Three-Dimensional BEM Analysis for Mixed Mode Interface Cracks Considering Crack Closure

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ABSTRACT

This paper presents an influence function method based upon a boundary element method by which the problem of mixed mode cracks can be analyzed effectively considering the crack closure . The method was applied to the problem of mixed mode interface cracks existing in the dissimilar materials.

1 INTRODUCTION

There are many studies about the problems of interface cracks between dissimilar elastic materials such as electric devices and metal-ceramics joints. The most simple problems of two-dimensional interface cracks have been analyzed from the viewpoint of the elastic fracture mechanics [1-7]. It is necessary to evaluate the complex stress intensity factor, K, of the mixed mode crack since a particular singularity exists in the interface crack tip. And two dimensional numerical analyses have been reported in which the complex stress intensity factors are calculated for elastic interface cracks [8, 9]. Generally, in the problem of mixed mode cracks, one has to consider the effect of the crack closure which has been neglected in the published papers. The authors have proposed an influence function method which is based upon the boundary element analysis [10-11]. In this paper the authors will present the basic idea for analyzing the contact problem with proposed influence function method. The method has been applied to the analyses of three dimensional mixed mode stress intensity factor K, and the effect of the friction of the contact crack on K has been simulated.

2 METHOD OF ANALYZING CONTACT PROBLEM

Let's consider three-dimensional crack problem. When the cracked surface is composed of n nodal points, the displacement of the cracked surface is expressed as follows by the influence function method.

$$\{U_i\} = \sum_{j=1}^{n} [A_{ij}]\{P_j\} \tag{1}$$

where Ui is the displacement at the nodal point i and Pj is the surface stress at the nodal point j . Aij is the influence coefficient which means the displacement of the nodal point i due to unit distributed loading at the

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nodal point j. The matrix [Aij] is obtained by the boundary element analysis. In Fig. 1 {P}, {P'}, {P"} show distributed stresses on the cracked surface or virtual cracked surface. They have the following relation [10]

$$\{P'\} = \{P\} + \{P''\} \tag{2}$$

The relation between displacement u" and stress p" on the cracked surface can be expressed by Eq. (3).

$$\begin{Bmatrix} u^{x''} \\ u^{y''} \\ u^{z''} \end{Bmatrix} = \begin{bmatrix} A^{xx} & A^{xy} & A^{xz} \\ A^{yx} & A^{yy} & A^{yz} \\ A^{zx} & A^{zy} & A^{zz} \end{bmatrix} \begin{Bmatrix} p^{x''} \\ p^{y''} \\ p^{z''} \end{Bmatrix} \tag{3}$$

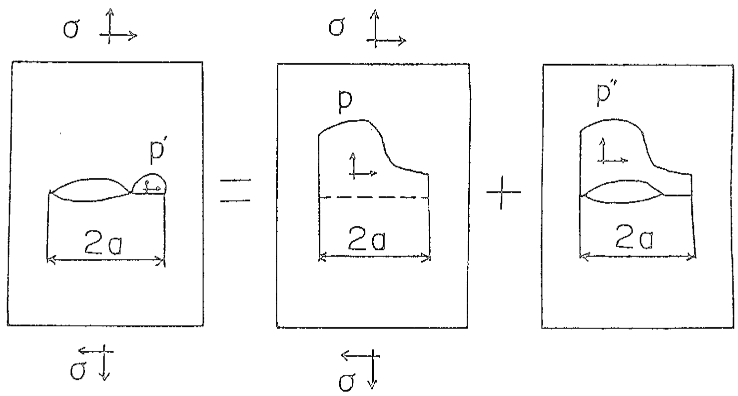

Fig. 1 Principle of superposition

When {P} is obtained through the analysis for the uncracked body , it is substituted into Eq. (2) which is transformed into Eq.(4) by taking account of the friction coefficient , μ .

