Assessing Leakage through Cracked Pressurized Reinforced Concrete Containment Structures

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ABSTRACT

Incremental leakage under a postulated severe accident may govern the functional failure mode in the safety analysis of reinforced concrete containment structures. However, while well established approaches and powerful finite element codes are commonly available to the analyst in assessing the ultimate capacity and structural integrity, there is a scarcity of practical information and guidelines as far as through-cracking leakage is concerned, particularly for box-type structural configurations. Such an approach is presented herein.

1 INTRODUCTION

Under postulated extreme accident conditions, such as excessive overpressure or pipe-whip impact, liner tearing as well as concrete shear cracking may occur at the junctions of adjacent wall and panels. This situation creates a potential leakage passage due to the dilation of the crack surfaces under slidding movements. The objective of this paper is to investigate through-cracking leakage and provide a procedure to calculate the expected amount of leakage, particularly for box-type structural configurations.

2 PRE-LEAKAGE ANALYSIS

2.1 Containment accident scenarios

The most critical accident scenario should be chosen in terms of overpressure, event duration and temperature. Rapid increase of pressure may cause dynamic amplification while a post-peak fast drop would cause de-amplification. Temperature effects are somewhat different. Due to concrete low conductivity and high specific heat as well as large thickness of Reactor panels (1.3-1.8 m), short duration of temperature changes would not be felt throughout concrete thickness.

2.2 Heat transfer analysis

Temperature distributions through concrete thickness of different structural components are a pre-requisite to stress analysis. While acceptable for operational conditions, a linear temperature distribution may result in excessive thermal stresses for a transient condition, such as LOCA, where the temperature inside the containment may be briefly much higher than outside. A one-dimensional heat transfer analysis is usually adequate for this case.
2.3 Nonlinear finite element analysis

A series of structural analyses may be required, using a general purpose finite element program such as ANSYS. The program material library contains constitutive models for reinforced concrete with separate models for reinforcement bars (rebar) and concrete, both with elastic-plastic-failure capabilities. The concrete plasticity is based on isotropic work hardening modified Chen and Chen's [1975] constitutive law.

Heat transfer analysis is performed if required, followed by a linear static analysis to identify areas of stress concentration. A linear dynamic analysis may be sufficient to simulate the transient nature of the loading by means of dynamic amplification factors applied on the peak overpressures.

A static non-linear analysis is usually load-history dependent, which requires correct loading sequence.

2.4 Shear friction and crack interface elements

For a box type heavily reinforced containment structure, shear failure becomes the governing factor. However, with commercially available nonlinear finite element programs, the maximum capacity of the structure may not be reached due to numerical difficulties caused by cracks. When a potential through-crack surface is identified, an alternative approach, based primarily on aggregate interlock considerations, may be required to obtain the maximum shear capacity and consequently the corresponding crack width.

Constitutive equations for cracks derived by Walraven [1981] or more recently by Yoshikawa et al. [1989], relating the interface concrete shear and normal stresses with the corresponding crack dilation and sliding movements, may be used to formulate crack interface elements. The effect of the local reinforcement distortions, such as "kinking" and dowel action, can be integrated as well. These elements may then be added between the two crack surfaces, following duplication of the contact nodes, allowing the extension of the nonlinear analysis into the large crack movements regime.

2.5 Average crack width and spacing

Where a clear crack line is not defined, one simple way to calculate possible crack width is using concrete strains. Ignoring the small elastic strain in the concrete between cracks, the crack width may be related to concrete strain \( \varepsilon_{cf} \) by the relation, \( \varepsilon_{c} = \frac{1}{s_{c}} \), where \( \varepsilon_{c} \) is the mean crack width and \( s_{c} \) is the mean crack spacing. Several empirical formulae exist in literature to obtain crack spacing and widths, such as CEB-FIP [1978], Gergely-Lucz [1968] and Bazant-Oh [1983].

