

Duality invariance of black hole creation rates

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Pair creation of electrically charged black holes and its dual process, pair creation of magnetically charged black holes, are considered. It is shown that the creation rates are equal provided the boundary conditions for the two processes are dual to one another. This conclusion follows from a careful analysis of boundary terms and boundary conditions for the Maxwell action. [S0556-2821(97)01214-9]

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I. INTRODUCTION

The classical Einstein-Maxwell equations are invariant under a duality transformation in which the electric and magnetic fields exchange roles. Recently the subject of electromagnetic duality has arisen in the context of black hole pair creation. One can consider the pair creation of magnetically charged black holes and its “dual process,” pair creation of electrically charged black holes.¹ Are the creation rates equal? The answer is yes,² as one might expect on physical grounds, but it is not immediately obvious from a technical standpoint. In fact, the factor $F_{\mu\nu}F^{\mu\nu} = 2(B^2 - E^2)$ in the Maxwell Lagrangian changes sign if the electric and magnetic fields are exchanged. Therefore it might appear that the semiclassical expressions for the two creation rates, which are both proportional to the exponential of the action, should include an exponential suppression in one case and an exponential enhancement in the other. However, as Hawking and Ross [4] recognized, boundary conditions play a crucial role in the analysis. One can see the importance of boundary conditions by recalling that the choice of boundary conditions affects the boundary terms that appear in the action. Hawking and Ross [4] did not use this observation in their primary argument for the equality of the pair creation rates, but they did give a brief, secondary argument based on the analysis of boundary terms in the Maxwell action. The main purpose of this paper is to elaborate on this argument by presenting a complete and detailed analysis of boundary terms and boundary conditions for the electromagnetic field action.

Section II begins with an analysis of the action, denoted $S_1[g, F]$, which is the integral of the Maxwell Lagrangian. I

¹Gibbons [1] first recognized that the pair creation rate for magnetically charged black holes can be computed in the semiclassical approximation (or, more precisely, the zero-loop approximation) using instanton techniques. The analysis was presented in Ref. [2]. The technical details of the calculation for electrically charged black holes were laid out in Ref. [3], although an absolute creation rate was not computed there—only the rate relative to those for charged matter distributions.

²An explicit demonstration was first given by Hawking and Ross [4] and more recently by Deser, Henneaux, and Teitelboim [5].

show that the boundary conditions for S_1 include fixation of the component B^+ of the magnetic field normal to the system boundary, and the components E^a of the electric field tangent to the system boundary. Let us assume that for a certain choice of boundary values B^+ and E^a , there exists an instanton (such as the magnetic Ernst solution) describing pair creation of magnetically charged black holes. The action S_1 can be used to compute the creation rate under the given set of boundary conditions. Now consider the pair creation of electrically charged black holes. An instanton (such as the electric Ernst solution) describing the pair creation of electrically charged black holes certainly exists for *some* choice of boundary values B^+ and E^a , and S_1 can be used to find the creation rate for electrically charged black holes under these conditions. However, we wish to compare the creation rates between the magnetic and electric cases, and for this comparison to be meaningful the boundary conditions must be related appropriately. In particular, the creation rate for magnetically charged black holes under the conditions of fixed B^+ and E^a should be compared to the creation rate for electrically charged black holes under the “dual conditions” of fixed E^+ and B^a . The action S_1 is not appropriate for these dual boundary conditions. Thus, we are led to consider another electromagnetic field action, $S_2[g, F]$, which differs from S_1 by a boundary term. The boundary conditions for S_2 include fixation of the normal component E^+ of the electric field and the tangential components B^a of the magnetic field.

