ABSTRACT

EDWIN, LIONEL ERNEST. Transforming Roving-Rolling Explorer (TRREx) for Planetary Exploration. (Under the direction of Dr. Andre P. Mazzoleni.)

All planetary surface exploration missions thus far have employed traditional rovers with a rocker-bogie suspension. These rovers can navigate moderately rough and flat terrain, but are not designed to traverse rugged terrain with steep slopes. The fact is, however, that many scientifically interesting missions require exploration platforms with capabilities for navigating such types of chaotic terrain. This issue motivates the development of new kinds of rovers that take advantage of the latest advances in robotic technologies to traverse rugged terrain efficiently. This dissertation proposes and analyses one such rover concept called the Transforming Roving-Rolling Explorer (TRREx) that is principally aimed at addressing the above issue.

Biologically inspired by the way the armadillo curls up into a ball when threatened, and the way the golden wheel spider uses the dynamic advantages of a sphere to roll down hills when escaping danger, the novel TRREx rover can traverse like a traditional 6-wheeled rover over conventional terrain, but can also transform itself into a sphere, when necessary, to travel down steep inclines, or navigate rough terrain. This work presents the proposed design architecture and capabilities followed by the development of mathematical models and experiments that facilitate the mobility analysis of the TRREx in the rolling mode.

The ability of the rover to self-propel in the rolling mode in the absence of a negative gradient increases its versatility and concept value. Therefore, a dynamic model of a planar version of the problem is first used to investigate the feasibility and value of such self-propelled
locomotion - ‘actuated rolling’. Construction and testing of a prototype Planar/Cylindrical TRREx that is capable of demonstrating actuated rolling is presented, and the results from the planar dynamic model are experimentally validated. This planar model is then built upon to develop a mathematical model of the spherical TRREx in the rolling mode, i.e. when the rover is a sphere and can steer itself through actuations that shift its center of mass to achieve the desired direction of roll. Case studies that demonstrate the capabilities of the rover in rolling mode and parametric analyses that investigate the dependence of the rover’s mobility on its design are presented.

This work highlights the contribution of the spherical rolling mode to the enhanced mobility of the TRREx rover and how it could enable challenging surface exploration missions in the future. It represents an important step toward developing a rover capable of traversing a variety of terrains that are impassible by the current fleet of rover designs, and thus has the potential to revolutionize planetary surface exploration.
Transforming Roving-Rolling Explorer (TRREx) for Planetary Exploration

by
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________________________________________________________________________
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DEDICATION

Dedicated to my family for their unconditional love and support.
BIOGRAPHY

Lionel Ernest Edwin was born on February 28, 1987 in Rayagada, India, to William and Swaroopa Edwin. He attended Timpany School in Visakhapatnam and then went on to earn his bachelor’s degree in Mechanical Engineering from Anna University, Chennai, in 2008. He came to North Carolina State University in 2009 and began working with Dr. Andre P. Mazzoleni. In May 2012 he received his Master of Science degree in Aerospace Engineering. He will complete the requirements at North Carolina State University for his Ph.D. degree in Mechanical Engineering in August 2014.
ACKNOWLEDGMENTS

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I would like to thank my parents William and Swaroopa, my sister Lucy, my grandparents and extended family who have all been a constant source of encouragement for me. I have been fortunate to have a great group of friends during my student life in Raleigh, Chennai and Visakhapatnam and wish to thank all of my friends for providing me an escape from the rigors of student life.

Finally, I would like to acknowledge the support from NASA Innovative Advanced Concepts (NIAC), North Carolina Space Grant Consortium and the Graduate Student Support Plan at NCSU towards completing this work.
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### NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>Center of mass of the chassis</td>
</tr>
<tr>
<td>$\bar{B}$</td>
<td>$B$ reference frame</td>
</tr>
<tr>
<td>$\hat{i}_B, \hat{j}_B$ and $\hat{k}_B$</td>
<td>Unit axes of frame embedded in chassis (chosen to be the principle axes of the chassis)</td>
</tr>
<tr>
<td>$C_j$</td>
<td>Center of mass of $j^{th}$ leg, where $j$ is the index for leg number</td>
</tr>
<tr>
<td>$\bar{C}_j$</td>
<td>$C_j$ reference frame, where $j$ is the index for leg number</td>
</tr>
<tr>
<td>$\hat{i}<em>{C_j}, \hat{j}</em>{C_j}$ and $\hat{k}_{C_j}$</td>
<td>Unit axes of frame embedded in $j^{th}$ leg (chosen to be the principle axes of each leg)</td>
</tr>
<tr>
<td>$O$</td>
<td>Origin of inertial reference frame</td>
</tr>
<tr>
<td>$\bar{O}$</td>
<td>Inertial reference frame</td>
</tr>
<tr>
<td>$\hat{i}_O, \hat{j}_O$ and $\hat{k}_O$</td>
<td>Unit axes of an inertial reference frame</td>
</tr>
<tr>
<td>$(\bar{T}<em>{B,sys})</em>{ext}$</td>
<td>Total external torque about $B$ acting on the system</td>
</tr>
<tr>
<td>$\frac{\partial}{\partial t}(\bar{r})$</td>
<td>Derivative of vector $\bar{r}$ with respect to the $\bar{O}$ frame</td>
</tr>
<tr>
<td>$\bar{h}_{B,sys}$</td>
<td>Angular momentum of the system about $B$ with respect to the inertial frame</td>
</tr>
<tr>
<td>$m_L$</td>
<td>Mass of each leg</td>
</tr>
<tr>
<td>$\bar{v}<em>{B/O} = \frac{\partial}{\partial t}(\bar{r}</em>{B/O})$</td>
<td>Velocity of point $B$ with respect to the $\bar{O}$ frame</td>
</tr>
<tr>
<td>$\bar{v}_{C_j/O}$</td>
<td>Velocity of point $C_j$ with respect to the $\bar{O}$ frame</td>
</tr>
<tr>
<td>$\bar{g}$</td>
<td>Gravity vector</td>
</tr>
</tbody>
</table>
\( g \)  
Scalar magnitude of gravity

\( \vec{F}_N \)  
Normal reaction force

\( \vec{F}_{fr} \)  
Frictional reaction force

\( \vec{F}_R \)  
Rolling resistance

\( \vec{r}_{p/}^{/B} \)  
Vector from center of mass of chassis \( B \) to point of contact on ground \( P \)

\( \vec{r}_{c/}^{/B} \)  
Vector from the center of mass of the chassis to the center of mass of the \( j^{th} \) leg.

\( \vec{h}_{B,\text{chassis}} \)  
Angular momentum of the chassis about \( B \) with respect to the inertial frame

\( \vec{h}_{B,\text{leg}(j)} \)  
Angular momentum of the \( j^{th} \) leg about \( B \) with respect to the inertial frame

\( \vec{h}_{C,\text{leg}(j)} \)  
Angular momentum of the \( j^{th} \) leg about its center of mass \( C_j \)

\( I_{xB}, I_{yB}, I_{zB} \)  
Principle moments of inertia of the chassis

\( \vec{\omega}_B \)  
Vector angular velocity of \( \vec{B} \) frame with respect to the \( \vec{O} \) frame

\( \omega_{xB}, \omega_{yB}, \omega_{zB} \)  
Angular velocity components of the \( \vec{B} \) frame (and hence the chassis) with respect to the inertial frame as expressed in the \( \vec{B} \) frame coordinates.

\( I_{xC_j}, I_{yC_j}, I_{zC_j} \)  
Principle moments of inertia of \( j^{th} \) leg

\( \vec{\omega}_{C_j} \)  
Vector angular velocity of \( \vec{C}_j \) frame with respect to the \( \vec{O} \) frame

\( \omega_{xC_j}, \omega_{yC_j}, \omega_{zC_j} \)  
Angular velocity components of the \( \vec{C}_j \) frame (and hence the leg \( j \)) with respect to the inertial frame as expressed in the \( \vec{C}_j \) frame coordinates.
\[
\frac{\partial}{\partial t}(\vec{r})
\]
Derivative of vector \(\vec{r}\) with respect to the \(\vec{B}\) frame

\[
\frac{\partial \vec{v}_{c/B}}{\partial t} = \frac{d}{dt}\left(\vec{v}_{c/B}\right)
\]
Velocity of center of mass of the \(j\)th leg with respect to the \(\vec{B}\) frame

\[
\dddot{a}_{c/O}
\]
Inertial acceleration of the center of mass of the \(j\)th leg

\[
\sum \vec{F}_{ext}
\]
Sum of external forces acting on the system

\[
M
\]
Mass of Chassis

\[
\dddot{a}_{B/O}
\]
Inertial acceleration of the center of mass of the chassis

\[
C_{rr}
\]
Coefficient of rolling resistance

\[
\bar{C}_{rr}
\]
Modified rolling resistance that includes a hyperbolic tangent function

\[
\theta
\]
Angular displacement of axis \(\hat{i}_B\) with respect to \(\hat{i}_O\) in planar model

\[
\dot{\theta} = \omega_{zB}
\]
Angular velocity of the chassis about the \(\hat{k}_B\) axis (or \(\hat{k}_O\) axis) in planar model

\[
\ddot{\theta} = \ddot{\omega}_{zB}
\]
Angular acceleration of the chassis about the \(\hat{k}_B\) axis (or \(\hat{k}_O\) axis) in planar model

\[
k_g
\]
Stiffness at contact point between ground and chassis in constraint relaxation (C.R.) technique

\[
c_g
\]
Damping at contact point between ground and chassis connections in C.R. technique

\[
p_d
\]
Penetration depth of chassis into ground in C.R. technique

\[
\hat{e}_n
\]
Unit normal vector coming out of the terrain plane in C.R. technique

\[
\mu
\]
Coefficient of friction in the continuous friction model adopted in C.R.
\( \hat{e}_b \) \hspace{1cm} Unit vector along the terrain plane in the direction opposing the direction of motion of the chassis

\( k_t \) \hspace{1cm} Translational spring stiffness at joints in C.R. technique

\( c_t \) \hspace{1cm} Translational damper damping coefficient at joints in C.R. technique

\( k_r \) \hspace{1cm} Torsional (rotational) spring stiffness at joints in C.R. technique

\( c_r \) \hspace{1cm} Torsional (rotational) damper damping coefficient at joints in C.R. technique

\( H_{jC} \) \hspace{1cm} Point on the \( j^{th} \) leg where it is hinged to the chassis

\( H_{jB} \) \hspace{1cm} Point on the chassis where the leg \( j \) is hinged to it in C.R.

\( \gamma_{jd} \) \hspace{1cm} Desired angle of the leg \( j \) with respect to the chassis

\( \gamma_{ja} = \gamma_j \) \hspace{1cm} Actual angle of the leg \( j \) with respect to the chassis

\( \vec{f}_{BC,j} \) \hspace{1cm} Restoring force acting on the chassis due to the \( j^{th} \) leg in C.R.

\( \vec{f}_{C,B} \) \hspace{1cm} Equal and opposite restoring force acting on the \( j^{th} \) leg due to chassis in C.R.

\( \hat{e}_{H_j} \) \hspace{1cm} Unit vector pointing from point \( H_{jB} \) to \( H_{jC} \) in C.R.

\( \vec{r}_{H_{jC}/H_{jB}} \) \hspace{1cm} Position vector from point \( H_{jB} \) to \( H_{jC} \) in C.R.

\( D_{H_j} = \left| \vec{r}_{H_{jC}/H_{jB}} \right| \) \hspace{1cm} Distance between points \( H_{jB} \) and \( H_{jC} \) in C.R.

\( f_{H_j} \) \hspace{1cm} Magnitude of the force between the chassis and the \( j^{th} \) leg in C.R.

\( \tilde{M}_{BC_j} \) \hspace{1cm} Restoring moment acting on the chassis due to the \( j^{th} \) leg in C.R.
\( \vec{M}_{C,B} \)  
Equal and opposite restoring moment acting on the \( j^{th} \) leg due to chassis in C.R.

\( M_{H,i} \)  
Magnitude of the moment between the chassis and the \( j^{th} \) leg in C.R.

\( \vec{f}_{C,j} ^{R} \)  
Resultant force on the \( j^{th} \) leg in C.R. technique

\( \vec{M}_{C,j} ^{R} \)  
Resultant moment on the \( j^{th} \) leg in C.R. technique

\( \vec{f}_{B} ^{R} \)  
Resultant force on the chassis in C.R. technique

\( \vec{M}_{B} ^{R} \)  
Resultant moment on the chassis in C.R. technique

\( \vec{r}_{H,c/\bar{C}_{i}} \)  
Position vector pointing from the center of mass of the \( j^{th} \) leg to the hinge point on the leg in C.R. technique

\( \vec{r}_{H,b/B} \)  
Position vector from the center of mass of the chassis to the hinge location on the chassis in C.R. technique

\( \{ \vec{r}_{B/o} \} _{\bar{O}} ^{O}, \{ \vec{r}_{C/o} \} _{\bar{O}} ^{O} \)  
Components of \( \vec{r}_{B/o} \) and \( \vec{r}_{C/o} \) when expressed in the inertial frame in C.R.

\( \{ \vec{r}_{H,b/B} \} _{\bar{O}} ^{O}, \{ \vec{r}_{H,c/\bar{C}_{i}} \} _{\bar{O}} ^{O} \)  
Components of \( \vec{r}_{H,c/\bar{C}_{i}} \) and \( \vec{r}_{H,b/B} \) when expressed in the inertial frame in C.R.

\( \{ \vec{r}_{H,b/B} \} _{\bar{B}} ^{\bar{C}_{i}}, \{ \vec{r}_{H,c/\bar{C}_{i}} \} _{\bar{C}_{i}} ^{\bar{C}_{i}} \)  
Components of \( \vec{r}_{H,c/\bar{C}_{i}} \) and \( \vec{r}_{H,b/B} \) when expressed in their respective body frames in C.R.

\( \bar{O} \{ C \} ^{\bar{B}} \)  
Rotation matrix that transforms coordinates in the \( \bar{B} \) frame to coordinates in the inertial \( \bar{O} \) frame
Rotation matrix that transforms coordinates in the $C_j$ frame to coordinates in the inertial $\bar{O}$ frame

$\theta_{Oj}, \theta_{Cj}$ Angular positions of chassis for opening and closing the $j^{th}$ leg

$\Omega_L$ Limiting angular velocity for dynamic ranges in controller

$\mu_s$ Coefficient of static friction

$\mu_k$ Coefficient of kinetic friction

$R_w$ Outer radius of the Planar/Cylindrical TRREx

$l_x, l_y$ $\hat{i}_{C_j}$ and $\hat{j}_{C_j}$ components of the location of the hinge in the frame of the $j^{th}$ leg $C_j$

$h_1, h_2$ $\hat{i}_{B}$ and $\hat{j}_{B}$ components of the location of the hinge in the frame of the chassis $B$

$\beta$ Slope of terrain

$t$ Time

$x_{B0}, y_{B0}, z_{B0}$ Inertial position components of the center of mass of the chassis

$\dot{x}_{B0}, \dot{y}_{B0}, \dot{z}_{B0}$ Inertial velocity components of the center of mass of the chassis

$\psi - \theta - \phi$ Euler angles for a 3-2-1 rotation sequence that represent the orientation of the $B$ frame embedded in the chassis with respect to the inertial frame in the spherical model

$q_{0B}, q_{1B}, q_{2B}, q_{3B}$ Coefficients of the orientation quaternion of the $B$ frame (and hence the chassis)

$g_{xB}, g_{yB}, g_{zB}$ Gravity vector components in the $B$ frame

$\hat{e}_p$ Unit vector in the direction opposite to the direction of velocity at the contact point
\( D_s \) Outer diameter of Spherical surface of the TRREx

\( x_{C,\bar{\bar{o}}}, y_{C,\bar{\bar{o}}}, z_{C,\bar{\bar{o}}} \) Inertial position components of the center of mass of the \( j^{th} \) leg

\( \dot{x}_{C,\bar{\bar{o}}}, \dot{y}_{C,\bar{\bar{o}}}, \dot{z}_{C,\bar{\bar{o}}} \) Inertial velocity components of the center of mass of the \( j^{th} \) leg

\( q_{0c_j}, q_{1c_j}, q_{2c_j}, q_{3c_j} \) Coefficients of the orientation quaternion of the \( \bar{C}_j \) frame (and hence the \( j^{th} \) leg)

\( \tau_{xB}, \tau_{yB}, \tau_{zB} \) External torques components acting on the chassis when written in the chassis frame in constraint relaxation (C.R.) technique

\( \tau_{xC_j}, \tau_{yC_j}, \tau_{zC_j} \) External torque components acting on the \( j^{th} \) leg when written in the leg frame in C.R. technique

\( q_{\_C,a} \) Orientation quaternion representing the actual orientation of the leg \( j \) in C.R. technique

\( q_{\_C,d} \) Orientation quaternion representing the desired orientation of the leg \( j \) in C.R. technique

\( \Delta q_{C_j} \) Rotation quaternion representing the rotation from the desired frame to the actual frame in C.R. technique

\( \hat{e}_{\rho_j}, \rho_j \) Equivalent ‘axis- angle’ representation of \( \Delta q_{C_j} \), where \( \rho_j \) is the magnitude of rotation between the desired and actual leg frames about a unit axis \( \hat{e}_{\rho_j} \) in C.R.

\( \gamma_{\text{min}} \) Angular position when a leg is fully closed

\( \gamma_{\text{max}} \) Angular position when a leg is fully open

\( T_c, T_s \) Time that it takes for the leg to go from a completely closed to a completely open position (or vice-versa) in the leg actuation model

\( f \) Fraction of time \( T_s \) for which a constant acceleration is maintained in the leg actuation model
$r_h$ Distance from a hinge to the point at which the linear motor’s pin is attached to the leg in the leg actuation model

$x_h$ Distance from the hinge to the point where the motor’s other pin is attached to the chassis in the leg actuation model

$\eta$ Angle between the two lines that form the distances $r_h$ and $x_h$ in the leg actuation model

$l_{motor}$ Pin to pin length of the extended motor in the leg actuation model
Chapter 1

Introduction and Review of Literature

The heavens have fascinated humankind for centuries, but it was not until the middle of the 20th century when rapid advances in computing and aerospace technology, finally made it possible for man to make the ‘giant leap’ and venture out into space. Since then numerous space exploration missions have been undertaken and a wealth of knowledge has been generated, helping answer some of the most profound questions about the cosmos. This dissertation contributes to furthering robotic space exploration technology by proposing and analyzing a new kind of transforming planetary surface exploration rover.

1.1 Motivation

For the last half a century there has been growing interest in the exploration of extra-terrestrial habitats in our solar-system, particularly of our closest neighbor, Mars. The scientific goals that drive these various missions are diverse but the underlying theme is to search for evidence of past or present life and assess the potential to harbor future human life [1]. Over the past decades, precursor missions have been using robotic exploration platforms to survey the
extraterrestrial environment and gather critical data that could serve in the design of future manned missions.

Although there have been a wide variety of vehicle architectures proposed in the literature for use in such precursor missions, all surface exploration platforms sent to Mars thus far have used traditional wheeled locomotion coupled with a particular passive suspension architecture called the “rocker-bogie suspension” [2, 3], the latest addition to the list being NASA’s MSL rover [4]. While this configuration is energy efficient and less complex, it significantly limits the types of terrain that the rovers can navigate safely. Apart from traction issues on slopes [5], traditional wheeled rovers face the risk of toppling over on steep slopes. The maximum tilt in any direction that the MER rovers can supposedly withstand without toppling over is limited to 45 degrees [3]. Their top speeds are also limited to below 0.01 m/s as it is desirable to operate in quasi-static equilibrium [6, 7].

Figure 1.1: Terrain features on Mayrs. (Source: ESA)
Thus while traditional wheeled exploration platforms can navigate moderately rough and flat terrain quite efficiently, they are not designed to traverse rugged terrain with steep slopes (see Figure 1.1- Source: ESA webpage [8]). The fact is, however, that most scientifically interesting missions require exploration platforms with capabilities of navigating such types of chaotic terrain. The science strategy for human exploration of Mars [1] identifies the area in Figure 1.2 as one that will give the most scientific returns. This area includes some of the most chaotic terrain features on the planet. It includes the Tharsis volcanoes, the Valles Marineris, and numerous craters and channels.

![Figure 1.2: Proposed region for maximum scientific return.](image)

This issue motivates the development of new kinds of rovers that take advantage of the latest advances in robotic technologies to traverse rugged terrain efficiently. This work analyses one such proposed rover concept called the *Transforming Roving-Rolling Explorer (TRREx)* [9]
that is principally aimed at safely navigating terrains that are combinations of flat areas, gentle gradients and steep rugged slopes.

Biologically inspired by the way the armadillo curls up into a ball when threatened, and the golden wheel spider uses the dynamic advantages of a sphere to roll down hills when escaping danger, the TRREx rover can traverse like a traditional 6-wheeled rover over conventional terrain, but can transform itself into a sphere (see Figure 1.3), when necessary, to travel down steep inclines, or navigate rough terrain.

1.2 Literature Review

There are five basic fields of knowledge that TRREx concept is founded on. A literature survey on the state of art knowledge in each field was conducted and brief reviews are presented below.
1.2.1 Extreme terrain exploration platforms (Single mode of locomotion)

This field is diverse with a variety of innovative exploration platforms, some of which are tabulated in Table 1.1 and shown in Figure 1.4.

Table 1.1: Extreme terrain exploration platforms and their locomotion principle

<table>
<thead>
<tr>
<th>No.</th>
<th>Exploration platform</th>
<th>Locomotion Principle</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.</td>
<td>ELMS[12]</td>
<td>Tracks</td>
</tr>
<tr>
<td>4.</td>
<td>LEMUR, gecko, LAD (locomotive Ameobic device)[14-16]</td>
<td>Legged</td>
</tr>
<tr>
<td>5.</td>
<td>ACM-III[17]</td>
<td>Body articulation (snakes)</td>
</tr>
<tr>
<td>7.</td>
<td>ARES, Entomopter [20, 21]</td>
<td>Flying</td>
</tr>
</tbody>
</table>

Figure 1.4: Extreme terrain exploration platforms (single mode of locomotion): From top left to bottom right - Axel rover, Elms concept, LEMUR, LAD, ARES, Single wheeled rover with arm, Co-operation in single wheeled rovers, ACM-III.
1.2.2 Transformation principles in design

Over the last decade, the advantages of being able to transform between multiple modes of operation is being systematically studied by the engineering design community. Products in the market, like a vacuum cleaner, sofa-bed, a convertible car or even a data-acquisition system are all seen to offer a choice of modes of operation (see Figure 1.5). Studies suggest a recurring set of fundamental design methodologies that are employed in all reconfigurable systems. These ‘transformation principles’ [22, 23] as referred to in the literature are, expand/collapse, fuse/divide, expose/cover, and reorientation.

