ABSTRACT

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Calculus is a gateway to the fields of science, technology, engineering, and mathematics (STEM), so as the need for STEM majors in the United States grows, so does the need for more students to successfully navigate calculus. Additionally, an increasing number of students are taking calculus in high school (in the Advanced Placement program), and this has resulted in an increase of students repeating calculus in college. This study explores the impact of the AP calculus experience on success in college calculus by identifying factors of success in both environments and how the two experiences compare.

Concurrent student interviews and test analyses were used to determine the factors of success in calculus and the challenges students encounter on calculus tests. Fourteen first-semester college students who were enrolled in calculus I in college and had taken AP calculus participated in interviews that were part semi-structured and part task-based. Six AP and six college calculus instructors provided tests and other course materials that were analyzed for multiple problem formats and required reasoning type.

Results reveal that students who have taken AP calculus enter calculus I in college with a strong sense of confidence about their understanding of calculus. This can decrease students' study motivation, but that may not be detrimental to students with a strong background in calculus from their AP course. For others, high study motivation and strong study skills and habits is essential and can be a deciding factor in the student's success. Students report that their study motivation is increased by high course structure and accountability and strong relationships with instructors and other students.

Other findings suggest that both AP calculus and college calculus tests are strongly dominated by problems that may be solved using imitative reasoning. There is no evidence to suggest that college calculus tests require more creative mathematical reasoning than AP tests. However, test problems in college calculus are more likely to require explanations or showing work and utilize different problem representations and function types than do problems in AP calculus.

The Transition from Advanced Placement to College Calculus: Factors Influencing Success

by
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BIOGRAPHY

Megan Ryals was born in Marion, North Carolina on October 3, 1981. She graduated from McDowell High School in 1999 and went on to study mathematics at North Carolina State University. While at NC State, she worked for the Undergraduate Tutorial Center as a peer tutor and Supplemental Instruction leader for math and physics courses and later as a lecture assistant for the math department. She was also able to participate in the Hewlett Campus Challenge with faculty members from the math department, a university wide initiative aimed at redesigning core courses in certain majors to be more inquiry oriented.

After completing her master's degree in mathematics at NC State, Megan worked for a year and a half at Wake Technical Community College in Raleigh as a mathematics instructor, where she taught courses in precalculus and calculus. She then accepted a position with the Undergraduate Tutorial Center (UTC) at NC State as the Coordinator of Supplemental Instruction, where she trains and supervises student math and science tutors. While working for the UTC, Megan has been able to tutor students in mathematics and to continue teaching calculus courses in the summers.

She began graduate work in mathematics education one year after returning to NC State. Her experience in teaching college mathematics and her work with first-year students has led to research interests in students' transition from high school to college mathematics. She hopes to pursue projects that will further communication between high school and college instructors and better equip students to adjust to the college math environment.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1: INTRODUCTION

Background

Over the next ten years, the United States is predicted to need one million more graduates in science, technology, engineering, and mathematics (STEM) fields than the education system is currently producing (Gates & Mirkin, 2012). While the number of Americans going to college is increasing, the number of them majoring in these fields is remaining constant. A 2012 report by the National Science Board explains that the percentage of students graduating from American universities with natural science and engineering degrees is far shy of the same percentages in other countries like China, Japan, Taiwan, and South Korea. Moreover, many U.S. graduates in the STEM fields are not U.S. citizens and do not enter the U.S. workforce upon graduation. Two things are clear; there is a growing need for students to be educated in the STEM fields, and currently there exists a significant gap in the supply of and demand for these students. There is a pressing need to discover the reasons for this gap and ways to close it.

Calculus is a gateway course into most of these disciplines and therefore it is imperative that students interested in STEM careers be able to successfully navigate calculus courses. In the last three decades we have seen steady growth in the number of students taking a first calculus course, particularly in high schools (Bressoud, 2009, 2010). The majority of students taking calculus in high school do so through the Advanced Placement (AP) program, which is administered by the College Board. AP Calculus has been heralded as a highly successful program, with multiple studies showing that students who receive

credit for calculus from the AP program are very successful in subsequent calculus courses, and in some cases more so than students who are introduced to calculus in college (Burton, 1989; Ferrini-Mundi, 1992; Morgan & Klaric, 2007). This is good news, considering the growth rate of AP Calculus. In 1985, approximately 46,000 took the AP Calculus exam, while over 360,000 took the exam in 2012 (Bressoud, Camp, & Teague, 2013).

As Bressoud explains, we should expect to see a similar increase in the number of students pursuing subsequent courses. Calculus I is not designed as a terminal course. Instead, it is most often offered as the first course of a 2 or 3 course sequence. Majors that require Calculus I typically also require Calculus II and Calculus III. However, in all but research intensive institutions, Calculus II enrollment has actually been declining (Bressoud, 2009). This concern has motivated a multi-faceted study by the Mathematical Association of America (MAA) to study successful college calculus programs (Bressoud, Carlson, Mesa, & Rasmussen, 2013; Ellis, Kelton, & Rasmussen, 2014; White, Mesa, & Blum, 2014). This study is reporting on the demographics of students taking calculus in college and the factors leading to their persistence in STEM fields. It will continue to highlight variables that may be changed to increase this persistence that are within the control of college mathematics departments and instructors. However, its focus is on the college course and therefore it does not offer connections between specific challenges or successes in college that may stem from previous classroom experiences. For example, Ellis, Kelton, and Rasmussen (2014) report finding a correlation between persistence in calculus and engagement during class. This must be studied further. To determine whether classroom engagement might actually

increase persistence, we should explore whether instruction that engages students frequently is part of the students' high school mathematics experience and how this experience shapes students' expectations, motivation, and decision making inside and outside of the college classroom.

Many college calculus students' previous mathematical experiences include calculus. Of the students enrolled in Calculus I at U.S. two and four year colleges and universities, 52% have taken some calculus course (though not necessarily AP calculus) previously in high school (Bressoud, Carlson, Mesa, & Rasmussen, 2013). This is not surprising, considering that one third of the students taking the AP exam do not pass and many others who take and pass the exam choose to repeat the course in college for various reasons. While previous exposure to calculus material certainly has its advantages, it is also cause for concern. A study by the College Board suggests that students who take AP Calculus and do not pass the AP test may be at a disadvantage in college calculus (Keng & Dodd, 2008). There is not yet explanation for this result, and it is a particularly curious result when compared with earlier findings about the positive impact of the AP experience for students who do receive credit from the exam. The question arises, how does taking calculus in high school affect a student's performance in the same course in college? Could taking AP Calculus actually be detrimental to some students' mathematical success in college? If not, what challenges do these students face as they transition from high school to college mathematics, and what may be done at each level to smooth this transition and ultimately increase students' persistence in STEM fields?

These are not new questions. The subject was initially investigated under somewhat dire circumstances. In the early 1980's, a large majority of students taking calculus in high school were not receiving college credit for the course and having to repeat it. Additionally, there was a large amount of anecdotal evidence from college and university mathematicians that this cohort of students was worse off than their previous students who had not studied calculus in high school and they were vocal about their disapproval of the perceived effect of the experience on their students. These concerns led to the creation of the Calculus Articulation Panel in 1983 by the Committee on the Undergraduate Program in Mathematics (CUPM Panel, 1987). The panel consisted of four high school teachers and three college instructors who were charged with conducting a three-year study of the problems of the high school to college transition and reporting back with recommendations. The committee reviewed research and met with administrators and teachers at the secondary and post-secondary levels. They reported that the major causes of difficulty in students' transition from high school to college calculus included both high school teacher and student qualifications and expectations, effects on students of repeating a course where previously experiencing success, college placement issues, and lack of communication between high schools and colleges (CUPM Panel, 1987). None of these came as a major shock to the mathematics or education communities. The problems however, remain. In 2006, Hill claimed "the joint between high schools and universities has arthritis" (p. 18). Unfortunately, rather than producing remedies to the problems, the CUPM report seemed to initiate a blame game that has persisted for decades. "Professors point to what they consider to be poor

mathematics instruction in the high schools. High school teachers point to outdated

pedagogical practices in many colleges…" (Bressoud, 2010, p. 80).   There is some truth to

both sides of the argument and to date, too little effort has been put towards coordinating the

high school and college calculus experience and examining the difficulties of transitioning

between the two.

Looking at Advanced Placement courses from a different perspective, Klopfenstein

and Thomas (2006, 2009) distinguish between what they term college preparatory and

college level courses. In college preparatory classes, students learn skills they need to be

successful in college level courses. They claim that AP courses (not only AP Calculus) have

jumped to being college level courses in terms of content and have bypassed the much more

needed preparatory work. "AP students should be challenged by advanced material, but

perhaps more important, they should learn study skills and habits of mind conducive to

success in college" (2006, p. 14). Colleges look to AP courses to prepare their students and

students look to these courses to prepare them, and this much needed preparation for college

clearly goes beyond understanding specific content to developing the necessary study

approaches and motivation and persistence needed in the much-less structured college

environment.

A successful coordination of the two experiences would require examining the

potential barriers to student success in both arenas and considering how both mathematical

content and student study approaches may differ across the two. Looking at either one of

these factors independently from the other in an effort to explain success or failure paints an

incomplete picture of the student experience and may lead to incomplete or inaccurate conclusions about the reasons for failure and about the best use of resources for improving college calculus success rates.

Study Significance

What creates difficulty for a student in a particular classroom may be related to design aspects of the course, such as the types of questions asked on tests. However, when assessing the demands placed on students on tests, considering only the test problems themselves does not necessarily allow for measuring the difficulty of the problem. The majority of previous studies attempting to classify calculus test problems have not compared these problems against the problems students have previously solved. This dissertation study is significant because when comparing the demands of AP and college calculus, it considers not only test problem characteristics, such as problem representation and response format, but also the type of reasoning that could be used to solve the problem, which is, by definition, dependent on the problems students have seen earlier in their coursework.

Difficulty for a student may also result from his/her characteristics such as background knowledge, level of effort or motivation, or knowledge of how to study appropriately. Studies suggest that these course and student-centered factors are interrelated and some of them often mediate the effects of others on student performance in college (Credé & Kuncel, 2008). While others have studied the relationships between study practices and success in college in general, there is little information available to explain why student academic behaviors are what they are or how they may need to change in calculus.

As appropriate study behaviors differ from one discipline to the next, it is essential to determine how calculus students are approaching their courses in high school and college and how their approaches are serving them or not. At this point, the student voice is glaringly missing from the research on calculus learning in general, and is particularly needed regarding the transition from AP to college calculus. Again, this study provides research to fill this gap.

This study explored students' perceptions of both their high school and college calculus experiences and how these experiences impact performance. More specifically, it identified differences in students' high school and college calculus experiences and how these differences might influence the success of students in their college course. The purpose of the study was to determine the impact of the AP Calculus experience on student success in college calculus. This was achieved by addressing two research questions:

1. What factors affect student success in calculus and how do these factors differ in AP and college calculus?

2. What challenges do students face on calculus tests and how do these challenges differ in AP and college calculus?

These questions were answered by analyzing concurrent student interviews and test analyses.

The interviews provided current first-semester college calculus students' perspectives about course design, instructor expectations, and student variables that may affect performance in both AP and college calculus, and how these things compare across the two

environments. The interviews also provided the opportunity to determine the types of reasoning students could have used on recently missed test problems and reasons for not having done so.

The test analyses determined how AP and college calculus tests differ and are similar in regards to the types of problems students are asked and the type of reasoning they are required to use to be successful. To determine the type of reasoning required in each classroom, test problems were compared to problems from course materials such as class notes, homework assignments, and textbooks. Comparisons were made across classrooms, to discover any pervasive differences in AP and college calculus testing patterns, regarding the types of problems posed and the type of reasoning required to solve test questions in each learning environment.

The methods used will be further outlined in Chapter 3, but first, Chapter 2 provides a review of the literature on the history of AP Calculus and variables that affect student performance and factors involved in student's transition from high school to college. Chapter 4 discusses the study's results, including the themes that emerged from the first part of the student interviews, a Success Factor Model for College Calculus that was developed from a comparative analysis of the interview data, and concludes with results of the tests analyses. In Chapter 5, conclusions based on comparisons of the different sections of results, implications for practice, and study limitations are discussed. Finally, Chapter 6 provides answers to the research questions and directions for future research.

CHAPTER 2: LITERATURE REVIEW

The History and Success of AP Calculus

The beginnings of Advanced Placement (AP) Calculus can be found in the middle of the twentieth century. In the post-World War II era, the Ford Foundation created the Fund for the Advancement of Education, in response to concern of a widening gap between American secondary and post-secondary quality of education. Authors of studies supported by this fund recommended that opportunities be created for high school students to "work at the height of their capabilities and advance as quickly as possible" (The College Board, 2003). The Advanced Placement Program was created as a result of these recommendations and by the late 1950's, the now well-known College Board had been asked to administer the program and AP Calculus was being offered in numerous schools across the country.

An increasing number of students are taking calculus for the first time in high school through the AP program. It is estimated that currently over 500,000 high school students enroll in AP Calculus each year (Bressoud, 2009). AP Calculus is generally regarded as a very strong, successful program. Multiple studies throughout several decades have shown that students who take AP calculus in high school and pass the AP test fare as well or better in subsequent calculus courses than their counterparts that take calculus for the first time in college. Burton (1989) looked at the test and final course grades of students enrolled in first semester calculus at the University of New Hampshire. She categorized students as either having no calculus background, a brief calculus introduction (one semester or less), one year of non-AP Calculus, or AP Calculus. Students in the latter two groups significantly

outperformed students in the former groups. While only 16.6% of students with a full year of high school calculus made a D or F in the first college calculus course, 56.5% of students with minimal or no calculus background made a D or F.

These results are supported by a study published several years later by Ferrini-Mundy (1992). Ferrini-Mundy tested for success in college calculus over the semester and found that students with a brief introduction to calculus in high school outperformed students with no previous exposure early in the college course. However, this effect quickly vanished; students with only a brief exposure did not receive better overall course grades. Final course grades did show that students who had had a yearlong calculus course in high school did significantly better in college than those who had not. Ferrini-Mundy further compared students who had taken an AP course versus a more general calculus course without the Advanced Placement title. Perhaps the most interesting part of these findings was that AP students did not significantly outperform the students in a yearlong non-AP calculus course. If high school courses that were not designed to get students college credit really were handicapping students, we would expect these results to be different.

More recent evidence suggests that AP Calculus experience also prepares students well for Calculus II. Morgan and Klaric (2007) found that college students who had received AP credit for Calculus I did significantly better in the second calculus course than students who had taken the first course in college. This result is not surprising, given the amount of effort exerted towards aligning the AP Calculus curriculum with college calculus courses. Every 5 to 6 years, a committee of college faculty and AP instructors survey college

mathematics departments to determine how to adjust the curriculum to better reflect the college course (Gollub, Bertenthal, Labov, & Curtis, 2002).

Positive reports from the program have led to a very successful campaign to increase the number of high schools offering AP Calculus. The title, Advanced Placement, brings with it a certain prestige. U.S. News and World Report uses AP *offerings*, as opposed to success rates, to determine national rankings of high schools (Lutzer, Rodi, Rodi, Kirkman, & Maxwell, 2007). Funding is sometimes tied to these *offerings*, with schools and school systems being enticed to offer AP classes regardless of the quality or outcomes of instruction, in Calculus and other content areas. As a result, many schools have offered an "AP calculus course" that does not closely resemble the course designed by the College Board. An interesting result of this is that the College Board now requires a course audit before schools can use the Advanced Placement title for a course.

Nevertheless, an increasing number of students continue to attempt to earn college credit for their AP Calculus courses. In the past three decades, the number of students taking the AP exam has increased from just over 40,000 students per year to well over 300,000 (Bressoud, 2009). This rapid expanse has created certain challenges. Schneider (2009) explains that because of the drastic increase in the number of AP exam takers and the fact the test is criterion-referenced, larger numbers of students are receiving passing scores. This has diminished the prestige of AP credit and has led to many universities that once accepted scores of 3 or 4 (out of 5) on AP exams no longer doing so. Certainly questions are being raised about the meaning of certain AP scores.

Another issue with this explosion of students taking the AP test is that approximately 100,000 of those who do take the exam do not pass it (Bressoud, 2009). Many of these students proceed to college calculus courses one to two years later and do not meet with success, halting or slowing their progress towards a STEM degree. A longitudinal study reported by the College Board found that students who took the AP Calculus test and did not place out of college calculus actually performed at a significantly lower level in their college calculus course than students who were taking calculus for the first time (Keng & Dodd, 2008). This is particularly curious in light of Morgan and Klaric's (2007) findings and it raises questions about the impact of AP Calculus on students who repeat the course in college. The Mathematical Association of America (MAA) and National Council of Teachers of Mathematics (NCTM) recently released a joint statement that includes a recommendation that high school students not rush to calculus, but focus on foundational material that will allow them to be successful in whatever mathematics they take in college. This is reminiscent of recommendations from thirty years ago. The Calculus Articulation Panel, developed in 1983 by the Committee on the Undergraduate Program in Mathematics, advocated that students only take Calculus I one time because of collected anecdotal evidence that suggested that students who were exposed to a course where they excelled and then repeated the course later were given a false sense of confidence about their knowledge about and ability in the subject (CUPM Panel, 1987).

This recommendation has not been heeded and more research about students in calculus has been started. In 2010 a National Science Foundation (NSF) funded study was

launched by the MAA to examine successful college calculus programs. The study reports that 52% of the approximately 300,000 students enrolled in Calculus I in colleges and universities across the United States in Fall 2010 had taken some version of calculus in high school. Even more surprising is that 16% of the 300,000 students had scored a 3 or higher on the AP exam (Bressoud, Carlson, Mesa, & Rasmussen, 2013). There are many students who are seeing this material in their college course for the second time.

College courses in general are associated with a greater volume and difficulty of work as well as less instructor-provided structure and support (Credé & Kuncel, 2008; Hong et al., 2009). Students will clearly state that they find their first college mathematics courses significantly more difficult than their high school mathematics classes. In Bready's (2000) study, 10 out of 100 students voluntarily and unprompted added a comment to a structured survey about their transition to college mathematics, saying that they found college mathematics to be more difficult than in high school. The ten were from different high schools; some had used reformed curricula (known for making use of technology and multiple representations and encouraging student invention and discussion) and others were from high schools that used a traditional curriculum (often assumed to more closely resemble the typical college instructional approach). Despite the different approaches they had experienced in high school, they still reported that their college math class was more challenging. Not only do students report that college mathematics is more challenging, but their grades actually reflect this. In a study comparing the college success of students coming from high school reform or traditional curricula, Smith and Star (2007) found that

students did significantly worse in their college course, whether coming from reform or traditional programs in high school. It is not surprising then that student attitudes towards mathematics are changing for the worse in college Calculus I courses. The MAA study (Bressoud, Carlson, Mesa, & Rasmussen, 2013) reports that both student confidence and enjoyment of mathematics decrease significantly between the beginning and end of the course. Since 52% of these students have taken calculus in high school, it is reasonable to question what may be different from their high school experience that could account for this change in attitude. The remainder of this chapter addresses both student-centered and classroom-centered challenges that students face in calculus and how these differ from high school to college environments.

<div align="center">**Student-Centered Challenges in Calculus**</div>

Pre-requisite Mathematical Knowledge

The secondary to tertiary transition requires many students to move from a closed system (a school system that designs its courses with its own subsequent courses in mind) to a college or university with students from numerous school systems that sets its own standards that may or may not align with those of the high schools of its students. Arguably one of the most prominent challenges facing college mathematics departments is that of teaching students coming from a variety of secondary schools and therefore a variety of levels of preparation.

Students in a first-semester calculus course for scientists and engineers are expected to have a working knowledge of trigonometry, familiarity with a library of functions and

their properties, and a mastery of algebraic manipulation skills. A student may pass an examination or placement test and yet have a weak background in any one of these areas, which may significantly impede their progress in learning calculus. College calculus instructors have expressed dissatisfaction for decades about the lack of mathematical preparation they see in their students (CUPM Panel, 1987). College instructors believe lack of background knowledge to be one of the most significant factors that contribute to student failure in college math courses and they stress that high schools should focus more on providing a strong foundation rather than rushing ahead to college-level mathematics (Hourigan & O'Donoghue, 2007; Stroumbakis, 2010). However, in Stroumbakis' survey (2010), high school teachers rated higher level topics as more important for success in college. This discrepancy may explain some of the tension students feel when transitioning to college mathematics.

Motivation and Effort

In Anthony's (2000) survey of college instructors and students, both groups indicated that a student's level of motivation was the most important factor that contributes to student success. Mayer (1998) discusses several components of motivation, including the role of self-efficacy and attribution theory. Self-efficacy could be defined as a belief in one's ability and self-efficacy theory correlates it with higher motivation and effort. It predicts that "students who improve their self-efficacy will improve their success in learning to solve problems" (p. 59). Attribution theory posits that a student's level of success is related to whether they attribute success or failure to ability or effort, and that students who attribute

success or failure to effort are more likely to succeed. This is particularly interesting in light of Anthony's study, which found that college students and instructors gave different reasons for failure. Students identified instructor or course controlled factors, such as boring lectures or irrelevant content, as having the greatest impact on student failure, while instructors attributed student failure primarily to student controlled factors, such as poor study techniques and insufficient practice.

The results of Anthony's (2000) study clearly indicate a discrepancy in the amount of practice that is expected from instructors and the amount that students believe is necessary. "Insufficient work" was the number one reason instructors gave for failure, while this was ranked 18th by students. Cerrito and Levi (1999) found that 75% of students surveyed felt that studying 3 hours outside of class per week was "unreasonably high." This belief apparently impacts students' study behavior. Of the students Anthony (2000) surveyed, 55% indicated they studied less than 4 hours per week for their mathematics course, which falls far short of the expectation set by most college mathematics departments.

It is likely that students bring with them to college an expectation that they can be successful with minimum effort outside the classroom, based on their experience in high school. The National Center for Education Statistics (NCES) reports that 71% of 12th graders study less than one hour each day, for all their classes combined (Gettinger & Seibert, 2002). While some of this 71% of 12th graders do not go on to college, the more successful students do, only to find that the demands of college academics are quite different, not only in the amount and difficulty of content, but in the structure of the course. The typical college

mathematics course requires much more independent studying from students than a comparable course taught in high school, partly because of difficulty but also because of the decreased amount of contact hours instructors have with students.

Study Approach

Students' lack of independent studying cannot be attributed solely to a lack of motivation. Study skills, which affect student success, certainly are mediated by motivation. That is, while a student may have well developed study skills and know when and how to use them, a lack of willingness to do so may negate the potential positive effects on performance. On the other hand, a willingness to study does not guarantee the effectiveness of a student's efforts.

Some students are unaware of the (sometimes unspoken) expectations of college mathematics departments. In many high school classrooms, students are able to be successful without a significant amount of independent effort. Study skills, attitudes, habits, and motivation are strongly correlated with success in college, but "exhibit near-zero relationships" to performance in high school (Credé & Kuncel, 2008, p. 442). Even high school teachers aren't recognizing a need for skills that college instructors might consider crucial. Stroumbakis (2010) found that college instructors rated "student-controlled" issues including time management, study skills, being proactive, effective use of textbook, and timely homework completion as more important for student success than did high school instructors, and there was a significant difference between the ratings of high school instructors who had taught in college before and those who had not.

In addition to students entering with the impression from high school that independent studying is not necessary for success, the lack of studying in college may be attributed to unawareness of what constitutes effective studying or an inability to evaluate the effect of one's study approach. Anthony (2010) explains that many college students are very motivated and put forth a lot of effort, but are not focusing that effort on activities that produce high grades. He calls these students "cue deaf," referring to the fact that they are not "cued into the kind of work that is necessary to achieve examination success" (p. 11). This is supported by the distinction that Gettinger and Seibert (2002) make between study tactics and study strategies. A study tactic is a "sequence of steps or a specific procedure, such as underlining or summarizing." A study strategy is "an individual's comprehensive approach to a task; it includes how a person thinks and acts when planning and evaluating his or her study behavior." (p. 351). These authors explain that a student may be able to apply certain study tactics, but not be successful because of the lack of an appropriate study strategy. For example, a student may be very adept at taking detailed class notes and know how to pick out main concepts and ideas and summarize these notes, yet not recognize his need to independently practice multiple problems from his class notes or assigned exercises. This is a potential area where AP Calculus can contribute to success of students in college mathematics.

Many high school courses, even AP Calculus, are designed to be college level courses; that is, they are designed so that success in the course produces students with equivalent knowledge as students who take the course in college (Howell 2007).

Klopfenstein and Thomas (2006, 2009) distinguish between what they term college preparatory and college level courses. They explain that if AP courses are truly "preparatory" then students who take these courses in high school should outperform those who don't. However, their research suggests this is not the case for AP courses in general. Preparation for college clearly goes beyond understanding specific content and we need to know whether AP calculus is only helping students with the short-term goals of introductory calculus content or whether it is also teaching students skills necessary for later success. One way to frame this study is that it investigated the degree to which AP Calculus serves as a college preparatory course by examining the behaviors and study skills that students bring with them from AP to college calculus.

Successful students do not study in one particular way; multiple studies have shown that successful students use a variety of study tactics (Credé & Kuncel, 2008; Snow & Lohman, 1984). That is, there is not a one-size-fits-all approach to studying. Students must navigate their particular learning environment and course requirements and determine, for themselves, in the particular course, what they need to do and how they need to study in order to perform well. However, the ability to use certain skills and to evaluate the effectiveness of these skills, while inherent for some students, may be learned by others. According to Pelligrino (2006), metacognitive skills, including noting failures to comprehend, activating background knowledge, and predicting outcomes, may be taught, and "can help students learn to take control of their own learning," which is essential in the college environment. Metacognitive skills are most effectively developed in context (Lester,

Garofalo, & Kroll, 1989) and may be developed by teaching students self-questioning strategies in problem solving (Montague 1992, Lucangeli, Cornoldi, Tellarini, Scruggs, & Mastropieri, 1998). Other more general study tactics, such as time management, note-taking, and reading texts are being taught within college mathematics courses with marked success (Seon & King, 1997; Taylor & Mander, 2003). Taylor and Mander argue, "Our experience and the responses of students indicate that it is now essential that strategies that were once thought to be acquired by osmosis are now an explicit component of first year university study" (p. 7). If the majority of secondary and tertiary mathematics courses are not teaching these skills, and they do not develop naturally in all students, it is imperative to discover if and how students are currently acquiring these skills or avoiding learning them. This study will add to the knowledge base about students' awareness and development of study approaches, specifically in calculus.

Course-Centered Challenges in Calculus

The previous section discussed research on student attributes that can contribute to or impede student success in a calculus course. It is also appropriate to report on research on aspects of course design that can create challenges for learners. The specific aspects of course design that will be discussed are the amount of cognitive load taken on by instructors, challenges specific to understanding the topics of limits and continuity, and finally problem classification schemes.

Instructors and Cognitive Load

Some instructors attempt to take on or assume the cognitive load for their students. Hourigan and O'Donaghue (2007) label this taking on of cognitive load a "reductionist orientation." A teacher with a reductionist orientation breaks down concepts and skills into small chunks that are each much more easily digestible for students. While the student is then capable of carrying out individual operations, she may not know why she is doing each operation, when to apply an appropriate strategy, or how all of them fit into the overarching concept. Even if the why and how are taught well, it leaves the student dependent on the teacher to provide this instruction explicitly for each new topic. Hourigan and O'Donaghue provide evidence that secondary school teachers are much more likely than college instructors to adopt this approach (p. 470). This difference may stem from philosophical differences, but it may also result from college instructors simply not having enough contact hours with students to provide this type of instruction. Because of this distinction, high school mathematics students may be ill-equipped to handle problems that differ significantly from those outlined by the teacher or those that require a depth of understanding beyond mimicking procedures (Cooney, 1999).

Arguably the amount of opportunity that a teacher has to "reduce" the material for the students depends in part on the content and format of problems students are required to solve on examinations. The following section immediately following describes specific challenges that students encounter when answering questions about the topics of limits and contuinity, and then the subsequent section describes research on ways problems may be classified.

Limits and Continuity

The research on students' understanding of limits and continuity has produced a fairly consistent collection of obstacles that students encounter. The following discussion describes certain misconceptions about limits and continuity, the challenge in confronting those misconceptions, and the language commonly used that sometimes leads to these misconceptions.

Conflicting Conceptions of Limit and the Connection with Continuity. Multiple studies have discovered that students often view limits as being unattainable (Juter, 2006, Elia, Gagatsis, Panaoura, Zachariades, & Zoulinaki, 2009). Students with this misconception may argue that continuous and more specifically, constant functions do not have limits, or that is ridiculous to discuss the limit of such functions (Williams, 1991). There also exists a tendency by some students to believe that functions continuous at a point *a* have a limit at point *a* but that cannot be attained. Lauten, Graham, & Ferrini-Mundy (1994) suspect that the use of the trace function on graphing calculators may contribute to such confusion. If the student has a very dynamic conception of limits and the calculator does not produce a *y* value for the *x* value *a*, it may be a very natural conclusion at which to arrive.

Interestingly, though it is common for students to view discussing limits of continuous functions as invalid or nonsensical, in other cases they deny the existence of a limit because of a discontinuity. In Duru's (2011) study, nearly 10% of students claimed that the limit did not exist at a specified point because the left and right hand limits were not equal to the function value at that point. It is possible that students such as these can

simultaneously hold both this view and that described above. Bezuidenhout (2010) gives

specific examples of students who made contradictory claims, such as a function

simultaneously being continuous yet undefined at a particular point.

Reasons for these contradictory beliefs include a lack of understanding of the

definitions of limit and continuity and a reliance on incorrectly memorized theorems. One

student in Bezuidenhout's (2010) study, when asked how he knew the function was

continuous at a point, claimed, "I say so, because I have learned that if a limit of a function

exists, then it means that the function is continuous at that point…" (p. 494). Other

contradictions stem from strong associations students make between procedures and specific

types of functions or specific representations of functions. Juter (2006) found that students

would use procedures they had learned for finding horizontal asymptotes for the following

problem.

$$\lim_{x\to1}\frac{x^2-1}{x-1}$$

Students so strongly associated rational functions with the process of dividing all terms by

the greatest power of x in the denominator, that they immediately resorted to this procedure

when asked about the limit at a point.

Ineffectiveness of Proof and Cognitive Conflict. It could be argued that *reality* to

students is often equivalent to *doing* calculus. Students often recognize conflict between

their dynamic technique and conceptual definition, yet still cling firmly to their established

procedure. Somehow "procedural knowledge…is largely separate from their conceptual

knowledge" and students are content with such a state (Williams, 1991, p. 233). One of the

students in Szydlik's (2000) study, whom Szydlik describes as having an external source of conviction, claimed that in calculus, "you're given some rules and some laws, and you go in and you plug numbers into it, and you kind of manipulate formulas and you get the right answer" (p. 266). Consider the following example from Tall and Vinner (1981). Many students develop an idea of a number that is infinitely small, yet larger than zero. This surfaces when students are asked prove that $0.999\ldots = 1$. One student constructed the following proof

$$0.999... = 3 \times 0.333... = 3 \times \frac{1}{3} = \text{rubbish}$$

The authors explain that the logic provided in his proof conflicted with his belief in the infinitesimally small difference between $0.999\ldots$ and 1 and ultimately he relied on his intuition (p. 158).

Sierpinska (1987) reports on "proving" to humanities students that the sum of the Grandi series: $1 - 1 + 1 - \ldots$ is equal to ½ by deriving the formula $1 + x + x^2 + \ldots = \frac{1}{1-x}$ and letting $x = -1$. Students certainly did not protest, and rarely even questioned the result. Though intuitively the students did not accept the result as being *true*, they trusted the proof as *valid*. The literature supports the idea that many students see a clear distinction between mathematical truth and validity. They accept the results of proof, even when in conflict with their intuition because, as one student claimed, "mathematics is completely abstract and far from reality," and "with those mathematical transformations you can prove all kinds of nonsense" (p. 375).

Language. Mathematicians often use terms such as approaches, tends to, and converges interchangeably. For students these terms often have meaning from outside the classroom that do not agree with the intended mathematical meanings. Students often hear "*n* goes to infinity," yet "the variable *n* is not 'going' anywhere, and within many mathematical systems 'infinity' is not any *place* you could go to (Davis & Vinner, 1986, p. 299).

Monaghan (1991) conducted a study specifically to examine problems created by the use of language. Students were asked to respond to true false statements about the limits of both sequences and functions. The same questions were asked using four different terms – limit, tends to, approaches, and converges. While mathematicians may view the different phrases as equivalents, students did not. For example, 74% of college bound subjects said that the sequence 0.9, 0.99, 0.999, … "tends to 1" but only 36% agreed that the limit of the sequence is 1.

Students were also asked to create sentences using each of the four terms. From these constructions Monaghan determined that students understand limit in terms of physical properties and in particular, boundaries. This notion is confirmed by 60% of the students agreeing with a statement that 0.9 is the limit of the sequence mentioned previously (p. 21). Oehrtman's (2009) study confirmed that students use "physical limitation metaphors" when reasoning about limits.

Students tended to associate the terms approaches and tends to with the idea of drawing nearer. Williams' interviews (1991) illustrate this. Students used approaching to

describe "the physical process of evaluating a function at different numbers, …or [to describe] the mental process of imagining the points on a graph moving closer and closer to the limit point" (p. 228). Researchers commonly accept now that this language may contribute to the belief that a limit may not be reachable or attainable.

Problem classifications

Previous research has explored different ways to classify problems and what types of problems are more difficult for students to solve. This body of work has primarily used problem classifications either as a means to compare two sets of problems (from two textbooks, for example) or as a framework for studying the difficulties students encounter in problem solving. Different studies have used different classification schemes and not surprisingly some of the classifications overlap. Discussed below are prominently studied classifications that have been shown to have an impact on the level of difficulty created for the student. This is far from an exhaustive list but includes those that are most likely to be measurable and significant for college calculus problems. As Tallman, Carlson, Bressoud, and Pearson (in press) explain, when attempting to apply an existing problem classification framework, challenges arise because of differences in the mathematical level and content studied. For example, a common classification used in research with elementary mathematics involves the number of steps or procedures required to solve the problem. While previous studies had found it was possible to determine the number of steps required in an arithmetic or algebra problem and that this result had a clear impact on the difficulty for the student (Muth 1992; Zhu & Fan, 2006), Tallman, Carlson, Bressoud, and Pearson (in

press) found it was quite difficult to determine this for calculus problems because of their complexity and was not as important as other problem features.

Format and representation of given information. One way that problems can differ is in the format or chosen representation of the given information. For example, a function that models the price of a cab ride may be given in symbolic or graphical form, or the cab's rates may be described verbally, or given in both pictorial and verbal representations. The mathematics education community has long advocated for introducing students to different representations of mathematical concepts and teaching students to fluently make connections and move between the representations (NCTM, 2000). With particular topics, certain representations are more challenging for students to grasp and students have preferences for one representation over another (Kerslake, 1981; Kieran, 1992). For example, when presented alone, visual representations are more difficult to use than verbal descriptions. However, presenting visual data alongside verbal descriptions decreases the difficulty of problems (Threadgill-Sowder & Sowder, 1982).

Amount of given information. This aspect of a problem's given information is the one most used in research comparing problems from different sources (Harding, 1995; Zhu & Fan, 2006). Research has shown consistently that problems that contain extraneous information are much more difficult for students to solve than those that have only the exact information needed (Suydam & Weaver, 1977). When students attempt problems with extraneous information, the most common tendency is to use all pieces of information, some incorrectly, rather than attempt to decipher what is needed and what is not. This may be due

to a lack of exposure to this type of problem. While much of this research has been done for elementary school students, Carpenter, Corbitt, Kepner, Lindquist, and Reys (1980) report that 25% of 13 year olds attempted to use all data on problems on the National Assessment of Education Progress (NAEP) test that contained extraneous information.

Response format. Response formats include "selecting from predetermined answers, providing short answers, and providing extended solutions including justifications" (Palm, Boesen, and Lithner, 2006, p. 16). The response format is often associated with whether there is a single correct answer and if not, the number of correct responses. Zhu and Fan (2006) define an open-ended problem as one with "several or many correct answers" (p. 613). They found that problems that are open-ended are much less common in textbooks and present students with greater challenges than closed-ended problems. However, as with other classifications, there is some evidence that the difficulty can be decreased by exposing students to problems of this type prior to examination.

