

## The Effect of Post-Cracking Constitutive Models on Crack Propagation and Ultimate Capacity of Reinforced Concrete Containments

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### ABSTRACT

The fixed crack model, as used in finite element analysis of reinforced concrete structures, assumes that once a crack forms, its direction remains fixed. A secondary crack is restricted to form perpendicularly to the initial crack. The "rotating crack" approach allows the concrete principal strain directions to change after the formation of initial cracks so that subsequent cracks can form at angles that are not necessarily normal to the initial crack. To assess the effect of the two assumptions on the crack propagation and ultimate capacity, concrete models based on these two approaches were used to predict the results of a test beam and a containment wall under a postulated pipewhip impact. The rotating crack approach was found to predict behaviour that is more consistent with experimental observations.

### 1 INTRODUCTION

Using finite elements to study the cracking behaviour of reinforced concrete members was first suggested by Ngo and Scordelis (1967), using a discrete formulation. Rashid (1968) pioneered a later approach that looked at the effect of cracking on the overall behaviour of the structure. In this model, which became known as the smeared crack approach, when a crack occurs and remains open, its direction is fixed and does not change under subsequent loading. This initial crack may close and a secondary crack is restricted to form orthogonal to the initial crack. This model is usually referred to as the fixed crack model.

Cope, et al (1980) and Gupta (1982) started the implementation of models that accounted for changing crack directions. Known as the "rotating crack" model, this approach assumes that the crack direction is always normal to the direction of the principal tensile strain or stress. Subsequent cracks are not restricted to form normally to the initial one.

This numerical study evaluates the two models in terms of their capability to predict the crack pattern, failure mode and ultimate load capacity of reinforced concrete structures under plane stress situations, such as beams and containment walls.

### 2 FIXED CRACK MODEL

The "freezing" of the directions of the primary and secondary cracks regardless of the actual orientations of the principal stresses under subsequent loading may of course result in violation of the cracking criterion within an element, which is likely to be more critical for non-isotropically reinforced regions. The fixed crack approach may also lead

to crack orientation that is not consistent with the ultimate limit state. The concrete representation in ABAQUS computer program, one of the two codes used in this study, is a plasticity based model with the fixed crack idealization. Cracking is assumed to occur when the principal stresses reach a crack detection surface, expressed as a Coulomb relationship in terms of the first and second stress invariants. A shear retention coefficient between 0 and 1.0 may be specified, as well as a decaying post-cracking stress-strain tensile curve.

### 3 ROTATING CRACK MODEL

In the rotating crack model, the crack direction is always perpendicular to the direction of the principal tensile strain and stress. The principal concrete strain directions are allowed to change after the initial cracking, so that secondary cracks can develop at other directions than normal to the initial cracks. The panel tests by Vecchio and Collins (1986) provide experimental evidence to support this mechanism. Vecchio and Collins observed that, in a non-isotropically reinforced panel, as the loading is increased the reinforcement yields in the weaker direction first, and the initial cracks become less prominent and new cracks are formed because the initial cracks retained partial shear transfer capacity. It is evident that such changes in the dominant cracks orientation should be simulated in an analytical model.

A constitutive model using a Cauchy type, non-linear elasticity approach was derived by Danay and Hathout (1985), based on the above mentioned experimental results. The principal stresses were expressed as two-dimensional surfaces in terms of the principal strains, resulting in a symmetric tangential stiffness matrix formulation. This rotating crack model, however, was not incorporated into a general purpose finite element program.

A concrete model based on the rotating crack approach and the Modified Compression Field Theory (1986) was implemented in the ADINA computer program by Adeghe and Collins (1986). This program was used to provide a numerical comparison with the results of the fixed crack approach in ABAQUS.

### 4 ANALYTICAL STUDY

The two cracking approaches provided by the concrete models in the ABAQUS and modified ADINA computer programs were used to predict the behaviour of a test beam under a uniformly distributed load and a containment wall-slab junction under a postulated pipewhip impact.

#### 4.1 Analysis of a uniformly loaded concrete beam

The simply-supported, uniformly loaded reinforced concrete beam, for which experimental test results were reported by Mailhot (1984), was analyzed twice, using the modified ADINA program and the ABAQUS computer code. The dimensions and reinforcement details are summarized in Fig. 1a. The finite element mesh used for the study is presented in Fig. 1b. The concrete was modelled by 4-node isoparametric elements. The total number of nodes and 2-D concrete elements were 231 and 192 respectively.

