Evaluation of Hydrodynamical Dumping Force Acting on a Plate in Motion in the Vicinity of Baffle Plate under Coolant Flow

Hisashi HISHIDA
Mitsubishi Atomic Power Industries, Inc., Tokyo, Japan

Katsuhiro SAKAI
Osaka Univ., Osaka, Japan

Kazumi SHIBATA
Wave Front Corporation, Yokohama, Japan

ABSTRACT

In order to investigate the characteristics of the flow induced vibration related to the integrity of reactor structures, excitation and dumping forces acting on the components under the fluid flow should be evaluated as a function of their motion and the influence of surrounding boundaries.

In this paper, the magnitude of hydrodynamical dumping force acting on a plate component in the extremely close vicinity of a wall parallel to the plate under the flow is analytically evaluated in terms of the velocity of the plate component in motion and its distance from the wall.

The results of evaluation are applicable to simulation studies on the flow induced vibration of specific reactor components under the flow condition.

1 INTRODUCTION

Flow induced vibration has been a subject of attention in connection with the prevention of small amplitude vibrations. Various experimental data and mathematical techniques have previously provided the designers with the necessary expertise and the requisite confidence in the reliability of the components (Bohm et al. 1981).

From the mathematical point of view, the fluid-structure interaction is mainly characterized with the structural stiffness, the excitation force due to turbulence and vortex shedding and the load produced by fluid pressure and fluid elasticity, which are usually evaluated based on experimental investigation using mockup facilities (Connors et al. 1982). However, for a small amplitude vibration of a plate component in the extremely close vicinity of a flat wall under the influence of fluid flow, the experimental investigation of hydrodynamical load exerted on a plate component involves considerable difficulties.

In the followings, the magnitude of hydrodynamical load being exerted on a plate component in motion in the extremely close vicinity of a flat wall which is parallel to the face of the plate component under fluid flow is evaluated in terms of the velocity of the plate component towards or away from the wall and the distance of their separation. The motion of the plate component is perpendicular to the wall. The resulting lift coefficients are to be introduced into the vibration analyses of a structure with more complicated configurations.

2 MATHEMATICAL FORMULATION

2.1 Governing equations

The Navier-Stokes equation and continuity equation which govern the flow fields in the physical domain including the plate component and the wall are written as follows:

\[ \rho \frac{dV}{dt} = -\nabla p + \frac{1}{Re}\nabla^2 V \]  \hspace{1cm} (1)

\[ \nabla \cdot V = 0 \]  \hspace{1cm} (2)

where \( \rho, Re, p \) and \( V \) are the density of fluid, the Reynolds number, the nondimensionalized pressure and the nondimensionalized velocity vector, respectively. The external force term is excluded from eq.(1) since it does not play a significant role in the present evaluation.

Taking the divergence of eq.(1), and introducing the continuity equation (2) into the resulting expression, the following Poisson equation for pressure (3) is obtained for the two-dimensional physical domain:

\[ \nabla^2 p = -\left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + 2 \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} \]  \hspace{1cm} (3)

Eq.(1) is solved with eq.(2) using the SOR scheme, where the Euler backward scheme is employed in reducing these equations into the difference equations except for the convection term involved in eq.(1). The convection term is linearized such that

\[ V \cdot \nabla V = V^a \cdot \nabla V^{a+1} \]  \hspace{1cm} (4)

where \( a \) stands for the value at \( t = t_n \). The third-order upwind method is employed for \( \nabla V^{a+1} \) of expression (4) (Kawamura, T., 1983).

The eq.(3) is expressed in the difference form as follows:

\[ \nabla^2 p^{a+1} = -\left( \frac{\partial u^a}{\partial x} \right)^2 + \left( \frac{\partial v^a}{\partial y} \right)^2 + 2 \frac{\partial u^a}{\partial x} \frac{\partial v^a}{\partial y} + \frac{D_p}{\Delta t} \]  \hspace{1cm} (3a)

where the continuity equation (2) is so reflected in deriving eq.(3a) that eq.(2) should be satisfied at the time step \( t = t_{n+1} \) through time marching.

2.2 Transformation of coordinate system

When the configuration of the boundaries in contact with fluid is not simple or the boundaries are in motion in fluid, it is generally less complicated to solve the governing equations in the generalized coordinate system (Thompson, J.F., et al. 1982).