Routine versus non-routine. Zhu and Fan (2006) define a routine problem as one "for which problem solvers can follow a certain known algorithm, formula, or procedure to obtain the solution, and, usually, the path to the solution is immediately evident" (p. 613). So a problem may be non-routine if 1) the solution path requires the problem to develop a new algorithm or procedure, or 2) if the solution uses a familiar procedure but the problem solver must select from a possible set of procedures and the problem does not make clear how to select the appropriate one. The first situation is actually quite complex for a learner and will be discussed below as "creative mathematical reasoning" (Lithner, 2008).

Schoenfeld (1980) likens the second scenario to trying to open a lock with a set of thirty keys in a limited amount of time. You may try three or four keys, but without something to help you narrow down the selection, you will likely be unsuccessful. According to Chartoff (1976), "when an individual first orients himself to a problem, he must 'attach' this problem to his existing cognitive structure" (p. 3). Research on expert and novice learners has shown that expert learners are good problem solvers because they are able to do this "attaching" process efficiently (Seufert, 2003). Sweller (1989) explains this in terms of schemas. As a learner encounters more and more problems of a certain type, he develops a schema for solving that type of problem that helps him know which "key to select." Expert and novice learners are differentiated by how well developed these schemas are. The remaining question is whether students can be *taught* to become expert learners. Silver (1997) differentiates between the classical and contemporary views of (mathematical) creativity. While the classical view sees creativity as an inherent ability of a select few, the contemporary position is that creativity is a teachable skill.

It may be argued that the encouragement and requirement of this type of problem solving in most high schools and some colleges is actually quite rare, likely because it is difficult to teach. The work of Hourigan and O'Donoghue (2007) suggests that students at the high school level are trained and conditioned to repeat procedures. Many mathematics courses are exam-driven, and AP Calculus is no exception. When the primary goal of instruction is having students passing a specific examination, instruction often focuses on specific types of problems and the algorithms needed to solve those problems. Students can

then be successful on a predictable test, arguably with a limited understanding of the underlying concepts that support those algorithms. Selinski (2012) found that in German universities, students were exposed to novel exercises in class and in homework assignments. In contrast, "gymnasium students were rarely exposed to novel exercises, and were neither expected – by the institution or in classroom-established norms – to persist in these situations" (p. 414). It is possible that non-routine problems are more common in college, but this has yet to be confirmed by research. Selden and Selden (2013) describe routine and non-routine problems more as a continuum and they hypothesize that "the ability to work moderately non-routine problems…is often considered part of the implicit curriculum [in college] and taken as equivalent to good conceptual grasp" (p. 306). Yet the ability or inability to work non-routine problems may have little to do with a strong conceptual understanding, and instructors may be testing for one when desiring the other. The literature has yet to show what is most important to instructors, whether this differs in high schools and colleges, and whether instructors are even testing students for those traits or skills they deem most important. This is another area where this study will attempt to expand the current knowledge base.

Strategies that help in a student's narrowing or selection process when solving problems have been a major topic of research for many years. Some, like Schoenfeld, have argued and presented evidence that teaching problem solving strategies or heuristics can increase students' success in selecting appropriate algorithms or procedures. However, there is conflicting data to suggest that these strategies do not work or have very limited success.

It may be that the heuristics that have been tried in classrooms and in these studies are too general. For example, telling students to "write down given information and what you need to find" is a very general heuristic that is not domain specific and will often leave students at a loss for which procedure to use to get from the given information to what they are supposed to find. Zweng, Geraghty, and Turner (1979) found this instruction to be the least effective of a number of hints they used and tested with students.

Nevertheless, there are other types of surface features, such as cues, hints, key words, and context of problems that do correlate strongly with students' probability of success. Sometimes this correlation is positive; other times it is negative. This depends on whether the surface feature in question is used in the problem the same way it has been used in other problems that require the same or different problem solving strategy. Zimmerman (1979) explains that students attend to surface features very naturally, and they map these features to specific algorithms, often when inappropriate. Palm, Boesner, and Lithner (2006) give the example of students' strong association of the word "maximum" with the process of taking a derivative and setting it equal to zero. This is an example of a key word serving as a "trigger" for an algorithm. This happened in Narayanan's (1984) study where the key word "gave" triggered subtraction in an addition word problem. This can also occur with problem context. Students may not recognize the mathematical structure of the problem situation, but correctly (or incorrectly) apply an algorithm simply because previous problems with the same context had a similar (or different) mathematical structure.

Similarly to how they use key words and problem context, students may use direct hints and problem assignments or directions to select their problem solving strategy (Palm, Boesen, & Lithner, 2006). Such hints as "use the derivative to" signal a specific process to the student. These are also surface features; however, these tend to improve students' chances of selecting an appropriate strategy more consistently than those discussed above.

Types of reasoning. The majority of research done on math problem classification treats problems independently; that is, it assumes that any one problem may be classified using only the components or elements of that problem, using a framework that consists one of more of the classifications discussed above. More recently, others have argued that determining the difficulty of a problem for a student requires examining the student's previous exposure to similar types of problems (Jonassen, 2000; Palm, Boesen, & Lithner, 2006). This may seem obvious when attempting to identify whether a problem is routine; however, researchers have attempted to classify the difficulty level of problems without examining all materials available to students (Mesa, Suh, Blake & Whittemore, 2013, Tallman, Carlson, Bressoud, & Pearson, in press; Zhu & Fan, 2006).

Lithner and his colleagues (Lithner, 2006; Palm, Boesen, & Lithner, 2006) have developed a conceptual framework that distinguishes between imitative and creative mathematical reasoning. Reasoning is defined by Lithner as "the line of thought adopted to produce assertions and reach conclusions in task solving" (p.257). Creative mathematical reasoning has four components – novelty, flexibility, plausibility, and mathematical foundation. Novelty requires the learner to produce a solution method they have never seen

or have forgotten. Flexibility requires a detachment from fixation on one solution method.

The plausibility requirement eliminates guesses and "vague intuitions," even if they are

novel. Finally, mathematical foundation holds that the argumentation be "founded on

intrinsic mathematical properties of the components involved in the reasoning." Imitative

reasoning, in contrast, is devoid of "attempts at originality" (Palm, Boesen, & Lithner, 2006,

p. 6). Reliance on imitative reasoning in problem solving can result from an educational

setting that operates under the reductionist approach. This prevents transfer of learning to

new problem types and settings.
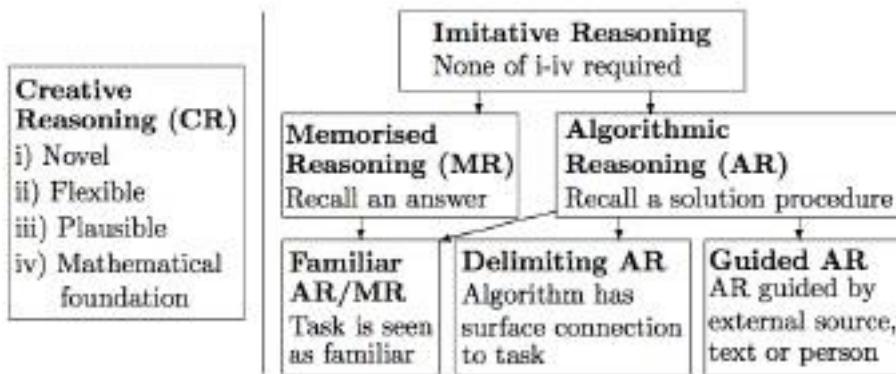
Figure 1. **Imitative versus Creative Reasoning Framework, Reprinted from A Framework for Analysing Creative and Imitative Mathematical Reasoning by J. Lithner, 2006. Retrieved from http://snovit.math.umu.se/forskning/Didaktik/Rapportserien/060705B4D2.pdf**

Imitative reasoning can be further categorized as either memorized or algorithmic

reasoning. Memorized reasoning requires simply recalling information to produce a

complete solution, such as a facts, definitions, or even proofs. Algorithmic reasoning is

similar to memorized reasoning in that in requires recalling something from memory.

However, what is recalled is not the full solution but rather a procedure or method that leads to a correct solution. A key feature of algorithmic reasoning is that once the problem solver recalls the algorithm, no justification for the choice of algorithm or understanding of why the algorithm works is required. In contrast to using creative reasoning, the problem solver may choose an algorithm based on its use in a previously seen task with similar directions or surface features where the algorithm was used successfully.

Algorithmic reasoning is further defined as one of three types. Familiar algorithmic reasoning is used when a problem solver associates a task or set of instructions with a particular algorithm. Once the task type is identified, the problem solver immediately knows to apply a specific procedure. This is akin to Zhu & Fan's (2006) definition of routine problems. Delimiting algorithmic reasoning, by contrast, is used when the task is not entirely familiar and requires the problem solver to select a set of algorithms. In choosing an algorithm, the problem solver does not reason through the decision making process using mathematical properties, but rather selects an approach based on surface property similarities with previous tasks. The final type, guided algorithmic reasoning, relies on using similar examples from written materials or assistance from another individual. In either situation, the burden of selecting an appropriate strategy is significantly reduced or altogether eliminated by the text or the individual. Sometimes problems have a small element which requires creative reasoning, but the overall problem does not. These problems may be solved using what Lithner terms *local* creative mathematical reasoning. Problems requiring local

creative reasoning may be difficult to distinguish from those solvable by imitative reasoning (Bergqvist, 2007).

Research to determine the type of reasoning required on mathematics tests based on previously seen problems is sparse. Students in Anthony's (2000) study reported looking for similar examples as their first approach when stuck on a particular problem, but it was not clear whether students were successful using this approach. Textbooks have been studied to determine whether exercises could be solved using imitative reasoning. Lithner (2004) first explored textbook problems to further refine the types of reasoning that could be used to solve these problems. Later, in a more focused attempt at determining required reasoning type on college tests, Bergqvist (2007) examined the types of problems on 16 calculus tests at four Swedish universities. The study found that 15 of the tests could be passed using only imitative reasoning.

Link Between Student-Centered and Course-Centered Challenges

If, as Lithner and Bergqvist's research suggests, there is little creative reasoning required on college calculus tests, it is reasonable to question why some students are still underperforming, particularly when having previously taken a calculus course. Credé and Kuncel (2008) have developed an Academic Performance Determinants Model that attempts to explain the factors of academic success and the connections between them (see Figure 2). This model suggests that certain *direct* factors are more proximal to success than others, and that some factors mediate the impact of certain factors on others. It links the type of

knowledge necessary for success with those student-specific factors discussed above, such as study habits and motivation.

The results of Credé and Kuncel's study show that study skills, habits, and attitudes are as effective predictors of college academic success as standardized tests and previous grades. Their model reflects this result. Certainly certain types of knowledge are direct factors of success, but study skills impact how much knowledge students will attain. This model is not specific to calculus or even mathematics and should be explored to determine whether the factors and the relationships presented in this model hold to be true for AP and/or college calculus students.
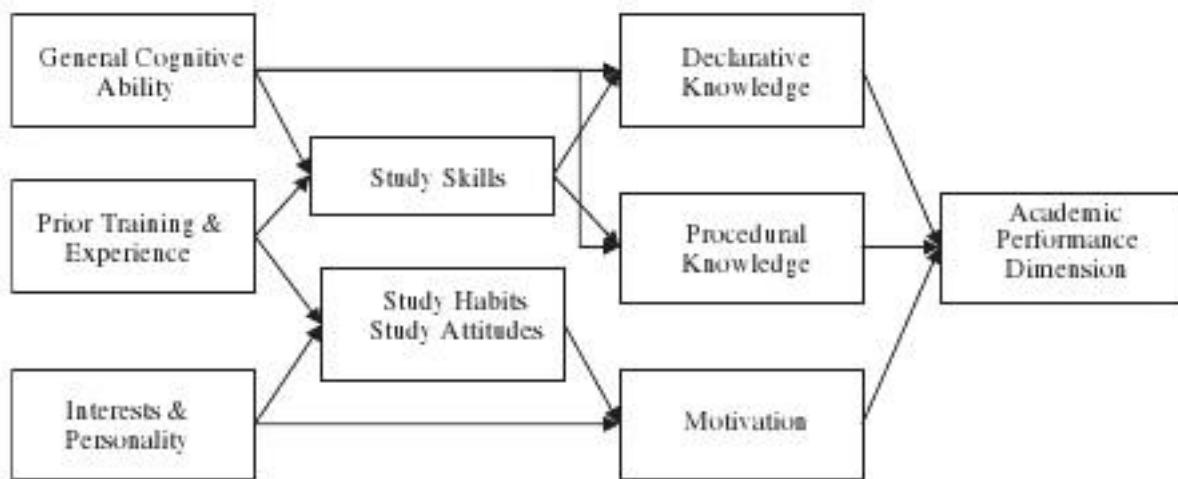
Figure 2. **Academic Performance Determinants Model.** Adapted from "Study Skills, Habits and Attitudes: The Third Pillar Supporting Collegiate Academic Performance by M. Credé and N. Kuncel, 2008, *Perspectives on Psychological Science, 3(6),* p. 386.

Summary

The heralded success of and increased enrollment of AP Calculus suggest it is poised to serve as a pipeline for future STEM graduates. However, a growing number of students are either not getting credit for AP Calculus or are voluntarily repeating the course in college, and many of them are underperforming in the college course. Previous research suggests that calculus students often struggle with insufficient mathematical background knowledge, motivational and study skill issues, and the demands of specific types of test problems. It is unknown at this point, and this study will explore to what extent these things vary from AP to college calculus and which of them have a greater impact on success in the college course.

CHAPTER 3: METHODS

Introduction

This study explored students' perceptions of their high school (Advanced Placement) and college calculus experiences and how students' experiences impact their performance. More specifically, it identified differences in students' high school and college calculus experiences and how these differences might influence the success of students in the college course. The overall purpose of the study was to determine the impact of the AP Calculus experience on student success in college calculus. This was achieved by addressing two research questions:

1. What factors affect student success in calculus and how do these factors differ in AP and college calculus?

2. What challenges do students face on calculus tests and how do these challenges differ in AP and college calculus?

Framework

Two conceptual frameworks, described in Chapter 2, informed the research design and guided the data analysis. The first, developed by Credé and Kuncel (2008) and referred to as the Academic Performance Determinants Model, was used as a starting point for developing a similar model specific to calculus. The second, developed by Lithner (2006) and colleagues, delineates the types of reasoning required to solve mathematical problems. These types of reasoning were used to code test problems in the data analysis and the types

of reasoning required on tests were used as factors to student success in creating the model

for calculus students.

Credé and Kuncel (2008) provide a model of academic performance determinants

(see Figure 2 above). This model explains how content knowledge and success are related to

and impacted by students' attitudes and study skills. The authors posit that performance on a

task is directly affected by three factors – declarative knowledge, or "knowledge of facts and

procedures," procedural knowledge, or the "skill to do what is required in a situation," and

motivation, or the "willingness to engage in and sustain a high level of effort in completing

the task" (p. 429). These factors are affected by other factors, which are affected by still

other factors. The effect of the more distant factors on performance is mediated by the more

direct factors. The model suggests that study skills, habits, and attitudes mediate the effect of

distant factors (such as cognitive ability) on the direct factors (such as procedural knowledge)

and on the success on the task.

Success is "directly a function of" the direct factors: Motivation, Declarative

Knowledge, and Procedural Knowledge; these are the factors most closely related to

performance. In contrast, other factors, which are study skills, habits, and attitudes, interests

and personality, prior training and experience, and general cognitive ability, must influence

one of these three direct factors to impact performance, and therefore may be unnecessary for

success. Suppose, for example, that a student has a significant gap in his procedural

knowledge. A task requiring procedural knowledge can prevent a problem solver from being

successful, regardless of his strengths in other areas, if he has deficiencies in his procedural

knowledge. Consider the contrasting situation with a non-direct factor. If a student has low

cognitive ability, he may have a tendency to underperform. However, this may be

compensated for with greater study skills, which could mediate the effect of his ability on his

acquisition of procedural knowledge and subsequently his performance on an exam. The

idea that some factors of success are more proximal to academic performance than others and

that some factors mediate the effect of others was adapted to develop a Success Factor Model

for calculus that will be discussed in Chapter 4. It should be noted that Credé and Kuncel's

definition of procedural knowledge does not clearly align with how the term is used in the

mathematics education community. Rather than declarative and procedural knowledge, the

distinction in the mathematics education literature is more often made between conceptual

knowledge and procedural knowledge. Hiebert (2013) defines conceptual knowledge as "a

connected web of knowledge, a network in which the linking relationships are as prominent

as the discrete pieces of information." Procedural knowledge includes "the formal language,

or symbol representation system, of mathematics," and "algorithms, or rules, for completing

mathematical tasks" (p. 6). This was considered when adapting the model for calculus.

The second framework used in this study was Lithner's (2006) categorizations of

types of mathematical reasoning (see Figure 1). As mentioned above, Lithner's approach to

categorizing reasoning differs from most problem classification systems in that it does not

categorize a problem or solution method independently, but rather considers which problems

students have seen previously when determining the reasoning required to solve the problem.

That is, two different types of reasoning may be required of two different problem solvers

because of the problems they have previously learned to solve. Lithner argues that reasoning used during mathematical problem solving can be classified as either creative or imitative. For the reasoning to be considered creative, it must meet four criteria; it must be novel, flexible, plausible, and have a mathematical foundation (see explanation in Chapter 2).

If a problem solver has not used creative reasoning to solve his problem, he has used some form of imitative reasoning. Imitative reasoning can be further classified as either memorized or algorithmic reasoning. Memorized reasoning requires either recalling information necessary to solve a problem, such as a multiplication fact, or recalling an entire solution, such as a proof. In calculus these may take the form of recalling parts of or entire definitions. For example, a problem may ask, "What conditions must be verified to show a function is continuous at point a?" As a reminder to the reader, algorithmic reasoning also requires recalling something from memory, but in contrast to memorized reasoning, the full solution is not recalled but rather a procedure or method that leads to a correct solution. In contrast to using creative reasoning, once the problem solver chooses the algorithm, no justification for the choice of algorithm or understanding of why the algorithm works is required. The problem solver may choose an algorithm based on its use in a previously seen task with similar directions or surface features where the algorithm was used successfully. It may also be guided by a written resource or another individual.

Lithner's framework is more often presented in terms of reasoning used by the problem solver rather than required reasoning type for a particular problem. It may be argued that the type of reasoning required is unique to the problem solver and therefore not

able to be determined for a problem. However, Lithner and colleagues have used these classifications to determine reasoning type of textbook exercises and university exam problem by deciding the type of reasoning that *could* be used based on the problems that students had available (Lithner, 2004, Bergqvist, 2007). As Bergqvist explains, the validity of this framework for classifying problems was verified by comparing reasoning students actually used to the type determined by the framework (Boesen, Lithner, & Palm, 2005).

Subjectivity Statement

In addition to the two frameworks described above, the research design and data analyses were influenced by my work over the past decade. I have worked with undergraduate students in tutoring and academic coaching environments and have taught undergraduate mathematics courses at two post-secondary institutions. I have worked with many motivated, intelligent students who were not achieving their academic goals in math and science courses. In my experience, students have not been required to use creative reasoning often in their math and science courses. Most exam problems are strikingly similar to problems students have seen in either class or homework assignments, yet students sometimes find themselves surprised by these questions on exams. I have seen these students drastically improve their performance not by getting help with the content but by significantly changing their study approach to their course, and particularly being intentional about learning how to solve particular types of problems. These experiences impacted the study design and likely how I interpreted the data.

Research Design Overview

This mixed-methods study consisted of two parallel parts – individual student interviews and test analyses. The phases were not dependent on one another, but because the interviews were conducted and analyzed prior to the test analyses, the test analyses were somewhat informed by the interview data. See Figure 3 for an outline of the design.

Research Question 1 was answered by analyzing two-part individual student interviews. Interview participants were college calculus students who were repeating Calculus I after taking AP Calculus in high school. (See Table 1 for a brief description of the sample of interview participants.) The first part of the interviews were semi-structured, conversational interviews (Drever, 1995, Smith, 1995) that addressed general challenges in the calculus experience. College, rather than high school calculus students were interviewed because they could speak to their experiences in both high school and college calculus. The second part of the interviews addressed Research Questions 1 and 2 and were designed to identify challenges specific to test-taking through structured task-based interviews (Kelly, A. & Lesh, R., 2000). In a structured task-based interview, participants are presented with "questions, problems, or activities" and they interact with these tasks in addition to the interviewer (p. 519). In this study, the tasks presented were missed test problems and similar problems from the students' course resources. The collective goal of the tasks was to identify whether the student could have used imitative reasoning (IR) to solve the missed test problem.
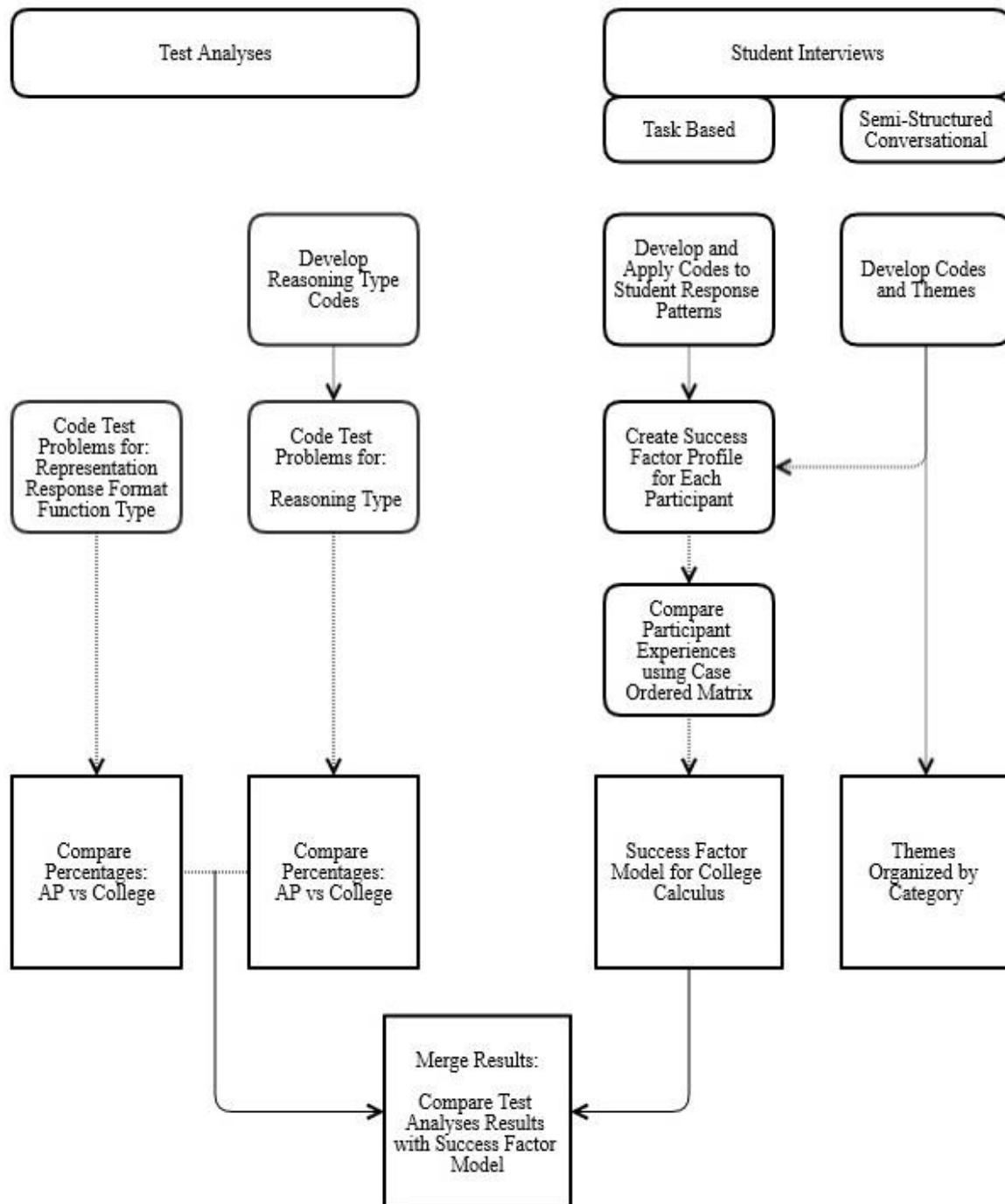
Figure 3. **Study Design Sequence.**

Research Question 2 was also addressed by the second part of the study, the test analyses. In this part of the study, test problems from both college and AP Calculus tests were classified for certain problem characteristics, including problem representation, response format, function type, and required reasoning type. The first three classifications could be made by considering each test problem directly. However, classifying a problem's reasoning type required comparing the test problems to similar problems in course resources that students had access to prior to the test (Lithner, 2006). The college tests analyzed were obtained from the same institutions as the student interview participants so that results from the interviews could be reasonably compared to results from the problem classification. See Table 1 below for a description of this connection.

Table 1

Selection of Interview Participants and Tests

	Part 1: Student Interviews (divided into two sections)	Part 2: Test Analyses
	(Research Questions #1 and #2)	(Research Question #2)
University	14 participants from 6 universities	6 tests from 6 instructors at the same 6 universities
High School	---	6 tests from 6 teachers at 6 high schools

Finally, to address the overarching research goal, the results from the two questions were merged. This process involved searching for any similarities and discrepancies in the results from the two phases as well as for explanatory potential of some results for others. Specifically, results from the test analyses were compared to the Success Factor Model to

identify any discrepancies between the challenges discovered from the class materials and those from the student participants' experiences. These comparisons are discussed at length in Chapter 5.

Interview Participant and Test Selection

The units of analysis for Parts 1 and 2 of the study were college calculus students and individual calculus test problems from high schools and universities, respectively. The processes of acquiring interview participants and finding tests were interdependent because of the design requirement of having the interview participants come from the same universities that provided tests for the analyses. Therefore I will describe the process of selecting participating universities and students from the universities and then the process of acquiring high school tests.

Universities and Students

I initially targeted six colleges and institutions for participation with the expectation of finding 15 student interview participants and six instructors who would share their tests and class materials. These institutions were targeted for convenience of proximity but also for diversity in size and whether they were public or private. I contacted one or more Calculus I instructors at each institution who agreed to help recruit participants and share course materials. Two of these instructors were prior acquaintances, but others were found by using university websites to identify current Calculus I instructors. I obtained IRB approval at each institution before advertising the opportunity to students.

Instructors made brief in-class announcements and distributed fliers to their students and I contacted the same students the same or next day via email with the same information. (See Appendices A and B for sample emails and fliers as well as the informed consent form.) To participate initially, a student must have been a first-year student enrolled in a Calculus I course and must have made a D or F on their exams up to that point in the semester. The student must have taken and not passed the AB Calculus test or taken and not passed the AB subscore of the BC test, by their institution's standards. For institutions who offered more than one version of Calculus I, I recruited students from the most rigorous course, outside of honors or majors sections, typically referred to as being for Scientists and/or Engineers.

These criteria proved to be too narrow to produce the desired number of participants, so they were broadened so that the recruited student was not required to have failed the AP exam and was only required to have made a C- or lower on at least one college calculus test. This still resulted in a total of only seven participants. I then made new instructor contacts at additional universities, obtained IRB approval, and advertised the opportunity to students, until I found an additional seven student participants at three of these universities using the second set of criteria. This resulted in a total of 14 interview participants from six universities. The universities included two small private and four large public universities in two states in the southeast. These six universities were all on the semester system and students were recruited anywhere from 8-14 weeks into the 16-week semester. Each of the six instructors who provided course materials, not necessarily the instructors of the

interviewees, was a faculty member of the math department at his or her institution and had
taught Calculus I previously. A description of student participants is shown in Table 2.

Table 2

Student Participant Descriptions

Student Participant	Institution	Passed AP test?	Passing Course?
Jeremy		No	Minimally
Allen	Large Public #1	No	Minimally
Frank		No	No
Michael		No	No
Albert	Large Public #2	Yes	Yes
Jeffrey		Yes	Yes
Wade	Large Public #3	No	Yes
Erin		Yes	Yes
Samuel	Large Public #4	No	No
Haley		No	Minimally
Katelynn		No	Minimally
Maggie	Small Private #1	No	Minimally
Isaac		No	Yes
Blake	Small Private #2	Yes	Yes

High Schools

To obtain AP Calculus test and course materials, I searched websites of school boards
and individual schools to find public high schools in the Southeast that offered AP Calculus
and to find the contact information of AP Calculus teachers. I used purposeful sampling to

find schools in different geographic regions and in both rural and suburban areas. Proximity

was not necessary for the high schools as it was with the universities; however, I limited my

scope to the two states where the six participating universities were located. AP instructors

were required to have been in at least their second year of teaching AP Calculus. This

produced a list of 25 instructors that I contacted by mail and email, asking if they would

share their test on limits and continuity and any corresponding class materials, including

class notes, homework assignments, practice tests, quizzes, etc. I copied school principals on

the mail correspondence. (See Appendix A for sample contact letter.) Four of these

instructors agreed to participate and sent materials either by mail or email. I then found an

additional two instructors through established contacts. See Table 3 for details about the

participating instructors and their high schools.

Table 3

Descriptions of Participating AP Calculus Classrooms

	Age of AP Calculus program (years)	Years Instructor Has Taught AP Calculus	Location
High School 1	5-10	5-10	Rural
High School 2	10+	5-10	Urban
High School 3	10+	10+	Urban
High School 4	10+	<5	Rural
High School 5	10+	10+	Rural
High School 6	10+	unknown	Rural

Data Collection

Collection of Interview Data

I conducted interviews in the fall semester of 2012 between mid-October and early December and scheduled them as soon as possible after the student agreed to participate. Each interview was conducted face-to-face in an office or group study room on the participant's campus and lasted approximately 90 minutes. Detailed field notes were taken and the interviews were videotaped unless the student requested to be audiotaped instead.

I used a semi-structured, conversational interview process for the first 45 minutes. The interview protocol consisted of 17 open-ended questions. Many of these questions asked the student to describe a particular aspect of their experience in both the high school and college setting (See Appendix C for complete protocol.) For example, one question posed was "What did your instructor expect from students?" and the students were asked this question about both their AP and college calculus instructors. The interview protocol was influenced by the Academic Performance Determinants Model which suggests that a student's study skills, habits, and attitudes influence their acquisition of knowledge.

I used a structured, task-based approach for the second portion of the interview. Unlike many task-based interviews, the goal was not primarily to gain understanding about the participants' knowledge of a particular concept. This portion of the interview was diagnostic – it served the purposed of determining the reason(s) the participants missed the problems they did on their test. More specifically, in light of Lithner's framework (2006), its purpose was to determine whether imitative reasoning could have been used to solve each

missed test problem. Participants were required to bring their course materials, including

their most recent exam. (Each participant had made a C- or below on their most recent

exam.) I first asked the student to identify two problems on which they had lost the most

points. Then the following process was completed for each of the two problems (with

exception of 2 students who took longer to complete the first problem).

Taking one of the two test problems at a time, I asked the participant to help me

locate similar questions to the test problem in the homework, textbook, class notes, or other

materials (such as practice tests and quizzes). To save time, we split the course materials to

locate these similar problems. (In some cases I was able to access the tests and course

materials prior to the interview and identify these similar problems ahead of time.) Once the

most similar problem was identified, I asked the participant to solve this problem. I provided

assistance when needed, first by showing the participant a worked out example or written

explanation from the resources. If this did not prove sufficient, I would then use questioning

to guide the participant to the solution. Once the participant had solved the similar problem,

I then asked him or her to solve the test problem. (The previous problem was no longer

accessible to the student; they were not able to look back while solving the test problem,

therefore making it more akin to a testing situation and making guided algorithmic reasoning

unavailable.) Any point where the participant indicated confusion or stopped to ask for help

was noted; that is, any reason for not being able to complete the test problem after working

the similar problem was documented. At the conclusion of this process I asked the

participant, "If you had known how to do this problem [from your resources] going into the test, would you have missed the test question?"

Collection of Course Materials

I collected course materials from twelve instructors – six from universities and six from high schools (see Table 1). I asked each instructor to provide a copy of his or her test(s) covering the topics of limits and continuity. Limits and continuity were chosen primarily because problem types dealing with these topics can vary so greatly. While some Calculus I topics such as Related Rates are inherently application-based, and others such as derivative "shortcuts" are primarily procedural, limits and continuity have both rich conceptual and procedural components and can utilize any representation format. Additionally, limits and continuity are typically taught early enough in the Calculus I course so that instructors would have already tested these topics at the point in the semester when the materials were requested.

Each instructor sent only one test. For some instructors, the test consisted of only limits and continuity; for others, these topics made up only a portion of the test. In addition to the test, I asked for materials about limits and continuity that students were exposed to prior to the test. This included class notes, homework assignments, textbooks (if students had individual access to them), practice tests, quizzes, and handouts or worksheets. It was necessary to collect these materials to determine the type of reasoning required for each test problem. In most cases instructors sent their materials directly by mail or via email. In

several cases most of the materials from one teacher could be accessed through a course website.

Data Analysis

The following is described below: Analysis of Part I of the interviews led to the emergence of 17 themes relating to students' experiences in AP and college calculus and how they compare. Analysis of Part II involved writing summaries for each student's attempts at solving missed test problems and determining what type of reasoning could have been used to solve these problems. The results from both parts of the interviews were then combined to produce individual success profiles for each student, and then a comparative analysis of the profiles illuminated the common factors of success in calculus and how they were related, which resulted in the Success Factor Model for College Calculus. The results of the test analyses were compared against the Success Factor Model.

Interview Data Analysis

Analysis of Part I, the Semi-structured Interviews. I completely read Part 1 of each of the 14 interview transcripts at least one time. During a second reading, I used line-by-line open coding to identify any experience potentially common to another participant. "Line by line coding forces the analyst to verify and saturate categories, minimizes missing an important category and ensures the grounding of categories the data beyond impressionism" (Glaser & Holton, 2004). Codes were highlighted and noted in the transcripts and upon a third reading they were recorded. Many of these codes were related in

some way and I listed the codes in an order that showed connections, but at this point they were still written as distinct codes directly from the transcripts.

My expectations of themes that would emerge were influenced by the Academic Performance Determinants Model (Credé & Kuncel, 2008), the protocol design, and personal experience. Most questions asked students to first describe a particular aspect of their high school calculus experience and then the same for college, so I expected themes to be differences in the two environments. I first grouped all codes that did describe a difference in the high school and college environment, and this left others that were specific to high school or to college. However some codes did not fit into any of these categories. For example, a majority of the participants discussed knowing one or both of their calculus instructors well and this having a positive impact on their motivation and performance and this was not specific to either high school or college. I then began to examine the codes for any other kinds of similarities, using axial coding to group the codes into themes (Strauss & Corbin, 1997). For example, a theme emerged around the course emphasis of calculus courses. Some students had discussed a shift from a concrete or procedural focus in high school to a conceptual or abstract focus in college. Others described their experience in terms of "doing' versus "understanding". Still other students highlighted the increased requirement of formal mathematical language and attention to detail on exams. However, all these codes were related to how content was presented or tested in the classroom and were therefore collapsed into a theme called course emphasis. I discussed decisions in the axial coding process at length with my advisor – in an effort to limit the resulting number of

themes while not losing important data. She both provided new options for ways to group or collapse the codes and provided her opinions of my suggestions for doing the same.

Once the codes were sufficiently collapsed into 17 themes, I was able to recognize four distinct categories of themes. These were 1) Course Content, 2) Class Format, 3) Approach to Studying, and 4) Confidence and Self-Assessment. The first two categories house themes that are controlled by a course or instructor student-specific while the latter two contain themes that are student-specific. I organized the 17 themes under these four categories. I then revisited the transcripts to code line-by-line again for each theme. If a theme was not found amongst at least 3 participants, it was eliminated. These themes and categories of themes will be elaborated in Chapter 4.

Analysis of Part II, the Task-based Interviews. After reading each interview transcript for the second portion of the interview, I wrote a 1-2 paragraph description of the student's problem solving process for the attempted test problems and similar problems. In this part of the interview, each participant had solved two test problems, so my summaries included two separate descriptions in cases where the participant's process varied across the two problems. These descriptions included the following three items: 1) whether the participant was able to solve the similar problem without assistance, 2) whether the participant was then able to solve the test problem after learning to solve the similar problem, and 3) if not, what barrier(s) the student encountered in solving the test problems (such as insufficient pre-requisite mathematical knowledge). Item 1 spoke to the student's preparation for the test, while item 2 revealed whether imitative reasoning could have been

used to solve the problem. Item 3 for each problem was compared across participants and commonalities were noted.

