Layout of Piping Systems and the Joint Forces and Moments Acting on the Components

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Summary:

The forces and moments occuring with the thermal expansion of a piping system acting on the metal containment, pressure vessels, pumps or other equipment are received just at the end of a finite element analysis of the system. If they cause excessive stresses in the components, it is necessary to repeat the finite element calculation to reduce the values of the above-mentioned forces and moments by trial and error. The new computer programme "Mathematica" allows us to solve algebraic equations and consequently use efficiently the Castigliano-method for simplified calculations of the joint forces and moments. In this paper the dimensional analysis has been additionally used to determine non dimensional numbers including forces and moments.

The examination of the stress analysis of different pipe systems has shown that their weak points frequently occur in the area of the link between the pipe and the adjacent components. For this reason we have investigated the joint forces and moments in different pipe systems. In this paper we have confined ourselves to the analysis of forces and moments occuring through free thermal expansion because the secondary stresses caused thereby change their values rapidly when small changes in the geometry take place and therefore their determination constitutes a delicate target for the first approximative dimensioning. For the analysis the Castigliano principle and the scale-up theory will be applied. The investigation will be conducted gradually from simple to complicated pipe lay-outs. First the joint forces and moments caused by temperature change in simple pipe system situated on the same plane (Fig. 1a) will be analysed. Its model is shown in Fig. 1b. With the change of ΔT point P_2 of the pipe system will change its position through free thermal expansion to P_2^*. The forces N, Q and the moment M arising at the fixed point will bring point P_2^* back to the inital position P_2.

Thus the deformation energy in the system will be

$$\mathscr{H} = \int_0^l \frac{[N(x)]^2}{2AE}\,dx + \int_0^l \frac{[M(x)]^2}{2EI_z}\,dx \qquad (1)$$

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where: A = The section area of the pipe
E = modulus of elasticity
I_z = moment of inertia
$N(x)$ = force
$M(x)$ = moment

The energy part due to the shear force Q was neglected.

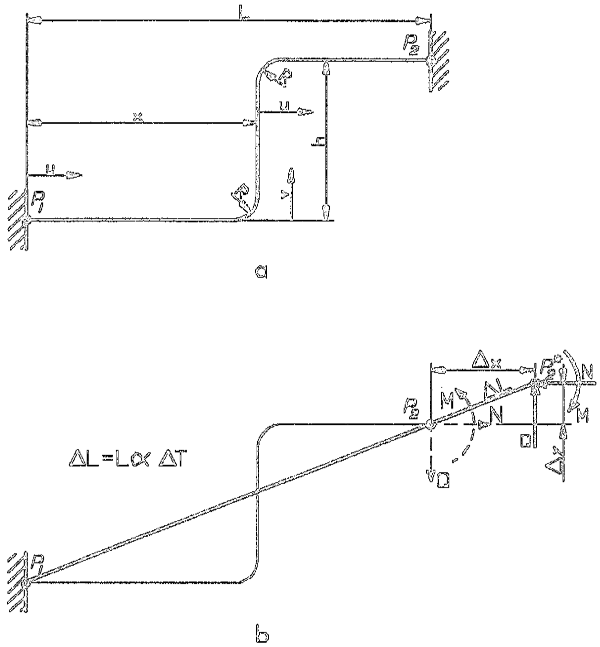

Figure 1. Pipe system with one vertical element(a) and its model(b)

For the calculation of the forces and moments in P_2 the following equations (Castigliano principle) are used:

$$\frac{\partial \mathscr{H}}{\partial N} = -L\alpha\Delta T \tag{2}$$

$$\frac{\partial \mathscr{H}}{\partial M} = 0 \tag{3}$$

$$\frac{\partial \mathscr{H}}{\partial Q} = h\alpha\Delta T \tag{4}$$

Where: α = coefficient of thermal expansion
ΔT = temperature interval

for $L << R$ will be assumed: R = 0 (5)

