A Probabilistic Fracture Mechanics Assessment Procedure

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ABSTRACT

The R6 procedure for assessing the integrity of structures containing defects is well-established and now in its third revision. Although the basic procedure has not changed for a number of years, further developments are continuing through the addition and revision of appendices giving advice on a number of topics. This paper describes the latest edition to R6: an appendix giving advice on probabilistic fracture assessments.

1 INTRODUCTION

R6 (Wilson et al. 1986) uses a deterministic fracture mechanics approach in which values for material properties, loadings and dimensions are used in an analysis to assess whether failure will be avoided. A conservative assessment is made by choosing so-called lower or upper bound values for each quantity according to whichever gives the more pessimistic result. However, in some instances it is necessary to depart from the use of bounding values. One of these is in leak-before-break where a more predictive approach is needed and a paper at the last SMiRT conference (Langston et al. 1989) described development of a leak-before-break procedure based on R6.

Probabilistic assessment is another area where bounding values are not used. Instead, probability density functions are assigned to the input parameters and, in effect, integrity assessments are performed for all possible combinations of parameter values. The development of probabilistic fracture mechanics has received considerable attention (Wilson 1977; Gates 1985, 1986). Probabilistic assessments are a useful supplement to deterministic analyses and can also be used as an aid to decision making in areas such as safety analyses, design studies and deployment of resources on maintenance, inspection and repair (Wilson and Haines 1989).

This paper describes a methodology for quantifying the risk of failure based on the R6 Revision 3 procedure. The methodology is designed to take account of pre-service proof testing, changes in mechanical properties during service and in-service fatigue crack growth, where necessary. Section 2 first outlines the background to the methodology. Then Section 3 presents the procedure with the following sections giving some details of suitable forms for the distribution of input parameters and associated computer programs for performing the necessary calculations.
The probabilistic fracture mechanics methodology is illustrated by considering that there are three main assessment parameters, defect size, fracture toughness \( K_R \), and flow stress \( \sigma \), which are distributed statistically and which are independent. However, in principle the methodology can handle a larger number of variables and allow for correlation between some parameters. Certain combinations of parameters may lead to failure under the relevant loading conditions. Using RA, two parameters, \( K_R \) and \( L_R \), are evaluated and the point \(( L_R, K_R)\) plotted on a failure assessment diagram. Failure is conceded for those combinations for which the assessment point lies outside the failure assessment curve as shown in Fig. 1.

The failure probability, \( P_f \), for the specified loading conditions and geometry of thickness \( w \) may be stated in the form

\[
P_f = \int_0^w f(a) \int_{A_{FAIL}} p_a(L, K_R) \, dK_R \, dL \, da
\]

where \( p_a(L, K_R) \) is the bivariate probability density function which expresses the variability of \( K_R \) and \( L \), arising from the variability of \( K_C \) and \( \sigma \) for a given defect size, \( a \). The quantity \( f(a) \) is the probability density function for defect size. The integration is performed over \( A_{FAIL} \) which is defined as the region on the assessment diagram where failure is conceded according to the RA criteria; this is simply the area outside the failure assessment curve as shown in Fig. 1.

Fig. 1 Schematic Failure Assessment Diagram
Equation (1) must be evaluated in all applications. For convenience in the calculations and computer programs referred to below, an alternative is to transform from the space of the R6 failure assessment diagram to the corresponding \((K_{TC}, \sigma)\) space using the R6 definitions

\[
K_T = K(a, P)/K_{TC}
\]

\[
L_T = P/P_L(a, \sigma) = P \sigma / \sigma_L P_L (a, \sigma)
\]

where \(K\) is the stress intensity factor for an applied load \(P\) and \(P_L\) is the corresponding limit load evaluated for the yield stress \(\sigma_y\) or flow stress \(\sigma\).

In practical applications, the methodology needs to take into account certain effects not included in eqn (1). These include fatigue crack growth, stable tearing, leak-before-break, fault loadings and pre-service proof tests. The incorporation of these may add considerably to the complexity of the actual expressions to be evaluated but the basic principles remain the same. Mathematical details for including these effects are omitted here but, for illustration, the influence of a pre-service proof test is considered qualitatively.

A pre-service loading or proof test serves to eliminate those combinations of defect size and material properties which would have led to failure at the proof test loading. This, therefore, reduces the calculated failure probability in service. However, a complication is that the material properties probability density functions under operating conditions may be different from those at the time of the proof test because of differences in temperature and changes in material properties due to thermal ageing or irradiation. The methodology has been developed to take this into account either assuming that there are certain relationships between the corresponding proof test and operating properties or assuming no correlation between properties. Thus, it is possible to calculate the in-service failure probability allowing for the effect of the proof test.

3 PROCEDURE

The procedure for evaluating failure probabilities is written as a number of well-defined steps in an appendix to R6. These steps are listed below along with references to sections of R6 Revision 3 (Milne et al. 1986) or sections of this paper where further information is given.

(a) Specify distribution types and parameters for the defect size, fracture toughness and flow stress distributions (see sub-sections 3.1-3.2).

(b) Specify using the R6 procedures as necessary

- Geometry.
- \(\sigma^P\) stresses for all loadings including fault and pre-service proof test if appropriate (see Section 5 of R6).
- \(\sigma^G\) stresses for all loadings including fault and pre-service proof tests if appropriate (see Section 5 of R6).
- Failure assessment diagram to be used (see Section 6 of R6).
Category of analysis (see Section 7 of R6).