Creating the Success Factor Model for College Calculus. The summaries from the Interview Analysis Part II were combined with some of the results from Interview Analysis Part I to develop a Success Factor Model for College Calculus that captured all factors affecting success of the fourteen participants and the relationships between these factors. I adopted the idea from the Academic Performance Determinants Model (Credé & Kuncel, 2008) that some factors are more closely related to student success than others and some factors mediate the effect of other factors on success. As I examined transcripts and my summaries, I began to change the proximity of some factors to success, add additional factors and remove others, as well as add and remove connections between factors. This was a long iterative process that involved going back and forth between the overall model and the individual participants' experiences.

In cases where participants were able to use imitative reasoning during the interview but had failed to practice the similar problem prior to the test, the first part of their interview transcripts were searched for potential explanations, such as a belief that they knew how to solve that type of problem or an expectation that it would not be on the test. As these barriers to success and explanations were discovered, they were used as factors for a first draft of a Success Factor Model for College Calculus that might be appropriate for all the students.

I developed the initial draft of the Success Factor Model after examining individually and then comparing the summaries of only four participants. This draft included all factors

that affected any of those four participants. Subsequently, I read additional summaries (in no particular order) and created individual maps for each participant that highlighted the particular path to success and barriers to success that student encountered. Each individual map is simply a version of the model that highlighted weak and strong areas for that participant. In cases where a participant's experience could not be thoroughly explained by factors on the model, I made changes to the model to account for these factors. This was an iterative process; when a change was made, I then revisited previously examined participants' summaries and maps to assure the new version of the model would represent their experiences.

I initially used certain success factors and terminology from the Academic Performance Determinants Model while leaving out others. Some terms from this model, such as cognitive ability, were eliminated because of my inability to objectively measure the quantity for each participant from the interview data. Other terms were combined with language more specific to the discipline to create new factors. For example, the factor "Prior Training and Experience" was changed to AP Calculus Experience.

One aspect of the Success Factor Model for College Calculus emerged as distinct from the Academic Performance Determinants Model. A factor was deemed "direct" only if it was a requirement for success on certain calculus problems. If there was an alternative route to success, meaning that students could bypass that particular factor and still solve all test problems, it was not included as a direct factor. This prevented, for example, motivation from being included as a direct factor in the calculus model, although it was a direct factor in

Credé and Kuncel's model. While motivation certainly aids in successful problem solving for many students on many problems, some students proved to be successful on problems without also demonstrating a great deal of motivation.

One particularly challenging aspect of the process of creating the Success Factor Model was determining which of the themes from Part I of the interview data could be considered as factors of success. For example, in response to one or more of the interview questions, many of the participants discussed the impact of one or more of their calculus courses being highly structured. This might include regular accountability provided by the instructor and/or a course design with regular assignments, quizzes, frequent tests, etc. The participants consistently suggested that this increased their motivation and subsequently their time spent on the course. This surfaced easily as a theme, and it also made sense to include it as a factor in the model since all students who mentioned it indicated a direct association between this factor and their success in the course.

This example of theme inclusion contrasts with other themes from Part I of the interview data such as class size. Many participants noted differences in class size from high school and college when asked about differences between the two. However, when further probed about the significance of the class size, reactions were mixed regarding whether this had an impact on their course grade. Many said that while they preferred smaller classes, they could have gotten the help they needed from the instructor or other sources in larger classes. Because of the lack of consistency among responses and because of a clear

association between class size and performance, this theme was not included on the Success Factor Model.

Another specific challenge was determining how to organize and name the direct factors of success. I initially attempted to use the distinctions procedural and conceptual knowledge as factors of success. The data showed that most problems required some type of procedural knowledge, but conceptual knowledge was not essential. However, procedural knowledge alone did not lead to success on all problems for all participants. Some students, for example, could not solve their problem(s) because of inability to recall information that should have been memorized. Others were halted in their progress by a trick or nuance to the problem that did not necessarily require calculus knowledge. This led to considering using creative and imitative reasoning as factors. However, like conceptual versus procedural knowledge, the reasoning framework could not, by itself, account for all the distinctive reasons that students missed test problems. Specifically, it did not allow for identifying whether conceptual knowledge or understanding played a role in the problem solving process.

At this time, I revisited the factors to success in light of an additional classification system for mathematical knowledge. Building on the work of Ryle (1949) and Skemp (1979), Mason and Spence (1999) distinguish between four types of knowing – knowing-THAT something is true, knowing-HOW to do something, knowing-WHY you do something or why something is true, and knowing-TO do something in a particular situation. They claim that classroom education focuses on teaching knowing-THAT, HOW, and WHY,

which amounts to knowing *about* a subject, but that this does not equate to knowing-TO. They also explain that while there are certainly connections between the types of knowing, and often one can facilitate another, they are distinct, in that one does not guarantee or precede another.

I found that knowing-HOW, TO and WHY did not perfectly align with any of the types of reasoning, but there were significant connections between them, such as knowing-THAT and memorized reasoning (MR). Including both creative (CMR) and algorithmic reasoning (AR) as well as knowing-HOW, WHY, THAT, and TO, as separate factors, I revisited the individual student profiles and found which of these factors always overlapped and was able to combine some of these. For example, if a participant used algorithmic reasoning, they had to know-HOW to perform a certain procedure and they also had to know-TO use that procedure from similar instructions or directions given in the problem. Other pieces of the terminology were included separately, such as knowing-THAT and knowing-WHY. There are also situations where knowing-THAT can be necessary, in addition to one of those types described above, and situations where knowing-WHY can impact knowing-TO using creative reasoning or knowing-THAT.

Determining Relative Impact of Success Factors. Once the model was complete with its factors and pathways, I set out to determine the relative importance or impact of each factor. For example, I was interested in which direct factors were most commonly required on calculus test problems, and which of the indirect factors students tended to rely on more or less often to aid in their success. To accomplish this, I created a meta matrix (Miles &

Huberman, 1994). (See Table 4.) Each student participant was listed on a separate row of the matrix. The column headings or variables of interest were found by revisiting interview transcripts and individual students' maps and looking for similarities and differences in both the AP and college experiences, such as whether the student was passing the college class, how they felt about their mathematical ability, whether they had found AP Calculus to be difficult, and what barriers they had encountered during the second portion of the interviews in solving their test problems. Note that not all information was available for all 14 participants.

I then used the meta matrix to create case-ordered matrices (Miles & Huberman, 1994). That is, I sorted the participants by certain variables such as their college calculus grade, their AP test score, or whether they felt they were good in math. Once I had sorted for a particular variable, I then looked across the other categories for similarities amongst cases with this same characteristic, to determine which variables might be associated with others.

I was able to group students by their relative success in college calculus and AP test score. Doing this illuminated which factors were consistent for successful students and which factors varied, or were not essential. For example, no student who made a 3 or higher on the AP test was failing his or her college course. This process illuminated the different pathways students could take to success and the common factors that hindered other students' progress. The results of this analysis are discussed in Chapter 4. However, most orderings did not reveal consistencies, but rather illuminated how different the individual student experiences were from others. Therefore, the results of the interviews in Chapter 4

Table 4

Student Profile Meta Matrix

	Grade in class	Problem 1 Barrier	Problem 2 Barrier	HS Difficult?	AP Score	Overconfident because of AP?	Lot of effort in AP?	Confident in math ability?	Recognizes problems in study approach?	I know this stuff; doesn't show on test	Tests have "curveballs"?	Don't know what else to do?
Albert	A-/B	Does w/out help	Notation/alg and unique	Yes (B)	3	yes	No	Sort of		Yes		
Allen	B/C	Taking derivative	---	Yes (B)	3			Yes, not calculus				Yes
Blake	B	Does w/out help	Algebra	YES (C+)	3		NO	yes				
Haley	C	Algebra	Algebra	Yes (C)	2		No	No				
Jeremy	C	Does w/out help	CMR	Yes (B)	2	Yes	Yes		Yes – unused resources	Yes		
Katelynn	C	Still needs a lot of help	Yes, with algebra	Yes (C)	1		NO	Not any more			Yes	
Michael	F	Algebra	---	No	1	Yes	No	Yes	Yes, low effort			
Maggie	C	Does w/out help	Does w/out help	No (A)	3	Yes		Not any more	Yes – use of hw			Yes
Isaac	A/B	CMR	Algebra	No (A)	3		Yes	Yes			Yes	
Samuel	F	CMR	Trig	No	2		Not until end	No	Yes – use of hw	Yes	Yes	Yes
Frank	F	trig	Algebra/trig	NO	1	Yes	No	Yes	No	Yes	yes	Yes
Wade	B-/C	Does w/out help	Does w/out help	No	2			Not any more				
Erin	B	Does w/out help	Does w/out help	Yes (C)	3	Yes	No	Yes	No		yes	yes
Jeffrey	B+	Memori-zation (trig)	Does w/out help	No (A)	4	Yes	No	Yes	Yes –too little effort			

will contain individual descriptions of the participants that note similarities between them, but are not separated into categories.

Test Analyses

In analyzing the class materials from the six college and six high school instructors, my first step involved reading and identifying which problems dealt with my chosen topics of limits and continuity and solving those problems. After solving the problems, I eliminated some problems from analysis. I eliminated limit problems dealing with end behavior because some instructors did not cover or fully cover these types until later in the course when they taught L'Hôpital's Rule. Problems were also eliminated from analysis if they required knowledge beyond limits and continuity. For example, some asked about both differentiability and continuity.

Coding for Imitative Reasoning versus Creative Mathematical Reasoning. I decided to classify problems as requiring either creative, memorized, or algorithmic reasoning. (Recall from Chapter 2 that memorized and algorithmic reasoning are the two types of imitative reasoning.) This is consistent with Palm, Boesen, and Lithner (2006) who compared teacher made tests to national tests in Sweden. These types can be further categorized as either familiar, delimiting, or guided reasoning, but two complications arose when trying to apply these distinctions in this setting. First, guided reasoning does not apply since the point of this exercise was to ascertain whether the participants could solve their test problems without assistance. Secondly, the distinction between familiar and delimiting reasoning is possible to discern when observing a student making problem solving choices,

but much more subjective when classifying problems independently from student work; it is a feature of the student's association of problem and procedure and is specific to the student, rather than the problem.

Once the qualified questions were identified on a particular test, the first step in classifying a problem as IR (solvable using imitative reasoning) or CMR (requiring creative mathematical reasoning) was determining whether a test problem could be solved by memorized reasoning (MR), which is a sub-type of IR. This did not require comparing the problem to other problems, but rather to information presented in the resources. These problems were rather easy to identify and usually took the form of recalling all or some part of a definition, such as continuity.

For the problems not classified as MR, I set out to determine if the problem could be solved using AR (the other sub-type of IR) or whether CMR was required. I examined the corresponding course materials for similar problems. Any problem that could be solved using a procedure that could also be used to solve a test problem was recorded as a match to that test problem. The reason for searching for these similar problems was that if the student had known how to solve the matching problem prior to taking the test, the student should have been able to use AR to solve the test problem. (Issues regarding this claim will be addressed in the limitations section of Chapter 5.) This process was completed for each of the six university and six AP Calculus tests. Note that textbook problems were only considered if 1) students were issued personal copies of the book, in high school, or were

considered a "required" part of the college course, AND either the problem was a worked out example in the text or was an assigned or suggested homework problem.

Any given test question might have multiple corresponding problems in the resources. Similar problems were sifted from multiple sources, such as textbooks, class notes, and homework assignments. In this way, this study purposes to be more accurate in classifying the type of reasoning required on tests, since any problem students have seen previously, in any source, has the potential to impact the reasoning type available to students on certain test problems. It is worth noting that this is different than prior studies where questions were classified as to reasoning type based on one source (Bergqvist, 2007, Palm, Boesen, & Lithner, 2006).

I reviewed each test problem and its corresponding problems and considered the following question, "If a student had known how to solve the similar problems going into the exam, is it possible she could still not be able to complete the test problem?" The answer to this question was somewhat subjective, often beginning with, "The student *should* know how to…" or "the student *should* be able to…" For a given problem, if I was unable to decide whether it could be soved using AR, I brought the problem and the most similar problems from the resources to my advisor. We discussed how the test problems compared to the most similar examples and what obstacles may still exist for a student after learning to solve the similar problems. These discussions led to more detailed codes intended to eliminate some of this subjectivity. The need for this is not surprising, considering the already established connection between imitative reasoning (IR) and local creative mathematical reasoning

(Bergqvist, 2007). Local CMR is required when the overall task may be familiar, but some small element requires creativity (Lithner, 2006). Differentiating between these two did prove to be challenging and led to the following process.

I began asking the question, "What is different about the test problem that makes it potentially challenging for a student who has completed these similar examples?" I compared these answers across test problems I thought may require CMR to find differences in how problems differed from previously seen examples. From the problems that were deemed to possibly require CMR, seven answers emerged in response to the question above. These 7 codes are presented in Chapter 4, but an example for one of the codes is provided here. Examples for the remainder of the codes are provided in Table 16 in Chapter 4.

Problems were labeled as Code 1 if they created knowing-TO difficulty because of the function type. These problems could be solved with a familiar procedure; however, knowing-TO choose this procedure may be difficult because this was the first problem students had seen requiring them to use this procedure with this function type. For example, in one classroom students had practiced problems where direct substitution could be used to find the limit of a polynomial function. However, there were no practice problems where direct substitution could be used with a rational function. There was a test problem which asked for the limit of a rational function where other common techniques (such as factoring the numerator and denominator and canceling like terms) would not prove possible or useful. While students likely would have been able to solve the problem if prompted to use direct substitution, they may not have known-TO use this procedure because of the function type.

Following the emergence of this and the six other codes, I revisited each test problem to verify that at least one of these codes would apply and at least a combination of these codes would account for any and all challenges students might face in solving this question after solving the similar problem(s). After this was confirmed, I coded each test problem – either as AR or as one of the 7 specific types of CMR. Multiple codes could be assigned to any given problem.

Coding for Other Problem Characteristics. All qualified test problems were then additionally coded for 1) problem representation, 2) response format, and 3) function type. Other classifications from the literature, such as the existence of extraneous information, were considered but not found to differentiate the problems that were reviewed. (For example, none of the 207 test problems provided extraneous information.)

Problem representation codes included symbolic, graphical, numerical, and verbal representations. These choices were informed by previous problem classification research but also confirmed by the initial reading of the test problems. I coded response format either as open-ended, multiple choice/open-ended, multiple choice, short answer, or true/false. Both problem representation and response format are common types of classifications in mathematics problem-classification studies. The final classification, function type, is not prevalent in the literature. The decision to code for this distinction was based on two factors. First, multiple student participants had discussed the presence of trigonometry-based test questions as being either a challenging factor for them in calculus overall or a distinguishing factor between their AP and college calculus courses. Second, the initial review of the

problems illuminated the fact that at least with the topics of limits and continuity, the

procedures needed to solve two seemingly similar problems can differ significantly based on

the function type used, even when the problem directions are identical. For example, a

common problem asks for the limit of a function as x approaches a certain number. When

the function is a rational function with factors common to the numerator and dominator, such

as

$$f(x) = \frac{x-3}{x^2-9},$$

the limit of the function at a point of discontinuity may be found by canceling the like factors

and then using direct substitution with the resulting function. However, this process is not

helpful for functions like the one below.

$$g(x) = \frac{\sqrt{x+1}-1}{x}$$

Reliability. After I had finished coding each test problem for problem representation,

response format, function type, and reasoning type, I then asked a second researcher (not my

advisor) to code approximately 20% of the problems for the same (42 problems). These

problems were not chosen completely at random. I had coded only 23 of the 207 problems as

requiring some type of CMR. I wanted the second researcher to view and code at least one

problem that I had coded as each of the 7 types of creative reasoning. So for example, I took

all the problems coded as #1 and randomly selected one of these. I did the same for code #2

and so on. This produced 7 problems. The remaining 35 problems to be coded were selected

at random from the 6 tests that contained at least one problem requiring CMR. (I pulled only

from these 6 tests to limit the amount of class materials the second coder would have to

review.) The inter-rater reliability for the coding of reasoning type was 74.5%. If a problem was coded by both researchers as creative reasoning but using different codes, then it was considered a disagreement, so 74.5% of the problems were coded exactly the same.

The inter-rater reliabilities for problem representation and response format were 90.5% and 95.2%, respectively. The inter-rater reliability for function type was only 59.5%. The second coder was allowed to code for more than one function type, and disagreements were almost all due to functions being of more than one type (for example, a piecewise function whose pieces were polynomial functions, but one coder would give priority to the piecewise definition and not code for polynomial). Because of this issue, I created more distinct codes so that each problem would receive only one code. After recoding, the agreement was 87.5%. These final codes for function type are part of the results presented in the results in Chapter 4.

Comparisons between AP and College Calculus. The analysis of the test problems in regards to each of the four problem classifications discussed above led to tables of results that contained counts (and percentages) of both AP and college test problems that fell into each category. These are presented in the results chapters. Since the study, and particularly research question 2, was designed to ascertain what differences exist among AP and college tests, it was important to determine whether differences existed regarding the four classifications and whether these were statistically significant.

I used chi-squared tests to determine whether differences between the AP and college tests were significant. Chi-squared is the appropriate statistical test to use since the data was

categorical and the comparison was between two distinct samples (Levine, Berenson, Krehbiel, & Stephan, 2010). In most cases of comparing the types, some categories had to be merged since they were so sparsely populated. Categories to merge were determined by both what made sense in similarities of types and comparing the percentages of AP and college calculus problems in each category to identify where the largest differences existed; these merged categories were the most likely place to compare for significance or non-significance. The merged data were compiled into contingency tables and these are displayed in Chapter 4. A significance level was not pre-determined; instead, p-values are reported for each test.

Summary

In this chapter I presented the research framework and design of the study. I also discussed how participants and course materials were selected as well as the processes of data collection and analysis. In Chapter 4 I will present the results of the interviews and test analyses, and Chapter 5 will discuss the comparisons among these results.

CHAPTER 4: RESULTS

This chapter is organized into three sections. The first section will discuss themes that emerged from the analysis of the first part of the student interviews. These themes address research question 1. The second section will present the Success Factor Model for College Calculus. This model was developed from a comparative analysis of both parts of the interviews and addresses both research questions. This section begins with definitions of factors of the model, followed by discussions of the individual participants' profiles, and finally it presents the relationships between the factors and discusses the relative impact of the factors. The third and final section contains the results of the test analyses which address research question 2. Chapter 5 will further discuss these findings and address the connections between them.

Interview Results (Part I)

This first part of this chapter presents the results of the first part of the student interviews. Seventeen themes emerged from the analysis and these themes may be categorized as either student-specific or course-specific themes. Student-specific themes are issues controlled by or true about individual students, while course-specific themes are those that are specific to or controlled by either the course or instructor. Course-specific themes could be further classified as either a course content issue or a class format issue. Student-specific themes were either related to confidence and self-assessment or study approach.

Course-specific Variables

Course Content. Five themes emerged that are related to the material instructors expect their students to learn or the ways in which they are required to learn or demonstrate this knowledge. They include memorization, precalculus knowledge, calculator use, course emphasis, and problem similarity. These themes are discussed below.

Memorization. Four of the participants discussed how the need for memorization negatively affected their motivation for studying the subject. Interestingly, this was especially true for those who were very confident in their mathematical ability. Albert explained, "The derivatives have a lot more to do with memorization rather than understanding how it works and I have a hard time memorizing them." A high demand for memorization tended to decrease these participants' motivation. Frank expressed frustration with not having memorized connections between calculus I material and other mathematics and practical applications. For example, "some of the trig identities, I mean yeah they're useful, they can save a lot of steps and a lot of hassle, but you don't *have* to know those."

The amount of memorization required in the students' prior math courses varied across the participants. Samuel expressed surprise – not that his college instructor expected him to know "the dreaded unit circle" - but that many of his classmates already did have it memorized from their precalculus course. He had been successful not only in precalculus but also in his AP Calculus course without having to memorize the unit circle and then felt overwhelmed when he had to learn this for his college calculus course.

Precalculus Knowledge. A majority of the specific information that the participants'

mentioned as being problematic to memorize was from precalculus, and more specifically

trigonometry. Nine of the 14 participants suggested that they struggled with calculus because

of some degree of difficulty with precalculus information. Some participants indicated that

trigonometry was much more prevalent on their college calculus tests than it had been in high

school. Jeremy explained how he avoided learning a lot of trigonometric information in high

school. "There were some questions [with trigonometry], but they weren't as hard, and even

if you didn't know the trig, there were a lot of other questions without trig, just normal, that

you could balance out, so you could still do well without knowing trig." Even some students

who did not discuss a lack of trigonometry in their high school experience still suggested it

was a challenging part of their college course. Erin, who discussed at length how most

material was already familiar to her in college calculus because of a strong AP background,

identified "integrating and taking the derivatives of trig functions" as one of only two topics

that gave her any trouble in her college course. Multiple participants were hindered in

solving their test problems, both during the test and during the interviews, by deficient

trigonometry knowledge. These deficiencies often prevented students from being able to

completely solve problems, even when they understood fundamental calculus principles; that

is, had they been asked the same questions using polynomial functions, for example, it is

quite possible they would have been able to complete these problems.

Calculator Use. It is possible that the trigonometry issue was confounded by the

accessibility of calculators (or lack thereof) on exams. The ability or inability to use

calculators on tests had the potential to change the mathematical content for which students are responsible and this proved to be problematic for some students. Three participants discussed how calculators were used much more frequently in their AP calculus courses than in college. No one suggested they used calculators more frequently in college, and many students indicated they did not use calculators at all in college on tests. Therefore, some symbolic techniques had to be learned for the first time in college because the calculator had been used for those techniques in the AP course. Wade explained "[in high school] a lot of the integrals we did, we had access to calculators for them, so that made everything a lot easier."

This difference in the high school and college expectations is exacerbated by the fact that in their college course most students were able to use their calculators in class and on homework assignments, but not on tests. This tended to produce a false sense of security in their knowledge of certain topics that was shattered when they were asked to complete a task on the test without the calculator that they had only practiced when they had access to it. Isaac gave a specific example. "When we were finding concavity - when you have two critical points and you have to test – if you have a calculator you can plug it in, but it's hard when he uses functions like sine." Apparently Isaac had used his calculator to calculate trigonometric values when practicing problems outside class. What is alarming is how unaware students were of the gaps in their knowledge because of not being confronted with this prior to the first exam.

Course Emphasis. A prominent difference in the AP course and college course for some participants was the focus on concrete and procedural material in high school versus abstract and conceptual material in college. While not all participants addressed this variable, none suggested things were reversed in their experience. A couple of students discussed the amount of time spent in class in college on derivations and proofs. For some, it went beyond a change in lecture emphasis to a change in requirements on exams. Katelynn explained how she had been required to memorize that $\lim_{x \to 0} \frac{\sin x}{x} = 1$ in high school but in her college course was required to know how to prove it.

This distinction is similar to how other participants described the difference in their high school and college courses as doing versus understanding, respectively, or "learning how" versus "learning why". Samuel explained than in preparing for tests in his high school class, he'd "know how to do the problem without ever understanding the theory behind it." Other students described their college courses as going more "in depth" than their AP course.

Another related difference that the students identified was an emphasis in the college setting on using proper mathematical language and notation. Erin described calculus as "nit-picky," and expressed frustration for losing points for not including parentheses that were in fact necessary to make her answer correct. Samuel described having difficulty expressing his answers completely and accurately, explaining that his college instructor often emphasized the importance of being able to "communicate mathematically." He explained that answers for which he lost points on his college tests would have been considered correct in his high school course.

Problem Similarity. The final theme that emerged relating to course content involved the degree of similarity between test problems and problems to which students were previously exposed. Participants made two main points regarding this issue. Firstly, the AP course focused on solving specific types of problems, rather than on concepts or general problem solving strategies. Four students recalled spending a significant amount of class time working old AP exams in preparation for their AP exam or their instructors regularly including these problems in lecture and on in-class tests. Students were divided in their appreciation for this approach. Some thought it prevented them from understanding the material at a level needed for college. Maggie, for example, said that if she could retake AP Calculus, the change she would make would be to "not learn it for each test and for the AP test - actually learn it to know calculus." Others explained that had they worked harder to learn how to do these specific types of problems, they would have learned the material better or at least have gotten credit for the course and not had to repeat it in college. Samuel said that he "talked to some other guys and they were like yeah we never did theory, just plug and chug, and they got 5's on the [AP] exam."

In contrast to this approach of "teaching-to-the-test" that some students saw in their AP course, by the students' accounts, the college courses were much more likely to test students on items that differed from those seen previously in lecture and assignments. Isaac explained that his study process, which was relatively thorough, did not allow him to prepare for all questions on his college instructor's tests. The analysis of the second portion of his interview confirmed his claim; working the most similar problems from his resources still did

not enable him to correctly solve two of his test questions. Another student described these problems that were in some way unique as "curveballs," indicating she did not know to expect these problems. However, unlike Isaac, this student was able to solve her test problems easily after studying solutions of problems from her assigned homework. Therefore, students' perceptions of how much test problems differ are related to other issues of the students' experience; this will be further discussed in Chapter 5.

Class Format. Five themes emerged around the category of class format. These included class size, interaction amongst and participation of students during class, the existence or absence of relationships between students and instructors and amongst students, the amount of structure and accountability provided by the instructor, and the type of homework requirements for the course.

Class Size. Not surprisingly, one of the differences that the participants' noted between their AP and college calculus courses was the class size. Four of the 14 participants stated that their college calculus class was significantly larger than their AP class and they saw this as a negative aspect of the course. No one suggested they might prefer a larger class size or mentioned any benefits that the larger size might afford.

Participants were not necessarily surprised by the large classes in college, but rather by the effect it had on them as students. Frank described being particularly surprised by how his role in the classroom changed as he moved to a class with over 90 students. He explained that he was an outgoing individual and very confident in his abilities, but still found himself being passive during lecture. Other students explained that they held their questions or

comments out of concern that they would be holding back other students or because of an understanding that there was not time for individual questions. They did not describe this as a concern in their high school courses. Interestingly, most of the participants described their college instructors as being very welcoming of students' questions. Furthermore, no one described having a college classroom where multiple students were competing for the chance to participate, so it is likely that their perception of there being no time for their questions was at least partially unfounded. Nevertheless, it is safe to say that students tended to become more passive learners in the classroom with increased class size, even though this may not have been necessary and certainly not the intent of the instructors.

Interactions. The passivity of students described above, which was largely associated with class size, was consistently viewed as a negative aspect of the college calculus experience. Eight of the 14 participants discussed the opportunity for interaction and engagement during class as being an important aid to their learning. Several students mentioned more student interaction as the primary change they would make if teaching their own college calculus course. Recalling his high school experience, Allen explained, "Everything would fall into place. And it was like now I understand it. Cause you hear it from other people instead of just from the teacher once. I'd find a way [in my college course] to incorporate everybody else." The predominance of this type of instruction in the high school environment is seen by the students as a strength of the AP course over college calculus.

However, not all participants had experienced less interactive college classrooms. While the majority of students were part of a traditional lecture-based course, there were a handful of students who described their college courses as very interactive. These students were just as vocal about the importance of classroom interaction as those who only experienced it in high school. Blake, in discussing how he would teach AP Calculus, described aspects of his college course: "interactive, very interactive. Work with the students, a lot of group work. Spend time with each of my students. Less time talking AT them, more time talking with them… [My college instructor] does a good job of talking WITH rather than AT." Keeping students engaged with the instructor and each other is viewed as a positive attribute of instruction both in the high school and college environments.

Instructor and Student Relationships. Closely related to the issue of interaction and engagement was the amount of familiarity between students and between the student and his or her instructor. Seven participants indicated that strong relationships tended to increase motivation in and outside of the classroom. Some described being in their AP class with other students they had known for many years and they identified several ways how these relationships positively affected their academics. They discussed how it was easy to work with other students because of knowing others so well and some formed study groups outside of class. Jeffrey even described being motivated by the long-standing competition between him and other students who had been in the same math classes for several years. These relationships were much more common in the high school setting, with a couple of notable exceptions. Katelynn described how she had been one of only a couple of senior students in

her AP course and did not have many friends in her class. However, in college she found it easy to form study groups with other girls in her residence hall who were taking the same course.

The students' relationships with their instructors were even more significant than students' familiarity with each other. While knowing other students tended to mostly affect how the participants behaved in class, having or not having a relationship with their instructor also impacted how they would study for the course outside of class. Frank suggested that he would have been content with a lower grade than he actually received in his AP course, but because he did not want his teacher to think poorly of him and because he did not want the teacher to think s/he was not an effective teacher, he worked harder than he would have, had this relationship not existed. So students were motivated by these relationships not only to meet their goals but to exceed them.

Strong relationships with instructors were much more common in the AP course than in college and the lack of them was viewed by the participants as a negative factor in their college experience. Haley described how she was not motivated as she had been in high school. "I don't know [my teacher] as well so I'm not motivated to do as well…It's terrible to get bad grades, but not because I know him and I'll be embarrassed." Other students made small references that echoed Haley's sentiment, such as "I don't think the teacher knows my name. I don't think she would recognize me." The participants were overwhelmingly understanding of this issue; they did not fault their college instructors for not knowing them, but rather seemed to have accepted it, like the class size issue, as how things have to be.

Most students had made little or no effort themselves to get to know their college instructors. One exception was Frank. He talked at length about having very recently gone to office hours and how much it had encouraged him to persist, despite his previous poor test scores. He expressed regret for having not done this earlier in the semester.

Course Structure and Accountability. A stronger relationship with instructors tended to be associated with greater accountability provided by the instructor and course design. This accountability took various forms, including frequent assessments, guided opportunities for practice, accessible materials (such as PowerPoint class notes posted on a website), and verbal or written reminders from instructors to students. Accountability was viewed positively by the students and was much more predominant in the AP course than college. Katelynn recalled about her AP teacher, "I definitely liked the fact that he stayed on me. I probably would have failed the class if he didn't." Allen described his teacher as "very strict" and explained that she "expected you to know how to do everything. She'd give us quizzes all the time trying to prepare us [for the AP exam]." Frequent assignments and assessments were typically more common in AP and also viewed positively by most of the participants. Isaac talked about "hav[ing] more room for error [in high school] than in college" because of having so many more sources of grades that contributed to the students' final grades.

Regular assignments and assessments provide a structure to a course that motivate and guide students' studying. The lack of regular assignments in a majority of the participants' college courses along with their instructor's lack of suggestions left students

struggling to determine how to focus their study efforts. Michael expressed a confidence in his mathematical ability but also uncertainty about how to approach his college course. "I think I can do great in math. I just need to study more, 'cause the material itself is not hard. So I maybe need slight clarification on what to do..."

An overwhelming majority of the participants were at a loss when asked about their college instructors' expectations of students, although they answered this question about their AP teachers with ease. Some even said that their instructor "didn't expect anything," which meant either that the instructor had not communicated his or her expectations (such as keeping up with reading or seeking help with needed) or was not holding the students accountable for adhering to these expectations. Students were consistently accepting of this fact. They explained that in college learning was their responsibility and therefore they did not expect their instructors to hold them accountable as some of their AP teachers had done. Unfortunately, an awareness and acceptance of the current state of things did not necessarily lead to increased effort on the part of the students.

Homework. The one aspect of college courses that was most noticeably more structured than AP courses was homework assignments. The most common type of homework in high school was assigned problems from the course textbook. Most participants said these assignments were never or rarely graded, and when they were they were graded for completion rather than accuracy. As a result, only 3 participants reported putting in a significant amount of effort on their homework on a regular basis in AP Calculus. In contrast, most participants had regular, graded online homework assignments in

their college course. These were not a large part of their course grade, but seemed to be significant enough to motivate the students to complete the assignments consistently.

Student-specific Variables

Approach to Studying. Themes emerged related to participants' descriptions of their study approaches to calculus. These included associating studying with doing homework, which resources students preferred, and the level of awareness of one's study approach. These themes are discussed below.

Studying Equals Doing Homework. Whether or not homework was required and graded, students still tended to see it as a significant aspect of their calculus courses. In a few cases, the participants immediately described their approach to homework when asked about their study approach and many did not distinguish between studying and completing assignments. This is interesting because many students described using homework assignments in ways they were not intended by instructors or course designers. As described next, specific cases of this were common.

First, in high school it was common for participants to quickly write down something to show their instructor for homework checks when they did not know how to solve the problems in their assignments. They were able to avoid ever learning how to solve these problems because their teachers distributed review sheets or practice tests that were very similar to the tests and students would only need to learn these problems to be successful on the tests. Allen reported not keeping up with his AP Calculus homework, and when asked whether he did it before tests, he explained, "she gave us reviews, stuff she thought would

prepare us. The next class we'd go over it." Samuel's experience mirrored Allen's. He claimed that "the single most damaging factor about our high school class was our homework was not graded…homework was not graded, so what's the incentive to do it? We would just do it in like 10 minutes and write random stuff." When asked how he did well on tests, he explained, "When it came time for the test, we got practice tests. I'd figure out how to *do* it, but I didn't understand it."

Second, in both high school and college courses, students were often given time to ask questions about homework problems, but students chose not to do so. In high school it was primarily because the students had not really attempted the problems and therefore did not have specific questions. Erin said her instructor "put the answers to the homework up and asked if anyone had questions and usually no one did the homework so no one had any questions." In college, the reason was less consistent, but some students felt uncomfortable asking questions for fear of holding back other students, while others did not feel that they had questions because they had received high scores on their online assignments. This issue will be further addressed in a section below.

Finally, some students reported doing their homework but, in college particularly, using resources to complete the assignments that removed the challenge and therefore did not allow them to really learn to solve the problems. Sometimes solutions were provided within the homework systems and students would simply copy and paste their numbers into the solution. Other times they would use computational tools to solve problems they were being asked to solve symbolically. The students who reported doing this had, by the point of the

interview, decided this was not effective and believed this approach had had a negative effect on their test grades.

Preferred Resources. There was a shift in the study resources that students preferred from AP to college calculus. In high school, students saw their teacher as their primary resource. Most participants reported finding it easy to ask questions of their teachers in class and many also took time to seek help outside of class, either before or after school or during lunch. The second most common resource in high school was other students, mostly classmates whom the participants had known for many years. The participants discussed having opportunities in class to ask questions of their classmates and some would call or meet with classmates after school to study.

In college, other students were the primary resource mentioned by the participants, but these students were not usually classmates. They were mostly friends or neighbors (from the same hallway, suite, or dormitory) who had either already taken the course or were currently enrolled in it. The second most prominent resource used in college was online websites. These ranged from structured instructional websites like Kahn Academy and Calcchat to search engines where students look up solutions to specific problems.

Common to students' experience in both high school and college was their lack of preference for written resources. Some students did mention using their class notes as a study aid, but it was used more as a reference when they were stuck rather than a starting place. The textbook was even more neglected. Most participants had their own copies of a

calculus textbook in both AP and college calculus but rarely if ever used it for anything other than assigned exercises.

(Lack of) Awareness of Study Approach. One way that participants varied was in their awareness of how they could study more effectively. Five students claimed to be at a loss for how they might change or add to their study approach, even though they might have only been attending class and completing homework assignments. For some, this likely meant that they did not know how to study in ways that would be effective for them that would take what they considered to be a reasonable amount of time. Others equated doing all they could do with doing only what was required. Samuel, for example, claimed he was out of ideas for becoming successful in his college course. However, at other points in the interview he talked about a university math center that he thought might be helpful and he pulled out a handout of suggested practice problems he had not attempted for a specific topic from a previous test. He mentioned that doing that many problems would have taken him far too long, so he did not attempt any of them. Erin's situation was similar. She had missed problems on her test that were extremely similar to problems from suggested homework assignments, yet she still claimed, "I literally could not have done anything else to get a better grade like I went to class, did all my [required] assignments, did all my homework…"

Other students were convinced they could have done better thus far in the course if they had put forth more effort. Some of these were able to identify specific resources they had not been using or using regularly that would have been helpful. These included office hours, emailing the instructor, working exercises from the textbook, and practice worksheets

from the instructor. Michael, for example, replied that success in the college course for him would be "having better study habits." When asked what that would entail, he explained, "looking at all the notes when you have free time that day…and also doing practice problems from the textbook even though she didn't require us to bring it to class…" These students did not have a clear explanation for why they had not used these resources up to this point, and most did not suggest they fully intended to use them in the future. Michael was the exception. When asked why he hadn't yet attended office hours, replied, "I always liked to work on my own so I figured I didn't need anybody's help, but calculus is kicking my ---. so I figured it can't hurt to ask somebody for help."

