly though, the HCM does not offer a method to predict the likelihood of drivers yielding as a function of site attributes, geographic location, or behavioral characteristics of drivers or pedestrians at the crosswalk.

3. METHODOLOGY

This is an empirical study which uses field observations to validate and improve the HCM pedestrian delay method and the mixed-priority pedestrian delay model. Observed driver yielding behavior and observed delay to pedestrians will be used in this effort. Critical gap, or headway, will be determined using the HCM method, maximum likelihood method, and the graphical method. SAS is used to develop a motorist yielding model based on site variables. This model is validated by comparing predicted yield rates with observed rates from field data. For pedestrian delay, the observed yielding rates and predicted yielding rates, as well as the critical gaps from the HCM, maximum likelihood, and graphical methods, will be entered into the HCM method and mixed-priority models. Both will then be compared to the field estimated delay. Figure 2 below shows the methodology as a flow chart.

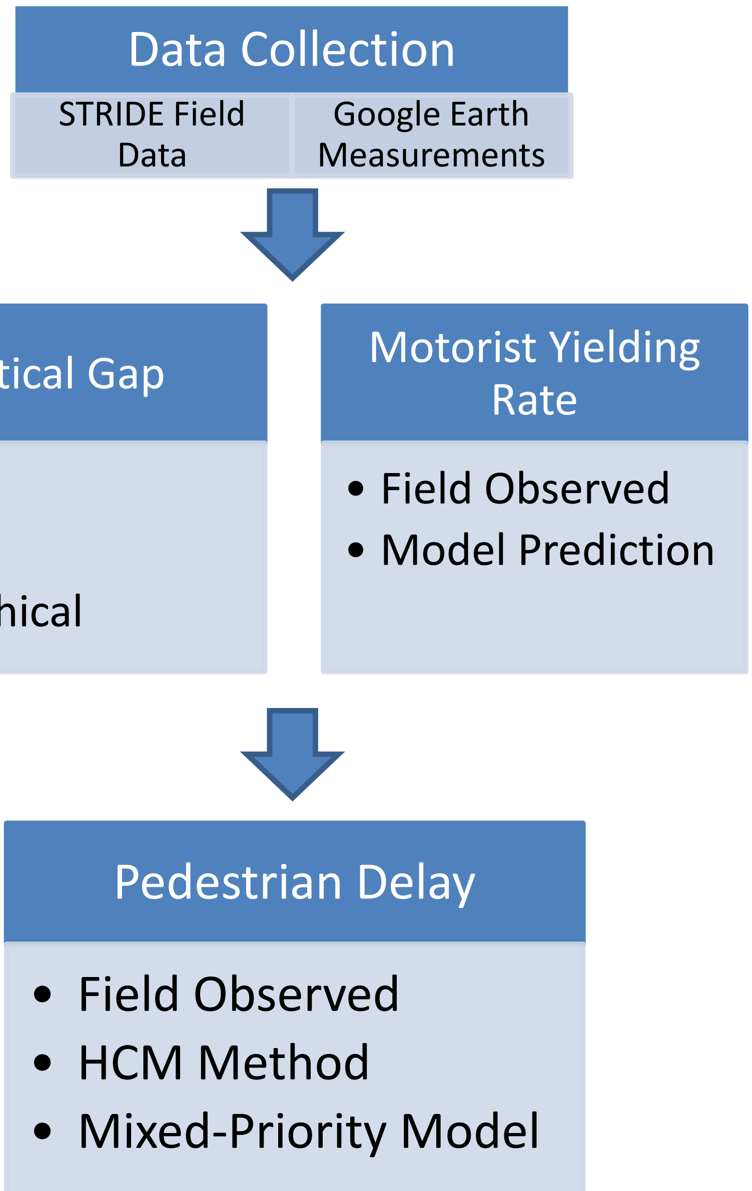


Figure 2: Methodology

The HCM 2010 method and the mixed-priority pedestrian delay model both require certain input data. Observational data on pedestrian-vehicle interaction was collected at the event level for midblock pedestrian crossings. Event level in this context refers to the interaction of one vehicle and one pedestrian. Vehicle volumes will be collected rather than assuming equal directional distribution by drivers at two-stage crosswalks. Non-yielding platooned vehicles will be accounted for in longer field delays. Driver yielding behavior and pedestrian gap acceptance may vary significantly at locations with varying land use and facility designs. In order to achieve greater heterogeneity in the study, various unsignalized midblock crossing locations were considered for the observational studies with varying pedestrian treatments, lane configurations, built-up environment, and travel activity from Alabama, Florida, and North Carolina. At sites with limited pedestrian activity, controlled experiments were performed. A total of 27 sites were selected for data collection.

The following definitions will be used through the remainder of the thesis:

- Event: The interaction of one vehicle and one pedestrian.
- Yield: A driver either coming to a complete stop or slowing down to create a crossing opportunity for the pedestrian(s)
- Gap: Time (in seconds) between two successive vehicle arrivals (front bumper to front bumper)
- Lag: Time (in seconds) between the pedestrian arrival at the crosswalk and the arrival of the first vehicle measured in seconds. For analysis purposes, lags were considered to be accepted/rejected gaps
- Critical Gap/Headway: Minimum average gap length accepted by half of all pedestrians to safely cross the street.

3.1 Data Needs for Analysis Methods

In order to use the HCM 2010 method and the mixed-priority pedestrian delay model to estimate the average pedestrian delay at several sites, input data must be gathered from field visits. The information in Table 1 is needed for the HCM 2010 method, where variables in bold must be gathered from field and video data in order to calculate the other values (TRB, 2010).

Table 1: HCM Pedestrian Delay Method Inputs

Variable	Meaning	Units
t_c	Critical headway for a single pedestrian	Secs.
L	**Crosswalk length**	**Feet**
$\mathbf{S_p}$	**Average pedestrian walking speed**	**Ft/s**
$\mathbf{t_s}$	**Pedestrian start-up time and end clearance time (assume default of 3 secs.)**	**Secs.**
N_p	Spatial distribution of pedestrians	Peds.
N_c	Total number of pedestrians in the crossing platoon	Peds.
$\mathbf{W_c}$	**Crosswalk width**	**Feet**
$\mathbf{v_p}$	**Pedestrian flow rate**	**Ped/s**
v	**Vehicular flow rate**	**Veh/s**
$t_{c,G}$	Group critical headway	Secs.
P_b	Probability of a blocked lane	Decimal
P_d	Probability of a delayed crossing	Decimal
L	**Number of through lanes crossed**	**Integer (+)**
d_g	Average pedestrian gap delay	Secs.
d_{gd}	Average gap delay for pedestrians who incur nonzero delay	Secs.
d_p	Average pedestrian delay	Secs.
i	Crossing event (i=1 to n)	Integer (+)
h	**Average headway for each through lane**	**Sec/veh/lane**
$P(Y_i)$	Probabilities that motorists will yield for a given crossing event	Decimal
n	Int(d_{gd}/h), average number of crossing events before an adequate gap is available	Integer (+)
$\mathbf{M_y}$	**Motorist yield rate**	**Decimal**

In the HCM method, the critical headway is calculated using the crosswalk length, average pedestrian walking speed, and the pedestrian start-up time and end clearance time. Default values are provided for average pedestrian walking speed and pedestrian start-up time and end clearance time in the HCM 2010 and can be used when field data is unavailable. Pedestrian start-up time and end clearance time was not collected in the field, so the default value of three seconds will be used. It is best to use as few default values as possible, since the accuracy becomes more approximate as the number of default values used in analysis increases. Maximum likelihood estimation and the graphical method will also be used to determine the critical gap from rejected and accepted gaps.

The mixed-priority pedestrian delay model requires the calculation of several probabilities; including encountering a yield, yield utilization, encountering a crossable gap, and gap utilization. These probabilities are used to calculate the probability of crossing. Probability of yield utilization was assumed to be 100%, which is a reasonable assumption when all pedestrians are sighted. Number and size of rejected gaps, critical gap, and whether the pedestrian crossed in a gap or yield are required to calculate the other probabilities. Other variables will be needed to develop an improved driver yielding model for the estimation of pedestrian delay. Number of lanes, crossing width, traffic volume, pedestrian volume, treatment in place, whether the site is one-way or two-way, on-campus or off-campus, and which state the site is located in will be used to develop the improved model.

3.2 Field Data Collection

The data collection methodology to evaluate the interaction of pedestrians and drivers at midblock pedestrian crossings at a microscopic or event level was borrowed from previous research by Schroeder (2008). Accepted and rejected gap sizes, pedestrian walking speed, observed pedestrian delay, and driver decision will be gathered from individual observations. Percentage of driver yielding and average pedestrian delay can be determined from the data. The data collection effort was performed in conjunction with a Southeastern Transportation Research, Innovation, Development and Education Center research project (STRIDE 2012-016S). A variety of empirical data on pedestrian-vehicle interaction was collected using observational studies. These variables must be collected accurately using reliable measurement devices and coded consistently between crossing events, sites, and locations to effectively evaluate the interaction. Observational data are obtained from trained observers with the help of tally sheets, video surveillance equipment, and radar detection devices. The beginning of an interaction is defined to begin when a pedestrian arrives in the crosswalk influence area or waiting location while a driver is on the approach of the crosswalk. An assumption is made that both the driver and pedestrian are consciously aware of each other's presence.

A three-pronged data collection approach that combined real-time observations by a trained observer on a tally sheet, video recording of the crosswalk, and LIDAR speed measurements of approaching vehicles was used. Multiple hours of observations were taken at each site. The LIDAR was used to record the speed and distance of the approaching vehicle once the pedestrian (waiting at the curb or in the crosswalk) entered the view of the driver. A video camera was set up on a tripod to capture the pedestrian-vehicle interactions so that the researchers could gather some data at a later point, rather than having to quickly gather everything immediately. The researcher taking the speed and distance measurements would read these values out loud, so that the interaction could be quickly identified in the video. Figure 3 shows a schematic of the data collection set-up.

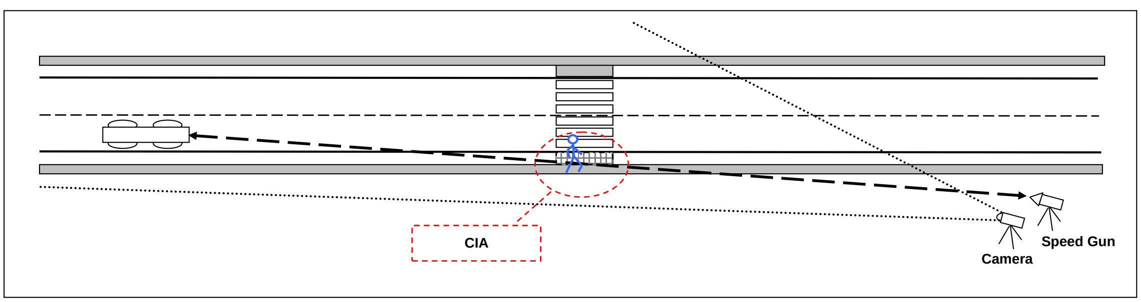


Figure 3: Field Data Collection Set-Up

In order to capture all relevant data, the video angle has to cover events concurrent to the interaction, such as the presence of multiple pedestrians. As shown in the diagram, the video camera angle is wide enough to cover the crosswalk influence area (CIA) or waiting location, and the approach to the crosswalk.

Site visits and/or using Google Earth served as a starting point to collect basic information about potential sites. A few sites generated little naturally occurring pedestrian interactions with vehicular movements. Controlled experiments comprising of staged crossings by research members while another member recorded detailed vehicular data were used at such locations to obtain pedestrian-vehicle interactions.

A total of 27 sites were selected for data collection and analysis (9 in Alabama, 10 in Florida, and 8 in North Carolina), where 11 of the sites were at university on-campus locations. An example of a study site from North Carolina is depicted in Figure 4. Posted speeds at these locations ranged from 15 mph to 40 mph. Single lane and multilane configurations were used. Some of the sites also had bike lanes and on-street parking.

Figure 4: Example of Study Site and Data Collection Setup

Table 2, Table 3, and Table 4 give details for each site, including streets, city, number of lanes, description, and campus type (on-campus or off-campus), as well as the naturally occurring and controlled event sample sizes.

Table 2: Data Collection Sites in North Carolina

	Crosswalk Location	City	Lanes	Description	Type	Ntrl.	Ctrl.
1	Fayetteville at Hargett & Martin	Raleigh	Two	Single stage, On-street parking	Off	11	33
2	Fayetteville at Morgan & Hargett	Raleigh	Two	Single stage, On-street parking	Off	14	36
3	Wilmington at Hargett & Martin	Raleigh	Two (one way)	Single stage, On-street parking	Off	79	8
4	S. Elm at Washington & February 1	Greensboro	Two	On-street parking	Off	19	167
5	S. Elm at Washington & McGee	Greensboro	Two	On-street parking	Off	6	91
6	Dan Allen at Thurman & Cates	Raleigh	Two	Single stage	On	37	0
7	Sullivan at Dan Allen & Varsity	Raleigh	Two	In-road sign, Single stage	On	15	0
8	Main Campus at Research & Campus Shore	Raleigh	Two	Single stage, On-street parking	On	19	0

Table 3: Data Collection Sites in Florida

	Crosswalk Location	City	Lanes	Description	Type	Ntrl.	Ctrl.
1	Gale Lemerand Dr.	Gainesville	Two	Flashing pedestrian sign, Bike lanes	On	35	82
2	Museum Rd. & Fraternity Row (EB)	Gainesville	Two	Single stage, Bike lanes	On	0	45
3	Museum Rd .& Fraternity Row (WB)	Gainesville	Two	Single stage, Bike lanes	On	0	47
4	Museum Rd. & Reitz Union Dr.	Gainesville	Two	Ped sign, Single stage, Bike lanes	On	30	0
5	Hull Rd.	Gainesville	Two	Single stage, Bike lanes	On	42	0
6	Museum Rd. & SW 13th St.	Gainesville	Two	Ped sign, Bike lanes	On	38	0
7	SW 2nd Ave. & SW 8th St.	Gainesville	Two	In-road sign, On-street parking, Bike lanes	Off	1	21
8	SW 2nd Ave. & SW 3rd St.	Gainesville	Two	In-road sign, On-street parking, Bike lanes	Off	0	29
9	SE 2nd Ave. & SE 6th St.	Gainesville	Two	Single stage, On-street parking, Bike lanes	Off	0	27
10	SW 2nd Ave. & SW 1st St.	Gainesville	Two	In-road sign, On-street parking, Bike lanes	Off	1	26

Table 4: Data Collection Sites in Alabama

	Crosswalk Location	City	Lanes	Description	Type	Ntrl.	Ctrl.
1	University Blvd. and Hackberry Lane	Tuscaloosa	Two	Single stage, Bike lanes	On	58	0
2	Richard Arrington Blvd. and 7th Ave N.	Birmingham	Three (one way)	Single stage, Curbside Parking	Off	220	9
3	Greens Springs Hwy. and 24th Ave S.	Birmingham	Three (one way)	Single stage	Off	0	66
4	10th St. S. and 10th Ave. S.	Birmingham	Four	Divided highway, only crossed half (single stage)	Off	0	248
5	7th Ave. and Campus Dr. (A)	Tuscaloosa	Two	Single stage, Bike lanes	On	0	39
6	7th Ave. and Campus Dr. (B)	Tuscaloosa	Two	Single stage, Bike lanes	On	0	50
7	University Blvd. and Colonial Dr.	Tuscaloosa	One (one way)	Single stage, Bike lane	On	1	62
8	Campus Dr. E. and 5th Ave.	Tuscaloosa	Two	Single stage, Bike lane	On	0	63
9	Ridge Rd. and Oxmoor Rd.	Homewood	Two	In-road sign, Single stage	Off	0	178

Table 5 summarizes all data collection elements used in pedestrian delay estimation by the HCM method and the mixed-priority method.

