A Simple Approach for Selecting the Most Severe Values of Failure Parameters among a Set of Materials

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I. INTRODUCTION.

Generic and conservative analysis of a large set of nuclear components or joints; e.g. all the welds of one line having potentially the same crack sizes, is performed, for convenience, with the worst possible fracture mechanics parameters for this set of welds. Considering the GE-EPRI or the R6 rev. 3 engineering rules, the two most sensitive parameters for a given crack length are :

- the fracture toughness J_{1C},
- and the yield strength σ_y, which has an influence on J_{app} computed from the GE-EPRI handbook using the limit load M_L and from R6 using L_R.

The usual way to perform a generic analysis of this set of components, is to consider a hypothetical material having the following characteristics:

- the lower bound yield strength value,
- the lower bound fracture toughness value.

Considering the real distribution of σ_y and J_{1C} (or KCU) of this set of materials, we note on figure 1 that this hypothetical material, (point A_0), is more conservative than any individual material in any component.

The aim of this paper is to propose a method, based on the R6 double criterion rule, to select the worst possible real material among a set of components, which envelops all the other components.

II. BASES OF THIS APPROACH.

The starting point is the material data base for the set of components. Each item represented by its yield strength σ_y and KCU values, is plotted on figure 1. Note that these two characteristics are systematically determined during nuclear component acceptance testing. A conservative linear relation already established between KCU and fracture toughness J_{1C} gives the same diagram of yield strength versus J_{1C}. Then, we observe that we have the same two variables: σ_y and J_{1C}, as in the R6 Failure Assessment Diagram (FAD):

- σ_y in L_R: $L_R^* = \sigma_e/Q\sigma_y$ σ_e elastic nominal stress on the component

- J_{1C} in K_R: $K_R^* = K_I/(J_{1C}E')^{1/2}$

The assumed material, used for the generic analysis, is represented by the point A_0. The four black dots selected on figure 1 (n° 1, 3, 4, 10) represent materials which are individually less conservative than the fictitious material A_0. The four σ_y and J_{1C} values are given in table 1.

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This paper tries to answer this question: which one of these four points can be used to perform a generic fracture mechanics analysis, envelopes all the other materials, and is less severe than the hypothetical material represented by point A_0 ?

II.1. Application of R6 rev. 3 approach to determine the critical material.

For each of the four black points on figure 1, we can plot the Failure Assessment Line (FAL) using σ_y and J_{1C} values. We assume that all these four materials have a stress-strain curve with basically the same shape, defined by the mean Ramberg Osgood parameters α and n.

(1) $$\varepsilon = \sigma/E + \alpha\ \varepsilon_y\ (\sigma/\sigma_y)^n \qquad \varepsilon_y = \sigma_y/E$$

The equation proposed by AINSWORTH [1] for the FAL is the following:

(2) $$K_R = [E\varepsilon_{ref}/\sigma_{ref} + L_R^{\,2}/2(1 + L_R^{\,2})]^{-1/2} \qquad \text{where} \quad L_R = \sigma_{ref}/\sigma_y$$

Equations (1) and (2), with $\varepsilon = \varepsilon_{ref}$ and $\sigma = \sigma_{ref}$, lead to the following expression of the FAL as a function of the Ramberg Osgood parameters:

(3) $$K_R = [\ 1 + \alpha L_R^{\,n-1} + L_R^{\,2}/\ 2(1 + L_R^{\,2})]^{-1/2}$$

This equation (3) gives the same FAL for the four materials. But positions of the assessment points of the same crack in the FAD, figure 2, under the same load, of these four materials, defined by (4) and (4') are different, (K_R^* and L_R^* denote the assessment points, K_R and L_R the FAL).

(4) $$K_{R\ n}^{*} = K_I/\ (E'J_{1Cn})^{1/2} \quad = \quad K_{R1}^{*}(J_{1C1}/J_{1Cn})^{1/2}$$

(4') $$L_{R\ n}^{*} = \sigma_e\ /Q\sigma_{yn} \quad = \quad L_{R1}^{*}\sigma_{y1}/\sigma_{yn} \qquad n = [1,\ 4]$$

In order to compare these materials in the FAD; we can use the following method: 1) we compute equation (3) for the FAL and we assume that it corresponds to material 1. This becomes the reference FAL. For the four components, we plot the assessment points of this crack with (4) and (4').
2) In order to get the same assessment point for the four components, we perform the following transformation on K_R and L_R of the reference FAL given by equation (3):

