The Circumferential Crack Problem in a Cladded Pressure Vessel under Thermal Shock

F. ERDOGAN, A. A. RIZK
Lehigh University, Bethlehem, PA USA

ABSTRACT

The thermal shock problem for a layered plate containing cracks perpendicular to its boundary and supported by an elastic foundation approximating a coated cylinder is considered and the stress intensity factors for various crack geometries are given.

1 INTRODUCTION

The main interest in this paper is in subcritical crack propagation and fracture of coated materials, specifically of cladded pressure vessels subjected to repeated thermal shock conditions. The study is restricted to relatively large part-through circumferential cracks. Thus, the problem is approximated by an axisymmetric one which provides an upper bound for the stress intensity factors in the actual three dimensional crack problem. The crack is assumed to initiate either on the clad surface or at the interface and grow in a plane perpendicular to the axis of the composite cylinder. The crack geometries studied cover all possible crack propagation modes. In thermal shock problems the standard boundary condition of a step change in surface temperature seems to give thermal stresses that are far in excess of the yield and the tensile strength of the material. In this study, therefore, an additional time constant is introduced to account for the cooling rate at the surface.

Despite the assumption of axisymmetry, the crack problem in the composite cylinder is still higly complicated. Thus, in this study to make the problem analytically tractable, we take advantage of relatively small thickness to radius ratio in the medium and represent the composite cylinder by a layered plate on an elastic foundation (Erdogan and Rizk, 1991). In the other limiting case in which the base material is very thick compared to the coating and to the size and distance of the crack from the surface, the problem may be approximated by an elastic layer on a semi infinite elastic substrate (Rizk and Erdogan, 1989). A similar problem was studied by Nied (1984) who assumed that the cylinder is homogeneous in elastic but nonhomogeneous in termal properties. The axisymmetric crack problem for an elastic cylinder under thermal shock was considered by Nied and Erdogan (1984) who provided the results that form the basis of comparison in developing the plate model used in this paper. In the type of thermal stress problems under consideration all coupling and inertia effects are known to be very small and hence, in this study all such effects are neglected.

2. THE TEMPERATURE DISTRIBUTION

The procedure to solve the crack problem for a composite cylinder under thermal shock is as follows: approximate the cylinder by a composite plate on elastic foundation (Fig. 1),

SMiRT 11 Transactions Vol. G (August 1991) Tokyo, Japan, © 1991

determine the transient temperature distribution, determine the transient thermal stresses in the plate on elastic foundation in the absence of cracks, and use the equal and opposite of these stresses as tractions on the crack surfaces to solve the crack problem. The composite medium is initially at a homogeneous temperature T_∞ and at t=0, x=0 is subjected to the boundary condition shown. Defining

$$\theta_i (x,t) = T_i (x,t) - T_\infty \ , \ (i = 1,2) \tag{1}$$

and referring to Fig. 1, the diffusion problem may be formulated as

$$\frac{\partial^2 \theta_i}{\partial x^2} = \frac{1}{D_i} \frac{\partial \theta_i}{\partial t} , \quad \theta_i (x,0) = 0 , \quad (i = 1,2) , \tag{2}$$

$$\theta_1 (h_1 , t) = \theta_2 (h_1 , t) , \ k_1' \frac{\partial \theta_1(h_1, t)}{\partial x} = k_2' \frac{\partial \theta_2 (h_1,t)}{\partial x} , \ \frac{\partial}{\partial x} \theta_2 (h_1 + h_2, t) = 0, \tag{3}$$

$$\theta_1 (0, t) = \theta_o H (t) , \quad \theta_o = T_o - T_\infty , \tag{4a}$$

$$\theta_1 (0, t) = \frac{\theta_o}{t_o} [tH(t) - (t-t_o) H(t-t_o)] , \quad \theta_o = T_o - T_\infty \tag{4b}$$

where k_i' and D_i , (i = 1,2) are respectively the coefficients heat conduction and diffusivity. Boundary condition (4a) represents standard thermal shock and (4b) corresponds to linear heating ($\theta_o > 0$) or cooling ($\theta_o < 0$) shown in Fig. 1. The problem is rather straightforward and its solution is given by Erdogan and Rizk (1991).

