Evaluation of Deformation Mode Change along 3-Dimensional Crack Front Line by the CED

Nobuhiko YOSHIIKA, Katsuhiko WATANABE
University of Tokyo, Tokyo, Japan

ABSTRACT

In an idealized 2-dimensional crack, the deformation constraint is kept constant along the crack front line. However, the deformation constraint and fracture mode change generally along the crack front line in a 3-dimensional crack, and this deformation mode change cannot be dealt with as far as conventional parameters such as $K$ or $J$-integral are applied. This paper purports to demonstrate that the deformation mode change above can be evaluated quantitatively when the CED (Crack Energy Density) is applied.

1. INTRODUCTION

The recent development of computational system has made it practical to carry out 3-dimensional crack analyses. However, there are some problems to be solved about crack parameters. That is, most conventional 3-dimensional crack parameters are defined as the extensions of 2-dimensional ones like stress intensity factor $K$ [1] and $J$-integral [2] and the same limitations in their applicabilities as in 2-dimensional problems cannot be helped since their meanings are clear only under special constitutive relations. Moreover, although the deformation constraint is kept constant along crack front line in an idealized 2-dimensional crack (so it is expected that fracture mode is also kept constant along crack front line), the deformation constraint and fracture mode change generally along crack front line in a 3-dimensional crack, and this deformation mode change cannot be dealt with as far as conventional parameters such as $K$ or $J$ are applied.

The CED (Crack Energy Density) [3]-[5] was proposed as a parameter which has no restriction on constitutive equation and is expected applicable to almost all kinds of crack problem. In this paper it is shown that the deformation mode change above can be evaluated quantitatively when the CED is applied. For this purpose at first it is shown that the CED can be divided generally into three components corresponding to three shear deformation types and one corresponding to volume change in an arbitrary orthogonal coordinate system. Then 3-dimensional elastic, elastic-plastic and elastic-creep finite element analyses of a center cracked body are carried out and it is demonstrated that the evaluation of deformation mode change along crack front line becomes possible through the evaluation of each component of CED above.
2. CED AND ITS COMPONENTS CORRESPONDING TO DEFORMATION MODES

The CED is conceptually defined as "the work done per unit area in the plane containing a crack front during deformation, that is, strain energy area density [3]." According to this definition, the CED is defined quantitatively for a concrete crack model. In this study Continuum Notch Model (Notch Model) [4] is adopted as a crack model. While the CED is defined at any point in the plane containing a crack front line, in general [6], the quantity concerned with crack tip is considered here for simplicity. In Notch Model an actual crack is modeled as a notch in continuum. In a 3-dimensional problem, an actual crack tip is modeled by a semicircle of radius \( \rho \) in the plane perpendicular to a crack front line at each tip as shown in Fig.1[6], and the CED at the time \( t \) of the point \( (x_1, x_2, x_3) = (x_1', 0, x_3') \) as

\[
\mathbf{g}(t, x_1, x_3) = \int_{\Gamma(x_1', x_3')} Wdx_2
\]

\[W = \int_0^{\infty} \sigma_{ij} d\varepsilon_{ij}; \quad d\varepsilon_{ij} = (\partial \varepsilon_{ij} / \partial t) dt\]

where \( \Gamma(x_1', x_3') \) is a semicircular path representing crack tip, \( W \) is strain energy density, \( \sigma_{ij} \) is stress and \( \varepsilon_{ij} \) is strain respectively. As the CED is strain energy area density, the value of CED is independent of coordinate system. The CED is expressed also by path independent integral. In deformation theory, the CED expressed by path independent integral exactly agrees with J-integral [6].

In an arbitrary orthogonal coordinate system, strain energy density is separated into the following four parts.

\[
W_{12} = \int_0^t \left\{ \frac{1}{3} (\sigma_{11} - \sigma_{22}) (d\varepsilon_{11} - d\varepsilon_{22}) + 2\sigma_{12} d\varepsilon_{12} \right\}
\]

\[
W_{23} = \int_0^t \left\{ \frac{1}{3} (\sigma_{22} - \sigma_{33}) (d\varepsilon_{22} - d\varepsilon_{33}) + 2\sigma_{23} d\varepsilon_{23} \right\}
\]

\[
W_{31} = \int_0^t \left\{ \frac{1}{3} (\sigma_{33} - \sigma_{11}) (d\varepsilon_{33} - d\varepsilon_{11}) + 2\sigma_{31} d\varepsilon_{31} \right\}
\]

\[
W_{V} = \int_0^t \frac{1}{3} (\sigma_{11} + \sigma_{22} + \sigma_{33}) (d\varepsilon_{11} + d\varepsilon_{22} + d\varepsilon_{33})
\]

---

Fig.1 Notch model

Fig.2 Shear components of strain energy density
\( W_{12}, W_{23} \) and \( W_{31} \) are considered as the shear deformation components of strain energy density in \( x_1-x_2 \), \( x_2-x_3 \) and \( x_3-x_1 \) planes respectively. \( W_\text{v} \) is considered as the volume deformation component of strain energy density. The meaning of shear deformation components is explained clearly in the case that the coordinate system is taken so that \( x_1, x_2 \) and \( x_3 \) axes correspond with the directions of principal stress and strain. In this case, \( W_{12} \) is represented in the form of Eq.(6).

\[
W_{12} = \int_0^1 \left( \frac{1}{3} (\sigma_1 - \sigma_2) (d\varepsilon_1 - d\varepsilon_2) \right)
\]

where \( \sigma_1 \) and \( \varepsilon_1 \) mean principal stress and principal strain components respectively. As shown in Fig.2, \( \sigma_1 - \sigma_2 \) means maximum shear stress in \( x_1-x_2 \) plane.

