A Nonlinear Constitutive Model for Concrete under High Rates of Loading

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ABSTRACT

A realistic nonlinear constitutive model that can describe the dynamic tensile behavior of concrete is presented. The model is obtained by generalizing rate-independent nonlinear tensile stress-strain relations for concrete. The affinity transformations are employed to model the effect of strain rate. The material parameters are characterized in terms of the strain-rate magnitude. The present theory, which can model the dynamic tensile behavior of concrete, is compared with the dynamic tensile test data available in the literature.

1 INTRODUCTION

The mechanical behavior of concrete under dynamic loads induced from earthquakes, impacts, air blasts, wind gusts, and ocean waves is very complicated. The strain-rate effect of concrete has been investigated extensively. Most studies, however, have been directed to the case of compressive loading and only a few studies are available for the dynamic tensile loads. Recently, Bazant and Oh(1982) have developed a model to predict the strain-rate effect of concrete in compression. However, no model of this type exists to describe the dynamic behavior of concrete in tension. The purpose of this paper is, therefore, to propose a realistic model which can describe the dynamic tensile behavior of concrete.

2 STATIC CONSTITUTIVE MODEL

To develop a model to predict the dynamic tensile behavior of concrete, it is first necessary to clarify the static tensile behavior. Recently, Bazant and Oh(1985) proposed a rate-independent nonlinear constitutive model that can describe the static tensile behavior of concrete. This model considers that certain weak planes exist within the material in which the stress relief due to microcracking takes place as a function of the stress and strain on each particular plane. The total strain tensor $\varepsilon_{ij}$ is considered as a sum of an elastic strain tensor $\varepsilon^{e}_{ij}$ and an inelastic strain tensor $\varepsilon^{i}_{ij}$ in which the subscripts refer to cartesian coordinates $x_i (i = 1, 2, 3)$. The rheological model for this theory is depicted in Fig. 1. The
relation between the macroscopic stresses $\sigma_{ij}$ and the microstresses $\tau$ on the weak plane may be determined from the virtual work equation

$$\delta W = \frac{1}{3} \pi q_{ij} \delta \sigma_{ij} = 2 \int_S \tau_n \delta e_n \, dS$$  \hspace{1cm} (1)

in which the factor $4 \pi/3$ corresponds to integrating over a sphere of radius 1, $S$ represents the surface of a unit hemisphere, $dS = \sin \phi d\phi d\theta$, and $n_i = \text{direction coines of the unit normal } n \text{ of the weak plane}$. The normal microstrain $e_n$ on a weak plane may be expressed as $e_n = n_i n_j e_{ij}$ in which the repeated indices indicate a summation. The microstress $\tau_{ij}$ on the weak plane may reasonably be determined by assuming that $\tau$ is a function of the normal microstrain $e_n$ on the same plane. That is, $\tau_n = (2\pi/3)f(e_n)$ in which the factor $2\pi/3$ is introduced to facilitate the integration. By using the above-mentioned properties, Eq. (1) may be simplified as

$$\sigma_{ij} = \int_0^{2\pi} \int_0^{\pi/2} f(e_n) n_i n_j \sin \phi d\phi d\theta$$  \hspace{1cm} (2)

Differentiation of Eq. (2) leads to

$$d\sigma_{ij} = B_{ijkm} d\varepsilon_{km}$$  \hspace{1cm} (3)

$$B_{ijkm} = \int_0^{2\pi} \int_0^{\pi/2} n_i n_j n_k n_m f'(e_n) \sin \phi d\phi d\theta$$  \hspace{1cm} (4)

in which $f'(e_n) = df(e_n)/de_n$, and $B_{ijkm}$ may be considered as the tangent stiffness of the weak plane system. Since the strain $\varepsilon_{ij}$ is a sum of $\varepsilon_{ij}$ (Fig. 1), the incremental stress-strain relation may be written

$$d\sigma_{ij} = D_{ijkm} d\varepsilon_{km}$$  \hspace{1cm} (5)

in which

$$D_{ijkm} = \left[C_{ijkm} + (B_{ijkm})^{-1}\right]^{-1}$$  \hspace{1cm} (6)

Here $C_{ijkm}$ = elastic compliances corresponding to $\varepsilon_{ij}$.

The function $f(e_n)$ (Eq. (2)) must be capable of describing the microcracking behavior of concrete. The simple and desirable expressions for this behavior are adopted as

$$\sigma_n = E_n e_n \text{ for } e_n > 0, \sigma_n = E_n e_n^p \text{ for } e_n < 0$$  \hspace{1cm} (7)

where $E_n, K,$ and $p$ are material parameters that are functions of concrete strength $f'$. It was found from this study that the average value of $E_n = 5.322 \sqrt{f'}$, $k = 4.07 \times 10^6/\sqrt{f'}$, and $p = 2$, where $f'$ is given in N/mm² (1 N/mm² = 145 psi). The expression in Eq. (4) must be evaluated numeri-
cally. The numerical integration formulas for this purpose have been
developed and described in detail in the paper of Bazant and Oh(1985).