Table 5: Data Collection for Pedestrian Delay Estimation

	Factor	Description	Value
First Vehicle Variables	SPD	The speed of the first vehicle (mph), at the time the pedestrian arrives at crosswalk influence area (waiting location), recorded from speed gun	Mph
	DIST	The distance from the first vehicle to the researcher recorded from the laser speed gun	Ft
	Y_NY	Whether the first vehicle yielded or not	Yield=1, Non-yield=0
	TRIG	If the first vehicle yielded, was it triggered (forced) by the pedestrian. In other words, if the yield happened before pedestrian stepping into the crosswalk (0) or after (1)	Triggered Yield, Yes=1, No=0
	STP	Whether the first vehicle had already stopped at the time that the pedestrian arrived. Observations where the vehicle was previously stopped were removed.	Stopped=1
	DIST_DEL	Delay between when speed should have been taken (at time pedestrian arrives at CIA) and when gun beeped	Seconds
	ADJDIST	Vehicle position at time of pedestrian arrival in CIA measured using LIDAR speed measurement device; ADJDIST calculated from measured distance, speed, distance delay and Distance to Crosswalk; ADJDIST= DIST+SPD*1.467*DIST_DEL-Distance to Crosswalk	Ft
Pedestrian Variables	CTRL	Whether crossing pedestrian was controlled (researcher) or random (observational study)	Controlled=1, Random=0
	CROSS	Whether pedestrian crossed in a gap or a yield	Gap/Yield (G/Y)
	TIME	Time from pedestrian stepping into crosswalk to reaching measured location (such as specific white crosswalk marking, center line, or opposite side of crosswalk). Used in calculating pedestrian walking speed.	Seconds
	W_SP	Pedestrian walking speed while crossing; W_SP=Crossing Distance/TIME	Ft/s
	GAP	Observed lag or gap time, calculated from SPD and ADJDIST	Seconds
	REJ1 – REJ15	Rejected gap sizes	Seconds
	MAX REJ	The largest rejected gap for a single pedestrian (maximum value of REJ1 – REJ15)	Seconds
	ACC	Accepted gap size	Seconds

Table 5 Continued

	Factor	Description	Value
	DELAY	Time from the pedestrian arriving at the crosswalk influence area (waiting location) to stepping into the crosswalk to cross	Seconds
Site Variables	Distance to Crosswalk	The distance from the researcher using the laser speed gun to the middle of the crosswalk along the curb.	Ft
	Crossing Distance	The distance from the curb to a measured location, such as a specific white crosswalk marking, a center line, or the opposite side of the crosswalk.	Ft
	Crosswalk Length	Equivalent to lane width multiplied by the number of through lanes for each stage of crossing measured from Google Earth	Ft
	Crosswalk Width	Width of the crosswalk measured from Google Earth	Ft
	M_y	Observed motorist yield rate	Decimal
Other	COUNT	If the first vehicle did not yield, how many vehicles went through before the pedestrian crossed	Number
Video	v_p	Pedestrian flow rate	Peds/s
	v	Vehicular flow rate	Peds/s

Free-flow speed was found by averaging a sample of unimpeded vehicle speeds for each site. For sites where the free-flow speed was not reported, speed limit was used. Pedestrian and vehicular flow rates were found either in the field or from the video at a later point in time. The crosswalk lengths used will not include on-street parking, bike lanes, or median refuges.

3.3 Analysis Methods

The first step for pedestrian delay estimation in the HCM method was to identify two-stage crossings. For two-stage crossings, the crosswalk length is represented by a single lane width for bidirectional two-lane roads or by the width of two lanes for four-lane roadways where two lanes are crossed in a single stage. Vehicular volumes were determined for each direction for two-stage crossings. The delay will be calculated using each direction of vehicular volume for that stage of crossing. The total volume was used for single-stage crossings. The next step is to determine the critical headway and group critical headway for sites with pedestrian pooling. Walking speeds were averaged for each site and then used to find the critical headway for the HCM method of determining critical gap (seen in Equation 1). Maximum likelihood estimation and the graphical method were also used and the results compared in Chapter 5. If the value for the rejected gap or accepted gap size is not provided, values of "3" and "30" were respectively used. Group critical headway was calculated from Equation 2 using the critical headway for a single pedestrian and the spatial distribution of pedestrians, which was calculated from Equation 3 and Equation 4.

Only observations with rejected and accepted gaps can be used in maximum likelihood estimation. The accepted gap and the largest rejected gap for each observation are entered into the spreadsheet created by Troutbeck (1992). A visual basic code then runs through as many iterations as are necessary to converge on a single value. This value is taken to be the average for all pedestrians observed. Observations with very large accepted gaps may need to be removed. The threshold to determine what is considered "large" will be decided during data analysis. Sample sizes for this method are greatly reduced, since only paired observations could be used and large values had to be removed. Other methods may provide more accurate results from the larger sample sizes. Sites are segregated by crossing lengths into three groups, 30+ feet, 15 – <30 feet, and <15 feet crossing lengths. It is assumed that critical gaps for sites within each group would be similar since crossing time is strongly dependent on the length of the crosswalk. The graphical method used all of the rejected gaps and accepted gaps. Distributions were created for the three groups identified above. The intersection of distributions for each graph is taken as the critical gap for the sites in the grouping. This method is easily biased by outliers, so "large" values were removed. Other critical gap methods, such as Ramsey-Routledge method, could also have been used. For the purpose of this research, the number of methods was limited to three.

The third step for the HCM method of pedestrian delay estimation is to estimate the probability of a delayed crossing (Equation 6) from the probability of a blocked crossing

(Equation 5) and the number of through lanes crossed. The fourth step is to calculate the average delay to wait for an adequate gap (Equation 7) and the fifth step is to estimate the delay reduction due to yielding vehicles (third term of Equation 9). The final step is to calculate the average pedestrian delay (Equation 9) from the delay in waiting for a yield, delay in waiting for an adequate gap, and the adjustment to delay in the event of a driver deciding to yield. Delays related to driver yielding rely on $P(Y_i)$, the probability that motorists yield to the pedestrian on a particular crossing event *i*. Equations 10, 11, and 12 calculate this probability at one-, two-, and three-lane crossings, respectively. The probability of a blocked lane, probability of a delayed crossing, and motorist yield rate are used to calculate $P(Y_i)$. Motorist yield rates were found for each site by taking the number of motorists who yielded divided by the number of events and will also be predicted by a regression model developed in this thesis. These two values will be compared to test the validity of the regression model.

For mixed-priority pedestrian delay estimation, probabilities of a pedestrian encountering a yield event, utilizing a yield, encountering a crossable gap event, and utilizing a crossable gap must be calculated. These are used to calculate the probability of crossing which is then used to calculate the average pedestrian delay (Equation 14). *P(*Y_ENC*)*, the probability of encountering a yield, is zero if the pedestrian crossed in a gap (Y_NY = 0 and CROSS = G) and is equal to one (the number of yields encountered) divided by the number of motorists encountered (COUNT + 1) if the pedestrian was able to cross in a yield (CROSS = Y). *P*(Y_ENC) will also be found by using equation 15 and the model predicted *P*(Yield). Triggered yields are counted as accepted gaps by pedestrians and as motorist yield events. Yield utilization, *P(*GO | Y*)*, was assumed to be 100%. Probability of encountering a crossable gap, *P(*CG_ENC*)*, is found by taking the number of crossable gaps encountered, gaps greater than the critical gap (from HCM method, maximum likelihood estimation, and graphical method), and dividing this by the total number of gaps encountered. *P(*GO | CG*)*, probability of utilizing a crossable gap, is zero if the pedestrian crossed in a yield and/or accepted a short gap triggering a yield (CROSS = Y) and is equal to one divided by the number of crossable gaps if the pedestrian was able to cross in a gap (number of rejected and accepted gaps greater than the critical gap). For average pedestrian delay at each site, where each site represents a data point, these probabilities are averaged before calculating P(Cross).

Average pedestrian delay will be estimated from the HCM method and mixed-priority method using three different critical gap values and two different values for motorist yielding rate for each site. This will result in twelve values for average pedestrian delay per site. These estimates will be compared to the observed average pedestrian delay. Pedestrian delay may vary between methods for estimation and critical gap values used.

3.4 Statistical Models

Mean yield rates are given for several crossing treatments, such as overhead flashing beacons, high-visibility signs, and rectangular rapid-flashing beacons, in the HCM pedestrian delay method. For the mixed-priority pedestrian delay model, the probability of encountering a yield event was defined at a microscopic scale as the number of yields divided by the total number of events encountered by a pedestrian. Only a few sites were studied for each of these methods. It is recommended that practitioners supplement mean yield rates with local knowledge and engineering judgment.

Regression models will be used to predict motorist yield rate (M_y) based on several explanatory variables. The model will be validated by comparing the predicted yield rates with the observed rates. For pedestrian delay, the observed and predicted yielding rates will be entered into the HCM method and mixed-priority models and then be compared to the field estimated delay.

SAS will be used to develop the motorist yielding model, which predicts the percentage of driver yielding expected at a site. Both categorical and numerical data will be used in modeling. The categorical data used will be nominal, where “1” represents true values and “0” represents false values.

Table 6 summarizes all variables considered for model development. This includes the expected impact of the variable on driver yielding, where "+" represents that higher or true values are expected to increase the chance of driver yielding and "–" represents the expectation of a decrease in the chance of driver yielding. Overall yielding rates were highest in Florida and lowest for North Carolina. Based on this, it was decided that dummy variables would be created for these two states and yielding in Alabama would be captured in the intercept value. Advance signing was not considered a treatment, unless the sign includes a flashing beacon, since all crosswalks included basic advanced signing. Treatments would include in-street pedestrian crossing signs, overhead flashing amber beacons, and HAWK beacon signals.

Table 6: Variables Considered for Driver Yielding Model

Variable	**Description**	**Value**	**Anticipated Effect on Yielding**
CAMPUS	This variable distinguishes sites on-campus (1) from those off-campus (0)	On-Campus=1	+
FLORIDA	This variable distinguishes sites in the state of Florida (1) from those in the other two states (0)	Florida=1	+
NCAROLINA	This variable distinguishes sites in the state of North Carolina (1) from those in the other two states (0)	North Carolina=1	–
TREATMENT	Whether a treatment was present	Treatment=1	+
TWO_WAY	Whether the road was two-way (bidirectional) or one-way	Two-Way=1	+
TWOLANE	Whether the site had two lanes	Two Lanes=1, One=0, Three=0	+
TWOSTAGE	Whether the pedestrian crossing could be completed in two stages (median or pedestrian refuge) or had to be completed in a single stage	Two-Stage=1, Single=0	–
PARKING	Denotes presence of on-street parking	Present=1	+
BIKELANE	Denotes presence of bike lane(s)	Present=1	+
L_WIDTH	Lane width	Feet	–
C_WIDTH	Crosswalk width	Feet	–
SPEED	Free-flow speed (if available) or speed limit on the roadway	Mph	–
PED_RATE	Total pedestrian flow rate	Ped/hr	+
VEH_RATE	Total vehicular flow rate (all lanes, directions summed)	Veh/hr	–

In order for the newly developed driver yielding model to be validated, observed and predicted pedestrian delays should be similar for observed and predicted motorist yielding rates.

4. YIELDING MODEL DEVELOPMENT

Multiple linear regression is used to develop a model for predicting driver yielding rates at individual sites. This type of regression shows the linear effects of independent variables on a continuous dependent variable (Moore, 2013). The general linear model is provided below in equation 17. Interaction terms of independent variables could also be used in model development, but were not considered as part of this research. Simple statistics and variable relationships, such as correlation, should be determined as an early step in the modeling process. Several potential models are created using SAS in order to determine a best fit final regression model, with variables included based on personal observations and statistical significance. The final step is to ensure that model assumptions are met.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 \ldots + \beta_k x_k + \varepsilon \tag{17}$$

Where: y = dependent variable

β_0 = intercept value

β = coefficients of independent variables

x = independent variables

ε = random error

Of the 27 sites that were studied, 24 were on two lane roadways and 22 had bidirectional traffic. Crosswalk treatments were present at 8 sites and 7 sites included median refuges. On-street parking and bike lanes were present at 11 and 15 sites, respectively. Observed lane widths ranged 10 to 17 feet and crossing widths ranged from 8 to 30 feet. Free-flow speeds and speed limits ranged from 15 to 45 mph. The overall motorist yielding rate was 43.0%, as can be seen in Table 7. Motorist yielding rates were higher at on-campus locations (69.9%) and at sites in Florida (79.9%). Figure 5 represents some of the motorist yielding rates in a bar chart. Florida is shown to have the highest motorist yielding rate for each of the displayed site characteristics. On-campus sites were shown to have higher rates than off-campus sites for each state. The effects of treatment, parking, and crossing width varied from state to state.

Table 7: Observed Motorist Yielding Rates

	Alabama	Florida	N. Carolina	Overall
On-Campus	0.563	0.870	0.531	0.699
Off-Campus	0.254	0.581	0.272	0.291
Treatment	0.667	0.776	0.253	0.568
No Treatment	0.285	0.836	0.363	0.388
One-Way	0.230	N/A	0.287	0.280
Two-Way	0.308	0.799	0.430	0.512
One Lane	N/A	N/A	0.365	0.365
Two Lane	0.308	0.799	0.328	0.450
Three Lane	0.230	N/A	0.384	0.342
Single Stage	0.421	0.846	0.343	0.425
Two Stage	0.184	0.760	N/A	0.444
Parking	0.273	0.581	0.384	0.344
No Parking	0.500	0.870	0.331	0.491
Bike Lane	N/A	0.799	0.531	0.695
No Bike Lane	0.295	N/A	0.272	0.282
Lane Width <=12	0.208	0.799	0.339	0.494
Lane Width >12	0.405	N/A	0.345	0.360
Crossing Width <=10	0.233	0.519	0.331	0.306
Crossing Width >10	0.400	0.818	0.384	0.598
FFS <=20	0.295	0.870	0.421	0.481
FFS >20	N/A	0.581	0.278	0.327
Overall	0.295	0.799	0.343	0.430

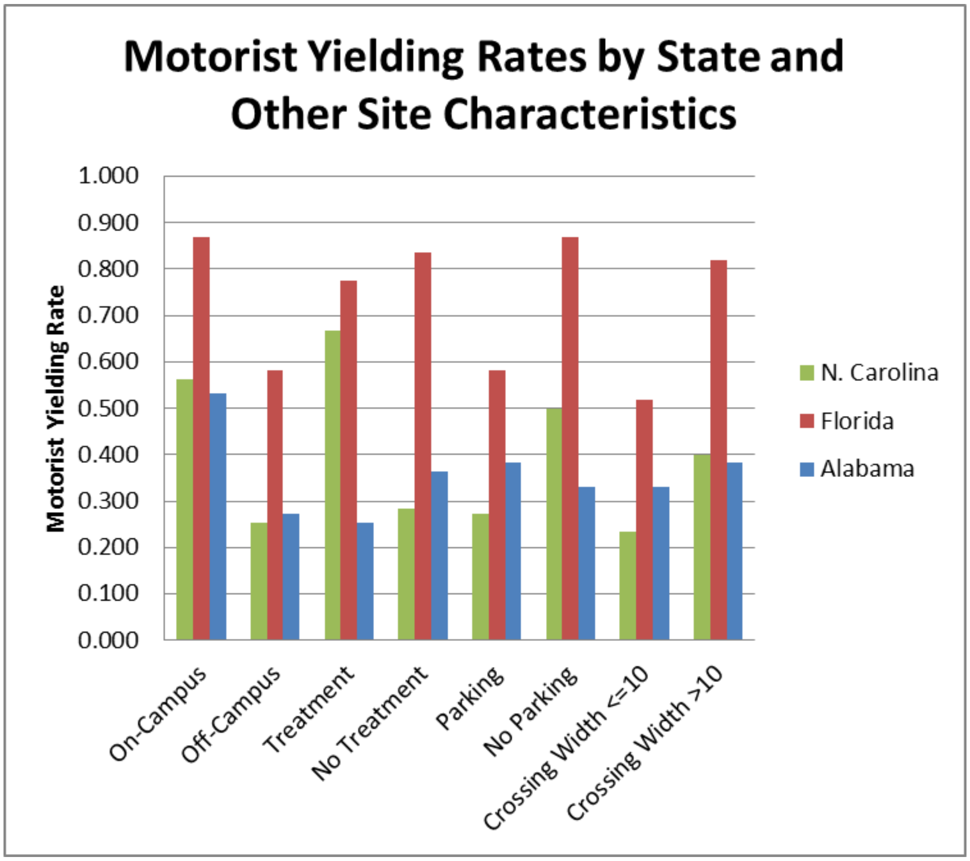


Figure 5: Motorist Yielding Rates by State and Other Site Characteristics

Table 8 shows the correlation coefficients and significance levels between all variables. The following independent variables are positively correlated to the dependent variable, suggesting an increase in yielding from larger and true values: CAMPUS, FLORIDA, TREATMENT, TWO_WAY, TWOLANE, TWOSTAGE, BIKELANE, C_WIDTH, and PED_RATE. The following are shown to be negatively correlated, suggesting a decrease in yielding: NCAROLINA, PARKING, L_WIDTH, SPEED, and VEH_RATE. Comparing these results to the anticipated impacts on yielding from

Table 6 shows that most variables affect yielding as expected, excluding TWO_STAGE, PARKING, and C_WIDTH. CAMPUS, FLORIDA, TWO_WAY, BIKELANE, and L_WIDTH were each seen to have a significant impact on driver yielding as evident by low p-values. It is expected that these variables will show up in many of the potential models. The high number of significant correlation coefficients shows that the yielding decision relies on several factors, rather than being the result of any single variable.