3 LEAKAGE ANALYSIS

3.1 General considerations

Once the cracks sizes are determined, the next step of leakage analysis is to determine the amount of expected flow through them. The leakage rate is the amount of gas mass that flows through the cracks during a specified period of time. This amount is then converted to a volume of gas per hour and compared with the total volume of the containment system. The nature of gas leakage through a crack depends upon a number of factors. A major one is whether the pressure difference across the crack faces \( \delta P \) is large enough to cause sonic flow (i.e. gas velocity equal to speed of sound). At low \( \delta P \), laminar flow is expected. Turbulent flow occurs at intermediate values of \( \delta P \). In turbulent flow, the inertia forces are so great that viscous forces cannot dampen out disturbances caused primarily by crack surface roughness which would create eddies resulting in gas velocity in both rotational and translational directions. Transition from laminar to turbulent is accompanied by a drop in leakage rate. At high pressure differential (\( \delta P > 100 \) kpa) gas velocity would become sonic and the flow rate reaches its maximum possible value (normally described as choked flow).
3.2 Flow equations

The following symbols are commonly used in the equations while other symbols are defined as they first arise. Crack geometry (see figure 1): D = concrete thickness or through crack depth, \( l = \) flow direction, \( P = \) crack length, \( W = \) average crack width. Gas properties: \( P = \) absolute pressure, \( \delta P = \) pressure drop across the crack, \( V = \) velocity, \( \rho = \) density, \( T = \) absolute temperature, \( \mu = \) dynamic viscosity, \( k = \) ratio of specific heats, \( C_p = 1.4 \) (air), \( R = \) gas constant, \( P/(\delta T) = \) perfect gas, \( M = \) Mach number, \( V/(\sqrt{RT}) = \) Reynolds's number, \( V/(2W\rho/\mu) = \) flow rate, \( C = \) mass flow per unit area of crack, i.e., mass/time/area, \( m = \) mass flow rate = G.W.L, \( q = \) volumetric leakage rate = volume/time. Suffixes: \( 2 & e \) = exit (at end of crack), \( 1 & i \) = inlet (at the beginning of crack), \( j \) = crack number.

U. Buss [1972], based on a series of tests on concrete slabs, developed the simple following equation to calculate volumetric leakage rate "q".

\[
q = \frac{[\delta P (2+\delta P/P_2) \cdot W^3 \cdot L \cdot g]}{[300 D (1+\delta P/P_2) \cdot r \cdot \Omega]}
\]

Where: \( g = \) acceleration due to gravity, \( r = \) air specific gravity = \( \rho / \rho_0 \), \( \Omega = \) gas kinematic viscosity = \( \rho / \mu \).

This equation, however, is derived for very low values of Reynolds's number \( \rho_2 \mu \) and consequently small pressure drop across crack sides, which may not be applicable for postulated accident conditions which are characterized by high pressures and temperatures.

Rizkalla [1984], assuming isothermal air flow, developed a mathematical expression for leakage rate through reinforced concrete members subjected to much higher differential pressure. The expression was refined by using experimental data to describe the air flow rate through any given crack pattern. The mass flow rate "q" through a crack of width "W" is given by:

\[
\frac{(P_1^2 - P_2^2)}{D} = \left(\frac{P_0^2}{2}\right) \left(\frac{W/2}{\Omega T^{\alpha-1}}\right) \frac{P_0 Q}{L} \left(1/W^2\right)
\]

Dimensionless wall roughness \( F \) and flow coefficient \( n \) are obtained experimentally as \( F = 2.907 \times 10^7 (W^3)^{0.428} \), \( n = 0.133/(W^3)^{0.083} \), with \( W \) in feet units for a member having many cracks, the total flow rate is obtained by replacing \( W \) in the previous equation with \( \Sigma W_j \), where \( W_j \) is the width of individual crack no. \( j \). The concept of equivalent crack width for given crack pattern was also introduced, in cases where a clear crack line is not defined, such as where using equations of crack spacing. In this case, if \( "N" \) is number of cracks and average crack width "W_ave", then \( \Sigma W_j \) in equation (2) should be replaced by \( \Sigma W_j^3 = 1.42 N W_ave^3 \).

Manning P.T. [1977] and Chivers & Manning [1977] reported a method to predict an upper and lower limit of gas flow rate through cracks. For an upper bound on leakage rate, the crack is assumed smooth, while for a lower bound the lower of the two possibilities is recommended:

(a) The crack is assumed smooth and the width is arbitrary reduced to \( W' \) which is given as:

\[
W' = 0.87 W \text{(in)} - 0.16
\]

(b) The crack is rough and the friction factor \( f \) is taken as:

\[
f = 1/[4.5 \log_{10} (2W/R_s)^{2.5} ]^2
\]

Where: \( R_s = \) surface roughness having same units as crack width.