The beam considered in this study was designed for a uniform load of 106.3 KN/m. The test specimen failed at a load of 129.7 KN/m. Failure was initiated in the experimental test by yielding of both the longitudinal and stirrup reinforcement. The rotating crack model predicted a failure load of 125.8 KN/m and both longitudinal and the stirrup yielded prior to failure. The fixed crack model predicted a failure load of 113.6 KN/m and only the longitudinal steel yielded before ultimate load was achieved.

This behaviour is illustrated in Fig. 2, which presents the variation of average stirrup strains along the beam span at ultimate load. The rotating crack model reproduces the form of the experimental curve very well, while the fixed crack model shows considerable underprediction.

Fig. 3 compares the load deflection curves for the analyses and the test. In spite of the considerable differences in stirrups strains, the fixed crack model, though slightly stiffer, is in good agreement with both the rotating crack model and the experimental test. This seems reasonable, since the deflections in this example are governed mostly by the longitudinal strains.

#### 4.2 Analysis of a containment wall-slab junction

A numerical study was carried out for a concrete containment wall-slab junction under the action of a postulated pipewhip impact. The dimensions and reinforcement details are presented in Fig. 4a and the finite element mesh in Fig. 4b. The non-isotropic reinforcement in this section was thought to be ideal for the comparative study of these two approaches. The frame width is 5.0m.

Figs. 5a to 5c present the amount of cracking at various load levels as predicted by the two approaches. The patterns are similar, though the rotating crack model shows more extensive cracking at every stage.

Figs. 6a and b show the variation of steel strains at some critical locations of the wall. Unlike the first example, the strain levels in the steel do not show a consistent pattern of prediction between the two approaches.

The failure load, however, could not be reached in the fixed crack model due to numerical difficulties encountered by ABAQUS in the severe cracking regime. The analysis terminated prematurely at about 32MN vs 40 MN for the rotating crack model, where yielding of the vertical wall reinforcement and severe cracking occurred.

The load deflections are rather similar, the maximum deflection at 32MN is about 1.5mm, compared with 1.8 mm for the rotating crack model.

### 5 SUMMARY AND CONCLUSIONS

A comparative study of the two most widely used approaches for accounting for crack propagation in finite element analysis of reinforced concrete has been presented.

The rotating crack approach was found to predict strains, crack pattern and failure modes that are more compatible with experimental results. Regarding crack propagation, steel strains and failure modes, caution should be exercised when interpreting the results of fixed crack model analyses.

### REFERENCES

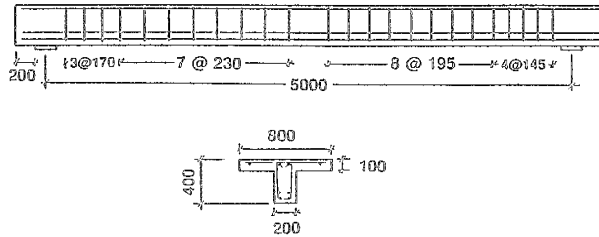
- Adeghe, L.N. and Collins, M.P., 1986, A finite element model for studying reinforced concrete detailing problems, Publication No. 86-12, Dept. of Civil Engineering, University of Toronto, Toronto, Canada.
- Cope, R.J., et al, 1980, Modelling of reinforced concrete behaviour for finite element analysis of bridge slabs, Numerical methods for non-linear problems, Pineridge Press, Swansea, UK, pp 457-470.
- Danay, A. and Hathout, I., 1985, Non-linear constitutive model for cracked concrete panels under in-plane and normal stresses, Trans. of 8th SMIRT Conf., Paper H3/5, Brussels, Belgium.
- Gupta, A.K. and Habibollah, A., 1982, Changing crack direction in reinforced concrete analysis, Civil Engineering Dept., North Carolina State University, Raleigh, North Carolina, USA.
- Mailhot, G., 1984, Experiments on the staggering concept for shear design, M.Eng. Thesis, Dept. of Civil Engineering and Applied Mechanics, McGill University, Montreal, Canada.

Ngo, D., and Scordelis, A.C., 1967, Finite element analysis of reinforced concrete beams, American Concrete Institute Journal, Vol. 64, No. 3.