Figure 1.5: Examples of Transformation in Engineering Design

Other works to explore the utility of transformation in designs have focused on parametric studies [24] and probabilistic models to predict when state modifications should occur [25].
Groups have also investigated transformation specifically in the context of planetary exploration vehicles, and case studies suggest enhanced performance compared to traditional designs [26, 27].

### 1.2.3 Reconfigurable Rovers (Multiple modes of locomotion)

These rovers can reconfigure to increase their operational capabilities. Typically the reconfiguration allows multiple modes of locomotion. Some examples of such rovers/robots in the literature are

**LEON** [28]: LEON is a hexapod that can fold two of its limbs to transform them into wheels (see Figure 1.6). This enables it to transform into a large-wheeled rover from a compact size.

![Figure 1.6: Reconfigurable Rover - LEON](image_url)

**RDB (Rolling Disk Biped)** [29]: The RDB has two modes of locomotion, walking and rolling as depicted in Figure 1.7. It is a three degree of freedom architecture consisting 4 links and 3 actuators. Continuous rolling is achieved by morphing the disk to move the center of gravity. Quasi-static and impulse generated (similar to push-actuation) rolling dynamics have been studied. The developers claim to apply the experience gained from RDB to a future spherical hexapod called Hex-a-Ball [30].
MorpHex: Although there is no technical publication on this robot, it is worth a mention as it is a completed prototype that is closely related to the TRREx concept that is in the public domain [31]. The MorpHex is a hexapod that can transform into a sphere and takes advantage of walking and rolling as its two modes of locomotion.

AZIMUT [32]: Developed in Université de Sherbrooke, AZIMUT is wheel-track-legged robotic platform that transforms between three modes of locomotion (see Figure 1.8). Having a symmetric configuration with four independent articulations it was designed as a commercial platform for robotic Earth applications.

ATHLETE [33]: The ATHLETE (All-Terrain Hex-Limbed Extra-Terrestrial Explorer) is high mobility wheel-limbed robot that is designed to carry heavy cargo in support of sustained
human presence on the moon. It can use walking or wheels as two modes of locomotion, and when it has to traverse slopes greater than 20 degrees it uses a tether.

Modular concepts [34, 35]: There are several proposed modular robotic concepts that have multiple possible modes of locomotion. These concepts comprise of many identical small modules that can combine to form larger platforms. These have been shown to be able to crawl, roll [36] and walk.

1.2.4 Dynamics and control of active suspension wheeled Rovers

The common trade-off adopted toward improving traction and mobility of the traditional passive suspension wheeled rover is to replace the suspension with an actively controlled suspension [37-39]. The dynamics of such active suspensions have been studied to optimize traction and stability of the wheeled platforms. These studies have been generic and are applicable to any wheeled rover with an actively articulated suspension.

1.2.5 Spherical rovers and their dynamics

Wind-propelled: The bouncing, sliding and rolling dynamics of tumbleweed rovers including atmospheric effects has been studied and their performance while negotiating craters, ravines and rock-fields has been analyzed [13, 40-43]. These models are for generic spherical rovers that do not have any form of control. With simplifying assumptions on the mass distribution properties of the TRREx, these models can be applied to ‘free rolling’ under the influence of gravity.
**Self-propelled** [44]: Several proposed spherical robots that are capable of self-propulsion referred to as Spherical mobile robots (SMR) can be found in the literature (see Figure 1.9). The means by which they achieve self-propulsion can be classified as

- Mechanisms to shift the center of mass - Rollo [45], Roball [46], BHQ-2 [47], August [48], Spherobot [49], Sphiricle [50], Kickbot [51]

- Using conservation of momentum (spinning disks) - Shu et al. [52], Joshi et al. [53], Bhattacharya et al. [54], Gyrover [55].

- Continually deforming the sphere/circle to achieve locomotion - Deformable Robot [56], RDB [29]

Each of these concepts are fundamentally very different from the working of the TRREx which is also shown to be able to self-propel in unique ways in the spherical configuration.

![Figure 1.9: Self-propelled Spherical robots: From top left to bottom right - August, Gyrover, Sphiricle, Rollo-3, Spherobot, Kickbot, Roball, Deformable robot.](image-url)
Chapter 2

Architecture of the Transforming Roving-Rolling Explorer (TRREx) Rover

From the literature review in the previous chapter it is observed that exploration platforms with multiple modes of locomotion tend to have higher mobility, however they also tend to have more moving parts and thereby are challenged by correspondingly increased complexity. This architectural challenge of having multiple modes of locomotion while minimizing the number of moving parts is addressed by using the principles of transformation [22, 23], i.e. Expand/Collapse, Expose/cover, Fuse/divide or Reorient parts to ‘transform’ from one mode of locomotion to another. The choice of base designs is made considering designs that best suit the target geographic mission profile. (A wheeled rover and a tumbleweed rover are chosen). Architectural choices to enable the transformation between these base designs are made trying to minimize the complexity of the transformations while maximizing the functionality of the moving parts (reducing and reusing). For example, the TRREx uses the same actuator in active suspension, transformation and actuated rolling.
To introduce the TRREx rover architecture and outline its capabilities, the following scenario is presented.

Picture a typically harsh extraterrestrial environment with moderately rough terrain - rocks, crevices, hummocks and depressions. In the corner of this image is an unobtrusive exploration rover, battling the extreme conditions and slowly inching towards its exploration target. It negotiates the rugged terrain with its *active suspension* (discussed later), distributing its weight evenly over all six wheels to extract maximum traction. As it moves steadily along, it senses an overall gradient in the direction of a mission target and triggers a decision making algorithm. Some quick computations and a couple of decision trees later, the result is a ‘negative’ to switch modes, so it trundles along until it reaches the edge of a steep descent. It has to now choose between roving down the slope or finding another path along which the decent is not as risky. Given inherent advantages in the design of the TRREx, such a decision is trivial, and the unobtrusive rover is finally given a chance to flaunt its hidden potential. Accompanied by the robotic humming of actuators the rover transforms (see Figure 2.1). True to its Acronym-name ‘TRREx’ the ‘Transforming Roving-Rolling Explorer’ transforms from an unassuming conventional rover into a sophisticated spherical rover and is ready to roll down the slope, undaunted by its inclination.
Figure 2.1: TRREx concept of operations

Only a few feet separate the rover from tumbling effortlessly down the edge, but how does it traverse these few feet of flat terrain? Just as it seems the rover has no mobility in this spherical configuration without the aid of a gradient, the active suspension begins to actuate cyclically...
around the sphere to propel the rover forward by creating and maintaining a virtual center of gravity in the direction of the required motion (see Figure 2.6).

Once the rover senses it is over the edge the harmonic ‘actuated rolling’ commands are stopped and the now spherical rover takes a free ride - ‘free rolling’- down the incline towards the objective. Towards the lower end of its ride the gradient is less steep, but the terrain is not as friendly and is covered with small rocks. The rover senses a rather large obstacle in its path, a boulder. It judges that a collision could be potentially harmful and thus performs a set of precisely computed and executed control actuations thereby steering around the obstacle. This is an example of the ‘controlled rolling’ capability of the rover, which is a combination of free rolling and intermittent actuated rolling.

When the incline levels off, the rover follows an algorithm that decides which mode to operate in, based on a variety of inputs: roughness of upcoming terrain, inclination, soil type, constraints on efficiency, time and safety. Apart from the standard parameters, the algorithm also has to accommodate facts such as, the scientific interest in certain parts of the path. For example, for the purpose of taking pictures or collecting soil samples, it might be more advantageous to operate in roving mode as opposed to rolling mode.

The rover design is presented here as a platform that payload and instruments can be mounted onto based on the mission requirements. The general mission types that TRREx would be suited for are areas that have a combination of flat areas, gentle gradients and steep slopes; areas that cannot be explored by traditional rocker-bogie rovers because of their limitation in maneuvering down slopes. The design architecture of the TRREx rover inherently mitigates
the risk of overturning that the traditional rovers face when traversing down a steep slope. The landing site could be chosen at a higher altitude than the rest of the planned mission path so that the rover is traversing downhill for most of the mission, in order to take advantage of potential energy in rolling. On Mars, some example exploration sites where such terrain with slopes and rapid variation of altitudes are found are the Hellas basin and the Tharsis region. The novelty of the TRREx could have applications in the field of robotics in general. Spin-offs of this architecture could have applications on Earth in remote-reconnaissance, exploration, defense or search and rescue operations.

2.1 Design Architecture

The design features of the TRREx mentioned in the above scenario will be discussed in further detail in this section. The TRREx rover operates in one of two modes that it can transform between - the roving mode and the rolling mode.

2.1.1 Roving mode

![Figure 2.2: TRREx in Roving mode](image)
While roving the rover is able to use the following design features:

Active suspension:

Given the generic terrain on an extraterrestrial body, it is advantageous to have some sort of suspension system for many reasons. Traditionally, an arrangement of mechanical linkages, called the Rocker-Bogie mechanism, has been used in planetary exploration rovers. This is a form of passive suspension where there is no actuation of any sort. It has the advantage of being power efficient, simple, and thus reliable and robust. From the view point of even distribution of weight on all wheels, however, it is not the ideal suspension system. For maximum traction it would be ideal to distribute the rover’s weight evenly over all its legs. This will give maximum possible normal force on each wheel and thus no slippage on any one wheel. There is no simple mechanical system that can accomplish this.

In the era of robotics, however, we can resort to closed loop sensing and actuation to achieve this. Considering that such systems are continually becoming more efficient and reliable, they are promising replacements for purely mechanical systems.

![Figure 2.3: Active suspension conforming to terrain](image)
The simple control loop employed in the active suspension works by continually sensing if any one of the wheels is bearing more weight than the other, and actuating that leg in such a way that this excess weight is shifted onto the other legs. This can be done with a combination of force sensors between each wheel and the leg that will monitor the weight borne at that wheel, and actuators that change the angle of the leg to relieve or increase the load on that wheel. The result is a suspension system that conforms to the underlying terrain (see Figure 2.3) with lag and tolerance a function of the control system design. Such a system distributes the rover’s weight evenly on the six wheels and thus enables maximum traction with minimum slippage.

![Figure 2.4: The flexible hip joint (left) and tethered operation (right)](image)

**Flexible hip joint:**

The hip joint of the TRREx as labeled in Figure 2.2, has two degrees of freedom. One of these degrees of freedom is shown in Figure 2.4. This is a rotational degree of freedom that has its axis out of the page. The rotation axis of the other is pointing upward. This design feature gives the rover a lot of redundancy and allows the hip to also contribute to the active suspension. Other advantages of such a joint are discussed later in the capabilities section.
**Detachable-Tethered Halves:**

Another unique feature of the TRREx is that the two halves can be detached while being connected by a tether (see Figure 2.4). This expands the capacities of the rover and allows the two halves to operate separately within the vicinity of each other. The tether also provides a method for recovery if one of the halves gets stuck.

### 2.1.2 Transformation

![Figure 2.5: Step by step transformation of the TRREx between the roving and rolling modes](image)

The step by step transformation between the two modes of locomotion is shown in Figure 2.5. Even though this step by step picture suggests discrete sequential actions are undertaken to change modes, the process is actually continuous where all actuations are simultaneous. In fact high speed sensor-actuator integration could be investigated such that the transformation between modes can happen as the rover is moving, i.e. through precise computations and actuation, the inertia of the rover can be maintained during the transformation.
2.1.3 Rolling mode

In the rolling mode the TRREx is in a spherical shape and the two hemispheres together have a total of eight independently actuated “quarters” (two of which do not serve as legs but still have the capacity to open and close). The different ways in which such a sphere can roll is detailed below.

*Free rolling:*

Free rolling is when the rover is rolling down an incline solely due to gravity and without any sort of interfering actuation. There is no control involved in this type of rolling.

*Actuated rolling:*

![Figure 2.6: TRREx actuated rolling](image)

Actuated rolling occurs when there is no gradient and yet a rolling motion is produced by continuous cyclic actuation and dynamic shifting of the center of gravity. This is shown in Figure 2.6, where initially (without any actuation) the center of gravity (CG) is at the center of
the sphere. As the legs actuate the center of gravity shifts as shown, and the sphere rolls trying to bring the CG to the lowest possible configuration. By cyclic actuation, however, the CG is not allowed to settle at the bottom and is dynamically kept offset.

![Diagram showing CG at center of Sphere and Top front and top right quarters open.]

Figure 2.7: Actuated rolling directional controllability

In this type of rolling a closed loop control scheme is employed that makes use of sensors, such as accelerometers, in tandem with dynamic models, to produce continual actuations that maintain a virtual center of gravity which is offset from the geometric center of the rover, resulting in the desired direction and magnitude of rolling. Figure 2.7 shows how the design allows for directional controllability during actuated rolling.

*Controlled rolling:*

Controlled rolling is when the rover is free rolling, but intermittent actuations are made to momentarily shift the center of gravity in an attempt to control the direction of rolling (for example to steer clear of an obstacle in the free rolling path). Such control is effective when the rover is in contact with the terrain and there is reasonable friction between the rover and the terrain.
2.2 Capabilities

_Gaits and steering possibilities:_

Owing to the many redundancies in its design, the TRREx rover can demonstrate many gaits and steering configurations. Some of them are discussed here. The rover can move forward and backward in the regular roving mode by turning its wheels. It can also move sideways i.e. 90 degrees to the left or right by turning the adaptors between the legs and the frame and by lifting the front and rear legs off the ground. Similarly it can also achieve any intermediate angle, and is thus capable of performing a “crab gait” (see Figure 2.8). To turn, the rover can use its flexible hip to form a turning circle of required radius.

![Figure 2.8: Crab steering in roving mode (left) and rolling using ‘push’ actuation (right).](image)

There might also be various other possibilities, such as a self-propelled rolling were the quarter touching the terrain opens up imparting an impulse that pushes the rover forward (see Figure 2.8).
Climbing over a ledge:

Figure 2.9: TRREx climbing over a ledge in roving mode

Figure 2.9 shows how the rover in the roving mode can take advantage of its flexible hip design to perform a maneuver where it can haul itself up a ledge.

Exploring Craters:

Impact Craters are of prime scientific interest on planetary bodies, and exploring their bases would grant access to invaluable data. Figure 2.10 shows how the TRREx can climb the crater sideways and how the detachable tethered halves allow for one half to anchor itself on the top of the crater edge and winch down the other half to explore the base. Once the exploration
mission is over the anchored half can haul the other half up out of the crater (an anchoring system would need to be developed for the rover-half perched on the edge of the crater).

### 2.3 Research Questions and Path

Designing an exploration platform requires a detailed systems and subsystems level analysis encompassing all aspects of the defined mission. Some common challenges for space exploration platforms are mass, volume and energy restrictions, mobility challenges, structural design challenges, communication challenges, Thermal challenges etc.

Since the USP (unique selling proposition) of the TRREx is its increased mobility compared to conventional rovers, among all the above listed sub-systems the study of the mobility of the TRREx needs to be prioritized. This could be investigated by developing a dynamic model that accurately captures how the TRREx interacts with its environment.

Understanding the dynamics of any rover is critical from its operational capabilities point of view. This answers questions about its performance on a given terrain and also determines what sort of terrains the rover can navigate safely. To underscore the importance of answering these questions, recall that on May 1st 2009, MER-A (Spirit) was trapped in loose soil on relatively flat ground, recovery attempts to free it failed and the mission was eventually ended on March 22, 2010 [57]. Also in 1992 the walking robot Dante II on its ascent trip from Mt. Spurr volcano overturned due to excess lateral tether forces [58].

The dynamics of the TRREx can be separated into three distinct studies:
- A dynamic model for the *roving mode*.

- A dynamic model during the *transformation*.

- A dynamic model for the *rolling mode*.

Since most of the productive time of the TRREx will be spent either in the roving or the rolling mode, the study of the dynamics during transformation is non-essential in preliminary analysis attempting to characterize the mobility performance of the rover.

The dynamics of a wheeled rover with passive suspension is a recurring subject in the literature [7, 59, 60] and so are studies on an active suspension, some of which are very general and can be applied to any rover [38, 39, 61]. Thus, there is a wealth of knowledge from which we can draw to understand the dynamics of the TRREx in the roving mode. In fact existing off-the-shelf dynamic modeling software [62] was used in the preliminary analysis of the roving mode of the TRREx [27].

The dynamics of rolling of a TRREx-like architecture, on the other hand, has not been examined in the literature. There are works that investigate the dynamics of self-propelled spherical rovers [28, 29, 45, 47, 49, 52, 54-56, 63] but, each of them are principally very different from the working of the TRREx and do not contribute to understanding its rolling dynamics. For this reason and also for the fact that the overall increased mobility of the TRREx is expected to be attributed to the novel (self-propelled) rolling mode, detailed study of dynamics of the TRREx in *rolling mode* has been prioritized.
This work will thus investigate the contribution of the rolling mode to the enhanced mobility of the TRREx by specifically focusing on the dynamics and controls problems posed during rolling. Topics such as structural design and power consumption are not considered yet. The philosophy is to highlight the increased capability and later seek the means to enable it [64].

The broad question is - “What sort of mobility does the TRREx have on chaotic terrain?”

Some of the more specific questions, when seeking the contribution of the rolling mode to increased mobility are:

- Can the TRREx self-propel rather than relying solely on gravity to move in the rolling mode?

- What sorts of speeds can be achieved in ‘actuated rolling’, and what are the factors that limit this maximum speed?

- How do the terrain characteristics (hard/loose terrain) affect performance of the rover in actuated rolling?

- Can this mode of locomotion be employed to roll uphill?

- How do the design parameters (dimensions and mass properties) affect the self-propulsion capabilities?

- To what extent can we maneuver or control the direction of roll during self-propelled rolling?
• How does this maneuverability depend on the design parameters?

• Can the rolling mode be the primary mode of locomotion of the TRREx?

The broader question stated above is sought to be answered by considering the above sub-questions.

The spherical TRREx in the rolling mode presents a complicated dynamics problem and a simplified planar (i.e. cylindrical) version of the TRREx is first considered. We note that although this is a simpler problem than the spherical TRREx, such a study is an important first step, as considerable insight regarding questions raised about the mobility of the TRREx in rolling mode can be gained by considering the planar problem. The milestones that the research is divided up into are shown in Figure 2.11.
Having presented the architecture of the TRREx in this chapter the following topics are discussed in the upcoming chapters.
Chapter 3

Modeling, Construction and Experimental Validation of the Cylindrical TRREx

3.1 Introduction

In this chapter the dynamics of actuated rolling is studied considering a Cylindrical/Planar version of the TRREx. Two very different modeling approaches are presented and the results from both approaches are shown to match. Feasibility of actuated rolling and the effect of the TRREx design on actuated rolling performance are investigated via simulations using the analytical model. Further, the construction and software development of a cylindrical prototype that is capable of demonstrating actuated rolling is presented. Finally, the dynamic model is validated by comparing simulation results with experimental data obtained from test runs conducted using the prototype.

3.2 Modeling the Cylindrical TRREx

Multi-body dynamical systems have been traditionally modeled using various analytical techniques, all of which incorporate some form of rigid constraints to facilitate an order-
reduction process that reduces the overall number of independent degrees of freedom, and thus the number of equations required to completely describe the system. This process forces the complexity into a smaller set of equations that can then be integrated to give the time response of the system. Typical examples of such analytical techniques are the Newton-Euler formulization and the Lagrange’s Equations approach [65, 66].

With fast paced advancements in computational capacities, traditional disadvantages associated with simultaneously solving a large number of equations are less relevant, and various novel computational techniques of dynamic modeling that are more suitable to apply in computer code have emerged [67-72]. These methods implement non-rigid constraints and thus do not focus on the order reduction process. Thus a large number of equations of motion are generated but each equation is very simple. A typical example of this class is the finite element approach [67] which has been widely applied and is shown to be particularly well-suited for flexible multi-body systems.

Each of these classes has their pros and cons. The traditional Newton-Euler approach which makes use of rigid constraints generates fewer equations and can yield considerable physical insight when put in symbolic form. The numerical model which used non-rigid constraints is easier to construct and implement in computer code (even though it involves more equations). In the course of this research, models using both techniques were developed and simulation results from the two were observed to match exactly.
3.2.1 Analytical Model of the cylindrical TRREx

System Description:

![Image](image.png)

Figure 3.1: Cylindrical/Planar version of the TRREx

The planar version of the TRREx (Solidworks® model shown in Figure 3.1) has four arms that are actuated by motors. In dynamically modeling this system we consider it to be a multi-body system with 5 bodies; one central frame or ‘chassis’ and four ‘legs’. The four contacts between the legs and chassis are constrained to be hinges with a single rotational degree of freedom. No slip is assumed at the contact between the ground and the cylindrical surface, and this assumption is checked and verified as part of the simulation. Each unconstrained body in the planar space has three independent degrees of freedom, but after applying the rigid constraints at the joint and applying the no slip constraint, we have from the Kutzbach-Gruebler's mobility equation [73] that such a system has five degrees of freedom. Out of these five degrees of freedom, four degrees of freedom, i.e. motion of the legs, are control inputs provided by the controller; so in actuality the system has one degree of freedom, namely the angular position.
of the chassis. The analytical derivation of the governing equation for this degree of freedom is presented below. The development is valid in the no slip regime (between the ground and the cylindrical surface) and assumes that the movements of the parts of the motor contribute a negligible amount to the overall dynamics of the system.

**Definition of frames:**

![Figure 3.2: Definition of Frames for the Cylindrical TRREx](image)

The first step in dynamical analysis of a system of rigid bodies is to define frames, each specified by its origin, unit axes, and the body that it moves with (i.e. is embedded in). For each body that is moving, a separate frame is defined with its origin at the center of mass of that body, with the unit axes aligned in the direction of that body’s principle axes. As shown in Figure 3.2, if the point $B$ is the center of mass of the chassis (excluding the legs) and the directions $\hat{i}_B$, $\hat{j}_B$ and $\hat{k}_B$ ($\hat{k}_B$ is into the plane of the paper) are the principle axes directions of
the chassis, then the $B$ frame, denoted by $\vec{B}$, is a frame with its origin at $B$ and unit axes $\hat{i}_B, \hat{j}_B$ and $\hat{k}_B$.