(5) $$K_{Rn} = K_R\ (J_{1Cn}/\ J_{1C1})^{1/2}$$

(6) $$L_{Rn} = L_R\ \sigma_{yn}/\ \sigma_{y1}$$

Using equations (5) and (6) with σ_y and J_{1C} values of materials 3, 4, and 10 we obtain the set of four FAL* on figure 3.
Equations (5), (6), and (3) give the equation of these four offset FAL, labelled FAL* :

(7) $$K_{Rn} = \left[\frac{J_{1C1}}{J_{1Cn}}\left(1 + \alpha\left(L_{Rn}\frac{\sigma_{y1}}{\sigma_{yn}}\right)^{n-1} + \left(L_{Rn}\frac{\sigma_{y1}}{\sigma_{yn}}\right)^{2}\Bigg/2\left(1 + \left(L_{Rn}\frac{\sigma_{y1}}{\sigma_{yn}}\right)^{2}\right)\right)\right]^{-1/2}$$

The FAL* given by equation (7), for materials 1, 3, 4 and 10 are shown on figure 3. In this new diagram, the same crack, in the four components, subjected to the same load, is represented by the same assessment point :

$$K_{R\ n}^{*} = K_{R\ 1}^{*} \qquad L_{Rn}^{*} = L_{R1}^{*}$$

We have also represented on this figure the FAL^* of the hypothetical material A_0, with the same procedure. We note on this figure:
1) this FAL^* (A_0) is indeed conservative as shown above,
2) the FAL^* of material 1 envelopes all the other materials in the domain of low L_R and the one of material 3 envelopes the other materials in the domain of L_R greater than 0.65. This last FAL^* is less severe than the one for A_0.

II.2. Relationship between σ_y and J_{1C} values leading to the same initiation load.

In this section, for material 1 (J_{1C1}, σ_{y1}), we endeavor to find another material, defined by J_{1Ci}, σ_{yi}, which is equivalent from a fracture mechanics point of view. For material 1, and for a given crack length, we can find a load $C(K_I, \sigma_e)$ which leads to crack initiation.

We obtain: $K_{R1}^* = K_I / (J_{1C1} \, E')^{1/2}$

$L_{R1}^* = \sigma_e / Q \, \sigma_{y1}$ (see section II.3 for K_I and σ_e)

This point (K_{R1}^*, L_{R1}^*) is on the FAL represented by equation (3) as shown on figure 4. For this demonstration, we consider that the crack is small in depth and in length ; then the shape factor Q is close to 1. We assume the same crack in another component (1') with a higher fracture toughness J_{1Ci} and with the same yield strength σ_{y1}, subjected to the same load. We get point (1') in the FAD of figure 4.

We have: (8) $K_{R1'}^* = K_{Ri}^* = K_{R1}^* \, (J_{1C1}/J_{1Ci})^{1/2}$ with $L_{R1'}^* = L_{R1}^*$

In order to get point (1') back to the FAL, at point i, we can decrease the yield strength of material (1') down to the value σ_{yi}, with the same K_R :

(9) $L_{Ri}^* = L_{R1}^* \, \sigma_{y1}/\sigma_{yi}$

Since the two points (1) and (i) lie on the FAL, we conclude that these two materials are equivalent for the risk of crack initiation under load C. For material (1), from the value of L_{R1}^*, we determine the value of K_{R1}^* with equation (3) for the FAL:

(10) $K_{R1}^* = [\, 1 + \alpha L_{R1}^{*n-1} + L_{R1}^{*2} / \, 2(1 + L_{R1}^{*2})]^{-1/2}$

and also K_{Ri} for the point (i):

(11) $K_{Ri}^* = [\, 1 + \alpha L_{Ri}^{*n-1} + L_{Ri}^{*2} / \, 2(1 + L_{Ri}^{*2})]^{-1/2}$

Substituting expressions for K_{Ri}^*(8) and L_{Ri}^*(9) in equation (11), we obtain:

(12)
$$K_{R1}^* \left(\frac{J_{1C1}}{J_{1Ci}}\right)^{1/2} = \left[1+\alpha\left(L_{R1}^* \frac{\sigma_{y1}}{\sigma_{yi}}\right)^{n-1} + \left(L_{R1}^* \frac{\sigma_{y1}}{\sigma_{yi}}\right)^2 \Big/ 2\left(1+\left(L_{R1}^* \frac{\sigma_{y1}}{\sigma_{yi}}\right)^2\right)\right]^{1/2}$$