Suppose \( (x, y, t) \) be the Cartesian coordinates and the time in the physical domain where the boundary configuration and the boundary values are specified and \( (\xi, \eta, t) \) be the generalized coordinates and the time in the computational domain, then the variables and the differential operators in the both domains are related such that

\[ \xi = \xi (x, y, t) \]  \hspace{1cm} (5a)

\[ \eta = \eta (x, y, t) \]  \hspace{1cm} (5b)

\[ r = r (x, y, t) \]  \hspace{1cm} (5c)
and

\begin{align}
\frac{\partial}{\partial x} & \begin{pmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial \tau} \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial \tau} \end{pmatrix} \\
\frac{\partial}{\partial y} & = \begin{pmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial \tau} \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial \tau} \end{pmatrix} \\
\frac{\partial}{\partial \tau} & = \begin{pmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial \tau} \end{pmatrix} \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial \tau} \end{pmatrix} \\
\end{align}

(6)

Since time variable \( t \) in the physical domain and \( \tau \) in the computational domain are treated identical and \( t \) is independent of \( \xi \) and \( \eta \), the following relations should be held:

\begin{align}
\frac{\partial \xi}{\partial t} & = 1 \\
\frac{\partial \eta}{\partial \tau} & = \frac{\partial \xi}{\partial \eta} = 0
\end{align}

(7a, 7b)

It then follows that the elements of the matrix associated with transformation (4) may be expressed in terms of those associated with the inverse of the transformation as follows:

\begin{align}
\begin{pmatrix} \xi_x & \eta_x & \tau_x \\
\xi_y & \eta_y & \tau_y \\
\xi_t & \eta_t & \tau_t \\
\end{pmatrix} & = \begin{pmatrix} \gamma & \eta & 0 \\
-x & x & 0 \\
x & -y & 1/J
\end{pmatrix}
\end{align}

(8)

where \( J \) is the Jacobian of the inverse transformation.

Equations (1) and (3) expressed in the physical domain are transformed into the computational domain based on relation (8) and are solved using the SOR scheme.

Here, it should be noted that the left-hand side of eq.(1) in the two-dimensional physical domain is transformed into the computational domain such that

\begin{align}
\frac{\partial \xi}{\partial \tau} + \frac{u}{\partial x} + \frac{v}{\partial y} & = u x + u \{ (u-x) \xi_x + (v-y) \xi_y \} + u v \{ (u-x) \eta_x + (v-y) \eta_y \}
\end{align}

(9a)

and

\begin{align}
\frac{\partial \eta}{\partial \tau} + \frac{u}{\partial x} + \frac{v}{\partial y} & = v y + v \{ (u-x) \xi_x + (v-y) \xi_y \} + v v \{ (u-x) \eta_x + (v-y) \eta_y \}
\end{align}

(9b)

where the convective velocity components in the \( \xi \) and \( \eta \) directions consist of the relative fluid velocity components with respect to the velocity of the grid points in the physical domain. Hence, the moving boundaries in the physical domain turn out to be stationary in the computational domain and with the non-slip condition for the flow velocity components, the second and the third terms on the right sides of equations (9a) and (9b) vanish on the boundaries.

3 BOUNDARY CONDITION

On the boundaries, the pressure is determined so as to satisfy the following boundary conditions:
\[ \frac{\partial p_B}{\partial z} = \frac{1}{Re} \nabla^2 u_B \quad (10a) \]
\[ \frac{\partial p_B}{\partial y} = \frac{1}{Re} \nabla^2 v_B \quad (10b) \]

For the velocity field, the non-slip condition is applied and for the remote boundaries, the undistributed flow condition is employed.