The key result, derived in Sec. II, is that the electromagnetic field actions S_1 and S_2 satisfy the relationship $S_1[\tilde{g}, \tilde{F}] = S_2[\tilde{g}, -*\tilde{F}]$. Here, $g_{\mu\nu} = \tilde{g}_{\mu\nu}$, $F_{\mu\nu} = \tilde{F}_{\mu\nu}$ is a given classical solution and $g_{\mu\nu} = \tilde{g}_{\mu\nu}$, $F_{\mu\nu} = -*\tilde{F}_{\mu\nu}$ is the dual solution. This observation solves the pair creation problem: The pair creation rate for magnetically charged black holes under the conditions of fixed B^+ and E^a is proportional to the exponential of S_1 evaluated at an appropriate classical solution \tilde{g} , \tilde{F} (such as the magnetic Ernst instanton). The pair creation rate for electrically charged black holes under the dual conditions of fixed E^+ and B^a is proportional to the exponential of S_2 evaluated at the dual solution \tilde{g} , $-*\tilde{F}$ (the electric Ernst instanton). From the relationship $S_1[\tilde{g}, \tilde{F}] = S_2[\tilde{g}, -*\tilde{F}]$, it follows that the pair creation rates are

equal in the zero-loop approximation.³

Section II concludes with an argument that shows the impossibility of fixing E^+ and E^a , or B^+ and B^a , as boundary data. Thus, one cannot speak of black hole pair creation (or any other physical process) under such conditions.

II. ELECTROMAGNETIC FIELD ACTIONS AND BLACK HOLE PAIR CREATION

Consider the spacetime manifold \mathcal{M} with metric $g_{\mu\nu}$. Assume for the moment that \mathcal{M} is the cross product of a space manifold Σ and the time line interval I , $\mathcal{M}=\Sigma\times I$. The boundary of Σ is $\partial\Sigma$, which need not be connected. The boundary of \mathcal{M} consists of the surfaces Σ' and Σ'' at the end points of the line interval and the history $\mathcal{T}=\partial\Sigma\times I$ of the spatial boundary.⁴

Let γ_i^μ denote the projection mapping $T_p^*\mathcal{M}\rightarrow T_p^*\mathcal{T}$, where p is a point in \mathcal{T} . [In terms of local coordinates X^μ on \mathcal{M} and x^i on \mathcal{T} , $\gamma_i^\mu=\partial X^\mu(x,r)/\partial x^i$ where \mathcal{T} is an $r=\text{const}$ surface.] Also define $\gamma_\mu^i=g_{\mu\nu}\gamma^{ij}\gamma_j^\nu$ where $\gamma_{ij}=\gamma_i^\mu g_{\mu\nu}\gamma_j^\nu$ is the induced metric on \mathcal{T} and γ^{ij} is its inverse. With these definitions the identity tensor at points $p\in\mathcal{T}$ becomes $\delta_\mu^\nu=n_\mu n^\nu+\gamma_\mu^i\gamma_i^\nu$ where n_μ is the spacelike unit normal of \mathcal{T} . Likewise, let h_i^μ denote the projection mapping $T_q^*\mathcal{M}\rightarrow T_q^*\Sigma''$, where q is a point in Σ'' . Define $h_\mu^i=g_{\mu\nu}h_j^i h_j^\nu$ where $h_{ij}=h_i^\mu g_{\mu\nu}h_j^\nu$ is the induced metric on Σ'' and h^{ij} is its inverse. The identity tensor at points $q\in\Sigma''$ is $\delta_\mu^\nu=-\bar{u}_\mu\bar{u}^\nu+h_\mu^i h_i^\nu$ where \bar{u}_μ is the timelike unit normal of Σ'' . Similar definitions hold for Σ' .

Now consider the action for the electromagnetic field,⁵

$$S_1[g,F]=-\frac{i}{16\pi}\int_{\mathcal{M}}d^4X\sqrt{-g}F^{\mu\nu}F_{\mu\nu}, \quad (1)$$

where $F_{\mu\nu}=2\partial_{[\mu}A_{\nu]}$. The notation $S_1[g,F]$ indicates only that S_1 is a functional of the metric tensor $g_{\mu\nu}$ and the electromagnetic field strength $F_{\mu\nu}$; it is not meant to imply that $F_{\mu\nu}$ is a fundamental variable in the variational principle. Rather, S_1 is to be varied with respect to the potential A_μ and the metric $g_{\mu\nu}$. The variation of S_1 with respect to A_μ yields the source free Maxwell equations $\nabla_\mu F^{\mu\nu}=0$ and the variation of S_1 with respect to $g_{\mu\nu}$ yields the stress-energy-momentum tensor for the electromagnetic field.