Corresponding to the separation of strain energy density into four parts above, the CED is separated into four parts as shown in the following.

\[
\varepsilon_{12}(t, X_1, X_3) = \int_{\gamma(X_1, X_0)} W_{12} dx_2
\]

\[
\varepsilon_{23}(t, X_1, X_3) = \int_{\gamma(X_1, X_0)} W_{23} dx_2
\]

\[
\varepsilon_{31}(t, X_1, X_3) = \int_{\gamma(X_1, X_0)} W_{31} dx_2
\]

\[
\varepsilon_{V}(t, X_1, X_3) = \int_{\gamma(X_1, X_0)} W_{\text{v}} dx_2
\]

\( \varepsilon_{12}, \varepsilon_{23}, \) and \( \varepsilon_{31} \) represent the shear deformation components of CED in \( x_1-x_2 \), \( x_2-x_3 \) and \( x_3-x_1 \) planes respectively. \( \varepsilon_{V} \) represents the volume deformation component of CED. Making use of these components of CED, it is expected that the deformation mode at a crack tip is evaluated quantitatively.

---

![Diagram](image)

Fig.3 Center cracked specimen

---
3. NUMERICAL EVALUATION OF DEFORMATION MODE AT CRACK TIP

In this chapter, the evaluations of deformation mode at a 3-dimensional crack tip are demonstrated by finite element analyses. Elastic, Elastic-Plastic and Elastic-Creep problems are treated. The specimen is a center cracked body under uniform tension. Figure 3 illustrates its dimensions. In Elastic-Plastic analyses, the constitutive law accords with $J_2$ flow theory characterized by Young's modulus $E=210.7$ GPa, Poisson's ratio $v=0.3$, yield stress $\sigma_Y=392.0$ MPa and strain hardening ratio $H'=E/100$. In Elastic-Creep analyses, Norton's Law characterized by exponent of power law creep $n=7.0$ is adopted. In each problem two specimens, $D=0.5\text{mm}$ and $60.0\text{mm}$, are analyzed.

The deformation mode evaluation is carried out by paying attention to the ratio of CED components mentioned in previous chapter. When crack front line is straight and parallel to $\sigma_3$ direction, $\varepsilon_{12}$ and $\varepsilon_{23}$ represent the shear deformations schematically shown in Fig.4(a) and (b) respectively. In other words, there is possibility that the degrees of plane strain type and plane stress type deformations can be evaluated through $\varepsilon_{12}$ and $\varepsilon_{23}$ respectively. As one measure of deformation mode, each component of 2-dimensional problem, under plane stress condition and plane strain condition, is also evaluated.

![Fig.4 Correspondence of shear components of CED to deformation mode](image)

![Fig.5 Ratio of shear components of CED (Elastic problem)](image)

- 132 -
3.1 ELASTIC PROBLEM

In Elastic problem uni-axial load $\sigma$ is set at 9.8MPa. The ratios of shear components of CED are shown in Fig.5. In the figure, the values of $\frac{\varepsilon_{12}}{\varepsilon}$ and $\frac{(\varepsilon_{12} + \varepsilon_{23} + \varepsilon_{31})}{\varepsilon}$ are indicated by black square and triangle respectively. In this case the value of $\varepsilon_{23}$ is small enough to neglect. $\varepsilon_{31}$ is substantially shown by the difference between square and triangle. The leftmost points are the results of plane stress analyses and the rightmost ones are of plane strain analyses hereafter. In the case of $D=0.5\text{mm}$, the ratio of each component of CED is almost the same with that of plane stress case at every point of crack front line. In the case of $D=60.0\text{mm}$, the ratio agrees with that of plane stress case at surface. On the other hand, the ratio agrees with that of plane strain case at center of the specimen. These results quantitatively evidence widely-accepted expectation about the deformation mode of 3-dimensional crack tip, that is, the mode of thin cracked specimen is similar to plane stress type and at the center of thick one plane strain constraint is kept.

![Fig.6 Ratio of shear components of CED (Elastic-Plastic problem)](image)

![Fig.7 Ratio of shear components of CED (Elastic-Creep problem)](image)
In Elastic problem, total value of CED must exactly agree with J-integral [6]. In this study numerical results don't violate it.

3.2 ELASTIC-PLASTIC PROBLEM

The results of evaluations of CED components when \( \sigma = 0.625 \gamma \) are shown in Fig.6. In each case the difference of ratios of components between at surface and center of specimen becomes greater. These results suggest that plastic deformation causes the difference of the deformation mode between at surface and center of specimen.

3.3 ELASTIC-CREEP PROBLEM

In Elastic-Creep problem, \( \sigma \) is set at 58.8MPa as step load. Time \( t \) is non-dimensionalized by \( t_0 \), which is the time when, in the case that non-cracked specimen is tensed by average ligament stress, the creep strain becomes equal to the elastic strain. The results at the time \( t/t_0 = 2.15 \times 10^{-3} \) are shown in Fig.7. Different from Elastic and Elastic-Plastic problem, the ratio of \( \Delta \gamma \) decreases much and plane strain type deformation is difficult to be kept even at the center of thick specimen D=60.0mm.

4. CONCLUDING REMARKS

It is clarified that the CED is separated into four components corresponding to shear deformations and volume deformation. Evaluation of deformation change along a crack front line which is one of the typical problems in 3-dimensional fracture mechanics is thought to be treated by using these components of CED. Finite element analyses of center cracked specimens evidence the possibility of evaluation of the deformation mode in view of these components of CED.

REFERENCES