Confidence and Self-Assessment. Other themes emerged that were related to either the student's confidence or the student's ability to assess his or her progress in calculus. These themes are discussed below. They include how calculus can impact students' confidence in mathematics, students' uncertainty of their level of success in college calculus, and beliefs by students that they understand more calculus than their tests reflect, while the opposite is true with online homework assignments.

Calculus Impacts Mathematical Confidence. Very few participants felt as confident about their mathematical ability after taking calculus as they had in previous courses. This led some students to make a distinction between math and calculus. When asked about his math ability, Allen replied, "Math or calculus?"

For some participants the change in confidence came in AP Calculus and for others in did not occur until their college course. This change tended to be associated with a

comparison of themselves with other students. Haley discussed struggling to keep up in her AP course. Katelynn admitted not putting forth a lot of effort in her high school course but also felt like she was at a disadvantage from other students. Both Haley and Katelynn suggested there were people in their AP courses who were just "really good at" math and they were not these people.

Samuel echoed this sentiment, but he did not experience this feeling of being left behind until college. When asked what he thought college instructors expect from their students, he replied, "I don't know. Studying their notes, doing practice problems, but I don't know, to me it seems like some kids just get it immediately…maybe I just don't have the mathematical wit." He, like 6 other participants, described having a sense of overconfidence from the knowledge he had gained in his AP course. While the AP experience did make some aspects of the college course easier, students tended to underprepare for at least their first or second exams in their college course. Many had intentions of making an A or high B in the course because of having seen the material before, but by the time of the interviews were expecting a much lower score. For some this was disappointing, but others had clearly changed their definition of success to match their current level of performance.

Uncertainty about Current Level of Success. Six participants were quite uncertain about their level of performance thus far in their college calculus course. While they could quickly recite their test scores, they seemed unsure whether the average of these scores accurately reflected the grade they could expect in the class. The main reason given for this

was their lack of knowledge of how other students were doing in the class. Their comments reflected an expectation that performance should be measured relatively, in comparison with other students. In Frank's case, this was based on his experience in his physics class, where his instructor announced class averages after each test. Because he was meeting or exceeding the average, he believed he was doing well, despite his physics instructor not mentioning curving grades. He reasoned that he could be doing well in calculus in comparison to other students, despite two failing test grades.

Students who were unsure of their current status in the class were predominantly hopeful about their ultimate success. While some had decreased their expectations of themselves since the start of the class, they still expected to pass, and some with high marks. Michael, who had failed both of his first two tests, was confident he would make a B in the course. When asked, "What are the chances of getting a B? What's the probability?" he responded, "85%." It should be noted, however, that Michael was interviewed approximately halfway through the semester. Samuel, by contrast, was another student that expressed uncertainty about his performance. He was interviewed towards the end of the semester and was much less convinced of his ability to pass the course.

Level of Understanding Not Accurately Measured by Test Performance. Some students were convinced their level of understanding of calculus was not reflected in their test grades. For Maggie, this was reflected most in her distinction between understanding and being able to solve problems. She explained, "I feel like I understand until I go to do problems. Like I understand concepts, but the problems just seem so difficult compared to

what I used to do." For some students, their prior successes in high school outweighed the impact of their college calculus test grades in determining how much they believed they understood the material, at least for a short period of time at the beginning of the semester. Frank had done poorly one both of his first two tests but was still very confident about his understanding of calculus at the midpoint of the semester. He explained that there was too much memorization and precalculus required in his college course, but he was very comfortable with topics like integration which he had learned in his AP course.

Lack of Understanding Not Reflected by Online Homework. Students' awareness of their level of preparation or understanding was not only affected by experiences in AP Calculus but also by aspects of the college course. Four participants reported being made overconfident about their test preparation in college by their homework assignments. All of these students had regular online assignments that were graded electronically for accuracy. Some of these assignments had solutions provided to very similar problems that could easily be manipulated or copied to obtain full credit. Wade explained, "They have 'show me how to do this problem. When you click this, it changes the numbers. But the 'show me an example,' if you look at the end, you can find the pattern and b.s. the answer." Similarly, Maggie said that she "looked at so many examples in order to figure it out that in the end I didn't really figure it out, I just looked at it and would substitute my own numbers in." Maggie explained that if she could start the semester over, "I wouldn't trick myself with my online homework. I'd actually take the time to learn how to do it."

Students struggled with knowing whether they really knew how to do a problem they received credit for in their online assignments. One reason was that they often had multiple submissions for these assignments. Frank said that he understood why this was setup like it was and understood its benefits for learning, but had mixed feelings about its results. He recommended having a companion section of homework where you only get one try because "it puts the pressure on you to really learn it rather than just try a couple things until something works." The other issue with the online homework was how the answer format requirements differed from exams. Samuel's instructor graded the students' work on tests along with their answers. He explained, "Even if you write something off to the side, if it's not exactly mathematically correct, you lost points." Samuel had completed his early homework assignments mostly "in his head," without writing down full solutions. He believed this set him up to fail. He said, "I theorize that students twenty years ago understood calculus ten times better than anybody in our class does…[online homework] will unfortunately never go away, but I think it's hurting our math students."

Summary

Seventeen themes emerged from the analysis of semi-structured interviews conducted with 14 first-semester college students enrolled in Calculus I who had taken AP calculus in high school. The themes address challenges the participants experienced in both courses and how their experiences in AP and college calculus compared. The themes are organized into four categories and are listed in Table 5.

Table 5

Summary of Interview Part I Results

Course-Specific Themes	
Course Content	Memorization
	Precalculus Knowledge
	Calculator Use
	Course Emphasis
	Problem Similarity
Class Format	Class Size
	Interactions
	Instructor and Student Relationships
	Course Structure and Accountability
	Homework
Student-Specific Themes	
Approaches to Studying	Studying Equals Doing Homework
	Preferred Resources
	(Lack of) Awareness of Study Approach
Confidence and Self-Assessment	Calculus Impacts Mathematical Confidence.
	Uncertainty about Current Level of Success
	Level of Understanding Not Accurately Measured by Test Performance
	Lack of Understanding Not Reflected by Online Homework

Interview Results (Parts I & II Combined)

This section of the chapter discusses the results of a comparative analysis of the

fourteen participants' interview data. The results take the form of a developed Success

Factor Model that illuminates factors of success in college calculus and the relationships

between them as a general model (see Figure 4) and a model for each of the participants.

This model may be used to identify barriers to success or paths to success for particular

students, but it also highlights how teaching practices may be adjusted to best increase

student success. The model was designed by iteratively revisiting individual student profiles

and modifying the model to see if it accounted for each participant's experience. For this reason, after presenting the model and defining the factors, this chapter will provide an individual profile for each participant that discusses which factors were most significant for him or her and how this compared with other participants. Finally, a discussion of the overall results from the interviews will follow the individual descriptions. The results include an explanation of how each of the factors affect student success, how they are related to one another, and the relative importance of the factors.
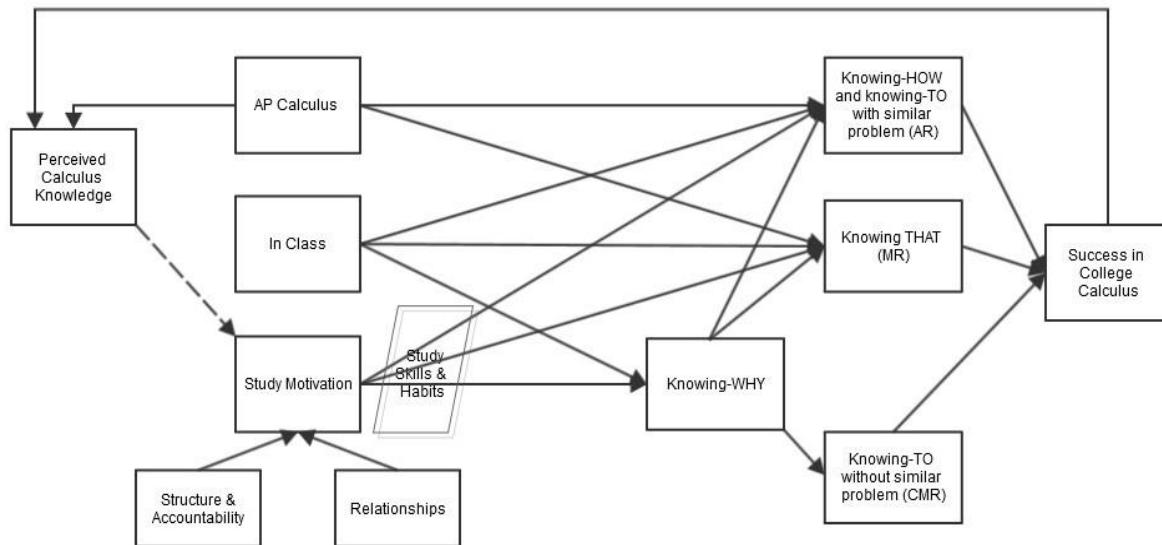
Figure 4. Student Success Factor Model for College Calculus (modified from Credé & Kuncel, 2008)

Figure 4 contains the observed factors influencing student success in calculus that my analysis revealed and illustrates the possible pathways and barriers to student success. Arrows connecting factors A and B indicate an impact of Factor A on Factor B. Bold arrows indicate a direct relationship; an increase in the factor A positively impacts factor B. A

dashed arrow indicates an inverse relationship; that is, an increase in factor A results in poorer results for factor B. Distance of a factor from success on the diagram does not determine relative impact of that factor; larger distance merely shows that there is a mediating factor that determines how much of an impact that factor will have on success.

There are likely multiple indirect factors which exist but were not observed. Therefore, for any given participant, the observed strength of each factor may not be fully accounted for by the factors in the diagram connected to it. For example, study motivation is affected by structure and relationships; however, a student may still have high study motivation even if their class has low structure and few relationships.

The three factors connected to success are subsequently referred to as *direct factors*. If a factor is identified as direct, then for at least some students, a weakness in this area would result in diminished success, regardless of strengths in other areas. All other factors impacted success indirectly through mediating factors and necessity for strengths in these factors could be bypassed or avoided by having strengths in other factors. Each factor may be seen both as a potential route to success as well as a potential barrier to success, depending on the strength of the factor for the particular student.

Definitions

Knowing-HOW and Knowing-TO with Algorithmic Reasoning. When using imitative reasoning (IR), students "copy or follow a model or example without any attempts at originality" (Palm, Boesen, & Lithner, 2006, p. 7). With the more specific memorized reasoning (MR), students recall a solution, whereas with algorithmic reasoning (AR), they

either select a procedure because of the task's familiarity or its similar surface features to previous tasks. To use AR, two types of knowing are required – knowing-TO and knowing-HOW. The student must know-TO choose the same algorithm as applied in the previous problem, and he must know-HOW to perform the algorithm. The key feature with AR is that the usual difficulty of knowing-TO is eliminated by the connection between the problems. Therefore the challenge of using AR to solve a problem is not knowing-TO select or perform a particular mathematical task or procedure; rather, it is identifying the problem type or making an appropriate association between problems; if an exam problem has very similar surface features or directions to previous similar problems, this difficulty may also eliminated.

Consider the following example. Two participants had missed a majority of points on the following test problem: Find $\lim_{x \to \infty} (\sqrt{x^2 + 7x} - x)$. This is an indeterminate form of type $\infty - \infty$. As such, L'Hôpital's Rule may not be applied unless the expression is first rewritten. Therefore, the problem solver must know-TO first multiply by the conjugate, $\sqrt{x^2 + 7x} + x$, as well as know-TO and know-HOW to apply L'Hôpital's Rule. During the interviews the following problem was discovered in one of the homework assignments of the participants: Find $\lim_{x \to \infty} (x - \sqrt{x^2 + 4x})$. Both participants immediately recognized this as being the same type of problem as their test question. Both were able to solve the homework problem after a short amount of time studying the provided solution and subsequently encountered no difficulty in solving the test problem independently. This strongly suggests

that had the students known how to solve this specific homework problem before the test, they would have been able to use AR to successfully complete the test problem.

Knowing-TO using Creative Mathematical Reasoning. Some problems or parts of problems may require the student to create a new or recreate a forgotten solution sequence that is flexible and justified, based on "intrinsic mathematical properties" (Palm, Boesen, & Lithner, 2006). These problems require creative mathematical reasoning (CMR). It is sometimes only required for a localized part of a problem, which can then further be solved using IR. CMR requires that the student know-TO select or devise an appropriate solution strategy for a part of a problem without there being a surface connection to a previously worked problem that connects the problem type to the strategy.

In the previous example, suppose the homework problem had not been assigned, nor any just like it. Suppose the most similar problem available to the students had been the following: Find $\lim_{x \to \pi}(\sin x - \tan x)$. This is another indeterminate form of type $\infty - \infty$ where rewriting the expression and then applying L'Hôpital's Rule is appropriate. However, the first step of rewriting this expression requires a different process from our previous problem. It is quite possible that students not exposed to one of the problems like the test question could have known they were supposed to apply L'Hôpital's Rule, but not known-TO multiply by the conjugate. This would have prevented the student from solving the problem using only AR. In this case, knowing-TO multiply by the conjugate would require using CMR.

Knowing-THAT using Memorized Reasoning. Solving problems often requires students to know facts as well as procedures. Knowing-THAT using memorized reasoning (MR) involves either the recollection or reconstruction of these facts. This recollection or reconstruction may consume an entire problem or it may be a small component of a larger problem that requires other types of knowing. The problem may be complex, but if the solution itself is familiar, the problem may be solved using only this type of knowing.

Knowing-WHY. Knowing-WHY is related to both facts and procedures. It is an understanding that goes beyond knowledge. A student can know (or not know) why something is true. He can also know (or not know) why a procedure works or is applicable in a particular situation. To know-WHY is to make connections among mathematical properties; it is to possess reasoning or justification for given information.

Study Motivation. Study motivation refers to a student's willingness to study and the "sustained and deliberate effort" they exert in studying (Credé & Kuncel, 2008, p. 428). It is separate from the student's "sense of responsibility for and value in one's own learning," which is defined as study attitude (p. 428). One can have a positive study attitude but yet poor study motivation. Study motivation is also distinct from study skills and habits, which are defined below.

Study Skills and Habits. Study skills and habits encompass the student's knowledge of appropriate studying techniques and the awareness of when and how to apply those techniques. The knowledge can be acquired, but is somewhat specific to the individual; that is, the student must be aware of what works best for him or her. This knowledge includes

skills such as note-taking, time management, self-testing, allocating resources, and selecting a conducive study environment.

Structure and Accountability. Structure refers to how a course is organized. The amount of structure in a given course may be affected by both departments and instructors. Structure exists on a continuum, rather than being a high-low dichotomy. Higher structure decreases the burden for independent learning and/or initiative on the student. (Note that this does not necessarily equate to reducing the required amount of effort.) Higher structure choices include more frequent assessments and guided opportunities for practice. Mandatory practice, such as regular graded homework, is an even higher form of structure. Another aspect of course structure is the accessibility of materials and information. Higher structure courses may use online course management systems to post class notes, for example, or instructors in these courses may send regular reminders to students via email.

Accountability is related to structure in that it also reduces the burden on the student for independent learning. Accountability provided by an instructor is an effort on the part of the instructor to motivate certain behaviors in the student that are expected to increase success. Most often this comes in the form of verbal reinforcement or encouragement such as reminders, but it may also be written, such as a letter to a parent. One way courses are differentiated in their level of accountability is whether certain actions or behaviors are required and graded; doing so is a factor of a higher structure course.

Relationships. A relationship may be between a student and the instructor or among students. It may refer to a certain level of trust or esteem one has for another or simply a

familiarity or comfort amongst those individuals. It is most often present because of longer

associations from previous courses, but may develop within a single course.

Individual Student Profiles

This section contains individual profiles for the fourteen student participants that

were created using data from both parts of the interviews. (The individual profiles were

created simultaneously with the Success Factor Model and were iteratively changed as more

participants were examined, so the profiles discussed below are the final models rather than

what was initially created.) This discussion of each of the profiles highlights the specific

factors associated with the particular student's success and explains how these factors

contributed to or hindered his or her overall success in the college course. The profiles are

not presented in the order of development but rather in a way that illustrates important

connections between the participants.

The relative strength of the factors in the diagrams that noticeably affected to the

student's success in the course are indicated with arrows. Each factor that was addressed by

the participant is labeled either with an up or down arrow of one of two sizes. Longer arrows

pointing up indicate the factor is a strength for the participant. Shorter arrows pointing up

indicate the factor is a moderate strength. Longer arrows pointing down indicate the factor is

a weakness for the student and shorter arrows pointing down indicate a moderate weakness.

Frank. At the time of his interview, Frank was failing his college calculus course.

Frank's success was being limited by two of the three direct factors - knowing-HOW and TO

with AR and knowing-THAT (MR); CMR was not required for the discussed problems on

Frank's test. He expressed frustration during his interview at being surprised by test

problems in his college course and suggested that he was not able to prepare adequately

because of not knowing what would be on the test. However, when shown solutions for

similar problems from his resources, he was able to immediately interpret both the HOW and

WHY of these solutions and then was able to apply the same procedures to his test problems.

After completing this process, he recognized that the problems were very similar, and that

had he known how to work the similar problems prior to the test, he would have gotten the

test problems at least mostly correct. This suggests that Frank has poor study habits, in

addition to having low study motivation.
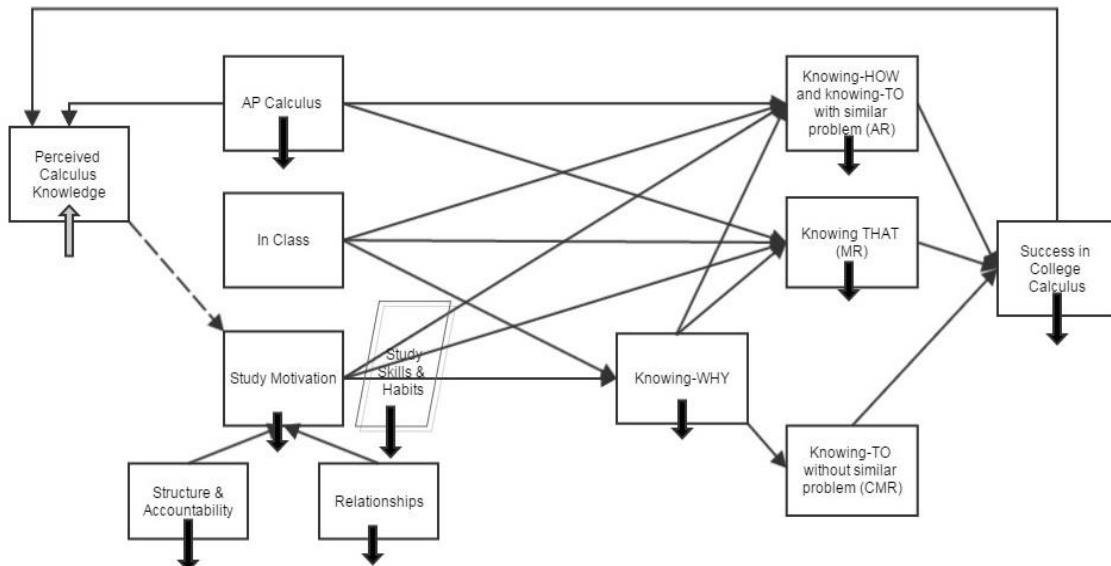
Figure 5. Frank's Success Profile

Of the fourteen participants, Frank was most likely the most confident in his

knowledge of calculus. He had made a very high grade in his AP class, despite making a 1

on the AP test. Despite multiple failing test grades in his college course, he was still

convinced that he had a firm grasp on the material; thus, his lack of success in the class was not able to overcome his confidence in motivating him to study. His motivation was also decreased by the lack of structure in his college course and the lack of relationships with his college instructor and other students in his class – things which he explained were very important to him about his AP course.

   Erin. Like Frank, the two direct factors that limited Erin's success were knowing-HOW and TO with AR and knowing-THAT with MR; CMR was not required for the discussed problems on Erin's test. However, she was ultimately being successful in her college course.
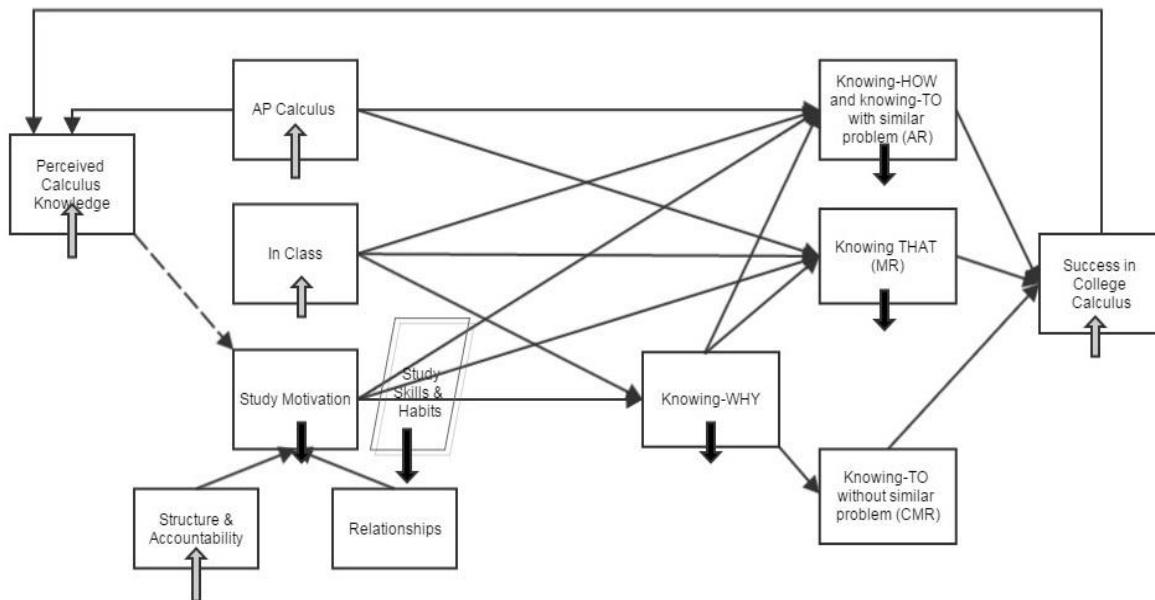
Figure 6. Erin's Success Profile.

   The knowledge Erin showed came primarily from her AP Calculus class; she explained that she only had to learn a couple of very specific topics in college; everything

else she recalled from her high school course. She did indicate that class in college was a good source of information, stating that attending class was very important and helpful. She demonstrated a lack of knowing-WHY. While reading through a worked example, she expressed some frustration with not knowing the reason for certain steps being taken in the provided solution. Unlike Frank, Erin was not immediately able to uncover the "why" of the procedures being used. However, this did not prevent her from being able to understand what steps were being taken and then apply those steps in the similar problem from her test.

The factors most lacking for Erin were her study motivation and study habits. Erin was very confident in her knowledge of calculus because of her AP experience and because she had been fairly successful on several of her other tests in her college course. These factors combined to decrease her study motivation; she was being as successful as she needed or wanted to be without having to study significantly outside of class.

The design of Erin's college calculus course worked in opposition to her perceived calculus knowledge, in respect to its effect on her study motivation. Erin's class was very interactive; she worked in small groups with classmates every day. Her course was one of the most structured of any of the participants, requiring regular participation in class and having multiple forms of assessment and practice. However, the positive effect of these classroom design features on Erin's study motivation was outweighed by the negative effects of her perceived knowledge of calculus and ultimately her motivation was among the lowest of the participants. She did suggest that had she not been successful, this would have

changed. However, an increase in study motivation alone would likely not have increased her success drastically because of her poor study habits.

Erin demonstrated a lack of knowledge for how to study appropriately that was very similar to Frank. She explained that some test problems were "curveballs" and that she was not able to prepare for them. However, the test problems she missed had almost identical examples in her written homework assignments, whose solutions were made available to students prior to the test. She explained that she had essentially ignored these assignments since they were not a graded part of the course. When comparing the problems to her test, Erin was surprised, exclaiming, "Who *knew* there were so many similar problems?!" So while she may have been more *motivated* to rework her online homework assignments or study her class notes if she had had a weaker AP background or poor first or second test grade, she may still have been unsuccessful on her last test because of not knowing to spend time practicing the examples from her written homework assignments.

Jeffrey. Jeffrey was being successful in his college course. The two direct factors that somewhat limited Jeffrey's success were knowing-HOW and TO with AR and knowing-THAT with MR. The only predominantly weak factor for Jeffrey was his study motivation. Like Erin, this seemed to stem from his perceived calculus knowledge. Jeffrey was able to pick up what he needed mostly while in class and his success deterred him from studying more. He differed from Erin in that while Erin struggled with knowing-WHY, Jeffrey did not. He seemed to learn WHY either simultaneously with or prior to learning HOW and TO and learning WHY was a pathway that made knowing HOW and TO easier. He also differed
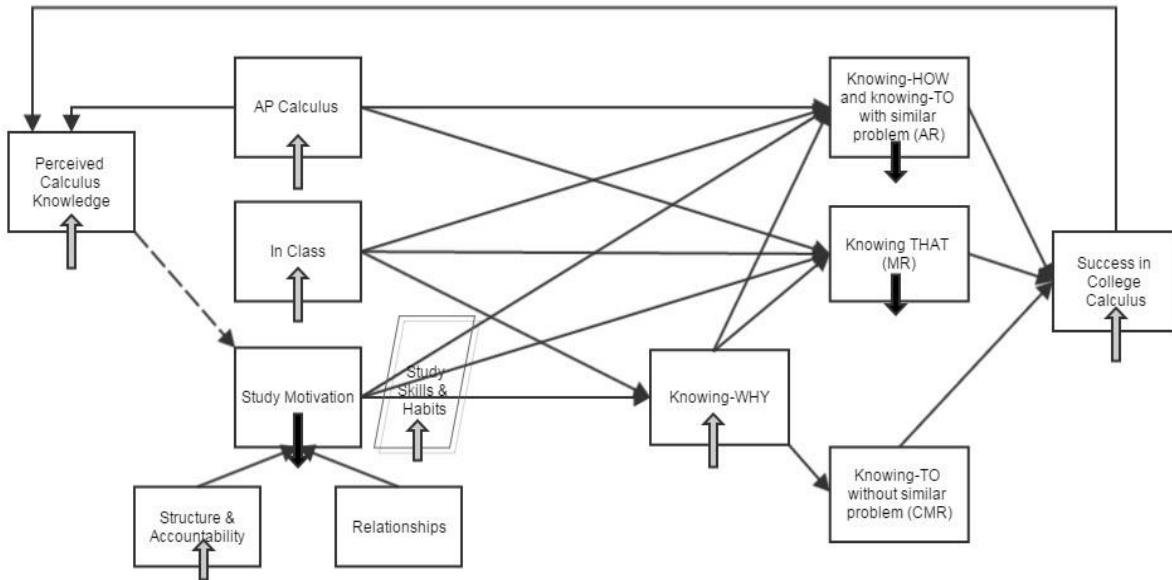
Figure 7. Jeffrey's Success Profile.

from Erin in his study habits. Jeffrey was one of the most vocal participants when it came to

acknowledging and identifying deficiencies in his study approach. He connected these

deficiencies to lost points on his test. He was not surprised when very similar problems to

the test problems were found in his course resources, nor was he surprised by his being able

to solve the test problems after studying the solutions of the similar problems.

     Michael. Michael's experience closely resembled that of Jeffrey's. He was also very

forthcoming about his lack of study motivation; he identified this himself as the biggest

barrier to his success. However, Michael was not being successful in his course like Jeffrey

was. Michael's course was less structured than Jeffrey's, which negatively affected his

motivation. He had regular online homework assignments, but was not required to

participate regularly in class and had no other regular assignments or assessments other than
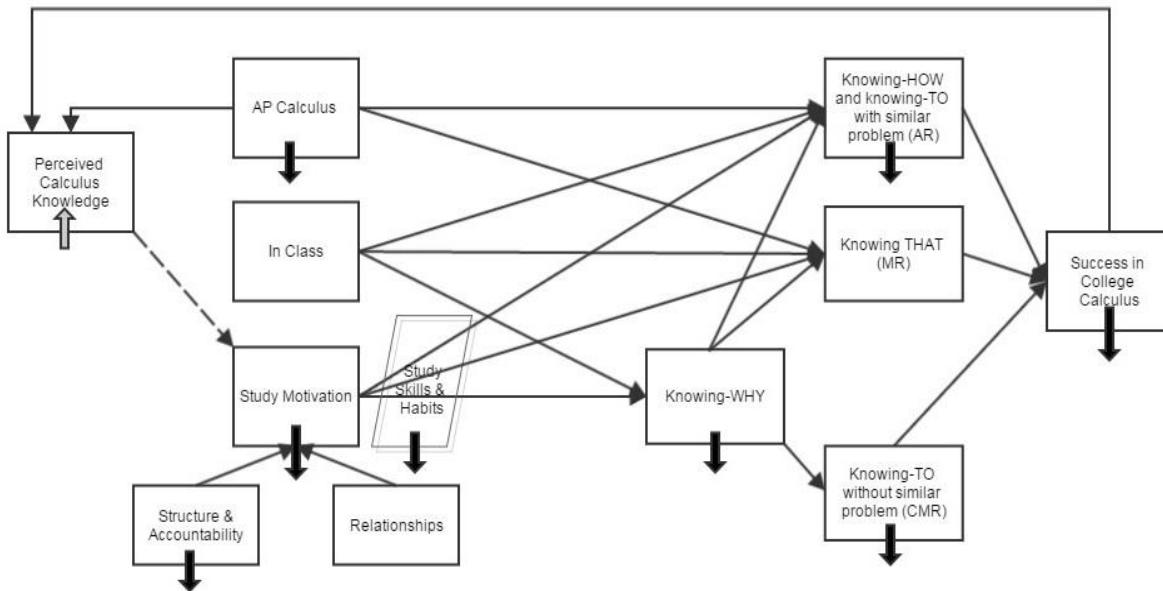
Figure 8. Michael's Success Profile.

tests. Another noticeable difference was his inability to quickly discern WHY certain

procedures were being used as Jeffrey was able to do. While this was not necessary for

solving Michael's test problems, it could have increased his ability to know-HOW and TO.

Michael also encountered the need to know-TO using CMR which kept him from being able

to finish solving his problem.   Finally, while Michael scored a 1 on the AP Calculus test,

Jeffrey scored a 4. This difference alone – the amount of knowledge gained from the AP

course - may account for the difference in these two students' performance in college

calculus, since their amounts of effort were so similar.

   Katelynn. Katelynn was just passing her college class. Katelynn had relatively good

study motivation and habits, although she described herself as being very unmotivated in her

high school course. She mentioned specific ways she had started studying in college that she

did not try in high school, including meeting with a study group, reading her notes after class,

and practicing unassigned problems. It is likely her motivation was lower than it could have been due to the lack of structure of her college course. Although she did not work hard in high school and did not pass the AP exam, she attributed the effort she did put forth to her teacher holding her accountable. She claimed, "I definitely liked the fact that he stayed on me. I probably would have failed the class if he didn't."
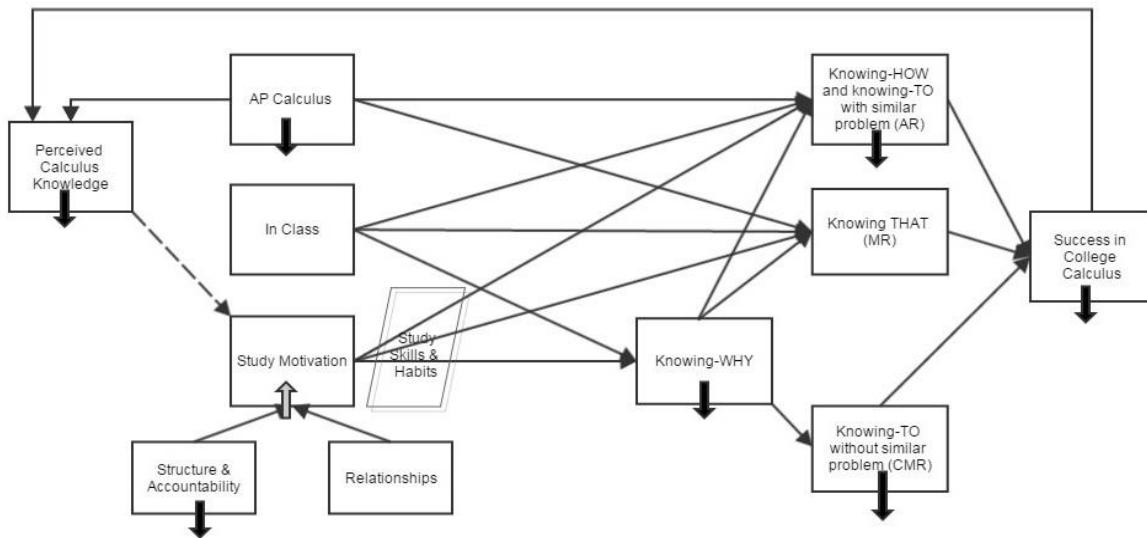
Figure 9. Katelynn's Success Profile

Katelynn's primary obstacle was knowing-TO using CMR; however, she also lacked some knowing-HOW and TO using AR as well as knowing-WHY. Both of Katelynn's missed test questions required significant amount of CMR. The first required CMR only for the second half of the problem where she had to use algebraic techniques to solve an equation. Her similar problem required the same, but the algebra was different enough so that she still was unable to complete the test problem, even after correctly solving the similar one. Her second problem dealt with related rates. There were parts of this problem that she

learned how to do after solving a similar problem, indicating there were gaps in her knowing-TO and HOW with AR. However, she was still unable to even set up the initial equation that needed to be differentiated because of not recognizing the relationship between the variables.

      Isaac. Isaac's difficulty with his test problems was similar to Katelynn. However, with only a small exception, he did not lose points for lack of knowing-HOW and TO with AR. Isaac easily worked the similar problems without assistance. His study motivation and habits were the strongest of the fourteen participants. His approach to the class included reading the textbook, understanding class notes, and practicing problems from the textbook. He demonstrated a belief that this should prepare him for tests.
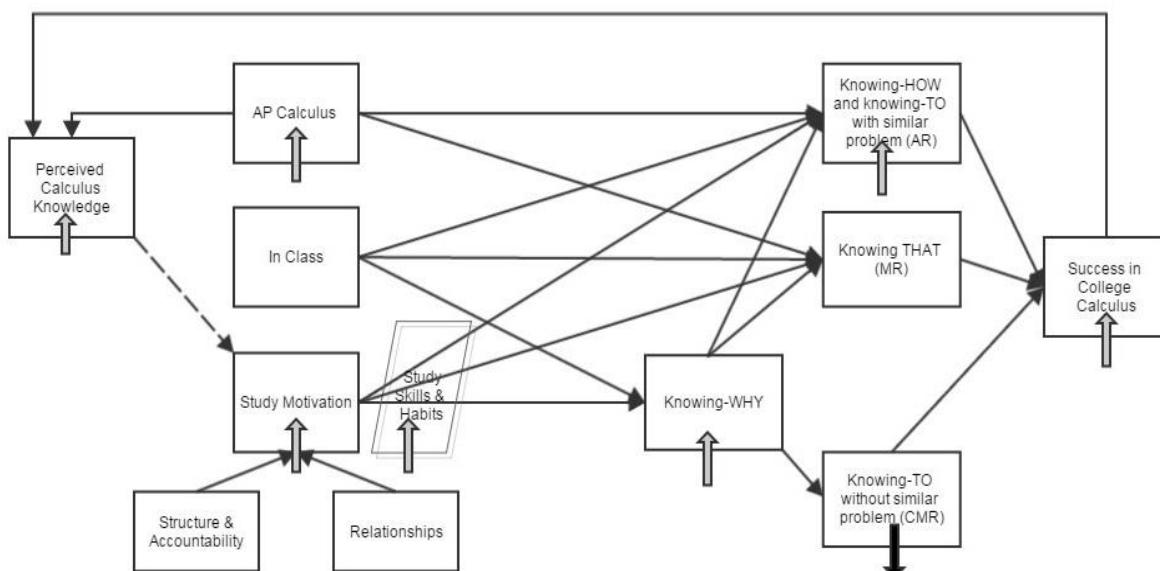
Figure 10. Isaac's Success Profile.

However, Isaac also expressed frustration that his methods left him short of meeting his goal of a solid A for the class because some test problems required him to "apply it to a different level." He claimed "you can't really study for that…" When he went back to solve

the test problems in the interview, he had the same difficulty he had during the actual exam.