Substituting equation (10) in (12) and $L_{R1}^* = \sigma_e/\sigma_{y1}$, we obtain:

(13)
$$J_{1Ci} = J_{1C1} \frac{1 + \alpha\left(\frac{\sigma_e}{\sigma_{yi}}\right)^{n-1} + \left(\frac{\sigma_e}{\sigma_{yi}}\right)^2 / \; 2\; 1 + \left(\frac{\sigma_e}{\sigma_{yi}}\right)^2)}{1 + \alpha\left(\frac{\sigma_e}{\sigma_{y1}}\right)^{n-1} + \left(\frac{\sigma_e}{\sigma_{y1}}\right)^2 / \; 2(1 + \left(\frac{\sigma_e}{\sigma_{y1}}\right)^2)}$$

All points (J_{1Ci}, σ_{yi}) satisfying relation (13) are equivalent for the initiation of this given crack under load $C(K_I, \sigma_e)$. To plot J_{1Ci} versus σ_{yi} on figure 1, we need to determine the value of σ_e (see section II-3). If load σ_e is given, the method to determine the critical crack length in this 2nd case is described in section II-4.

II.3. 1st case : determination of the critical nominal stress σ_e for given crack size and loading mode.

For a unit load C_u (considered as primary), we assume that the corresponding stress distribution in the thickness (t) is parabolic:

$$\sigma_u = \sigma_{0u} + \sigma_{1u}\frac{a}{t} + \sigma_{2u}\left(\frac{a}{t}\right)^2$$

and we can easily compute the corresponding membrane stress σ_{mu}. For any load $C = \beta C_u$, we compute the stress intensity factor at the crack tip using the influence function technique :

(14) $$K_I = \beta\sqrt{\pi a}\,[\sigma_{0u}i_0 + \sigma_{1u}i_1\frac{a}{t} + \sigma_{2u}i_2\left(\frac{a}{t}\right)^2]$$

with i_j: influence coefficients, and $\sigma_j = \beta\sigma_{ju}$

(15) $$\sigma_e = \beta\,\sigma_{mu}$$

(14) and (15) give a linear relation between K_I and σ_e:

$$K_I = \sqrt{\pi a}\,[\sigma_{0u}i_0 + \sigma_{1u}i_1\frac{a}{t} + \sigma_{2u}i_2\left(\frac{a}{t}\right)^2]\sigma_e/\sigma_{mu}$$

With (15') $$\gamma = \sqrt{\pi a}\,[\sigma_{0u}i_0 + \sigma_{1u}i_1\frac{a}{t} + \sigma_{2u}i_2\left(\frac{a}{t}\right)^2]/\sigma_{mu}$$

we obtain: (16) $$K_I = \gamma\,\sigma_e$$

Substituting K_I and σ_e in (16) along with (17) and (18)

(17) $$K_R = K_I/(J_{1C1}E')^{1/2}$$

(18) $$L_R = \sigma_e/\sigma_{y1}$$

we obtain the well-known straight line:

(19) $$K_R = \gamma\;\sigma_{y1}\;L_R/(J_{1C1}E')^{1/2}$$

This curve is the loading line: when the load increases, K_R and L_R increase linearly, as shown on figure 2. Equations (3) and (19) allow us to obtain σ_e corresponding to crack initiation, i.e. intersection of the loading line with the FAL. σ_e is solution of (20), obtained by substituting (18) and (19) in (3) :

(20) $$[\gamma\sigma_e]^2 = J_{1C1}E'/[1 + \alpha\left(\frac{\sigma_e}{\sigma_{y1}}\right)^{n-1} + \left(\frac{\sigma_e}{\sigma_{y1}}\right)^2/\,2(1 + \left(\frac{\sigma_e}{\sigma_{y1}}\right)^2)]$$

γ being determined by (15'). This solution may be obtained numerically or graphically. The critical value of σ_e being determine by equation (20), its introduction in equation (13) gives the relation between J_{1Ci} and σ_{yi}, for a given crack size and loading mode, which is drawn on figure 1.