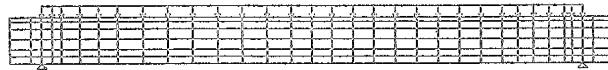
Rashid, Y.R., 1968, Analysis of prestressed concrete pressure vessels, Nuclear Eng. Design, Vol. 7, No. 4, pp.334-344.

Vecchio, F.J. and Collins, M.P., 1982, The response of reinforced concrete to in-plane shear and normal stresses, Publication No. 82-03, Dept. of Civil Engineering, University of Toronto, Canada.

Vecchio, F.J., and Collins, M.P. 1986, , The modified compression field theory for reinforced concrete elements subjected to shear, Journal of the American Concrete Institute, Vol. 83, No.2, pp. 219-231.



a: Details of test specimen (dimensions are in mm)



b: Finite element mesh

Fig. 1 : Experimental and analytical beam models

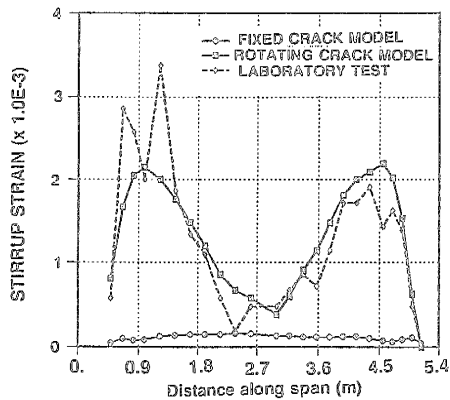


Fig. 2 : Variation of average stirrup strain along span

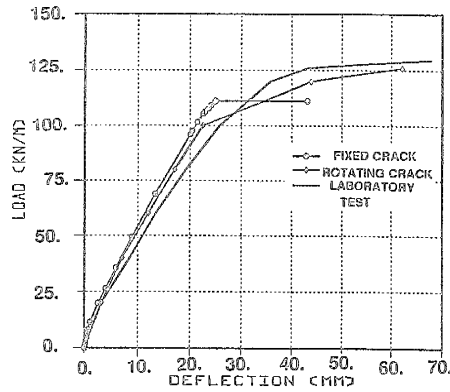
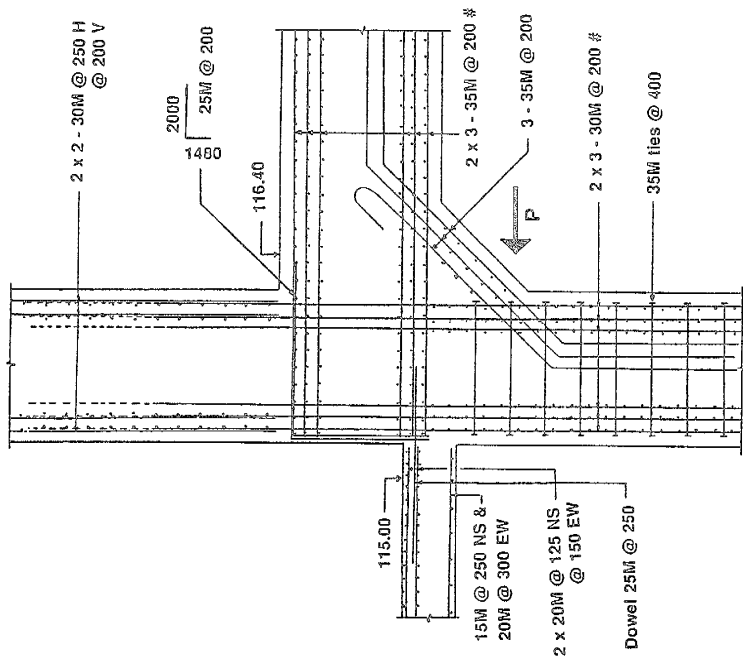
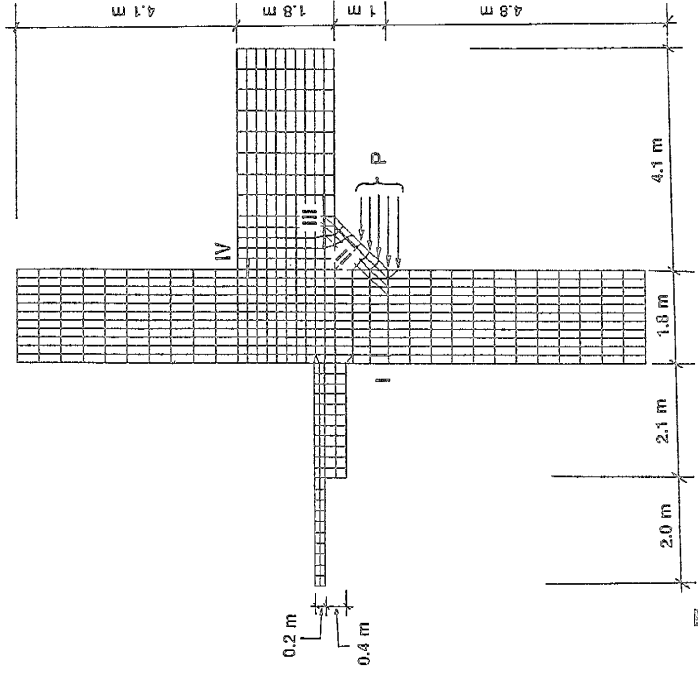