That is, working the most similar problems in his resources did not sufficiently prepare him

for these questions on his test. Isaac's test problems required CMR, and though he was doing

well in the course in comparison to other participants, this prevented him from achieving the

level of success he desired.

Maggie.   Maggie was being marginally successful in the course. The direct factors

that impacted Maggie's success negatively were knowing-HOW and TO with AR and

knowing-TO with CMR. Maggie's knowing-HOW and TO with AR was impacted

negatively by the lack of interaction in her college class and her poor study skills and habits.
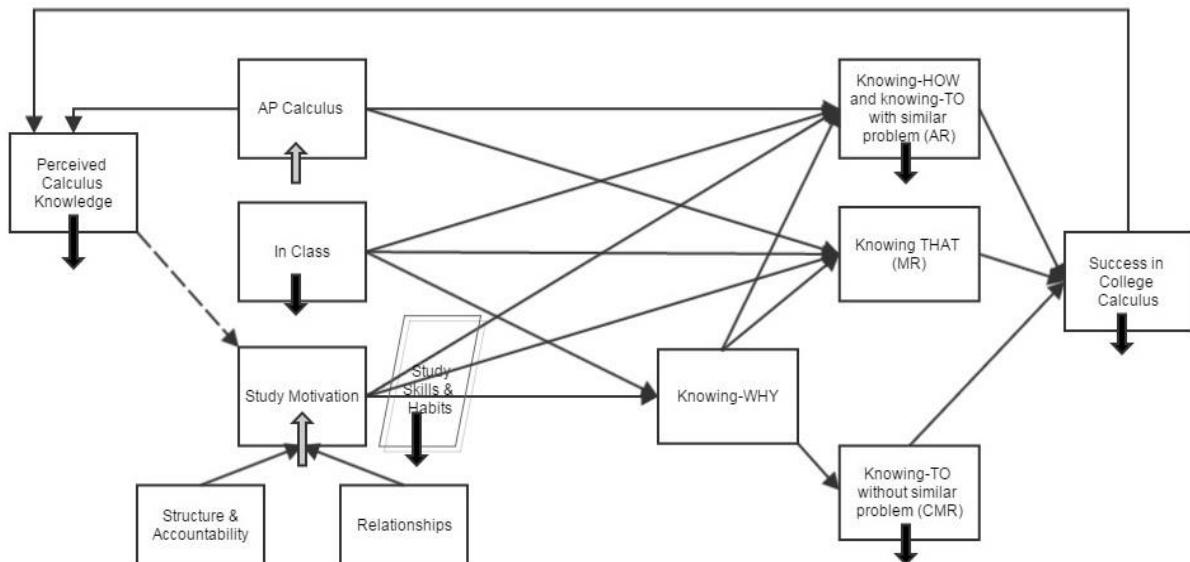
Figure 11. Maggie's Success Profile.

Maggie was unique amongst the participants in that she had very high study

motivation but poor study skills and habits. She was also the only participant who had

sought out help from her instructor regarding how to study appropriately and had made

changes to her approach to the course. As a result, her study skills had improved, but unfortunately the changes occurred too late in the semester to prevent multiple poor test scores. Even after making the changes her instructor recommended, Maggie demonstrated a lack of awareness of other ways to study appropriately. When asked how she might make sure she meets her minimum goal of a C in the course, she exclaimed, "I will probably read my whole textbook if I have to!"

Albert. Albert was being successful in his college course. Like Erin, he attributed this largely to having learned a lot of the material in his AP course. (He had made a 3 on the AP exam.) Albert had been studying, on average, one hour per week in his college course and felt this was sufficient. His studying was limited to doing his homework and referencing his class notes when needed. He showed no indication of an awareness of other things he could be doing to study. When asked about the textbook, he explained, "I've only looked
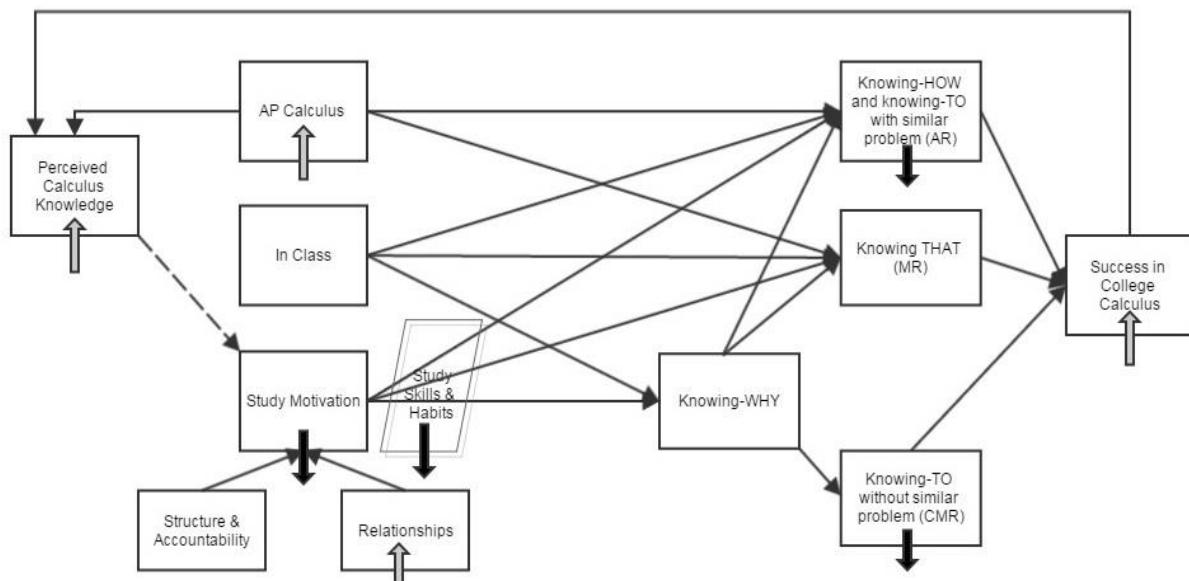
Figure 12. Albert's Success Profile.

through it 2 or 3 times and that's more for like definitions." This was apparent when the

most similar problem to one of his missed test problems was found as a worked example in

the book. Albert was able to interpret the solution and subsequently was able to easily follow

the same process to solve his test problem. Albert's success was limited by knowing-HOW

and TO with AR because of poor study motivation, but his grades were not suffering

sufficiently to increase his motivation.

Albert's success was also limited somewhat by knowing-THAT and knowing-TO

with CMR. Knowing-THAT issues arose during the second part of the interview, but he also

brought up this challenge himself during the first part when asked about his confidence in the

course. He explained that there were things he just had to memorize "rather than

understand," and this was difficult for him.

Blake.
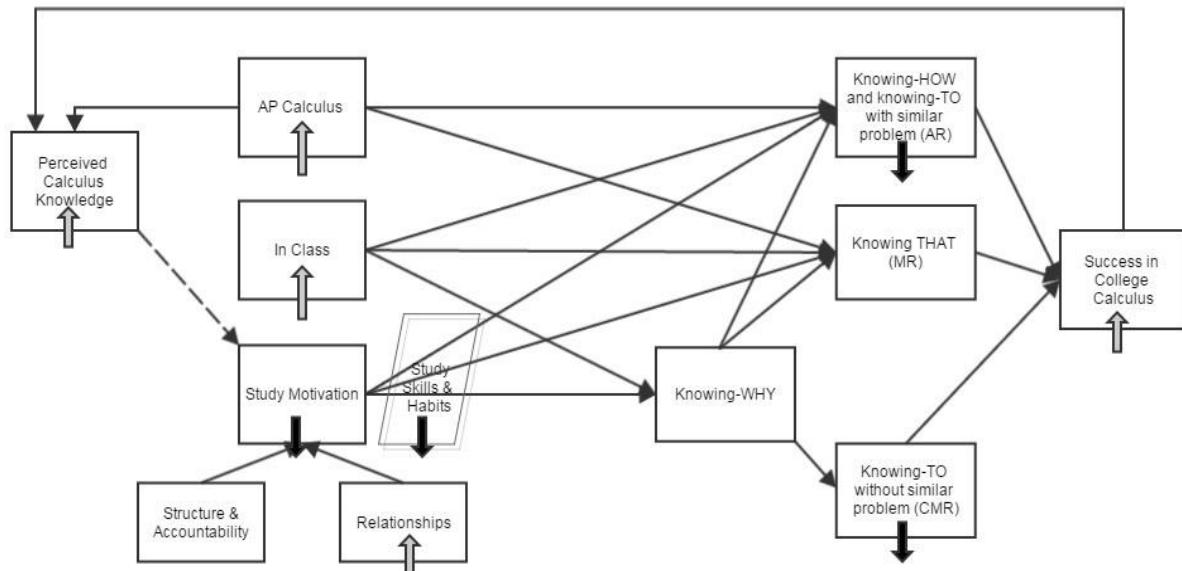
Figure 13. Blake's Success Profile.

Blake described his AP calculus course as very demanding. He made a 3 on the AP test, despite having "senioritis" and not studying appropriately. His knowledge from AP Calculus was a significant factor in his success in college. Classroom interaction and relationships were also important for Blake. He felt he learned what he needed in class, which he described as very interactive. He made a point of praising his college instructor for being approachable and encouraging. Blake was on track to receive a B in his college course. This had happened without much effort outside of class. When asked about his study approach, he replied, "Yeah, I couldn't…that's the other problem. I don't really have a specific study, uh, … I go through the Webassign." For Blake, like Albert, the problems from his resources that were similar to his test problems came from his textbook, which he had not been using. He studied the worked out examples for the first time during the interview, and with some help was able to interpret the solutions. He was then able to use AR to apply the calculus principles to his test problems. However, Blake was still very hindered by CMR with prerequisite material. He was not able to completely solve either of his test problems because of algebra requirements in the test problems that were different from the similar problems he had studied in his text.

Allen. Although Allen was on track to make at least a high C in his college course, he had expected to receive an A, since he had taken the AP course and made a 3 on the exam. Until the third exam, he had experienced success with little effort, and because of this he had not felt the need to do much studying besides his homework. However, later in the course he began experiencing greater challenges. He described the college course material as being

familiar but "deeper" than what he had seen in his AP course. As an example, he kept

referring to optimization and related rates problems. His study habits were better than many

of the participants. For example, Allen used the provided practice tests as a diagnostic tool

rather than simply as extra practice problems. He said, "If I can't figure one out, I try to not

look at the answer and just look at the steps that she's doing and figure it out from there, or

go look in my notes and see if there's something I missed. I try and do it like the test."
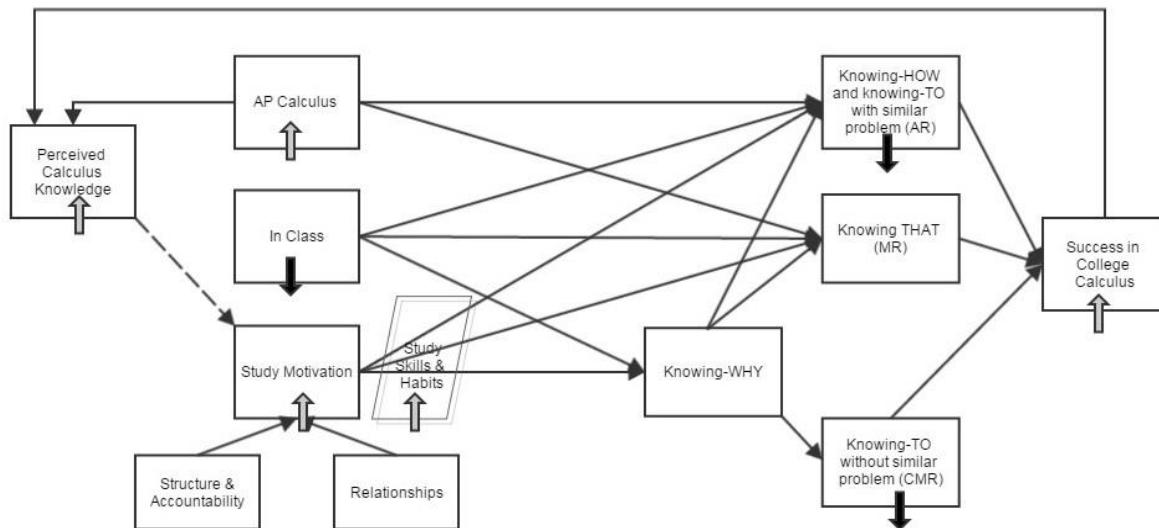
Figure 14. Allen's Success Profile.

But Allen's methods failed him when it came to the topics of related rates and optimization.

He said, "I felt like I had gone through every problem I could possibly on related rates and

optimization that she put up. I felt like I kind knew it to the point where I could get by but

then I was like I suck at these…I don't know what to do from here…every one was

different." Knowing-TO with CMR was a significant barrier for Allen, and not only with

pre-requisite material as with some other participants. In the optimization problem from his

test, he struggled to find the correct derivative even after being able to construct the correct function and putting it in terms of only one variable. Knowing-HOW and TO with AR was also a barrier for him, likely as a lasting effect from his poor study motivation at the beginning of the course. Before studying a similar example from his class notes, he was not able to create a second equation and use it to write surface area in terms of only one variable, but he did this easily for his test problem after studying the solution in his notes.

Haley.
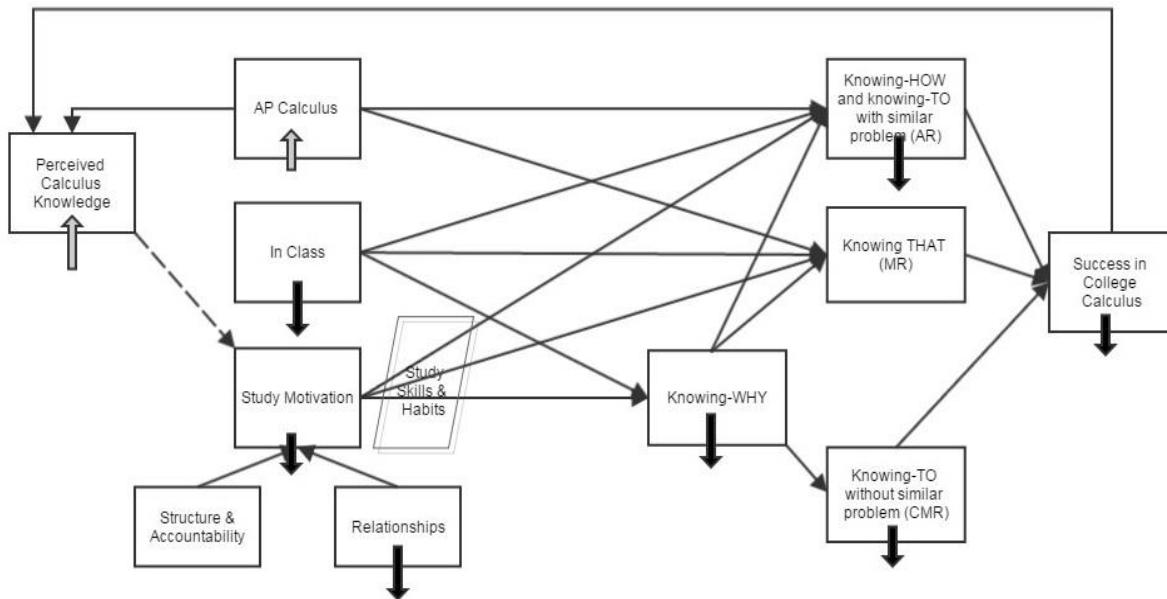
Figure 15. Haley's Success Profile.

Haley did not positively experience many of the student success factors. The only factor that seemed to really contribute to her success was her AP background. She mentioned that she preferred how her AP teacher taught and was confused by her college instruction because it was different from the way the material was presented in high school.

Haley scored a 2 on the AP exam. She explained that what she gained from high school was procedural, memorized information. "It helps knowing the background behind…I'll just remember some tricks. Things we had to practice a lot in high school to remember, here, I remember them easily." However, material that was new to Haley was difficult for her to process in her college class. Two factors – not knowing her instructor and not interacting during class – were very significant to her. These led to problems such as not feeling comfortable asking questions in class.

Haley's study motivation was low and this, combined with the difficulty of learning during class, led to significant issues with knowing-HOW and TO with AR. Both of her missed test problems discussed during the interview were almost identical to worked out examples in her textbook. She met with no difficulty in following the provided solutions. She did, however, express frustration with not knowing-WHY some steps of the solution were being taken. It did not prevent her from being able to use the process on her test problems but knowing-WHY could have been an asset to Haley. She also found difficulty in completing the test problems, even after studying the similar problems, because of algebra issues – specifically in solving equations. In this way, knowing-TO with CMR was also a barrier for Haley, though not to the same extent as knowing-HOW and TO with AR.

Jeremy. Like Haley, Jeremy did not positively experience many of the success factors. He was also frustrated by the lack of interaction in class and was demotivated by the lack of structure and by not knowing his college instructor. He entered college calculus with

a belief that he would be successful, and this negatively impacted his study motivation, particularly at the beginning of the semester.

Jeremy struggled with knowing-HOW and TO with AR, knowing-THAT (MR), as well as knowing-TO with CMR. His knowing-THAT deficiencies, in particularly, were affected by his AP experience. He explained that in his AP course, he avoided learning "trig stuff" because while there might have been one or two problems on a test with trigonometric functions, he was able to score high enough without knowing how to solve those problems. Jeremy's success in the course certainly could have increased with an increase in study motivation, because he indicated knowing specific ways to better prepare for tests, and part of his missed test problems were issues of not knowing-HOW and TO with AR. However, Jeremy did also struggle with knowing-TO with CMR.
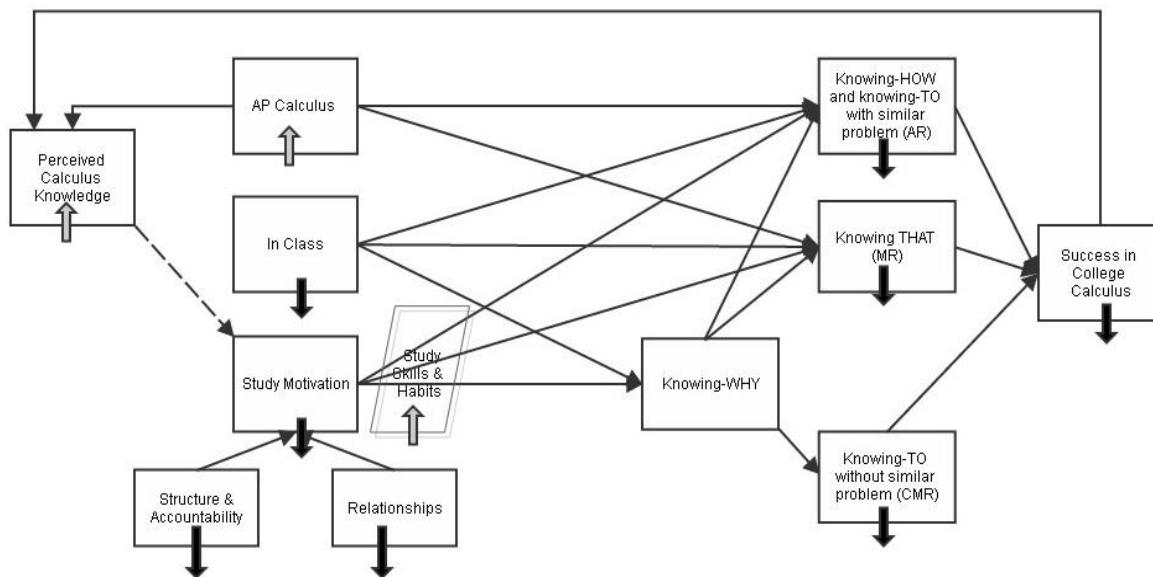
Figure 16. Jeremy's Success Profile.

Samuel. Samuel did not positively experience any of the student success factors.

Samuel was very vocal about his lack of pre-requisite knowledge. This affected both his

knowing-THAT and his knowing-TO with CMR, although his CMR problems also involved

calculus topics. However, despite these both being significant negative factors, Samuel

could have performed much better on his test with only an improvement in knowing-HOW

and TO with AR. For example, one of his missed test problems dealt with related rates. He

did not demonstrate a basic understanding of the concept or know a basic solution strategy

for these types of problems. He had with him a worksheet of over 20 example problems his

instructor had provided, but he had not worked these problems. One of them was similar

enough that after working it during the interview, he was able to make significant progress on

his test problem.
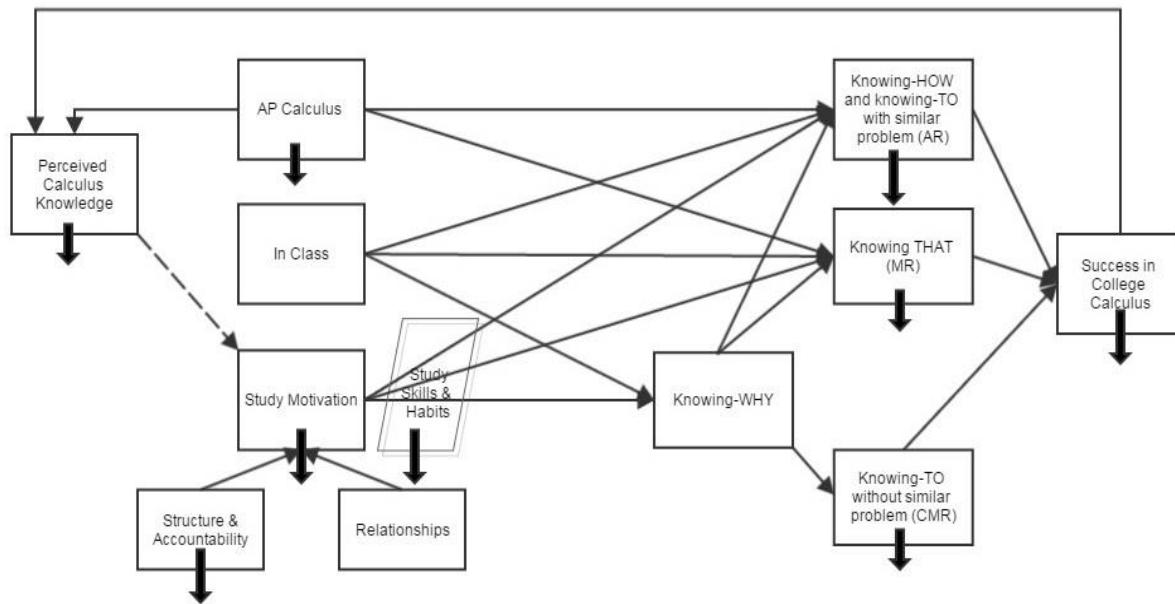
Figure 17. Samuel's Success Profile.

Samuel did not seem to think that working these problems was a realistic expectation and he was frustrated by his lack of success by the time of the interview. His study motivation was very low because he felt he had fallen too far behind and he needed to dedicate his time to other classes. He did not have knowledge for how to study appropriately, saying he was out of ideas. He talked more about using resources not directly tied to his class (such as Kahn Academy and getting help from a roommate) and largely ignored suggested practice assignments and assistance available from the university math center and office hours.

Samuel was also very vocal about how a lack of course structure had negatively impacted him. Multiple times he stressed that he let himself get behind in the course and that it was impossible to catch up. He found that even required online homework assignments were not sufficient to force him to learn the material well because he could use sources like Wolfram Alpha to complete it without knowing how to solve the problems like he would need to for an exam. He said that under ideal circumstances, students would have to submit homework with all work provided and have it graded by hand.

Wade. Wade also discussed how he had let himself get good homework grades without learning how to solve problems, particularly as the semester progressed. He was aware, however, that this was not effective and knew what he could be doing to better test himself. Wade had decent study habits but generally poor study motivation, which was decreased by his surface familiarity with the early material from his AP course and his success on the first exam.
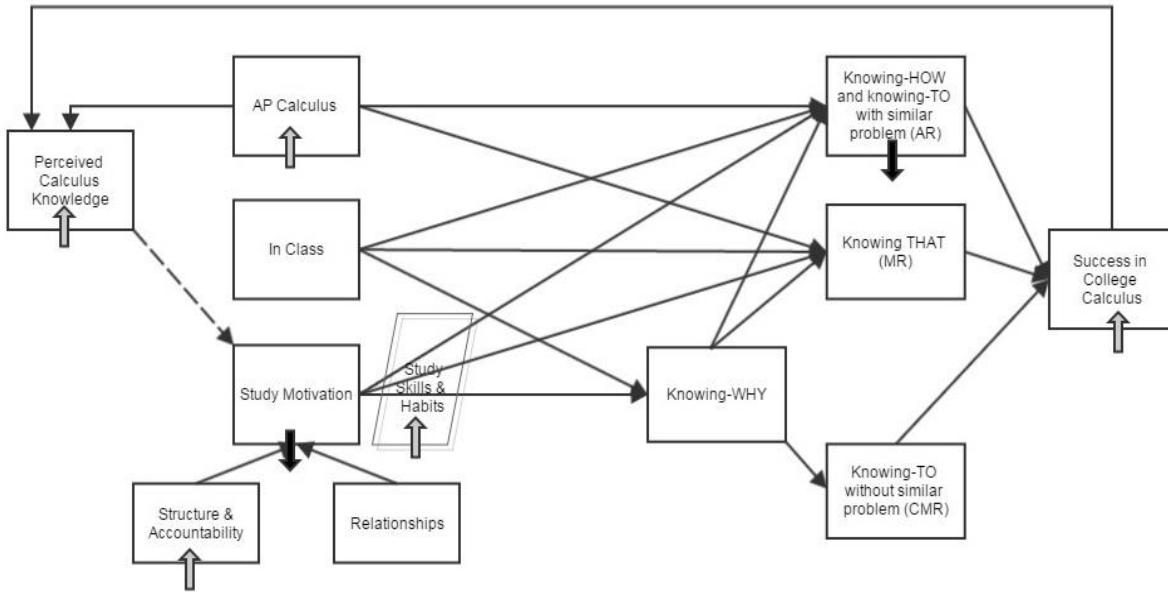
Figure 18. Wade's Success Profile.

Wade's primary obstacle to success later on in the course was knowing-HOW and TO with AR, and specifically with recognizing a problem as a certain type. Knowing-HOW did not present as problematic on the test discussed during the interview; once Wade identified the necessary procedure, he was able to carry it to completion. Wade was given help in identifying similar problems in his resources to his missed test problems, thereby reducing the knowing-TO burden. He had no trouble following the solutions given and being able to reproduce the procedures used, and subsequently was able to solve the test problems with no help. However, he claimed he would not have recognized the test problems as being the same type (requiring the same "tricks" or procedures) if he had worked the similar problems during his studying and not had the similarities pointed out to him. Thus it is unclear whether his primary obstacle is knowing-HOW and TO with AR or knowing-TO with CMR.

This is particularly interesting because his test problems and similar problems from his resources were at least as similar, if not more so, than all the other participants.

Wade's similar examples came from his homework solutions and his class notes, both of which were posted online. He was not familiar with either similar problem before the interview; he was unable to solve the similar problems before studying their posted solutions. So while he may be correct in claiming that he would not have been able to recognize the problem type of his missed test problems, he arguably would have increased his chances of doing so by being exposed to these specific similar problems ahead of time.

Putting it All Together: Factors of Student Success

This section discusses findings from the comparative analysis (See chapter 3 for further explanation) of the individual student profiles described above. As shown in the previous section, there was significant variation amongst the participants regarding which factors were significant in either impeding or facilitating their success in college calculus. Therefore, it is not possible to categorize or group the pathways taken by the individuals into a small number of common experiences. Instead, what will be presented is a discussion of the relative importance of the individual factors and the ways in which they impact other factors and ultimately success. The discussion will begin with the direct factors and then move to indirect factors. Illustrations are provided throughout that highlight where on the diagram the discussion is focusing.

The Three Direct Factors. Three factors emerged as direct factors affecting success on college calculus tests (see Figure 19). These factors were identified primarily from

analysis of the second portion of the student interviews where students attempted to solve a previously missed test question, after first working through a similar problem from one of their class resources. These factors are knowing-HOW and TO solve a problem using algorithmic reasoning (AR), knowing-TO use a particular procedure using creative mathematical reasoning (CMR), and knowing-THAT with memorized reasoning (MR), or recalling mathematical facts or solutions to problems from calculus and previous mathematics courses. They are termed direct factors because all missed points on the discussed test questions resulted from insufficiencies in one of more of these three factors. Alternatively, it may be said that if a student had no deficiencies in any of these three areas, the student would consistently make perfect scores on the types of college calculus tests the 14 participants were given.
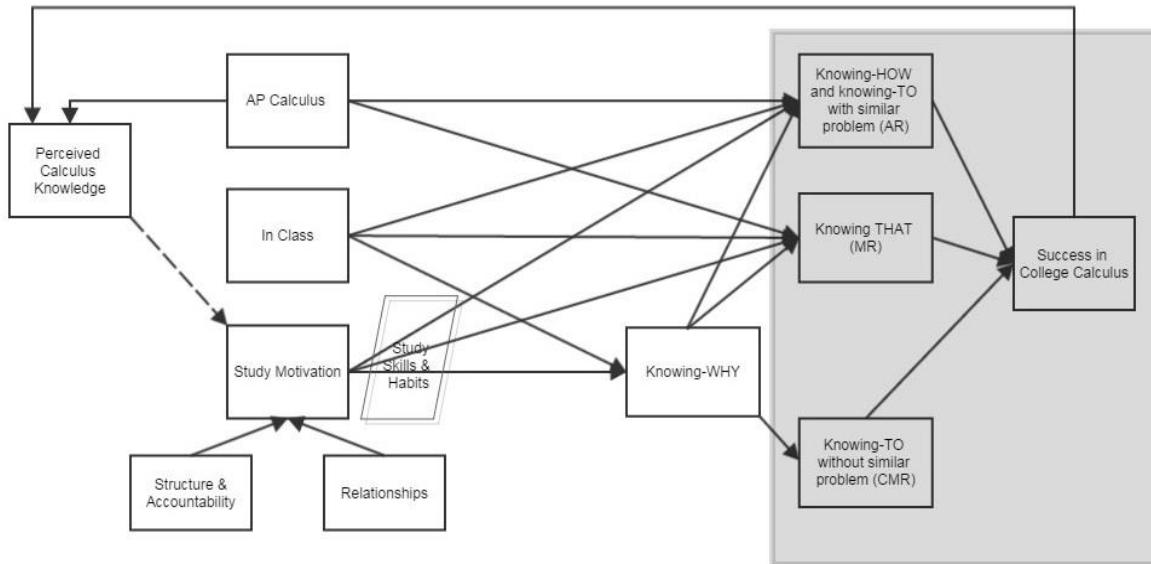
Figure 19. Success Factor Model: Direct Factors.

Knowing-HOW and TO with AR. The second part of the interviews were designed to determine whether it was possible for the student to use AR to solve particular test questions. To achieve this goal, the following two things were established. First, were there practice problems in the student's resources which contained similar instructions and required the use of the same algorithms? Second, was the student capable of learning to apply the algorithms? The interviews showed that the majority of the discussed test questions had a corresponding problem somewhere in the resources whose directions were similar and which required the same algorithm to be applied, and in the majority of cases, participants were able to learn the associated algorithms before attempting the test question. Mastering the algorithm during the interview required the participant to either read a conceptual explanation in the textbook, study a worked out example, or verbally discuss the algorithm. Twelve of the 14 participants were able to make more progress on at least one of their two discussed test problems during the interview than they had made during the test after first learning to solve a similar problem from their resources. After working the similar problem, they saw a connection between it and the test problem and subsequently applied the same algorithm successfully. This involved knowing-TO choose that algorithm and then knowing-HOW to carry it out. Very little variation was found in the directions for the problems from the tests and class resources; therefore, recognizing the problem type did not prove to be problematic for the majority of participants. The implication is that students who missed test problems that could be solved with AR from a similar problem that was available in the course resources could have improved their grade on the test by having worked these

similar problems prior to taking their test. This is an issue of either inadequate study motivation or study habits, both of which will be discussed. It should be noted, however, that in the interviews the participants were given assistance in identifying suitable similar problems and therefore had an immediate and appropriate association when attempting to work the test problem. Moreover, no time elapsed between the participants working the similar problems and test problems, while when actually taking a test, time most certainly has passed since working similar problems from the resources. Both of these factors have the potential to create a more challenging problem situation for students than was observed during the interviews.

Knowing-TO with Creative Mathematical Reasoning (CMR). Ten of the participants needed to apply some degree of CMR to successfully complete at least one of their discussed test problems. Perhaps the most striking finding is that very few of the problems required CMR for calculus material; rather, most elements of problems requiring CMR dealt with pre-requisite knowledge or skills. In these cases the appropriate solution sequence for that element was one the students had likely encountered in their previous mathematical experience; however, it had not been associated with the calculus topic of the problem at hand (or at least it was not observed in the student's course materials during the interview.) Therefore, not knowing-TO use that particular solution method was a barrier to the student. In these cases, the local element requiring CMR came at the end of the problem and therefore did not cause the students to miss the entire problem. For example, when Blake was asked to find the absolute minimum and maximum of a function, he was able to

take a derivative and knew to set it equal to zero, but was then unable to solve the equation. In a limited number of cases, the element requiring CMR came at the beginning of the problem and did prevent the student from being able to continue, thereby costing the students all or most of the points for the problem. For example, in a related rates problem, Isaac did not recognize a right triangle relationship between elements in his problem that would have led to an equation he should implicitly differentiate. He knew that he should find an equation and what to do with it, but was unable to create the equation and could not proceed. These cases were limited, however. No student had enough CMR required on his or her test so as to prevent passing the test without it. CMR was a barrier for some to making an A or B on the test, but all fourteen tests could have been passed only by improving the other two direct factors, knowing-HOW and TO with AR and knowing-THAT. The following section describes the specific variations of CMR that were observed.

Variations of CMR. To be more specific about this construct, three problem variations will be described that required the use of local CMR. What the student had to know-TO do was different in each of these elements. The first and most widespread of these was solving algebraic equations. Five of the 14 participants had at least one problem where they were able to use AR to solve the calculus portion of the test problem during the interview, but then were not able to finish the problem when required to solve an equation. For example, Michael's test problem and similar example in the textbook both stated, "Find where the tangent line is horizontal [for a given function]." Michael learned from the textbook example that he should take the derivative of the function, set it equal to zero, and

solve. He attempted to repeat this process for the test problem. He successfully took the derivative and set it equal to zero. He wrote $e^x(-\sin x) + e^x \cos x = 0$ and stopped here; he was unable to solve this equation. Michael did not know-TO factor out e^x and set both e^x and $-\sin x + \cos x$ equal to zero and solve. While it is unlikely that Michael had never before been exposed to an algebraic equation of this type in his previous mathematical experience, it is possible that in his study of derivatives, he had never encountered a problem that required him to draw on this specific solution technique, or at least not involving both exponential and trigonometric functions. It is important to note here that not every single example in Michael's course materials were examined to determine whether he had seen a similar equation; the similar problems for the interviews were identified based on calculus content rather than on the algebraic processes required within. It is quite possible that Michael's materials did contain such an example. If that had been identified in the interview process, then this case would not be classified as being an issue of knowing-TO using local CMR.

A second variation of problems that required local creative reasoning was recognizing relationships between variables. Two students, Isaac and Katelynn, had missed the majority of points on a related rates problem. They both recognized the problem as a related rates problem and had solved other problems of this type correctly, but they were unable to construct the equation that they were supposed to implicitly differentiate. The problem stated that two trees, located 12 feet apart, were growing at two different rates (these rates were provided). The problem asked how fast the distance between the tips of the trees was

growing after a specified amount of time. Neither student recognized the right triangle

relationship formed by the line connecting the tips of the trees, the constant horizontal

distance between the trees, and the part of the taller tree extending beyond the shorter tree.

This prevented them from constructing the equation $x^2 + y^2 = z^2$ and then using their

knowledge of implicit differentiation with respect to time. Interestingly, Isaac, demonstrated

right before attempting this test question that he could solve completely, without assistance, a

related rates problem that also used the Pythagorean Theorem relationship. But in this

previous problem, the right triangle relationship was apparent to him. So although he knew-

HOW to solve a related rates problem with a right triangle relationship, he did not know-TO

use this relationship for this test problem.

