Stress Intensity Factors for Cracks in Arbitrary Stress Fields: The Case of Residual Stresses in Welded Pipes

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ABSTRACT

An analytical method is presented for determination of the stress intensity factor (SIF) for cracks in arbitrary stress fields, assuming that no relevant effects (bounding error) occur when it is assumed that a load distribution may be broken down into load steps acting on a part of the crack and that the contribution made by each such step to the overall SIF may be calculated by eliminating the material contained in the non-loaded part of the crack.

1. INTRODUCTION

For more or less complicated geometries and for simple stress fields, such as constant load distributions or linear load distributions, formulae exist which allow us to estimate the stress intensity factor (SIF) for a given crack in one of these geometries.

In many problem cases, it may be conservatively assumed that the load present can be estimated on the basis of a constant load distribution plus a linear distribution, with subsequent application of the solutions reported in SIF literature. In other cases, this assumption is excessively conservative, particularly when the actual load gradients are very pronounced, e.g.:

residual stresses in welded pipes. If, as in this case, these types of stresses are also initially responsible for crack propagation, formulae must be available for evaluation of the SIFs in complex load fields, without constraint.

The objective of this paper is to present a method for estimation of the SIF for any type of load, with variation restricted to crack depth.

Although this methodology might be generally applied, it has been developed particularly for stress intensity factor calculation in the case of three-dimensional surface cracks on the inner walls of pipes in the presence of residual stress fields.

Through the application of Linear Elastic Fracture Mechanics, an attempt is made to calculate the SIF for arbitrary load distributions by adding the SIFs produced by a series of elementary load distributions.

The problem of calculating the SIF is thus reduced to estimating this factor for a step load acting on a part of the crack.

Assuming that the SIF for a step load acting along the entire crack (membrane stress) is known, the aim is to estimate this same parameter assuming that the material found in the area where the step is not applied can be eliminated parallel to the surrounding area, thus generating an equivalent geometry allowing the step load to be considered as a stress membrane. (Figure 1).

By adding (overlapping) all the SIFs for each of the specific problems, we obtain an estimate of the SIF for the overall complex field of stresses, with a generally acceptable margin of error.


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2. EDGE CRACKS IN A SEMI-INFINITE PLATE

The case of the continuous crack in a semi-infinite solid might initially serve to gain clearer insight than has been accomplished in the introduction into the methodology proposed, as well as to determine the suitability of the method for cases in which the "exact" solution is known.

Let us consider the parabolic load distribution:

\[ \sigma(x) = \frac{\sigma_T}{a^2} \cdot x^2 \]

Breaking down the load distribution into \( n \) steps, in accordance with what is specified in Figure 1, implies calculating the stress intensity factor as the sum of \( n \) stress intensity factors, each of them for the case of a constant value load, for step \( i \):

\[ \sigma_i = \frac{\sigma_T}{a^2} \cdot (2i-1) \]

and a crack depth:

\[ a_i = a \cdot \left(1 - \frac{i-1}{n}\right) \]

The error involved may be estimated in accordance with the following expression, where calculation of the SIF by the method proposed is shown in the numerator and calculation of the SIF for the parabolic load distribution using a conventional method is shown in the denominator.

\[
\xi = 1 - \frac{\sum_{i=1}^{n} \int_{a_i}^{a} \frac{1.12}{(\pi a_i)^{1/2}} \sigma_i \left( \frac{a_i + x}{a_i - x} \right)^{1/2} + \left( \frac{a_i - x}{a_i + x} \right)^{1/2} \, dx}{\int_{a}^{a} \frac{1.12}{(\pi a)^{1/2}} \frac{\sigma_T}{a^2} x^2 \left( \frac{a + x}{a - x} \right)^{1/2} + \left( \frac{a - x}{a + x} \right)^{1/2} \, dx}
\]
By resolving the integrals we obtain the following:

$$
    \xi = 1 - \frac{2}{n^2} \sum_{i=1}^{n} \left( 2i - 1 \right) \left( 1 - \frac{i-1}{n} \right)^{1/2}
$$

Table 1 shows the error involved in using the proposed method for different values of \( n \) and for different load cases. It may be appreciated that in no case does the error exceed 10\%, for a number of steps of more than 100, and in any case is always conservative.

<table>
<thead>
<tr>
<th>Load distribution</th>
<th>Number of steps considered</th>
<th>( n=10 )</th>
<th>( n=100 )</th>
<th>( n=1000 )</th>
<th>( n&gt;&gt;&gt; )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma (x) = \frac{\sigma_T}{a} \cdot x )</td>
<td>-11%</td>
<td>-5.5%</td>
<td>-4.8%</td>
<td>-4.8%</td>
<td></td>
</tr>
<tr>
<td>( \sigma (x) = \frac{\sigma_T}{a^2} \cdot x^2 )</td>
<td>-18%</td>
<td>-8.0%</td>
<td>-6.8%</td>
<td>-6.7%</td>
<td></td>
</tr>
<tr>
<td>( \sigma (x) = \frac{\sigma_T}{a^3} \cdot x^3 )</td>
<td>-23%</td>
<td>-9.0%</td>
<td>-7.9%</td>
<td>-7.7%</td>
<td></td>
</tr>
<tr>
<td>( \sigma (x) = \frac{\sigma_T}{a^4} \cdot x^4 )</td>
<td>-11%</td>
<td>-8.5%</td>
<td>-8.6%</td>
<td>-8.4%</td>
<td></td>
</tr>
</tbody>
</table>

3. TWO-DIMENSIONAL SURFACE CRACKS IN A SEMI-INFINITE PLATE

The continuous crack having been analyzed, it is now necessary to analyze the case of a semi-elliptic surface crack.