$$\begin{aligned} \{p^{X''}\} &= -(\ \{p^{X}\} - \mu\ \{p^{Z}\}\ ) + \mu\ \{p^{Z''}\} \\ \{p^{Y''}\} &= -(\ \{p^{Y}\} - \mu\ \{p^{Z}\}\ ) + \mu\ \{p^{Z''}\} \end{aligned} \tag{4}$$

By substituting Eq. (4) into Eq. (3), the following equation is obtained.

$$\begin{Bmatrix} u^{x''} \\ u^{y''} \\ u^{z''} \end{Bmatrix} = \begin{bmatrix} A^{xx} & A^{xy} & A^{xz} + \mu\ (A^{XX} + A^{XY}) \\ A^{yx} & A^{yy} & A^{yz} + \mu\ (A^{YX} + A^{YY}) \\ A^{zx} & A^{zy} & A^{zz} + \mu\ (A^{ZX} + A^{ZY}) \end{bmatrix} \begin{Bmatrix} -(p^{X} - \mu\ p^{Z}) \\ -(p^{Y} - \mu\ p^{Z}) \\ p^{Z''} \end{Bmatrix} \tag{5}$$

When we define the region A and B as those including the upper and lower cracked surfaces , respectively , the following conditions must be satisfied in each region.

$$\begin{aligned} & n_{\nu i}{}^{(A)} p_i{}^{(A)} + n_{\nu i} p_i{}^{(B)} = 0 \qquad (\nu = 1, 2, 3) \\ & -\mu\ n_{1i}{}^{(A)} p_i{}^{(A)} + n_{2i}{}^{(A)} p_i{}^{(A)} = 0 \\ & n_{\nu i}{}^{(A)} u_i{}^{(A)} = n_{\nu i}{}^{(B)} u_i{}^{(B)} + A_\nu \qquad (\nu = 1, 3) \end{aligned} \tag{6}$$

where n_i is direction cosine, A is initial value of the distance between corresponding points on the contact surface, $\nu = 1$ normal direction of cracked surface, $\nu = 2$ slipping direction, $\nu = 3$ vertical direction for slipping direction. In Eq. (5) and Eq. (6) ,the displacement of the z-direction has the following relation.

$$\{u^{z(A)}\} - \{u^{z(B)}\} = \{0\} \tag{7}$$

After $\{P_z\}$ is eliminated from Eq. (5), the influence coefficients , in which the contact of cracked surface is considered, are obtained,

$$\begin{Bmatrix} u^{x''} \\ u^{y''} \\ u^{z''} \end{Bmatrix} = \begin{bmatrix} A^{xx''} & A^{xy''} \\ A^{yx''} & A^{yy''} \\ A^{zx''} & A^{zy''} \end{bmatrix} \begin{Bmatrix} -(p^X - \mu\, p^Z) \\ -(p^Y - \mu\, p^Z) \end{Bmatrix} \tag{8}$$

Thus we can get $\{U''\}$ and $\{P''\}$. Since contact zone is unknown in contact problem, displacement $\{U''\}$ and traction $\{P''\}$ are judged from contact condition [13], which is shown as Eqs. (9), (10)

a) judgment from the traction

$$n_{1i}^{(A)} p_i^{(A)} \quad \begin{cases} > 0 & \text{free boundary} \\ \leqq 0 & \text{slipping} \end{cases} \tag{9}$$

b) judgment from the displacement

$$n_{\nu i}^{(A)} u_i^{(A)} - n_{\nu i}^{(B)} u_i^{(B)} - \Delta_\nu \quad \begin{cases} > 0 & \text{free boundary} \\ \leqq 0 & \text{slipping.} \end{cases} \tag{10}$$

Until all nodes satisfy the contact condition, calculation is iterated and stress intensity factor is determined by means of displacement extrapolation method.