II.4. 2^{nd} case: determination of the critical crack a_c under a given load C.
Here, the load $C(K_I, \sigma_e)$ is known, and we have to determine the critical crack size. Locations of pairs (J_{1Ci}, σ_{yi}) on figure 1 are always given by equation (13), with σ_e applied on the crack. To obtain the critical crack size under this load C, we use equation (20) which gives γ:

$$\gamma^2 = J_{1C1}E'/[\sigma_e{}^2[\ 1 + \alpha\left(\frac{\sigma_e}{\sigma_{y1}}\right)^{n-1} + \left(\frac{\sigma_e}{\sigma_{y1}}\right)^2/\ 2(1 + \left(\frac{\sigma_e}{\sigma_{y1}}\right)^2)]$$

by applying relation (15') between γ and a, we obtain a_c and then K_I.

III. NUMERICAL APPLICATION.
The selected component is a straight pipe, with an axial internal surface crack, loaded in pressure. Dimensions and stresses as a function of pressure P (obtained from the LAME equations) are given on figure 5.

For material (1): $\sigma_{y1} = 210$ MPa, $J_{1C1} = 28.5$ KJ/m^2

Using (14), with $i_0 = 1.175$ and $i_1 = 0.67$ we obtain $K_I = 2.861$ P

and we obtain (21) $K_R = K_I/(J_{1C}E')^{1/2} = 0.0381$ P with E = 180 000 MPa

and (22) $L_R = 9.5P/210 = 0.0452$ P with $Q \approx 1$

(21) and (22) give the same loading line as (16): $K_R = 0.843\ L_R$

Intersection of this curve with the FAL given by (3), figure 6, gives a pressure $P_C = 18.8$ MPa; then $\sigma_e = 178$ MPa. With this value of σ_e, we compute J_{1Ci} in function of σ_{yi} with (13). This relation is the one plotted on figure 1, (curve C). This value of σ_e is the lowest possible for all materials of curve C, for this crack size. For this application, materials 1 & 3 give the same initiation load ; the material 4 allows a higher initiation load.

IV. CONCLUSIONS.
This study addresses the case of a generic crack stability analysis for scattered σ_y, J_{1C} material properties. A lower bound limit curve may be drawn in the σ_y, J_{1C} space based on R6 rev. 3 option 2 fracture analysis method. This simple relation has been derived, for a given crack shape, to determine the equivalent pairs of σ_y, J_{1C} values leading to the same and lowest initiation load. If the load is given, the same methodology can be used to deduce the pair of σ_y-J_{1C} values leading to the smallest critical size. This material is the most severe material regarding crack initiation. Further work needs to be performed to test the limits of the method and the effect of the shape factor Q.

Reference.
[1] R. AINSWORTH. "The assessment of defects in structures of strain hardening material". Eng. Frac. Mech. Vol. 19, N° 4, pp 633-642, 1984.

TABLE 1 : YIELD STRESS AND FRACTURE TOUGHNESS PROPERTIES

Points	σ_y (MPa) 343°C	KCU (daJ/cm^2) 20°C	J_{IC} (kJ/m^2) 300°C
1	210	1.26	28.5
3	184	1.4	33.0
4	172	1.67	42.1
10	155	3.47	102.2

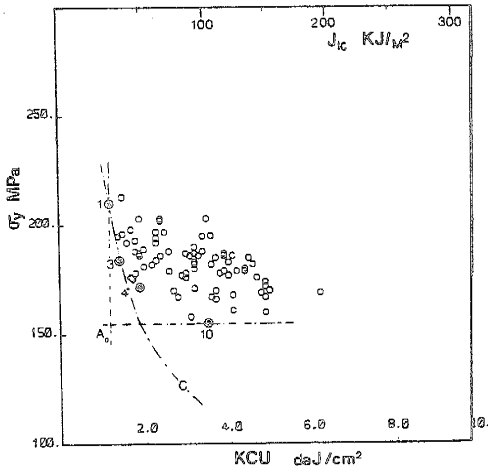

Figure 1: σ_y and J_{IC} data base for the set of components.