³The argument presented in Ref. [5] for the equality of the pair creation rates is rather different. There, the authors use Hamiltonian methods to construct an action in which both the electric charge and the magnetic charge are fixed at spatial infinity, and only the transverse parts of the electric and magnetic fields are dynamical. They then show that this action is invariant under a duality transformation in which, simultaneously, the transverse electric and magnetic fields and the electric and magnetic charges are transformed.

⁴The spatial boundary can be chosen at asymptotic infinity for those cases in which asymptotic infinity exists. Here the spatial boundary is left unrestricted, so the analysis applies whether or not an asymptotic region exists.

⁵For simplicity, I define the action as the argument of the exponential in the path integral, hence the factor of i in Eq. (1).

The boundary conditions suitable for the variational principle based on the action S_1 are determined from the variation

$$\begin{aligned} \delta S_1 &= (\text{EOM's}) + \frac{i}{4\pi}\int_{\Sigma'}d^3x\sqrt{\bar{h}}\bar{u}_\mu F^{\mu\nu}\delta A_\nu \\ &\quad - \frac{i}{4\pi}\int_{\mathcal{T}}d^3x\sqrt{-\gamma}n_\mu F^{\mu\nu}\delta A_\nu. \end{aligned} \quad (2)$$

Here, EOM's are terms that yield the classical equations of motion. Inserting the identity tensor between the factors $F^{\mu\nu}$ and δA_ν , we find

$$\begin{aligned} \delta S_1 &= (\text{EOM's}) + \frac{i}{4\pi}\int_{\Sigma'}d^3x\sqrt{\bar{h}}\bar{u}_\mu F^{\mu\nu}h_\nu^i\delta A_i \\ &\quad - \frac{i}{4\pi}\int_{\mathcal{T}}d^3x\sqrt{-\gamma}n_\mu F^{\mu\nu}\gamma_\nu^i\delta A_i, \end{aligned} \quad (3)$$

where $A_i=A_\nu h_\nu^i$ and $A_i=A_\nu\gamma_\nu^i$. (This derivation uses the result $\delta\bar{\gamma}_i^i=0$ for variations in the dynamical variables $g_{\mu\nu}$ and A_μ . Also note that δn_μ and δh_μ^i are each proportional to n_μ . Similar results hold for h_μ^i , h_μ^i , and \bar{u}_μ .) Equation (3) shows that, for the action S_1 , the boundary conditions consist of fixed A_i on Σ' and Σ'' , and fixed A_i on \mathcal{T} .

Let us see what these boundary conditions imply for the electric and magnetic fields. First consider Σ' and Σ'' . The electric and magnetic fields measured by observers with four-velocities \bar{u}^μ at Σ' or Σ'' are $E^\mu=F^{\mu\nu}\bar{u}_\nu$ and $B^\mu=-*F^{\mu\nu}\bar{u}_\nu$, respectively. Here, $*F^{\mu\nu}=(1/2)\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ is the dual of the field strength $F^{\mu\nu}$, where $\epsilon_{\mu\nu\rho\sigma}$ is the spacetime volume form. Expressed as a vector in the tangent space to Σ' or Σ'' , the magnetic field $B^i=h_\mu^i B^\mu$ becomes

$$B^i=\epsilon^{ijk}\partial_j A_k, \quad (4)$$

where ϵ_{ijk} is the volume form on Σ' or Σ'' . (This derivation uses the result $h_\mu^i h_\nu^j \partial_{[\mu} h_{\nu]}^k=0$ and the fact that each term in $\partial_{[\mu}\bar{u}_{\nu]}$ is proportional to either \bar{u}_μ or \bar{u}_ν .) Recall that for S_1 , A_i is fixed on the boundary elements Σ' and Σ'' . The result (4) shows that the magnetic field B^i is fixed on Σ' and Σ'' . (It is assumed that the induced metric h_{ij} and hence also the volume form ϵ_{ijk} are fixed on Σ' and Σ'' .)