Table 8: Correlation Table

Pearson Correlation Coefficients, N = 27
Prob > |r| under H0: Rho=0

	YIELD	CAMPUS	FLORIDA	NCAROLINA	TREATMENT	TWO_WAY	TWOLANE	TWOSTAGE	PARKING	BIKELANE	L_WIDTH	C_WIDTH	SPEED	PED_RATE	VEH_RATE
YIELD	1.00000	0.61055 0.0007	0.65006 0.0002	-0.30157 0.1263	0.31142 0.1138	0.50595 0.0071	0.30295 0.1245	0.02985 0.8825	-0.27376 0.1671	0.61329 0.0007	-0.48213 0.0109	0.07219 0.7205	-0.22657 0.2558	0.37274 0.0555	-0.25424 0.2006
CAMPUS	0.61055 0.0007	1.00000	0.12507 0.5342	-0.18638 0.3519	-0.02405 0.9052	0.30390 0.1233	0.13104 0.5147	-0.27565 0.1640	-0.70960 <.0001	0.48067 0.0112	-0.11590 0.5648	-0.25192 0.2049	-0.41801 0.0300	0.33905 0.0836	-0.23985 0.2282
FLORIDA	0.65006 0.0002	0.12507 0.5342	1.00000	-0.49767 0.0083	0.51011 0.0066	0.36564 0.0607	0.27116 0.1713	0.42133 0.0286	-0.01156 0.9544	0.68599 <.0001	-0.68644 <.0001	0.06955 0.7303	0.16771 0.4031	0.09681 0.6310	-0.08389 0.6774
NCAROLINA	-0.30157 0.1263	-0.18638 0.3519	-0.49767 0.0083	1.00000	-0.24342 0.2211	0.10054 0.6178	-0.02868 0.8871	-0.01371 0.9459	0.45243 0.0178	-0.72548 <.0001	0.33236 0.0903	0.06256 0.7566	-0.45040 0.0184	0.11976 0.5518	-0.18821 0.3472
TREATMENT	0.31142 0.1138	-0.02405 0.9052	0.51011 0.0066	-0.24342 0.2211	1.00000	0.30934 0.1164	0.22942 0.2497	0.54155 0.0035	-0.04280 0.8321	0.25392 0.2012	-0.37609 0.0532	0.06256 0.7566	0.11668 0.5622	-0.07731 0.7015	0.16808 0.4020
TWO_WAY	0.50595 0.0071	0.30390 0.1233	0.36564 0.0607	0.10054 0.6178	0.30934 0.1164	1.00000	0.74162 <.0001	0.28204 0.1541	0.00719 0.9716	0.34112 0.0816	-0.22105 0.2678	-0.18505 0.3555	-0.40052 0.0384	0.07699 0.7027	-0.23832 0.2313
TWOLANE	0.30295 0.1245	0.13104 0.5147	0.27116 0.1713	-0.02868 0.8871	0.22942 0.2497	0.74162 <.0001	1.00000	0.20917 0.2951	-0.18655 0.3515	0.15811 0.4309	-0.26687 0.1784	-0.40152 0.0379	0.05086 0.8011	-0.08833 0.6613	-0.06653 0.7416
TWOSTAGE	0.02985 0.8825	-0.27565 0.1640	0.42133 0.0286	-0.01371 0.9459	0.54155 0.0035	0.28204 0.1541	0.20917 0.2951	1.00000	0.36949 0.0578	0.18898 0.3451	-0.41467 0.0315	0.05945 0.7683	0.15563 0.4383	-0.15121 0.4515	0.03947 0.8450
PARKING	-0.27376 0.1671	-0.70960 <.0001	-0.01156 0.9544	0.45243 0.0178	-0.04280 0.8321	0.00719 0.9716	-0.18655 0.3515	0.36949 0.0578	1.00000	-0.32024 0.1034	0.04064 0.8405	0.41844 0.0298	0.07591 0.7067	-0.09728 0.6293	-0.18881 0.3456
BIKELANE	0.61329 0.0007	0.48067 0.0112	0.68599 <.0001	-0.72548 <.0001	0.25392 0.2012	0.34112 0.0816	0.15811 0.4309	0.18898 0.3451	-0.32024 0.1034	1.00000	-0.38579 0.0469	-0.21415 0.2835	0.01287 0.9492	0.04819 0.8114	-0.27694 0.1620
L_WIDTH	-0.48213 0.0109	-0.11590 0.5648	-0.68644 <.0001	0.33236 0.0903	-0.37609 0.0532	-0.22105 0.2678	-0.26687 0.1784	-0.41467 0.0315	0.04064 0.8405	-0.38579 0.0469	1.00000	-0.10053 0.6178	-0.36610 0.0604	0.03722 0.8538	0.03840 0.8492
C_WIDTH	0.07219 0.7205	-0.25192 0.2049	0.06955 0.7303	0.06256 0.7566	0.06256 0.7566	-0.18505 0.3555	-0.40152 0.0379	0.05945 0.7683	0.41844 0.0298	-0.21415 0.2835	-0.10053 0.6178	1.00000	0.02731 0.8924	0.22375 0.2619	0.04780 0.8128
SPEED	-0.22657 0.2558	-0.41801 0.0300	0.16771 0.4031	-0.45040 0.0184	0.11668 0.5622	-0.40052 0.0384	0.05086 0.8011	0.15563 0.4383	0.07591 0.7067	0.01287 0.9492	-0.36610 0.0604	0.02731 0.8924	1.00000	-0.35314 0.0708	0.19053 0.3411
PED_RATE	0.37274 0.0555	0.33905 0.0836	0.09681 0.6310	0.11976 0.5518	-0.07731 0.7015	0.07699 0.7027	-0.08833 0.6613	-0.15121 0.4515	-0.09728 0.6293	0.04819 0.8114	0.03722 0.8538	0.22375 0.2619	-0.35314 0.0708	1.00000	0.07188 0.7216
VEH_RATE	-0.25424 0.2006	-0.23985 0.2282	-0.08389 0.6774	-0.18821 0.3472	0.16808 0.4020	-0.23832 0.2313	-0.06653 0.7416	0.03947 0.8450	-0.18881 0.3456	-0.27694 0.1620	0.03840 0.8492	0.04780 0.8128	0.19053 0.3411	0.07188 0.7216	1.00000

It can also be seen that some independent variables are intercorrelated. The following variable combinations had the highest intercorrelation values: CAMPUS to PARKING

(-0.70960), NCAROLINA to BIKELANE (-0.72548), and TWO_WAY to TWOLANE (0.74162).

Several multiple linear regression models were then created to predict the likelihood of yielding, represented by the dependent variable YIELD. Several variable selection processes are used (SAS, 2011):

- Full Model – uses all independent variables regardless of their p-value.
- Forward selection – successively adds variables to the model at a $p<0.05$ inclusion threshold.
- Backward elimination – starts with a full model and then removes variables starting with the highest p-value, until all remaining variables are at $p<0.05$.
- Max R^2 – forward selection to fit the best one-variable model, the best two-variable model, and so on, where variables are switched so that R^2 is maximized.
- Manual selection – a custom model that is informed by the first four modeling results, but considers practical significance (as opposed to just being motivated by statistical fit).

The full model was used to determine which variables were significant and to what extent they were significant when compared to the other variables. It is expected that variables seen as significant in the correlation table will also be significant in the full model. Results for forward selection and backward elimination processes should result in similar models, but may produce slightly different results, especially when independent variables are intercorrelated. Various test statistics will be evaluated for each model, including variable p-values and R-squared. P-value indicates the confidence level, where 95% confidence is represented by $p<0.05$. R-squared describes the amount of variability in data that is explained by the model, where higher values indicate better model fit. This statistic can be inflated with the addition of variables. The adjusted R-squared value penalizes the model for inclusion of additional variables, and is thus a better measure for models with many independent variables.

PROC REG and PROC GLM were used for the full model. Both of these procedures provided the same results, so PROC REG is used for the other potential models. The full model showed that when all of the variables are used, none of them are significant at the $p<0.05$ level. Forward selection showed only CAMPUS and FLORIDA to be significant (both with p-values of <0.0001). Backward elimination and the best two-variable model (max R-squared method) were identical to the forward selection model. The three- and four-variable models found using the max R-squared method included variables with p-values < 0.15. Specifically, both models included CAMPUS, FLORIDA, and C_WIDTH, and the four-variable model also included TWO_WAY. Models with five variables and more included at least one variable with a p-value > 0.15, so these models were not explored any further. The five-variable model included the same variables as the four-variable model, with the addition of TWOSTAGE. The six-variable included the same with the addition of PED_RATE. Statistics for the potential models are shown below in Table 9.

Table 9: Potential Models and Statistics

Model	p-value	R-squared	Adjusted R-squared
Full	0.0083	0.8317	0.6353
Forward	<0.0001	0.7071	0.6827
Backward	<0.0001	0.7071	0.6827
2-variable Max R^2	<0.0001	0.7071	0.6827
3-variable Max R^2	<0.0001	0.7373	0.7030
4-variable Max R^2	<0.0001	0.7682	0.7261
5-variable Max R^2	<0.0001	0.7830	0.7313
6-variable Max R^2	<0.0001	0.7891	0.7259

The chosen model is the four-variable model from the max R^2 selection method shown below as equation 18. The overall model fit is significant ($p<0.0001$) and has an overall adjusted R^2 value of 0.7261. Only the five-variable model resulted in a higher adjusted R^2 value, but as previously mentioned, this model included a variable that was not seen to be significant. The model shows that motorist yielding rate is predicted by CAMPUS, FLORIDA, TWO_WAY, and C_WIDTH.

$$M_y = 0.04972 + 0.28046CAMPUS + 0.26527FLORIDA + 0.13311TWO\_WAY + 0.01251C\_WIDTH \quad (18)$$

The model shows that on-campus sites add 28 percent to the motorist yielding rate, while Florida sites add about 27 percent. Having two-way traffic at a site would add 13 percent to the motorist yielding rate, suggesting that motorists are less likely to yield to pedestrians if the roadway has one-way traffic. Figure 6 shows the effect of campus, two-way traffic, and crossing width on the motorist yielding rate. It can be seen that the motorist yielding rate is expected to be higher for on-campus bidirectional traffic with wide crosswalks. The widest crosswalk examined was 30 feet, so this model should not be used for wider crosswalks.

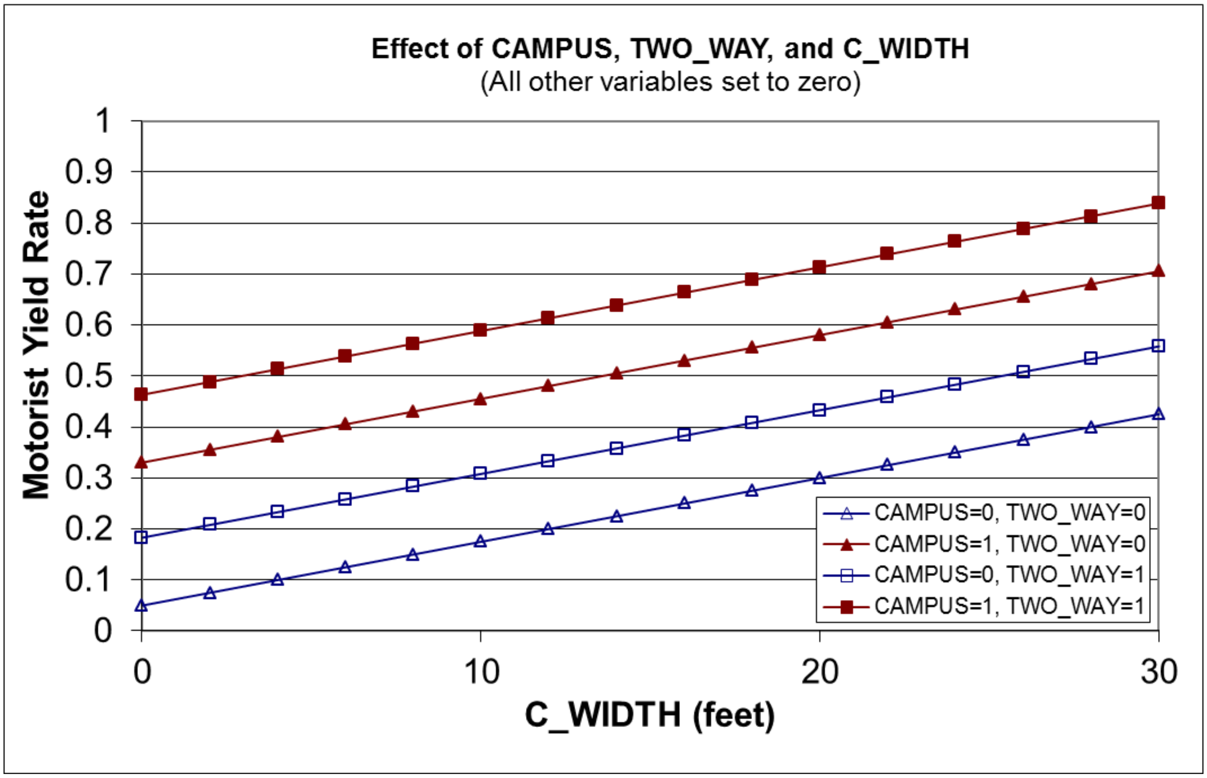


Figure 6: Model Probability Plots

The following assumptions are made in multiple linear regression:

- Linearity of the relationships between the dependent and independent variables
- Independence of the errors
- Normally distributed errors
- Homoscedasticity, constant error variance

SAS returns many plots for fit diagnostics of the dependent variable, YIELD. One plot used to determine linearity shows the predicted value for the motorist yielding rate on the x-axis and the residual for each value on the y-axis. Other similar plots show actual values for the independent variables versus the residual. As long as these plots lack curvilinear trends, then it can be said that the regression model is correctly specified as linear and that the relationships between variables are linear (Washington, et al. 2003). None of the plots displayed curvilinear trends, so the linearity assumption is acceptable.