![Rheological model for concrete (a) and spherical coordinate system (b)](image)

Fig. 1 Rheological model for concrete (a) and spherical coordinate system (b)

3 CONSTITUTIVE THEORY FOR DYNAMIC LOADINGS

The rate-independent constitutive model needs to be generalized to include
the effect of strain-rate. Recently, Bazant and Oh(1982) developed a strain
-rate dependent nonlinear constitutive theory for concrete in compression.
This theory is extended here to model the dynamic tensile behavior of con-
crete.

The shape of the uniaxial stress-strain curve of concrete largely depends
on the strength. Generally, the peak portion of the curve is flatter for
a lower strength concrete and becomes sharper for a higher strength con-
crete. This nature may be characterized by the parameter \( r = \frac{E_p}{\sigma_0} \) in
which \( \sigma_0 \) = peak stress, \( \varepsilon_0 \) = strain at peak stress, and \( E = \text{initial elastic modulus} \). The parameter \( r \) represents the ratio of strain at peak stress to
the elastic strain corresponding to this stress. The general stress-strain
curve may, therefore, be characterized by three basic parameters, i.e., \( \sigma_0 \),
\( E \), and \( r \).

The parameter \( r \) depends on the strength \( f_c' \). This relation may be written
as \( f_c' = f_1(r) \). Since the elastic modulus \( E \) of concrete is known as a func-
tion of \( f_c' \), it may reasonably be written that \( E = f_2(r) \).

Since the constitutive equation expressed in Eq.(5) is a function of current
strain and concrete strength, this relation may be rewritten as

\[
d\sigma_{ij} = D_{ijk}\varepsilon_{1} \varepsilon_{2} f_1(r) \varepsilon_{3} \varepsilon_{4} \varepsilon_{5} \varepsilon_{6}
\]

For the given value of \( r = r^* \), these constitutive relations will yield the
peak stress \( f_1(r^*) \), and initial tangent modulus \( f_2(r^*) \).

The affinity transformations may be applied to preserve the value \( r^* \) but
allow changes in the peak stress and the initial elastic modulus. The
strain values \( \varepsilon \) are replaced by \( a\varepsilon \) and the stress values \( \sigma \) by \( b\sigma \). Therefore, Eq. (8) may be written as

\[
b d\sigma_{ij} = D_{ijk}[a\varepsilon, b\sigma, f_1(r^*)] ad\varepsilon km
\]

The parameter \( r \) is not a affected by such transformations. However, the
peak stress \( f_1(r^*) \) and the initial elastic modulus \( f_2(r^*) \) will be trans-
formed to the values \( \sigma^* = f(r^*)/b \) and \( \varepsilon^* = af_2(r^*)/b \). The transformation
coefficients \( a \) and \( b \) are determined as

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b = \frac{f_1(r^*)}{\sigma_p} \tag{10}

a = \frac{E^*}{\bar{I}_2(r^*)} b \tag{11}

The following formulas have been obtained by fitting the available dynamic tensile test data

\[ g(\dot{\varepsilon}) = \frac{1 - \dot{\varepsilon}^{1/8}}{2.2 + 3.2 \dot{\varepsilon}^{1/8}} \tag{12} \]

\[ r^* = 2.09 - 0.0215 \sigma_{po} + g(\dot{\varepsilon}) \tag{13} \]

\[ \sigma^* = [1.95 - 3.32 g(\dot{\varepsilon})] \sigma_{po} \tag{14} \]

in which \( \dot{\varepsilon} \) = strain rate, given in strain per second, and \( \sigma \) = uniaxial static strength in N/mm\(^2\) (1 N/mm\(^2\) = 145 psi). The formulas in Eq. (12) through (14) differ from those for dynamic compression because the effect of strain rate is more sensitive in dynamic tension. This is probably due to the fact that the concrete cracking greatly influences the strain-rate sensitivity. The relation \( f_c^* = f_1(r) \) is formalized from the test data as follows

\[ f_1(r^*) = (110 - 46r^*) \tag{15} \]

\[ f_2(r^*) = 4,740 \sqrt{f_1(r^*)} \tag{16} \]

in which \( f_1(r^*) \) and \( f_2(r^*) \) are expressed in N/mm\(^2\), and \( f_1(r) \) is obtained from the relation between the elastic modulus and the strength.

4 COMPARISONS WITH TEST DATA

The rate-dependent tensile stress-strain relation derived in the previous section has been compared with existing dynamic thesile test data. Fig. 2 shows the comparisons of the uniaxial tensile stress-strain curves obtained for different strain rates by Hatano in which the solid lines indicate the results from the present theory and the dashed lines indicate the test data. It can be seen that the tensile strength of concrete is greatly increased with the increase of strain rate.
5 CONCLUSION

Rate-dependent nonlinear constitutive relations for concrete in tension are proposed. The model is obtained by generalizing recently developed, rate-independent nonlinear tensile constitutive relations for concrete. The model adequately predicts the dynamic tensile properties of concrete and allows more realistic dynamic analysis of concrete structures.

REFERENCES