Another student, Maggie, missed the following problem on her test: "Use a

linearization to approximate the value of $\frac{1}{\sqrt{8}}$." Maggie encountered no difficulty in finding a

linearization for the function $f(x) = \frac{1}{\sqrt{x}}$ at the point a=9 once given the function and the a

value, but she did not recognize during the test that she was supposed to use this function at

this point. That is, she did not know-TO use $f(x) = \frac{1}{\sqrt{x}}$ and the point a=9. In this case as

well as the previous one, students missed the entire problem because the part of the problem

requiring local CMR came at the beginning. Without using local CMR successfully, the

students could not demonstrate that they *knew-how* to complete the required procedure. The

degree to which the need for local CMR affected the student's performance on problems

varied. If local CMR was required for an element of the problem that was necessary for and

preceded other parts of the problem, it was much more likely to cost the student more points on the test.

The final variation of problems requiring local CMR involved applying or choosing between multiple calculus rules in one problem. Consider this problem from Samuel's test: "An oil tanker leaks oil and the spill creates a circle. The radius of the spill is given by r(t)=2t+1. Find the rate at which the area of the spill is growing when the radius is – ft." After studying a similar problem, Samuel was able to set up and implicitly differentiate the equation $A = \pi r^2$ with respect to time, giving him $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$. From his experience with similar problems, he expected to be given a value for $\frac{dr}{dt}$. Instead, the problem provided a specific value of the radius and an equation for radius. Samuel did not know-TO find the derivative of *r(t)*.

Jeremy experienced a similar issue when one of his test problems asked him to find the derivative of $y = sin^2(3x^2) + 5^{1-2x}$. He was able to correctly find the derivative of $sin^2(3x^2)$ but not 5^{1-2x} because in previous problems he had been able to use logarithmic differentiation to differentiate exponential functions; he was unaware of rules for finding derivatives of exponential functions (or their compositions). Jeremy did not know-TO apply the logarithmic differentiation procedure to this problem because of the addition of two functions – a nuance to this problem he had not previously encountered.

Knowing-THAT. Jeremy's barrier to success with this problem was classified as needing local CMR because of the specific solution strategy he used. However, the discussion showed that had he attempted a different solution strategy he would still have

been hindered by the third factor directly related to success – knowing-THAT. Jeremy did

not know-THAT the derivative of $5^{u(x)}$ is $5^u \frac{du}{dx} ln5$. This example was somewhat unusual,

in that the majority of knowing-THAT type mistakes made by the participants tended to

involve trigonometry or some pre-requisite piece of mathematical knowledge, which is

consistent with what was seen with local CMR mistakes. Frank, for example, attempted the

same problem as Michael, described earlier. Unlike Michael, Frank knew-TO factor out e^x

and set both e^x and $\sin x + \cos x$ equal to zero, but he was then unable to solve the equation

$\sin x = \cos x$ because he did not know the trigonometric values of special angles.

The fact that knowing-THAT issues are dominated by pre-requisite material is

supported by participants' comments during the first portion of the interviews. For example,

in answer to a question about his expectations for success in his college course, Albert

explained, "The only thing I've really struggled with, and I also struggled with…is

trigonometry. The derivatives have a lot more to do with memorization rather than

understanding how it works and I have a hard time memorizing them." It is important to note

that even for pre-requisite issues, it is likely that many instructors review these topics during

class and/or explicitly taught them; if that were the case for these students, it may be argued

that the problem could have been solved with AR and since it was not, the barrier to success

was a study habit issue. However, without class observation, this cannot be known.

Relative Impact of Direct Factors. Knowing-HOW and TO with AR, Knowing-TO

with local CMR, and Knowing-THAT all directly affect success on college calculus tests.

However, the impact of these three factors does not appear to be equal. The question is

which of the factors may be bypassed, and success still attained? The amount of impact of each is determined by the problems on the test and the degree of similarity to previously seen problems, and this varies from classroom to classroom. Knowing-HOW is absolutely essential for success in the classrooms of all participants. This may seem obvious; however, a test could be constructed that predominantly requires the recollection of facts, or the explanation of concepts. None of the interview participants brought up a problem of this type when asked for the problems on which they had lost the most points, and in the test analyses, each test had either no questions like this or very few; rather, the test problems required the application of procedures.

The impact of knowing-TO using AR versus knowing-TO using Local CMR varied across classrooms. The analysis of the data for some interview participants showed no evidence of the need for local CMR. In other cases, the need for local CMR was a minor requirement, in that local CMR was required for only small parts of problems and particularly at the end, so that students could obtain partial credit for these problems even if they did not know-TO do a certain part at the end.   Knowing-THAT issues aside, 8 of the 14 students could have received full credit for at least one of the two problems they missed by using only AR (apart from knowing-THAT issues) and another 4 participants could have received partial credit for at least one of their problems, thereby increasing their grade on this test, using AR. Only 2 participants could not have gained a significant amount of credit on either of their missed problems using AR. Neither of these students were failing the course, and one of the two had a B average. While they could have been *more* successful with the

application of local CMR, they were still able to pass with deficiencies in applying local

CMR. This finding is consistent with that of Bergqvist (2007) who found that 15 of 16

Swedish calculus tests could be passed using only imitative reasoning.

While the necessity of knowing-TO using local CMR varied across classrooms, the

relative impact of knowing-THAT tended to be a much more student-specific barrier. This is

because the knowing-THAT issues dealt most often with pre-requisite material. However,

none of the participants were hindered primarily by knowing-THAT issues. While 4 of the

participants could have increased their success somewhat by overcoming a knowing-THAT

barrier, all of these were already or could have been successful without changing their

amount of knowing-THAT. This does not mean that knowing-THAT is not a significant

direct factor; in fact, most if not all test problems required some element of knowing-THAT,

but students knew much of this information, resulting in the conclusion that by comparison,

knowing-THAT is not a significant barrier to success.

Indirect Factors. The following factors impact success indirectly by influencing one

or more of the direct factors described above. The discussion that follows describes the

relationships amongst indirect factors and between indirect and direct factors.

Knowing-WHY. As discussed previously, Frank could not finish his problem that

asked for the location of a horizontal tangent line because he did not know how to solve the

equation $\sin x = \cos x$; he had not memorized the trigonometric values of special angles that

his instructor required. Obviously this is not only an issue of knowing-THAT; this equation

can (perhaps even more easily) be solved by reasoning: since sine corresponds to *y*-
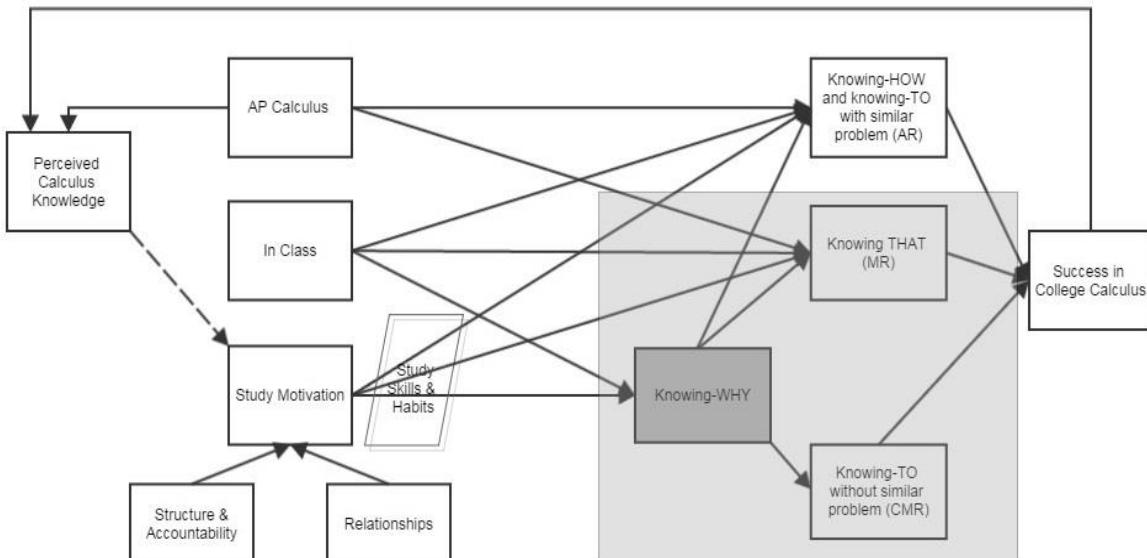
Figure 20. Success Factor Model: Knowing-WHY.

coordinates of points on the unit circle and cosine to *x*-coordinates of the same, these values

will be equal only at multiples of 45 degrees in the first and third quadrants. This is an

example of how knowing-WHY has the potential to impact knowing-THAT. Knowing-

WHY can also influence knowing-TO using local CMR. When initially attempting this

problem, Frank studied the same similar example from the textbook that Michael did. He

focused on the textbook's discussion rather than the solution. After approximately thirty

seconds, he quickly attended to the explanation that the derivative is equal to zero when the

tangent line is horizontal. He did not proceed to study the provided solution and therefore

did not need to use AR. Frank demonstrated an understanding of the connection between the

concept of a horizontal tangent line having a slope of zero and the process of setting a

derivative equal to zero. He no longer needed an association with a particular type of

problem or set of instructions to be able to solve his test question; he now knew TO because

he knew WHY. In contrast, when Michael looked at the textbook, he focused on the solution

rather than the discussion. He explained that in the provided solution, "they would set it

equal to zero and get their x values. They then plugged those back into the original equation

to get the y's." When asked why they did that, he replied, "I'm not exactly sure why."

However, this did not prevent him from being able to then solve the corresponding test

question. This happened with several other students. Two students, Haley and Erin, showed

frustration with not understanding WHY but were successful with solutions to their test

questions using AR. Haley, for example, explained, "I guess probably the reason I'm

confused is I never really understood why you took the derivative of the equations to figure

out how to minimize things in general, but I understand how they did what they're doing; I

just don't understand why." In contrast, Blake did not seem concerned that he did not

understand the reasoning behind the process he was now comfortable using. The analysis of

the data shows that some students prefer to arrive at knowing-TO through knowing-WHY,

while others are content to bypass knowing-WHY, at least in particular problem situations.

However, all students were able to be successful without knowing-WHY. Knowing-WHY

does not appear to be a necessary factor in student success on college calculus tests. This

finding is consistent with the claim of Mason and Spence (1999); understanding is related to,

but not dependent on or necessary for knowing-TO. "You can 'understand' but not know-

TO act,…you can know-TO act and yet not fully understand" (p. 140). This finding raises

questions about the type of knowing instructors most value and test for and this issue will be

discussed in Chapter 5.

AP Calculus.   Many of the participants made some indication that the calculus knowledge they gained in their high school AP course had been helpful to them in their college course.   None suggested that the topics covered differed significantly and several mentioned they expected to do well because of their familiarity with the content.   When asked about differences between his high school and college courses, Wade said the "the material is the same. When we learn something in here, I was like, 'oh, I remember doing that.'" Albert explained, "I really feel like I need to make an A [in college calculus] since a lot of it has been review." Erin even claimed that she had not learned anything new in her college course, except in the areas of exponential and logarithmic functions. One exception was notable; one student stated that he saw the topics of optimization and related rates for the first time in his college course; his AP calculus teacher had omitted these topics from the curriculum.
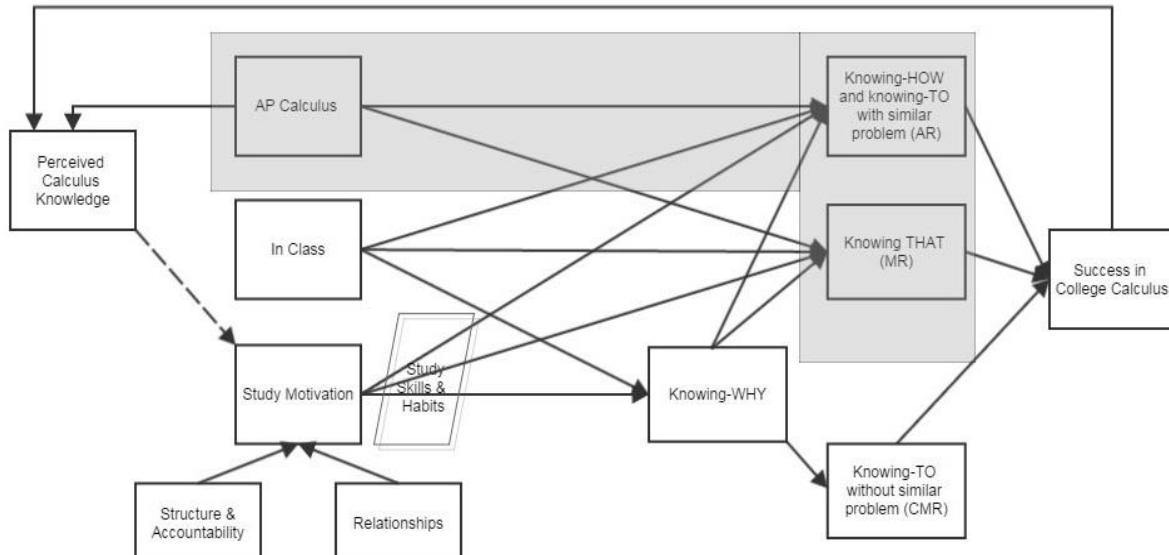
Figure 21. Success Factor Model: AP Calculus.

There is also evidence (See Table 4) to suggest that AP Calculus knowledge contributes to success in college calculus. Only one of the six students who scored a 3 or 4 on the AP test were in danger of not passing their college course. Moreover, of all three students who were failing the course, all of those had made either a 1 or 2 on the AP test. Of course the correlation between AP grade and college calculus success cannot necessarily be attributed to knowledge gained in the AP course, but previous studies do lend credence to this hypothesis (Burton, 1989; Ferrini-Mundy, 1992; Morgan & Klaric, 2007). However, none of these studies explored what type of knowledge or understanding students gained in AP Calculus that contributed to their success in college.

The data suggests that the most likely impact that AP Calculus has on student success is via the acquisition of knowing-HOW and TO with AR. There is also evidence that it impacts knowing-THAT. The other pathway to effect on success that may be expected is through knowing-WHY. Instruction related to knowing-WHY, however, was not discussed by any of the participants as a significant aspect of their AP calculus class. In contrast, the presence of this type of instruction was noted by several participants as a key distinction (and challenge) of college calculus. Katelynn, who clearly articulated that her high school course was very challenging, explained that her college course was still more challenging because of this knowing-HOW component. She stated, "I'd definitely say the big difference I've noticed then to now – we learned more the how to do things as opposed to why you do them. Like I remember learning like sin x over x equals one, but I had to learn how to derive it this year. That was a lot harder." Other participants made reference to the material in college

going more "in depth." Haley's description of her AP course resonates with Katelynn's and

it also suggests that AP Calculus may have impacted her knowing-THAT. She explained,

"I'll just remember some tricks. Things we had to practice a lot in high school to remember,

here, I remember them easily."

AP Calculus and Perceived Knowledge of Calculus.
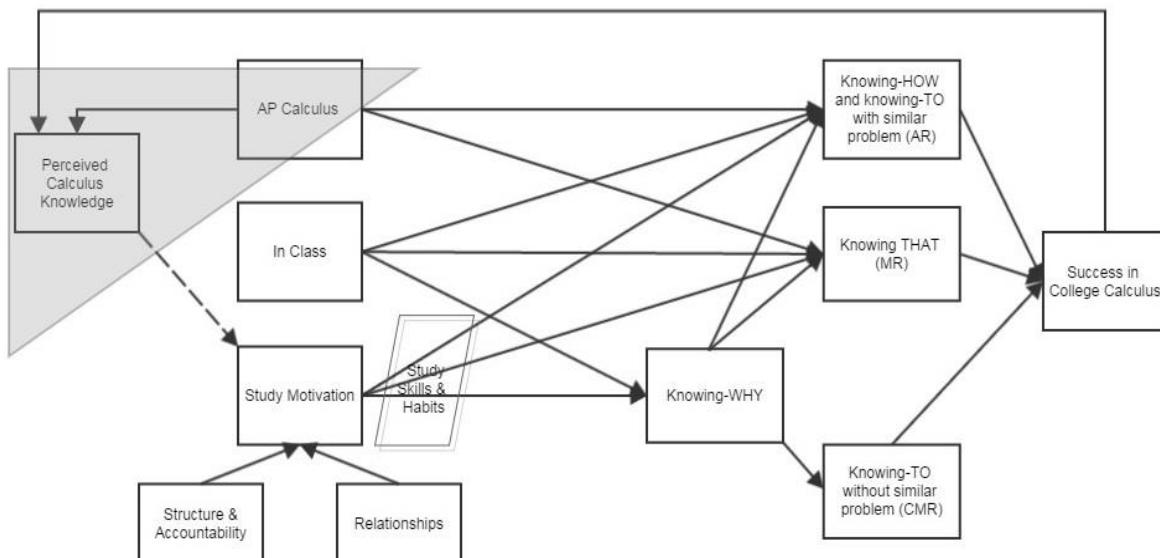
Figure 22. Success Factor Model: AP Calculus and Perceived Calculus Knowledge.

The acquisition of knowing-HOW and TO with AR and knowing-THAT from AP

Calculus tended to increase students' perception of their understanding of calculus as well as

their confidence in their ability to succeed in college calculus. Almost all of the participants

made some reference to what they had learned in the high school course having been helpful

in college calculus, which suggests a consistency between at least some of the course

material. This is not surprising, since the AP curriculum is regularly revised and updated

with the input and feedback of college calculus instructors across the country (Gollub,

Bertenthal, Labov, & Curtis, 2002). However, 7 of the participants suggested or stated explicitly that having taken AP Calculus made them overconfident going into the college course. Many discussed a relatively recent change in their expectations for their grade in their college course – they went in expecting A's or high B's because of their previous exposure to the subject, and by the time of the interview had changed their goal to a C or passing. Jeremy, a student who was on track to make a low B or C in the course explained, "I think I just, since I already took calculus in high school, I thought it would be a lot easier that it was." This trait was not specific to students who were struggling to pass. Albert's grade at the time of the interview was a high B and his comments about his belief in his ability to perform well echoed that of Jeremy. "I was overconfident at first because I [thought I] remembered it all." Jeffrey, who was expecting to make a high B or low A, explained that if he could start the semester over, he would take the first test more seriously. He said, I didn't even look at notes. I was like I already took this class, it's gonna be a breeze."

Perceived Calculus Knowledge and Study Motivation. The confidence from knowledge gained in AP Calculus had a negative impact on participants' study motivation – which is defined by Credé and Kuncel (2008) as "sustained and deliberate effort" (p. 428). That is, study motivation was often lower than it needed to be to achieve success because of an elevated sense of confidence that stemmed at least partially from the perceived knowledge gained in AP Calculus.
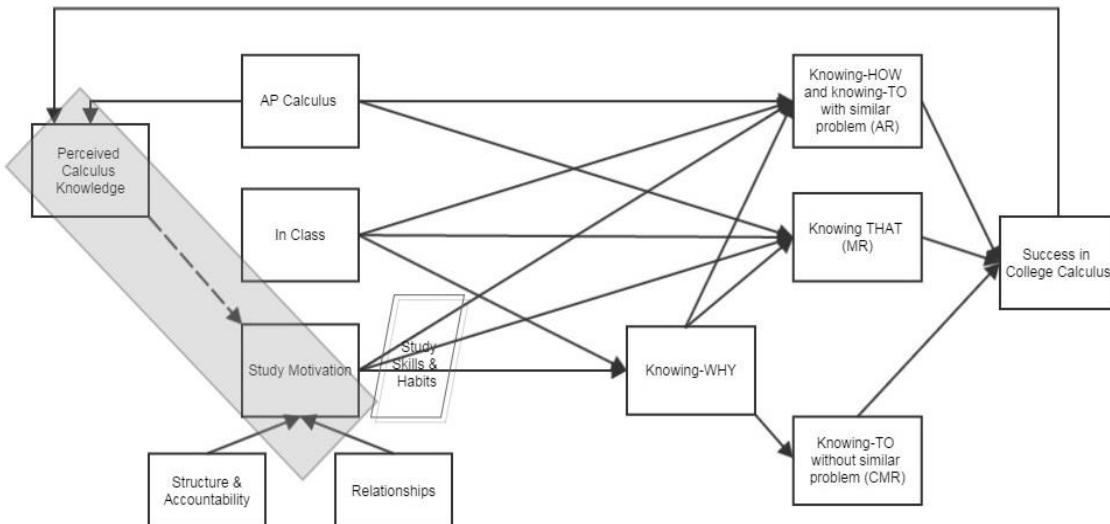
Figure 23. **Success Factor Model: Perceived Calculus Knowledge and Study Motivation.**

This negative effect of perceived calculus knowledge on study motivation was sometimes tempered by poor grades on college calculus tests, but only for some participants. Participants' responses to lack of success varied tremendously. Maggie immediately began seeking help from her instructor for her study approach after her first bad test grade. She described changes she had made at various stages throughout the semester that she implemented because she was unhappy with her grades. In contrast, Frank had made no changes to his approach to the course after getting two poor test scores and continued to ignore a majority of his resources. His lack of success on his college tests did not change his belief in his calculus knowledge and ability, and therefore did not positively impact his study motivation. For other students, the relationship between perceived knowledge and motivation was difficult to ascertain because the student's overall grade in the course was still at or above the minimum standard they have set for themselves. That is, they had not

reached a point where more effort was required. Both Jeffrey and Erin, for example, indicted that despite one poor test grade, they were still making B's in the course, and if this had not been the case, they would have put forth more effort. Jeffrey said that had he gotten lower scores on his first two tests, he would have studied more. When asked, "So you're willing to do it, but you just don't have to do it?" he replied, "Correct. I'm just lazy."

Study Skills and Habits. The impact of study motivation on knowledge acquisition for the participants was mediated by their study habits, which include "self-regulation, ability to concentrate, [and] self-monitoring" (Credé & Kuncel, 2008, p. 428). That is, the effect of a student's willingness or effort to study was either amplified or tempered by the student's ability to study appropriately. Maggie, for example, had very high study motivation, but poor study habits aside from the suggestions given directly by her instructor. When asked if she would be able to make at least a C in the course, she declared, "I'll MAKE it happen!" But when she was asked to describe *how* she would do this, she referred only to doing more of the same activities she had already been doing that had not led to success. For example, she said "I will probably read my whole textbook, if I have to." She was willing to put in more study time, but did not know how to appropriately direct this study time. Isaac, in contrast, had high study motivation and also good study habits. The effect of these study habits was significant. Comparison of Maggie's and Isaac's test problems indicated that Isaac's course required more local CMR and therefore should be more challenging; however, Isaac was doing much better in the course than Maggie, despite them receiving comparable AP scores. Isaac had figured out how to be prepared for test questions that did not require

local CMR: "…first you have to read the textbook and then make sure you understand the notes in class, and then do practice problems from the book." He noted, however, that this approach only helped him with certain problems on his test (those not requiring local CMR). For others, he claimed, "you can't really study for that, it's more just like understanding, so just like basic concepts but you have to apply it to a different level. So that part's tricky."

Allen relayed a similar experience in studying related rates and optimization problems. He exhibited high study motivation with these topics, saying "that was probably the hardest I've ever worked." But while his motivation to and knowledge of how to study appropriately made him successful with other topics, when it came to related rates and optimization problems where "everyone was different," these methods failed him. He stated, "I could never get it down just the process of doing it and figuring it out and every problem being different – I could never get it." Thus it seems that high study motivation, mediated by appropriate study habits, can lead to greater success in knowing-HOW and knowing-TO using AR, but there is no evidence to support them impacting knowing-TO using local CMR.

Study Motivation and Study Skills and Habits. A student can overcome poor study motivation and study habits by having either 1) a strong AP experience or 2) the ability to pick up material quickly and easily in the college classroom. That is – study motivation and habits are not necessarily required for success in college calculus. A strong AP experience means that the student gained knowing-HOW and TO with AR, as well as the knowing-THAT that was essential for success in the college course. The ability to pick up material quickly in college could be affected by both student-controlled factors, such as a strong

mathematical background or a strong interest in mathematics, but also by instructor or classroom-controlled variables such as the amount of interaction and engagement required of students during class.
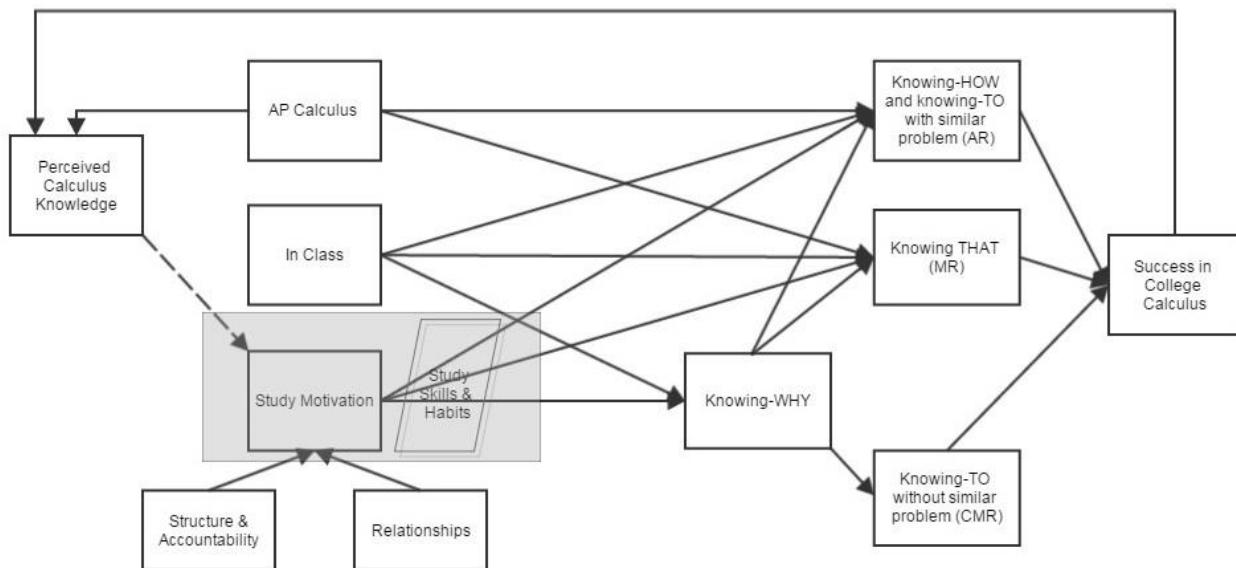
Figure 24. Success Factor Model: Study Motivation and Study Skills and Habits.

Interestingly, while the need for good study motivation and habits may be circumvented by other factors and they do not guarantee students will meet their own goals, the data suggests that having strong study motivation and habits can be the deciding factor in whether some students succeed. Analysis from the second portion of the interviews showed that students at four of the six institutions would likely have been making A's on tests with these improvements. Students at the other two institutions were limited somewhat by the presence of multiple problems requiring local CMR. However, even these students could have been making at least a B- in their course with these improvements. No test from any of the 14 participants required so much local CMR as to prevent students from being successful

if their knowing-HOW and TO with AR and knowing-THAT knowledge was complete.

Isaac, for example, went in to his college course expecting to easily make an A or high B in

the course. He presented with the most impressive study habits of all 14 participants,

although he was still not quite meeting his goal at the time of the interview. The test

problems of Isaac's that were discussed during the interview required a significant amount of

knowing-TO with local CMR. However, not all of his test questions required this. He was

able to do very well on most problems and ultimately be successful in the course due to his

knowledge from his AP course and what he learned during his study time, despite being

frustrated that his grades did not meet his own high expectations.

> ### *Structure and Accountability and Student Motivation*.
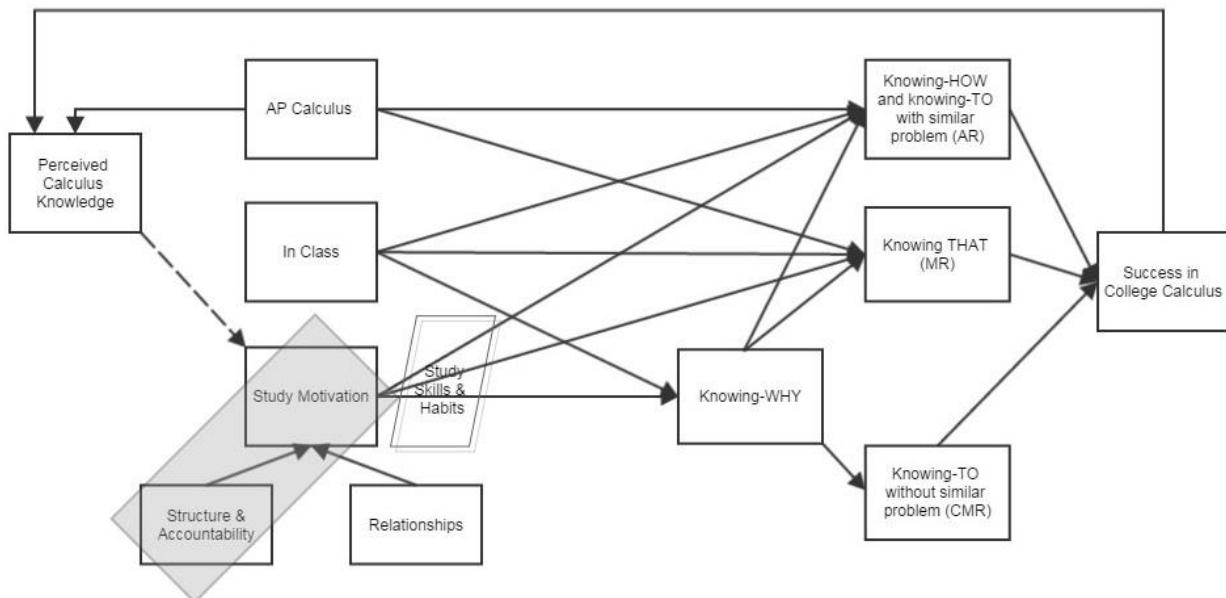
Figure 25. Success Factor Model: Structure and Accountability and Study Motivation.

While the participants generally all accepted responsibility for their own learning, they also indicated that they were motivated by external structure and accountability provided by both instructors and course design. Students discussed examples of this throughout the first part of the interviews, often when comparing their high school and college experiences. The most commonly discussed element of course structure was the grading of homework, or lack thereof. Most participants had no regularly graded assignments in high school, but did have this in college, and most claimed it being graded was the only factor that would motivate them to complete it. This was an exception to an important pattern - in most cases, participants described their high school course as much more structured and indicated this was a positive aspect of the high school course. College instructors were not thought to have many expectations of students. Albert explained, "her job is to teach us, not make sure we care about class." Maggie's comments echoed this sentiment. She said , "I think it's like, we're supposed to do it, be responsible for ourselves. It's college, so…" Students seemed to understand and validate the responsibility put on them in the college environment, but they offered no evidence that it made them more successful. In contrast, Katelynn discussed her AP teacher "staying on top of [the students]" and claimed she would not have passed the course if he hadn't. AP courses tended to provide more regular examination, frequent reminders from teachers, and accessibility of help from teachers.

Relationships and Study Motivation. The relationship between teachers and the participants was one of the most widely discussed differences between high school and
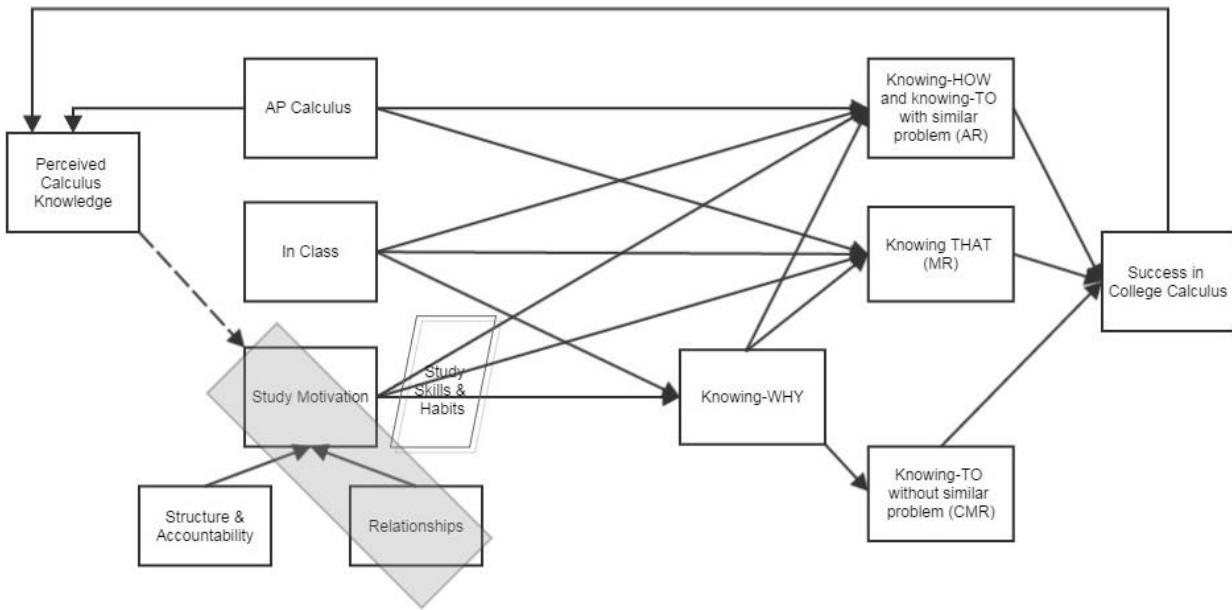
Figure 26. **Success Factor Model: Relationships and Study Motivation**

college calculus courses. Multiple participants volunteered descriptions of their relationship with their high school calculus teacher. Some indicated that their relationship was not just academic but also personal – one student had attended the wedding of his teacher and another described his teacher as "involved and relatable, explaining "I actually went out to dinner with him after I graduated, like me and four friends." Several participants had taken previous courses with their AP Calculus teacher. One explained that "it was almost like we had become a family because everyone had had [this instructor] for so long." The impact of these relationships was significant because the students did not want to "let down" their teachers. Multiple participants increased their amount of studying simply because of their relationship with their teacher. Jeffrey and Frank went so far as to say they would have been content with a B in their AP courses, but they did not want their teachers to think they weren't trying, or that he or she was a bad teacher, so they put it the extra effort to get an A.

There was a glaring void of these kinds of stories from the college courses. Some participants volunteered that they did not believe their instructor knew them by name. They were not critical of these instructors, but indicated this lack of relationship was significant to them. Just as the relationship motivated the students in high school, the lack of relationship in the college course was demotivating. Haley explained, "I don't know him as well [as my high school teacher] so I'm not motivated to do as well because I know him. It's terrible to get bad grades, but not because I know him and I'll be embarrassed." When students had a relationship with their teacher, they were motivated to work harder because of how their teacher would perceive them; when the relationship was absent, so was the extra motivation.

Some participants' study motivation was also affected by relationships with their classmates. Overall the participants indicated they had many more relationships with other students in high school than in their college calculus class. In the AP classroom, many participants were in class with students whom they had been in math courses with for several years prior. This familiarity led to students being comfortable asking each other for help both inside and outside the classroom. Some formed study groups that met outside of school and ultimately increased their study time and many of them indicated that in-class interaction with other students was crucial to their success.

In Class. The final factor or source impacting students' calculus knowledge was the college classroom. Students described two distinct sources of their learning and knowledge acquisition in the classroom. The first was interaction. This included interaction with the instructor and other students, and time spent engaged in practice or discussion. Eight of the

14 participants indicated that the opportunity to be engaged with practice or discussion during class had, or would have had, a positive impact on their success. Ten participants described their AP classrooms as much more interactive than their college ones, and indicated this was a strength of their AP class over their college class. Many of these ten spoke much more favorably of their AP class in general than their college class and made either explicit or indirect links between the interactive design and their favorable view of the class. The four participants whose college classrooms were interactive spoke positively of the design of their college classrooms and (perhaps as a consequence) did not express a strong preference for their high school course.

The second source of knowledge acquisition was class instruction. Analysis of the interview data provided evidence that college class instruction impacted knowing-HOW, and for some students, knowing-WHY. Many participants volunteered that one of the most important things they could do to be successful was attend class regularly, or indicated regret over not doing so. No one suggested that class was a waste of time or that class instruction was not closely related to material being tested. Some students indicated that their primary form of studying, after working assigned problems, was reading class notes, which suggests that the students had found a connection between the material discussed in class and that on their exams. Albert explained that whenever he became stuck on a problem, he would always use his class examples as a resource. He said, "I haven't really had any problems that I wasn't able to get through otherwise," suggesting that the knowing-HOW required for at least his homework problems was fully covered by the examples his instructor worked during

class. Another student, Jeffrey, explained, "just looking over the lectures and just really paying attention has really been a strategy of mine, even if that means putting down my pencil and paying attention to what he's saying versus taking notes…" A couple of students described things about classroom instruction that led to knowing-WHY. Katelynn specifically stated that her college class focused more on knowing-WHY something worked, whereas her AP course had focused more on knowing-HOW.