The forces N, Q and the moment M depend on the following values:

$$N = f_1(x, h, L, R, A, E, I_z, \alpha, \Delta T) \tag{6}$$

$$Q = f_2(x, h, L, R, A, E, I_z \alpha, \Delta T) \tag{7}$$

$$M = f_3(x, h, L, R, A, E, I_z, \alpha, \Delta T) \tag{8}$$

The functions in the equations (6)-(8) are transformed to dimensionless expressions:

$$\frac{NL^2}{EI\alpha\Delta T} = f_4\left(\frac{x}{L}, \frac{h}{L}\right) \tag{9}$$

$$\frac{ML}{EI\alpha\Delta T} = f_5\left(\frac{x}{L}, \frac{h}{L}\right) \tag{10}$$

$$\frac{QL^2}{EI\alpha\Delta T} = f_6\left(\frac{x}{L}, \frac{h}{L}\right) \tag{11}$$

The values of the functions f_4, f_5 and f_6 are determined by the programme "Mathematica" and presented here graphically in Fig. 2, 3, 4.

The forces N and Q have the same values in point P_1 as in point P_2. The curves for the calculation of the moments in point P_1 are symmetrical to the curves for the calculation of the moments in point P_2 (Fig. 4). The axis of symmetry is the vertical line x/L=0,5. With these considerations and from the Fig. 2-4 we find that for x/L≃0,7 and x/L≃0,3 the moments in the points P_2 and P_1 respectively are almost zero and the forces N and Q show reasonable values. The middle position (x/L=0,5) of the vertical pipe in Fig. 1 which is expected to be advantageous is in fact unfavourable concerning the joint forces and moments to adjacent components. In the calculations of joint forces and moments R/L=0 has been assumed. Consequently for $R/L > 0$ correction factors for the calculated values of N, Q and M must be introduced. The correction factors for different x/L-ratios and different L/R are calculated with the programme "Mathematica".
In Fig. 5 the correction factor

$$\varphi = \frac{N_{R/L}}{N_{R/L=0}} \text{ for } x/L = 0{,}7$$

is shown as an example.

Because of the limited scope of this paper it is not possible to present here all calculated values of the correction factors for forces and moments. The following conclusions however are based on the analysis of all correction factors already calculated in [2]:

- For L/R values over 8 the value 1 for the correction factors of N, Q, and M may be used.
- For the calculation of N, Q and M in short pipe systems (L/R < 8) the correction factors determined in [2] must be used.

The pipe system in Fig. 6 with two vertical elements will be calculated in the same way as the simple system in Fig. 1. The dimensionless equations (6) to (8) will change to

$$\frac{NL^2}{EI\alpha\Delta T} = f_{13}\left(\frac{x_1}{x}, \frac{h_1}{h}, \frac{x}{L}, \frac{h}{L}\right) \tag{12}$$

$$\frac{ML}{EI\alpha\Delta T} = f_{14}\left(\frac{x_1}{x}, \frac{h_1}{h}, \frac{x}{L}, \frac{h}{L}\right) \tag{13}$$

$$\frac{QL^2}{EI\alpha\Delta T} = f_{15}\left(\frac{x_1}{x}, \frac{h_1}{h}, \frac{x}{L}, \frac{h}{L}\right) \tag{14}$$