Now turn to the boundary element \mathcal{T} with spacelike unit normal n^μ . For the considerations that follow the spacetime manifold \mathcal{M} need not be a product $\Sigma\times I$. The boundary element \mathcal{T} is the interface between the system and the exterior universe. It is here that boundary conditions are imposed for such problems as black hole pair creation. Conceptually, the boundary values at \mathcal{T} are considered to be fixed and/or monitored as the system evolves in time by a fleet of observers located at the system boundary. These observers must synchronize their measurements in time, which implies the existence of a foliation of \mathcal{T} into $t=\text{const}$ hypersurfaces \mathcal{B} . (If \mathcal{M} is a product $\Sigma\times I$, we can identify \mathcal{B} with $\partial\Sigma$.) I will

assume that the observers are at rest in these time slices.⁶ Let us denote the four-velocities of the \mathcal{T} observers by u^i ; as vectors in $T_p\mathcal{M}$ with $p \in \mathcal{T}$, the four-velocities are $u^\mu = u^i \gamma_i^\mu$. Let σ_a^i denote the projection mapping $T_p^*\mathcal{T} \rightarrow T_p^*\mathcal{B}$. [In terms of local coordinates x^i on \mathcal{T} and z^a on \mathcal{B} , $\sigma_a^i = \partial x^i(z, t) / \partial z^a$ where \mathcal{B} is a $t = \text{const}$ surface.] Also define $\sigma_a^i = \gamma_{ij} \sigma^{ab} \sigma_b^j$ where $\sigma_{ab} = \sigma_a^i \gamma_{ij} \sigma_b^j$ is the induced metric on \mathcal{B} and σ^{ab} is its inverse. With these definitions the identity tensor on \mathcal{T} becomes $\delta_i^j = -u_i u^j + \sigma_a^i \sigma_a^j$.

The electric and magnetic fields measured by the observers u^i are $E^\mu = F^{\mu\nu} u_\nu$ and $B^\mu = -{}^*F^{\mu\nu} u_\nu$, respectively. A short calculation shows that the component of B^μ orthogonal to \mathcal{T} , namely $B^\dagger = B^\mu n_\mu$, is

$$B^\dagger = \epsilon^{ab} \partial_a A_b, \quad (5)$$

where ϵ_{ab} is the volume form on \mathcal{B} and $A_a = A_i \sigma_a^i = A_\mu \gamma_i^\mu \sigma_a^i$ are the components of the electromagnetic potential tangent to \mathcal{B} . Recall that for the action S_1 , A_i (and hence also A_a) is fixed on the boundary element \mathcal{T} . The result (5) shows that the fixed boundary data includes the component B^\dagger of the magnetic field that is orthogonal to the system boundary \mathcal{B} as measured by the observers u^i . (It is assumed that the induced metric γ_{ij} is fixed on \mathcal{T} .) Another short calculation shows that the components of the electric field tangent to \mathcal{B} , namely $E^a = E^\mu \gamma_\mu^i \sigma_i^a$, are given by

$$E^a = 2 \sigma_a^i \gamma^{jj} u^k \partial_{[j} A_{k]}. \quad (6)$$

Thus the fixed boundary data for S_1 also includes the components E^a of the electric field tangent to the system boundary \mathcal{B} as measured by the observers u^i .

The magnetic charge Q_m for the system is defined by the magnetic field flux through a surface \mathcal{B} :

$$4\pi Q_m = \int_{\mathcal{B}} \mathbf{F} = \int_{\mathcal{B}} d^2z \sqrt{\sigma} B^\dagger. \quad (7)$$

The result (5) shows that Q_m is fixed as boundary data in the variational principle for S_1 . Of course, there are no local magnetic field sources in the system since the Maxwell equations imply the vanishing of the (covariant in space Σ) divergence of B^μ . However, the charge Q_m need not be zero because \mathcal{M} can have a nontrivial topology. In particular, black holes can carry magnetic charge.

The action S_2 for the electromagnetic field is defined by

$$\begin{aligned} S_2[g, F] &= S_1[g, F] + \frac{i}{4\pi} \int_{\mathcal{M}} d^4X \partial_\mu (\sqrt{-g} F^{\mu\nu} A_\nu) \\ &= \frac{i}{16\pi} \int_{\mathcal{M}} d^4X \{ \sqrt{-g} F^{\mu\nu} F_{\mu\nu} + 4 \partial_\mu (\sqrt{-g} F^{\mu\nu}) A_\nu \}. \end{aligned} \quad (8)$$

⁶This is not a necessary assumption, just a convenient one. Alternatively one can consider observers who are boosted relative to the constant time surfaces [6].