3 THERMAL STRESSES IN THE UNCRACKED COMPOSITE PLATE

We observe that if the uncracked infinite composite plate is elastically supported as shown in Fig. 1, because of symmetry it would remain flat under the self-equilibrating thermal stresses. Thus, from the conditions of equilibrium and symmetry and the Hooke's law we obtain

$$\sigma_{iyy}^T (x, t) = \frac{E_i}{1-\nu_i} [\epsilon_o(t) - \alpha_i \theta_i (x, t)] , (i = 1, 2) , \tag{5}$$

$$\epsilon_o(t) = \frac{E_1\alpha_1 h_1 (1-\nu_2) \bar{\theta}_1(t) + E_2\alpha_2 h_2 (1-\nu_1) \bar{\theta}_2(t)}{E_1 h_1(1-\nu_2) + E_2 h_2(1-\nu_1)} \tag{6}$$

where ϵ_o is the inplane strain, α_i the coefficient of thermal expansion, E_i , ν_i , the elastic constants and $\bar{\theta}_i$ the thickness average of the temperature in the respective layer.

In the axisymmetric radial deformation of a cylindrical shell by observing that $\epsilon_\theta = \Delta R/R$, $N_\theta = Eh\epsilon_\theta$ and considering the radial equilibrium of an element $\Delta s = R\Delta\theta$, it can be shown that the force per unit area resisting the transverse displacement ΔR due to the shell curvature may be expressed as

$$F_R = N_{\theta\theta}\Delta\theta/\Delta s = Eh\Delta R/R^2. \tag{7}$$

Thus, locally the slice of the shell may be considered as a plate on an elastic foundation having the stiffness $\chi = F_R/\Delta R = Eh/R^2$. For the composite shell it may then easily be shown that

$$\chi = (E_1 h_1 + E_2 h_2)/R_n^2 , \quad R_n = R_1 + \frac{h_1}{2} + \frac{E_2 h_2(h_1 + h_2)}{2 (E_1 h_1 + E_2 h_2)} \tag{8}$$

where R_1 is the inner radius of the composite cylinder.

4 THE CRACK PROBLEM

The general crack problem under plane strain conditions is described in Fig. 1. In addition to the boundary conditions at x=0, the continuity conditions at $x=h_1$ and the symmetry conditions at y=0, the two dimensional elasticity problem must be subject to

$$\sigma_{2xy}(h_1+h_2, y) = 0, \quad \sigma_{2xx}(h_1+h_2, y) = -\chi\, u_2(h_1+h_2, y), \quad 0<y<\infty; \tag{9}$$

$$\sigma_{1yy}(x, 0) = -\sigma_{1yy}^T(x, t), \; a_1<x<b_1; \quad v_1(x, 0) = 0, \; 0<x<a_1, \; b_1<x<h_1; \tag{10}$$

$$\sigma_{2yy}(x, 0) = -\sigma_{2yy}^T(x, t), \; a_2<x<b_2; \quad v_2(x, 0) = 0, \; h_1<x<a_2, \; b_2<x<h_1+h_2, \tag{11}$$

where u_i and v_i are the components of the displacement vector. The mixed boundary value problem under consideration is rather complicated but may be solved in a straightforward manner by using the Fourier transforms. Thus, by defining the unknown functions

$$f_i(x) = \frac{\partial}{\partial x} v_i(x, +0), \quad (i = 1, 2) \tag{12}$$

the problem may be reduced to a system of singular integral equations of the form

$$\sum_1^2 \left(\int_{a_j}^{b_j} \left[\frac{\delta_{ij}}{r-x} + k_{ij}^f(x, r) + k_{ij}^s(x, r)\right] f_j(r)\, dr = \frac{\pi(1+\kappa_i)}{4\mu_i} \sigma_{iyy}^T(x, t), \; a_i<x<b_i, \; (i=1,2) \tag{13}$$