3 K value for an interface crack

Fig. 2 shows a crack lying along the interface of dissimilar materials and the definition of Cartesian and polar coordinates. μ and ν are shear modulus and poisson's ratio. In Fig. 2, relative displacement between the cracked surface can be expressed as,

$$\delta_y + i\,\delta_x = \frac{K_1 + iK_2}{2(1+2i\varepsilon)\cosh(\varepsilon\pi)} \times \left[\frac{\kappa_1+1}{\mu_1} + \frac{\kappa_2+1}{\mu_2}\right]\left[\frac{r}{2\pi}\right]^{1/2}\left[\frac{r}{l}\right]^{i\varepsilon} \tag{11}$$

$$\kappa_i = \begin{cases} (3-\nu_i)/(1+\nu_i) & \text{plane stress} \\ 3-4\nu_i & \text{plane strain} \end{cases}$$

$$\varepsilon = \frac{1}{2\pi}\ln\left[\frac{\kappa_1/\mu_1 + 1/\mu_2}{\kappa_2/\mu_2 + 1/\mu_1}\right]$$

In the above equation , K_1 and K_2 are stress intensity factors for an interface crack, and these have different definition from those for homogeneous material. This form has oscillation singularity which produce overlapping at the crack tip. But it is known that this overlapping zone is very small compared to crack length, so, this oscillation phenomena can be neglected. Stress intensity factors can be calculated by the following equation through the displacement extrapolation method. In Eq. (12) the value of $\sqrt{K_1^2+K_2^2}$ is defined as Ki,

$$\lim_{r \to 0} \frac{\sqrt{\delta_y^2+\delta_x^2}}{2\pi r} = \frac{\sqrt{K_1^2+K_2^2}}{4\pi\sqrt{(1+4\varepsilon^2)}\cosh(\varepsilon\pi)} \times \left[\frac{\kappa_1+1}{\mu_1} + \frac{\kappa_2+1}{\mu_2} \right] \qquad (12)$$

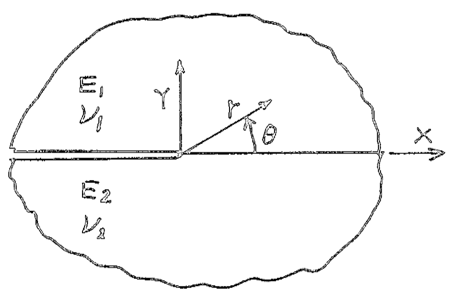

Fig. 2 Interface crack between two bonded dissimilar materials

4 RESULT

The procedure in the previous chapter has been applied to some sample analyses. The parameter F in the vertical axis in Figs. 4, 5, 7 and 8 show the normalized stress intensity factors . Fig. 4 shows the result of rectangular flat plate with a slant through crack , as shown in Fig. 3, subjected to the uniform tension and the uniform compression. In the analysis against the uniform compression the crack closure is taken into account with zero friction coefficient. The results are compared with those of Yuuki(2-dimension) [14], and good coincident is obtained. Fig. 5 is the result of the same problem but subjected to the uniform compression with the coefficient of friction at the contact cracked surface changing from 0.0 to 0.3. Fig. 7 shows the result of the rectangular flat plate with a through interface crack, as shown in Fig. 6, subjected to the uniform tension , and the ratio of Young's modulus between region 1 and region 2 is changed. The result is compared with that of Yuuki(2-dimension)[10]. Fig. 8 shows the result for problem of rectangular flat plate with a slant through interface crack , as shown in Fig. 3, subjected to the uniform compression. The ratio of Young's modulus between region 1 and region 2 is 2.0 and the coefficient of friction is changed from 0.0 to 0.3.

5 CONCLUSIONS

The authors have presented an influence function method by which the stress intensity factors, K, of three dimensional cracks can be calculated effectively by taking account of crack closure. The method has been applied to analyze K for mixed mode cracks, including an interface crack, and the effect of friction of the cracked surface on K has been investigated.