Individual observations of dependent variables must be independent from each other for the errors to be independent (Washington, et al. 2003). The variables that were seen to be significant for the model are independent of each other, so the error independence assumption is satisfied. There are several methods to determine if the errors are normally distributed, including histograms and quantile-quantile (Q-Q) plots (Washington, et al. 2003). The histogram for the model will reveal a bell-shaped normal curve and the Q-Q plot will plot a nearly perfect straight line if the errors are normally distributed. It can be seen in Figure 7 below that the errors are normally distributed. Mean squared error, MSE, is used to estimate error variance. MSE will be larger for heteroscedastic regression (Washington, et al. 2003). Since the MSE for this model is 0.0191, it can be said that the model displays homoscedasticity. All of the assumptions were met, which means that multiple linear regression is an appropriate way to model the data.

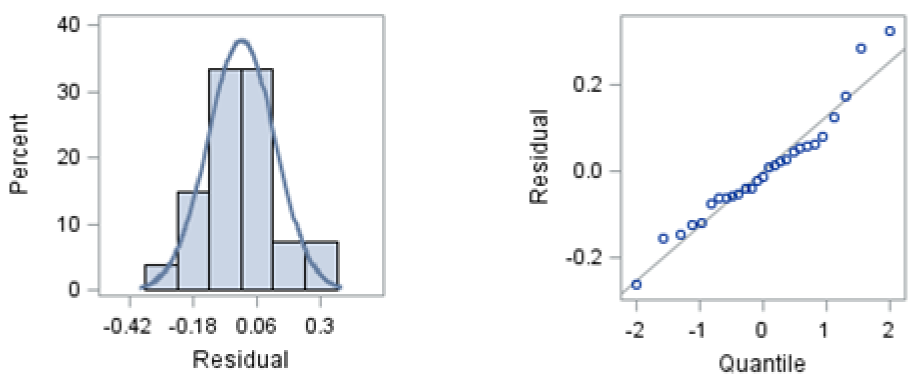


Figure 7: Histogram (left) and Q-Q Plot (right) to determine Error Distribution

5. DATA ANALYSIS

Results for critical gap, motorist yielding rate, and average pedestrian delay are discussed and compared below.

5.1 Critical Gap

5.1.1 HCM Method

In the HCM, critical gap for a single pedestrian is calculated from the crosswalk length, average pedestrian walking speed, and pedestrian start-up time and end clearance time, which is assumed to be the default value of 3 seconds. Group critical headway is found if pedestrian platooning is seen in the field. Spatial distribution of pedestrians is “1” when there is no pedestrian platooning, which was the value found from the data collected. Table 10 shows the number of pedestrians observed, average walking speed, crosswalk length, and critical gap for each site.

Table 10: Critical Gap from HCM Method

	Peds. (#)	S_p (ft/s)	L (ft)	t_c (sec)
NC1	28	4.63	32	9.911
NC2	31	4.63	32	9.911
NC3	33	4.80	42	11.750
NC4	89	4.62	12	5.597
NC5	70	4.66	12	5.575
NC6	33	5.28	28	8.303
NC7	15	5.00	20	7.000
NC8	18	5.76	28	7.861
UF1	108	5.36	10	4.866
UF2	45	5.20	20	6.846
UF3	42	5.47	20	6.656
UF4	29	5.53	22	6.978
UF5	40	4.94	22	7.453
UF6	37	4.94	11	5.227
UF7	21	4.59	10	5.179
UF8	21	5.70	11	4.930
UF9	27	6.73	22	6.269
UF10	23	4.71	11	5.335
UAB1	51	4.50	26	8.778
UAB2	81	4.29	33	10.692
UAB3	12	4.97	24	7.829
UAB4	51	4.74	26	8.485
UAB5	20	4.57	26	8.689
UAB6	24	4.8	26	8.417
UAB7	38	4.80	17	6.542
UAB8	14	4.11	28	9.813
UAB9	22	4.80	32	9.667

It can be seen that the sites with longer crosswalk lengths have longer critical gaps, which supports the decision to divide the sites into groups based on crosswalk length for maximum likelihood estimation and the graphical method. The groups are as follows:

- 30 + feet: NC1, NC2, NC3, UAB2, and UAB9
- 15 – < 30 feet: NC6, NC7, NC8, UF2, UF3, UF4, UF5, UF9, UAB1, UAB3, UAB4, UAB5, UAB6, UAB7, and UAB8
- < 15 feet: NC4, NC5, UF1, UF6, UF7, UF8, and UF10

5.1.2 Maximum Likelihood Estimation

Maximum Likelihood Estimation uses paired observations of the largest rejected gap and the accepted gap to converge on a single value for the critical gap. Critical gap distribution is assumed to be log-normal and the pedestrian population is assumed to be consistent and homogeneous. The studied sites showed homogeneous populations, especially on-campus where most of the pedestrians were students. MLE is commonly used for drivers at TWSC intersections, where the driver is assumed to need to wait for an acceptable gap before moving. When translating this assumption to pedestrians, it means that pedestrians are assumed to encounter some delay before being able to cross, which may not be the case for some of our sites with low vehicle volumes. For the “30+ feet” group, the code was able to converge without error, so no values were removed. The sample size was 104 pairs and the critical gap for this group was taken to be 6.764 seconds. Some observations needed to be removed for the “15 – < 30 feet” group to converge on a single value. Observations where the accepted gap was more than twenty times greater than the maximum rejected gap were removed resulting in a sample size of 77 pairs and a critical gap of 6.339 seconds. Estimates from MLE should not be biased and are reliable if the coefficient of variation of the estimate is small (Troutbeck, 1992). For the “15 − < 30 feet” group, the coefficient of variation is 0.281, which is considered small. For the “< 15 feet” group, accepted gaps greater than three times the required crossing time were removed resulting in a sample size of 41 pairs and a critical gap of 6.252 seconds.

5.1.3 Graphical Method

All rejected gap values are used for the graphical method, not just the maximum rejected gap. The distributions of rejected and accepted gaps were plotted and the intersection was taken to be the critical gap. Accepted gap values greater than three times the required crossing time were removed for all groups, leaving the following sample sizes:

- 30 + feet: 139 accepted gaps and 388 rejected gaps
- 15 – < 30 feet: 149 accepted gaps and 423 rejected gaps
- < 15 feet: 99 accepted gaps and 285 rejected gaps

Figure 8 below shows the full graph for the "15 – <30 feet" group and Figure 9 shows close-up views of the intersections for each group.

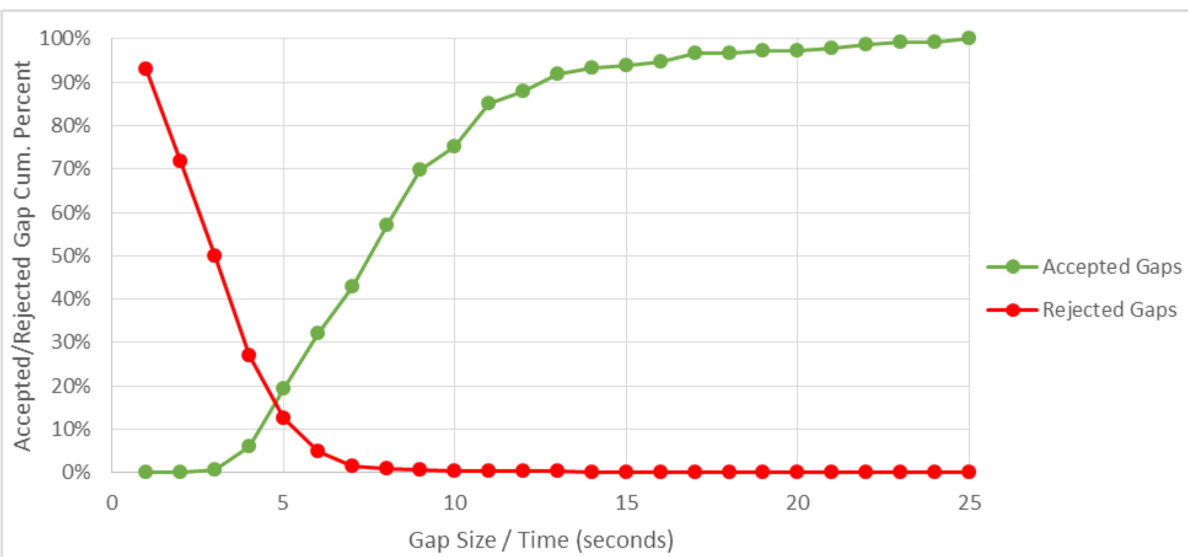
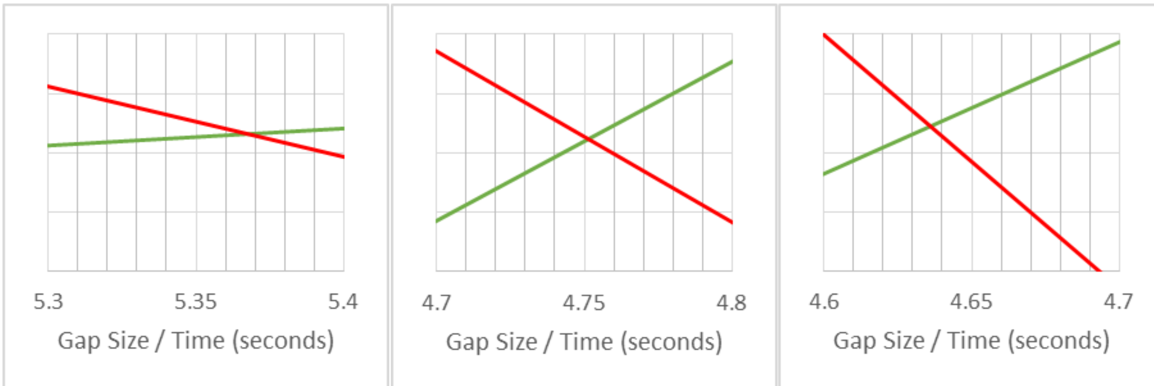


Figure 8: Graphical Method for "15 - < 30" Group



(a) “30 +” (b) “15 – < 30” (c) “< 15”

Figure 9: Graphical Method Close-ups

It can be seen that the critical gap is approximately 5.365 seconds for the “30 +” group, 4.750 for the “15 – < 30” group, and 4.635 for the “< 15” group.

5.1.4 Comparison

It can be seen from the values in Table 11 that the longest critical gap values are observed at sites with longer crosswalk lengths, such as NC3 and UAB2, and shorter critical gap values are observed at sites with shorter crosswalk lengths, such as UF1 and UF8. Maximum likelihood estimation results in longer critical gaps than the graphical method. The HCM method resulted in the longest critical gap values for the “30 +” group and the “15 – < 30” groups when compared to the other two methods (except for UF9). Some of the critical gap values from the HCM method seem very large. These large values correspond with low walking speeds and/or long crosswalks. Low walking speeds may be more likely to be observed at sites with higher yielding, where pedestrians are not trying to rush across the roadway in a gap before a vehicle arrives. This relationship should be examined in further research. Other reasons for the difference in values may be due to smaller sample sizes and assumed distributions.

Table 11: Critical Gap Values by Site and Method

Site	t_c (HCM)	t_c (MLE)	t_c (GM)
NC1	9.911	6.764	5.365
NC2	9.911	6.764	5.365
NC3	11.750	6.764	5.365
NC4	5.597	6.252	4.635
NC5	5.575	6.252	4.635
NC6	8.303	6.339	4.750
NC7	7.000	6.339	4.750
NC8	7.861	6.339	4.750
UF1	4.866	6.252	4.635
UF2	6.846	6.339	4.750
UF3	6.656	6.339	4.750
UF4	6.978	6.339	4.750
UF5	7.453	6.339	4.750
UF6	5.227	6.252	4.635
UF7	5.179	6.252	4.635
UF8	4.930	6.252	4.635
UF9	6.269	6.339	4.750
UF10	5.335	6.252	4.635
UAB1	8.778	6.339	4.750
UAB2	10.692	6.764	5.365
UAB3	7.829	6.339	4.750
UAB4	8.485	6.339	4.750
UAB5	8.689	6.339	4.750
UAB6	8.417	6.339	4.750
UAB7	6.542	6.339	4.750
UAB8	9.813	6.339	4.750
UAB9	9.667	6.764	5.365

5.2 Motorist Yield Rate

5.2.1 Observed Yielding Rates

Each site has an observed yielding rate found by counting the number of yield events and dividing by the total number of vehicle events. Motorist yielding rates, or M_y, were also found for groups of sites based on site characteristics, such as campus type, state, and number of lanes.

5.2.2 Driver Yielding Model

Predicted values for each site and various groups were found using a driver yielding model developed in SAS. The model can be seen in chapter 4. The predicted motorist yielding rate is given as a decimal. TWOLANE, TWOSTAGE, PED_RATE, and VEH_RATE were not used in the motorist yielding model, but these variables are used in the HCM pedestrian delay estimation method.

5.2.3 Comparison

Figure 10 and Table 12 show the observed and predicted yielding rates by site.