In a similar fashion, let the legs be numbered 1 through 4 and have centers of masses $C_1$ through $C_4$ respectively. Let the principle axes directions of legs 1 through 4 be $\hat{i}_{C_1}, \hat{j}_{C_1}, \hat{k}_{C_1}$ through $\hat{i}_{C_4}, \hat{j}_{C_4}, \hat{k}_{C_4}$ respectively. Then, $\vec{C}_j$ is a frame with its origin at $C_j$ and unit axes $\hat{i}_{C_j}, \hat{j}_{C_j}$ and $\hat{k}_{C_j}$ for $j=1$ to 4. In addition to this, an inertial fixed reference frame $\vec{O}$ is defined, whose origin is arbitrarily placed at the point of contact between the sphere at the ground at a given instant (e.g. $t=0$), and whose axes are aligned as shown in Figure 3.2.

*Derivation of the Governing Equation:*

Following a standard Newton-Euler approach (e.g. as described in Meirovitch [65]), the total external torque about $B$ acting on the system $(\vec{T}_{B,sys})_{ext}$ is related to the change in angular momentum of the system about $B$ as follows:

$$
(\vec{T}_{B,sys})_{ext} = \frac{d}{dt}(\partial h_{B,sys}) + \partial \vec{v}_{B/\vec{O}} \times m_L \sum_{j=1}^{4} \partial \vec{v}_{C_j/\vec{O}}
$$

(3.1)

In this equation, $\partial h_{B,sys}$ is the angular momentum of the system about $B$ with respect to the inertial frame, $m_L$ is the mass of each leg, $\partial \vec{v}_{B/\vec{O}}$ is the inertial velocity of the center of mass of the chassis and $\partial \vec{v}_{C_j/\vec{O}}$ is the inertial velocity of the center of mass of the $j^{th}$ leg. The term
\[ \frac{\partial \vec{V}_{B/O}}{\partial t} \times m_L \sum_{j=1}^{L} \frac{\partial \vec{V}_{C/O}}{\partial t} \] appears because the torques and angular momentum are written about a point \( B \) which is not the center of mass of the entire system [65].

**Figure 3.3. External forces on the Cylindrical TRREx**

Next, in Equation (3.1) the external torques acting on the system are written in terms of the forces they are derived from, and the right hand side is written in terms of inertias, angular velocities and correction terms (which arise due to the fact that \( B \) is not the center of mass of the entire system, as discussed above). The external forces acting on the system are forces due to gravity (vector denoted by \( \vec{g} \)) and interaction forces between the ground and cylindrical surface. The ground is modeled as flat terrain (although it can be sloped with respect to the horizontal) and \( \vec{F}_N \) is the normal reaction, \( \vec{F}_{fr} \) is the frictional reaction and \( \vec{F}_R \) is the rolling
resistance. These forces are illustrated in Figure 3.3, where for ease of illustration the terrain is portrayed as horizontal.

The sum of external torques about B acting on the system is:

\[
\left( \mathbf{T}_{B,\text{sys}} \right)_{\text{ext}} = \mathbf{r}_{P/B} \times (\mathbf{F}_{fr} + \mathbf{F}_{N}) + m_l \sum_{j=1}^{4} \left( \mathbf{r}_{C_j/B} \times \mathbf{g} \right)
\]

(3.2)

Where \( \mathbf{r}_{P/B} \) is a vector pointing from the center of mass of the chassis B to the point of contact with the ground \( P \), and \( \mathbf{r}_{C_j/B} \) is the vector from the center of mass of the chassis B to the center of mass of the \( j \)th leg (i.e. the point \( C_j \)). Since the system consists of the chassis and the four legs (we are considering the angular momentum contributions of the other components to be negligible in comparison), the total angular momentum of the system can be written as

\[
\overrightarrow{\mathbf{h}}_{B,\text{sys}} = \overrightarrow{\mathbf{h}}_{B,\text{chassis}} + \sum_{j=1}^{4} \overrightarrow{\mathbf{h}}_{B,\text{leg}(j)}
\]

where \( \overrightarrow{\mathbf{h}}_{B,\text{chassis}} \) is the angular momentum of the chassis about B with respect to the inertial frame and \( \overrightarrow{\mathbf{h}}_{B,\text{leg}(j)} \) is the angular momentum of the \( j \)th leg about B with respect to the inertial frame.

It can be shown [65] that the angular momentum of a body about a point other than its center of mass can be written as a function of its velocity and its angular momentum about its center
of mass. Therefore, following [65] we have that the angular momentum of each leg about point \( B \) can be expressed as follows:

\[
\vec{h}_{B,\text{leg}(j)} = \vec{h}_{C_j,\text{leg}(j)} + \vec{r}_{C_j/B} \times m_l \vec{v}_{C_j/O}
\]

Therefore,

\[
\vec{h}_{B,\text{sys}} = \vec{h}_{B,\text{chassis}} + \sum_{j=1}^{4} \left( \vec{h}_{C_j,\text{leg}(j)} + \vec{r}_{C_j/B} \times m_l \vec{v}_{C_j/O} \right)
\]

(3.3)

where \( \vec{h}_{C_j,\text{leg}(j)} \) is the angular momentum of the \( j \)-th leg about its center of mass \( C_j \).

If the moments of inertia of a body are computed about its principle axes then the products of inertia will be zero. For the chassis, its reference frame axes \( \hat{i}_B, \hat{j}_B \) and \( \hat{k}_B \) were by definition chosen to be the principal axes, thus let the principle inertia components computed about these axes be \( I_{x_B}, I_{y_B}, I_{z_B} \), respectively. Writing the angular velocity of the \( B \) frame with respect to the \( O \) frame as, \( \vec{\omega}_B = \omega_{x_B} \hat{i}_B + \omega_{y_B} \hat{j}_B + \omega_{z_B} \hat{k}_B \), it can be shown [65] since \( B \) is the center of mass of the chassis that

\[
\vec{h}_{B,\text{chassis}} = I_{x_B} \omega_{x_B} \hat{i}_B + I_{y_B} \omega_{y_B} \hat{j}_B + I_{z_B} \omega_{z_B} \hat{k}_B
\]
Similarly for each leg if \( I_{x_C}, I_{y_C}, I_{z_C} \) are the inertia components about the principle axes \( \hat{i}_C, \hat{j}_C, \hat{k}_C \) respectively and the angular velocity of the \( C_j \) frame with respect to the \( O \) frame is \( \frac{\partial \vec{\omega}}{\partial \vec{c}} = \omega_{x_C} \hat{i}_C + \omega_{y_C} \hat{j}_C + \omega_{z_C} \hat{k}_C \), then it can be shown [65] that

\[
\frac{\partial \vec{h}_{C_j, leg(j)}}{\partial \vec{c}} = I_{x_C} \omega_{x_C} \hat{i}_C + I_{y_C} \omega_{y_C} \hat{j}_C + I_{z_C} \omega_{z_C} \hat{k}_C
\]

Differentiating Equation (3.3) with respect to the \( O \) frame yields Equation (3.4),

\[
\frac{\partial}{\partial t} \left( \frac{\partial \vec{h}_{B,sys}}{\partial \vec{c}} \right) = \frac{\partial}{\partial t} \left( \frac{\partial \vec{h}_{B, chassis}}{\partial \vec{c}} \right) + \sum_{j=1}^{4} \frac{\partial}{\partial t} \left( \frac{\partial \vec{h}_{C_j, leg(j)}}{\partial \vec{c}} \right) + \frac{\partial}{\partial t} \left( \vec{r}_{C_j/\beta} \times m_L \vec{\omega}_{C_j/\beta} \right)
\]

Rewriting the first term on the right hand side by using transport theorem [66] we obtain:

\[
\frac{\partial}{\partial t} \left( \frac{\partial \vec{h}_{B, chassis}}{\partial \vec{c}} \right) = \frac{\partial}{\partial t} \left( \frac{\partial \vec{h}_{B, chassis}}{\partial \vec{c}} \right) + \frac{\partial}{\partial t} \left( \int_{B}^{\partial} \vec{h}_{B, chassis} \right) = \left( I_{x_B} \omega_{x_B} - (I_{y_B} - I_{z_B}) \omega_{y_B} \omega_{z_B} \right) \hat{i}_B
\]

\[+ \left( I_{y_B} \omega_{y_B} - (I_{x_B} - I_{z_B}) \omega_{x_B} \omega_{z_B} \right) \hat{j}_B
\]

\[+ \left( I_{z_B} \omega_{z_B} - (I_{x_B} - I_{y_B}) \omega_{x_B} \omega_{y_B} \right) \hat{k}_B
\]

Since our system is restricted to planar motion about the \( \hat{k}_B \) axis, we have

\( \omega_{x_B} = \omega_{y_B} = \omega_{z_B} = 0 \). Thus, the above expression reduces to,

\[
\frac{\partial}{\partial t} \left( \frac{\partial \vec{h}_{B, chassis}}{\partial \vec{c}} \right) = I_{z_B} \omega_{z_B} \hat{k}_B
\]

Following a similar process for each leg, we obtain,
Using the product rule and transport theorem [66] the second term in the summation in (3.4) can be re-written as:

\[
\frac{\partial}{\partial t} \left( \bar{R}_{c/\ell} \times m_L \bar{V}_{c/\ell} \right) = m_L \left( \bar{V}_{c/\ell} \times \bar{V}_{c/\ell} + (\bar{\omega}_{\ell/\ell} \times \bar{R}_{c/\ell}) \times \bar{V}_{c/\ell} + \bar{R}_{c/\ell} \times \bar{A}_{c/\ell} \right)
\]

(3.7)

where \( \bar{V}_{c/\ell} = \frac{\partial}{\partial t} \left( \bar{R}_{c/\ell} \right) \) is the velocity of center of mass of the \( j \)th leg with respect to the \( \bar{B} \) frame and \( \bar{A}_{c/\ell} \) is the inertial acceleration of the center of mass of the \( j \)th leg.

Using (3.5), (3.6) and (3.7) in Equation (3.4) gives:

\[
\frac{\partial}{\partial t} \left( \bar{R}_{B,sys} \right) = I_{3B} \ddot{\omega}_{B} \hat{k}_B + \sum_{j=1}^{4} I_{c_i} \ddot{\omega}_{c_i} \hat{k}_{c_i}
\]

(3.8)

Now, using (3.2) and (3.8) in Equation (3.1), we obtain:

\[
\bar{r}_{f/\ell} \times (\bar{F}_{f/\ell} + \bar{F}_{\ell}) + m_L \sum_{j=1}^{4} \left( \bar{r}_{c/\ell} \times \bar{g} \right) = I_{3B} \ddot{\omega}_{B} \hat{k}_B + \sum_{j=1}^{4} I_{c_i} \ddot{\omega}_{c_i} \hat{k}_{c_i}
\]

(3.9)
It is now noted that for the planar system \( \hat{k}_B = \hat{k}_{C_i} = \hat{k}_{O} \), and that the \( \hat{k}_O \) component in the vector Equation (3.9) will ultimately yield the equation of motion for the degree of freedom associated with the chassis.

The unknown forces in equation (3.9) can be found when Newton’s second law is applied to the system:

\[
\sum \vec{F}_{ext} = M \vec{a}_{b/0} + m_L \sum_{j=1}^{4} \vec{a}_{c_j/0}
\]

In the above equation, \( \sum \vec{F}_{ext} \) is the sum of external forces acting on the system, \( M \) is the mass of the chassis, \( m_L \) is the mass of each leg, and \( \vec{a}_{b/0} \) and \( \vec{a}_{c_j/0} \) are the inertial accelerations of the center of mass of the chassis and the acceleration of the center of mass of the \( j^{th} \) leg respectively. External forces acting on the system are gravity, static friction, the normal reaction of the ground acting on the rover and rolling resistance forces. Hence we have:

\[
Mg + 4m_Lg + \vec{F}_r + \vec{F}_N + \vec{F}_R = M \vec{a}_{b/0} + m_L \sum_{j=1}^{4} \vec{a}_{c_j/0}
\]  

(3.10)

Note that rolling resistance is modeled as a force opposing motion that is proportional to the magnitude of the normal force.

\[
\vec{F}_R = -C_r \left| \vec{F}_N \right| \hat{i}_O \quad \text{if motion is in the positive } \hat{i}_O \text{ direction}
\]

\[
\vec{F}_R = C_r \left| \vec{F}_N \right| \hat{i}_O \quad \text{if motion is in the negative } \hat{i}_O \text{ direction}
\]
Numerically integrating the governing equations:

In developing computer code to numerically integrate the analytical equations listed above, the dependence of the direction of rolling resistance force on the direction of motion is implemented by inserting a hyperbolic tangent function in the evaluation of a modified rolling resistance coefficient $\overline{C}_{rr}$. This was done to avoid the integrator taking inordinate amounts of time to integrate when the velocity of the chassis becomes extremely small (close to zero). When the system is approaching rest, the direction of the rolling resistance force keeps changing almost every time-step, which can lead to significant numerical errors over time. The hyperbolic tangent function mitigates this issue by providing a margin between a small positive velocity and a small negative velocity during which the opposing force tapers off to zero at zero velocity.

Thus, if $\vec{F}_r = -\overline{C}_{rr} |\vec{F}_n| \hat{j}_o$ then $\overline{C}_{rr}$ is set to be: $\overline{C}_{rr} = C_{rr} \tan \left( \frac{\dot{\theta}}{\theta_{\text{trigger}}} \right)$ where $\theta_{\text{trigger}}$ is some small value of angular velocity. The value of the hyperbolic tangent function is unity and takes the sign of $\dot{\theta}$ at all values within the function except when $|\dot{\theta}| < \dot{\theta}_{\text{trigger}}$. In this range, when the velocity of the system is practically zero, the hyperbolic tangent function rapidly approaches zero and thus makes the rolling resistance force zero.

The $\hat{j}_o$ component in Equation (3.10) is used to first solve for the normal force. From this, the rolling resistance can be determined and used in the $\hat{i}_o$ component to solve for the
frictional reaction. This allows for the quantity \((\vec{F}_f + \vec{F}_N)\) to be solved for in Equation (3.10) and plugged into Equation (3.9). The \(k_{\theta}\) component of Equation (3.9) is in terms of system geometric and mass constants and the control inputs of the legs, which are all known. The only unknown is the variable describing rotary motion of the chassis about the \(\overline{B}\) frame. Labeling this variable as \(\theta\), we have that \(\omega_{zB} = \dot{\theta}\) and \(\omega_{zB} = \ddot{\theta}\). Thus, Equation (3.9) yields a second order differential equation in \(\theta\) that can be numerically integrated to give the motion of the chassis as a function of time.

### 3.2.2 Constraint Relaxation Model for the Cylindrical TRREx

#### Treatment of connections:

An alternate approach to numerical integration of the analytical model described above is to use a “constraint relaxation” technique [69], as will be described below. The constraint relaxation technique preserves all the degrees of freedom of each body due to the fact that now non-rigid (visco-elastic) constraints replace the rigid constraints at the joints between bodies in the multi-body system. Thus for our planar multi-body system with 5 bodies, since each body has 3 independent degrees of freedom, the overall system has \(5 \times 3 = 15\) degrees of freedom. The dynamic behavior of a non-rigid connection tends to the behavior of the rigid connection as stiffness increases, and in the limit the two are equivalent [69, 70].
In the cylindrical TRREx the connections between each leg and the chassis are modeled via springs and dampers (linear and rotational) as shown in Figure 3.4, where the stiffness parameters are chosen to be sufficiently high to approximate rigid connections. More precisely, the stiffness is chosen based on the required resolution of the displacement response at the connections. If $\varepsilon$ is the required resolution and $F_{\text{peak}}$ is the peak force occurring at the connection, then the stiffness that will produce small enough maximum displacement of $\varepsilon$ is given by $k = F_{\text{peak}} / \varepsilon$. The un-stretched length of the linear spring at each connection is zero and the un-stretched angle of the torsional spring $\gamma_d(t)$ is a control input provided by the controller. Viscosity parameters are chosen such that the system is critically damped.
Treatment of ground interaction:

Ground interaction in this technique is treated very differently than the traditional analytical methods. Consistent with the idea of ‘constraint relaxation’ the ground is now assumed to be a non-rigid surface with visco-elastic parameters that sufficiently represent the dynamic interaction of the body with the ground. The stiffness $k_g$ is chosen based on the required resolution of the displacement response at the contact point and damping $c_g$ is found in terms of the coefficient of restitution between the bodies [69]. Thus the normal force equation is given by,

$$
\bar{F}_N = \begin{cases} 
\bar{0} & \text{when } p_d \leq 0 \\
 (k_g p_d + c_g \dot{p}_d) \hat{e}_n & \text{when } p_d > 0 
\end{cases}
$$

where $p_d$ is the penetration depth and $\hat{e}_n$ is the unit normal vector coming out of the ground surface. Note that $p_d$ is measure positive in the $-\hat{e}_n$ direction. This treatment is similar to treatment of collisions in [69, 70, 72].

To model static and kinetic friction, a continuous friction model is adopted, where instead of two distinct equations in cases of slip and roll there is now only one equation $\bar{F}_{fr} = \mu |\bar{F}_N| \hat{e}_p$ (replaces both kinetic and static friction laws), where $\hat{e}_p$ is a unit vector along the direction opposing to the direction of velocity at contact point. Similar models have been used in the literature [69, 71]. Rolling resistance is modeled the same way as would be done in analytical
modelling, $\vec{F}_g = C_r \vec{F}_n \hat{e}_b$, where $\hat{e}_b$ is a unit vector along the direction opposing the direction of motion of the chassis (opposite to x component of inertial chassis velocity).

Modeling ground interactions in this way eliminates the need for discrete sets of equations for rolling, sliding and bouncing and thus is more efficient to implement in computer code.

*Forces and Moments on each body:*

Let $H_{jC}$ be the point on the $j^{th}$ leg where it is hinged to the chassis and let $H_{jB}$ be the corresponding point on the chassis. Let there be a translational spring (of stiffness $k_r$) and
damper (with damping coefficient $c_t$) between the two points. Also let there be a torsional spring (of stiffness $k_t$) and damper (with damping coefficient $c_r$) between the two bodies. Note that the un-stretched length of the translational spring is zero and the un-stretched length of the torsional spring is $\gamma_{jd}(t)$ for the $j^{th}$ leg.

If at a given instant the $j^{th}$ leg is displaced as shown in Figure 3.5, then the restoring force acting on the chassis is given by $\hat{f}_{bc_j} = f_{H_j} \hat{e}_{H_j}$ and the force acting on the leg is equal and opposite given by $\hat{f}_{C,B} = -f_{H_j} \hat{e}_{H_j}$, where $\hat{e}_{H_j}$ is the unit vector pointing from point $H_{jb}$ to $H_{jc}$ and can be written as,

$$\hat{e}_{H_j} = \frac{\vec{r}_{H_j}/r_{H_j}}{D_{H_j}}$$

and $f_{H_j}$ is the magnitude of the force between the two bodies given by $f_{H_j} = k_t D_{H_j} + c_t \dot{D}_{H_j}$.

Similarly moments are generated when at the given instant the actual angle $\gamma_{ja}$ of the leg with respect to the chassis is different from the desired angle $\gamma_{jd}$. The moment exerted on the chassis is $\hat{M}_{bc_j} = M_{H_j} \hat{k}_{C_j}$ and an equal and opposite moment is exerted on the leg $\hat{M}_{C,B} = -M_{H_j} \hat{k}_{C_j}$, where the magnitude of the moment is $M_{H_j} = k_r (\gamma_{ja} - \gamma_{jd}) + c_r (\dot{\gamma}_{ja} - \dot{\gamma}_{jd})$. Note that $\gamma_{ja}$ is the
degree of freedom of the leg while $\gamma_{jd}$ is an input to system, also, in the planar system

$$\dot{k}_o = \dot{k}_B = -\dot{k}_C = -\dot{k}_C = -\dot{k}_C$$ for all time.

Thus computing the resultant forces on each body, the leg has a force acting on it at the connecting point and the force of gravity acting at its center of mass. Thus the resultant force on the $j^{th}$ leg is

$$f^R_{C_j} = f_{C_j,B} + m_l g$$

(3.11)

and the resultant moment on the $j^{th}$ leg is

$$M^R_{C_j} = \overrightarrow{r}_{H,C_j} \times f_{C_j,B} + M_{C_j,B}$$

(3.12)

where $\overrightarrow{r}_{H,C_j}$ is the position vector pointing from the center of mass of the $j^{th}$ leg to the hinge point on the leg.

The chassis has four forces due to the connections at each leg, the gravitational force acting through its center of mass, frictional, normal reaction and rolling resistance forces acting on it. Thus the resultant force on the chassis is given by

$$f^R_B = Mg + F_{fr} + F_{N} + F_{R} + \sum_{j=1}^{4} f_{B,C_j}$$

(3.13)

and the resultant moment is
\[ \ddot{M}_b^r = \vec{r}_{p/b} \times (\vec{F}_n + \vec{F}_k) + \sum_{j=1}^{4} \vec{r}_{H_B/j} \times \vec{f}_{BC/j} + \vec{M}_{BC/j} \]  

(3.14)

where \( \vec{r}_{H_B/j} \) is the position vector from the center of mass of the chassis to the hinge location on the chassis.

**Writing unknowns in terms of knowns:**

At each time-step the unit vector \( \hat{e}_{H_i} \) is computed from the states at that time as follows,

\[
\hat{e}_{H_i} = \frac{r_{H/C_i} - \left( \vec{r}_{B/O} - \vec{r}_{C/O} \right) \vec{r}_{H/B} \vec{r}_{H/B}}{D_{H_i}}
\]

Here \( \vec{r}_{B/O} \) and \( \vec{r}_{C/O} \), when expressed as components in the inertial frame as \( \{ \vec{r}_{B/O} \}_{O} \) and \( \{ \vec{r}_{C/O} \}_{O} \), directly correspond to states at that time-step. Vectors \( \vec{r}_{H/C_i} \) and \( \vec{r}_{H/B} \) can be written from geometry in their respective body frames as \( \{ \vec{r}_{H/C_i} \}_{C_i} \) and \( \{ \vec{r}_{H/B} \}_{B} \) and can then be transformed to the inertial frame using the rotation matrices that define the orientations of the chassis (i.e. \( ^O \{ C \}^B \)) and the leg (i.e. \( ^O \{ C \}^C \)) with respect to the inertial frame as follows:
Again the rotation matrices are computed directly from the states corresponding to angular positions of the chassis and $j^{\text{th}}$ leg.