where σ_{iyy}^T is given by (5), κ_i and μ_i are the elastic constants, and the kernels k_{ij}^f are bounded functions. For disconnected cracks embedded in homogeneous materials shown in Fig. 1, the kernels k_{ij}^f are also bounded. However, for a crack tip crossing the free boundary (that is for the edge crack) or terminating at the interface the functions $k_{ij}^s(x, r)$ become unbounded as x and r approach such limiting values and contribute to the singular behavior of the stresses at the corresponding crack tips (Erdogan and Rizk, 1991). In general the solution of (13) has the form

$$f_j(x) = g_j(x)/[(x-a_j)^{\gamma_j}(b_j-x)^{\beta_j}], \quad a_j<x<b_j, \quad 0<(\gamma_j, \beta_j)<1, \; (j = 1, 2) \tag{14}$$

where $g_j(x)$ is bounded and for embedded crack tips $\gamma_j = 1/2 = \beta_j$. The characteristic equations for γ_j and β_j at the crack tips $a_1 = 0$, $b_2 = h_1+h_2$, $b_1= h_1$, $a_2 = h_1$, and for the crack crossing the interface $b_1 = h_1 = a_2$ are given by Erdogan and Rizk (1991) who also give the expressions determining the stress intensity factors. We note that γ_j and β_j are also the power of stress singularities at the crack tips. For example, at a given crack tip $x = b_1$ the mode I stress intensity factor is defined by

$$k_1(b_1) = \lim_{x\to b_1} \sqrt{2(x-b_1)}\, \sigma_{1yy}(x, 0, t) = -\frac{4\mu_1}{1+\kappa_1} g_1(b_1) / \sqrt{(b_1-a_1)/2}, \text{ for } b_1<h_1, \tag{15}$$

$$k_1(h_1) = \lim_{x\to b_1} \sqrt{2}(x-h_1)^{\beta_1} \sigma_{2yy}(x, 0, t), \text{ for } a_1<h_1, \; b_1 = h_1. \tag{16}$$

Similarly, for a crack crossing the interface, i.e., for $b_1 = h_1 = a_2$ we have

$$\sigma_{ij}(r,\theta,t) \simeq \frac{k_o}{r^\beta} F_{ij}(\theta), \; 0<\theta<\pi, \; k_{xx} = k_o F_{xx}(\pi/2), \; k_{xy} = k_o F_{xy}(\pi/2), \; (i, j = x, y) \tag{17}$$

where $\beta=\gamma_2= \beta_1$ and r, θ are the polar coordinates with r = 0 corresponding to $x = h_1$, y = 0.

5 RESULTS AND DISCUSSION

First we examine the validity of the assumption of representing a relatively thin walled cylinder by a plate on an elastic foundation in solving axisymmetric crack problems. To do this we compare the stress intensity factors obtained from the continuum elasticity solution of a hollow cylinder (given by Nied and Erdogan, 1984) and the plane strain solution of a plate on an elastic foundation both under transient thermal stresses or static tension (Table 1). The thermal stress results are given for various values of the dimensionless time τ. The plate results shown under Pl., $\chi \neq 0$ and $\chi = 0$ are obtained by using the formulation given in this paper. The unconstrained plate results corresponding to $\chi = 0$ are also given in the table for comparison. Note that the differences between the cylinder and plate results are small enough to justify the approximation which simplify the problem quite considerably.

Some sample results for the thermal shock problem in a composite cylinder approximated by a plate on elastic foundation are given in Figures 2-7. Thermoelastic properties of the materials used in the examples are given in Table 2. Material pair A corresponds to a stainless steel layer welded on a ferritic base simulating cladded pressure vessels. In pair B material 1 is a ceramic and 2 is steel. The support stiffness χ is calculated by assuming that $R_1/L = 9$, R_1 and L being the inner radius and the thickness ($h_1 + h_2$) of the cylinder. The nominal stress used in normalizing the stress intensity factors is $\sigma_0^T = - E_1 \alpha_1 \theta_0/(1-\nu_1)$. The dimensionless time which is a measure of the surface cooling rate is given by $\tau_0 = t_0 D_1/h_1^2$. In interpreting the results given in the figures it should be noted that in each case the normalizing stress intensity factor is an increasing function of the crack length ℓ, that is $k_0 = \sigma_0^T \sqrt{\ell}$.