Note that S_2 differs from S_1 by a boundary term. Like S_1 , the fundamental variables in S_2 are A_μ and $g_{\mu\nu}$. The variation of S_2 is given by

$$\begin{aligned} \delta S_2 &= (\text{EOM's}) - \frac{i}{4\pi} \int_{\Sigma'} d^3x A_\nu \delta(\sqrt{h} \bar{u}_\mu F^{\mu\nu}) \\ &\quad + \frac{i}{4\pi} \int_{\mathcal{T}} d^3x A_\nu \delta(\sqrt{-\gamma} n_\mu F^{\mu\nu}) \\ &= (\text{EOM's}) - \frac{i}{4\pi} \int_{\Sigma'} d^3x A_\nu \delta(\sqrt{h} \bar{u}_\mu F^{\mu\nu} h_\nu^i) \\ &\quad + \frac{i}{4\pi} \int_{\mathcal{T}} d^3x A_i \delta(\sqrt{-\gamma} n_\mu F^{\mu\nu} \gamma_\nu^j). \end{aligned} \quad (9)$$

[The derivation uses the results $\delta(\sqrt{-h/g} \bar{u}_\mu) = 0$ and $\delta(\sqrt{\gamma/g} n_\mu) = 0$.]

The electric field on Σ' or Σ'' , as measured by the observers \bar{u}^μ , is

$$E^i = -\bar{u}_\mu F^{\mu\nu} h_\nu^i. \quad (10)$$

The result (9) shows that in the variational principle for S_2 the electric field E^i is fixed on Σ' and Σ'' . (Again, it is assumed that the induced metric on the boundary of \mathcal{M} is fixed.) Now consider the boundary element \mathcal{T} . Equation (9) shows that $n_\mu F^{\mu\nu} \gamma_\nu^i$ is fixed on \mathcal{T} . Using the decomposition $F^{\mu\nu} = 2u^{[\mu} E^{\nu]} + \epsilon^{\mu\nu\rho\sigma} u_\rho B_\sigma$ for the field strength tensor, we find

$$n_\mu F^{\mu\nu} \gamma_\nu^i = -(E^\dagger) u^i + \epsilon^{ab} \sigma_a^i B_b, \quad (11)$$

where $E^\dagger = E^\mu n_\mu$. Therefore the fixed data for S_2 includes the component E^\dagger of the electric field orthogonal to the system boundary \mathcal{B} and the components B^a of the magnetic field tangent to \mathcal{B} . The electric charge Q_e for the system, defined by

$$4\pi Q_e = \int_{\mathcal{B}} {}^* \mathbf{F} = \int_{\mathcal{B}} d^2z \sqrt{\sigma} E^\dagger, \quad (12)$$

is fixed as well. In this paper only the sourceless Maxwell equations are considered; however, a nonvanishing electric charge (like magnetic charge) can arise through the presence of charged black holes.

Now turn to the derivation of the duality relationship $S_1[\tilde{g}, \tilde{F}] = S_2[\tilde{g}, -{}^* \tilde{F}]$. Consider a solution $g_{\mu\nu} = \tilde{g}_{\mu\nu}$, $A_\mu = \tilde{A}_\mu$ of the classical Einstein-Maxwell equations. The potential yields the field strength $\tilde{F}_{\mu\nu} = 2\partial_{[\mu} \tilde{A}_{\nu]}$. The dual classical solution is defined by $g_{\mu\nu} = \tilde{g}_{\mu\nu}$, $A_\mu = {}^D \tilde{A}_\mu$, where the potential ${}^D \tilde{A}_\mu$ yields the dual of $\tilde{F}_{\mu\nu}$; that is, ${}^* \tilde{F}_{\mu\nu} = 2\partial_{[\mu} {}^D \tilde{A}_{\nu]}$.⁷ We can evaluate the action $S_1[g, F]$ at the solution $\tilde{g}_{\mu\nu}$, \tilde{A}_μ simply by substituting \tilde{g} for g and \tilde{F} for F into Eq. (1). When the action $S_2[g, F]$ is evaluated at