Equations of motion and integration in the constraint relaxation technique:

The equations of motion related to the motions of the centers of mass are given by,

$$ M^o \ddot{a}_{B/O} = \ddot{f}^R_B $$

and

$$ m_l^o \ddot{a}_{C_j/O} = \ddot{f}^R_{C_j} \text{ for } j=1 \text{ to } 4 $$

The $\hat{i}_O$ and $\hat{j}_O$ components in equations (3.15) and (3.16) represent equations of motion corresponding to the 10 translational degrees of freedom (DOF) in the system. The rest of the five equations of motion (EOMs for the rotational DOFs are represented by the $\hat{k}_O$ components of equations (3.17) and (3.18).

$$ \frac{\partial}{\partial t} (\vec{\alpha} \vec{h}_{B, \text{chassis}}) = \vec{\dot{M}}_B^R $$

and

$$ \frac{\partial}{\partial t} (\vec{\alpha} \vec{h}_{C_j, \text{leg}(j)}) = \vec{\dot{M}}_{C_j}^R \text{ for } j=1 \text{ to } 4 $$

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Note that when writing the LHS of equations (3.17) and (3.18) in terms of body inertias, these inertias should be represented in the inertial frame.

The above 15 EOMs, each of which is a 2nd order ordinary differential equation, yield 30 state equations, the states being 15 positions (3 for each body, two translational and one rotational position) and 15 velocities. Given these initial conditions, and given the environmental, geometric and mass properties of the system, the equations can be integrated to produce the time response of the system.

Caution should be exercised in setting the maximum step size for the integrator. It needs to be much less than the smallest natural-time-period of vibration in the system (i.e. about 1/100th the time period corresponding to the highest natural frequency in the system). This presents a trade-off between the resolution of the displacements required (based on which we set the stiffness parameters at each connection) and the time-steps taken to complete the integration. Thus the higher the resolutions required, the higher the stiffness’s are set, but this introduces higher natural frequencies in the system and thus smaller maximum step sizes of integration should be chosen and the longer the integration will take. Simulation results presented using the constraint relaxation technique employed a Runge-Kutta integrator [74] written in MATLAB® [75].

### 3.2.3 Control Inputs to Models

Recalling Figure 3.2, if $\gamma_{jd}$ is the desired angle at a given instant of time between the $\hat{k}_{c_j}$ and $\hat{i}_b$ axes in the positive $\hat{k}_{c_j}$ direction, then the desired input motions of the legs (as functions of
time) are given by $\gamma_{jd}, \dot{\gamma}_{jd}, \ddot{\gamma}_{jd}$ for the four legs i.e. $j=1, 2, 3, 4$. In the analytical technique these are inputs into the equation of motion yielding a second order differential equation with the only unknown being the chassis angular position $\theta$. This differential equation can be numerically integrated using the ode45 suite in MATLAB® [75] to obtain the time evolution of the chassis angle. An assumption here is that the actuators (linear motors) generate the desired motion exactly. When using the constraint relaxation technique this assumption is relaxed and the actual motion of the legs ($\gamma_{ja}, \dot{\gamma}_{ja}, \ddot{\gamma}_{ja}$) are allowed to be slightly different from the desired motion ($\gamma_{jd}, \dot{\gamma}_{jd}, \ddot{\gamma}_{jd}$), where the motion is dictated by the rotational stiffness and damping at the connection.

Initial studies assumed that the angular acceleration of each leg followed a cubic polynomial (in time) from rest position to rest position. But it was observed from experiments on the prototype that a piecewise constant acceleration of the piston (of the linear motor between the leg and the chassis), when transformed to the angle of the leg, better characterized the opening and closing motion of the legs. (For more details see Appendix A)

One of the intended features of the TRREx is that self-propulsion (actuated rolling) can be achieved by controlling the timing of opening and closing of legs such that continuous rolling is generated, and we will demonstrate that this intended feature is a reality through both analysis and experiment. As the TRREx rolls the legs have to be retracted in time so that the system does not roll over an opened leg (i.e. so that ‘ground interference’ does not occur). Each leg could be set to open and close at pre-determined values of the angular position (for range
of operation, see Figure 3.6), but as the angular velocity increases the legs will have less and less time to completely retract. Thus, the maximum speed of actuated rolling is directly limited by the capacity of the actuators which move the legs, so to avoid ground interference: (1) the actuators are set to retract the legs at maximum capacity; and (2) the ranges of operation are made to be a function of chassis angular velocity, i.e. the controller starts closing the legs earlier and earlier as the speed of rolling increases.

Looking at Figure 3.6, we note that if leg 2 is set to open at a particular chassis angle \( \theta = \theta_{o2} \) (note that here “O” in the subscript stands for “open”), and close at an angle \( \theta = \theta_{c2} \) (note that here “C” stands for “closed”), as the angular velocity of the system increases, this closing angle will move closer to \( \theta_{o2} \) (proportionally) and is equal to \( \theta_{o2} \) at some limiting velocity \( \Omega_L \).

<table>
<thead>
<tr>
<th>Leg</th>
<th>Open angle</th>
<th>Close angle</th>
<th>Range of operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \theta_{o1} )</td>
<td>( \theta_{c1} )</td>
<td>( \theta_{c1} - \theta_{o1} )</td>
</tr>
<tr>
<td>2</td>
<td>( \theta_{o2} )</td>
<td>( \theta_{c2} )</td>
<td>( \theta_{c2} - \theta_{o2} )</td>
</tr>
<tr>
<td>3</td>
<td>( \theta_{o3} )</td>
<td>( \theta_{c3} )</td>
<td>( \theta_{c3} - \theta_{o3} )</td>
</tr>
<tr>
<td>4</td>
<td>( \theta_{o4} )</td>
<td>( \theta_{c4} )</td>
<td>( \theta_{c4} - \theta_{o4} )</td>
</tr>
</tbody>
</table>

Figure 3.6: Controller - Dynamic ranges of operation
Therefore the range of operation of the leg decreases linearly and tends to zero as the system velocity tends to the limiting velocity.

Thus, in our closed loop scheme, control inputs to the legs are functions of both of the current states - angular position and velocity. Since the control inputs are discontinuous functions of the states, they are implemented in code by detecting ‘event crossings’ (as opposed to substituting the inputs as a function of states in the equations of motion). During integration of the equation of motion, when an event is detected (i.e. when the chassis position crosses an open or close angle of a leg) the input to the system is changed appropriately and integration is then continued. It should be noted that, at the occurrence of an event, ranges of all the legs except the one that just finished actuation are updated (this avoids jitter).

3.2.4 Comparison of Analytical and Constraint Relaxation Model Results

Figure 3.7: Comparison of Analytical and Constraint Relaxation Results.
The results from numerical integration of the analytical and constraint relaxation models are compared for various test cases and they were observed to match exactly (within the tolerances of our numerical integration methods). One particular set of results with a specific set of physical and environmental parameters, in Table 3.6 and Table 3.7, respectively, is presented in Figure 3.7. Actuations were stopped after 5 seconds and the system was then allowed to coast. It can be seen that both models yield essentially the same response.

3.3 Simulations Investigating the of Effect of Cylindrical TRREx Design on Performance

Having developed a dynamic model, it is put to use in exploring the effect of the system’s design on its ability to perform actuated rolling. The terrain difficulty for these simulations is characterized by the coefficients of rolling resistance \( C_r \) and static friction \( \mu_s \). For example, an ‘easy terrain’ would be one that offers low rolling resistance and large enough traction to avoid slip between the rover and the terrain. A ‘difficult terrain’ would be one in which \( C_r \) is high and \( \mu_s \) is low. The analytical model is employed for these simulations and as noted before, it is only valid in the no slip regime (i.e. when \( |\vec{F}_r| < \mu_s |\vec{F}_N| \)). Thus, after each simulation, it was verified that the system was constantly operating within this regime. Also for simulations in this section it is assumed that the angular acceleration of the leg follows a cubic polynomial (in time) from rest position to rest position. (See Appendix A for details)
3.3.1 Earth Simulations

Anticipating experimental work on a physical prototype that demonstrates actuated rolling, using the model to explore reasonable designs for an Earth prototype was considered. The dynamical model was thus used to examine candidate designs performing actuated rolling in various Earth environments. A starting candidate design was obtained by modeling the system in SolidWorks® and assigning material properties to the components. The design parameters for this first candidate design are listed in Table 3.1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric parameters</td>
<td>Outer radius of cylindrical surface</td>
<td>$R_w$</td>
<td>0.3750</td>
<td>Meters</td>
</tr>
<tr>
<td></td>
<td>Location of center of mass of leg w.r.t. hinge</td>
<td>$l_x$</td>
<td>0.1961</td>
<td>Meters</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$l_y$</td>
<td>0.0313</td>
<td>Meters</td>
</tr>
<tr>
<td></td>
<td>Location of hinge of leg w.r.t. center</td>
<td>$h_1$</td>
<td>0.3393</td>
<td>Meters</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$h_2$</td>
<td>0.1275</td>
<td>Meters</td>
</tr>
<tr>
<td>Mass properties</td>
<td>Mass of ‘chassis’</td>
<td>$M$</td>
<td>11.3000</td>
<td>Kg</td>
</tr>
<tr>
<td></td>
<td>Mass of each ‘leg’</td>
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<td>Kg</td>
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<td>Kg.m$^2$</td>
</tr>
<tr>
<td></td>
<td>Inertia about the rotation axis of each leg</td>
<td>$I_{zC_j}$</td>
<td>0.0394</td>
<td>Kg.m$^2$</td>
</tr>
</tbody>
</table>

The first simulation of this system was focused on an easy Earth terrain characterized by the parameters listed in Table 3.2. A controller was used to actuate the legs cyclically, and the evolution of system states - such as chassis angular position and velocity - were recorded.
Table 3.2: Simulation 1 Environmental Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Environment parameters</td>
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<td>9.81</td>
<td>m/s$^2$</td>
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<tr>
<td></td>
<td>Rolling resistance</td>
<td>$C_{rr}$</td>
<td>0.005</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Coefficient of static friction</td>
<td>$\mu_s$</td>
<td>0.5</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Slope of terrain</td>
<td>$\beta$</td>
<td>0</td>
<td>Degrees</td>
</tr>
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</table>

Figure 3.8 shows the simulation results for this scenario, where the system was started at rest position and achieves continuous rolling motion. The top subplot depicts the actuations of the four legs as generated by the controller. When the value of $\gamma$ for a particular leg is minimum, then that leg is fully closed; when it is maximum, it is in the fully open position.

The middle and bottom subplots represent the chassis’ angular position and angular velocity, respectively. For this simulation, the controller was switched off at the 10 second mark, all legs were brought to the fully closed position, and the system was allowed to coast to a halt. The low rolling resistance allows the system to roll without actuation for a significantly longer distance. It is more of a challenge to tune the controller for terrains that provide little or no opposition to rolling to avoid ground interference. This is because in such scenarios the system builds up to a higher rotation speed and the legs have less time before they have to be pulled back to prevent the rover from rolling over an open leg.
The next simulation scenario placed the same design configuration on terrain where it would be difficult to produce continuous actuated rolling. Here, values of $C_{rr} = 0.1$ and $\mu_s = 0.15$ are used to simulate terrain that is equivalent to the rover rolling on sand [76, 77]. The other environmental parameters remain the same as the previous simulation. The results in Figure 3.9 show that the system starts to roll, but is not able to roll enough for the next leg in sequence to constructively contribute to the perpetuation of rolling motion. Thus this design configuration experiences ‘stall’ and is not able to achieve a continuous actuated rolling motion on difficult terrain.
As the system configuration described in Table 3.1 was unable to achieve continuous rolling motion on difficult terrain, the next simulation explored if changes to system configuration would address this limitation. The effect of changing various parameters in the design space on the response was observed and the highlighted cells in Table 3.3 denote the system parameters updated. Essentially, these changes (1) pushed each leg’s center of mass further from the system center to produce more torque, (2) increased the inertia of the chassis to reduce the tendency for stall, and (3) increased the overall mass of the system to reduce the tendency to slip.
Table 3.3: Candidate 2 Design Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric parameters</td>
<td>Outer radius of cylindrical surface</td>
<td>$R_w$</td>
<td>0.3750</td>
<td>Meters</td>
</tr>
<tr>
<td></td>
<td>Location of center of mass of leg w.r.t.</td>
<td>$l_x$</td>
<td>0.3000</td>
<td>Meters</td>
</tr>
<tr>
<td></td>
<td>hinge</td>
<td>$l_y$</td>
<td>0.0313</td>
<td>Meters</td>
</tr>
<tr>
<td></td>
<td>Location of hinge of leg w.r.t. center</td>
<td>$h_1$</td>
<td>0.3393</td>
<td>Meters</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$h_2$</td>
<td>0.1275</td>
<td>Meters</td>
</tr>
<tr>
<td>Mass properties</td>
<td>Mass of ‘chassis’</td>
<td>$M$</td>
<td>15</td>
<td>Kg</td>
</tr>
<tr>
<td></td>
<td>Mass of each ‘leg’</td>
<td>$m_l$</td>
<td>5</td>
<td>Kg</td>
</tr>
<tr>
<td></td>
<td>Inertia about the rotation axis of</td>
<td>$I_{z\bar{z}}$</td>
<td>1.5</td>
<td>Kg.m²</td>
</tr>
<tr>
<td></td>
<td>chassis</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Inertia about the rotation axis of each</td>
<td>$I_{z\bar{c}_j}$</td>
<td>0.0394</td>
<td>Kg.m²</td>
</tr>
<tr>
<td></td>
<td>leg</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

Figure 3.10: Simulation 3 - Actuated rolling of Candidate 2 on difficult terrain on Earth

The behavior of this system is shown in Figure 3.10. The results in these figures show that the updated design produces a continuous rolling motion without stalling or slipping on difficult
terrain. As in the previous simulations, actuation is stopped after 10 seconds. Here, however, the system stops quickly because of the high rolling resistance of the terrain.

To further test the updated design, a gradually inclined ‘moderate terrain’ was generated. The terrain characteristic coefficients for moderate terrain were chosen to be equivalent to a dirt road or relatively hard sand [76]. Values of environmental parameters that are different from the previous simulation are highlighted in Table 3.4.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Environment parameters</td>
<td>Gravity</td>
<td>$g$</td>
<td>9.81</td>
<td>m/s$^2$</td>
</tr>
<tr>
<td></td>
<td>Rolling resistance</td>
<td>$C_{rr}$</td>
<td>0.05</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Coefficient of static friction</td>
<td>$\mu_s$</td>
<td>0.3</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Slope of terrain</td>
<td>$\beta$</td>
<td>3</td>
<td>Degrees</td>
</tr>
</tbody>
</table>

Figure 3.11 shows the results of this simulation, while the ground reaction forces that were monitored for slip and are shown in Figure 3.12. The top and middle subplots in Figure 3.12 plot the variation of $|\vec{F}_r|$ and $|\vec{F}_N|$ with time, respectively. The bottom subplot shows the ‘slip ratio’ $\frac{|\vec{F}_r|}{\mu_s|\vec{F}_N|}$, where the system is in the no slip regime while this ratio is less than 1. The top sub-plot in Figure 3.11 shows the input leg actuations with time and the bottom four plots show the response of the system in time-lapse in four separate time windows. The movement of the center of mass of each leg is tracked using a circular marker. The observer in these plots moves along with the system. The top left plot shows the system starting from rest, then leg 4
opening and closing before it interferes with the ground. The remaining plots depict how the controller cyclically actuates the legs based on the angular position and velocity of the system.

Figure 3.11: Simulation 4 - Actuated rolling of Candidate 2 up a slope on moderate terrain on Earth (Time Lapse Plots)
Figure 3.12: Simulation 4 – Ground reaction forces monitor

From these figures, it is observed that the rover successfully uses actuated rolling to traverse a gradual incline on intermediate terrain without stalling or slipping. After 10 seconds, when actuation of the legs are stopped, the rover coasts to a halt and then begins to roll back down the slope under the influence of gravity.

3.3.2 Mars Simulations

In the previous simulations it was observed that the capability of the system is governed both by the design and the terrain. In the following simulations, design candidates are subjected to Mars gravity and the ability to perform actuated rolling is explored.

To begin this study, the updated design (Candidate 2) from the previous section is simulated using difficult terrain parameters in a Mars environment, as shown in Table 3.5. The plots in Figure 3.13 show the results of the simulation, where it is observed that the rover has
difficulties maintaining a continuous rolling motion. This difficulty arises from the inability of
the third leg to change the angular position of the chassis in such a way that the next leg in the
sequence can constructively contribute to the rolling motion. Using the design parameters
listed in Table 3.3, the system stalls in approximately 6 seconds.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Environment</td>
<td>Gravity</td>
<td>$g$</td>
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<td>m/s$^2$</td>
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<td>Rolling resistance</td>
<td>$C_{rr}$</td>
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<td>-</td>
</tr>
<tr>
<td>Coefficient of static friction</td>
<td>$\mu_s$</td>
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<td></td>
<td>-</td>
</tr>
<tr>
<td>Slope of terrain</td>
<td>$\beta$</td>
<td>0</td>
<td>Degrees</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3.13. Simulation 5 - Actuated rolling of Candidate 2 on difficult terrain on Mars

Initial efforts to overcome stall focused on scaling the model so that it would have the same
weight on Mars that the ‘Candidate 2’ configuration had on Earth. Trial simulations of this
scaled up model were only slightly better at generating actuated rolling, but were marred by an increased occurrence of slipping. Similar results were found when the masses of the legs were increased, or when the center of mass of each leg was moved further out from the center of the system.

It was found that increasing only the inertia of the rover chassis had three positive effects. First, the system was better able to maintain actuated rolling motion. Second, the tendency to slip was reduced because the inertial accelerations of all the masses were reduced. This caused a reduction in the sum of external forces on the system in such a way that Equation (3.10) held. While the peak of the quantity $\mu |\bar{F}_N|$ remained similar, the peak frictional reaction force $|\bar{F}_r|$ was reduced, meaning that the system was less prone to slip. The third positive effect was that the legs had more time to retract before interfering with the ground because the overall motion of the system was slower.

A third candidate design was created where all the design parameters in Table 3.3 remain fixed, except the mass of the chassis $M$ is now 17.5 Kg and the inertia $I_{\bar{b}}$ is 2 kg/m². Simulation results for this system are shown in Figure 3.14. Here, the design is able to maintain continuous actuated rolling motion without stalling or slipping. When actuation is stopped at 15 seconds, the system reaches a rest state almost immediately.
3.3.3 Discussion

The candidate designs proposed in the above simulations demonstrate that rover performance under actuated rolling is a function of rover design and the environment in which it operates. These configurations were established through iterative trial and error, and represent only a sampling of the capabilities that such an architecture can achieve. Finding the performance bounds associated with actuated rolling needs to involve a formal design of experiments study to fully sample and characterize the design space.

The linear speeds of the rover in the above simulations under actuated rolling are significant in the context of rover design. The results in Figure 3.14 correspond to an average linear speed of 0.19 m/s, which suggests that actuated rolling could be used as a mode of locomotion in conducive environments. In the context of the spherical TRREx design, actuated rolling might
be used to roll small distances between slopes to limit the need of transforming between rolling and roving mode. Further, simulations in this section suggest that it could also be used to travel flat or slightly inclined terrain if needed.

3.4 Construction and Development of the Prototype and Microcontroller for the Cylindrical TRREx

3.4.1 Hardware

Structure:

A cylindrical prototype of the TRREx was constructed to facilitate the experimental validation of simulation results obtained from the dynamic model. The prototype, like the multi-body system modeled above, is a 5-body system with a central chassis and four legs hinged to this chassis. The physical prototype differs, however, from the conceptual model seen in Figure 3.1 in that the physical prototype rolls on circular discs, rather than on the curved surfaces of its legs (see Figure 3.15). This was done to avoid machining curved surface areas; however, given the assumptions made in creating the mathematical model of the conceptual cylindrical TRREx, (i.e. we assumed that the surface of contact is always a continuous circular shape) the equations of motion for the physical prototype will be the same as for the conceptual model after the mass and moment of inertia properties of the disks are incorporated into the mass and moment of inertia properties of the chassis. Hence, both the conceptual and the physical cylindrical TRREx models are completely described by the same set of physical parameters (see Table 3.6) namely, the outer radius of the cylindrical surface($R_o$), the location of the center of mass of the leg with respect to the hinge($l_z$ and $l_y$), the location of a hinge with respect
to the center of mass of the chassis ($h_1$ and $h_2$), the mass of the chassis ($M$) and each leg ($m_l$) and the inertia about the axis of rotation of the chassis ($I_{zB}$) and each leg ($I_{zC_j}$) about the respective points $B$ and $C_j$.

Figure 3.15: Prototype (left) in comparison with the dynamically modeled system (right)

Also provided for in the physical prototype is the ability to modify the mass properties of the legs between experimental runs by adding lead weights at the ends of the legs (shown as small grey circular disks in Figure 3.15). This enables the validation of the results from the dynamic models for different points in the design space.

**Electronics:**

The legs are driven by high speed linear motors with potentiometer feedback and run on a 12 volt DC supply (see Figure 3.16). They are driven by pulse-width-modulation through an H-bridge. A multi-axis accelerometer is used as an orientation sensor. It determines the gravity vector direction in the chassis frame from which the micro-controller is able to compute the
orientation of the chassis. A National Instruments micro-controller, the Sb-RIO 9611 [78] is used to implement the control system. It has a 266 MHz processor running a RTOS (real-time operating system), 128 MB storage, 64 MB RAM and has a 1M gate Xilinx Spartan FPGA. The board is powered by a 24 volt DC supply. There are two battery packs: a 12 V 10,000 mAh battery to run the motors and a 24 V 3800 mAh battery to run the controller. It should be noted that care was taken while mounting all the electronic components to ensure that the center of mass of the entire system when all the legs are closed remained at the center of the chassis.

Figure 3.16: Prototype and Hardware (Top-left: System, Top-middle: Legs and actuators, Top-right: Controller, Bottom-left: H-bridge, Bottom-middle: Accelerometer, Bottom-right: Battery packs)
3.4.2 Software

The microcontroller can be programmed using the National instruments proprietary graphical programming software known as LabVIEW [79]. The processor onboard runs a RTOS which interacts with the FPGA module which in turn interfaces with hardware I/O. As the role of the microcontroller (target) is to perform simultaneous control and data acquisition while the system is in motion, a ‘headless’ architecture was chosen for the embedded system on the TRREx prototype, i.e. all data acquisition, control and storage is performed on the target and there is no need for interaction with a computer (host). Once the target memory is full, data can be transferred to another storage device using standard FTP protocol over an Ethernet connection.