The numerical procedure followed is iterative. The Mach number of the gas at inlet \( M_i \) is assumed. The mass flow rate per unit area of the crack is calculated according to the following relation:

\[
G = [(P_1 + P_2) + (0.5 M_i^2)] / [1 + 3/4 M_i^2]
\]

The friction factor is calculated first as:

\[
f = 24/R_s, \quad \text{if} \quad R_s (\text{Reynold's number}) < 2048
\]

\[\downarrow 2048\]
\[ f = 0.079/(R_n)^{0.25}, \quad \text{if} \quad R_n > 2048 \] (7)

The friction factor is calculated again from the following equations:

\[ \frac{fD}{W} = \left\{ \begin{array}{ll}
\frac{1}{4} \ln \left( \frac{M_i}{P_o} \right), & \text{if} \quad M_i > P_o/P_i \\
\frac{1}{4} \ln \left( \frac{M_i}{P_o} \right)^2 + \ln \left( \frac{P_o}{P_i} \right), & \text{if} \quad M_i < P_o/P_i
\end{array} \right. \] (8)

The value of \( M_i \) is then adjusted until both values of friction factor obtained from equations (6), (7) and (8), (9) become very close. For the calculation of lower limit of flow rate using the same crack width, the friction factor for a very rough surface is assumed constant at high Reynolds numbers. The friction factor is obtained from equation (4) and substituted in equations (8), (9) to obtain \( M_i \). The mass flow rate is obtained by substituting \( M_i \) in equation (5).

Ewing, D.J.F. [1989] presented closed-form approximate analytical formulae to predict gas flow rate through cracks. The crack is idealized as a rough, tapering, wedge shaped channel and the fluid is idealized as an isothermal or polytropically-expanding perfect gas. The mass flow rate per unit area of the crack is calculated as:

\[ Q = C_0 \sqrt{\frac{P_1}{P_2}} \] (10)

Here: \( C_0 \) is a non-dimensional discharge coefficient, depending on the non-dimensional friction number \( F = fD/W \), in which \( f \) is the friction coefficient.

An approximate expression for \( C_0 \) is given for three flow regimes:

(a) Very low friction ( \( F \leq F_1 \)). Choked flow occurs at entry and flow is independent of friction.

\[ C_0 = 0.4(1-d) \] (11)

(b) Intermediate friction ( \( F_1 < F < F_2 \)). Choked flow occurs at exit.

\[ C_0 = (1-d^2)/(1+\sqrt{1+F/2}) \] (12)

(c) High friction ( \( F > F_2 \)). Choked flow does not occur at all.

\[ C_0 = \frac{(1-d^2)(1-(P_o/P_1)^{(n+1)/2})}{(1+\sqrt{1+(1+n)/F})^2} \] (13)

Here:

\[ F_1 = \frac{(1.5+2.5d)^2/(n+1)}{F_2 = [(1-d)(P_2/P_o)^{(n+1)/2}-(n+1)]/(n+1)} \]

\( n \) is gas polytropic index \( (P=kT) \), 
\( d \) is divergence parameter \( (\omega_1 - \omega_1)/(\omega_1 + \omega_1) \), and
\( h = (1-d) \left( (P_1/P_2) - (P_1/P_2)^{(n-1)/2} \right) \)

J. Tinkler, et. al. [1987] re-analyzed the measured data by Rizkalla [9] to include the compressibility effects on the air flow. Assuming adiabatic flow of compressible gas along a constant area duct with surface friction, the leakage through a crack may be obtained by solving the following three non-linear equations using Newton-Raphson method to obtain the three unknowns \( C_0, M_i \) and \( M_e \).

\[ \frac{W}{2KD} \left( 1-M_i^2 - (K-1)/2 \right) \ln \left( \frac{M_i^2}{1 + M_i^2} \right) = 0 \] (14)

\[ -(a+b)\frac{\mu D}{G} = 0 \]

\[ M_i \left( 1 + M_e^2 \right) \frac{(K-1)/2}{(K-1)/2} = 0 \] (15)

\[ M_e \left( 1 + M_e^2 \right) \frac{(K-1)/2}{(K-1)/2} = 0 \] (16)

Here:

\[ a = 28.1 - 10.5x10^3 \left( \epsilon/L \right) + 0.987x10^6 \left( \epsilon/L \right)^2, \]

\[ b = 677,000 - 223,000 \times \left( \epsilon/L \right)^3 + 20,000 \times \left( \epsilon/L \right)^2, \]

in which \( \epsilon \) = crack roughness.