For $h_2/h_1 = 9$ and $\tau_0 = 10$ the variation of the stress intensity factor with crack length and time in material pair A is shown in Fig. 2. The effect of τ_0 may be seen in Fig. 3. The results for an underclad crack and for a crack crossing the interface are shown in Figs. 4 and 5. Figures 6 and 7 show some results for the elastically dissimilar material pair B. Referring to (16) in Fig. 6 the power of singularity is $\beta_1 = 0.552538$, whereas in Fig. 7 we have $\gamma_2 = 0.451242$ at $x = a_2 = h_1$ and $\beta_2 = 0.5$ at $x = b_2$.

REFERENCES

Erdogan, F. and Rizk, A.A. (1991). Fracture of Coated Plates and Shells under Thermal Shock, Int. J. of Fracture (in press).

Nied, H.F. (1984). Thermal Shock in a Circumferentially Cracked Hollow Cylinder with Cladding, J. Engrg. Fracture Mechanics, Vol. 20, pp. 113-137.

Nied, H.F. and Erdogan, F. (1984). Transient Thermal Stress Problem for a Circumferentially Cracked Hollow Cylinder, Journal of Thermal Stresses, Vol. 6, pp. 1-14.

Rizk, A.A. and Erdogan, F. (1989). Cracking of Coated Materials under Transient Thermal Stresses, Journal of Thermal Stresses, Vol. 12, pp. 125-168.

Table 1. Normalized stress intensity factors $k_1/\sigma_0^T\sqrt{b}$ due to transient thermal stresses and $k_1/\sigma_0\sqrt{b}$ due to uniform tension σ_0 in a cylinder with an internal circumferential crack and in a plate on elastic foundation containing a plane strain surface crack of depth b; $\tau = tD/h^2$, $\sigma_0^T = -E\alpha\theta_0/(1-\nu)$, $R_i/h = 9$, $\chi h/E = 0.01108$

	$\tau = 0.01$		$\tau = 0.05$		$\tau = 0.1$		$\tau = 0.5$		Tension σ_0		
b/h	Cyl.	Pl.	Cyl.	Pl.	Cyl.	Pl.	Cyl.	Pl.	Cyl.	$\chi\neq0$	$\chi=0$
0.01	0.962	0.957	0.833	0.821	0.724	0.709	0.277	0.261	1.122	1.122	1.122
0.1	0.657	0.653	0.701	0.691	0.633	0.621	0.247	0.233	1.158	1.157	1.189
0.2	0.426	0.432	0.589	0.592	0.560	0.559	0.224	0.218	1.253	1.273	1.367
0.3	0.300	0.307	0.501	0.509	0.502	0.507	0.206	0.200	1.392	1.427	1.660
0.4	0.238	0.240	0.432	0.435	0.453	0.437	0.190	0.183	1.568	1.598	2.111
0.5	0.205	0.207	0.378	0.379	0.408	0.408	0.174	0.168	1.779	1.814	2.825
0.6	0.185	0.187	0.334	0.337	0.367	0.367	0.158	0.153	2.025	2.077	4.033

Table 2. Thermoelastic properties of the material pairs used in the examples

Material Pair	k_2^l/k_1^l	D_2/D_1	α_2/α_1	E_2/E_1	ν_2/ν_1	$\chi L/E_2$
A	3	3	0.75	1	1	0.01108
B	3.385	4.070	2.294	0.611	1	0.01185