A headless architecture has two top level ‘Virtual Instruments’ (VI’s) running asynchronously; one on the RTOS and the other on the FPGA. An architecture diagram showing the processes and interconnecting communication paths is depicted in Figure 3.17. The Data acquisition

![Figure 3.17: Embedded system architecture](image)
loop/process on the FPGA records the values at the analog inputs of the target which are connected to the potentiometers and the accelerometer. This data along with a timestamp generated by the 40 MHz FPGA clock is streamed through a single DMA (dynamic memory allocation) FIFO (first-in-first-out) stream in an interleaved fashion and is unpacked on the data logger process on the Real-time VI (RT VI) using decimation before it is saved onto the onboard memory. Simultaneously this data is also made available to the control loop process on the FPGA (via inter-process communications), which uses the data to compute the orientation of the chassis and the positions of the legs. This information is shared via current value tags with the Dynamic range computation loop on the RT VI that further computes the rotational velocity of the chassis and the dynamic ranges. This information is again relayed back to the control loop on the FPGA VI (via tags) which then makes control decisions based on the position of the chassis and dynamic ranges of operation of each leg and sends out control signals to the motors.

While to ensure high throughput at high acquisition rates a DMA FIFO stream was the preferred communication path for data transfer in data acquisition processes, to ensure low latency (at high execution rates) in control, current value tags were the preferred communication path between control processes on the RT VI and the FPGA VI. Messages/Commands were also used to relay infrequent data that facilitate lossless data transfer and storage (acknowledgment flag, buffer overflow flag etc.). The dynamic ranges computation process was offloaded onto the Real-Time processor to take advantage of floating-point math which is more efficiently executed by a processor compared to the FPGA.
Additional logic in the control loop on the FPGA provided for an initiation time for the legs to go to their initial positions (and for filter stabilization), for software implemented limit switches with locking torques to hold the leg in position when fully open or closed and also for hard stops in cases of emergency. The dynamic ranges logic in the RT VI was implemented in exactly the same way as in the mathematical model of the system, i.e. the ranges of operation.
of each leg get updated (as a function of current rotational velocity) every time an event is detected.

### 3.5 Comparison of Simulations versus Experiments

#### 3.5.1 Determination of parameters

<table>
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<tr>
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<th>Description</th>
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<td>Meters</td>
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<td>Location of hinge of leg w.r.t. center</td>
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<td>Meters</td>
</tr>
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<td></td>
<td>$h_2$</td>
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<td>Meters</td>
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<td>$l_y$</td>
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<td>Meters</td>
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<td>Kg</td>
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<td>Mass of each ‘leg’</td>
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<tr>
<td></td>
<td>Inertia about the rotation axis of chassis</td>
<td>$I_{\theta\theta}$</td>
<td>1.2650</td>
<td>Kg.m$^2$</td>
</tr>
<tr>
<td></td>
<td>Inertia about the rotation axis of each leg</td>
<td>$I_{\theta\theta_j}$</td>
<td>0.0082</td>
<td>Kg.m$^2$</td>
</tr>
</tbody>
</table>

In order to compare experimental results with the results produced by the mathematical model, the physical parameters (dimensions, masses and inertias) of the prototype must be determined accurately and input into the model. The geometric and mass properties can be measured directly from the prototype but the inertias are extracted from Solidworks® data. Care was taken to set relatively strict tolerances during the fabrication processes such that the deviation of the mass properties of the parts from the Solidworks® model to the physical prototype were
minimal. The physical parameters for the prototype (with a particular ‘leg design A’) are listed in Table 3.6.

Figure 3.19: Rolling Resistance determination

The determination of the coefficient of rolling resistance was done by experimentally measuring the deceleration during a free coast, assuming this to be a constant value, and using the computational model to find the value of rolling resistance that yields this deceleration [80]. We used two different surfaces in our experiments, as is discussed below. Experimental results showing how rolling resistance was calculated for ‘Surface 1’ is shown in Figure 3.19. The sub-plots starting from the top are position, velocity and acceleration respectively. The system is accelerated to a certain speed and then allowed to freely coast to a halt. In this case a rolling resistance of 0.0035 in the analytical model corresponds to the average deceleration
experienced by the experimental prototype on this surface. Static friction values for the testing surfaces are taken from the literature [81-83]. All environmental parameters for ‘Surface 1’ are listed in Table 3.7.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Environment parameters</td>
<td>Gravity</td>
<td>( g )</td>
<td>9.81</td>
<td>( \text{m/s}^2 )</td>
</tr>
<tr>
<td></td>
<td>Rolling resistance</td>
<td>( C_r )</td>
<td>0.0035</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Coefficient of static friction</td>
<td>( \mu_s )</td>
<td>0.5</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Slope of terrain</td>
<td>( \beta )</td>
<td>0</td>
<td>Degrees</td>
</tr>
</tbody>
</table>

Table 3.7: Environmental Parameters of Surface 1

3.5.2 Comparison of responses

To validate the results from the models, several different cases are presented each representing a different point in the parameter space of the problem. Two different surfaces are considered, ‘Surface 1’, represented by parameters shown in Table 3.7, was a terrazzo floored hallway and ‘Surface 2’, represented by parameters shown in Table 3.8 was a carpeted hallway.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Environment parameters</td>
<td>Gravity</td>
<td>( g )</td>
<td>9.81</td>
<td>( \text{m/s}^2 )</td>
</tr>
<tr>
<td></td>
<td>Rolling resistance</td>
<td>( C_r )</td>
<td>0.0175</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Coefficient of static friction</td>
<td>( \mu_s )</td>
<td>0.75</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Slope of terrain</td>
<td>( \beta )</td>
<td>0</td>
<td>Degrees</td>
</tr>
</tbody>
</table>

Table 3.8: Environmental Parameters of Surface 2
Also, three different mass distributions for the legs were considered (achieved by varying the masses attached to the ends of the legs). The parameters for these, i.e. ‘leg design A’ (extracted from data in Table 3.6), ‘leg design B’ and ‘leg design C’, are shown in Table 3.9.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$l_i$</th>
<th>$l_y$</th>
<th>$m_L$</th>
<th>$I_{zC_i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>Meters</td>
<td>Meters</td>
<td>Kg</td>
<td>Kg.m$^2$</td>
</tr>
<tr>
<td>Leg Design A</td>
<td>0.1967</td>
<td>0.0364</td>
<td>0.8720</td>
<td>0.0082</td>
</tr>
<tr>
<td>Leg Design B</td>
<td>0.2419</td>
<td>0.0500</td>
<td>1.4240</td>
<td>0.0137</td>
</tr>
<tr>
<td>Leg Design C</td>
<td>0.2624</td>
<td>0.0529</td>
<td>1.9510</td>
<td>0.0162</td>
</tr>
</tbody>
</table>

*Case 1: Leg Design A on Surface 1*

First, the system with Leg design A is allowed to perform actuated rolling on surface 1. Data was acquired using the physical prototype and simulation results were generated for this scenario using the mathematical model. It should be noted that for all the simulations, the ground interaction forces were monitored after the integration and checked to ensure that at every time-step $\left| \vec{F}_{gs} \right| < \mu \left| \vec{F}_N \right|$, i.e. that slip did not occur in the run. The plots in Figure 3.20 show the experimental data acquired in comparison with the simulation results. Leg actuations are started at the 5 second mark (true for cases 2-5, below, as well) and the system is allowed to perform actuated rolling for 20 seconds, then actuations are stopped at the 25 second mark. The top subplot shows the leg actuations, the middle subplot shows the chassis position and the bottom subplot shows the velocity of the chassis with time. It can be seen from this
comparative plot that there is good correspondence between experimentally observed data and simulation results with the system using Leg design A while rolling on surface 1. (Note that the difference in leg positions versus time between the experiment and the simulation is because of the fact that the opening and closing angles of a particular leg (see Figure 3.6) were reached at slightly different instances of time in the physical system as compared to the simulation.)

Case 2: Leg Design A on Surface 2

Next the same system is placed on ‘Surface 2’ which has a higher rolling resistance than ‘Surface 1’. The actuation period is identical to Case 1, and results of both the experiment and simulation are shown in Figure 3.21. It is seen in this case that the system starts to roll, but is
not able to roll enough for the next leg in sequence to constructively contribute to the perpetuation of rolling motion. Thus the system with this design configuration (Leg design A) experiences ‘stall’ and is not able to achieve a continuous actuated rolling motion on ‘Surface 2’. It can be seen from the comparative plot that this ‘stall’ is also predicted by the simulation.

![Cylindrical TRREx – Comparison of Numerical Simulation and Experimental results](image)

Figure 3.21: Experimental Validation - ‘Leg Design A’ on ‘Surface 2’.

**Case 3: Leg Design B on Surface 2**

In the next case ‘Leg design B’ is mounted on the system and the system is now allowed to roll on ‘Surface 2’. The actuation period is identical to Case 1, and the results are shown in Figure 3.22. Since Leg design B increases the mass of the leg and moves the center of mass further out compared to Leg design A, it now enables the system to achieve continuous actuated...
rolling without stalling. Again, simulation predictions are consistent with experimental observations.

![Graph showing comparison of numerical simulation and experimental results.](image)

**Figure 3.22**: Experimental Validation - 'Leg Design B' on 'Surface 2'.

**Case 4: Leg Design B on Surface 1 with Dynamic ranges**

For the next case, the system with Leg design B is now allowed to roll on surface 1. Simulations and experiments showed that the system tends to reach higher rolling velocities quickly and the legs could not be retracted in time to avoid the system rolling over an opened leg (i.e. ‘ground interference’ occurs). To avoid this, dynamic ranges with a limiting velocity of \( \Omega_L = 4\pi \text{ rad/sec} = 720^\circ/\text{sec} \) were used (refer section 3.2.3). (Note that for all previous simulations static ranges were used, i.e. the range of operation of the legs did not depend on
velocity. Equivalently in the simulation $\Omega_L = 2\pi \times 10^{16}$ rad/sec was used). Actuated rolling is allowed for 10 seconds and then the system is allowed to coast. The comparative results of experimental data and simulation are shown in Figure 3.23.

![Cylindrical TRREx – Comparison of Numerical Simulation and Experimental results](image)

Figure 3.23: Experimental Validation - 'Leg Design B' on 'Surface 1' with Dynamic Ranges

**Case 5: Leg Design C on Surface 2**

In this case Leg design C is mounted on the system which in comparison to Leg design B further increases the mass of the leg and moves the center of mass out towards the tip of the leg. Actuated rolling is allowed for 10 seconds and the system is allowed to coast to a halt. The comparative plots of simulation predictions and experimental data are shown in Figure 3.24 and a good correspondence between the two is observed.
When the system with Leg Design C was placed on Surface 1 (a surface with lesser rolling resistance than Surface 2) it was observed from both simulations and experiments that the system is more prone to ground interference and dynamic ranges need to be employed to avoid it.

Figure 3.24: Experimental Validation - 'Leg Design C' on 'Surface 2'

By comparing the results at different points in parameter space with multiple experimental runs and observing that the simulation results match well with the experimental data in all cases, the dynamic model of the cylindrical TRREx is validated. The developed model can thus be used to predict accurately enough the behavior of the actual physical system and has applications in analysis, design and control optimization of the system for a given target environment.
For example, some general/characteristic trends observed as we go from leg design A to leg design B to leg design C (assuming the same motor capacity) on a given surface were that the tendency to stall decreases, but the tendency for ground interference increases. These same trends were observed when rolling resistance decreases while keeping a constant leg design. So in a target environment with a higher rolling resistance, a leg design with more mass set out at the tip will tend to roll without stall, but a controller incorporating dynamic ranges with a lower limiting velocity $\Omega_L$ needs to be employed to avoid ground interference.

The experimentally validated mathematical model for the cylindrical TRREx presented in this chapter could also be applied to study actuated rolling performance by defining performance parameters. One example performance parameter that captures both stall tendency and ground interference tendency is the peak velocity achieved before the second actuation. This is an indicator of starting acceleration and (from Figure 3.21, Figure 3.22 and Figure 3.24) increases from 20 deg/sec to 50deg/sec to 65deg/sec as we go from leg design A to B to C, respectively, on Surface 2. Another interesting performance parameter that can be found by letting the simulations run until a steady state is observed is the maximum steady state velocity that is attainable by the system on a given surface.

3.5.3 Discussion on deviations between model and experiments

Although the simulation results exhibit a good match with experimental data, the match is not exact. The differences between the two can be divided up into two main categories: (1) deviations of the mathematical model from the true response of the system due to inaccurate modeling assumptions, and (2) deviations of the mathematical model from the true response
due to experimental errors. Some modeling assumptions that are bound to introduce a deviation are due to the fact that the movements of the parts of the linear motor were ignored, and the motors were assumed to exactly generate the desired motion of the arms. Inaccuracies could also be due to the relatively simple friction and rolling resistance models used. Errors introduced into the experimental data could be due to inaccurate estimation of the physical parameters (both design and environmental) and due to construction challenges such as reducing the free-play of the legs, balancing the system so that the center of mass is exactly at the center of the chassis, and fabricating perfectly circular contact surfaces.

3.6 Conclusions

Mathematical models that describe the ‘actuated rolling’ dynamics of a cylindrical version of the TRREx were developed using the Newton-Euler and the constraint relaxation approaches. An ad hoc control strategy to avoid ‘ground interference’ using ‘dynamic ranges’ was discussed. Initial simulations suggested that actuated rolling could be used as a mode of locomotion in conducive environments and also on slightly inclined terrain if needed. Further, the construction and software development of a physical prototype that facilitates model validation was presented. The results generated by the model were then validated by comparing them with data acquired on experimental runs using the prototype. The comparison was done for several different configurations on two different surfaces, and in each case good correspondence was observed between simulation results and experimental data.
The experimentally validated dynamic model presented in this chapter can thus be used to predict accurately enough the behavior of the actual physical system and can be used to answer questions concerning the capabilities and limitations of the actuated rolling of a TRREx. It not only has direct applications in guiding the design and control optimization of a Spherical TRREx but also provides a foundation for further work on dynamically modeling a Spherical TRREx.
Tumbleweed locomotion relies on the inherent dynamics of a spherical body. The spherical TRREx, designed as a self-propelling rover, can roll, bounce, slip, and interact with its environment through the actuation of its legs. This chapter presents a mathematical model that captures the dynamics of the spherical TRREx in the rolling mode, emphasizing its ability to steer itself through coordinated leg movements and center of mass adjustments. Three case studies illustrate the rover's self-propelling and maneuvering capabilities on flat and sloped surfaces. Furthermore, two parametric analyses reveal how the rover's performance, including its ability to self-propel uphill and maneuver downhill, is influenced by its design features.
complicated dynamical system due to the fact that it is a multi-body system which accepts control inputs that can be used to dictate to a certain extent the motion of the rover. In this section the governing equations for the Spherical TRREx during rolling are developed from first principles following the classical Newton-Euler formulization. Both slip and no slip situations are considered. (For details on a model of the Spherical TRREx using the constraint relaxation technique refer Appendix B)

4.2.1 System Description

The Spherical TRREx during rolling is considered as a multi-body system with nine bodies; one central frame or ‘chassis’ and eight ‘legs’ that are each connected to the chassis via a single-degree-of-freedom hinge. The central chassis has six degrees of freedom in three dimensional space (three rotational and three translational). The corresponding six second-order ordinary differential equations of motion form the basis for this analysis. Three of these
equations of motion come from the application of conservation of angular momentum, and the
three remaining equations result from applying Newton’s second law to the system.

The legs are labeled 1 thorough 8 as shown in Figure 4.1 and are assumed to have identical
mass properties. Single point contact is assumed at all times between the spherical surface of
the TRREx and the terrain and also it is assumed that the contribution of the movements of the
parts of the motors that actuate the legs to the overall dynamics of the system can be considered
to be negligible.

4.2.2 Definition of Frames

The following convention is adopted to assign reference frames for each body in the multi-
body system - the origin of the body reference frame is placed at the center of mass of that
body and its unit axes are aligned in the direction of that body’s principle axes. Thus, looking
at Figure 4.1, if the point $B$, denoted by the diamond marker, is the center of mass of the
chassis (excluding the legs) and the directions $\hat{i}_B$, $\hat{j}_B$ and $\hat{k}_B$ are its principle axes directions,
then the ‘B frame’ denoted by $B$, is a frame with origin at $B$ and unit axes $\hat{i}_B$, $\hat{j}_B$ and $\hat{k}_B$.

Similarly, if the point $C_j$ is the location of the center of mass of the $j^{th}$ leg (denoted in Figure
4.1 by circular markers) and $\hat{i}_{C_j}$, $\hat{j}_{C_j}$ and $\hat{k}_{C_j}$ are its principle axes directions, then $C_j$ is a frame
with its origin at $C_j$ and unit axes as $\hat{i}_{C_j}$, $\hat{j}_{C_j}$ and $\hat{k}_{C_j}$ (for $j=1$ to 8). Apart from these nine
reference frames (see Figure 4.2), an inertially fixed reference frame $O$ is defined, whose
origin is arbitrarily placed at the point of contact between the sphere and the ground at a given

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 instantaneous of time (e.g. \( t = 0 \)) and whose \( \hat{k}_O \) axis is normal and pointing out of the terrain surface (the \( \hat{i}_O - \hat{j}_O \) plane is parallel to the terrain surface). The chassis frame (i.e. \( \bar{B} \) frame) in Figure 4.2 is aligned with inertial frame \( \bar{O} \) at the instant shown.

![Figure 4.2: Frame definitions for the Spherical TRREx (Left - looking along the \( \hat{i}_B \) axis, Right – looking along the \( \hat{j}_B \) axis)](image)

### 4.2.3 Derivation of Governing Equations

Using a Newton-Euler approach, along with Euler angles to represent the spatial orientation of the system, a set of 12 first order ordinary differential equations describing the dynamics of the chassis can be generated (the motion of the legs relative to the chassis will be treated as control inputs), where the 12 variables are: the positions of the center of mass of the chassis (}
in the \( \hat{i}_O, \hat{j}_O, \hat{k}_O \) directions with respect to \( O \) frame, the corresponding 3 inertial velocities of the center of mass of the chassis \( (\hat{x}_{BO}, \hat{y}_{BO}, \hat{z}_{BO}) \), the 3 Euler angles that represent the orientation of the \( B \) frame embedded in the chassis with respect to the inertial frame \( (\psi - \theta - \phi) \) and the 3 angular velocities \( (\omega_{x_B}, \omega_{y_B}, \omega_{z_B}) \) of the \( B \) frame (and hence the chassis) with respect to the inertial frame as expressed in the \( B \) frame coordinates.

The 3 kinematic equations relating the Euler angles \( (\psi - \theta - \phi) \) to the angular velocities for a 3-2-1 rotation sequence are given by:

\[
\begin{bmatrix}
\phi \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
1 & \sin(\phi) \tan(\theta) & \cos(\phi) \tan(\theta) \\
0 & \cos(\phi) & -\sin(\phi) \\
0 & \sin(\phi) \sec(\theta) & \cos(\phi) \sec(\theta)
\end{bmatrix}
\begin{bmatrix}
\omega_{x_B} \\
\omega_{y_B} \\
\omega_{z_B}
\end{bmatrix}
\]  

(4.1)

It should be noted, however, that the matrix in equation (4.1) becomes singular when \( \theta \) is an odd multiple of \( \pi / 2 \) and can lead to numerical instabilities.

A more robust method is to represent the orientation of the chassis using quaternions [84]. Employing this technique replaces the 3 equations in (4.1) with 4 equations that propagate the quantities \( q_{0B}, q_{1B}, q_{2B} \) and \( q_{3B} \). The relationship between \( q_{0B}, q_{1B}, q_{2B}, q_{3B} \) and the angular velocities are given by:
\[
\begin{bmatrix}
\dot{q}_{0B} \\
\dot{q}_{1B} \\
\dot{q}_{2B} \\
\dot{q}_{3B}
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
0 & -\omega_{\gamma B} & -\omega_{\zeta B} & -\omega_{\xi B} \\
\omega_{\beta B} & 0 & -\omega_{\gamma B} & -\omega_{\zeta B} \\
\omega_{\gamma B} & -\omega_{\beta B} & 0 & -\omega_{\xi B} \\
\omega_{\zeta B} & -\omega_{\gamma B} & -\omega_{\zeta B} & 0
\end{bmatrix} \begin{bmatrix}
\dot{q}_{0B} \\
\dot{q}_{1B} \\
\dot{q}_{2B} \\
\dot{q}_{3B}
\end{bmatrix}
\]  
(4.2)

Thus when using quaternions to represent orientations, there are a total of 13 equations and 13 states that completely describe the system at any given instant of time \((x_{B0}, y_{B0}, z_{B0}, x_{BO}, y_{BO}, z_{BO}, q_{0B}, q_{1B}, q_{2B}, q_{3B}, \omega_{\beta B}, \omega_{\gamma B}, \text{ and } \omega_{\zeta B})\).

The 3 state equations corresponding to the states \(x_{B0}, y_{B0}, z_{B0}\) are obtained directly from the definitions of the states, and equation (4.2) gives the state equations corresponding to the states \(q_{0B}, q_{1B}, q_{2B}, q_{3B}\). Newton’s second law and the relationship between external torque and the angular momentum of the system can then be used to generate the rest of the state equations (corresponding to the states \(x_{BO}, y_{BO}, z_{BO}, \omega_{\beta B}, \omega_{\gamma B}, \omega_{\zeta B}\)). The relationship between external torque and the angular momentum of the system is presented first, followed by the application of Newton’s second law.

From the Newton-Euler method described in Meirovitch [65], the total external torque about \(B\) acting on the system \(\vec{T}_{B,sys}\) is related to the change in angular momentum of the system about \(B\) as follows:

\[
\left(\vec{T}_{B,sys}\right)_{ext} = \frac{d}{dt}\left(\vec{h}_{B,sys}\right) + \vec{\omega}_{B/O} \times m_L \sum_{j=1}^{8} \vec{v}_{c/O}^j
\]  
(4.3)
In this equation, \( \vec{h}_{B,\text{sys}} \) is the angular momentum of the system about \( B \) with respect to the inertial frame, \( m_L \) is the mass of each leg, \( \vec{v}_{B/O} \) is the inertial velocity of the center of mass of the chassis and \( \vec{v}_{C/O} \) is the inertial velocity of the center of mass of the \( j \)th leg. The term \( \vec{h}_{B/O} \times m_L \sum_{j=1}^{s} \vec{v}_{C/O} \) appears because the torques and angular momentum are written about a point \( B \) which is not the center of mass of the entire system [65].