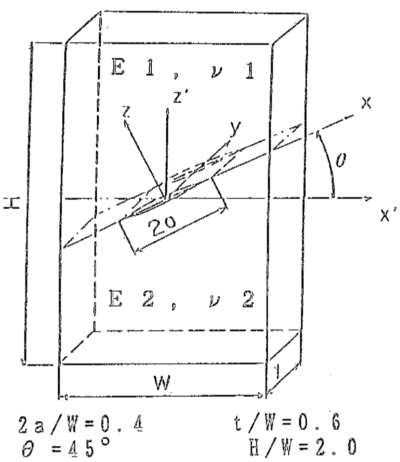

Fig. 3 Rectangular flat plate with a slant through crack

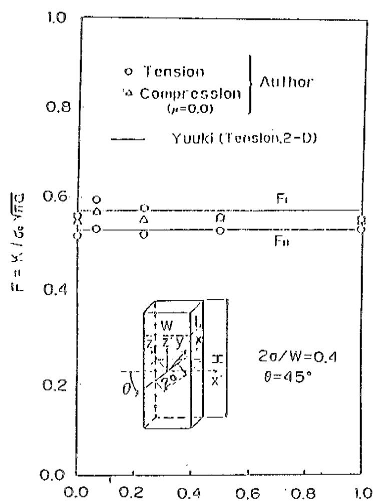

Fig. 4 Distribution of the normalized stress intensity factor along the leading edge of crack

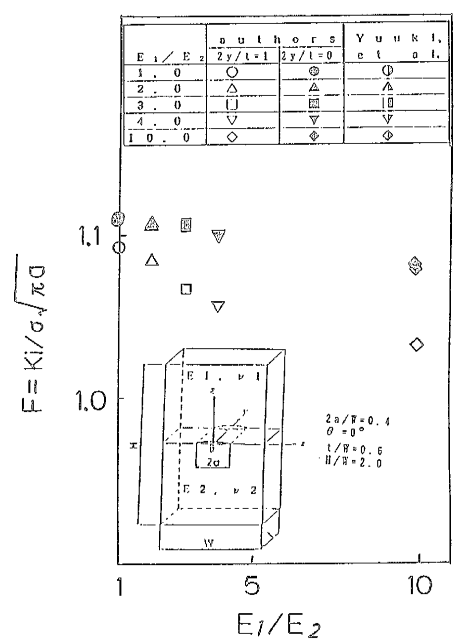

Fig. 7 Comparison of the normalized stress intensity factors under various rate of Young's modules

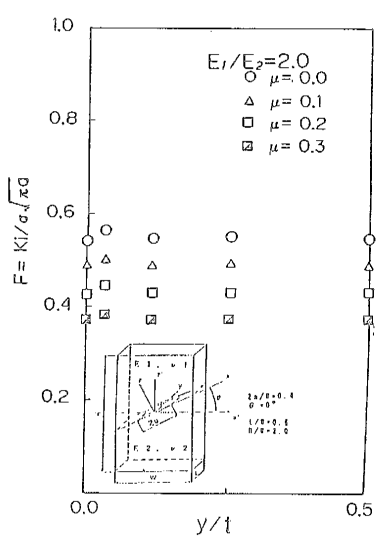

Fig. 8 Effects of the friction coefficient on the k value

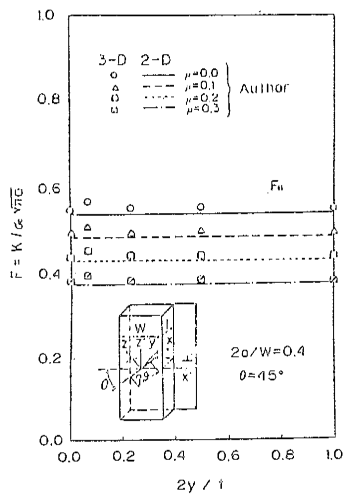

Fig. 5 Effects of the friction coefficient on the k value

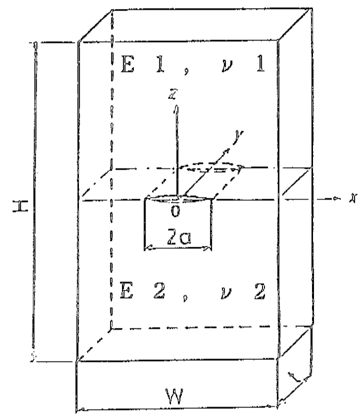

Fig. 6 Rectangular flat plate with a through interface crack

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