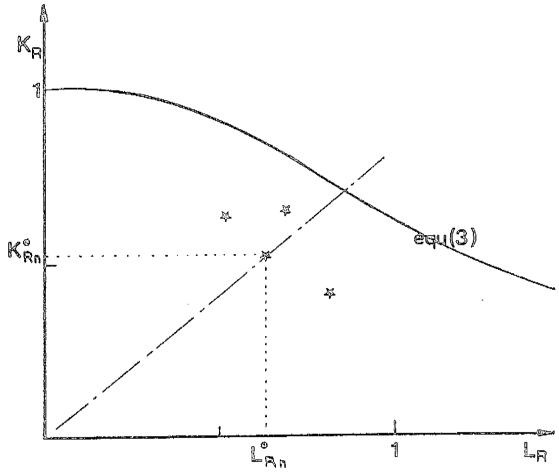

Figure 2: Assessment points of the crack for the four materials.

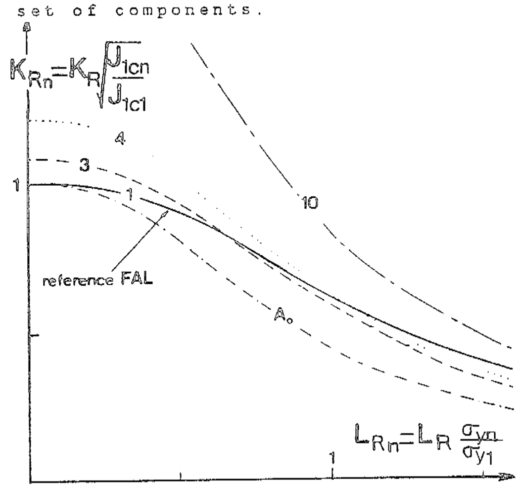

Figure 3: Offset FAL* for the four materials plus A_o.

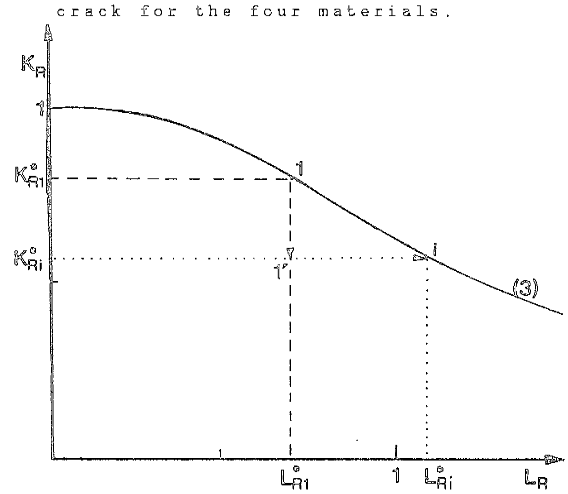

Figure 4: Variations of σ_y and J_{IC} to obtain the same crack initiation.

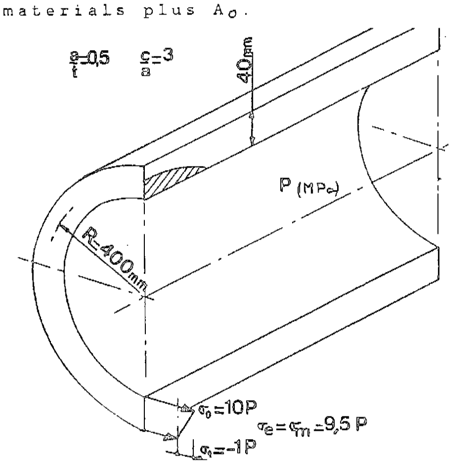

Figure 5: Geometry, material properties and stresses under pressure P.

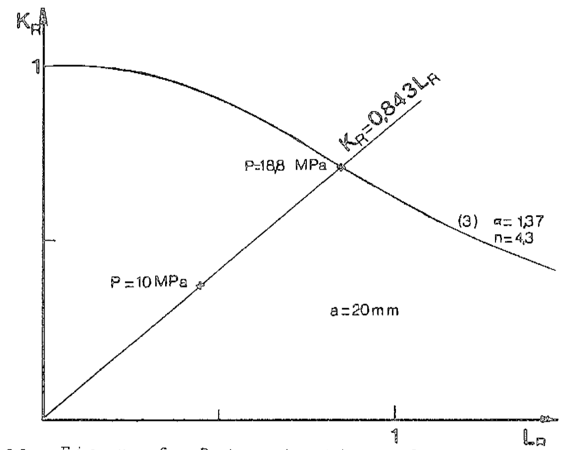

Figure 6: Determination of σ_e and critical pressure to initiation.