![Figure 4.3: External forces on the Spherical TRREx.](image)

Next, in equation (4.3) the external torques acting on the system are written in terms of the forces they are derived from, and the right hand side is written in terms of inertias, angular velocities and correction terms arising from kinematic relations [65]. The external forces acting on the system are forces due to gravity and interaction forces between the ground and the
spherical surface. The ground is modeled as flat terrain (although it can be sloped with respect to the horizontal), and \( \vec{F}_N \) is the normal reaction, \( \vec{F}_f \) is the frictional force and \( \vec{F}_r \) is the rolling resistance force. These forces are illustrated in Figure 4.3, where for ease of illustration the terrain is portrayed as horizontal.

The sum of external torques about \( B \) acting on the system is:

\[
\left( \vec{T}_{B,sys} \right)_{ext} = \vec{r}_{p/B} \times (\vec{F}_f + \vec{F}_N) + m_l \sum_{j=1}^{s} \left( \vec{r}_{c_{Bj}/B} \times \vec{g} \right)
\]

where \( \vec{r}_{p/B} \) is a vector pointing from the center of mass of the chassis to its point of contact with the ground and \( \vec{r}_{c_{Bj}/B} \) is the vector from the center of mass of the chassis to the center of mass of the \( j \)th leg. The gravity vector is denoted here by \( \vec{g} \).

The first term on the right hand side of equation (4.3) can be written as:

\[
\frac{\partial}{\partial t} \left( \vec{\omega}_{B,sys} \right) = \frac{\partial}{\partial t} \left( \vec{\omega}_{B,chassis} \right) + \sum_{j=1}^{s} \frac{\partial}{\partial t} \left( \vec{\omega}_{B,leg(j)} \right)
\]

where \( \vec{\omega}_{B,chassis} \) is the angular momentum of the chassis about \( B \) with respect to the inertial frame and \( \vec{\omega}_{B,leg(j)} \) is the angular momentum of the \( j \)th leg about \( B \) with respect to the inertial frame.
But it can be shown [65] that the angular momentum of a body about a point other than its center of mass can be written as a function of the velocity of the center of mass and the angular momentum of the body about its center of mass; therefore the above equation can be rewritten as follows:

\[
\frac{d}{dt} (\vec{J}_{B,sys}) = \frac{d}{dt} (\vec{J}_{B,chassis}) + \sum_{j=1}^{\mathcal{S}} \left( \frac{d}{dt} (\vec{J}_{C,j,leg(j)}) + \frac{d}{dt} \left( \vec{r}_{C,j/B} \times m_l \vec{v}_{C,j/O} \right) \right)
\]

Using the product rule and transport theorem [66] to write the quantities in appropriate body frames we have:

\[
\frac{d}{dt} (\vec{J}_{B,sys}) = \frac{d}{dt} (\vec{J}_{B,chassis}) + \frac{\partial}{\partial \vec{x}} \times \vec{J}_{B,chassis} + \sum_{j=1}^{\mathcal{S}} \left( \frac{\partial}{\partial \vec{x}} \times \vec{J}_{C,j,leg(j)} + \frac{\partial}{\partial \vec{x}} \times \vec{J}_{C,j,leg(j)} \right)
\]

\[
= \sum_{j=1}^{\mathcal{S}} \left( \frac{\partial}{\partial \vec{x}} \times \vec{J}_{C,j,leg(j)} + \frac{\partial}{\partial \vec{x}} \times \vec{J}_{C,j,leg(j)} \right) + \sum_{j=1}^{\mathcal{S}} \left( m_L \left( \vec{v}_{C,j/B} \times \vec{v}_{C,j/O} \right) + \left( \frac{\partial}{\partial \vec{x}} \times \vec{r}_{C,j/B} \right) \times \vec{v}_{C,j/O} + \vec{r}_{C,j/B} \times \vec{v}_{C,j/O} \right)
\]

\[
= \sum_{j=1}^{\mathcal{S}} \left( m_L \left( \vec{v}_{C,j/B} \times \vec{v}_{C,j/O} \right) + \left( \frac{\partial}{\partial \vec{x}} \times \vec{r}_{C,j/B} \right) \times \vec{v}_{C,j/O} + \vec{r}_{C,j/B} \times \vec{v}_{C,j/O} \right)
\]

(4.5)

where \( \vec{v}_{C,j/B} = \frac{d}{dt} \left( \vec{r}_{C,j/B} \right) \) is the velocity of center of mass of the \( j \)th leg with respect to the \( \vec{B} \) frame and \( \frac{\partial}{\partial \vec{x}} \vec{a}_{C,j/O} \) is the inertial acceleration of the center of mass of the \( j \)th leg.

Substituting equations (4.4) and (4.5) into equation (4.3) we obtain:
\[
\frac{b}{d} \left( \frac{\partial \vec{h}_{B,chassis}}{\partial t} + \vec{\omega} \times \vec{h}_{B,chassis} \right) + \sum_{j=1}^{N} \left( \vec{c}_j \frac{d}{dt} \left( \frac{\partial \vec{h}_{C,j,leg(j)}}{\partial t} + \vec{\omega} \times \vec{h}_{C,j,leg(j)} \right) \right) + \sum_{j=1}^{N} \left( \vec{v}_{C,j} \times \vec{a}_{C,j,leg(j)} + \left( \vec{a}_{C,j,leg(j)} \times \vec{v}_{C,j} \right) \times \vec{h}_{C,j,leg(j)} \right)
\]

(4.6)

Recall that the chassis reference frame axes \( \hat{i}_B, \hat{j}_B \) and \( \hat{k}_B \) were by definition chosen to be the principal axes, thus let the principle inertia components computed about these axes be \( I_{xB}, I_{yB} \) and \( I_{zB} \) respectively. Writing the angular velocity of the \( \vec{B} \) frame with respect to the \( \vec{O} \) frame as \( \vec{\omega} = \omega_{xB} \hat{i}_B + \omega_{yB} \hat{j}_B + \omega_{zB} \hat{k}_B \), it can be shown [65] that the angular momentum can be written in the body frame as, \( \vec{h}_{B,chassis} = I_{xB} \omega_{xB} \hat{i}_B + I_{yB} \omega_{yB} \hat{j}_B + I_{zB} \omega_{zB} \hat{k}_B \) and consequently the quantity \( \frac{b}{d} \left( \frac{\partial \vec{h}_{B,chassis}}{\partial t} + \vec{\omega} \times \vec{h}_{B,chassis} \right) \) in equation (4.6) can be written in the body frame as,

\[
\frac{b}{d} \left( \frac{\partial \vec{h}_{B,chassis}}{\partial t} + \vec{\omega} \times \vec{h}_{B,chassis} \right) = \left( I_{xB} \omega_{xB} - (I_{yB} - I_{zB}) \omega_{yB} \omega_{zB} \right) \hat{i}_B \\
+ \left( I_{yB} \omega_{yB} - (I_{zB} - I_{xB}) \omega_{xB} \omega_{zB} \right) \hat{j}_B \\
+ \left( I_{zB} \omega_{zB} - (I_{xB} - I_{yB}) \omega_{xB} \omega_{yB} \right) \hat{k}_B
\]

Similarly, let \( I_{xC_j}, I_{yC_j} \) and \( I_{zC_j} \) be the principle inertia components about the \( \hat{i}_{C_j}, \hat{j}_{C_j} \) and \( \hat{k}_{C_j} \) axes, respectively, of the \( j^{th} \) leg, and let \( \gamma_{jd} \) be the angle at a given instant of time between the
\( \hat{e}_j \) axis and \( \hat{i}_\mathcal{B} - \hat{j}_\mathcal{B} \) plane measured in the positive \( \hat{k}_\mathcal{C} \) direction (see Figure 4.2). Then the angular velocity of the \( \mathcal{C}_j \) frame with respect to the \( \mathcal{O} \) frame can be written as 
\[ \dot{\omega}_{\mathcal{O}\mathcal{C}_j} = \dot{\omega}_{\mathcal{O}\mathcal{B}} + \omega_{\mathcal{B}\mathcal{B}} \hat{i}_\mathcal{B} + \omega_{\mathcal{B}\mathcal{B}} \hat{j}_\mathcal{B} + \omega_{\mathcal{B}\mathcal{B}} \hat{k}_\mathcal{C}_j \] (note that the single degree of freedom hinge between each leg and the chassis restricts the angular motion of each leg to motion about a single axis, i.e. \( \omega_{\mathcal{B}\mathcal{B}} \hat{i}_\mathcal{B} + \omega_{\mathcal{B}\mathcal{B}} \hat{j}_\mathcal{B} + \omega_{\mathcal{B}\mathcal{B}} \hat{k}_\mathcal{C}_j \)) and the quantity 
\[ \frac{\mathcal{C}_j d}{dt} \left( \hat{e}_{\mathcal{C}_j, \mathcal{B}} \right) + \hat{e}_{\mathcal{C}_j, \mathcal{B}} \times \hat{e}_{\mathcal{C}_j, \mathcal{B}} \] in equation (4.6) can also be written in the leg frame in terms of inertias, chassis angular velocities, chassis angular accelerations and leg angular velocities and leg angular accelerations (\( \dot{\gamma}_j \) and \( \ddot{\gamma}_j \)).

Thus, equation (4.6) is a vector equation that can be written with the system mass parameters, geometric parameters, environmental parameters, leg motion inputs and the 13 states which are all known at the initial time on the right hand side and the terms containing the six first derivatives of the states (\( \dot{\mathcal{X}}_\mathcal{B}, \dot{\mathcal{Y}}_\mathcal{B}, \dot{\mathcal{Z}}_\mathcal{B}, \dot{\omega}_{\mathcal{B}}, \dot{\omega}_{\mathcal{B}}, \dot{\omega}_{\mathcal{B}} \)) on the left hand side. Thus the vector equation (4.6) represents 3 equations and 6 unknowns. Note that a rotation matrix computed from the orientation quaternion of the chassis [85, 86] will need to be used to transform the required vector quantities in equation (4.6) into the chassis frame, when formulating these equations in a form that can be solved numerically. For example, in the process of preparing equation (4.6) for numerical integration, \( \vec{g} = -g \hat{k}_\mathcal{O} \) (when the terrain is horizontal) needs to be expressed in the \( \mathcal{B} \) frame as, \( \vec{g} = g_{\mathcal{B}\mathcal{B}} \hat{i}_\mathcal{B} + g_{\mathcal{B}\mathcal{B}} \hat{j}_\mathcal{B} + g_{\mathcal{B}\mathcal{B}} \hat{k}_\mathcal{B} \) using:
This is accomplished by using the relation [85, 86]:

\[
\begin{bmatrix}
g_{xB} \\
g_{yB} \\
g_{zB}
\end{bmatrix} = ^b [C]^O \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}
\]

\[
\bar{b} [C]^O = \begin{bmatrix}
(q_{0B}^2 + q_{1B}^2 - q_{2B}^2 - q_{3B}^2) & 2(q_{1B}q_{2B} + q_{0B}q_{3B}) & 2(q_{1B}q_{3B} - q_{0B}q_{2B}) \\
2(q_{1B}q_{2B} - q_{0B}q_{3B}) & (q_{0B}^2 - q_{1B}^2 + q_{2B}^2 - q_{3B}^2) & 2(q_{2B}q_{3B} + q_{0B}q_{1B}) \\
2(q_{1B}q_{3B} + q_{0B}q_{2B}) & 2(q_{2B}q_{3B} - q_{0B}q_{1B}) & (q_{0B}^2 - q_{1B}^2 - q_{2B}^2 + q_{3B}^2)
\end{bmatrix}
\]

Depending on which frame is originally used to compute \( \vec{F}_{fr} \) and \( \vec{F}_N \), these force vectors may need to be treated similarly when preparing equation (4.6) for numerical integration.

Now, applying Newton’s second law to the system yields:

\[
M \ddot{\vec{a}}_{y/O} + m_L \sum_{j=1}^{8} \ddot{\vec{a}}_{c_j/O} = M\ddot{\vec{g}} + 8m_L\ddot{\vec{g}} + \vec{F}_{fr} + \vec{F}_N + \vec{F}_R
\]  

(4.7)

In the above equation, \( M \) is the mass of the chassis, \( m_L \) is the mass of each leg, \( \ddot{\vec{a}}_{y/O} \) and \( \ddot{\vec{a}}_{c_j/O} \) are the inertial accelerations of the center of mass of the chassis and the acceleration of the center of mass of the \( j \)th leg, respectively, and the right hand side of the equation is the sum of all of the external forces acting on the system (gravity, friction, the normal reaction of the ground acting on the rover, and rolling resistance forces). At this point the development splits into two cases (i) when there is no slip and (ii) when there is slipping. Note that in both the cases the rolling resistance is modeled as \( \vec{F}_R = C_{rr} |\vec{F}_N| \hat{e}_b \), where \( C_{rr} \) is the coefficient of rolling
resistance between the terrain and the surface of the TRREx and \( \hat{e}_b \) is a unit vector opposing the motion of the chassis.

**No Slip:**

When there is no slip an additional kinematic constraint given by, \( \Omega_{\mathcal{O}} = \omega_{\mathcal{B}} \times \mathbf{r}_{\mathcal{B}/\mathcal{P}} \) is satisfied and therefore the inertial acceleration of the center of mass of the chassis can be written as,

\[
\frac{\partial}{\partial \mathcal{O}} \mathbf{a}_{\mathcal{B}/\mathcal{O}} \bigg|_{\mathcal{O}} = \frac{d}{dt} \left( \omega_{\mathcal{B}} \times \mathbf{r}_{\mathcal{B}/\mathcal{P}} \right)
\]

(4.8)

However in this case the frictional reaction force \( \bar{F}_f \) becomes an unknown that needs to be solved using equation (4.7) and monitored to determine when slipping starts to occur. The condition for no slip is given by \( |\bar{F}_f| < \mu_s |\bar{F}_N| \) where \( \mu_s \) is the coefficient of static friction between the terrain and the surface of the TRREx.

Thus the vector equation (4.8) represents three equations in the same set of 6 unknowns \( \bar{x}_{\mathcal{B}/\mathcal{O}}, \bar{y}_{\mathcal{B}/\mathcal{O}}, \bar{z}_{\mathcal{B}/\mathcal{O}}, \bar{\omega}_{\mathcal{B}/\mathcal{O}}, \bar{\omega}_{\mathcal{B}/\mathcal{O}}, \bar{\omega}_{\mathcal{B}/\mathcal{O}} \). Therefore for the no slip case, equations (4.8) and (4.6) together give 6 equations and 6 unknowns that can be solved for by standard linear algebra techniques.

**Slip:**

When the no slip condition is violated then the frictional force is known and takes on a value given by, \( \bar{F}_f = \mu_k |\bar{F}_N| \hat{e}_p \), where \( \hat{e}_p \) is a unit vector in the direction opposite to the direction of
velocity at the contact point, and equation (4.7) now represents 3 equations in the 6 unknowns
\( \ddot{x}_{BO}, \ddot{y}_{BO}, \ddot{z}_{BO}, \dot{\omega}_{iB}, \dot{\omega}_{jB}, \dot{\omega}_{kB} \). Thus for the slip case, equations (4.7) and (4.6) together give 6 equations in 6 unknowns that can be solved by standard linear algebra techniques.

Thus by switching between two sets of 13 state equations (3 from the definition of the states, 4 from equation (4.2) and 6 from either, (4.8) and (4.6) for the no slip case or (4.7) and (4.6) for the slip case) the system dynamics during both slip and no slip is modeled.

4.2.4 Integration of state equations

A flowchart illustrating the algorithm used to monitor slip and switch between the two sets of equations is shown in Figure 4.4. At the first time-step no slip is assumed and the ground reactions \( \vec{F}_F \) and \( \vec{F}_N \) are computed using equation (4.7). Then the no slip assumption is checked by verifying that \( |\vec{F}_F| < \mu |\vec{F}_N| \). Also the initial conditions are checked to ensure that

\( ^O\gamma^\frac{\vec{r}}{B/\gamma} = ^O\hat{\omega}^B \times \vec{r}_{B/\gamma} \). If both of these are true then the states are propagated using the 13 state equations corresponding to the no slip case; otherwise the 13 state equations that model slipping are used to propagate the states. The state equations are switched back from the slip to the no slip equations when \( ^O\gamma^\frac{\vec{r}}{B/\gamma} \) becomes equal to \( ^O\hat{\omega}^B \times \vec{r}_{B/\gamma} \) (within a specified tolerance).

Note that the implementation of quaternions typically requires a periodic normalization of the quaternion such that it remains of unit magnitude [84, 87]. Thus, given the initial conditions, these equations can be integrated to find the time evolution of the 13 states that completely define the system. The ‘ode45’ integrator in MATLAB® [75] was used to perform the integration.
Figure 4.4: Flowchart of algorithm that switches between slip and no slip cases.

Initial Conditions

Assume no slip and compute \( \vec{F}_p \) and \( \vec{F}_N \)

If

\[
\vec{v}_d / \gamma = \vec{\omega} \times \vec{r}
\]

and

\[
|\vec{F}_p| < \mu |\vec{F}_N|
\]

True (no slip)

Propagate states with ‘no slip’ equations

False (slip)

Propagate states with ‘slip’ equations

If

\[ t \geq t_{\text{end}} \]
4.2.5 Control inputs to the model

Recalling Figure 4.2, if \( \gamma_{\theta, j} \) is the desired angle at a given instant of time between the \( \hat{i}_{C_i} \) axis and \( \hat{i}_B - \hat{j}_B \) plane measured in the positive \( \hat{k}_{C_i} \) direction, then the desired input motions of the legs (functions of time) are given by \( \gamma_{\theta, j}, \gamma_{\theta, j}' \) and \( \gamma_{\theta, j}'' \) for the eight legs i.e. \( j=1 \) to 8. It is assumed in the model that the actuators (linear motors) generate the desired motion exactly.

Initial studies assumed that the angular acceleration of a leg \( \gamma_{\theta, j}'' \) followed a cubic polynomial (in time) from rest position to rest position. But it was observed from experiments on a cylindrical prototype that a piecewise constant acceleration of the piston (of the linear motor between the leg and the chassis), when transformed to the angular acceleration of the leg \( \gamma_{\theta, j}'' \), better characterized the opening and closing motion of the legs (see Appendix A).

4.3 Case Studies

In the following sections, various case studies are presented that demonstrate different capabilities of the TRREx while performing actuated rolling. The quantities that serve as inputs to the mathematical model include geometric parameters (see Figure 4.5), mass parameters, environmental parameters, and the motions of the legs as a function of time as dictated by the rover’s control system. The physical design parameters (geometric and mass) used in the case studies are listed in Table 4.1.
Table 4.1: Physical/Design Parameters of the Spherical TRREx used for case studies.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric parameters</td>
<td>Outer diameter of Spherical surface</td>
<td>$D_s$</td>
<td>1.0000</td>
<td>Meters</td>
</tr>
<tr>
<td></td>
<td>Location of hinge w.r.t. center</td>
<td>$h_1$</td>
<td>0.4705</td>
<td>Meters</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$h_2$</td>
<td>0.1275</td>
<td>Meters</td>
</tr>
<tr>
<td>Mass parameters</td>
<td>Location of COM of leg w.r.t. hinge</td>
<td>$l_x$</td>
<td>0.3317</td>
<td>Meters</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$l_y$</td>
<td>0.0690</td>
<td>Meters</td>
</tr>
<tr>
<td></td>
<td>Mass of ‘chassis’</td>
<td>$M$</td>
<td>47.4224</td>
<td>Kg</td>
</tr>
<tr>
<td></td>
<td>Mass of each ‘leg’</td>
<td>$m_L$</td>
<td>6.0341</td>
<td>Kg</td>
</tr>
<tr>
<td></td>
<td>Inertias of the chassis</td>
<td>$I_{x_B}$</td>
<td>3.6273</td>
<td>Kg.m²</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$I_{y_B}$</td>
<td>3.6273</td>
<td>Kg.m²</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$I_{z_B}$</td>
<td>6.6036</td>
<td>Kg.m²</td>
</tr>
<tr>
<td></td>
<td>Inertias of each leg</td>
<td>$I_{x_C_j}$</td>
<td>0.0773</td>
<td>Kg.m²</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$I_{y_C_j}$</td>
<td>0.2385</td>
<td>Kg.m²</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$I_{z_C_j}$</td>
<td>0.1836</td>
<td>Kg.m²</td>
</tr>
</tbody>
</table>

Figure 4.5: Geometric parameters of Spherical TRREx.
4.3.1 Case study I: End over end Actuated rolling.

In this case study, the rover is placed on a terrain type on Mars (that has the environmental properties listed in Table 4.2) and is made to roll along a straight line. It is initially started at rest with an initial orientation such that opening legs 1 and 5 simultaneously starts to make the system roll in the positive \( \hat{j}_0 \) direction. As the TRREx begins to roll appropriate legs are sequentially actuated to maintain a continuous rolling motion in the desired direction. These leg motions are inputs to the system provided by the controller and are shown in the top sub-plot of Figure 4.6. Note that in this case study, pairs of legs (i.e. legs 1 and 5, 2 and 8, 3 and 7, and 4 and 6) are chosen to be actuated simultaneously.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Environment parameters</td>
<td>Gravity</td>
<td>( g )</td>
<td>3.711</td>
<td>m/s(^2)</td>
</tr>
<tr>
<td></td>
<td>Rolling resistance</td>
<td>( C_{rr} )</td>
<td>0.05</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Coefficient of static friction</td>
<td>( \mu_s )</td>
<td>0.15</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Coefficient of Kinetic friction</td>
<td>( \mu_k )</td>
<td>0.12</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Slope of terrain</td>
<td>( \beta )</td>
<td>0</td>
<td>Degrees</td>
</tr>
</tbody>
</table>

Using the mathematical model developed in the earlier section to simulate the system dynamics given all the inputs, the time response of the 13 states that describe the motion of the chassis can be generated. The time evolution of the 6 states that correspond to translational and angular velocities of the chassis frame are presented in the lower two sub-plots of Figure 4.6. The remaining states give its position and orientation information and this is presented in the form
of a time-lapse plot of the system in 3-dimensional space in Figure 4.7. It can be seen from these figures that by actuating the legs as shown in this case study the TRREx can be made to roll end-over-end, self-propelling itself in a straight line and gradually building velocity.