Test Analyses Results

In this section results are presented from the test analyses. Problems on limits and continuity from six AP calculus and six college calculus tests were analyzed for four aspects – 1. Representation used in the problem, 2. Response format, 3. Function type, and 4. Reasoning type. The problems could be classified for each of the first three aspects by considering the test problems alone. In contrast, to classify a problem for reasoning type, it was compared to problems from class resources such as class notes, homework, and textbooks. Included below are results for both the AP and college classrooms and a discussion of the comparison between the two. 149 AP test problems and 58 college test problems were analyzed.

Representation used in problem

Each test problem was classified as either containing a symbolic, graphical, numerical, or verbal representation, or some combination of these. The large majority of limit and continuity problems on the tests asked about the limit or continuity of a particular function; therefore, it was the representation of the function itself that was coded. Many

problems that presented a function graphically or numerically gave instructions partially in

symbolic form rather than verbally (see Table 6 for examples), but these problems were

coded based on the function representation rather than the format of the instructions. This

decision was made because the solution method for limit and continuity problems are

typically determined by how the function is presented.

Table 6

Examples of Problem Representation

Symbolic	Let $f(x) = \frac{x^2-1}{x-1}$. Find $\lim_{x \to 1} f(x)$
Graphical	Find $\lim_{x \to 1} f(x)$
Numerical	Find $\lim_{x \to 1} f(x)$. <table><tr><td>x</td><td>0.9</td><td>0.99</td><td>0.999</td><td>1.001</td><td>1.01</td><td>1.1</td></tr><tr><td>$f(x)$</td><td>3.0514</td><td>3.0072</td><td>3.0003</td><td>2.9933</td><td>2.9837</td><td>2.9578</td></tr></table>
Verbal	Which of the following conditions must be true if $\lim_{x \to 1} f(x)$ does not exist? a) $\lim_{x \to 1^+} f(x)$ does not exist b) $f(1)$ does not exist c) $f(x)$ has a removable discontinuity at $x = 1$. d) $f(x)$ is discontinuous at $x = 1$ e) the limit of $f(x)$ as x approaches 1 from the left and right must be equal

If a problem did not ask about a specific function or did not present a function in either

symbolic, graphical, or numerical (or tabular) form, it was coded as having a verbal

representation. Problems coded as having a verbal representation were not required to be devoid of all mathematical symbols.

Only the problem as presented was coded; the required representation of the answer or justification was not considered. The representation needed for an acceptable answer was not considered because student work and instructor solutions were not provided. Also some tests' written instructions were not clear about expectations for the answer format. These expectations were likely assumed based on class discussions but could not be verified.

Table 7

Problems by Representation

	Symbolic	Graphical	Numerical	Verbal	Combination
AP	112(75.2%)	28(18.8%)	1(0.7%)	2(1.3%)	6(4.0%)
College	38(65.5%)	19(32.8%)	0	1(1.7%)	0
Total	150(72.4%)	47(22.7%)	1(0.5%)	3(1.4%)	6(2.9%)

The analyses showed that limit and continuity problems are most likely to present symbolically, and are almost always guaranteed to present either symbolically or graphically (see Table 7). Over 72% of all the 207 test problems were presented symbolically and another 23% were shown graphically. Problems with a verbal or numerical representation were very uncommon on both AP and college tests. Additionally, only 6 problems (all AP) presented both symbolically and graphically or symbolically and numerically. However, it is

worth noting that the difficulty of the problems which used a combination of representations was likely reduced for students because students could use either representation or both to solve the problem, yet the problem instructions were not more demanding than other problems that presented functions only one way.

No college test problems were presented numerically or verbally or used a combination of representations, but a noticeably greater percentage of college problems were presented graphically than on the AP tests and there were a greater percentage of symbolic problems on the AP tests. These differences were tested with chi-squared tests. The difference in problems using a graphical representation was found to be significant at the 0.05 level ($p = .031$) (see Table 8 below).

Table 8

Problems by Representation:
Graphical versus Other Representations

	Graphical	Other
AP	28(18.8%)	121(81.2%)
College	19(32.8%)	39(67.2%)

It is unknown whether this difference makes college tests more challenging for students, but it would suggest that college calculus students coming from an AP course may not anticipate so many test problems requiring them to interpret and use graphs on their tests. One reason for this significance might be that college calculus uses calculators much less in

their instruction and thus they give more graphical "pictorially" to allow students to show their understanding without calculators.

The chi-square test for the use of symbolic representations (see Table 9) did not give a significant result. ($p = .163$).

Table 9

Problems by Representation:
Symbolic versus Other Representations

	Symbolic	Other
AP	112(75.2%)	37(24.8%)
College	38(65.5%)	20(34.5%)

Response Format

Each problem was next coded as having one of the following response formats: open-ended, multiple-choice/open-ended, multiple-choice, short answer, or true/false. These are described below and examples are provided. The types are disjoint; that is, each problem was coded with only one response format. The response formats used were somewhat determined prior to analysis but were certainly refined after examining the test problems for similarities and differences in the required responses. For example, the true/false category was not originally included but was added after reviewing a particular test that contained true/false questions.

The response format for each problem proved more difficult to determine than the representation, primarily because answer keys and graded student work were not available.

For example, for some tests it was clear that answers needed to contain full solutions to receive credit; in these cases, problems were coded as *open-ended*. On other tests, the same kind of problems might not contain specific directions about answer format, and therefore were coded as *short answer*. Low inter-rater reliability with a second coder led to a refinement of the classification system used for response format and especially to more clearly distinguishing between short answer and open-ended items. This refinement was that items which were not multiple-choice or true/false were re-coded as short-answer unless directions specifically asked that all work be shown or an explanation be provided. Additionally, in some cases, questions which provided blanks for students to write were coded as short answer. In other cases, if students were required to write on the test paper and there was clearly not space available to show work, the question was coded short answer.

Problems were classified as having a multiple choice response format if answer choices other than true and false were provided. Multiple choice was originally included as one of the response format possibilities, but multiple choice/open-ended was added after reviewing particular problems that provided answer choices but required work or explanation, since the goal was to code each problem as having only one response format. Answer choices typically took the form of providing 4 or 5 distinct answers; however, there were a few problems which took a choose-all-that-apply approach. For example, one problem provided 3 functions and asked which of the three were continuous at a particular x value; each answer choice was some combination of the three functions.

For a problem to be classified as open-ended, it either required the problem solver to show their work or to provide a written explanation. Correct solutions to open-ended problems could vary from student to student based on the particular steps taken in the solution process or in the ways their work was explained. In contrast, the solutions to short-answer problems could be graded as either right or wrong and students with correct responses would all have provided the same answer. An example is provided in the Table 10 that distinguishes between these two types.

Table 10

Examples of Response Formats

Open-ended	Determine the value of k that will make $f(x)$ continuous at $x=4$. Show your work. $$f(x) = \begin{cases} 1, & x = 4 \\ -3x + k, & x \neq 4 \end{cases}$$
Multiple Choice/Open-ended	Determine the value of k that will make $f(x)$ continuous at $x=4$. Explain your answer using the definition of continuity. $$f(x) = \begin{cases} 1, & x = 4 \\ -3x + k, & x \neq 4 \end{cases}$$ a) 1 b) 3 c) 4 d)11 e) 13
Multiple Choice	Determine the value of k that will make $f(x)$ continuous at $x=4$. $$f(x) = \begin{cases} 1, & x = 4 \\ -3x + k, & x \neq 4 \end{cases}$$ a) 1 b) 3 c) 4 d)11 e) 13
Short Answer	Determine the value of k that will make $f(x)$ continuous at $x=4$. $$f(x) = \begin{cases} 1, & x = 4 \\ -3x + k, & x \neq 4 \end{cases}$$
True/False	Answer true or false. There are multiple values of k that will make $f(x)$ continuous at $x=4$. $$f(x) = \begin{cases} 1, & x = 4 \\ -3x + k, & x \neq 4 \end{cases}$$

Problems requiring work or explanation were much less common than those asking only for an answer, both on AP and college tests. AP problems were predominantly (52%) short answer. There were no true/false questions or multiple-choice questions that required work or justification. College problems were more evenly distributed between formats. Approximately 29% of the items were open-ended, while 26% were multiple-choice and 38% were short answer. There were an additional 2 multiple choice questions requiring justification and another 2 true/false questions.

Table 11

Problems by Response Format

	Open-ended	MC/Open-ended	Multiple Choice	Short Answer	True/False
AP	23(15.4%)	0	48(32.2%)	78(52.3%)	0
College	17(29.3%)	2(3.4%)	15(25.9%)	22(37.9%)	2(3.4%)
Total	40(19.3%)	2(1.0%)	63(30.4)	100(48.3%)	2(1.0%)

Combining some of these categories, it becomes more obvious that college problems are more likely than AP problems to require showing work or providing explanation (see Table 12). Only 15.4% of AP problems fit this description, while 32.7% of college problems did. A chi-squared test revealed this difference to be significant at the .01 level ($p = .005$). This means that the observed data is very unlikely if there is really no difference between the proportion of AP and college tests that require open-ended responses. The result suggests that students in AP calculus are not required as often to explain their answers or show their

work as they are in college. This could be an important challenge for students navigating the

transition from AP to college calculus. If college instructors are particular about the types of

justification students provide and notation they use and students are not accustomed to

providing this type of information in AP calculus, they may not know what instructors are

looking for, or they may have developed methods of getting correct answers that do not work

when asked to provide full solutions.

Table 12

Problems by Response Format:
Open-ended versus Other Formats

	Open-ended	Other
AP	23(15.4%)	126(84.6%)
College	19(32.7%)	39(67.3%)

Function Type

Tables 13 and 14 show how the problems from AP and college tests were categorized

based on the function type used in the problem. Problems were coded as only one of the

disjoint types shown in Table 14. For example, a problem using the function $f(x) = \sin \sqrt{x}$

was coded as square root and trigonometric (S/T) rather than separately as S and T. Four AP

and 3 college test problems did not ask a question about a particular function; these were not

coded.

Test problems on both AP and college tests were dominated by rational functions (or

ratios of functions) and piecewise functions. This is not surprising, given that the test

problems coded were testing for limits and continuity. Rational functions, for example,

present certain continuity issues that are of interest, and therefore may be expected to be

tested more often than say, exponential functions, which are continuous on the real number

line. Similarly, if problems testing derivative rules had been analyzed for function type, it is

expected that there would be more problems using trigonometric functions and fewer

piecewise functions. These results, therefore, are more useful for the comparison between

AP and college calculus rather than overall totals.

Table 13

Function Type Codes

P	Polynomial
R	Rational or Ratio[a]
PW	Piecewise or Split-Domain
T	Trigonometry
L	Logarithmic or Exponential
A	Absolute Value
S	Square Root
O	Other*

Table 14

Problems by Function Type

	P	R	PW/P	T	L	S	P/T	L/T	A/R	T/R	S/R	L/R	R/PW	L/PW	S/PW	O
AP	12 (8.3)	50 (34.4)	36 (24.8)	1 (0.7)	1 (0.7)	3 (2.1)	2 (1.4)	2 (1.4)	5 (3.4)	11 (7.6)	11 (7.6)	1 (0.7)	5 (3.4)	3 (2.1)	0 (0.0)	2 (1.4)
College	1 (1.8)	11 (20.0)	28 (50.9)	1 (1.8)	1 (1.8)	0 (0)	0 (0)	0 (0)	3 (5.5)	4 (7.3)	3 (5.5)	0 (0)	0 (0)	0 (0)	3 (5.5)	0 (0)
Total	13 (6.5)	61 (30.5)	64 (32.0)	2 (1.0)	2 (1.0)	3 (1.5)	2 (1.0)	2 (1.0)	8 (4.0)	15 (7.5)	14 (7.0)	1 (0.5)	5 (2.5)	3 (1.5)	3 (1.5)	2 (1.0)

a. Functions were classified as "ratio" if they consisted of one function (not necessarily polynomials) divided by another

The categories were condensed to more easily view the differences. (See Table 15.)

Table 15

Problems by Function Type:
Comparison of Rational/Ratio and Piecewise Functions

	R (not PW)	PW (not R)	Other
AP	77 (53.1)	39 (26.9)	29 (20.0)
College	21 (38.2)	31 (56.4)	3 (5.5)

Fifty-three percent of AP questions were found to use rational functions (or a ratio of two functions). In contrast, only 38% of college questions used these functions. This difference is more than accounted for by piecewise functions (PW). Fifty-six percent of the college questions used piecewise functions while only 27% of the AP problem functions were defined piecewise. Looking at Table 14 and the individual codes, the use of trigonometric, square root, absolute value, and logarithmic and exponential functions do not appear different from AP to college calculus. All four of these function types were used infrequently. However, in the condensed Table 15, we see that a much greater percentage of AP problems fall under the "other" category. Many of these problems contained trigonometric, square root, absolute value, and logarithmic and exponential functions. A chi-squared test revealed that the differences shown in Table 15 were significant at the .01 level ($p = .0002$). This means that if there were no difference on AP and college tests in the three categories of function types in Table 12, it is highly unlikely that these results would have been observed.

Interpreting what this means for students is somewhat challenging. Problems with one particular function type are not necessarily more difficult for students, so we must consider how the familiarity with certain types of functions in AP calculus impacts students in future courses. In calculus, methods are often very specific to the function type. With limits and continuity specifically, a student can have a firm conceptual grasp and still not be able to solve a particular problem because of the function type. If a student works many problems that test a particular concept and the majority of these problems use a certain type of function, it is likely the student may associate the concept with the procedure specific to that function. Disassociating the topic or concept from the procedure may likely prove challenging in a later course.

Reasoning Type

In order to develop reasoning types, individual test problems were first compared to problems from corresponding class materials such as class notes, homework assignments, and textbooks. The most similar problem or problems from the class materials were then used to determine whether imitative reasoning could be used to solve the test problem. First it was determined whether memorized reasoning could be used. Problems coded as memorized reasoning asked students to recall the definition of continuity, but they was uncommon; only 3% of the AP and 3% of the college test problems could be solved using memorized reasoning.

If the reasoning type was not found to be memorized reasoning, the problem was then classified as requiring either algorithmic reasoning or creative reasoning. Determining

Table 16

Creative Reasoning Codes

	Description	Example
Code 1	Student may not know-TO choose a procedure or may incorrectly apply a procedure because of function family. Procedure is familiar, but student has never had to use this technique with this function family. May need to know something about the function to solve.	Test Problem: Find $\lim_{x\to 5} \frac{x-5}{x^2+5}$. Need to know to use direction substitution. Similar Problem: Find $\lim_{x\to 3} 3x^2 - 2x + 7$.
Code 2	New approach or reasoning is required because of the function family. Have been asked the same question before, but the process for solving is new because of the function family. May need to know something about the function to solve.	Test Problem: Determine whether the function is continuous at x = 1: $f(x) = \ln(x-1)$ Must know that $\lim_{x\to 1} \ln(x-1)$ does not exist or that $f(x) = \ln(x-1)$ is undefined at x = 1. Similar Problem: Determine whether the function is continuous at x=3: $f(x) = \begin{cases} 5, & x > 3 \\ x-2, & x < 3 \end{cases}$
Code 3	New procedure reasoning required for reasons other than function type.	Test Problem: If $f(x) = 2x^2 + 1$, find $\lim_{x\to 0} \frac{f(x)-f(0)}{x^2}$ Previous Problems: All previous problems asked for the limit of a particular function, rather than the limit of an expression
Code 4	Using parameter instead of only numbers (but could be solved with AR if student plugs in a number)	Test Problem: Find $\lim_{x\to a} \frac{x^2-a^2}{x^4-a^4}$. Multiple choice problem. Must know how to factor $x^4 - a^4$ unless plugging in a specific number and checking against provided answers. Similar Problem: Find $\lim_{x\to 3} \frac{x^2-9}{x-3}$
Code 5	Student may not know-TO use specific technique from pre-requisite material	Test Problem: Find $\lim_{x\to 3} \frac{3-x}{x-3}$ Need to factor -1 from either numerator or denominator. Similar Problem: Find $\lim_{x\to 3} \frac{x^2-9}{x-3}$
Code 6	Must combine procedures from 2 or more previously-seen problems	Test Problem: Determine for which values of c the function is continuous at x=9: $$f(x) = \begin{cases} \frac{3-\sqrt{x}}{9-x}, & x \neq 9 \\ c^2, & x = 9 \end{cases}$$ To determine the limit at x = 9, must multiply by the conjugate, $3 + \sqrt{x}$. Must set two pieces of the function equal to solve for c. Similar Problems: 1) Determine for which values of c the function is continuous at x=1: $$f(x) = \begin{cases} c+6, & x \neq 1 \\ c^2, & x = 1 \end{cases}$$ 2) Find $\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x}$
Code 7	Can use AR to solve but requires the student to provide explanation or justification for the first time	Test Problem: Given that $f(x) = \begin{cases} 3x^2 + 5, & x < -2 \\ 17, & x = -2 \\ 10 - 3x^2, & x > -2 \end{cases}$ determine whether $f(x)$ is continuous at $x = -2$ and explain why or why not. Similar Problems: Given that $f(x) = \begin{cases} 3x^2 + 5, & x < -2 \\ 17, & x = -2 \\ 10 - 3x^2, & x > -2 \end{cases}$ Determine whether $f(x)$ is continuous at $x = -2$.

whether a problem could be solved using algorithmic reasoning proved to be difficult and

therefore codes were created to specify ways that a problem might differ from previous

problems and therefore require some degree of creative reasoning. Then, if a problem did

not meet the conditions for one of the seven codes developed to further classify creative

reasoning, it was coded as algorithmic reasoning. See Table 16 for a description of the 7

creative reasoning codes.

A total of 23 problems were found to require some degree of creative reasoning.

Table 17 shows how many of the 23 problems were coded as each of the 7 types. The

distribution amongst the 7 codes was fairly even, with the exception of Code 6. The most

common type of problem that was not solvable using algorithmic reasoning on the AP tests

required the student to combine his knowledge of procedures from two different previous

problems.

Table 17

Number of Problems by CMR Code

	AP	College
Code 1	2	2
Code 2	2	0
Code 3	2	0
Code 4	2	1
Code 5	1	1
Code 6	6	1
Code 7	1	2

Table 18

Problems by Reasoning Type

	IMITATIVE		CREATIVE
	Memorized	Algorithmic	
AP	5(3.3%)	128(85.9%)	16(10.7%)
College	2(3.4%)	49(84.5%)	7(12.1%)

Results (see Table 18) showed that most problems did not fit the description of any of

the 7 codes and could therefore be solved using algorithmic reasoning, and there was very

little difference between AP and college tests. Only 10.7% of the AP test problems and

12.1% of the college problems required some type of creative reasoning.

Due to a small number of memorized reasoning problems, the memorized and

algorithmic reasoning problems were combined under the title Imitative Reasoning (see

Table 19).

Table 19

Problems by Reasoning Type:
Imitative versus Creative Reasoning

	IMITATIVE	CREATIVE
	---	---
AP	133(89.3%)	16(10.7%)
College	51(87.9%)	7(12.1%)

A chi-squared test revealed that the difference in proportions of imitative versus creative

reasoning problems on AP and college calculus tests was not statistically significant ($p =$

.784). There is no evidence to suggest that either AP or college calculus tests require a

higher level of reasoning than the other. This result is important because this suggests that a

higher level of reasoning is likely not a challenge for students in their transition from AP to

college calculus. This result will be discussed in comparison to interview results in Chapter

5.

Summary of Test Analyses

Most limit and continuity problems in both AP and college calculus contain questions

about particular functions and these functions are overwhelmingly presented either

symbolically or graphically. The results show that college calculus tests contain a greater

percentage of problems whose functions are represented graphically, and this difference is

statistically significant.

Open-ended problems, or those requiring explanation or showing work, are less

common on calculus tests than problems that require answers only, both in AP and college

calculus. There is a difference between AP and college, however. While 32.7% of college

test problems required either an explanation or showing work, only 15.4% of AP problems

did this. This result is also statistically significant and important because students may not be

developing the reasoning and communication skills necessary to succeed in college calculus

while in the AP course.

AP Calculus tests were more likely to contain questions about rational functions,

whereas college problems were more likely to ask about piecewise functions. This result was

statistically significant. Collectively, the use of logarithmic or exponential functions,

trigonometric functions, and functions containing either absolute value or square roots,

appeared more often in AP classrooms. The use of these types of functions was relatively low both places; however, this is likely specific to the topics of limits and continuity. This data does not explain which situation (AP or college calculus) may be more challenging for students, but it does suggest students are being exposed to problems with specific functions more often in AP calculus and this may impact their success in the college course.

Finally, to code for algorithmic versus creative reasoning, codes were developed to identify specific challenges of certain problems in how they differed from other problems in course materials. Analysis of the problem reasoning type showed that a small portion of calculus test problems require some degree of creative reasoning. Nearly 90% of test problems may be solved with imitative reasoning. The large majority of these utilize algorithmic, rather than memorized reasoning, meaning that the problem solver must construct, rather than recall, a solution. This suggests that students who take tests knowing how to solve problems from their course resources should be able to solve approximately 90% of their test problems without using any creative mathematical reasoning. Moreover, this result did not differ between AP and college calculus. That is, college test problems did not differ more substantially from problems in the resources than did AP test problems, and vice versa. These results suggest that in regard to the type of reasoning required, college calculus is not more challenging than AP calculus.

CHAPTER 5: Discussion

The purpose of this study was to determine the impact of the AP Calculus experience on student success in college calculus. The results discussed in Chapter 4 answered the study's two research questions:

1. What factors affect student success in calculus and how do these factors differ in AP and college calculus?

2. What challenges do students face on calculus tests and how do these challenges differ in AP and college calculus?

In this chapter, I address the overall goal by explaining the connections between the results from the student interviews and the test analyses, discussing agreements and disparities between the data, and providing potential explanations for the disparities. I also connect this study to the literature, showing how the results of this study support, refine, or challenge the findings of prior research, and offer implications for practice.

Discussion of Results

CMR is Not a Requirement for Passing College Calculus

Two themes emerged from the first part of the student interviews that suggest that college tests might require more creative mathematical reasoning than AP Calculus, and therefore explain why students could be successful in their AP Calculus class but not their college course. First, some students explained that their college calculus material was more abstract, conceptual, or formal versus more concrete, procedural, or informal in high school. Secondly, students noted a closer similarity of test questions to previously seen material in

their AP course, versus college where their teachers threw "curveballs" for which they were unable to prepare. However, these claims were not supported by the second part of the interviews or the test analyses.

In the second part of the interviews, multiple participants struggled with knowing-TO with CMR. However, these issues were typically dealing with pre-requisite material rather than calculus and they usually came towards the end of the problem and therefore did not prevent the student from being able to make at least some progress. When comparing the student profiles to develop the Success Factor Model for Calculus, I found that while knowing-TO with CMR could limit some students' success, it did not prevent students from passing calculus. In the test analyses, differences in reasoning type (imitative versus creative) between AP and college tests did not prove to be statistically significant. Both AP and college tests were dominated by problems that very closely resembled those students had seen previously in their class notes, homework, and textbooks. Collectively, the interview and test data suggest that although CMR is a factor in being able to solve certain calculus problems, CMR is not a requirement for passing college tests. This is consistent with prior research by Berqvist (2007) that found that 15 of 16 college tests at four Swedish universities could be passed without CMR.

This difference in student perception and test results may stem from students' study habits and lack of exposure to certain practice problems. For those participants who were not passing their college calculus course, the second part of their interviews indicated that these students had gone into their tests not knowing how to solve problems from their resources

that were strikingly similar to the missed test problems and that resolving this problem

(increasing their knowing-HOW and TO with AR) would have led to a passing grade.

Moreover, these students demonstrated a strong capability for learning how to solve these

problems. In many cases, they were able to learn how to solve the similar problems quickly

and without assistance.

It should be emphasized that the data suggests only that students can *pass* calculus

without CMR. Several students had problems they could not solve because the problems

required local CMR. The test analyses confirm this – while only a small percentage of

problems required CMR, there were enough in both AP and college calculus to prevent a

student from making an A if he could not solve any of the problems requiring CMR. It is

unclear whether this design aspect of the tests is a result of a conscious decision by

instructors. College instructors may expect students to use CMR in calculus or they may

simply not recognize how the problems on their tests differ from previously seen examples.

The question remains – why did some students perceive a need for CMR to be

successful in their college calculus class? First observe that this study considered only test

questions themselves and not graded students' work on tests. It is possible that students

experienced difficulties related to the rigor involved with answer format or required

justification on tests that was not captured by this study. For example, during his interview,

one student described losing points on his college test because he wrote $\lim_{x \to 3} \frac{x^2-9}{x-3} = x + 3$

instead of $\lim_{x \to 3} \frac{x^2-9}{x-3} = \lim_{x \to 3} x + 3$. He explained that this would not have mattered on

his high school tests, as long as his final answer was correct. This difficulty is supported by

the test analyses. The test analyses showed that college tests were more likely than AP tests to contain open-ended problems that require showing work or providing explanations.

It should also be noted that test problems may not completely reflect what an instructor emphasizes in class, which obviously impacts students' perceptions of requirements. While class notes were provided by many instructors for this study, they were not reviewed except to locate example problems. Two instructors may both test students on using the fact $\lim_{x \to 0} \frac{\sin x}{x} = 0$, but one may spend class time proving this fact while the other shows two examples of how it is used. While a more abstract or formal focus in class does not necessarily imply a more challenging course, it certainly might impact students' perceptions of the importance of certain aspects of the course in comparison to their previous experience.

Further complicating this issue is that the number of available practice problems and test questions differed substantially in AP versus college classrooms. The AP classrooms had a greater number of problems on their tests and a greater number of practice problems. Of the 207 test problems that were analyzed, 149 were from six AP tests and only 58 were from six college tests. The six college tests examined for this study had a range of 6-14 problems on limits and continuity. In contrast, the six AP tests had anywhere from 13 to 40 problems. This is not surprising, considering that most college Calculus I courses have anywhere from one third to one half of the contact hours of the AP course. While this does not guarantee that any given problem on a college test is more likely to require CMR, it does suggest that students may have only seen a similar problem to this problem once before, in

only one of his or her resources. In contrast, in high school, students were much more likely

to have seen that type of problem repeatedly in practice.   It could be that students assume

from their high school experience that they will only be tested on types of problems which

they have been exposed to multiple times. However, college instructors may hold students

accountable for any type of problem that is available in the course resources and may not

state this explicitly. This could account for some students' surprise at some questions which

were solvable by AR, and subsequently for the discrepancy in the data from the interviews

and test analyses.

Another aspect of the study design must be considered in interpreting the results. In

the second part of the interviews, students were given assistance in finding the most similar

problems from the course materials. So while the study showed whether the students were

subsequently able to solve test problems after learning to solve these similar problems, it did

not determine whether they were able to make the appropriate association between problems

in an actual testing situation. This challenge would be amplified in a testing situation

because time would lapse between practicing the similar problem and seeing the problem on

the test, and students would likely solve other practice problems requiring other procedures

in between practicing the similar problem and taking the test.

Pre-requisite Knowledge: Must Know-THAT and Know-TO

This student interviews indicated that even if a student is able to identify the

appropriate calculus procedure for a particular problem, there is still a good chance she may

not be able to completely solve the problem, and a small chance she may not be able to begin

the problem, because of not knowing-TO perform certain procedures learned in previous

math courses. For example, she may know that a problem requires her to take a derivative

and find where the derivative is zero and undefined, but still not be able to do so because of

not knowing which procedures from her previous math courses to employ to find where the

derivative is zero and undefined. This is not a deficiency in her calculus knowledge, but a

lack of knowing-TO use certain procedures from pre-requisite knowledge. Often these

procedures are reviewed in class lectures or at least in individual practice, but students still

encounter some of these issues on their calculus tests for the first time, requiring them to use

some degree of CMR. The data from this study shows that in many of the cases where

calculus problems require CMR, the part of the problem requiring CMR has to do with

previously learned mathematical knowledge rather than calculus material. This is significant

because by the time students take calculus, they have 10-13 years of previous mathematical

experiences from which to pull information. While this may be an advantage, it is also a

challenge; the more information available, the more difficult the task of sorting through it to

identify what is needed.

   This was not confirmed by the test analyses, as only 11% and 12% of the problems on

the AP and college tests, respectively, required some type of CMR. While the codes

developed to identify CMR included the possibility of not knowing-TO use procedures from

pre-requisite material, this was not a commonly found requirement on either the AP or

college tests. This seeming contradiction between the two phases of the study can likely be

explained by the difference in topics tested on the tests for the test analyses and those on the

participants' tests used for the interviews. As stated above, while many of the interview participants did run into problems *finishing* their test problems because of knowing-TO with CMR, none of them *failed* their test because of this issue.

In addition to knowing-TO use procedures from previous math courses, students must bring with them certain memorized information to be successful in calculus. Knowing-THAT did not present as a major barrier to success for the interview participants, but for those that did lose points on their college tests or mention struggling with knowing-THAT, their issues tended to be more with pre-requisite knowledge and specifically trigonometry, rather than current material. These difficulties may be influenced by experiences where students were successful in their AP course without knowing the same facts that they were then held accountable for in their college course. This may lend support to arguments suggesting that students should not be accelerated through college level courses in high school but rather given additional time in high school to gain a stronger foundation in topics they will need for college level courses (Bressoud, 2012).

Knowing-WHY is not Essential for Success in Calculus

The interview results showed that knowing-WHY could be of assistance to some students in knowing-TO using CMR or knowing-THAT but was not a direct factor of success. Some students showed a strong desire for reasoning for certain procedures and expressed frustration when it was lacking. However, those same students displayed an ability to successfully solve test problems without first gaining this understanding. Other students did not express this preference, but again were able to be successful without

knowing-WHY. This finding supports Mason and Spence (1999) who make a point of distinguishing knowing-TO from knowing-WHY, explaining that one is not necessarily dependent upon the other.

Knowing-WHY is not essential for success on calculus tests, either in AP or college. A majority of the problems on both AP and college tests did not require explanation or showing work, although college tests were more likely to do so. It is unclear whether this was the intent of instructors when designing the tests. It is possible that both AP and college instructors are unaware of how successful students can be without knowing-WHY. That is, when designing a test, the instructor may not realize that the majority of problems may be solved using procedures that have no conceptual basis in the minds of their students.

The Role of Self-Efficacy

One factor of success often discussed in the literature that was considered when analyzing the data but not included as a factor in the Calculus Success Factor Model is self-efficacy. Self-efficacy has been defined as "one's perceived capabilities to learn or perform behaviors at designated levels" (Schunk & Pajares, 2005, p. 85) and it has strong ties to motivation and thereby success. Much of the literature suggests that students with greater self-efficacy tend to outperform students with lower self-efficacy (Chemers, Hu, & Garcia, 2001; Mayer, 1998; Pajares & Graham, 1999). This is often explained by a link with persistence; students who believe they are capable of completing a task stick with it despite early failures, trying different approaches, rather than giving up, thereby increasing their chances of success. The claim here is that self-efficacy increases motivation. There was

some evidence to support this theory from the interview data. Some participants were not satisfied with their current grades but were confident they could still meet their goals for the course and had already made significant changes to their study approach in an effort to meet their goals. Other students with low self-efficacy had all but given up. These were the students who had decided they were not as mathematically gifted as some of their classmates and could not overcome their earlier failures in the course.

However, there was also evidence to suggest the opposite – that is, that high self-efficacy leads to decreased success. This situation may be explained by a link with metacognition and perceived need for studying. That is, if a student has high self-efficacy and low metacognition, he may believe that he does not need to study much to accomplish a task and thereby put his chances for success at risk. This was very evident with a couple of students, one who was succeeding and another who was not. Both were demotivated to study by their confidence in their ability. One's student's confidence was appropriately place and the other's was not, but the result of this confidence on motivation (and success) was negative for both.

The Role of Study Attitude

Credé and Kuncel (2008) define study attitude as "a student's positive attitude toward the specific act of studying and the student's acceptance and approval of the broader goals of a college education" (p. 427). Study attitude is a factor included on the Academic Performance Determinants Model provided by Credé and Kuncel and was considered as a potential factor for the Success Factor Model for Calculus. Credé & Kuncel's data and other

studies suggest that a more positive study attitude positively affects motivation, which increases success (Maree, Pretorius, & Eiselen, 2003). The interview data from this study, however, did not suggest that study attitude has a clear impact on student success. A large majority of the interview participants had a very positive study attitude. Overwhelmingly, students accepted responsibility for their own learning and expressed belief that college instructors should not be responsible for providing a lot of structure or accountability. Those who were not happy with their performance did not place blame on their former or current instructors and most indicated they had the power to control their level of success in the class. However, this belief system did not noticeably impact their study behavior and could not be shown to impact student success. Though students believed it was their responsibility to study appropriately and seek help when needed, many indicated a pattern of not doing either and did not identify any forces beyond their control that had prevented them from doing these things.

Implications for AP Calculus

This study provides some evidence that some AP Calculus courses are not being administered according to College Board guidelines. Courses may deviate from either content or administrative guidelines. For example, some students discussed material that was omitted from their AP course, such as related rates, and others talked about using a calculator indiscriminately.   This is not surprising; rather, this concern has been voiced for many years, so much so that recently the College Board has begun requiring an approval process, termed a course audit, before schools can use the Advanced Placement title with a course. However,

this process is considered by some to be easy to complete while still circumventing the intended content and/or methods of the program. This is particularly concerning in light of finding how strongly students' perceptions of their calculus knowledge are impacted by their success in AP Calculus. If, for example, a student succeeds in an AP Calculus course that does not require a thorough understanding of trigonometry and then repeats calculus in college, he may not only be unprepared for, but also surprised by the examples and problems posed by questions in his college course involving trigonometric functions. In deciding how to concentrate their study efforts for tests in college, some participants gave more weight to what material had been tested in their AP class than they did to what material was presented in their college class and homework assignments. Therefore, ignoring topics or not testing them fully may have even more negative and long-lasting consequences than AP teachers might anticipate.

One of the differences noted by the participants between AP and college calculus was that AP was test-driven. They believed they were being taught how to solve certain types of problems because those problems could be expected on the AP exam, and that doing this meant sacrificing time for gaining understanding of calculus concepts. There is some evidence from the test analyses to support this. While this wasn't a specific area of analysis for this study, some of the tests from the AP classrooms contained problems that looked different from problems in the course textbook yet very similar to problems from released AP exams. While it may not be the intent of the College Board, it is commonly known in the mathematical community that some AP teachers use old tests to guide their instruction and

assessments throughout the course and also likely that students in these courses may score higher on the AP exam because of the similarity of questions from one year to the next. Some study participants who were passing college calculus but had not passed the AP test indicated that these practice problems had been available to them and they regretted not practicing more because they believed it would have led to them passing the AP test and not having to repeat Calculus I. However, participants who were struggling in their college course believed this approach by their AP Calculus teachers had hurt their chances of success in college. That is – they believed learning to do the specific kinds of problems in the AP course had not given them the advantage in college that having spent more time reviewing concepts would have provided. It should be noted that the latter were students whose tests contained problems requiring CMR.

This speaks to the larger issue of whether AP Calculus is currently designed and implemented as a college level or college preparatory course. Klopfenstein and Thomas (2006) distinguish between the two, saying that while college-level courses include the same material as an equivalent college course, college preparatory courses develop the skills and habits needed to be successful in subsequent college courses. The success of students in later Calculus courses suggest that AP is at least a college level course; that is, students who do well demonstrate that they have the necessary knowledge base to succeed in later courses (Morgan & Klaric, 2007). However, the results of this study suggest that students who repeat the course cannot expect to be better prepared for how to approach new or unfamiliar material in college. They may have the advantage of already having a certain memorized

body of knowledge and specific techniques or procedures, but when having to learn new material, the students' approach is not better developed as a result of their experience in AP. In fact, it may be hindered. Significantly more class time in high school and well-intentioned efforts on the part of AP teachers (such as providing practice tests and review sheets) to reduce the demand of the course can lead to a reduced necessity for individual effort outside of class by students. When arriving in the college classroom, as a result of the increased content knowledge, confidence is sometimes inflated which leads to a decreased perception of the need to study. This lends support and explanation for documented recommendations from college instructors as far back the 1980's that students should not be exposed to material in high school if they were not to be challenged to learn it fully the first time (CUPM Panel, 1987).