Fig. 3 : Load - deflection response of specimen



a: Details of Reinforcement for Wall-Slab Connection



b: Finite element model

Fig. 4 : Dimensions and analytical model of wall-slab junction

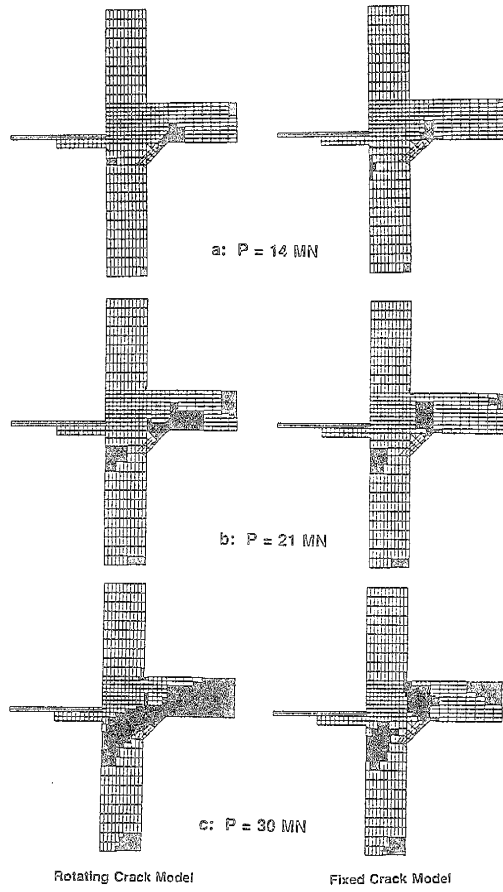
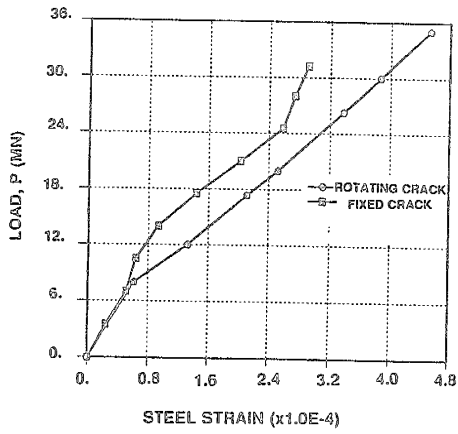
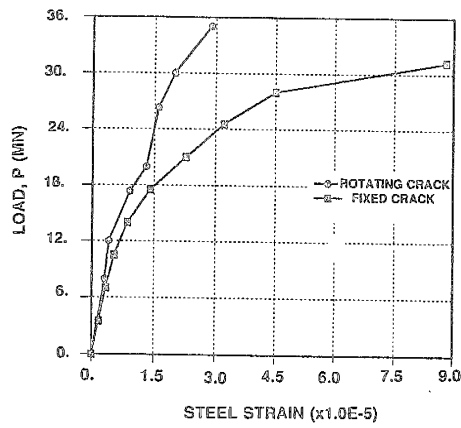


Fig. 5 : Extent of cracking at various load increments



a: LOCATION II



b: LOCATION IV

Fig. 6 : Variation of steel strain with load at some critical locations