Figure 4.6: Case Study I – Leg inputs and time evolution of states corresponding to translational and angular velocities of the chassis.
4.3.2 Case study II: Actuated Rolling on chassis demonstration

Another technique to self-propel is to make the TRREx roll on its chassis as opposed to end-over-end actuated rolling. Simulations show that this increases the rolling capacity of the rover in certain situations. This is demonstrated here by placing the same TRREx design as the previous case study on terrain that has a higher rolling resistance ($C_n = 0.1$, used to simulate sandy terrain [76, 77]) and keeping the rest of the environmental parameters the same as in Table 4.2 (the environmental parameters for Case Study II are summarized in Table 4.3). On this terrain, self-propulsion using end-over-end actuated rolling as shown in Case Study I was not achievable, i.e. the TRREx ‘stalled’. In other words the previous actuations were not able to rotate the chassis sufficiently such that the next actuations in sequence could constructively contribute to perpetuate the rolling motion. Actuated rolling on the chassis however

![Figure 4.7: Case Study I - Time-lapse plot showing the evolution of states (orientations and positions) in 3D space.](image)
successfully self-propelled the TRREx and the results of this simulation are shown in Figure 4.8 and Figure 4.9.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravity</td>
<td></td>
<td>( g )</td>
<td>3.711</td>
<td>m/s(^2)</td>
</tr>
<tr>
<td>Rolling resistance</td>
<td></td>
<td>( C_r )</td>
<td>0.1</td>
<td>-</td>
</tr>
<tr>
<td>Coefficient of static friction</td>
<td></td>
<td>( \mu_s )</td>
<td>0.15</td>
<td>-</td>
</tr>
<tr>
<td>Coefficient of Kinetic friction</td>
<td></td>
<td>( \mu_k )</td>
<td>0.12</td>
<td>-</td>
</tr>
<tr>
<td>Slope of terrain</td>
<td></td>
<td>( \beta )</td>
<td>0</td>
<td>Degrees</td>
</tr>
</tbody>
</table>

Table 4.3: Environmental Parameters for Case Study II.

To maintain rolling on the chassis, the inputs provided by the controller (top sub-plot of Figure 4.8) are such that both the legs on either side of the chassis actuate symmetrically. From the lower two sub-plots of Figure 4.8 and from Figure 4.9, it can be seen how the TRREx starts at an initial orientation such that it is resting on its chassis, and as the actuations start it begins to roll on its chassis. The velocity builds and quickly reaches a steady-state with a mean of about 0.45 m/s. Note that the direction of motion, and the velocity of the center of mass, is in the \( \hat{j}_\theta \) direction, since the initial orientation of the chassis (see Figure 4.9) is such that the \( \hat{i}_B - \hat{j}_B \) plane is parallel to the \( \hat{j}_\theta - \hat{k}_\theta \) plane. When the actuations are stopped there is not much coasting and the system quickly halts because of the high rolling resistance of the terrain. Note that the maximum steady state velocity that can be achieved is dependent on the capacity of the motors that actuate the legs, since the motors have to be capable of retracting the legs with sufficient speed to keep them from hitting the ground as the rover rolls. For the case studies
presented in this chapter, the motor specifications used to generate the input leg motions were based on the motors used in our cylindrical TRREx prototype experiments.

Figure 4.8: Case Study II – Leg inputs and time evolution of states corresponding to translational and angular velocities of the chassis.
4.3.3 Case Study III: Controlled Rolling/Maneuverability demonstration

In this case study a TRREx design with parameters as listed in Table 4.1 is placed on the same terrain as in Case Study II except now the slope is $\beta = -5^\circ$ (negative sign implies the TRREx rolls downhill in the positive $\mathbf{\hat{j}}_O$ direction as shown in Figure 4.11) and there are obstacles on the terrain that it needs to avoid (see Table 4.4 for a summary of the environmental parameters for Case Study III). Note that in all the simulations where the terrain is inclined, the inertial frame is defined such that the $\mathbf{\hat{i}}_O - \mathbf{\hat{j}}_O$ plane is parallel to the terrain plane (as shown in Figure 4.11).

Upon obstacle detection, optimal evasive maneuver decisions need to be made by the controller in real time to generate the leg motions that will serve as inputs to the mathematical model.

Figure 4.9: Case Study II - Time-lapse plot showing the evolution of states (orientations and positions) in 3D space.
developed. Vision systems and sensors for obstacle detection and algorithms for optimal control for avoidance are not discussed here, but this case study demonstrates that by implementing a particular set of leg motion inputs (via the controller) the design architecture of the TRREx allows for ‘controlled rolling’ down a slope. This capability could be combined with a vision system to create an obstacle avoidance system for the TRREx.

Table 4.4: Environmental Parameters for Case Study III.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Environment parameters</td>
<td>Gravity</td>
<td>$g$</td>
<td>3.711</td>
<td>m/s$^2$</td>
</tr>
<tr>
<td></td>
<td>Rolling resistance</td>
<td>$C_r$</td>
<td>0.1</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Coefficient of static friction</td>
<td>$\mu_s$</td>
<td>0.15</td>
<td>-</td>
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<tr>
<td></td>
<td>Coefficient of Kinetic friction</td>
<td>$\mu_k$</td>
<td>0.12</td>
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</tr>
<tr>
<td></td>
<td>Slope of terrain</td>
<td>$\beta$</td>
<td>-5</td>
<td>Degrees</td>
</tr>
</tbody>
</table>

The inputs to the eight legs are shown in the top two sub-plots of Figure 4.10. The simulation results generated by the model are shown in the lower two subplots of Figure 4.10 and in Figure 4.11. At $t = 0$ seconds the TRREx is placed on the obstacle field and begins to roll downhill as leg 1 is opened, but directly in its path 3 meters ahead is an obstacle that it needs to avoid. The evasive maneuver begins by actuating leg 6 at 4.4 seconds and then following that up by actuating legs 4 and 1 to veer to the right to avoid the obstacle. This puts it on a collision course with another obstacle which it avoids by veering to the left by actuating legs 7, 3 and 6. A third obstacle on the path is similarly avoided by continuing to veer left by actuating legs 7, 3, 1 and 8. (Note that the maneuvers for this case study were achieved via an iterative process; an
optimal control scheme will need to be developed before the TRREx can be successfully deployed on Mars, but this is beyond the scope of this study.) This case study demonstrates the maneuverability of the particular TRREx design on a sloped obstacle field, but more importantly it demonstrates the capability of the TRREx to change direction and avoid obstacles while it is rolling.

Figure 4.10: Case Study III – Leg inputs and time evolution of states corresponding to translational and angular velocities of the chassis.
Figure 4.11: Case Study III - Time-lapse plot showing the evolution of states (orientations and positions) in 3D space.

4.4 Parametric Studies

The ability of the spherical TRREx to self-propel on flat terrain and to roll down a slope while maneuvering itself around obstacles was demonstrated above using case studies that employed a particular set of TRREx design parameters. In this section, parametric exploration of the design space is undertaken in order to investigate how the mobility of the TRREx (self-propulsion capacity and maneuverability) depends on its design parameters. The first study
explores the effect of the rover design on its performance up an incline and the second investigates the effect of the rover design on its maneuverability down an incline.

Table 4.5: Three sets of chassis design parameters used in parametric analysis.

<table>
<thead>
<tr>
<th>Chassis Design Parameters</th>
<th>Design A (A1, A2, A3)</th>
<th>Design B (B1, B2, B3)</th>
<th>Design C (C1, C2, C3)</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric parameters</td>
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<td></td>
</tr>
<tr>
<td>$D_s$</td>
<td>0.7500</td>
<td>1.0000</td>
<td>1.2500</td>
<td>Meters</td>
</tr>
<tr>
<td>$h_1$</td>
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<td>0.5991</td>
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</tr>
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<td>$h_2$</td>
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<td>0.1275</td>
<td>Meters</td>
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<tr>
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<td></td>
</tr>
<tr>
<td>$M$</td>
<td>31.7883</td>
<td>47.4224</td>
<td>58.8744</td>
<td>Kg</td>
</tr>
<tr>
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<td>3.6273</td>
<td>6.8847</td>
<td>Kg.m$^2$</td>
</tr>
<tr>
<td>$I_{z\bar{b}}$</td>
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<td>6.6036</td>
<td>12.9550</td>
<td>Kg.m$^2$</td>
</tr>
</tbody>
</table>

For these studies, a total of 9 TRREx designs that cover a wide range of the design parameter space were employed. The chassis mass properties were varied by selecting three different rover diameters and scaling their mass and inertia properties using a Solidworks® model. For each rover with a specific diameter, leg mass properties were varied by adding masses at the ends of the legs, and using a Solidworks® model to calculate their new center of mass locations, principle axes, masses and mass moments of inertia information. To aid in the comparison of the performance of the different designs, the masses added at the ends of the legs were chosen as proportions of the total mass of the rover i.e. for each of the designs with different diameters, masses that were 0.3%, 2.5% and 5% of the total mass of the rover were added at the ends of the legs. Adding a mass to the leg affects all of its design parameters, thus
in total there were 3 distinct chassis parameters (listed in Table 4.5) and 9 different leg parameters (listed in Table 4.6) that were used in this study.

Table 4.6: Nine sets of leg design parameters used in parametric analysis.

<table>
<thead>
<tr>
<th>Leg Design</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>B1</th>
<th>B2</th>
<th>B3</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_x$ (m)</td>
<td>0.1683</td>
<td>0.2231</td>
<td>0.2590</td>
<td>0.2488</td>
<td>0.3317</td>
<td>0.3913</td>
<td>0.3301</td>
<td>0.4327</td>
<td>0.5148</td>
</tr>
<tr>
<td>$l_y$ (m)</td>
<td>0.0335</td>
<td>0.0518</td>
<td>0.0555</td>
<td>0.0418</td>
<td>0.0690</td>
<td>0.0748</td>
<td>0.0531</td>
<td>0.0869</td>
<td>0.0953</td>
</tr>
<tr>
<td>$m_L$ (Kg)</td>
<td>1.9249</td>
<td>3.2076</td>
<td>5.3690</td>
<td>3.8746</td>
<td>6.0341</td>
<td>9.6234</td>
<td>6.4201</td>
<td>9.4166</td>
<td>14.4652</td>
</tr>
<tr>
<td>$I_{xc}$ (Kg.m$^2$)</td>
<td>0.0168</td>
<td>0.0195</td>
<td>0.0216</td>
<td>0.0706</td>
<td>0.0773</td>
<td>0.0826</td>
<td>0.2002</td>
<td>0.2127</td>
<td>0.2260</td>
</tr>
<tr>
<td>$I_{yc}$ (Kg.m$^2$)</td>
<td>0.0319</td>
<td>0.0521</td>
<td>0.0654</td>
<td>0.1441</td>
<td>0.2385</td>
<td>0.3054</td>
<td>0.4246</td>
<td>0.6858</td>
<td>0.8942</td>
</tr>
<tr>
<td>$I_{zc}$ (Kg.m$^2$)</td>
<td>0.0179</td>
<td>0.0404</td>
<td>0.0551</td>
<td>0.0838</td>
<td>0.1836</td>
<td>0.2521</td>
<td>0.2530</td>
<td>0.5258</td>
<td>0.7347</td>
</tr>
<tr>
<td>Total Mass of System (Kg)</td>
<td>47.2</td>
<td>57.4</td>
<td>74.7</td>
<td>78.4</td>
<td>95.7</td>
<td>124.4</td>
<td>110.2</td>
<td>134.2</td>
<td>174.5</td>
</tr>
</tbody>
</table>

4.4.1 Parametric Study I: Effect of Design on Ability to Scale Inclines

The goal of this study was to investigate how the slope climbing capacity of the TRREx depends on its design. This was done by finding the maximum slope $\beta_{max}$ that each of the 9 designs could successfully climb on 6 different terrain types on Mars, each having a distinct combination of surface properties as listed in Table 4.7. ‘Terrain IV’, with $C_{rr} = 0.005$, $\mu_s = 0.3$ and $\mu_k = 0.24$, represents the most favorable terrain, which is hard and rough (like a rock surface), while ‘Terrain III’ with $C_{rr} = 0.1, \mu_s = 0.15$ and $\mu_k = 0.12$, represents the most
unfavorable terrain, which is slippery and soft (like sand). For all the simulations, gravity was set at \( g = 3.711 \text{m/s}^2 \) (to simulate gravity on Mars) and the coefficient of kinetic friction was set to be 80% of the coefficient of static friction, i.e. \( \mu_k = 0.8\mu_s \) [77]. Also, in this parametric study, actuated rolling on the chassis was used to self-propel as it was observed in the case studies presented earlier in this chapter that this technique out-performs end-over-end actuated rolling.

<table>
<thead>
<tr>
<th>Terrain Parameters</th>
<th>( \mu_s = 0.15, \mu_k = 0.12 )</th>
<th>( \mu_s = 0.3, \mu_k = 0.24 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{rr} = 0.005 )</td>
<td>Terrain I</td>
<td>Terrain IV</td>
</tr>
<tr>
<td>( C_{rr} = 0.05 )</td>
<td>Terrain II</td>
<td>Terrain V</td>
</tr>
<tr>
<td>( C_{rr} = 0.1 )</td>
<td>Terrain III</td>
<td>Terrain VI</td>
</tr>
</tbody>
</table>

For each design, the procedure followed was to start by placing the design on one of the terrain types with a slope \( \beta \) that the rover could successfully climb via self-propulsion. Then, in subsequent runs the slope was incrementally made steeper until a maximum slope \( \beta_{\text{max}} \) was found beyond which the design failed to be able to self-propel up the slope. This procedure was then repeated for the rest of the terrain types for that design, and this whole process was repeated for all 9 designs. In all, 54 values of maximum slope \( \beta_{\text{max}} \) are found in this study, since there were 9 designs studies over 6 different terrains.

Three main types of failure to scale the slope were observed. On terrains where the rolling resistance was low, and the coefficient of static friction was high enough so that no slipping
occurred, the designs tended to fail because the steepness of the slope sent the TRREx rolling backward in such a way that it could not recover into a forward rolling motion. On terrains that had a higher rolling resistance, and on which the coefficient of static friction was high enough so that no slipping occurred, the designs tended to fail because of ‘stall’ i.e. the rover would reach a halt and get stuck because the previous actuations did not sufficiently turn the chassis such that the next actuation in sequence could contribute to rolling up the slope. On terrains in which the coefficient of static friction was not high enough to prevent slipping, the designs tended to fail because too much slipping led to a lack of appreciable progress up the slope.

Thus, for the purposes of this study a rover design was said to have successfully scaled the given sloped terrain if, starting from rest, it was able to complete more than one revolution of self-propelled rolling up the slope and in the process had not slipped more than 75%. More specifically, if at least $1 \frac{1}{4}$ revolutions of actuated rolling on the chassis was achieved and a distance of at least $\left(0.25 \times 1.25 \times \pi D \right)$ along the slope was covered during that time, the rover was said to have successfully scaled that slope.

The results are presented in Figure 4.12 in the form of bar graphs for each terrain. On terrains IV, V and VI the static friction was high enough that none of the rover designs exhibited slip during self-propulsion and an almost linear correlation was observed between the maximum slope the design was able to climb and end mass that was attached to the leg. Results suggest that if the terrain is favorable, the TRREx design ‘C3’ could roll up an incline as steep as 9 degrees. For many of the terrains studied, however, slipping was found to occur. This was
observed with rover designs A3, B3, C2 and C3 on terrains I, II and III which had a lower value of static friction. It should be noted here that only when slip occurs do the values of static and kinetic friction flow into the governing equations of the system. For example, if on two terrains the values of kinetic and static friction are different, but slipping doesn’t occur on either terrain, then the dynamics of the system on both terrains will be exactly identical. This explains the exact same maximum slope values obtained for designs A1, A2, B1, B2 and C1 on terrains I, II and III and terrains IV, V and VI respectively. On the high rolling resistance terrains III and VI, designs A1 and B1 were not able to self-propel even on flat ground, while C1 was able to do this with difficulty but was unsuccessful at scaling even a 0.1 degree slope.

Thus, from the performance of the various designs on the different terrain types it is clear that having a mass distribution of the leg such that there is more mass near the end is advantageous for the rover to be able to scale steeper slopes as long as slip does not occur. On unfavorable terrain with a lower value of static friction, however, the designs with a higher end mass tend to slip more, thereby nullifying the advantage gained by adding the end mass. In some cases when slip is occurring the rover with a higher end mass might even do worse than a rover with lower end mass. For example, the maximum slope that C3 could climb on ‘Terrain I’ \( \beta_{\text{max}} = 5.6^\circ \) was significantly less than what C2 was able to achieve \( \beta_{\text{max}} = 6.9^\circ \) on the same terrain.

Similarly, the larger diameter chassis designs fared better overall than their smaller diameter counterparts, except in the cases when slip occurred; for example, on ‘Terrain I’, design A3 was able to climb a steeper slope \( \beta_{\text{max}} = 6.4^\circ \) than the larger diameter design C3 \( \beta_{\text{max}} = 5.6^\circ \)
In making this comparison between rovers of different diameters, it should be recalled that the ratio of end mass to total mass was kept constant across the different diameters (and not the end mass itself). Note that it is important to consider the total mass of the rover (as found in Table 4.6), in addition to performance metrics, when selecting a design. For example, rover B2 outperforms rover B3 for terrains I, II and III, while B3 outperforms rover B2 for terrains IV, V and VI, but B3 has 30% more mass than B2; so, unless the characteristics of the terrain to be explored are known to be similar to IV, V and VI, and the improved performance on these terrains is essential, B2 may be a better choice, given the high cost per kilogram of sending rovers to Mars.
Figure 4.12: Results of the parametric study that investigates the effect of the design of the TRREx on its ability to scale inclines (only designs able to scale a slope greater than 0.1 degree are shown).
4.4.2 Parametric Study II: Dependence of Maneuverability down a slope on design

This parametric study attempts to characterize the dependence of maneuverability on the rover’s design as it is rolling down a slope. The 9 rover designs (with chassis and leg parameters shown in Table 4.5 and Table 4.6, respectively) are each initially placed atop the sloped terrain such that the rover’s $\hat{i}_B - \hat{j}_B$ plane is oriented perpendicular to the terrain surface and the $\hat{k}_B$ axis points up along the slope (see Figure 4.13). The terrain has the same surface properties as ‘Terrain III’ used in the previous study, and three different values of slope, $\beta = -10^\circ, -20^\circ, -30^\circ$, are considered here. As the rover begins to roll down the slope, one actuation of Leg 5 (opened at $t=0$ seconds and closed just in time such that the leg does not hit the ground) is performed causing the path of the rover to deflect to the right. By monitoring the deflection of the path of the center of mass of the chassis, caused by this one actuation of a leg, for each of the different designs, a quantitative comparison of the maneuverability of the various rover designs can be performed. A total of 27 runs were performed, consisting of simulations of 9 different designs rolling down the 3 different slopes.

Figure 4.14 shows the paths traced on the terrain plane by the center of mass of the chassis for each of the 9 different rovers as they roll down a 10 degree slope as leg 5 is actuated as described above. Similar plots for the 20 degree and 30 degree slopes are presented in Figure 4.15 and Figure 4.16, respectively.
Figure 4.13: Initial orientation of the designs for Parametric Study II.

Figure 4.14: Paths followed by the chassis center of mass for the different TRREx designs after one actuation at the top of a 10 degree slope. (As viewed perpendicular to the terrain plane).
Figure 4.15: Paths followed by the chassis center of mass for the different TRREx designs after one actuation at the top of a 20 degree slope. (As viewed perpendicular to the terrain plane).
These results indicate that for a given rover diameter, the higher the mass distribution near the end of the leg, the more an actuation is able to deflect the path of the rover. For example, in Figure 4.14, A1, A2 and A3 (which all have the same diameter, but different mass distributions), the design A3 (which has the most end mass) is able to deflect more than A2, which in turn deflects more than A1 (which has the least end mass on the leg); the same observations can be made for designs B1, B2, B3 and designs C1, C2, C3. A similar phenomenon is observed as the diameter of the rover is increased, provided the end masses are...
increased proportionally. For example in Figure 4.14, out of the three designs A1, B1 and C1 all having the same ‘end mass’ to ‘total mass’ ratio but having different diameters, the design C1 with the largest diameter is observed to deflect most and the design A1 the least; the same observations can be made for designs A2, B2, C2 and designs A3, B3, C3. These trends are consistently observed across all three slopes considered (see Figure 4.15 and Figure 4.16). Note however, from Figure 4.15 and Figure 4.16, that the magnitude of deflection at a given distance down the slope keeps decreasing as the slope increases; this is due to the fact that greater slopes lead to (1) higher velocities in the direction of the slope, and hence less time for a perpendicular displacement to occur over a given distance traveled down the slope and (2) more slip (due to reduced normal force) and hence lesser traction to cause perpendicular displacement.

Thus, in general, the greater the mass distribution at the end of the leg, and the larger the diameter of the rover, the more of a deflection one leg actuation is able to produce. This greater magnitude of path deflection, however, does not necessarily mean higher maneuverability because of the fact that a larger diameter rover needs to also deflect more to avoid an obstacle of a given size compared to a rover with smaller diameter. For further insight on whether the gain in deflection capacity overcomes the disadvantage of increased size while maneuvering around an obstacle, the bar charts in figures 4.17, 4.18 and 4.19 are presented.

These charts give the distance down the slope a particular design travels before its path deflects enough to avoid a spherical obstacle of a given diameter. Obstacles of diameters 0.5 m, 1m and 1.5m are considered and are represented at three discrete locations along the horizontal axis. The distances along the slope traveled before enough deflection has occurred for the rover
to avoid the obstacle are represented on the vertical axis, and a bar corresponding to each
different design is plotted on the graph (grouped by obstacle diameter). Note that designs which
were not successful in avoiding the given obstacle, even after rolling 50m down the slope, were
omitted from the charts. For example, looking at Figure 4.17 we note that all of the designs
successfully avoided the 0.5m obstacle, but designs A1 and B1 were not able to avoid the 1.5m
obstacle within 50m.

It is observed from the results presented in the three charts (in figures 4.17, 4.18 and 4.19) that
the designs with higher end mass require significantly less distance to avoid an obstacle. We
also see that the ability to avoid an obstacle increases with increased diameter, although we
note that due to the proportional increase in the end masses for the larger diameter rovers (due
to scaling, as discussed above) we are not comparing different diameter rovers with identical
absolute end-mass values.