Whether leak-before-break is to be considered (see Appendix 9 of R6; Langston et al. 1989).

Fatigue loading.

Frequency of fault loading.

(c) Specify the methods to be employed to calculate the relevant limit loads and stress intensity factors (see Appendices 2 and 3 of R6).

(d) Select a suitable computer program for use (see Section 4).

(e) Run the selected program with the input data for the cases of interest to calculate the failure probabilities.

(f) Perform sensitivity studies (see Section 5).

3.1 Defect size distributions

Defect depth distributions are normally difficult to estimate reliably for any given application. Therefore, wherever possible, sensitivity studies should be performed as part of an assessment to investigate the dependency of the calculated failure probability on the assumed defect distribution (see Section 5). Three basic types of defect distribution that have been used in probabilistic fracture assessments are considered below.

Where a single defect is known or postulated to exist, uncertainty in the defect depth may be taken into account using a normal distribution. The form of the probability density function \( f(a) \) for defect depth, \( a \), is

\[
f(a) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left\{ -\frac{1}{2} \left( \frac{a - \mu}{\sigma} \right)^2 \right\}
\]

where \( \mu \) is the mean depth and \( \sigma \) is standard error.

Another type of distribution which describes the probability of defects of a particular size being present in a sample volume is

\[
f(a) = R \exp \left\{ -(\gamma a) \right\}
\]

where \( 1/\gamma \) is the mean defect depth and \( R/\gamma \) is the mean number of defects in the sample volume. The so-called Marshall distribution is a particular case of this exponential distribution.

The above distributions consider only defect depth and do not include defect length explicitly. Wilson and Haines (1989) described a distribution which considers both depth and length: variability of extreme defect depth is represented by a Gumbel Type II distribution and variability of aspect ratio is represented by a Weibull distribution. The form of the resulting bivariate probability density function is given in Wilson and Haines (1989).

3.2 Fracture toughness and tensile property distributions

Normal, log-normal and Weibull distributions have been used to describe
variations in fracture toughness. A disadvantage of a normal distribution is that the mathematical transformations carried out with this distribution may result in negative values for fracture toughness. Experience within Nuclear Electric suggests that the log-normal and Weibull distributions are suitable, the choice being influenced by whichever gives a better fit to experimental data. Where uncertainty exists, a sensitivity analysis is recommended.

R6 uses the yield stress in the evaluation of $L_\text{y}$ and a flow stress, normally taken to be the arithmetic mean of yield stress and ultimate tensile stress, in defining the cut-off on the failure assessment diagram, Fig. 1. Thus, the distribution of flow stress can be derived from distributions of yield and ultimate tensile stress. The same three distributions as used with fracture toughness are usually used for these quantities. The preferred distribution shapes for tensile properties are Weibull and log-normal but when uncertainty exists a sensitivity analysis should be carried out.

4 COMPUTER CODES

Two computer programs have been developed with Nuclear Electric to carry out the calculations required to follow the methodology described in Section 2. The code STAR6 uses a combination of analytical and numerical integration over the probability distributions to evaluate failure probabilities. The code PROF employs Monte Carlo simulation with importance sampling to carry out the integrations. Each program considers defect depth, fracture toughness and flow stress as the stochastic variables. STAR6 is limited to these variables but PROF can consider more variables, if necessary. The three types of defect distribution and material property distribution referred to in sub-sections 3.1 and 3.2 are all available as options in STAR6 and PROF. Comparison of the results produced with STAR6 and PROF has been of great value in the verification of the two codes.

5 SENSITIVITY ANALYSIS

The absolute value of a calculated failure probability is based on a set of assumptions about failure modes and distributions of various parameters. Where input data are sparse or of poor quality, it is necessary to build pessimisms into the figures used. In that case the results obtained are likely to be upper bounds to the real failure probabilities of the components.

However, it is recommended that a sensitivity study is performed to assess the significance of particular assumptions. The results of the study may be used to identify those parameters which have the greatest influence on the risk of failure and to perhaps justify further work to provide more detailed information on those parameters. In this context, it is worth remarking that calculated failure probabilities are generally found to be much more dependent on the choice of defect depth distribution than the choice of distributions for material properties.

6 DISCUSSION

This paper has described a methodology which has been incorporated as a new Appendix in R6 and which has been developed as a means of quantifying the integrity of defective structures. The absolute values of failure
probability can be used as part of an overall plant safety analysis to make a
case for new plant or continued operation of old plant. Similarly the
results can be used in plant design or modification studies to investigate
the effect of changing various design parameters and thereby optimise the
design. In other instances the relative values of the failure probabilities
may be used as a guide to the most economic deployment of resources on
maintenance, inspection and repair. The results could be used, for example,
to concentrate an ultrasonic inspection of a vessel on the locations at
highest risk, thus gaining maximum benefit from the inspection. If it is
found that a particularly high risk is associated with a particular operating
condition, for example where transient stresses are involved, then
consideration could be given to modifying the method of operation.

The probabilistic approach described here has been applied to a number
of components within Nuclear Electric. Wilson and Haines (1989), for example,
illustrated its use by application to welds in straight sections and mitre
bends of Magnox reactor ducts. More recently, Ainsworth (1991) has used
relative values of failure probability to address the effect of operating
temperature on the integrity of Magnox reactor pressure vessels. It is
expected that the methodology will be further applied and developed and, as
with other appendices in R6, the approach will be updated in the future.

ACKNOWLEDGEMENT

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