Experimental GH Singularity Field around Growing Crack-Tip

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ABSTRACT

The displacement fields u_x, u_y at growing crack tip of LY12–M specimens with double edge cracks are measured using moire method. The experimental singularity fields are compared with GH theoretical field [12–14]. The size and shape of the experimental GH singularity fields are obtained. The error in both the experimental and theoretical evaluations is controlled with in ± 10%. The experiments show that there is $(ln\frac{A}{r})^{\bar{\alpha}+1}$ singularity dominant around a growing crack tip. The shape of this dominant region ranges from butterfly wing to then oblate and circular. Inside GH–field, there is a 3–D deformed damage zone, where no GH singularity exists.

Key Words: crack tip, singularity field, damage zone, moire method

I .INTRODUCTION

The investigation of the deformation and singularity field at a crack–tip is of central concern in fracture mechanics. For brittle materials, it is generally recognized that the crack tip field is characterized by the stress intensity factor K with a $1/\sqrt{r}$ singularity [1]. In elastic plastic materials under small scale yielding the crack tip may be represented by the J integral[2] and the singularity field is a function of the strain hardening index n, which was first given by Hutchinson[3], Rice and Rosengren[4] and is commonly referred as HRR–field. HRR–field applies to the case of stationary cracks only and is not valid in the region near crack tip where three dimensional deformation dominates as inves-

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tigated experimentally by Chiang, Hareesh and Liu[5–6].In the initial stage of crack–growth, the singularity field at the crack–tip was studied by Rice[7], Drugan et.al.[8], Hermann & Rice[9], and Sorensen[10] for elastic perfectly plastic materials and by Amazigo & Hutchinson [11], Gao & Hwang [12–14] for power hardening materials.

In this paper, we investigated the displacement and strain fields of growing crack for strain hardening material using moire and laser projected grating method. The experimental singularity fields are compared with those predicated by GH–fields and mapped out the zones of agreement at different stages of crack growth. The error between them is controlled wtih in ± 10%. In addition, 3–D deformed damage zone is found inside the GH–field at the very near of a crack–tip. In this zone GH singularity does not exist.

II .EXPERIMENTAL PROCEDURES

The test matrial is an aluminum alloy LY12–M, with strain hardening index n = 6.5. The properties of the material is determined by tensile tests. A typical stress–strain curve is shown in Fig.1. It is fitted with the following stress–strain relation:

$$\frac{\varepsilon}{\varepsilon_o} = \alpha\left(\frac{\sigma}{\sigma_o}\right)^n \tag{1}$$

where σ_o is the yield stress; ε_o the yield strain; α material constant and n strain hardening index. Both n and α are directly obtained from the experimental curve. For LY12–M material, $\sigma_o(\sigma_{0.2}) = 81.99MPa$, $\varepsilon_o(\varepsilon_{0.2}) = 3162.5\mu\varepsilon$, $n = 6.5, \alpha = 1.132, E = 0.706 \times 10^4 GPa$ and $v = 0.348$.

The specimens were thin plates with double edge cracks. The geometry of the specimen is shown in Fig.2. The cracks were machined through EDM (electro–discharge machining) process with a typical width of 0.1mm.

In the surrounding area of crack–tip, the crossed grating of 20 or 40 lines per mm was printed. The specimen was placed in a special machine, subjected to displacement loading, and illuminated by a optical system[5] as shown in Fig.3. When the specimen is deformed the printed crossed grating distorts in response to the in–plane deformation. The deformed gratings are photographed and superimposed with an initial reference grating,. Moire fringes representing contours of displacement components u_x, u_y can be generated respectively through optical spatial filtering technique [16–17], and the equations governing these fringes are simply:

$$u_x = NP$$

$$u_y = N'P \tag{2}$$

where P is the grating pitch. Usually the printed crossed gratings have constant grating pitch along x_1 and x_2 directions. N, N′ are fringe orders. Typical set of fringe pattern are shown obtained in Fig.4 which indieates the displacement fields u_x and u_y in x_1 and x_2 directions under different loadings ($\frac{\sigma}{\sigma_o}$ $= 0.941, 0.990, 1.041$)

Ⅲ.EXPRIMENTAL AND THEORETICAL ANALYSIS AT THE TIP OF A GROWING CRACK.

In the experiments, displacement fields u_x *and* u_y in x_1 *and* x_2 directions are directly obtained. From those displacement fields we can obtained the displacement distributions under different angles ($\theta_i = 0°, 15°, 30°, 45°, 60°, 75°, 90°$) and $(\frac{u_y}{r})^{\frac{n-1}{n}}|_{\theta_i}$ vs log r curves.

At the tip of a growing crack under mode I, according to Gao & Hwang [8–9], the dominant singularity field for displacements, strains and stresses are governed by the following equations, respectively:

$$u_i = r(ln\frac{A}{r})^{\tilde{\alpha}+1}\tilde{u}_i(\theta,E,v,n,\alpha,F)$$

$$\varepsilon_{ij} = (ln\frac{A}{r})^{\tilde{\alpha}+1}\tilde{\varepsilon}_{ij}(\theta,E,v,n,\alpha,F) \tag{3}$$

$$\sigma_{ij} = (ln\frac{A}{r})^{\tilde{\alpha}}\ \tilde{\sigma}_{ij}(\theta,E,v,n,\alpha,F)$$

where $\tilde{\alpha} = \frac{1}{n-1}$, n and α material constants, E Young's modulus, v Poisson's ratio and F a constant. A is an integral constant depending on the loading and r, θ are polar coordinates at the growing crack tip, $\tilde{u}_i, \tilde{\varepsilon}_{ij}$ and $\tilde{\sigma}_{ij}$ are angle distribution functions having different expressions in various regions as reported in Refs. [10][13]. The singularity field at the growing crack tip is divided into four regions as shown in Fig.5, In region I

$$u_x = -\varepsilon_{y_0} r(ln\frac{A}{r})^{\tilde{\alpha}+1}cos\theta, \quad u_y = \varepsilon_{y_0} \quad r(ln\frac{A}{R})^{\tilde{\alpha}+1}sin\theta,(0 \leqslant \theta \leqslant \frac{\pi}{4}) \qquad (4)$$

