Some remarks on the unified characterization of reproducing systems

KANGHUI GUO

Department of Mathematics, Southwest Missouri State University
Springfield, Missouri 65804, USA
E-mail: kag026f@smsu.edu

DEMETRIO LABATE

Department of Mathematics, North Carolina State University
Campus Box 8205, Raleigh, NC 27695, USA
E-mail: dlabate@math.ncsu.edu

Received June 24, 2005. Revised September 7, 2005

ABSTRACT

The affine systems generated by $\Psi \subset L^2(\mathbb{R}^n)$ are the systems

$$A_A(\Psi) = \{D_A^j T_k \Psi : j \in \mathbb{Z}, k \in \mathbb{Z}^n\},$$

where $T_k$ are the translations, and $D_A$ the dilations with respect to an invertible matrix $A$. As shown in [5], there is a simple characterization for those affine systems that are a Parseval frame for $L^2(\mathbb{R}^n)$. In this paper, we correct an error in the proof of the characterization result from [5], by redefining the class of not-necessarily expanding dilation matrices for which this characterization result holds. In addition, we examine the connection between the eigenvalues of the dilation matrix $A$ and the characterization equations of the affine system $A_A(\Psi)$ that are Parseval frames. Our observations go in the same directions as other recent results in the literature that show that, when $A$ is not expanding, the information about the eigenvalues alone is not sufficient to characterize or to determine existence of those affine systems that are Parseval frames.

Keywords: Affine systems, characterization equations, tight frames, wavelets.

MSC2000: 42C15, 42C40.