It should be emphasized that this study was formulated in order to provide a basis for the
comparison of maneuverability of different designs, and although it gives a qualitative
assessment of the capabilities of a particular design, it does not give its performance limits. It
is possible that an obstacle of a given diameter could be avoided in a shorter distance than
suggested by the bar graphs presented here by performing multiple actuations along the way
and thereby increasing the magnitude of the path deflection.
Figure 4.17: Distance down a 10 degree slope a particular design travels before its path deflects enough to avoid a spherical obstacle of a given diameter (only designs able to avoid the obstacle in <50m are shown).
Figure 4.18: Distance down a 20 degree slope a particular design travels before its path deflects enough to avoid a spherical obstacle of a given diameter (only designs able to avoid the obstacle in <50m are shown).
Figure 4.19: Distance down a 30 degree slope a particular design travels before its path deflects enough to avoid a spherical obstacle of a given diameter (only designs able to avoid the obstacle in <50m are shown).

4.5 Conclusions

This chapter presented a dynamic model of the system in rolling mode, and conducted parametric studies using this model which demonstrated how the rolling mode contributes to the enhanced mobility of the TRREx. A mathematical model capturing the dynamics of the multi-body system in the spherical rolling mode was developed considering both slip and no slip during rolling. This model, via case studies, was used to demonstrate the rover’s capacity
to self-propel and maneuver itself by actuating the appropriate legs in sequence during rolling. Two distinct techniques of self-propulsion ‘end-over-end actuated rolling’ and ‘actuated rolling on the chassis’ were demonstrated on flat terrain. Also the ability of the rover to avoid obstacles while rolling down a slope was demonstrated. It was observed in general that ‘end-over-end actuated rolling’ allowed for more maneuverability while ‘actuated rolling on the chassis’ allowed for higher self-propulsion capacities of the TRREx.

Two parametric studies were presented that investigated the dependence of the mobility of the TRREx on its design. The first study focused on the correlation between the capacity of the TRREx to self-propel up a slope and its design. It was found that a leg design with more mass at the end of the leg aided the slope scaling capacity as long as the terrain was favorable and no slip occurred. When slip began to occur the advantage of having more mass on the end of the leg was nullified by the increased tendency to slip and in some slip cases the rover with lesser mass on the end of the legs was found to do better that a one with more mass. The second parametric study examined the effect of design on the maneuvering capabilities of the rover as it is rolling down a slope. It was found that a greater mass distribution at the end of the legs and a larger radius rover allowed for the actuations imparting sharper deflections to the path thereby being able to avoid bigger obstacles at shorter distances.

The dynamic model developed in this chapter has applications in picking a design for a spherical TRREx prototype, optimization of the design and in developing optimal control schemes to navigate terrain rugged terrain.
Chapter 5

Conclusions

This dissertation proposed and analyzed a novel transforming planetary exploration rover called the ‘Transforming Roving-Rolling Explorer’ (TRREx) rover that is principally aimed at safely navigating terrains that are combinations of flat areas, gentle gradients and steep rugged slopes.

The architecture of the rover was presented including its operational capabilities followed by the modeling, construction and experimental validation of a planar version of the TRREx. The knowledge gained was used to model the rolling dynamics of the spherical TRREx and analyze its mobility in the rolling mode. Results showed that the TRREx architecture allowed for designs that were able to maintain sustained self-propulsion achieving reasonable linear speeds even on sandy terrain. Simulations also demonstrated the ability of the TRREx to scale gradual inclines using actuated rolling on the chassis. Further, specific designs also demonstrated good maneuverability and obstacle avoidance capabilities. Thus ‘actuated rolling’ in the rolling mode is not only useful to roll small distances between slopes to limit the need of transforming between rolling and roving modes, but could also serve as the primary mode of locomotion of the TRREx when the terrain is conducive.
Overall, this work has highlighted the contribution of the spherical rolling mode to the enhanced mobility of the TRREx rover and how it could enable challenging surface exploration missions in the future. Work following this dissertation will focus on the construction of a spherical TRREx prototype that is able to demonstrate actuated rolling and will be used to experimentally validate the analytical model developed in this dissertation.

Beyond this, the unique design features of the TRREx allow for a lot more interesting mobility possibilities that could be studied. For example, the optimization of controls in actuated rolling, using leg actuations to push against the ground to roll (‘push actuation’) and possibly even jump over obstacles, optimal transformation dynamics, crater exploration with tethered halves, different types of steering control in roving mode (including crab steering), different special gaits in roving mode (including various walking gaits) and climbing a high ledge using the articulated hip joint.
REFERENCES


APPENDICES
Appendix A

Leg Actuation Model

Initial simulations (section 3.3) assumed that the angular acceleration of the leg followed a cubic polynomial (in time) from rest position to rest position. If a leg started at an angular position $\gamma_{\text{min}}$ (fully closed position) and reaches a position $\gamma_{\text{max}}$ (fully open position) in $T_c$ seconds, then the desired path was chosen such that the initial and final velocities and accelerations were zero. The value of $T_c$ was adjustable based on the speed of the linear actuator used. The coefficients could be solved for using the boundary conditions, and the polynomial describing the desired path of a leg versus time $t$ was,

$$\gamma_{jd}(t) = \gamma_{\text{min}} + 10 \left( \gamma_{\text{max}} - \gamma_{\text{min}} \right) \left( \frac{t}{T_c} \right)^3 - 15 \left( \gamma_{\text{max}} - \gamma_{\text{min}} \right) \left( \frac{t}{T_c} \right)^4 + 6 \left( \gamma_{\text{max}} - \gamma_{\text{min}} \right) \left( \frac{t}{T_c} \right)^5$$

But it was observed from experiments on the prototype that a piecewise constant acceleration of the piston (of the linear motor) between the leg and the chassis, when transformed to the angle of the leg using the law of cosines, better characterized the opening and closing motion of the legs. An example of the translational motion of the piston generated in this way is shown in Figure A.1, in which, $a_{\text{max}}$ is the maximum value of constant acceleration produced by the actuator, $T_c$ is the time that it takes for the leg to go from a completely closed to a completely
open position (or vice-versa) and $f$ is the fraction of time $T$, for which a constant acceleration is maintained.

![Diagram](image_url)

Figure A.1: Position, velocity and acceleration variation of the piston of the linear motor as modeled.

Looking at the geometry of the linear motor mount (see Figure A.2), if $r_h$ is the distance from a hinge to the point at which the linear motor’s pin is attached to the leg, $x_h$ is the distance from the hinge to the point where the motor’s other pin is attached to the chassis and $\eta$ is the angle between the two lines that form these distances, then the transformation between the length of the third side of the triangle $l_{\text{motor}}$ i.e. the pin to pin length of the extended motor (related to the extension of the linear motor via a constant $l_{\text{man}}$ ) and $\eta$ (related to the angular motion of the leg via a constant $\gamma_{\eta=0}$) is given by the law of cosines as,
Using relation (A.1) the translational motion of the piston was transformed into the angular motion of the leg (example shown in Figure A.3). A comparison of leg actuations generated by this model with the motion of the leg experimentally observed on the cylindrical prototype is shown in Figure A.4 for two actuations that are generated at arbitrary times and for arbitrary durations. For the experimental validation of the cylindrical TRREx (section 3.5) and the studies on the Spherical TRREx (sections 4.3 and 4.4) such a model to generate the actuations was used.

\[ l_{\text{motor}}^2 = r_h^2 + x_h^2 - 2r_h x_h \cos(\eta) = r_h^2 + x_h^2 - 2r_h x_h \cos(\gamma_{jd} - \gamma_{j=0}) \]  

(A.1)
Figure A.3: Angular position, angular velocity and angular acceleration of the leg frame after transformation from piston motion.

Figure A.4: Comparison of experimentally measured and model generated leg motion.
Appendix B

Constraint Relaxation Model of the Spherical TRREx

As discussed in section 4.2.1, the Spherical TRREx is a multi-body system with 9 bodies in 3 dimensional space. Since in the constraint relaxation method all the degrees of freedom of each body are preserved, we have a total of $9 \times 6 = 54$ degrees of freedom. As explained in section 4.2.3, quaternions are best suited to represent the orientations of each body in 3 dimensional space, therefore each body requires 13 states (3 positions and 3 velocities, 1 quaternion i.e. 4 coefficients for orientation and 3 angular accelerations) to completely describe it. Thus in all, to model the Spherical TRREx using the constraint relaxation technique $9 \times 13 = 117$ first order state equations are required. The 13 states for the chassis are, $(x_{BO}^B, y_{BO}^B, z_{BO}^B)$ its 3 inertial positions, $(\dot{x}_{BO}^B, \dot{y}_{BO}^B, \dot{z}_{BO}^B)$ the 3 inertial velocity components, $(q_{0B}, q_{1B}, q_{2B}, q_{3B})$ the 4 coefficients of the orientation quaternion, and $(\omega_{xB}, \omega_{yB}, \omega_{zB})$ the 3 body angular velocities of the chassis with respect to the inertial frame, written in the $\bar{B}$ frame. Similarly $(x_{Cj,\bar{B}}, y_{Cj,\bar{B}}, z_{Cj,\bar{B}}, \dot{x}_{Cj,\bar{B}}, \dot{y}_{Cj,\bar{B}}, \dot{z}_{Cj,\bar{B}}, q_{0Cj}, q_{1Cj}, q_{2Cj}, q_{3Cj}, \omega_{xCj}, \omega_{yCj}, \omega_{zCj})$ are the 13 states corresponding to each leg $j$. 
6 state equations for each body come from the application of Newton’s 2nd law (see equation (B.1) for chassis and (B.2) for leg \( j \), 4 come from the kinematic relationship that propagates the quaternion (see equation (B.3) for chassis and (B.4) for leg \( j \)) and 3 come from Euler’s equations for the body (see equation (B.5) for chassis and (B.6) for leg \( j \)).

\[
M^O \ddot{a}_{/O} = \vec{f}_B^R
\]  
(B.1)

\[
m_L^O \ddot{a}_{/O} = \vec{f}_C^R \quad \text{for } j = 1 \text{ to } 8
\]  
(B.2)

\[
\begin{bmatrix}
\dot{q}_{0B} \\
\dot{q}_{1B} \\
\dot{q}_{2B} \\
\dot{q}_{3B}
\end{bmatrix} = \frac{1}{2}
\begin{bmatrix}
0 & -\omega_{xB} & -\omega_{yB} & -\omega_{zB} \\
\omega_{xB} & 0 & \omega_{zB} & -\omega_{yB} \\
\omega_{yB} & -\omega_{zB} & 0 & \omega_{xB} \\
\omega_{zB} & \omega_{yB} & -\omega_{xB} & 0
\end{bmatrix}
\begin{bmatrix}
q_{0B} \\
q_{1B} \\
q_{2B} \\
q_{3B}
\end{bmatrix}
\]  
(B.3)

\[
\begin{bmatrix}
\dot{q}_{0C_j} \\
\dot{q}_{1C_j} \\
\dot{q}_{2C_j} \\
\dot{q}_{3C_j}
\end{bmatrix} = \frac{1}{2}
\begin{bmatrix}
0 & -\omega_{xC_j} & -\omega_{yC_j} & -\omega_{zC_j} \\
\omega_{xC_j} & 0 & \omega_{zC_j} & -\omega_{yC_j} \\
\omega_{yC_j} & -\omega_{zC_j} & 0 & \omega_{xC_j} \\
\omega_{zC_j} & \omega_{yC_j} & -\omega_{xC_j} & 0
\end{bmatrix}
\begin{bmatrix}
q_{0C_j} \\
q_{1C_j} \\
q_{2C_j} \\
q_{3C_j}
\end{bmatrix}
\quad \text{for } j = 1 \text{ to } 8
\]  
(B.4)

\[
\begin{bmatrix}
\dot{\omega}_{xB} \\
\dot{\omega}_{yB} \\
\dot{\omega}_{zB}
\end{bmatrix} = \frac{\tau_{xB} + (I_{yB} - I_{zB})\omega_{xB}\omega_{zB}}{I_{xB}}
\]  
(B.5)
\[
\begin{align*}
\begin{bmatrix}
\dot{\theta}_x c_j \\
\dot{\theta}_y c_j \\
\dot{\theta}_z c_j \\
\end{bmatrix} = & \begin{bmatrix}
\tau_{xc_j} + (I_{yc_j} - I_{zc_j})\omega_{yc_j} \omega_{zc_j} & I_{xc_j} \\
\tau_{yc_j} + (I_{xc_j} - I_{zc_j})\omega_{xc_j} \omega_{zc_j} & I_{yc_j} \\
\tau_{zc_j} + (I_{xc_j} - I_{yc_j})\omega_{xc_j} \omega_{yc_j} & I_{zc_j}
\end{bmatrix} \\
\text{for } j=1 \text{ to } 8 \quad (B.6)
\end{align*}
\]

In the above equations, \(M\) and \(\mathbf{\ddot{a}}_{B/O}^O\) are the mass and the inertial acceleration respectively of the chassis and \(m_L\) and \(\mathbf{\ddot{a}}_{C/O}^O\) are the mass and the inertial acceleration respectively of the \(j^{th}\) leg. \(\mathbf{\ddot{f}}_B^R\) and \(\mathbf{\ddot{f}}_C^R\) are the sum of external forces acting at the center of mass of the chassis and the \(j^{th}\) leg respectively. \((\tau_{xB}, \tau_{yB}, \tau_{zB})\) are the components of the external torques acting on the chassis when written in the chassis frame and \((I_{xB}, I_{yB}, I_{zB})\) are the principle inertia components of the chassis. Similarly, \((\tau_{xcj}, \tau_{ycj}, \tau_{zcj})\) are the components of the external torques acting on the \(j^{th}\) leg when written in the leg frame and \((I_{xcj}, I_{ycj}, I_{zcj})\) are the principle inertia components of the \(j^{th}\) leg.

Ground interactions and joint connections are treated in exactly the same way as explained in section 3.2.2. The formulation of the restoring forces generated by the translational springs and dampers can be used unaltered from section 3.2.2, however, the formulation of the vector directions of the moments generated by the rotational springs and dampers (not always perpendicular to the plane of the paper) need to generalized further to apply to rotations in 3 dimensional space.
Restoring moments are generated when at the given instant the actual orientation of the leg (represented by an orientation quaternion $q_{C,\alpha}$) is different from the desired orientation (represented by a different orientation quaternion $q_{C,\delta}$). The direction of the restoring moment of the rotational spring and damper can be extracted from the quaternion ($\Delta q_{c_j}$) that represents a rotation from the desired frame to the actual frame. This is found using quaternion multiplication [84] as,

$$\Delta q_{c_j} = \tilde{q}_{C,\alpha} \otimes q_{C,\delta}$$

where $\otimes$ denotes quaternion multiplication and $\tilde{q}_{C,\alpha}$ denotes the conjugate of $q_{C,\alpha}$.

By representing this rotation quaternion as an equivalent ‘axis – angle’ ($\hat{\rho}_j-\rho_j$) representation using,

$$\Delta q_{c_j} = \cos\left(\frac{\rho_j}{2}\right) + \left(\hat{u}_j\hat{\iota}_{C,\delta} + \hat{u}_j\hat{\jmath}_{C,\delta} + \hat{u}_j\hat{k}_{C,\delta}\right)\sin\left(\frac{\rho_j}{2}\right)$$

the magnitude of rotation $\rho_j$ between the desired and actual leg frames and the appropriate unit vector for the direction of restoring forces $\hat{e}_{\rho_j}$ is found.

$$\hat{e}_{\rho_j} = \frac{\hat{u}_j\hat{\iota}_{C,\delta} + \hat{u}_j\hat{\jmath}_{C,\delta} + \hat{u}_j\hat{k}_{C,\delta}}{\sqrt{\hat{u}_j\hat{\iota}_{C,\delta} + \hat{u}_j\hat{\jmath}_{C,\delta} + \hat{u}_j\hat{k}_{C,\delta}}}$$
Note that this unit vector is given in the desired leg frame \((\hat{i}_{C,d}, \hat{j}_{C,d}, \hat{k}_{C,d})\) and since the Euler equations are simplest when written in the body frame, these need to be transformed into the respective body frames (i.e. chassis frame \((\hat{i}_B, \hat{j}_B, \hat{k}_B)\) for moments acting on the chassis and the actual leg frame \((\hat{i}_C, \hat{j}_C, \hat{k}_C)\) for moments acting on the \(j^{th}\) leg).

Thus the moment exerted on the chassis due to the \(j^{th}\) leg is \(\vec{M}_{BC,j} = M_{H_j} \hat{e}_{\rho_j}\) and an equal and opposite moment \(\vec{M}_{C,B,j} = -M_{H_j} \hat{e}_{\rho_j}\) is exerted on the leg by the chassis. The magnitude of the moment is given by \(M_{H_j} = k, \rho_j + c, \rho \dot{j}\). Note that \(q_{C,d}\) comes from the integration of the four states \((q_{0C}, q_{1C}, q_{2C}, q_{3C})\) in (B.4) while \(q_{C,d}\) is derived from the desired angular position of the leg \(\gamma_{jd}\), which is an input to system.

The resultant forces and moments on the chassis \((\vec{F}_B, \vec{M}_B)\) and each leg \((\vec{F}_C, \vec{M}_C)\) are found in exactly the same way as section 3.2.2,

\[
\vec{F}_B = M\ddot{g} + \vec{F}_R + \vec{F}_N + \vec{F}_R + \sum_{j=1}^{8} \vec{F}_{BC,j} \quad \text{(B.7)}
\]

\[
\vec{F}_C = \vec{F}_{C,B} + m_L\ddot{g} \quad \text{(B.8)}
\]

\[
\vec{M}_B = \vec{r}_{/B} \times (\vec{F}_R + \vec{F}_N) + \sum_{j=1}^{8} \vec{r}_{/B} \times \vec{F}_{BC,j} + \vec{M}_{BC,j} \quad \text{(B.9)}
\]
\[ \hat{M}_{C_i}^B = \hat{h}_{\ell/C_i} \times \vec{f}_{C_i,B} + \bar{M}_{C_i,B} \]  

(B.10)

where all the quantities are consistent in notation with section 3.2.2.

The first derivatives of the six states in equations (B.1) - \((\dot{x}_{BO}, \dot{y}_{BO}, \dot{z}_{BO}, \ddot{x}_{BO}, \ddot{y}_{BO}, \ddot{z}_{BO})\) and (B.2) - \((\dot{x}_{C_0}, \dot{y}_{C_0}, \dot{z}_{C_0}, \ddot{x}_{C_0}, \ddot{y}_{C_0}, \ddot{z}_{C_0})\) are unknowns and can be solved for after using equations (B.7) and (B.8) to substitute in for the external forces on the bodies (all the quantities are written in a consistent frame - here, inertial frame). When equation (B.9) is written in the chassis frame then the three torque components are directly \((\tau_{xB}, \tau_{yB}, \tau_{zB})\) which are used on the right hand side of equation (B.5). Similarly when equation (B.10) is written in the actual leg frame then the three torque components are \((\tau_{xC}, \tau_{yC}, \tau_{zC})\) which can be used on the right hand side of equation (B.6). Given the initial conditions, environmental, geometric and mass properties of the system and desired leg input motions, these state equations can be integrated to produce the time response of the system. As noted in section 3.2.2, caution should be exercised in setting the maximum step size for the integrator.

The results produced by the constraint relaxation technique exactly matched the analytical model results. As an example, simulation results using both the techniques for the design B2 performing end-over-end rolling on ‘Terrain V’ (see section 4.4 for corresponding parameters) are shown in the figures below (Figure B.1, Figure B.2, Figure B.3 and Figure B.4).
It should be reiterated here that because a ‘continuous friction model’ (see section 3.2.2) was adopted in the constraint relaxation technique to model ground interactions, equivalently the coefficient of kinetic friction was set equal to the coefficient of static friction in analytical model.
Figure B.1: Comparison of Analytical and Constraint Relaxation Model Results - Leg inputs (top subplot) and (lower three subplot) Position of center of mass of Chassis.
Figure B.2: Comparison of Analytical and Constraint Relaxation Model Results – Velocity components of center of mass of Chassis.
Figure B.3: Comparison of Analytical and Constraint Relaxation Model Results - Coefficients of the orientation quaternion of Chassis.
Figure B.4: Comparison of Analytical and Constraint Relaxation Model Results - Body angular velocity components of the Chassis.
Appendix C

Spherical TRREx Parametric Study I: Additional Plots

Parametric Study I (see section 4.4.1) presented an investigation of the effect of the design of the Spherical TRREx on its ability to scale inclines. In the study the relationship between kinetic and static friction was taken as $\mu_k = 0.8\mu_s$. In Figure C.1, results are presented for another higher coefficient of kinetic friction case ($\mu_k = \mu_s$ in comparison with the $\mu_k = 0.8\mu_s$ case). The higher coefficient of kinetic friction enables certain rover designs on Terrains I, II and III to scale higher slopes than in the lower kinetic friction case.
Figure C.1: Parametric Study I results for two cases of the coefficient of kinetic friction, i.e.\[ \mu_k = \mu_s \quad \text{and} \quad \mu_k = 0.8\mu_s. \]
Appendix D

Spherical TRREx Parametric Study II: Additional Plots

Attached in this appendix are results pertaining to Parametric study II (section 4.4.2), when the values of slope are $\beta = \{-40', -50'\}$. Significant slip was observed in these steep slope simulations and consequently the deviation in path caused by the one actuation at the top is much less. Figure D.1 and Figure D.2 show the paths traced on the terrain plane by the centers of masses of the chassis of the 9 different rovers as they roll down the 40 and 50 degree slopes after the one actuation at the start. Figure D.3 gives the distance down the 40 degree slope a particular design travels before its path deflects enough to avoid a spherical obstacle of a given diameter. It is observed in this plot that with the one actuation at the top, the best performing designs even to avoid the smaller obstacles travel significant distances down the slope before they can avoid them. On the 50 degree slope it was observed that none of the designs could deflect enough to avoid even the smallest (0.5 m diameter) obstacle within 50 meters of rolling down-hill.
Figure D.1: Paths followed by the chassis center of mass for the different TRREx designs after one actuation at the top of a 40 degree slope. (As viewed perpendicular to the terrain plane).
Figure D.2: Paths followed by the chassis center of mass for the different TRREx designs after one actuation at the top of a 50 degree slope. (As viewed perpendicular to the terrain plane).
Figure D.3: Distance down a 40 degree slope a particular design travels before its path deflects enough to avoid a spherical obstacle of a given diameter (only designs able to avoid the obstacle in <50m are shown).