In region II

$$u_x = -r(ln\frac{A}{r})^{\tilde{\alpha}+1}[\varepsilon_{y_0} cos\theta + \frac{2(1-v^2)F}{(\tilde{\alpha}+1)E}(sin\theta + cos\theta)]$$

$$u_y = r(ln\frac{A}{r})^{\tilde{\alpha}+1}[\varepsilon_{y_0} sin\theta + \frac{2(1-v^2)F}{(\tilde{\alpha}+1)E}(sin\theta - cos\theta)],(\frac{\pi}{4} \leqslant \theta \leqslant \pi - \tilde{\beta})$$

In the experiments, for elastic plastic material of mode I growing crack under plane stress, , the displacement singularity in regions I and II is shown the following form:

$$u_x = -\varepsilon_{y_o} r(ln\frac{A}{r})^{\tilde{\alpha}+1}cos\theta$$

$$u_y = \quad \varepsilon_{y_o} r(ln\frac{A}{r})^{\tilde{\alpha}+1}sin\theta \qquad (5)$$

where A is a undetermined constant depending on the loading. These equations can be converted into the following form:

$$\left(\frac{u_x}{r}\right)^{\frac{1}{\tilde{\alpha}+1}} = (lnA - lnr)(-\varepsilon_{y_o} cos\theta)^{\frac{1}{\tilde{\alpha}+1}}$$

$$\left(\frac{u_y}{r}\right)^{\frac{1}{\tilde{\alpha}+1}} = (lnA - lnr)(\varepsilon_{y_o} sin\theta)^{\frac{1}{\tilde{\alpha}+1}} \qquad (6)$$

These equations are called GH theoretial solution and are compared with the experimental results.

IV.EXPERIMENT DATA ANALYSIS AND RESULTS

Based on experimental fringe patterns representing contours of u_x, u_y *in* x_1, and x_2 directions, the displacement values were plotted against lnr along various directions except $\theta = 0°$, and strains (obtained by differentiating the experimental displacement data) were plotted as $\left(\frac{u_y}{r}\right)^{\frac{1}{\tilde{\alpha}+1}}$ vs lnr curves along various directions ($\theta = 0°, 15°, 30°, 45°, 60°, 75°, 90°$), too.

According to the experimental data, constants A and ε_{y_o} were determined in most loading conditions (for example, for specimen No.1, $\frac{\sigma}{\sigma_0} = 0.941$, 0.986,0.990, 1.036,1.041,1.061,1.067,....). The experiment shows that constant A increases with r and θ. We determine constant A and control the error within about 5% in the whole displacement field in each loading case. Then ε_{y_0} is determined. So GH solution along different angles can be calculated. The experimental values $(\frac{u_y}{r})^{\frac{n-1}{n}}$ in the whole field are compared with GH theoretical values $(\frac{u_y}{r})^{\frac{n-1}{n}}$, and typical $\frac{experimental}{theoretical}$ $(\frac{u_y}{r})^{\frac{n-1}{n}}$ values vs r curves along different angles are shown in Fig.6. The experimental GH regions with error about ± 10% of the theoretical solution are obtained. The experiments show that there is a GH dominant region around growing crack–tip (the form of singularity is $(ln\frac{A}{r})^{\bar{n}+1}$) for strain hardening material, called GH–field. Fig 7(1–10) exhibits that very near to the crack–tip there is a three–dimensional fracture process zone the size of which increases with the increasing of the load. Beyond that, there exists a zone dominated by the GH singularity field. The shape of GH–field is changed from butterfly wing to oblate and circular with the increasing of the load. If the load is lower, there is also a elastic zone outside GH–field. In the experiments, P vs $\triangle$ curve and COA (or CTOA) vs $\triangle a$ curve are obtained as shown in Fig.8,9.

V. CONCLUSION AND DISCUSSION

Using moire grating technique, the deformation fields around a growing crack tip for strain hardening material are obtained successfully. From experiments we show that very near to the crack tip there is a three–dimensional fracture process zone the size of which increases with the load.

Beyond that there exists a zone dominated by the so–called GH singularity field. It's shape changes from butterfly wing to oblate and circular with the increasing of load. If the loading is low, there is an elastic zone outside GH–field as well.

The constants A and ε_{y_0} are determined by experiments. For each loading, A is nearly constant, and ε_{y_0} increases with angle (θ) in the whole field. We can assume that this ε_{y_0} is the corrective function of the angle distribution $\sin\theta$ for plane stress.

From the experiments, the resostance curve, crack opening angle COA (or crack tip opening angle CTOA) vs the length of crack growth $\triangle a$ is obained.

From the curve in Fig.9, we can see that the crack opening angle COA has a constant value at initiation growth. COA decreases with the crack propagation, then approches to a horizontal line; but the crack tip opening angle CTOA is random.

In P vs $\triangle$ curve, after crack initiation there is a stable crack growth and then follows the unstable growth.

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