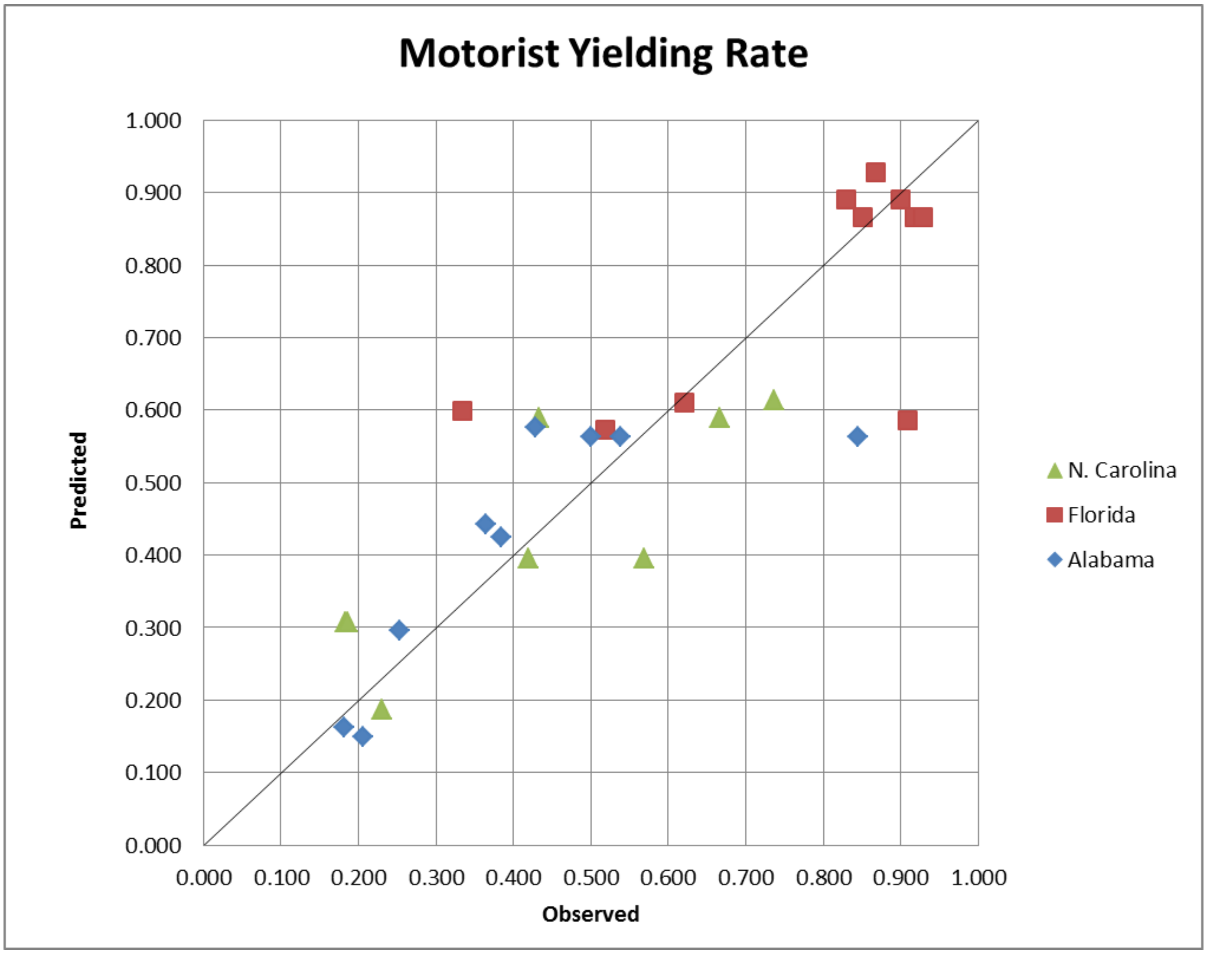


Figure 10: Observed and Predicted Motorist Yielding Rates by State

Table 12: Yielding Rates by Site

	Observed	Predicted	% Error
NC1	0.568	0.396	-33.32%
NC2	0.420	0.396	10.60%
NC3	0.230	0.187	-10.73%
NC4	0.183	0.308	-27.16%
NC5	0.186	0.308	4.63%
NC6	0.432	0.588	12.67%
NC7	0.667	0.588	21.28%
NC8	0.737	0.613	34.37%
UF1	0.829	0.891	16.86%
UF2	0.918	0.866	7.49%
UF3	0.851	0.866	-5.68%
UF4	0.900	0.891	1.77%
UF5	0.929	0.866	-0.98%
UF6	0.868	0.929	-6.72%
UF7	0.909	0.586	6.94%
UF8	0.621	0.611	-35.57%
UF9	0.519	0.573	-1.60%
UF10	0.333	0.598	10.55%
UAB1	0.845	0.563	79.47%
UAB2	0.384	0.425	-30.39%
UAB3	0.182	0.162	-5.83%
UAB4	0.206	0.150	-18.51%
UAB5	0.538	0.563	68.46%
UAB6	0.500	0.563	65.94%
UAB7	0.365	0.443	36.07%
UAB8	0.429	0.576	-11.74%
UAB9	0.253	0.295	-16.75%

It can be seen from Figure 10 that the model is more accurate when the predicted motorist yielding rates are above 0.8 and below 0.2. The observed yielding rates when the predicted rate is 0.6 vary from about 0.3 to 0.9. These values were examined to determine if they all shared a common variable that affected the prediction, but no overall commonality was found. Predicted yielding rates for UF7, UF8, and UF10 were between 0.58 and 0.62, but the observed rates were 0.909, 0.621, and 0.333, respectively. The site variables for these sites were all the same, or very close for numerical variables. This suggests that another untested variable may be affecting these sites. All of the other predictions are fairly accurate. As can be seen in Table 12, predicted values were within 10% of the actual values for a third of the sites and were within 15% for 14 of the 27 sites. Only 3 of the sites were off by more than 50%.Table 13 shows the motorist yielding rates for different groups. When the motorist yielding rates are averaged into groups with similar characteristics, the results become more accurate. For all but one of the groups, the observed and predicted rates differed by no more than 0.15 and none of the percent errors were greater than 50%. The largest errors are observed at sites with narrow crossing widths and at sites in North Carolina.

Table 13: Yielding Rates by Group

	Observed	Predicted	% Error
On-Campus	0.699	0.701	0.28%
Off-Campus	0.291	0.384	32.16%
Alabama	0.343	0.416	21.18%
Florida	0.799	0.768	-3.92%
North Carolina	0.295	0.423	43.25%
Treatment	0.568	0.674	18.61%
No Treatment	0.388	0.495	27.78%
One-Way	0.280	0.273	-2.32%
Two-Way	0.512	0.611	19.31%
One Lane	0.365	0.443	21.28%
Two Lane	0.450	0.573	27.31%
Three Lane	0.342	0.306	-10.42%
Single Stage	0.425	0.529	24.47%
Two Stage	0.444	0.604	36.18%
Parking	0.344	0.455	32.17%
No Parking	0.491	0.613	24.72%
Bike Lane	0.695	0.692	-0.33%
No Bike Lane	0.282	0.368	30.59%
Lane Width <=12	0.494	0.631	27.88%
Lane Width >12	0.360	0.445	23.46%
Crossing Width <=10	0.306	0.437	42.78%
Crossing Width >10	0.598	0.652	9.02%
FFS <=20	0.481	0.585	21.72%
FFS >20	0.327	0.444	35.58%

Overall, the driver yielding model provided relatively accurate results and it is recommended that this model be used when field data is unavailable.

5.3 Pedestrian Delay

For pedestrian delay estimation, the HCM method and the mixed-priority model were tested. Critical gaps and motorist yielding rates are used in both of these methods. There are twelve pedestrian delay estimates in total, using both estimation methods, three critical methods, and predicted and observed yielding rates. Each of these pedestrian delay estimates will be compared with the average observed delay at each site. Figure 11 below shows the distribution of observed delays. It can be seen that 70% of the observations were five seconds and less, with 20% being less than one second.

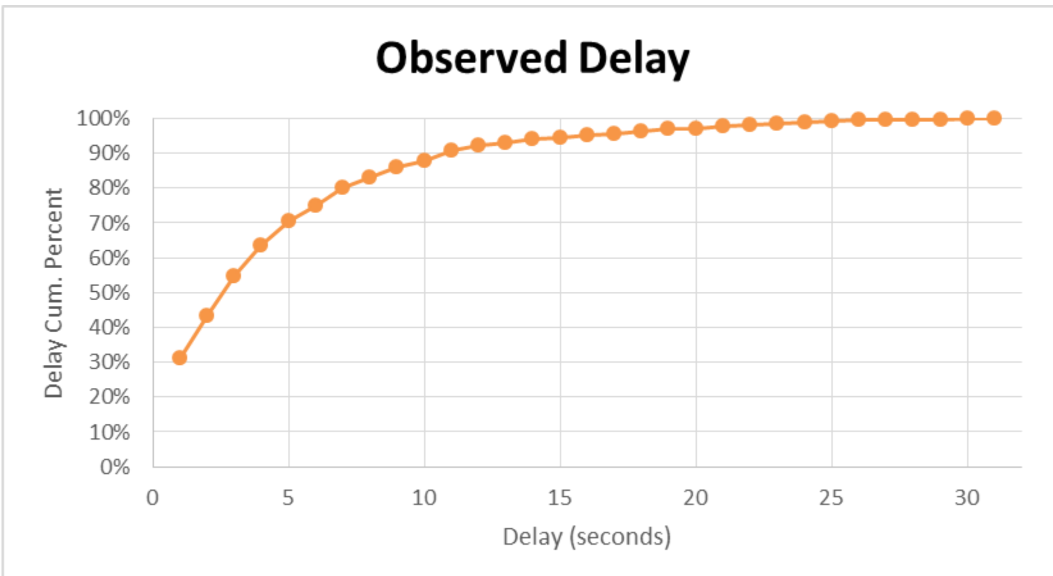


Figure 11: Distribution of Observed Delay

5.3.1 HCM Pedestrian Delay Estimation

A spreadsheet was created to run the pedestrian delay estimation calculations for all of the sites using the HCM method. Six different tabs were used to keep the critical gap and motorist yielding rates separated. The results for the six combinations are given in Table 14, as well as the observed field delay for each site. Pedestrian delay is given in seconds.

Table 14: HCM Pedestrian Delay by Yielding and Gap Acceptance Method

	Using Observed M_y (sec)			Using Predicted M_y (sec)			(sec)
Critical Gap	HCM	MLE	Graphical	HCM	MLE	Graphical	Field Delay
Site							
NC1	2.946	1.568	1.116	3.360	1.648	1.112	3.176
NC2	2.331	1.261	0.896	2.344	1.253	0.884	3.795
NC3	9.616	3.399	2.143	10.906	3.599	2.234	6.406
NC4	3.308	4.060	2.363	3.488	4.185	2.590	6.615
NC5	2.655	3.235	1.952	2.964	3.528	2.264	6.042
NC6	2.969	1.773	1.073	2.586	1.628	1.046	2.160
NC7	1.506	1.323	0.926	1.521	1.322	0.900	0.758
NC8	1.935	1.437	0.999	2.147	1.524	1.005	0.904
UF1	3.832	4.868	3.658	4.040	5.099	3.861	1.573
UF2	1.418	1.305	0.974	1.572	1.425	1.016	1.078
UF3	1.481	1.396	1.007	1.455	1.375	1.001	2.605
UF4	1.397	1.396	0.999	1.425	1.426	1.011	0.586
UF5	1.349	1.212	0.960	1.575	1.384	1.052	0.283
UF6	4.055	4.778	3.631	4.168	4.862	3.753	0.959
UF7	4.344	5.166	3.919	3.227	3.962	2.865	0.805
UF8	3.211	4.153	3.007	3.180	4.121	2.977	5.624
UF9	1.034	1.048	0.738	1.078	1.092	0.779	4.081
UF10	2.590	3.225	2.146	3.424	4.080	2.939	4.830
UAB1	1.958	1.491	1.040	3.448	2.151	1.281	1.053
UAB2	5.485	3.000	2.038	4.808	2.772	1.924	5.360
UAB3	7.519	4.834	2.700	7.817	4.968	2.738	13.760
UAB4	6.022	3.286	1.746	6.436	3.416	1.800	12.101
UAB5	1.903	1.211	0.829	1.897	1.220	0.841	5.101
UAB6	1.663	1.119	0.769	1.679	1.154	0.808	5.300
UAB7	2.021	1.928	1.275	2.098	2.008	1.368	4.299
UAB8	5.298	2.811	1.558	3.828	2.396	1.392	7.301
UAB9	10.687	6.733	4.545	8.692	5.873	4.130	5.375

Longer delays were estimated using the critical gap from the HCM method and shorter delays were estimated using the graphical method. For 15 of the 27 sites, the predicted motorist yielding rate provided more accurate pedestrian delay estimates. This was an interesting finding that these results were more accurate than using the observed motorist yielding rate. Using the HCM critical gap provided more accurate results for 10 of the 27 sites, while the MLE method was more accurate for 8 sites and the graphical method was more accurate for 9 sites.

The pedestrian delay trends are shown in Figure 12 below by plotting the observed delays against the predicted delays and fitting trendlines for each method, where “Obs-HCM” refers to using the observed yielding rate and the HCM method of determining critical gap, and so on. The black line represents the expected trend, where predicted delay equals the observed delay. All of the trendlines show that the HCM pedestrian delay estimation method underestimates the average pedestrian delay, with a majority of the individual sites showing underestimated delays as well. None of the intercepts are close to zero; they are all greater than 1.5 seconds. The slope values are all less than 0.5 and the ideal slope value is 1. The best fitting line, with an R^2 value of 0.3305, is found from the predicted yielding rate and the critical gap calculated from the HCM equation.

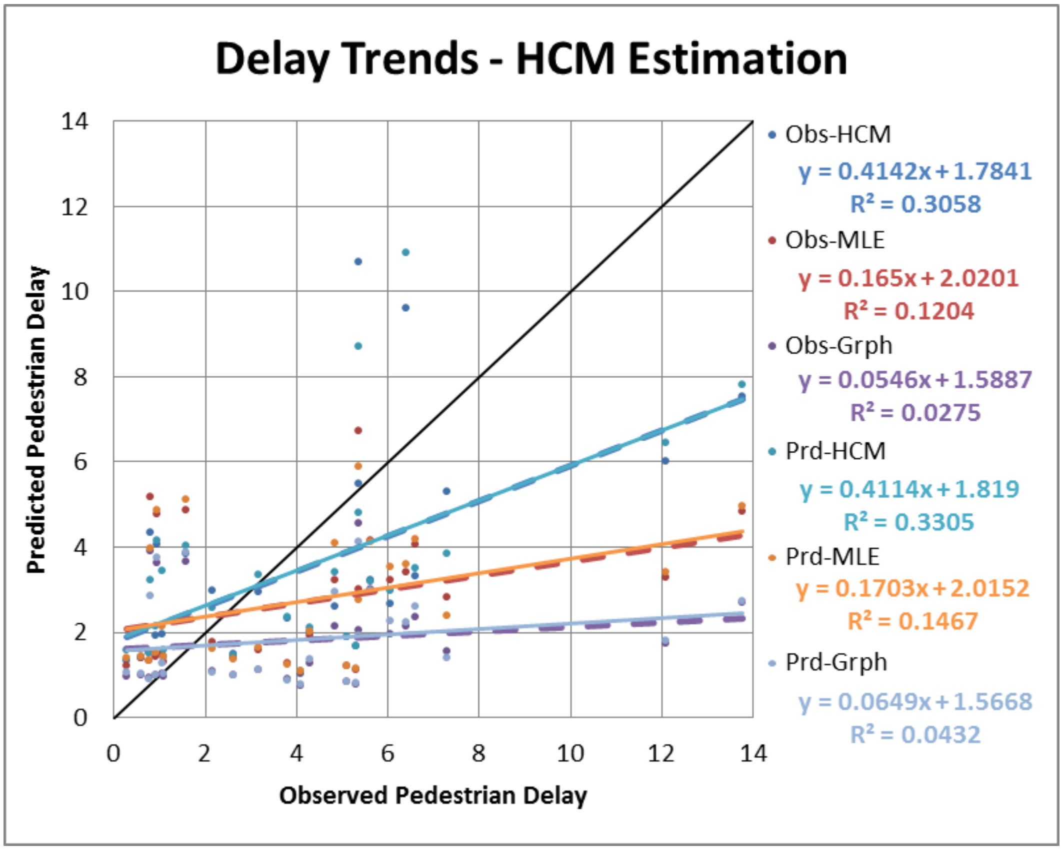


Figure 12: Pedestrian Delay Trends using HCM Estimation

5.3.2 Mixed-Priority Model

The mixed-priority model used the probability of crossing to determine average pedestrian delay. Probability of critical gap occurrence and utilization was calculated for each observation using the three critical gap values found. Probability of encountering a yield was found for each observation and was also calculated based on the predicted probability of motorist yielding. Probability of yield utilization was assumed to be “1” for all sighted pedestrians. These probabilities were averaged for the site level and then the probability of crossing was found for each site. Pedestrian delay was found for each site for all three critical gap values and observed and predicted motorist yielding rates. The results are provided with the observed pedestrian delay in Table 15.