4 HYDRODYNAMICAL FORCE ACTING ON BOUNDARIES

With the stress tensor \( \Pi \), the hydrodynamical force acting on the segment \( ds \) of boundary \( S_0 \) is
\[ F = \int_{S_0} \Pi \cdot \vec{n} \, ds \quad (11) \]
where \( \vec{n} \) is the outward normal vector on the boundary. As illustrated in Fig. 1, \( \vec{n} \) is expressed with its \( x \) and \( y \) components \( (n_x, n_y) \) such that
\[ \vec{n} = \frac{dy}{ds} \hat{\imath} - \frac{dx}{ds} \hat{j} = n_x \hat{\imath} + n_y \hat{j} \quad (12) \]

Fig. 1 Correlation of \( ds \) and \( \vec{n} \)

With relation (12), the hydrodynamical force \( F \) given by expression (11) may be written as follows:
\[ F_x = -\Phi_{B} \beta dy + \frac{1}{Re} \Phi_{B} \left( \sigma_{xx} dy + \sigma_{xy} (-dx) \right) \quad (13a) \]
\[ F_y = -\Phi_{B} \beta (-dx) + \frac{1}{Re} \Phi_{B} \left( \sigma_{yy} (-dx) + \sigma_{xy} dy \right) \quad (13b) \]
where \( \sigma_{xx}, \sigma_{yy} \) and \( \sigma_{xy} \) are the normal stresses in the directions of \( x \) and \( y \) axes and the tangential stress parallel to \( x \) plane and in \( y \) direction, respectively.

Fig. 2 Schematic presentation of the plate component in the computational domain

Fig. 2 shows schematically the cross section, in the computational domain, of the plate component and the wall which are sufficiently long in the direction perpendicular to the cross-sectional plate so as to adopt the two-dimensional treatment. Since \( v_x = u_t = 0 \) and \( dT = 0 \) on boundaries \( Q_1Q_2 \) and \( Q_3Q_4 \) and \( v_x = u_t = 0 \) and \( dT = 0 \) on boundaries \( Q_2Q_3 \) and \( Q_4Q_1 \) respectively, \( F_x \) and \( F_y \) given by expressions (13a) and (13b) may be expressed for those
boundaries in the computational domain as follows:

\[
\begin{align*}
F_x &= -\delta_B p\eta_2 d\xi + \frac{1}{Re} \delta_B (\eta_2 v_\eta - \eta_3 u_\eta) x_\xi d\xi \\
F_y &= \delta_B p x_\xi d\eta + \frac{1}{Re} \delta_B (\eta_3 v_\eta - \eta_3 u_\eta) y_\eta d\xi \\
& \quad \left\{ \begin{array}{c}
B = Q_1 \rightarrow Q_2 \\
Q_3 \rightarrow Q_4
\end{array} \right. \\
\end{align*}
\] (14a)

and

\[
\begin{align*}
F_x &= -\delta_B p x_\eta d\eta + \frac{1}{Re} \delta_B (\xi_2 u_\xi - \xi_3 u_\eta) x_\eta d\eta \\
F_y &= \delta_B p x_\eta d\eta + \frac{1}{Re} \delta_B (\xi_3 u_\xi - \xi_3 u_\eta) y_\eta d\eta \\
& \quad \left\{ \begin{array}{c}
B = Q_2 \rightarrow Q_3 \\
Q_4 \rightarrow Q_1
\end{array} \right. \\
\end{align*}
\] (14c)

5 COMPUTATIONAL RESULTS

The velocity vector field between the plate and the wall in the vicinity of the left edge of the plate component is shown in Fig. 3.

Lift coefficients \( C_L \)'s corresponding to the motion of the plate towards or away from the wall with constant velocity \( V_G \)'s of several different magnitudes are shown in Figures 4 and 5 as a function of the gap distance between the plate and the wall, where the fluid velocity on the left side boundary of the physical domain is 4 m/sec flowing into the domain across the boundary with \( Re = 1.23 \times 10^6 \) and the dimension of the cross-section of plate component taken into the computation is 0.43mm X 43.0mm. In Fig. 4, computational cases 1, 2, 3 and 4 correspond to \( V_G = -0.05, -0.10, -0.15 \) and -0.20 m/sec, while in Fig. 5, cases 5, 6, 7 and 8 correspond to \( V_G = 0.05, 0.10, 0.15 \) and 0.20 m/sec, respectively. Negative velocities correspond to the motion of the plate towards the wall.

![Fig. 3 Velocity vector field around the left end of the plate component corresponding to Case 2 with gap distance of 0.12mm](image-url)
ACKNOWLEDGMENT

The authors express their greatest gratitude to Mr. Masahito Sato for his participation in programming the grid system generator and his computational assistance.

REFERENCES