On breaking down the load into steps and attempting to calculate the effect of each step, it is observed that if the material preceding the step is eliminated, the geometry of the new crack does not correspond to a semi-elliptic surface. The geometry may be readjusted to give a new ellipse, but in this case the eccentricity will be different from that of the initial ellipse. (Figure 2)

The impact of this consideration may be established on the basis of the work carried out by Core and Burns, which analyzes irregular cracks subjected to normal stress fields.

![Figure 2. The effect of cutting an elliptic crack](image)
In the case of a semi-circular crack loaded on two symmetrical steps, the stress intensity factor is given by the following expression:

\[ K_I = 1.12 \int \int_A \sigma(x,y) K_{I'd} \, dx \, dy \]

where \( \sigma(x,y) \) represents the unit amplitude distribution of the symmetric steps and

\[ K_{I'd} = \frac{1}{\pi a^{1/2}} \frac{(a^2 - r^2)^{1/4}}{a^2 + r^2 - 2ra \cos \theta} \]

The non-loaded part of the crack having been eliminated, the stress intensity factor would be given by the following expression:

\[ K_I = 1.12 \frac{\pi a^{1/4}}{(3\pi/8) + (\pi/8) \cdot (a/c)^{1/4}} (a/c)^{1/4} \]

which, when compared to the expression obtained above, makes it possible to estimate errors of less than 9% for values of shift of between 0 and 75% of crack depth.

In the case of originally semi-elliptic cracks, the errors involved are of the order of those indicated for semi-circular cracks.

4. SURFACE CRACKS IN BODIES OF FINITE WIDTH

Up to now, the cracks analyzed have been in semi-infinite bodies. The redistribution of stresses occurring as a result of the front-end cut of the material, made in order to eliminate the non-loaded part of the crack, is altered by the dorsal surface effect.

In order to quantify the effect of these dorsal surfaces, the finite elements method was used to study the redistributions of stresses and associated stress intensity factors for the geometries shown in Figure 3.
Results of $K_t$ obtained for the geometries shown in Figure 3 can be seen in Table 2.

Table 2. Differences for $K_t/\sigma$ values before and after cutting

<table>
<thead>
<tr>
<th>a/t</th>
<th>$K_t/\sigma$ Before cutting</th>
<th>After cutting</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>4.69</td>
<td>4.97</td>
</tr>
<tr>
<td>0.5</td>
<td>9.92</td>
<td>10.03</td>
</tr>
<tr>
<td>0.875</td>
<td>75.2</td>
<td>75.3</td>
</tr>
</tbody>
</table>

5. CRACKS IN RESIDUAL STRESS FIELDS IN WELDED PIPES

As has been pointed out in the introduction, development of the proposed method originated from the need for a simple method allowing the stress intensity factor to be obtained for stress-corrosion cracks in piping with stress fields resulting from welding processes. (Figure 4)

![Graph showing residual stress distribution in a welded pipe]

Generally speaking:

$$\frac{K}{\sigma_0(\pi a)^{1/2}} = f \left( \frac{C}{a}, \frac{\sigma}{\tau}, \frac{R}{t}, \frac{\sigma(x/t)}{\sigma_0} \right)$$

where:
- $a$, is the depth of the crack
- $c$, is the semi-length of the crack
- $t$, is the thickness of the pipe
- $R$, is the characteristic radius of the pipe
- $\sigma(x)$, is the distribution of stresses, and
- $\sigma_0$, is the stress value characteristic of the distribution.

Given the great variety of cases which may be addressed (in fact too numerous to address in reality), all simplifications possible must be accomplished in order to provide a truly feasible methodology for calculation of the stress intensity factor.

In the case in hand, the Newman-Raju methodology was selected for $t/R = 0$ (flat wall) and for membrane and bending stress.

K. Kashima et al. have demonstrated that use of the Newman-Raju methodology for the case of a crack in a pipe ($t/R > 0$) is correct.

It remains, therefore, only to bring the applicability of the method for the case of far profile stress distributions into line with the profiles characteristic of the membrane or bending stresses for which the Newman-Raju formula is correct.
This is accomplished by applying the method proposed in this paper, since it allows the sufficiently contrasted Newman-Raju formula to be used for calculation of the stress intensity factor, the error involved being perfectly acceptable for the ultimate aim pursued.

In other words, and in summing up, what is proposed for stress-corrosion cracking in the welded parts of piping is application of the Newman-Raju formula for membrane and bending stresses, and breaking down of the remaining stress distributions into steps acting on the part of the crack under consideration. Following elimination of the unloaded part of each step, the Newman-Raju formula will be used once more for the membrane stresses.

Finally, and by summing all the stress intensity factor values thus determined, the SIF corresponding to the crack in question, which is immersed in a complex stress field, will be obtained.

REFERENCES


ACKNOWLEDGMENTS

The author would like to thank F. Estéban and J.M. García for their decisive contribution to development of this method, which they initiated.