Table 15: Mixed-Priority Pedestrian Delay by Yielding and Gap Acceptance Method

	Observed M_y			Predicted M_y			
Critical Gap	HCM	MLE	Graphical	HCM	MLE	Graphical	Field Delay
Site							
NC1	5.211	5.036	3.907	13.590	13.678	14.070	3.176
NC2	8.276	6.223	5.648	12.409	11.264	11.501	3.795
NC3	18.185	12.788	10.700	24.511	19.555	17.518	6.406
NC4	11.077	11.913	10.147	12.978	13.638	12.684	6.615
NC5	10.395	10.867	9.400	11.615	11.848	11.251	6.042
NC6	11.536	4.724	2.705	8.762	6.153	5.144	2.160
NC7	6.517	5.594	4.649	8.089	7.700	7.232	0.758
NC8	7.491	6.334	3.805	8.976	9.019	7.834	0.904
UF1	2.603	2.636	2.603	3.460	3.243	3.460	1.573
UF2	0.415	0.361	0.307	2.556	2.827	3.451	1.078
UF3	1.454	1.454	1.344	3.247	3.247	3.857	2.605
UF4	0.259	0.259	0.259	1.350	1.350	1.350	0.586
UF5	Invalid	Invalid	Invalid	1.495	1.495	1.495	0.283
UF6	0.831	0.845	0.831	1.415	1.000	1.415	0.959
UF7	2.262	2.578	2.262	10.710	9.718	10.710	0.805
UF8	4.473	4.473	4.541	9.544	9.544	9.851	5.624
UF9	5.537	5.537	5.149	8.114	8.114	10.107	4.081
UF10	7.609	8.044	6.137	8.389	7.753	8.086	4.830
UAB1	0.104	0.080	0.021	7.987	8.540	10.019	1.053
UAB2	8.750	8.590	8.557	12.818	14.328	14.760	5.360
UAB3	21.329	21.329	21.329	27.825	29.326	31.591	13.760
UAB4	17.157	17.157	17.157	28.128	28.308	30.330	12.101
UAB5	7.198	7.198	7.198	7.822	9.322	12.865	5.101
UAB6	8.02	7.954	7.894	8.369	9.327	13.374	5.300
UAB7	6.828	6.828	6.070	11.510	11.510	11.763	4.299
UAB8	8.616	8.616	8.616	8.251	10.225	14.724	7.301
UAB9	12.548	12.488	12.206	17.281	17.394	17.698	5.375

The graphical method and observed yielding rates (used yield encounter rate) resulted in more accurate estimates. The mixed-priority method was applied for each observation and then the probabilities were averaged for each site before calculating the average pedestrian delay. This may explain why the predicted yielding rate was much less accurate for this method. Predicted yielding rate was found for each site and yield encounter rate was calculated for each site, rather than each observation. A microscopic motorist yielding model for individual observations using pedestrian and driver characteristics would be expected to provide more accurate results for this method. Using the observed yield encounter rate for UF5 resulted in negative delay values, so these were invalid. This is likely due to the high yield encounter probability of 0.9625 for this site. Pedestrians at this site experienced little to no delay.

Pedestrian delay trends are shown for the mixed-priority model in Figure 13. The trendlines show that the mixed-priority model overestimates the average pedestrian delay. The slope values are all greater than 1.4. The best fitting line, with an R^2 value of 0.894, is found from the observed yield encounter probability and the critical gap from the graphical method. Data points were more consistent using the mixed-priority model for pedestrian delay estimation. Almost all of the data points showed overestimation of pedestrian delay. Compared to the graphical method for critical gap, HCM and MLE predicted greater delay using the observed yield encounter probability and lesser delay using the predicted yield encounter probability.

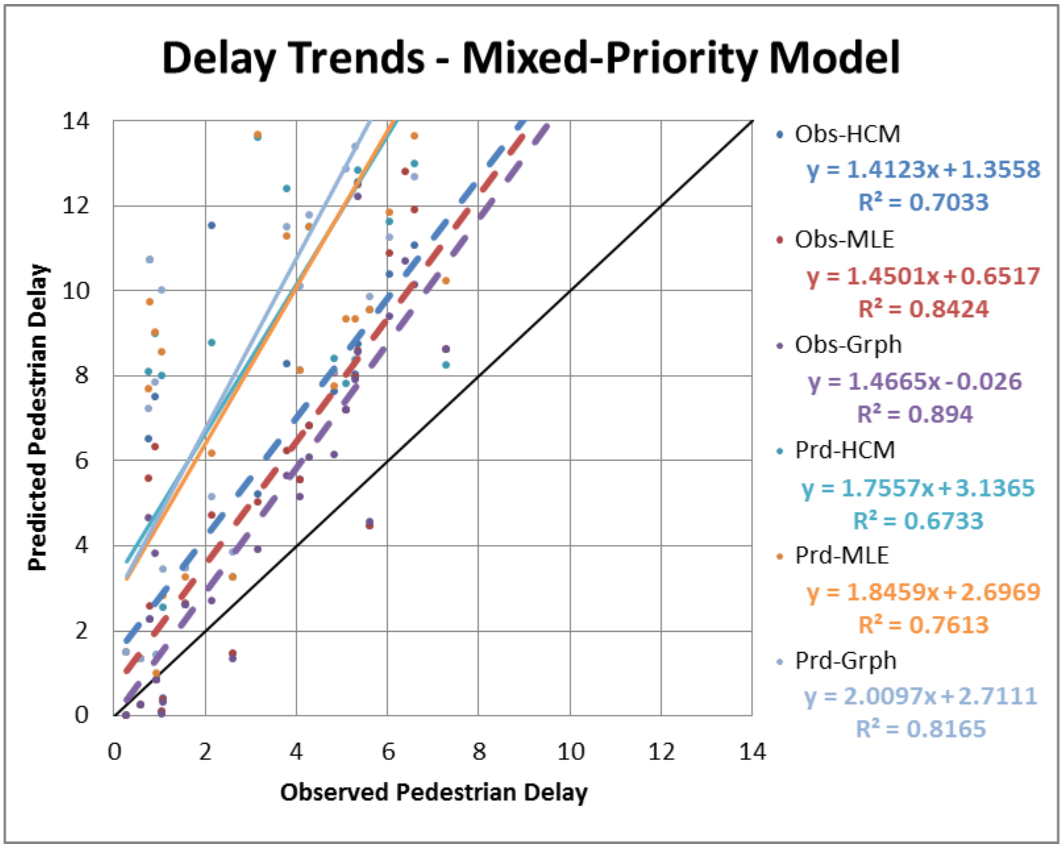


Figure 13: Pedestrian Delay Trends using Mixed-Priority Model

The mixed-priority pedestrian delay model was developed using behavioral crossing data of blind pedestrians for single-lane roundabouts. Yielding behavior at roundabouts is comparable to midblock crossings, due to the low speed required to enter and exit the roundabout and the fact that both are unsignalized. The model was designed to apply to any pedestrian population, where perfect yield utilization would be assumed for sighted pedestrians. The translation from a blind pedestrian model to a sighted pedestrian model may not be that simple. Mixed-priority pedestrian delay modeling was developed for single lanes at roundabouts, either entry or exit. An additional conversion factor may be necessary to account for these differences. A driver yielding model developed from behavioral data may result in more accurate results using this method, but would not be reasonable to implement in the HCM. The mixed-priority pedestrian delay model was shown to overestimate the average pedestrian delay. Using the critical gap from the graphical method and the observed yield encounter probability resulted in the most accurate pedestrian delay estimates, with the trendline for observed versus predicted average pedestrian delay having an R^2 value of 0.894. If the predicted average delay values are multiplied by a factor of 0.68, the trendline closely matches the expected relationship, as shown in Figure 14. This correction factor may reflect the major differences between the original use of the model for blind pedestrians at roundabouts to the current use for sighted pedestrians at unsignalized midblock crossings.

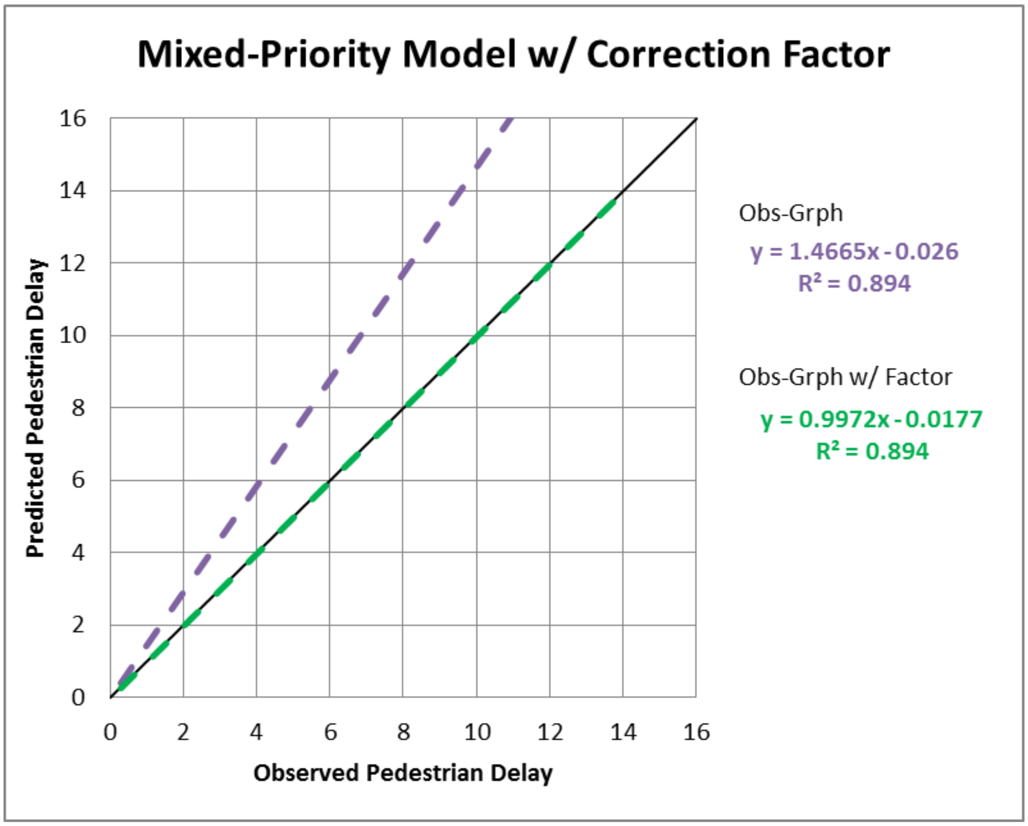


Figure 14: Mixed-Priority Pedestrian Model - Correction Factor

6. CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

Findings of this research may not apply to all regions and states. An effort was made to increase the heterogeneity by selecting midblock pedestrian crossings with varying pedestrian treatments, lane configurations, built-up environment, and travel activity in Alabama, Florida, and North Carolina. It is recommended that further research be performed in other states to validate the results of this study.

Sample size of individual observations for each site is significant. Walking speeds and critical gaps were averaged for each site when used in the HCM method and mixed-priority model. Pedestrian populations were fairly uniform, so this is likely an accurate representation. Sites where controlled crossings were necessary may not show an accurate representation of the pedestrian population. By averaging the walking speeds and calculating population critical gaps, any bias from controlled crossings should be cancelled out.

A motorist yielding model was developed using field observations and site-specific variables in order to more accurately predict pedestrian delay. This model does not address driver behavior and vehicle characteristics that may have an effect on the motorist yielding decision. The goal was to develop a model that could be implemented in the HCM to estimate motorist yielding at the site level. Average yield rates for some pedestrian treatments were provided in the HCM, but these rates were found from a limited number of sites and crosswalk geometry was not considered. This empirical study used field observations in 27 sites to estimate motorist yielding rates.

The chosen model had an adjusted R^2 value of 0.7261 and showed CAMPUS, FLORIDA, TWO_WAY, and C_WIDTH to be the significant variables. All of these variables were shown to increase the likelihood of yielding. Overall, the driver yielding model provided relatively accurate results and it is recommended that this model be used when field data is unavailable.

For pedestrian delay estimation in the HCM, the delay equation is broken into the following three terms: delay in waiting for a yield, delay in waiting for an adequate gap, and adjustment to delay in the event of a driver yielding. The delay estimates using the HCM method were consistently lower than the observed delay. This could be due to a single term in the equation or could be from a combination. These terms should be evaluated separately in the future to determine if the underestimation is due to a single term. The third term was added in the HCM 2010, so it is important to see if this term is causing the error. Definitions in the HCM were weak, terms were double-used, and both were difficult to comprehend. The method provided in the HCM 2010 had not been empirically developed or calibrated to field observations. An effort was made to validate the model, but even after using three different critical gap estimation methods and two different yield values, the estimates never matched the observed values. Using the HCM pedestrian delay estimation method with the critical gap calculated from the HCM equation and the motorist yielding rate predicted from the model resulted in a best-fit line with an R^2 value of 0.3305. Some of the observations showed zero delay, which cannot be predicted by either the HCM method or the mixed-priority method. Future research may consider removing these values. The method was very complicated to use and did not work for the data collected in this research. It is strongly recommended that a new method, or at least an edited method, be developed for the next edition of the HCM.

The mixed-priority pedestrian delay model was shown to overestimate average pedestrian delay. Using the critical gap from the graphical method and the observed yield encounter probability resulted in the most accurate pedestrian delay estimates, with the trendline having an R^2 value of 0.894. Both models have some issues and more research is needed.

6.2 Recommendations for Future Research

Further research should be performed in other states to validate the findings of this research. The sample size of naturally occurring pedestrians at some sites was too low, so controlled pedestrian crossings were required. Pedestrian delay may differ for naturally occurring pedestrians. Some sites showed low average delays, so it is recommended that more research be conducted at sites that have higher observed delays. These sites may be hard to find though, since sites that have demonstrated high delay would likely be converted from unsignalized to signalized.

Three critical gap methods were used in this thesis. Other methods, such as Ramsey-Routledge, could be used in future research. Walking speeds may differ for pedestrians crossing in gaps versus those crossing in yields. Future research could calculate average walking speed for pedestrians crossing in gaps separately from those crossing in yields to determine if these are significantly different.

The motorist yielding model developed as a part of this research uses four site variables to determine the rate of yielding and should be used when field data is unavailable. Interaction terms, as well as exponential and non-linear terms, can be considered in future research. A microscopic model should also be developed to examine the impacts of observation level behavior and characteristics on the motorist yielding rate. This model could then be compared to the macroscopic model.

The HCM method underestimated average pedestrian delay. Additional research should be performed to determine if this is due to one of the three terms in the equation or if there are other issues with the method. It is recommended that the mixed-priority pedestrian delay model be used over the HCM method and that a conversion factor of 0.68 be applied to the estimates of this model.

