Hydromechanical Analysis of a Primary Pipe(1D) 
Coupled to a Reactor Vessel(3D) during a Depressurization 

M. LEPAREUX, A. COMBESCURE, H. MAHIL 
CEA/DRN/SEM1. CEN Saclay. Gif sur Yvette, France

ABSTRACT

In order to analyze the consequences of the depressurization of a PWR, the water has been 
modelized by means of a non-equilibrated homogeneous diphasic material. An application to 
HDR reactor is also presented for validation purpose.

1 INTRODUCTION

For the safety of Pressurized Water Reactors, it is necessary to analyze the consequences of 
the hypothetical rupture of a primary pipe. In such an occurrence, the depressurization arises 
in two steps. At first, an acoustic phase, comprising water expansion and ejection in single 
liquid phase. Pressure waves propagate inside the circuit, then damp down after having 
speed up the water. Afterwards, the diphasic regime becomes general, the fluid increases 
its speed to reach a flow rate limited by the critical flow at the break. In the meantime, 
structures react and disturb the flowing.

All these phenomena being cross-coupled, it appears wise to own a digital tool allowing 
complete system analyses. Such a tool has to be able to carry out the 1D computation of 
piping circuit connecting the break to the reactor, and to process the 3D effects at the 
connection level. It is necessary to simultaneously perform the 3D computation of the reactor 
with its internal structures, taking into account the fluid-structure couplings, in order to 
evaluate the mechanical consequences of the depressurization. This action has been 
undertaken at CEA-DMT, and took place in two stages.

At first, the effort was focussed on the development of a 3D-1D junction, allowing to 
simultaneously couple the fluid and the structures between a 1-D domain (circuit) and a 3D 
domain (reactor). This first stage has been carried out in 1988, and was the subject of a 
presentation to the SMIRT 10 (Lepareux et al., 1988). An application, with a simplified 
behaviour law for the fluid, was also done to analyze the HDR tests of Karlsruhe (Germany) 
(HDR report, 1980) and gave encouraging results.

The second stage hereby described highlights the fluid behaviour. In fact in a 
depressurization, the water vaporization is not instantaneous, and the vapour bubbles which 
appear follow a particular dynamics which modifies the progress of phenomena. Then we 
carried out again the HDR calculation, taking into account this new model, and we were able 
to find back the behaviour highlighted during the tests, so validating the method used.

2 PLEXUS

PLEXUS is a general program for structure computation with finite elements method, pertaining 
to CASTEM system of CEA-DMT (Lepareux et al., 1988). It is well suited for analysis of rapid 
transient phenomenae. The time integration is explicit, and the formulation can be Lagrangian, 
Eulerian, or ALE. For a given step of time, the result is simply obtained by integration, in each element volume, of equations for conservation of mass, of energy, and of motion quantity.

Properties of materials appear through the status law giving the stress tensor (or pressure), from element local quantities calculated by Plaxis. One of the advantages of Plaxis is the great convenience for introducing new behaviour laws, allowing the user to entirely devote himself to the model he wishes to develop.

3 EQUILIBRATED HOMOGENEOUS MIXTURE

Among the usual materials entered in the library (today about sixty), we have access to water tables. So, assuming additionally that the bi-phase mixture water-vapour is at thermal equilibrium, it is possible, knowing the density and the mean mass enthalpy within an element, to infer the pressure, the temperature, and the void fraction, of the mixture. That is the material which was used in 1988 for the first HDR calculations. It has also to be noted that, once the law is chosen for the fluid, the behaviour at break is perfectly determined, specially the critical flow rate, if any. In fact, Plaxis will compute the transient regime starting from the fluid at rest, and aiming towards a stationary state, depending on local characteristics of the fluid at the break level. Of course, it is always possible to locally leave the equilibrated homogeneous fluid assumptions to make other choices. In particular, it is possible to keep the thermal equilibrium between the two phases, and to adopt a shifting coefficient, as Moody or Para using. A discussion about this definite subject is presented in an associated paper (Sperandio, 1991). In the HDR calculation presented in 1988, the equilibrated homogeneous model was also kept for the break.