Implications for Calculus Course Design

This study showed that the amount of structure provided by instructors varies across calculus classrooms, but typically is greater in AP Calculus than in college courses. Highly structured courses may include frequent assessments, graded homework assignments, and instructor-provided support and accountability. Participants reported that AP courses tend to provide more frequent assessments and more instructor-provided support and accountability; college courses tend to have more regularly graded homework assignments. Participants were consistent in their assertions that increased structure in a calculus course increases the chances of student success in the course. Only a few students spoke more positively about their college experience than their AP experience, and the majority of these students came

from college classrooms that were highly structured. These classrooms had frequent quizzes, multiple forms of homework assignments, and in-class work. The most significant aspect of structure from the students' perspective seemed to be accountability, which was very much tied to the existence of relationships between instructors and students. Not only did students who knew their instructors well report being encouraged and held accountable by their instructors, but in some cases they suggested their personal expectations of themselves actually increased because of their desire to please their instructors.

Course structure also affected student success by directing (or not directing) students' study habits. Overwhelmingly, participants demonstrated a lack of knowledge regarding their college calculus instructors' expectations of them. This may be partly attributed to a lack of reminders and accountability, but another part was due to the course design. With the exception of homework assignments which were a graded part of the course, students did not recognize any requirements of the course or instructor. There was little awareness of activities that instructors might have assumed they knew to do, such as reading before class or finding and completing extra practice problems. What is particularly interesting about this finding is that students could be very motivated yet still not be aware of, let alone using many study strategies that could have greatly impacted their success. Students were not simply choosing not to do these things; they were unaware that they *should* be doing them. This is consistent with Toms' (2013) study of first-semester college students which found that even motivated students often do not complete tasks that would improve their success if not explicitly told and reminded to do them by their instructor.

Not all students were unaware of other study strategies they could have been using in their college course. For example, several mentioned not having used office hours and some had access to practice tests, worksheets, and even assignments they had chosen not to complete. What students seemed to imply, though none stated this directly, is that there was a cost-benefit problem with these approaches; that is, they perceived too little return for the amount of time invested. One student suggested it was simply unrealistic to complete the worksheet of 20 related rates problems his instructor had provided. Consequently, he did not attempt any of the problems and missed a question on his test that closely resembled one of the problems from the worksheet. In contrast, many students described the opposite situation in their AP class. Students would avoid doing homework, since it was not graded or only graded for completion, but then would complete a review sheet or practice test that closely resembled the test. The review sheet or practice test took far less time to complete than the homework would have, and for the most part, using that resource only gave the students the results they desired. The challenge in transitioning to college calculus then becomes in either motivating oneself to commit a much greater amount of individual study time to the course and/or accurately identifying those study approaches that will give the greatest return for the time invested.

Unfortunately, students will not all be able to make this transition successfully without some type of assistance. Herein lies an incredible opportunity for both the AP and college calculus communities to partner in assisting their students. AP Calculus does not currently operate as a college-preparatory course, but it has the potential to do this and

thereby prepare students well regardless of which calculus course they first take in college. Similarly, while college instructors have not traditionally seen study skills as part of their curriculum, it is quite possible that integrating this type of instruction into their classrooms may have as great an impact on the performance of their students as providing quality instruction on calculus topics themselves. Doing these things will require an intimate knowledge of the specific misconceptions students have about how they should study, and specifically for AP instructors, a knowledge of how requirements differ in the college setting.

Completing homework is the one expectation of instructors that all participants were at least aware that they should be doing. Each of the fourteen interview participants had regular homework assignments in both their AP and college courses, and each of the 6 AP and 6 college courses whose materials were reviewed for the test analyses also had regular assignments. Across classrooms there are differences in the amount, format, and grading of assignments, but homework seems to be a very integrated, accepted way for students to learn calculus. However, the study results suggest that the way homework is administered and the changes in the way it is administered from high school to college may be significantly diminishing its potential for positive impact on student success.

Students reported having homework assignments in high school, but the majority of them claimed that homework was either not graded or graded only for completion. As a result, the students did not spend a lot of time trying to learn how to solve these problems. Yet, many of these same students were successful on the in-class tests. Some attributed this to review worksheets or practice tests that instructors distributed. Completing all of the

homework was seen as unnecessary because simply completing the review materials prepared the students sufficiently. What is not clear from this study is how much of an impact this practice had on students' habits in college calculus, but it is reasonable to suspect that students might have carried over an assumption that knowing how to solve problems from homework assignments was not a necessary condition for success.

This problem is conflated by the change of format of homework in college. Most participants either had online homework assignments or some combination of online and written assignments in their college course, and most or all of these assignments contributed to a small part of their course grade. Studies have shown that adding online homework assignments to college math and science courses can increase performance (Zerr, 2007, Richards-Babb, Drelick, Henry, & Robertson-Hanecker, 2011), but the effect on performance is not significantly different than that of traditional homework assignments (Allain & Williams, 2006, Bonham, Beichner, & Deardorff, 2001). With this result in mind and the fact that online systems decrease the grading load for instructors, it may be argued that these systems should be used frequently.

In this study, students gave mixed reviews of their views of their college homework systems, but had some particular concerns about their effects. (Some of these are specific to online systems and others speak more generally about homework assignments.) Generally participants reported having high homework averages; however, those grades were often achieved in ways that did not foster learning that would help students on tests and often led to students having an inflated sense of understanding of the material. Some mentioned that

on their homework assignments, they were able to use resources that were not necessarily

allowed on tests, such as calculators. For example, a student may not have needed to know

that $\sin\frac{\pi}{2} = 1$ while using his calculator to complete his homework; yet, on his test, a

calculator may not have been available. Another resource students used frequently was the

internet. Some students discussed Googling answers to specific problems and others using

educational websites such as Kahn Academy (www.kahnacademy.org) for explanation when

stuck on a problem. Calculators and websites can both serve as tools for learning and it can

be argued that homework is itself a tool for learning rather than assessment. Some students

explained that their online homework was set up to show them a similar example if they

could not automatically complete a problem, and most instructors had set up the assignments

so that students had multiple submissions to produce the correct answer. These factors

combined led to students not having to go back to their resources, such as written

explanations and worked examples from the class notes and textbook, to be able to complete

homework assignments. Furthermore, many online homework systems only asked students

for final answers, while some of their tests required work and justification. Students

suggested that these features led to them being able to get high scores without truly

understanding how to solve the problem or why the method was appropriate. Even some

students who were attempting to learn how to solve the problems ran into issues on tests

because they had been using homework as a learning tool but then used their homework

scores as a predictive measure of future success on tests which led to overconfidence and

subsequently insufficient study and practice.

Limitations

This section describes the limitations of the student interviews and the test analyses.

Interview Limitations

The sample and selection of interview participants was limited to specific regions of the country and specific schools and only fourteen students were interviewed. These students opted to participate in this study and therefore brought with them self-selection bias (Berk, 1983). They were from only six different universities in two neighboring states. Additionally, no participants attended community colleges, large private institutions or small public institutions. This aspect of the sample could have greatly impacted the data about class size and course structure.

The analysis of the first part of the student interviews relied, by design, on student reporting. The interviews were conducted with students at different points in the semester. Students' motivation and perspective certainly change throughout the course and student reporting of these traits was likely affected. Some of the information students presented was merged with results from the second part of the interviews or the test analyses. However, descriptions of things like classroom environments and instructor expectations could not be confirmed by the data from this study. It would be helpful to triangulate the data from the interviews with classroom observations and instructor interviews.

The second part of the interviews considered only one test for each student and because of time limitations, was further restricted to analyzing two missed test problems. While the problems were chosen intentionally by the number of missed points, it is possible

that the reasons for mistakes on other problems were different, and possible that the reasons for mistakes on other tests could be different as well. Moreover, the assessment of possible reasoning types was likely affected by the lack of time between when the student learned to work problems in their resources and when they reattempted their test problems, as well as the fact they had assistance in identifying these similar problems. In reality, students cannot practice all their problems immediately before taking tests and they will have to make associations between problems on their own when taking tests. However, it should be noted that with only one exception, students whose primary issue was something other than knowing-TO with CMR indicated that if they had learned to solve the similar problem prior to their test, they would not have missed as many points as they did.

Test Analyses Limitations

As with the interview participants, the size and diversity of the sample of instructors who shared their course materials was limited. The twelve tests came from six AP teachers and six college instructors. Both of these were a convenience sample, and in particular the AP teachers self-selected, since a larger population of teachers were invited to participate. AP and college instructors, like student participants, were from only two neighboring states. The college instructors were at the same institutions as the student participants, so the same limitations apply here.

In addition to issues with the sample, the design of the test analyses presents some challenges. First, the test analyses were restricted to the topics of limits and continuity. While these topics were chosen purposefully, it is quite possible that results about the type of

reasoning required to solve these particular problems may differ from problems on other topics in the course. This is important to note in interpreting comparisons between the interview data and the test analyses, as the interviews did not restrict the topics discussed. The restricted topics on the test analyses also likely affected the results regarding problem representation, response format, and function format. Second, even when only considering the topics of limits and continuity, the analysis assumed that students had access only to the materials that the instructor provided. It is possible that students had access to other materials that would change how test problems are coded. Finally, in coding test problems, only the problems themselves, and not the graded students' work, was analyzed. The difficulty of a problem can certainly be affected by the required format of the response and how strictly the problem is graded. This was not accounted for in this study.

CHAPTER 6: CONCLUSION

Summary

The purpose of the study was to determine the impact of the AP Calculus experience on student success in college calculus. This was achieved by addressing two research questions:

1. What factors affect student success in calculus and how do these factors differ in AP and college calculus?

2. What challenges do students face on calculus tests and how do these challenges differ in AP and college calculus?

The research questions were addressed through concurrent student interviews and test analyses. The first part of the interviews addressed the first research question. Seventeen themes regarding the factors of success in AP and college calculus emerged. The second part of the interviews illuminated the type of reasoning required on problems the participants had missed on a recent test. Results of this part of the interview were combined with the themes from the first part to produce a Success Factor Model for College Calculus. This model illustrates factors important to calculus success and the relationships between the factors. Test analyses explored the types of problems on AP and college calculus tests and compared them. The results of these analyses were then compared against the results of the Success Factor Model. This summary presents answers to the research questions and concludes with how the AP calculus experience influences student success in college calculus. It also provides direction for areas of future research based on the results of this study.

Answers to Research Question 1

Student success in calculus is impacted by what they experience both in and out of the classroom, and by factors that are specific to the student as well as those specific to the classroom or instructor. In the classroom, students overwhelmingly prefer instructors who engage them and allow them to interact with other students. Participants in this study found this beneficial in both the AP and college setting, and tended to speak more highly of whichever class was more interactive. This study suggests that while most AP calculus classrooms allow for class discussion and practice, these opportunities are less common in college. Most participants recognized the importance of attending class. They reported that their college instructors were open to questions, yet those in classrooms with more traditional lectures and larger class sizes found the transition difficult; they value the opportunity to participate, but rarely did in college classrooms that were not designed to facilitate interaction.

In addition to classroom interaction, increased structure and accountability tend to increase student success. Highly structured courses have more frequent assessments, guided opportunities for practice, and accessible instructional materials. Participants were much more aware of instructor expectations in highly structured courses. College courses were much less likely to provide structure and participants in these courses were unsure what they should be doing to succeed. Participants in these less structured courses often spent the vast majority of their study time outside class doing homework (when it was a graded part of the course) and equated doing homework with studying.

One of the most striking differences that students reported experiencing between their AP and college classrooms was regarding accountability provided by instructors. AP teachers "stayed on" the students, giving them frequent reminders and encouragement. Many students sought help from their AP teachers on a regular basis and saw their teacher as their primary source of help. In contrast, the study found that little accountability is provided by college calculus instructors. Students reported that they were not surprised by this and did not expect their college instructors to provide accountability, but it would have motivated them to work harder, as it had in high school.

Outside of class, students' willingness to study calculus and the ways in which they study vary greatly, but there are common factors which contribute to their motivation (or lack thereof). Relationships amongst students and between students and instructors contribute positively to student success and were much more common in AP calculus than in college. Participants reported being willing to work with other students they had known well for years in AP calculus and actually increasing their effort level beyond what it would take to achieve their own personal goals because of not wanting to disappoint their AP teachers.

Another difference between the AP and college calculus experience was the necessity of appropriate study habits and skills. Likely due to decreased structure and accountability of the course, students in most college courses were at a loss for how to study appropriately in order to achieve success, and this was even true for some participants who were being successful. The results of this study suggest that motivation alone does not produce success; students can be willing to put forth a lot of effort and still be unsuccessful.

The ways in which students use their resources and homework assignments pose significant challenges, particularly in college calculus. Participants reported all but ignoring their calculus textbooks, and even class notes were somewhat of a last resort. AP calculus provides an arena where students can get almost immediate assistance from teachers or other students. When this is not easily accessible in college, students tend to resort to online resources that are not specifically associated with their class or even their college. These online resources are often used to complete online homework assignments, resulting in high homework grades, an inflated sense of confidence about understanding, and subsequently low grades on tests.

Answers to Research Question 2

The results of this study show that three types of knowing or reasoning are required for success on calculus tests. Challenges can arise for students in any one or more of these areas. The first and most significant in both AP and college calculus is knowing-HOW and TO with AR. Knowing-HOW and TO with AR requires the problem solver to know how to apply a certain procedure and to recognize when to solve it because of its association with a previously worked problem. The large majority of problems on calculus tests reviewed in this study may be solved using knowing-TO with AR. Furthermore, the participants who were not passing their college course demonstrated during the interviews that had they increased their practice of certain problems prior to the test, they would have been able to solve the tests problems they missed and therefore could have been successful (though not

necessarily making a perfect score). A primary finding of this study is that an increase in this strategy alone is sufficient for succes.

The other two direct factors in success on calculus tests are knowing-THAT with MR and knowing-TO with CMR. These proved to be barriers for some students on some problems, but not significant enough to prevent success in calculus. That is, while these sometimes prevent students from attaining perfection on college tests, students can still pass their courses while struggling in these areas.

Memorization is a challenge for some students while not so much for others. Some participants discussed how memorization requirements decreased their motivation in the course. Problems relying primarily on memorization are sparse. Less than 5% of AP and college tests problems were found to be solvable using memorized reasoning. Problems requiring recall of certain facts were much more common, but this recall did not prove to be a primary barrier to success for any participant.

Test analyses showed that only 11 and 12% of problems in AP and college calculus, respectively, required knowing-TO with CMR. Despite the lack of problems requiring CMR, some students are still at a disadvantage because of their lack of familiarity with problems from their resources (which is related to study skills and habits, addressed below) and insufficient pre-requisite knowledge. In the second part of the student interviews, both knowing-THAT and knowing-TO with CMR difficulties presented most often as insufficient pre-requisite knowledge. This echoed what multiple students had said in the first part of the interviews; precalculus, and specifically trigonometry, is one of the most challenging aspects

of calculus. Some students claimed there was more trigonometry tested in college calculus than had been in the AP course; however, this was not confirmed by the test analyses. With the topics of limits and continuity, AP and college tests both utilized trigonometric functions sparsely. This result is likely very specific to the topics of limits and continuity and thus should not be generalized to the overall course. The analyses did show that regarding function type, college tests used piecewise functions more often than AP, and AP problems were more likely to use rational functions or functions with a ratio of two other functions.

The final type of knowing that emerged as a challenge for some students on AP and college calculus test was knowing-WHY. Knowing-WHY did not prove to be a direct factor of success on college calculus problems. Very few problems on the analyzed tests asked for explanations (most used either a short-answer or multiple answer format), and most that did require explanation were in courses where the same question had been asked of students prior to the test. For some students, knowing-WHY was never an issue; most students tests did not require explanations and they could use AR to be solve most of their problems. However, some students specifically expressed frustration with not knowing-WHY they were using certain procedures. This is particularly curious finding when comparing with the claim of multiple students that their college instructors required more depth, abstraction, and conceptual understanding of them than did their AP teacher. It seems while this is valued by some college instructors, it is not necessarily tested.

The Impact of AP Calculus on College Calculus Success

The results of this study suggest that AP Calculus positively affects students' performance in college calculus in two specific ways. It increases their knowing-THAT knowledge and increases their knowing-HOW and TO with algorithmic reasoning (AR). This is significant because a large majority of questions on the college tests could be solved using only these types of knowledge. Therefore, for students who do well in AP Calculus and whose college courses do not require a significant amount of creative mathematical reasoning (CMR), they may have an advantage over their classmates in not having to learn much of the material for the first time. For several participants, their AP experience allowed them to put very little effort into Calculus I in college and still meet their goals in the course.

In contrast, there is no evidence to suggest that AP Calculus increases students' knowing-WHY knowledge or knowing-TO using CMR. Some students indicated that knowing-WHY was more important in college calculus and they specifically contrasted this with their AP experience. They explained that it would be more helpful in their AP Calculus courses had focused on understanding concepts versus learning to work specific types of problems for the AP exam. These participants' perceptions were supported by the test analyses which found that college test problems were more likely to require students to show their work or provide explanations.

In addition to not increasing students' knowing-WHY, evidence suggests it does not increase knowing-TO with CMR. While this is certainly the most difficult types of knowing and reasoning to teach (Mason & Spence, 1999), the result is concerning, considering that so

much of students knowing-TO difficulties are with pre-requisite material. It seems students are making good grades in some AP Calculus classes without knowing, for example, how to solve certain algebraic equations. When these students then retake calculus in college, they may have memorized a list of derivative rules, but cannot consistently finish solving problems that require them to recognize when to pull specific techniques from their previous mathematical experience when needed.

It seems that if this particular issue arises for a student in college calculus, it would also have presented itself in the AP course. Yet most of the participants, and particularly those doing very poorly in their college calculus course, had made A's in AP calculus. While this study did not specifically compare the number of contact hours or the amount of practice problems in AP and college calculus, it appears from the collected course materials and participants' comments that AP calculus exposes students to many more examples than they see in the college course. AP calculus students are typically in class much more than college calculus students, and therefore teachers have more opportunities to introduce students to multiple types of problems and revisit the same types of problems. This study did not attempt to discover the *number* of similar problems available to each test problem, but only the existence of one. It is likely that where college test problems may have only one or two similar problems available in the resources, AP tests may have many. If this is the case, then students may expect from AP calculus that problems will only be tested if they have been seen in class or assignments multiple times, and may fail to practice or study certain problems in the college course that only show up once in a single resource.

Furthermore, the results show that the knowledge that students do gain in AP Calculus tends to decrease their study motivation. Therefore, if a student finds himself repeating Calculus I in college, he is at somewhat of a disadvantage. Although there are certain things he does not have to learn for the first time like his classmates, he may have an inflated sense of what he knows that may decrease his study efforts, and if he has not learned effective study and self-assessment techniques, he may not acquire the missing knowledge he will need on his college calculus tests. Students vary greatly in whether poor grades on college calculus tests can counteract this overconfidence. For some students, a good grade in AP calculus translates to their believing they truly understand the subject well and this takes priority. For other students, they become motivated to study when these poor grades are realized. Unfortunately for many, the overconfidence takes too long to overcome; students do not begin making appropriate changes until far too late into the course.

Students who are willing to make changes to their study approach do not necessarily know how to do so. This study showed that AP calculus does not appear to increase students' knowledge of how to study appropriately in a college calculus course and may instead inhibit the development of study habits and skills necessary for success in college mathematics courses. The evidence suggests AP calculus is a college-level but not college-preparatory course (Klopfenstein & Thomas, 2006). In many cases, the AP course created learning environments that (perhaps unintentionally) reduced the need for independent study and practice. The participants consistently reported that completing assigned homework was not required and as a result they did not complete it; yet, the majority received high grades in

the course. Some were able to do this because instructors provided practice tests or review

materials that eliminated the need to learn to solve all the homework problems.

Some students are extremely dependent on their AP teachers. The study results

indicate that AP calculus accustoms students to highly structured classroom environments

with a great deal of accountability provided by teachers. This structure and accountability,

such as frequent testing, reminders, and encouragement, increase student success in the short-

term. When students enter the college calculus classroom without these supports, even when

is anticipated, the students do not acclimate quickly or easily.

<div align="center">**Future Research**</div>

The Impact of Not Requiring CMR

This study suggests that university level Calculus I does not currently require students

to develop and use CMR to be successful. If the ultimate goal is to help more students

continue in calculus and pursue STEM degrees, the question arises as to whether students

will need CMR in later courses. Moreover, if it is found that CMR is necessary for success

in these courses, we need to know whether developing it in Calculus I might lead to greater

success in these courses. Future studies might compare the success rates of students from

Calculus I courses requiring more CMR with those requiring less in subsequent courses such

as Calculus II.

Instructor Perceptions of Their Tests

While CMR is not required for success in Calculus I, there are problems on Calculus I

tests that require CMR. These problems make up a small percentage of test problems and it

is not clear whether instructors intend to or believe they are requiring CMR of their students. Do they, as expert learners, overlook the nuances in their test problems that pose challenges for their students? If not, and they do intentionally test for the ability to use CMR, would they claim that their courses develop this ability? Research should be conducted to ascertain what AP and college calculus instructors believe about how much CMR students should be required to do on calculus tests and what type of problems actually require CMR. On a related note, we need to know more about whether instructors believe that knowing-WHY is important in Calculus I and whether their tests are assessing students' knowing-WHY. Future studies might explore how students' abilities to explain their problem solving strategies compare with instructors' predictions of those abilities based on test scores.

Formality

This study was limited in its assessment of calculus tests by not looking at graded students' work. Some participants suggested that college instructors required more detailed answers and more formal notation than their high school teachers. If this is the case, there is an extra level of rigor in the college course that is not accounted for in this study. Future research could examine graded work by AP and college instructors in comparison, or it may interview instructors about their beliefs about the importance of using formal or precise mathematical notation or language.

Repetition and Recentness of Problems

This study defined a test problem as not requiring CMR if the student had at least one problem in their resources that was similar enough to use FAR. Many participants

demonstrated that they had not known how to solve these similar problems going into their test and some seemed very surprised by their existence. The reason for this may be due to the lack of repetition of certain kinds of problems in college. Repetition of problems seems to be common in AP Calculus and is likely common in other high school mathematics courses where class time is more abundant. Future studies should explore how much more repetition exists in the high school classroom and how much this affects students expectations about what constitutes a fair test question on a college test.

Another issue not addressed by this study was how a time lapse between practice and the test can affect performance. Students in this study worked a similar problem and then immediately attempted their missed test question, which led to success on the test question in most cases. However, the results may be altered significantly if students were to wait hours or days before attempting the test question or if they worked multiple other types of problems in between learning the similar problem and attempting the test question. The relative impact of these factors should be studied to determine whether simply learning to work the appropriate problems prior to the test is sufficient for success as this study suggests.

Homework

Further work must be done to determine under what conditions homework can and should be used as a learning tool versus a predictive assessment tool for students, and how instructors can help students use it accordingly. We need to know more about how students are using specific features of online systems in particular since their use in college calculus has increased so rapidly in the last decade. Future studies should examine the claims of

participants that multiple submissions, worked examples, and short answer formats of these online systems lead to a sense of overconfidence that actually hurts student performance.

Resources

The majority of participants saw people (either instructors or other students) as their primary resource for help in calculus. In high school, students went first to their teachers and then to other students in their class. In college, peers were the number one source for information; however, peers did not necessarily mean classmates. College calculus students are seeking help from friends and roommates more often than other students in the class, and certainly more often than from instructors. Interestingly, for both the high school and college environment, written resources such as class notes and textbooks seem to be a last resort. Class notes might be referenced when completing homework assignments if a person is not available to or capable of explaining how to solve the problem, but many students said they rarely, if ever, used their textbook for anything other than practice problems. This was the case for both high school and college. This raises the question of whether students believe they will be held accountable for information provided in these resources, and if they do not, what experiences create and foster this belief. Moreover, is it problematic or are we at a place where students have real alternatives to these resources? At the very least, we must explore any disparities between instructor and student expectations about the use of written resources, and between instructors at the AP and college level.

Concluding Remarks

There is still much work to be done to determine how students' experiences in AP calculus affect their performance in college calculus. This study has shown how AP calculus is currently serving as college level, but not college preparatory course and provided potential reasons why students may struggle in the transition from AP to college calculus. This information is useful for both AP and college instructors, and future research has the potential to serve as a much needed bridge between high school and college mathematics instructors and to facilitate change in both environments that will positively impact student success in calculus. This success is crucial if we are to increase the persistence of students pursuing the STEM fields and ultimately meet the demands of the new workforce.

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APPENDICES

Appendix A: Sample Correspondence

Email #1 to College Students

Dear MA --- students,

I am a graduate student at NC State University. I am conducting a research study on the transition from AP to college calculus. During the week of October --, I will be on campus conducting student interviews for my research, and I am hoping to recruit students such as yourself to participate in these interviews. This is an opportunity to share your experience first-hand as a student and help inform the mathematics and mathematics education communities about the challenges students face as they transition from high school to college math.

To participate in this study, you must be a first-semester freshmen enrolled in MA --. You must have taken AP Calculus in high school and taken the AP Calculus test. I am looking for participants who have found calculus in college to be significantly more difficult than their high school calculus course.

If you fit these criteria and would be interested in sharing your experience with me, please email me at Megan_Ryals@ncsu.edu by October --. We will then schedule a time for a 60-90 minute interview at a time that is convenient for you. There are no other requirements to participate in the study besides the interview. You will be paid $20 in compensation for your time, upon completion of the interview.

Note that while the results of the study will be published as a dissertation, no names or other information that may be used to identify you will be used. The information you provide will remain confidential and your participation or lack thereof will not be shared with your instructor. Participation in this study will in no way affect your grade in this course.

Again, if you are interested in participating, or if you have questions, please email me at Megan_Ryals@ncsu.edu. Thank you for your time, and I look forward to hearing from you!

Email #2 to College Students

Dear Student,

Thank you for your interest in participating in this study. Please read the information below and reply.

This purpose of this study is…
Because I am interested in discovering the challenges students face in the transition from AP to college calculus, research participants must meet the following criteria
1. Took AP Calculus in high school
2. Took the AP Calculus test and made a 1 or 2 on the AB test or a 1 or 2 on the AB subscore of the BC test
3. Are currently enrolled in MA ---.
4. Have either a D or F on both calculus tests so far this semester

Please let me know if you fit all 4 criteria. If so, please also let me know which of the times below you are available for the interview.
Oct -- 11:00-12:30pm
Oct -- 1:00-2:30pm
Oct -- 2:30-4:00pm

You will need to bring the following materials with you to the interview:
-   Your most recent calculus test
-   Your class notes and homework for the material on the last test
-   Your textbook

Please let me know if you have any questions about the selection criteria or the requirements of participating. Thank you!

-Megan Ryals-

Sample Class Announcement (to be given by instructors)

"I am handing out a flier that has information about an upcoming research study. This study is about the challenges students face in their transition from high school to college calculus. The researcher is looking for students to participate in 60-90 minutes interviews on campus, the week of October --. This would be an opportunity to share your experience and contribute to the knowledge base about how we organize and teach calculus classes. If you took AP Calculus in high school, make sure to take one of these fliers and read it for more information. To participate, the first step is to email the researcher and her contact information is on the flier. I won't know who does or doesn't participate, and this does not affect your grade in this class in any way. Please email the researcher directly with any questions."

Letter to AP Calculus Teachers

Dear [Name of Teacher],

My name is Megan Ryals. I work at and am a graduate student in mathematics education at NC State University. I am conducting my dissertation research on students' transition from high school to college calculus. I have taught calculus courses at the college level and am particularly interested in the experience of students who take AP Calculus and then repeat the course in college.

I saw from [name of high school] website that you are currently teaching AP Calculus. I am writing in the hope that you will join me in my research. I am looking for several experienced instructors to participate in the document analysis portion of my study. I will be looking at problems and questions from tests, homework sets, class notes, and textbooks from both AP Calculus and college Calculus classrooms and analyzing the similarities and differences in these problems.

I expect that participation will require approximately 30 minutes of your time. The role of instructor participants will be to provide copies of their class notes, homework assignments, and tests that cover the topics of limits and continuity. (Because of time, I will limit the number of problems that I analyze to those dealing with these topics.) I can send a pre-addressed, stamped envelope to you for these materials. I am hoping to collect these by mid-November.

When I conducted a small pilot study last spring, one teacher, understandably, expressed concern about his test questions being released. As I explained to him, this information will not be accessed by anyone besides me and my doctoral committee. When I publish my dissertation findings, if any specific problems are published, the instructor and his or her school will not be named or identified.

I hope this is something you will consider. I believe that my research will add to the knowledge base in mathematics education and will be helpful to calculus instructors at both the high school and college levels as they seek to provide the best instruction possible for their students. Please let me know what questions I can answer for you and what concerns you may have. I can be reached at Megan_Ryals@ncsu.edu or (919) 602-8767. I will follow up with you soon via email and/or phone, but please feel free to contact me first.

Thank you for your time, and I look forward to hearing from you!

Sincerely,

Megan Ryals
Assistant Director, Undergraduate Tutorial Center
Ph.D. Candidate, Mathematics Education
North Carolina State University

cc: [Name of Principal]

Recruitment of Student Participants: Sample Flier

Share your experience as a calculus student!

I am looking for students who have taken AP Calculus and are now in MTH 111 to participate in a research study about the difficulty of transitioning from high school to college calculus.

If you:

- took the Calculus AP test

- are finding college calculus more difficult than high school calculus

- are available for a 60-90 minute interview the week of Nov 5
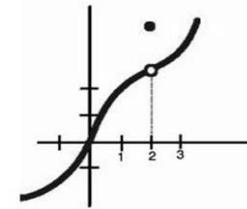
Please contact **Megan_Ryals@ncsu.edu** for more information!
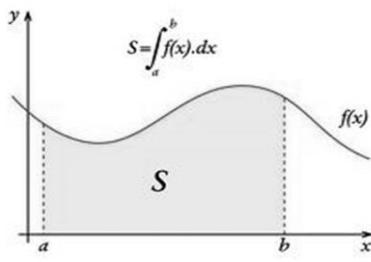
WHAT CAN YOU EXPECT?

- You will receive $20 in compensation for your time

- You can opt out of participation at any point

- Your name and identifying information will not be published or shared with anyone

Appendix B: Consent Form.

North Carolina State University
INFORMED CONSENT FORM for RESEARCH

<u>Title of Study</u> The Transition from AP to College Calculus: A Comparison of the Cognitive and Non-Cognitive Requirements and Challenges Students Encounter

<u>Principal Investigator</u> Megan Ryals   <u>Faculty Sponsor (if applicable)</u> Karen Keene

What are some general things you should know about research studies?

You are being asked to take part in a research study. Your participation in this study is voluntary. You have the right to be a part of this study, to choose not to participate or to stop participating at any time without penalty. The purpose of research studies is to gain a better understanding of a certain topic or issue. You are not guaranteed any personal benefits from being in a study. Research studies also may pose risks to those that participate. In this consent form you will find specific details about the research in which you are being asked to participate. If you do not understand something in this form it is your right to ask the researcher for clarification or more information. A copy of this consent form will be provided to you. If at any time you have questions about your participation, do not hesitate to contact the researcher(s) named above.

What is the purpose of this study?

The purpose of this study is to identify the most significant obstacles to student success in college calculus and how these may be linked to their experiences in high school calculus. More specifically, the study will attempt to discover potential reasons why students who have taken AP Calculus and repeat calculus in college under perform in the college course.

What will happen if you take part in the study?

If you agree to participate in this study, you will be asked to participate in a 60-90 minute interview that consists of two parts. During the first part of the interview you will be asked questions that pertain to your experiences in AP and college calculus. In the second part of the interview you will be asked to discuss your study habits and preparation for your most recent test and to help identify what caused you to make mistakes on certain problems on your test. The interview will be video-recorded with your permission. If you are not comfortable being videotaped, the interview may be audio-recorded.

Risks

No social, physical, financial, or legal risks are anticipated. You will be asked to discuss your study habits. There is potential for this to produce some anxiety and/or stress as you contemplate your current status in your current calculus course. You may share as much or as little as you feel comfortable. At any time you may opt out of participating in the interview and the study.

Benefits

Through the second part of the interview you may discover new approaches to studying for calculus and other courses that have the potential to help you be more successful. The data collected from this research will add to the mathematics and mathematics education communities' understanding of the difficulties students encounter as they transition from high school to college calculus and may serve to inform them in how to ease this transition.

Confidentiality

The information in the study records will be kept confidential to the full extent allowed by law. Video data will be stored electronically on a secure network and interview notes will be stored in a locked file cabinet. No

reference will be made in oral or written reports that could link you to the study. Original transcripts and handwritten notes will contain personal identifiers of students and direct quotes from the interview may be used in reports about the research. If your comments are referred to directly in the study, a pseudonym will be used.

Compensation
For participating in this study you will receive $20. If, after completing the interview, you withdraw from the study prior to its completion, you will be able to keep the $20.

What if you are a NCSU student?
Participation in this study is not a course requirement and your participation or lack thereof, will not affect your class standing or grades at NC State.

What if you have questions about this study?
If you have questions at any time about the study or the procedures, you may contact the researcher, Megan Ryals, at megan_ryals@ncsu.edu or 919-515-5620.

What if you have questions about your rights as a research participant?
 If you feel you have not been treated according to the descriptions in this form, or your rights as a participant in research have been violated during the course of this project, you may contact Deb Paxton, Regulatory Compliance Administrator, Box 7514, NCSU Campus (919/515-4514).

Consent To Participate
"I have read and understand the above information. I have received a copy of this form. I agree to participate in this study with the understanding that I may choose not to participate or to stop participating at any time without penalty or loss of benefits to which I am otherwise entitled."

Subject's signature_____ **Date** _____
Investigator's signature_____

Appendix C: Interview Protocol

Part I

Thank you so much for being willing to participate in this study. First I just want to clarify – you took AP Calculus in high school? And you're currently enrolled in (courses name and number)? And not doing as well as you'd like?
I am interested in learning more about how students experience the transition from AP to college calculus. I'll ask questions that are intended to inform me about this topic. Just as a reminder – all your responses will remain confidential. Your name won't be used and any identifying information will be removed. I am planning to video-record our discussion so I can review your responses afterward; is that okay with you?

1. First, can you tell me when you took AP Calculus? AB or BC? Semester or year long?

2. Tell me about a couple of memorable experiences from each class (AP and college).

3. Tell me about a typical day in each class.

4. What are the biggest differences between your experiences in high school calculus and college calculus?

5. What did/does your teacher expect from students?

6. What did/do you need to do to be successful in this course?

7. Did/you ever think about the possibility of not being successful?

8. How do you define success in this course?

9. How did you learn calculus in high school? How do you learn it now?

10. What did/do you do when you can't understand a topic or can't figure out how to solve a problem?

11. Describe how you study(ied) for math outside of class.

12. How did/do you feel about your math ability?

13. How would you describe yourself as a student, compared to others in your class?

14. If you were going to teach hs or college calc, how would you do it? What advice would you give your AP Calculus teacher?

15. If you could do AP Calculus over again, would you do anything differently? What advice would you give a high school student getting ready to take the class?

16. If you could start this semester over, would you do anything differently?

17. Is there anything else that would be helpful for me to know?

Part II

Now we're going to take a look at your most recent test. Go ahead and get that out, as well as your textbook and class notes and homework. Let's start by identifying the two or three problems where you lost the most points. Pick one for us to start with.

1. Did this question seem familiar to you?
2. Did you believe, going into the test, that you were prepared to answer questions like this one? If "no," what prevented you from being prepared?
3. When you finished this question on the test, did you think you had solved it correctly?

4. One of the things we want to identify is what caused you to miss the questions you did.
5. Here is a problem from your (notes, homework, or textbook) that is similar to this test question. Take a few moments and see how much of this one you can work without using your resources. That is, attempt it just like you would if this were a testing situation. Just let me know when you're finished and have done as much as you can.
6. "So how did it go?"
7.   Point the student to a place in the notes or textbook that is related to this problem. Ask the student to read for a few minutes, then reattempt the problem.
8. Now reattempt the test question.
9. Repeat steps 2-5 with a different test question.
10. So how do the test questions compare to the problems in your notes/hw/book?
11. If you had known how to work these similar examples from your resources before the test, how many points do you believe you would have gotten on these questions on the test?