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APPENDICES

Appendix A: Troutbeck's Maximum Likelihood Estimation Spreadsheet

	A	B	C	D	E	F	G
1	Difference between largest rejected gap						
2	and the accepted gap			0.5	seconds		
3	Difference between iteration results						
4	for convergence			0.00001			
5							
6							
7	Check data and evaluate						
8							
9	Number of Iterations			46			
10	Mean Critical Gap			6.339	seconds		
11	SD of the Critical Gap			1.784	seconds		
12							
13	Number	Accepted	Largest	Logarithm of	Logarithm of		
14		gap (s)	rejected	accepted gap	largest		
15			gap (s)		rejected gap		
16	1	25	4	3.2189	1.3863		
17	2	22.26	2	3.1028	0.6931		
18	3	18.4	4.5	2.9124	1.5041		
19	4	13	3	2.5649	1.0986		
20	5	16.6	1.9	2.8094	0.6419		
21	6	112.2	5.5	4.7203	1.7047		
22	7	5	6.4	1.6094	1.5041	Inconsistent driver	
23	8	83.1	5.1	4.4200	1.6292		
24	9	21.4	5.8	3.0634	1.7579		
25	10	36.5	5.6	3.5973	1.7228		
26	11	30	4.05	3.4012	1.3987		
27	12	30	4.28	3.4012	1.4540		
28	13	4.08	5.23	1.4061	1.2754	Inconsistent driver	
29	14	4.4	3.66	1.4816	1.2975		
30	15	10.58	2.6	2.3590	0.9555		
31	16	4.59	6.2	1.5239	1.4085	Inconsistent driver	
32	17	7.14	3.18	1.9657	1.1569		
33	18	7.32	4.87	1.9906	1.5831		
34	19	20.37	4.82	3.0141	1.5728		
35	20	10.01	3.11	2.3036	1.1346		

Figure 15: Troutbeck's Maximum Likelihood Estimation Spreadsheet

Appendix B: Graphical Method Spreadsheet

	A	B	C	D	E	F	G	H	I	J	K	L	M
1													
2				removed gaps 3x the required crossing time found through HCM									
3	Accepted Gap	Rejected Gap					Accepted Gap			Rejected Gap			
4	25	13.38			Bin		Cum Fre	Percent	Cum %	Cum Fre	Percent	Cum %	1-Cum
5	22.26	12.67			0	1	0	0%	0%	29	7%	7%	93%
6	21.4	9.3			1	2	0	0%	0%	119	21%	28%	72%
7	20.37	8.49			2	3	1	1%	1%	212	22%	50%	50%
8	18.4	7.79			3	4	9	5%	6%	309	23%	73%	27%
9	16.6	7.03			4	5	29	13%	19%	370	14%	87%	13%
10	16.03	6.71			5	6	48	13%	32%	402	8%	95%	5%
11	16	6.67			6	7	64	11%	43%	417	4%	99%	1%
12	15	6.67			7	8	85	14%	57%	419	0%	99%	1%
13	14.53	6.59			8	9	104	13%	70%	420	0%	99%	1%
14	13.9	6.4			9	10	112	5%	75%	421	0%	100%	0%
15	13	6.4			10	11	127	10%	85%	421	0%	100%	0%
16	12.99	6.37			11	12	131	3%	88%	421	0%	100%	0%
17	12.9	6.31			12	13	137	4%	92%	422	0%	100%	0%
18	12.89	6.28			13	14	139	1%	93%	423	0%	100%	0%
19	12.65	6.2			14	15	140	1%	94%	423	0%	100%	0%
20	12.6	6.19			15	16	141	1%	95%	423	0%	100%	0%
21	12.38	6.1			16	17	144	2%	97%	423	0%	100%	0%
22	11.56	6.1			17	18	144	0%	97%	423	0%	100%	0%
23	11.41	6.03			18	19	145	1%	97%	423	0%	100%	0%
24	11.08	6.03			19	20	145	0%	97%	423	0%	100%	0%
25	11	5.92			20	21	146	1%	98%	423	0%	100%	0%
26	10.91	5.8			21	22	147	1%	99%	423	0%	100%	0%
27	10.78	5.7			22	23	148	1%	99%	423	0%	100%	0%
28	10.75	5.67			23	24	148	0%	99%	423	0%	100%	0%
29	10.73	5.67			24	25	149	1%	100%	423	0%	100%	0%

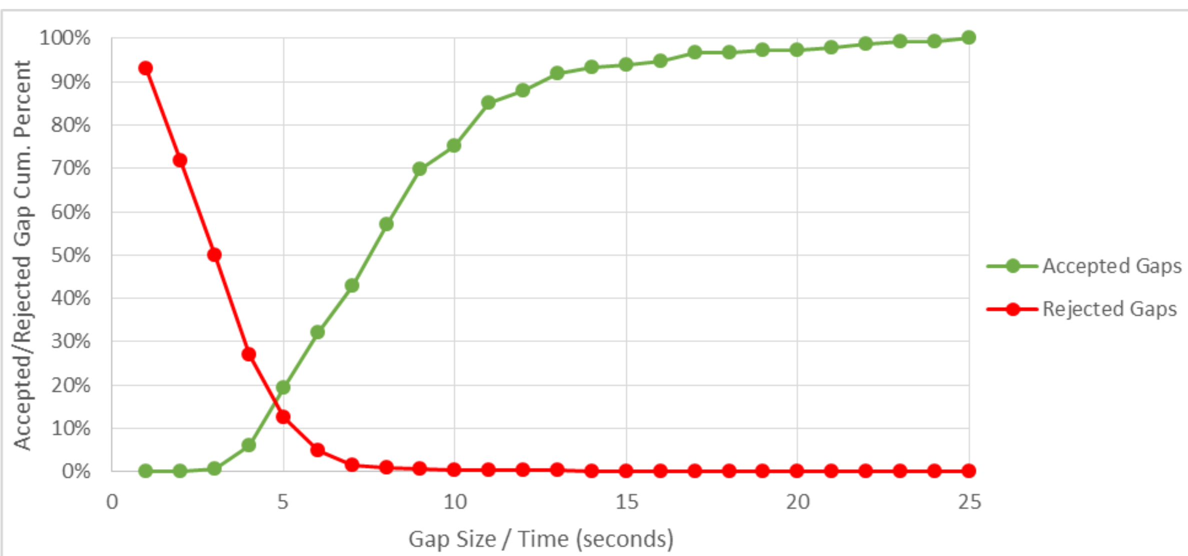

Figure 16: Graphical Method Spreadsheet and Result Example

Appendix C: Model Development for Driver Yielding Rate

	CAMPUS	FLORIDA	NCAROLINA	TREATMENT	TWO_WAY	TWOLANE	TWOSTAGE	PARKING	BIKELANE	L_WIDTH	C_WIDTH	SPEED	PED_RATE	VEH_RATE
NC1	0	0	1	0	1	1	0	1	0	16	17	15	56	244
NC2	0	0	1	0	1	1	0	1	0	16	17	15	236	160
NC3	0	0	1	0	0	0	0	1	0	14	11	15	208	504
NC4	0	0	1	0	1	1	1	1	0	12	10	20	60	596
NC5	0	0	1	0	1	1	1	1	0	12	10	20	8	448
NC6	1	0	1	0	1	1	0	0	0	14	10	20	132	316
NC7	1	0	1	1	1	1	0	0	0	10	10	20	92	208
NC8	1	0	1	0	1	1	0	1	0	14	12	20	220	288
UF1	1	1	0	1	1	1	1	0	1	10	13	20	16	300
UF2	1	1	0	0	1	1	0	0	1	10	11	20	16	428
UF3	1	1	0	0	1	1	0	0	1	10	11	20	12	356
UF4	1	1	0	1	1	1	0	0	1	11	13	20	180	516
UF5	1	1	0	0	1	1	0	0	1	11	11	20	528	536
UF6	1	1	0	1	1	1	1	0	1	11	16	20	384	556
UF7	0	1	0	1	1	1	1	1	1	10	11	30	4	300
UF8	0	1	0	1	1	1	1	1	1	11	13	30	12	356
UF9	0	1	0	0	1	1	0	1	1	11	10	30	24	108
UF10	0	1	0	1	1	1	1	1	1	11	12	30	16	356
UAB1	1	0	0	0	1	1	0	0	1	13	8	20	196	444
UAB2	0	0	0	0	0	0	0	1	0	11	30	30	132	576
UAB3	0	0	0	0	0	1	0	0	0	12	9	45	4	696
UAB4	0	0	0	0	0	1	0	0	0	13	8	30	8	492
UAB5	1	0	0	0	1	1	0	0	1	13	8	20	64	168
UAB6	1	0	0	0	1	1	0	0	1	13	8	20	68	140
UAB7	1	0	0	0	0	0	0	0	1	17	9	20	64	240
UAB8	1	0	0	0	1	1	0	0	1	14	9	20	48	504
UAB9	0	0	0	1	1	1	0	0	0	16	9	20	4	1040

Figure 17: Variable Inputs for All Sites

Pearson Correlation Coefficients, N = 27
Prob > |r| under H0: Rho=0

	YIELD	CAMPUS	FLORIDA	NCAROLINA	TREATMENT	TWO_WAY	TWOLANE	TWOSTAGE	PARKING	BIKELANE	L_WIDTH	C_WIDTH	SPEED	PED_RATE	VEH_RATE
YIELD	1.00000	0.61055 0.0007	0.65006 0.0002	-0.30157 0.1263	0.31142 0.1138	0.50595 0.0071	0.30295 0.1245	0.02985 0.8825	-0.27376 0.1671	0.61329 0.0007	-0.48213 0.0109	0.07219 0.7205	-0.22657 0.2558	0.37274 0.0555	-0.25424 0.2006
CAMPUS	0.61055 0.0007	1.00000	0.12507 0.5342	-0.18638 0.3519	-0.02405 0.9052	0.30390 0.1233	0.13104 0.5147	-0.27565 0.1640	-0.70960 <.0001	0.48067 0.0112	-0.11590 0.5648	-0.25192 0.2049	-0.41801 0.0300	0.33905 0.0836	-0.23985 0.2282
FLORIDA	0.65006 0.0002	0.12507 0.5342	1.00000	-0.49767 0.0083	0.51011 0.0066	0.36564 0.0607	0.27116 0.1713	0.42133 0.0286	-0.01156 0.9544	0.68599 <.0001	-0.68644 <.0001	0.06955 0.7303	0.16771 0.4031	0.09681 0.6310	-0.08389 0.6774
NCAROLINA	-0.30157 0.1263	-0.18638 0.3519	-0.49767 0.0083	1.00000	-0.24342 0.2211	0.10054 0.6178	-0.02868 0.8871	-0.01371 0.9459	0.45243 0.0178	-0.72548 <.0001	0.33236 0.0903	0.06256 0.7566	-0.45040 0.0184	0.11976 0.5518	-0.18821 0.3472
TREATMENT	0.31142 0.1138	-0.02405 0.9052	0.51011 0.0066	-0.24342 0.2211	1.00000	0.30934 0.1164	0.22942 0.2497	0.54155 0.0035	-0.04280 0.8321	0.25392 0.2012	-0.37609 0.0532	0.06256 0.7566	0.11668 0.5622	-0.07731 0.7015	0.16808 0.4020
TWO_WAY	0.50595 0.0071	0.30390 0.1233	0.36564 0.0607	0.10054 0.6178	0.30934 0.1164	1.00000	0.74162 <.0001	0.28204 0.1541	0.00719 0.9716	0.34112 0.0816	-0.22105 0.2678	-0.18505 0.3555	-0.40052 0.0384	0.07699 0.7027	-0.23832 0.2313
TWOLANE	0.30295 0.1245	0.13104 0.5147	0.27116 0.1713	-0.02868 0.8871	0.22942 0.2497	0.74162 <.0001	1.00000	0.20917 0.2951	-0.18655 0.3515	0.15811 0.4309	-0.26687 0.1784	-0.40152 0.0379	0.05086 0.8011	-0.08833 0.6613	-0.06653 0.7416
TWOSTAGE	0.02985 0.8825	-0.27565 0.1640	0.42133 0.0286	-0.01371 0.9459	0.54155 0.0035	0.28204 0.1541	0.20917 0.2951	1.00000	0.36949 0.0578	0.18898 0.3451	-0.41467 0.0315	0.05945 0.7683	0.15563 0.4383	-0.15121 0.4515	0.03947 0.8450
PARKING	-0.27376 0.1671	-0.70960 <.0001	-0.01156 0.9544	0.45243 0.0178	-0.04280 0.8321	0.00719 0.9716	-0.18655 0.3515	0.36949 0.0578	1.00000	-0.32024 0.1034	0.04064 0.8405	0.41844 0.0298	0.07591 0.7067	-0.09728 0.6293	-0.18881 0.3456
BIKELANE	0.61329 0.0007	0.48067 0.0112	0.68599 <.0001	-0.72548 <.0001	0.25392 0.2012	0.34112 0.0816	0.15811 0.4309	0.18898 0.3451	-0.32024 0.1034	1.00000	-0.38579 0.0469	-0.21415 0.2835	0.01287 0.9492	0.04819 0.8114	-0.27694 0.1620
L_WIDTH	-0.48213 0.0109	-0.11590 0.5648	-0.68644 <.0001	0.33236 0.0903	-0.37609 0.0532	-0.22105 0.2678	-0.26687 0.1784	-0.41467 0.0315	0.04064 0.8405	-0.38579 0.0469	1.00000	-0.10053 0.6178	-0.36610 0.0604	0.03722 0.8538	0.03840 0.8492
C_WIDTH	0.07219 0.7205	-0.25192 0.2049	0.06955 0.7303	0.06256 0.7566	0.06256 0.7566	-0.18505 0.3555	-0.40152 0.0379	0.05945 0.7683	0.41844 0.0298	-0.21415 0.2835	-0.10053 0.6178	1.00000	0.02731 0.8924	0.22375 0.2619	0.04780 0.8128
SPEED	-0.22657 0.2558	-0.41801 0.0300	0.16771 0.4031	-0.45040 0.0184	0.11668 0.5622	-0.40052 0.0384	0.05086 0.8011	0.15563 0.4383	0.07591 0.7067	0.01287 0.9492	-0.36610 0.0604	0.02731 0.8924	1.00000	-0.35314 0.0708	0.19053 0.3411
PED_RATE	0.37274 0.0555	0.33905 0.0836	0.09681 0.6310	0.11976 0.5518	-0.07731 0.7015	0.07699 0.7027	-0.08833 0.6613	-0.15121 0.4515	-0.09728 0.6293	0.04819 0.8114	0.03722 0.8538	0.22375 0.2619	-0.35314 0.0708	1.00000	0.07188 0.7216
VEH_RATE	-0.25424 0.2006	-0.23985 0.2282	-0.08389 0.6774	-0.18821 0.3472	0.16808 0.4020	-0.23832 0.2313	-0.06653 0.7416	0.03947 0.8450	-0.18881 0.3456	-0.27694 0.1620	0.03840 0.8492	0.04780 0.8128	0.19053 0.3411	0.07188 0.7216	1.00000

Figure 18: Correlation Table

- Independent variables positively correlated to dependent variable, suggesting an increase in yielding from larger and true values: CAMPUS, FLORIDA, TREATMENT, TWO_WAY, TWOLANE, TWOSTAGE, BIKELANE, C_WIDTH, and PED_RATE
- Negatively correlated, suggesting a decrease in yielding: NCAROLINA, PARKING, L_WIDTH, SPEED, and VEH_RATE
- Significant impact on driver yielding as seen by low p-values: CAMPUS, FLORIDA, TWO_WAY, BIKELANE, and L_WIDTH
- Expected that these variables will show up in many of the potential models

- Variable combinations with highest intercorrelation values: CAMPUS to PARKING (-0.70960), NCAROLINA to BIKELANE (-0.72548), and TWO_WAY to TWOLANE (0.74162)

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	0.36945	0.69031	0.54	0.6023
CAMPUS	1	0.32178	0.17821	1.81	0.0961
FLORIDA	1	0.16065	0.15217	1.06	0.3119
NCAROLINA	1	-0.03929	0.30095	-0.13	0.8983
TREATMENT	1	0.12595	0.10516	1.20	0.2541
TWO_WAY	1	-0.18842	0.29279	-0.64	0.5320
TWOLANE	1	0.37131	0.31252	1.19	0.2578
TWOSTAGE	1	-0.17183	0.12180	-1.41	0.1837
PARKING	1	0.22754	0.20079	1.13	0.2792
BIKELANE	1	0.10003	0.32435	0.31	0.7631
L_WIDTH	1	-0.02257	0.02607	-0.87	0.4036
C_WIDTH	1	0.01020	0.01277	0.80	0.4401
SPEED	1	-0.00974	0.01083	-0.90	0.3859
PED_RATE	1	0.00018736	0.00033512	0.56	0.5864
VEH_RATE	1	0.00002164	0.00029163	0.07	0.9421

Figure 19: PROC REG Full Model Result

- Overall model p-value: 0.0083
- R-square: 0.8317, Adjusted: 0.6353
- None significant at p<0.05, only CAMPUS (0.0961) significant at p<0.10
- PROC GLM Full Model provided the same results

Variable	Parameter Estimate	Standard Error	Type II SS	F Value	Pr > F
Intercept	0.28799	0.04513	0.89958	40.72	<.0001
CAMPUS	0.27864	0.05770	0.51517	23.32	<.0001
FLORIDA	0.31252	0.05971	0.60534	27.40	<.0001

Figure 20: PROC REG Forward Selection Results

- Overall model p-value: <0.0001
- R-square: 0.7071, Adjusted: 0.6827
- significant positive: CAMPUS (<.0001) and FLORIDA (<.0001)
- Backward elimination provided the same results, as did the Max R^2 2-variable model

Variable C_WIDTH Entered: R-Square = 0.7373 and C(p) = -0.2703

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	3	1.33481	0.44494	21.51	<.0001
Error	23	0.47566	0.02068		
Corrected Total	26	1.81047			