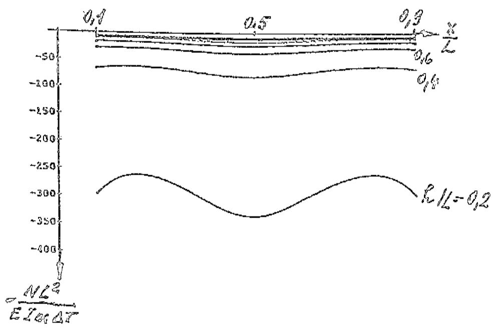

Figure 2. Axial force (N) curves

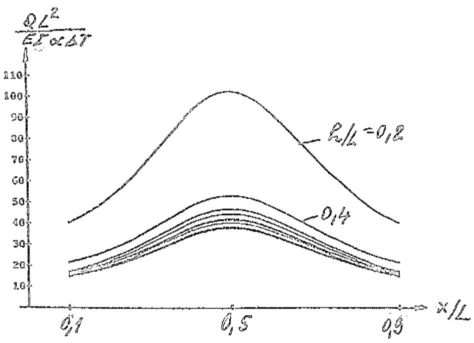

Figure 3. Shear force (Q) curves

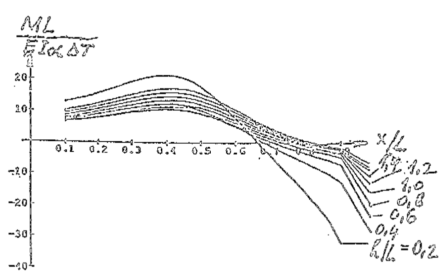

Figure 4. Curves of bending moment(M) in P_2

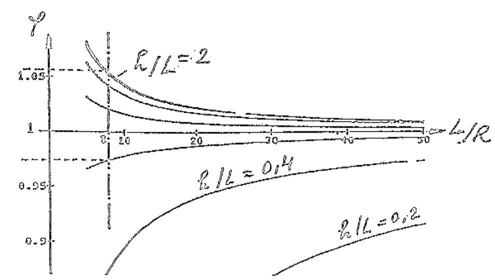

Figure 5. Correction factor for the axial force
Curves for different values of h/L for
x/L=0,7

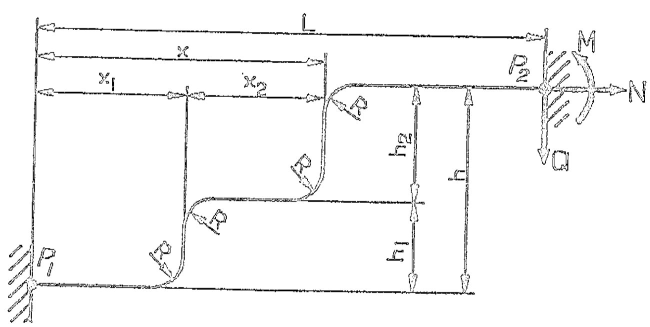

Figure 6. Pipe system with two vertical elements

The comparison of the values of forces and moments of two basis pipe models (Fig. 1 and Fig. 6) indicates:

- For every x_1/x and h_1/h and the same x/L and h/L the value of the axial force N in the model in Fig. 6 is higher than the corresponding value N for the model in Fig. 1. Consequently the lay-out in Fig. 1 is to be preferred to the lay-out in Fig. 6 regarding force N.
- For certain values h/L and x_1/x the model in Fig. 6 yields lower values for Q than the model in Fig. 1, namely for $h_1/h = 0{,}7$ for x/L = 0,1 to 0,6 and for $h_1/h = 0{,}2$ for x/L = 0,7 and 0,8 (see Fig. 7).
  The difference between the corresponding Q-values for the two given exceptions is so small, that we can conclude for the Q-force, too , that the configuration in Fig. 1 is to be preferred.
- The pipe lay-out in Fig. 1 yields lower bending moments than the lay-out in Fig. 6, namely for $x/L \leq 0{,}8$ for the bending moment in point P_2 and for $x/L \geq 0{,}2$ for the bending moment in point P_1 (see the comparison of the corresponding bending moments of Fig. 1 and Fig. 6 in [2]). These restrictions are of minor importance because the regions $x/L \leq 0{,}2$ and $x/L \geq 0{,}8$ will be avoided also from considerations of constructive design.