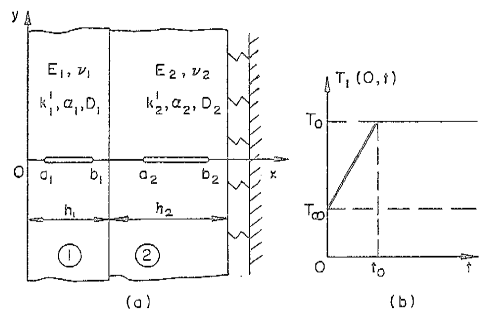

Fig. 1 Crack geometry and temperature boundary condition

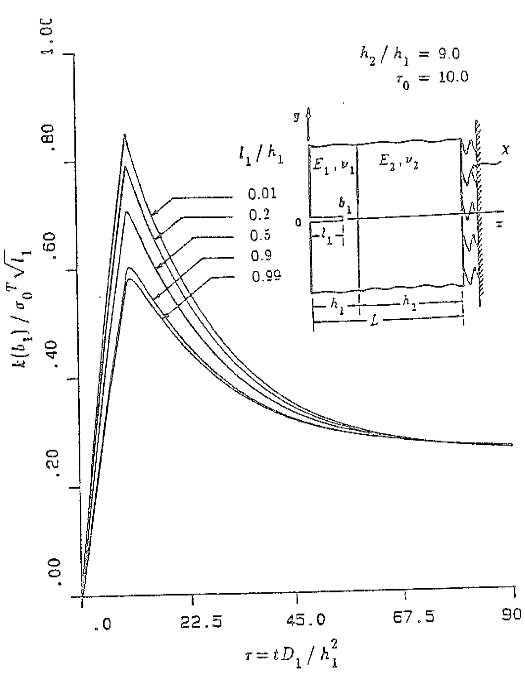

Fig. 2 Stress intensity factor vs. time for a surface crack, mat. pair A.

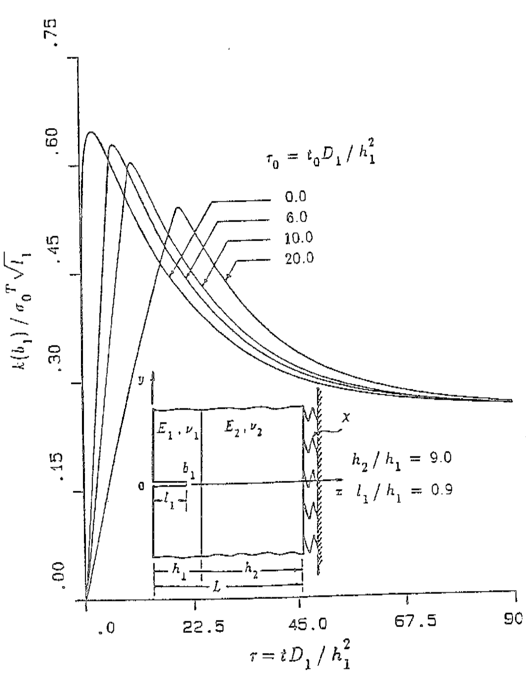

Fig. 3 Stress intensity factor vs. time for a surface crack, mat. pair A.

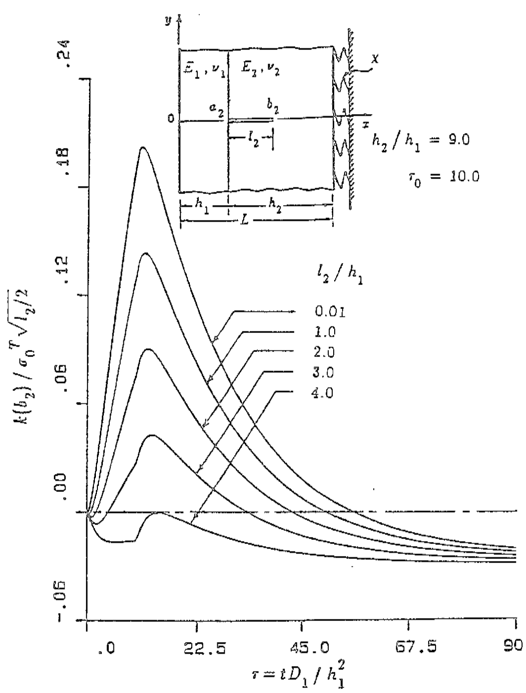

Fig. 4 Stress intensity factor vs. time for an underclad crack, mat. pair A.