Variable	Parameter Estimate	Standard Error	Type II SS	F Value	Pr > F
Intercept	0.15371	0.09348	0.05592	2.70	0.1137
CAMPUS	0.30340	0.05787	0.56843	27.49	<.0001
FLORIDA	0.30259	0.05809	0.56119	27.14	<.0001
C_WIDTH	0.01069	0.00658	0.05458	2.64	0.1179

3-variable model

Variable TWO_WAY Entered: R-Square = 0.7682 and C(p) = -0.4782

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	1.39088	0.34772	18.23	<.0001
Error	22	0.41959	0.01907		
Corrected Total	26	1.81047			

Variable	Parameter Estimate	Standard Error	Type II SS	F Value	Pr > F
Intercept	0.04972	0.10834	0.00402	0.21	0.6508
CAMPUS	0.28046	0.05716	0.45913	24.07	<.0001
FLORIDA	0.26527	0.05988	0.37429	19.62	0.0002
TWO_WAY	0.13311	0.07763	0.05607	2.94	0.1005
C_WIDTH	0.01251	0.00641	0.07266	3.81	0.0638

4-variable model

Figure 21: PROC REG Max R^2 3- and 4-Variable Results

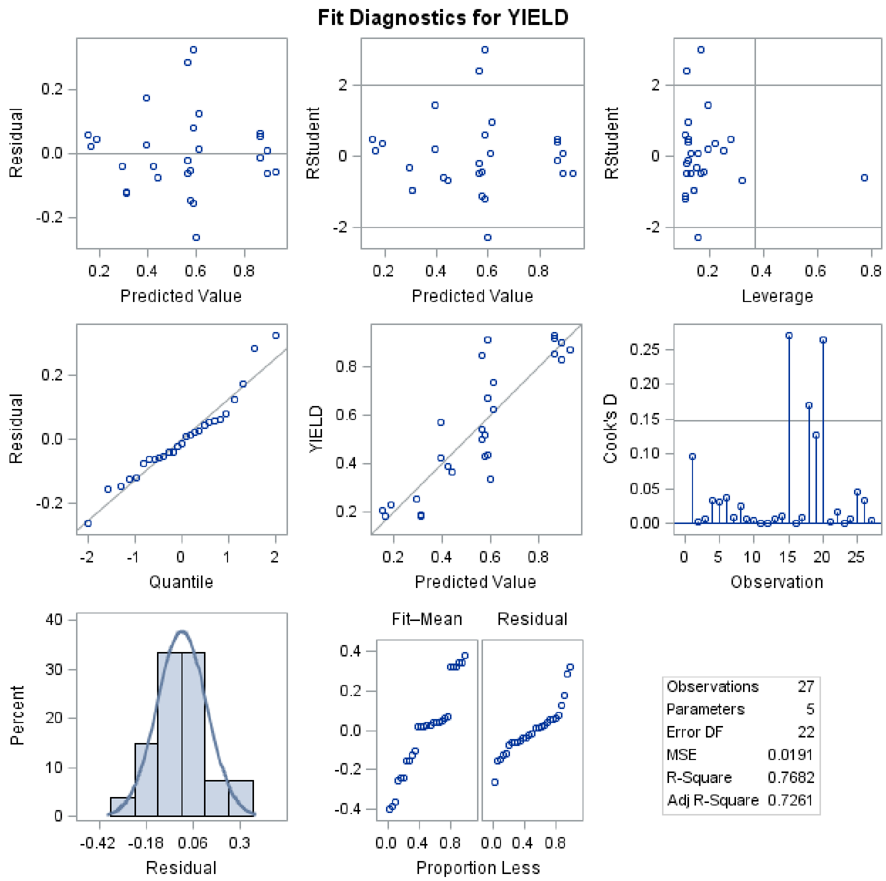

Figure 22: 4-Variable Model Fit Diagnostics

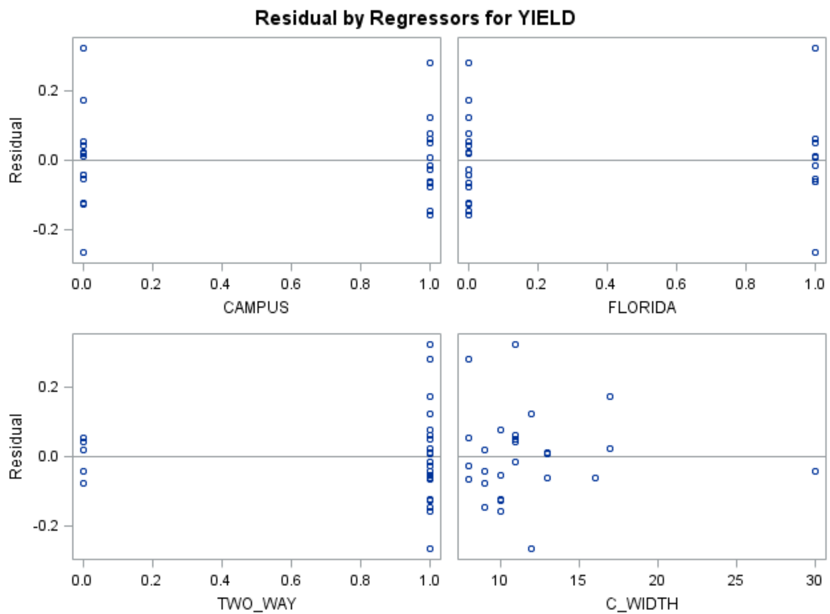

Figure 23: 4-Variable Model Residuals

Variable	Parameter Estimate	Standard Error	Type II SS	F Value	Pr > F
Intercept	0.05854	0.10756	0.00554	0.30	0.5920
CAMPUS	0.24871	0.06254	0.29585	15.81	0.0007
FLORIDA	0.29560	0.06452	0.39275	20.99	0.0002
TWO_WAY	0.15955	0.08001	0.07439	3.98	0.0593
TWOSTAGE	-0.08864	0.07421	0.02669	1.43	0.2457
C_WIDTH	0.01232	0.00635	0.07049	3.77	0.0658

Figure 24: PROC REG Max R^2 5-Variable Results

Variable	Parameter Estimate	Standard Error	Type II SS	F Value	Pr > F
Intercept	0.06974	0.10963	0.00773	0.40	0.5319
CAMPUS	0.23130	0.06717	0.22635	11.86	0.0026
FLORIDA	0.29284	0.06527	0.38426	20.13	0.0002
TWO_WAY	0.15781	0.08085	0.07272	3.81	0.0651
TWOSTAGE	-0.08342	0.07527	0.02345	1.23	0.2809
C_WIDTH	0.01060	0.00680	0.04646	2.43	0.1344
PED_RATE	0.00018407	0.00024123	0.01111	0.58	0.4543

Figure 25: PROC REG Max R^2 6-Variable Results

	Full	Forward	Backward	2-variable	3-variable	4-variable	5-variable	6-variable
Intercept	0.36945	0.28799***	0.28799***	0.28799***	0.15371*	0.04972	0.0584	0.0694
CAMPUS	0.32178*	0.27864***	0.27864***	0.27864***	0.30340***	0.28046***	0.24871**	0.23130**
FLORIDA	0.16065	0.31252***	0.31252***	0.31252***	0.30259***	0.26527**	0.29560**	0.29284**
NCAROLINA	-0.03929	---	---	---	---	---	---	---
TREATMENT	0.12595	---	---	---	---	---	---	---
TWO_WAY	-0.18842	---	---	---	---	0.13311*	0.15955*	0.15781*
TWOLANE	0.37131	---	---	---	---	---	---	---
TWOSTAGE	-0.17183	---	---	---	---	---	-0.08864	-0.08342
PARKING	0.22754	---	---	---	---	---	---	---
BIKELANE	0.10003	---	---	---	---	---	---	---
L_WIDTH	-0.02257	---	---	---	---	---	---	---
C_WIDTH	0.0102	---	---	---	0.01069*	0.01251*	0.01232*	0.01060*
SPEED	-0.00974	---	---	---	---	---	---	---
PED_RATE	0.00018736	---	---	---	---	---	---	0.00018407
VEH_RATE	0.00002164	---	---	---	---	---	---	---
R^2	0.8317	0.7071	0.7071	0.7071	0.7373	0.7682	0.7830	0.7891
Adjusted R^2	0.6353	0.6827	0.6827	0.6827	0.7030	0.7261	0.7313	0.7259

Where: *** represents $p<0.0001$, ** represents $p<0.05$, * represents $p<0.15$

Figure 26: Results for All Considered Models

Appendix D: HCM Pedestrian Delay Estimation Spreadsheet

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
1		# Lanes	S_p	L	t_c	N_p	N_c	W_c	v_p	v	$t_{c,G}$	P_b	P_d	L	d_g	d_{gd}	d_p	i	h	$P(Y_i)$	n	M_y	h(i-0.5)P(Yi)
2			ft/s	ft	sec	ped	ped	ft	ped/s	veh/s	sec	decimal	decimal		sec	sec	sec		s/veh	decimal		decimal	sec
3	NC1	2	4.63	32	9.911	1	1.063	17	0.016	0.068	9.911	0.285	0.489	2	4.219	8.624	2.946	1	7.377	0.258	1	0.568	
4	NC2	2	4.63	32	9.911	1	1.137	17	0.066	0.044	9.911	0.198	0.356	2	2.542	7.135	2.331	1	11.250	0.140	0	0.420	
5	NC3	3	4.80	42	11.750	1	1.873	11	0.058	0.140	11.750	0.422	0.807	3	18.114	22.447		1	2.381	0.114	9	0.230	0.136
6																		2		0.098			0.351
7																		3		0.084			0.502
8																		4		0.072			0.603
9																		5		0.062			0.665
10																		6		0.053			0.698
11																		7		0.046			0.708
12																		8		0.039			0.701
13																	9.616	9		0.034			0.681
14	NC4 (1)	1	4.62	12	5.597	1	1.023	10	0.017	0.079	5.597	0.357	0.357	1	1.440	4.033	1.590	1	12.676	0.065	0	0.183	
15	NC4 (2)	1	4.62	12	5.597	1	1.026	10	0.017	0.087	5.597	0.384	0.384	1	1.607	4.180	1.718	1	11.538	0.070	0	0.183	
16	NC5 (1)	1	4.66	12	5.575	1	1.002	10	0.002	0.052	5.575	0.253	0.253	1	0.896	3.549	1.179	1	19.149	0.047	0	0.186	
17	NC5 (2)	1	4.66	12	5.575	1	1.003	10	0.002	0.072	5.575	0.331	0.331	1	1.290	3.891	1.476	1	13.846	0.062	0	0.186	
18	NC6	2	5.28	28	8.303	1	1.131	10	0.037	0.088	8.303	0.305	0.518	2	3.917	7.569	2.969	1	5.696	0.201	1	0.432	
19	NC7	2	5.00	20	7.000	1	1.039	10	0.026	0.058	7.000	0.183	0.333	2	1.627	4.892	1.506	1	8.654	0.214	0	0.667	
20	NC8	2	5.76	28	7.861	1	1.163	12	0.061	0.080	7.861	0.270	0.467	2	3.083	6.604	1.935	1	6.250	0.330	1	0.737	
21	UF1 (1)	1	5.36	10	4.866	1	1.003	13	0.004	0.046	4.866	0.199	0.199	1	0.581	2.924	1.908	1	21.951	0.165	0	0.829	
22	UF1 (2)	1	5.36	10	4.866	1	1.002	13	0.004	0.038	4.866	0.168	0.168	1	0.476	2.834	1.924	1	26.471	0.139	0	0.829	
23	UF2	2	5.20	20	6.846	1	1.016	11	0.004	0.119	6.846	0.334	0.557	2	3.725	6.689	1.418	1	4.206	0.503	1	0.918	
24	UF3	2	5.47	20	6.656	1	1.009	11	0.003	0.099	6.656	0.280	0.482	2	2.762	5.728	1.481	1	5.056	0.400	1	0.851	
25	UF4	2	5.53	22	6.978	1	1.226	13	0.050	0.143	6.978	0.394	0.632	2	5.014	7.931		1	3.488	0.555	2	0.900	0.968
26																	1.397	2		0.068			0.354
27	UF5	2	4.94	22	7.453	1	1.674	11	0.147	0.149	7.453	0.426	0.670	2	6.205	9.256		1	3.358	0.610	2	0.929	1.025
28																	1.349	2		0.055			0.275
29	UF6 (1)	1	4.94	11	5.227	1	1.097	16	0.107	0.070	5.227	0.306	0.306	1	1.084	3.539	2.043	1	14.286	0.266	0	0.868	
30	UF6 (2)	1	4.94	11	5.227	1	1.121	16	0.107	0.084	5.227	0.357	0.357	1	1.344	3.765	2.012	1	11.842	0.310	0	0.868	

Figure 27: HCM Pedestrian Delay Estimation Spreadsheet

Appendix E: Mixed-Priority Pedestrian Delay Model Spreadsheet

	A	B	C	D	J	K	L	M	N
1		Observed					MLE		
2	SITE	P(Y_ENC)	P(Yield)	P(GO|Yield)	P(CG_ENC)	P(GO|CG)	delay (O)	P(Y_ENC)	delay (P)
3	UF1 Total	0.750	0.891	1	0.194	0.238	2.636	0.718	3.243
4	UF2 Total	0.918	0.866	1	0.102	0.082	0.361	0.778	2.827
5	UF3 Total	0.840	0.866	1	0.142	0.149	1.454	0.743	3.247
6	UF4 Total	0.931	0.891	1	0.029	0.069	0.259	0.866	1.350
7	UF5 Total	0.963	0.866	1	0.008	0.025	-0.210	0.859	1.495
8	UF 6 Total	0.892	0.929	1	0.050	0.108	0.845	0.883	1.000
9	UF 7 Total	0.714	0.586	1	0.298	0.286	2.578	0.411	9.718
10	UF 8 Total	0.569	0.611	1	0.399	0.339	4.473	0.367	9.544
11	UF 9 Total	0.467	0.573	1	0.367	0.517	5.537	0.363	8.114
12	UF 10 Total	0.360	0.598	1	0.380	0.513	8.044	0.371	7.753
13	UAB 1 Total	0.942	0.563	1	0.050	0.038	0.080	0.535	8.540
14	UAB 2 Total	0.528	0.425	1	0.157	0.043	8.590	0.358	14.328
15	UAB 3 Total	0.229	0.162	1	0.173	0.000	21.329	0.134	29.326
16	UAB 4 Total	0.302	0.150	1	0.041	0.000	17.157	0.144	28.308
17	UAB 5 Total	0.587	0.563	1	0.095	0.000	7.198	0.510	9.322
18	UAB 6 Total	0.554	0.563	1	0.103	0.038	7.954	0.506	9.327
19	UAB 7 Total	0.523	0.443	1	0.183	0.430	6.828	0.362	11.510
20	UAB 8 Total	0.534	0.576	1	0.167	0.000	8.616	0.480	10.225
21	UAB 9 Total	0.373	0.295	1	0.127	0.312	12.488	0.258	17.394
22	NC 1 Total	0.628	0.396	1	0.163	0.308	5.036	0.331	13.678
23	NC 2 Total	0.435	0.396	1	0.353	0.543	6.223	0.256	11.264
24	NC 3 Total	0.272	0.187	1	0.334	0.397	12.788	0.125	19.555
25	NC 4 Total	0.266	0.308	1	0.288	0.566	11.913	0.219	13.638
26	NC 5 Total	0.232	0.308	1	0.340	0.669	10.867	0.203	11.848
27	NC 6 Total	0.268	0.588	1	0.652	0.652	4.724	0.205	6.153
28	NC 7 Total	0.417	0.588	1	0.438	0.542	5.594	0.331	7.700
29	NC 8 Total	0.467	0.613	1	0.406	0.383	6.334	0.365	9.019

Figure 28: Mixed-Priority Pedestrian Delay Model Spreadsheet