4 HOMOGENEOUS METASTABLE MIXTURE

This model is primarily intended to deal with effects related to beginning of vaporization, when the void fraction is low. The mechanical equilibrium between the two phases will thus be kept, they will have the same pressure and the same velocity (no shifting). On the contrary, phases will be thermally non-equilibrated, i.e. the liquid temperature can be higher than the saturation temperature T_{sat}, while the vapour is at equilibrium at T_{sat}. The energy transfer from a phase to the other depends on liquid-vapour interface. It is mainly due to conduction in the liquid phase.

4.1 Mixture equations

In each element, the concentration of each phase allows to know the homogeneous mixture composition. There is no more separation surface. Let write with an index the quantities related to each phase \( i = \{ \text{l, v} \} \). Quantities without index are related to homogeneous mixture. Let \( m, V, \rho, h, x \) be the masses, volumes, densities, enthalpies, and concentrations. Let also \( \alpha \) be the void fraction and \( a \) the density of a phase related to total volume of the mesh. It comes :

\[
\begin{align*}
\alpha_i &= \frac{\rho_i x_i}{V} = \frac{m_i}{V} & (i = \{ \text{l, v} \}) \\
\alpha &= \frac{V_v}{V} = \frac{a_v}{\rho_v} \\
\rho &= a_i \div a_v & \rho h = a_i h_i \div a_v h_v
\end{align*}
\]

(1) (2) (3)

To simplify the writing of these equations, let consider a 1D case. Afterwards, it will be easy to generalize to 2D and 3D. The velocity of the two phases is \( U \), their pressure \( \rho \), the specific internal energy \( e \) . The conservation equations are :

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} &= 0 & \text{(mixture mass)}
\end{align*}
\]

(4)
\[
\frac{\delta n}{\delta t} + \frac{\delta (n u)}{\delta x} = \Gamma \quad \text{(vapour mass)} \tag{5}
\]
\[
\frac{\delta (\rho u)}{\delta t} + \frac{\delta}{\delta x} (P + \rho u^2) = -\zeta \quad \text{(momentum)} \tag{5}
\]
\[
\frac{\delta (\rho e)}{\delta t} + \frac{\delta}{\delta x} (\rho e u) = 0 \quad \text{(energy)} \tag{7}
\]

At the second member appear \( \Gamma \) and \( \zeta \), the vapour production rate and the friction on the walls, for 1D case.

4.2 Resolution

We have to calculate the mixture pressure taking into account the density and the mean enthalpy, using the water tables. Previously, we have to determine the vapour production rate \( \Gamma \), with the help of new assumptions. We will assume to be in Plesset and Zwick's conditions (1954), where dispersed vapour bubbles grow within a superheated liquid.

Let consider the growing of a spherical bubble, and neglect the very short initial inertial effects, to consider the instant when the radius increases is controlled by the conduction:

\[
\frac{dR}{dt} = \sqrt{\frac{3}{\pi}} \chi_l \frac{T_1 - T_v}{L} \frac{1}{\rho_v \sqrt{k_v}} \quad \text{with} \quad \chi_l = \frac{C_v}{\rho_v} k_v \tag{3}
\]

where \( \chi_l \), \( k_v \), \( C_v \), and \( L \) are respectively the conductivity, the diffusivity, the specific heat capacity, and the latent heat. Let eliminate the time \( t \) by integration, neglecting the initial small radius of the bubble:

\[
\frac{dR}{dt} = \frac{6}{\pi} \frac{1}{R} \left( \frac{\rho_v}{\rho_l} \right)^2 k_v \left( \frac{C_v}{L} \left( T_1 - T_v \right) \right) \tag{3}
\]

On the other hand, assuming that all the bubbles of a given mesh are identical, the radius \( R \) is related to the void fraction. If the number \( n \) of bubbles per volume unit is constant:

\[
\frac{d\alpha}{dt} = \frac{3\alpha}{R} \frac{dR}{dt} \tag{10}
\]

moreover, if \( \alpha \) is small, the equation (5) allows to infer \( \Gamma \) at the first order, from \( \alpha \):

\[
\Gamma = \rho_v \frac{d\alpha}{dt} \tag{11}
\]

grouping the formulas (9) to (11):

\[
\Gamma = \frac{16}{\pi} \alpha^{1/3} \left( \frac{4mn}{3} \right)^{2/3} \frac{\rho_v^2}{\rho_l} k_l \left( \frac{C_v}{L} \left( T_1 - T_v \right) \right)^2 \tag{12}
\]