Total mass flow = \( -\Sigma C_j L_j Q_j \) (17)
3.3 Sample calculation of leakage

Assume a containment of total volume of 17,000 Cu. meters, the internal pressure increased to a hypothetical value of 350 kpag, which would produce a through 0.8 mm crack, 3.0 m long, at the junction of two 1.8 m thick walls.

Crack data: W=0.8 mm, D=1.80 M, L=3.0 M, roughness $\varepsilon$ & $R_{n}=0.76$ mm.
Containment data: $p=350$ kpag, $t=100$ °C
Gas properties:($a_1$) $R=287$ J/kg/K, $\mu=2.5 \times 10^{-8}$ N.s/m², $k=1.4$, $T_1=573$ °K, $P_0=651$ Kpa, $P_1=101$ Kpa, $P_0=4.21$ kg/m³ (inside containment).

Using Manning [1977]:
Maximum flow, assume smooth crack: $m=0.5285$, Volumetric flow rate=653 cu.M/hr. Flow as % of containment volume=453x100/17000= 2.66 %
Minimum flow, assume rough crack: $m=0.0393$, Volumetric flow rate=33.68 cu.M/hr. Flow as % of containment volume=33.68x100/17000= 0.20 %

Using Bums [1972] method:
$q=1771$ cu.M/hr. Flow as % of containment volume= 10.10 % (much higher than that calculated by Manning method assuming smooth crack, $R_{n}$ is out of range of experimental data of this method).

Flow as % of containment volume=187x100/17000= 1.08 %

Using Ewing [1989] formulations:
Total mass flow $m=0.05045$ Kg/s=181.6 Kg/hr.
Flow as % of containment volume=181.6x100/(4.21x17000)= 0.25 %

Using Tinkler et al. [1987] method:
$m=0.00261$, $K_0=0.0108$ and $G=3.983$ Kg/sec.sq.m. Total mass flow rate (GLH)=34.41 kg/hr.
Flow as % of containment volume=34.41x100/(4.21x17000)= 0.05 %

This method leads to flow rate less than the minimum suggested by Manning and Ewing methods. However it should be noted that equation (4) is limited to roughness less than 20% of crack width. This may not be the case in concrete cracks. It should also be noted that Tinkler method requires high accuracy calculation as the three non-linear equations are very sensitive and could lead to smaller values of flow rate.

3.4 Liner effect on leakage:

A liner fracture forms an orifice in series with one or more of the concrete cracks and the analysis may follow that of pipe network. If the liner fracture is large, its pressure loss will be relatively small and its flow characteristics will be as for an incompressible flow orifice. Otherwise, the compressibility effect should be considered, becoming more and more pronounced as the fracture becomes smaller, until the pressure ratio across the liner fracture is sufficient to produce choking and the leakage becomes independent of the downstream concrete cracks. Further study is required for this problem.

4 CONCLUSIONS AND RECOMMENDATIONS

(1) In performing leakage assessment analysis, the following steps are recommended: The critical containment accident scenarios are selected in terms of overpressure, temperature, energy, duration of the event as well as of peak pressure and how fast it decays. Heat transfer analysis may be required to obtain more realistic temperature distribution through concrete thickness. Based on the stress analysis results, identify the critical locations where through cracking may develop and assess the cracks length and their opening. Carry out a gas flow analysis through the identified cracks, using either thermodynamic equations or empirical relations based on experimental data. Finally establish whether the total release is acceptable, otherwise the analysis may be repeated with more refined model and less conservative assumptions.

(2) Ewing method [1989] is recommended as first estimate of flow rate, while Tinkler method [1987] is recommended for more accurate estimate. A software program however, is required to obtain accurate results from that
method. Method suggested by Rizkalla [1984] is useful to calculate leakage in case of a group of small cracks instead of a definite crack line.

(3) A study of liner fracture effect on leakage is recommended.

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Figure 1. Through crack idealization.