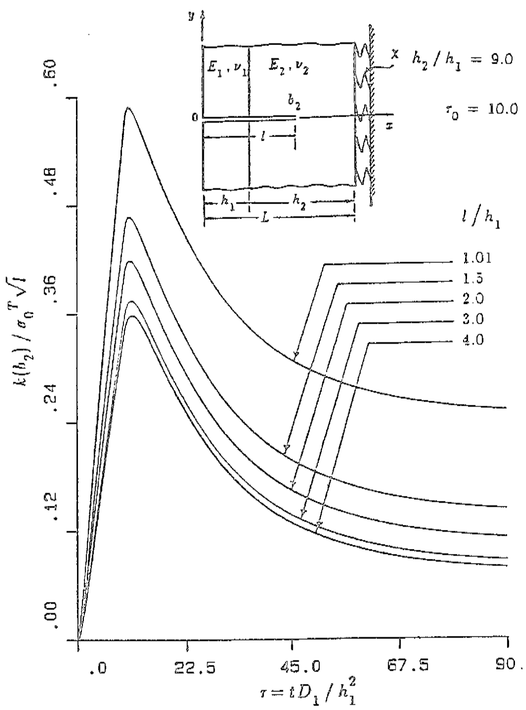

Fig. 5 Stress intensity factor vs. time for a surface crack, mat. pair A.

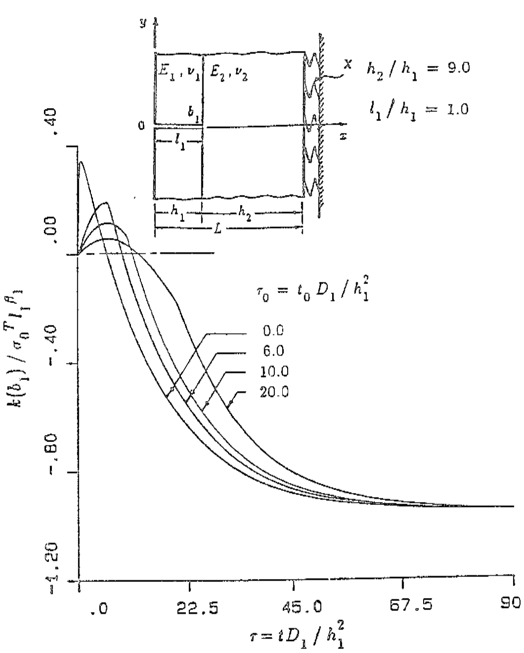

Fig. 6 Stress intensity factor vs. time for a broken clad, mat. pair B.

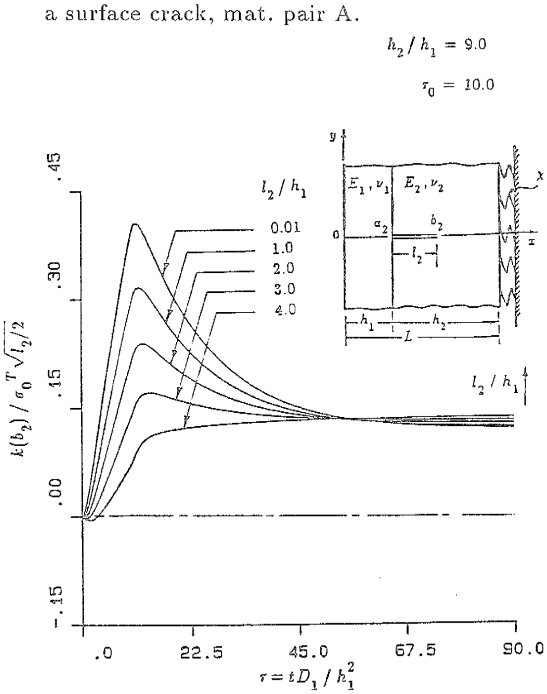

Fig. 7 Stress intensity factor vs. time for an underclad crack, mat. pair B.