The values of the forces and moments for the pipe lay-out in Fig. 6 are calculated from the already known corresponding values (the same x/L and h/L) of the forces and moments of the lay-out in Fig. 1 using a correction coefficient. E.g. the correction coefficient Φ_1 for the axial force N is:

$$\Phi_1 = \frac{\left(\dfrac{NL^2}{EI\alpha\Delta T}\right)_{Figure\ 6}}{\left(\dfrac{NL^2}{EI\alpha\Delta T}\right)_{Figure\ 1}} = \frac{N_{Figure\ 6}}{N_{Figure\ 1}} \tag{15}$$

The correction coefficients are determined from the known values of the forces and moments calculated with the computer programme "Mathematica" using the method of least squares and dimensionless functions

$$\Phi_1\left(\frac{x_1}{x}, \frac{h_1}{h}, \frac{x}{L}\right) = t\left(\frac{h_1}{h}\right)^u \left(\frac{x_1}{x}\right)^v \left(\frac{x}{L}\right)^w \tag{16}$$

The numerical values of the constant coefficients t, u, v and w in the equation (16) are given in Table 1.

h/L	0,2	0,4	0,6	0,8	1,0	1,2	1,4	1,6	1,8	2,0
t	2,260	2,380	2,480	2,580	2,680	2,760	2,840	2,900	2,970	3,030
u	0,200	0,193	0,189	0,187	0,186	0,186	0,187	0,188	0,190	0,192
v	-0,280	-0,249	-0,219	-0,191	-0,167	-0,146	-0,130	-0,117	-0,104	-0,094
w	0,318	0,356	0,382	0,400	0,414	0,423	0,429	0,434	0,437	0,439

Table 1. Correction factor for the axial force
Values for the constant coefficients t, u, v and w

The application of the least squares method in calculating t, u, v and w may, in exceptional cases, yield differences of up to 100% between the calculated values of N with eq.(15) and (16) and the corresponding exact values. The calculated values are always higher than the exact values.

The correction coefficients Φ_2 and Φ_3 for the force Q and the moment M respectively are calculated in [2] in the same way as for Φ_1.

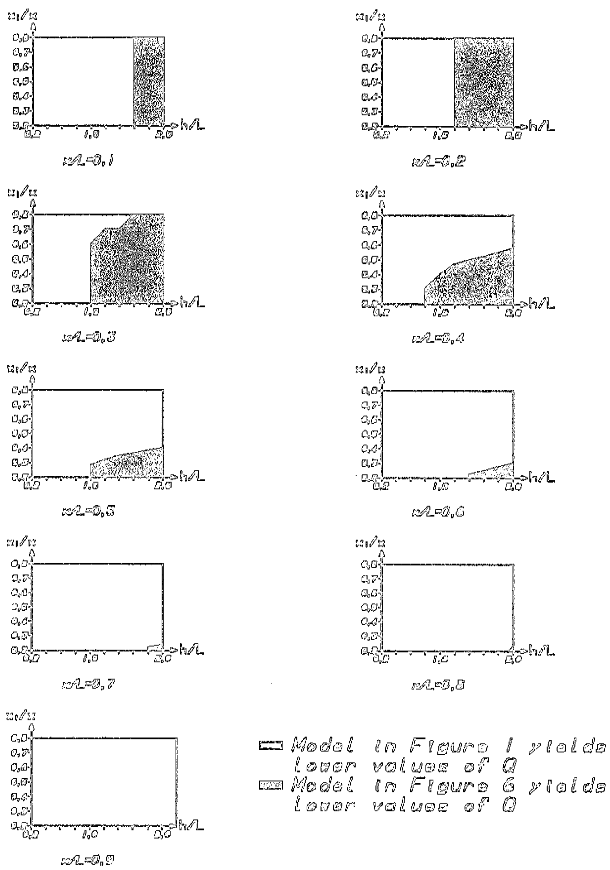

Figure 7. Comparison of the values of shear force Q in model (Fig. 1) and model (Fig. 6)

[1] L. Baraf — Optimisation of Pipe Systems in the Neighbourhood of Metal Containment, Pressure Vessels or Other Equipment Statement F8/4 at the 5th International Conference on Structural Mechanics in Reactor Technology, Berlin, August 1979

[2] S. Schreiber — Optimierung der Rohrleitungsauslegung in Bezug auf die Dyname in den Anschlusskomponenten. Forschungsbericht 1/BA-90, ETH Zuerich, 1990.

[3] H. A. Becker — Dimensionless Parameters. Theory and Methodology. Applied Science Publishers, London, 1976.