This formula is only depending on two adjustable parameters: \( n \) (number of bubbles) and \( \alpha \) (initial void fraction). All the others are coming from the water tables. It has to be noted in (12) that if \( \alpha \) is null, \( \Gamma \) is also null. Thus, there is an \( \alpha \) threshold to have the process triggered. So this parameter allows to ensure continuity between the monophasic behaviour law and the biphasic law. The number \( n \) is related to nucleation process, and depends on water properties and purity, but it can be estimated either experimentally or by a theoretical approach.
As the vapour is created, the void fraction increases and the temperatures of the two phases become closer. Then there is an evolution towards the equilibrated homogeneous model to process the mass vaporization.

5 ANALYSIS OF HDR TESTS WITH THE METASTABLE MODEL

The purpose of these tests was to highlight the thermohydraulic effects and the fluid-structure couplings. Figures 1 and 2 show the geometry and the main properties of the materials. The meshing used by Plexus (3282 elements) is on fig. 3, on which the fluid was oriented for clarity. Limit conditions and support stillness have been adjusted to find the frequencies of fundamental modes. The calculation has been carried out up to 100 ms, sufficient duration to highlight the most important fluid-structure effects.

Results are shown as curves versus time, where are reported the test results and the ones obtained with both fluid models (equilibrated and non-equilibrated). At first, an excellent agreement between the metastable model and the experiment has to be noted, as well for the break pressure as for the flow rate in the pipe (fig. 4 and 5). It is also noted that the equilibrated model stops at the saturation pressure (50 bars), while the metastable model (and the test) go down until 25 bars. We also note the equilibrated model has a tendency to underestimate the break flow, this is not conservative.

The fluid behaviour in the ring space and in the internal region is also well reproduced, while the equilibrated model gives a very different result (fig. 6, 7, and 9). The structure motion is also correctly reproduced, but the deviations with the equilibrated model are less significant. In fact, the structure excitation comes for a great account from the loss of bottom effect, which is identical for both models.

6 CONCLUSIONS

The developments implemented in Plexus allow to process in an unique pass the whole phenomena of fluid-structure interaction, taking into account the in-dimensional effects related to the connection with the rest of the circuit. The homogeneous non-equilibrated model taken for the fluid allowed to reproduce the physical phenomena associated to the depressurization, and to correctly analyze the transient regime. The correct interpretation of HDR tests shows that this tool is operational for industrial problems.

REFERENCES


Fig. 1 - NBR REACTOR

Fig. 2 - Mesh of the reactor (the fluid is not represented)

Fig. 3 - Initial State

- Pressure vessel (Steel)
  \[ \rho = 7850 \text{ kg/m}^3 \] (Density)
  \[ E = 212 \text{ GPa} \] (Young)
  \[ v = 0.29 \] (Poisson)
  \[ \sigma = 372 \text{ MPa} \] (Elastic limit)

- Core barrel (Steel)
  \[ \rho = 7900 \text{ kg/m}^3 \] (Density)
  \[ E = 190 \text{ GPa} \] (Young)
  \[ v = 0.26 \] (Poisson)
  \[ \sigma = 285 \text{ MPa} \] (Elastic limit)

- Fluid (Water)
  \[ P_b = 91 \text{ bar} \] (Pressure)
  \[ T_b = 273 \text{ °C} \] (Temperature)
  From the tables:
  \[ P_f = 787 \text{ kg/m}^3 \] (Density)
  \[ C_p = 1680 \text{ m/s} \] (Sound velocity)
Fig. 4 - Pressure at the ruptured end of the pipe

Fig. 5 - Water mass-flow in the pipe

Fig. 6 - Pressure in the ring region (up)

Fig. 7 - Pressure in the ring region (down)

Fig. 8 - Pressure in the inner region

Fig. 9 - Relative displacement of the core barrel