⁷In differential forms notation, $\tilde{\mathbf{F}} = d\tilde{\mathbf{A}}$. Since $\tilde{g}_{\mu\nu}$, \tilde{A}_μ satisfies the Maxwell equations, we have $d{}^* \tilde{\mathbf{F}} = 0$. Then the Poincaré lemma guarantees the local existence of ${}^D \tilde{\mathbf{A}}$ such that ${}^* \tilde{\mathbf{F}} = d{}^D \tilde{\mathbf{A}}$.

the dual solution $\tilde{g}_{\mu\nu}$, ${}^D\tilde{A}_\mu$, we find that the term in Eq. (8) involving $\partial_\mu(\sqrt{-\tilde{g}}*\tilde{F}^{\mu\nu})$ vanishes identically.⁸ Then using the result $*\tilde{F}^{\mu\nu}*\tilde{F}_{\mu\nu}=-\tilde{F}^{\mu\nu}\tilde{F}_{\mu\nu}$, we obtain the relationship

$$S_1[\tilde{g},\tilde{F}]=S_2[\tilde{g},-*\tilde{F}]=-\frac{i}{16\pi}\int_{\mathcal{M}}d^4X\sqrt{-\tilde{g}}\tilde{F}^{\mu\nu}\tilde{F}_{\mu\nu}. \quad (13)$$

As discussed in the Introduction, this result shows that the rate of creation of magnetically charged black holes under a given set of boundary conditions is equal to the rate of creation of electrically charged black holes under the dual set of boundary conditions.

It should be noted that the analysis presented here applies to both Lorentzian and Euclidean geometries, as well as to more general complex geometries. In particular, the relationship (13) holds for any metric signature. Of course, $\sqrt{-\tilde{g}}$ is imaginary if $\tilde{g}_{\mu\nu}$ is Euclidean. Likewise, the volume form $\epsilon_{\mu\nu\rho\sigma}$, which appears in the definition of the dual field strength $*\tilde{F}_{\mu\nu}$, is imaginary if $\tilde{g}_{\mu\nu}$ is Euclidean. However, in general, the analysis is completely self-consistent and there is no need to construct separate arguments for the Lorentzian and Euclidean cases.

⁸The equation $\partial_\mu(\sqrt{-\tilde{g}}*\tilde{F}^{\mu\nu})=0$ is equivalent to $d\tilde{\mathbf{F}}=0$, which holds because $\tilde{\mathbf{F}}=d\tilde{\mathbf{A}}$.

As a final remark, let me point out that no action principle for electrodynamics has the full electric field, E^\dagger and E^a , for its boundary conditions on \mathcal{T} . Likewise, no action principle has the full magnetic field, B^\dagger and B^a , for its boundary conditions on \mathcal{T} . The easiest way to see this is to recognize that fixation of $E^1\sim\partial_1A_0-\partial_0A_1$, $E^2\sim\partial_2A_0-\partial_0A_2$, and $E^3\sim\partial_3A_0-\partial_0A_3$ on \mathcal{T} is analogous to fixation of $B^2\sim\partial_1A_3-\partial_3A_1$, $B^1\sim\partial_2A_3-\partial_3A_2$, and $E^3\sim\partial_0A_3-\partial_3A_0$ on Σ' and Σ'' . Here, for simplicity, I have assumed flat Minkowski spacetime with coordinates X^μ . Also, \mathcal{T} coincides with an $X^3=\text{const}$ surface and Σ' and Σ'' coincide with $X^0=\text{const}$ surfaces. Now, the quantities fixed at the end points in time (that is, at Σ' and Σ'') are the canonical coordinates. Therefore the fixed quantities must have vanishing Poisson brackets among themselves. This is not the case for B^2 , B^1 , and E^3 . From the point of view of canonical transformations, this argument shows that there is no boundary term which, when added to the action S_1 (or S_2), leads to B^2 , B^1 , and E^3 as the fixed data on Σ' and Σ'' .⁹ Analogously, we find that there is no boundary term which, when added to the action S_1 (or S_2), leads to E^\dagger , E^a as fixed data on \mathcal{T} . A similar argument shows that fixation of B^\dagger , B^a is not possible on \mathcal{T} .

⁹From the quantum mechanical point of view, this argument shows that a specification of B^2 , B^1 , and E^3 would violate the uncertainty principle.

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