One Dimensional vs. Analysis into Normal Modes Solutions to Flow Induced Vibrations in a Square Cylinder Array

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1 INTRODUCTION

There are many engineering applications dealing with fluid cross flow over tubes or bundles of tubes. Examples are steam generators in nuclear power plants and various forms of heat exchangers. In standardized engineering designs it is usually assumed that tubes act as rigid bodies when subjected to the fluid flow. Fact of the matter is that during solid-fluid interaction feedback effects become important. One of the mannifestations of this effect is in the form of flow induced vibrations.

It is generally accepted (Blevins, 1990) that cross flow induced vibrations are subject to three general forms of excitation mechanisms, depending on ratio of streaming and ordinary Reynolds numbers (denoted also as nondimensional velocity).

- In the lowest range, the source of vibrations is random noise. This vibrations are not important as far as structural integrity of the tubes is concerned, they result in small scale amplitudes at random fluctuating frequencies.

- In the middle range, vortices start to form behind the tubes. At a particular value of the Reynolds number ratio the vortices start to separate from the tubes resulting in a phenomena known as vortex shedding. If the frequency of vortex shedding coincides with the natural frequency of the tubes (Strouhal number close to 1), a resonant instability may occur. This mechanism was considered as particulary dangerous until the late 1970's when the importance of a third excitation mechanism became apparent.

- High values of the nondimensional velocity result in rapidly increasing amplitudes. The reason for this instability is a strong feedback mechanism between the fluid and the structure, denoted as fluidelastic instabilities. It is now the general view that fluidelastic instabilities will result in the most damaging vibrations.

This work will address fluidelastic instabilites resulting from single phase cross flow over cylindrical structures. Single phase cross flow occurs at the bottom of steam generator, where the fluid is introduced into tube bundles. The main direction of the flow can be assumed as perpendicular to the bundle, thus justifying the term "cross flow".

2 STABILITY CRITERION

First question one must answer is: when does the solid-fluid structure become unstable? There are different possibilities; 1) the fluid is stable and the structure unstable, 2) the solid is stable and the fluid unstable, 3) both are unstable or 4) some combination in which both participants are in a stable mode yet the whole structure is in an unstable mode.

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Let us examine this. The single phase flow field can be assumed to be incompressible as pressure variations are too small to cause significant density variations. Unstable fluid would therefore vibrate with uniform frequency and impose that frequency on otherwise stable solid parts of the structure. Chen (1987) describes that frequency spectrum of the solid vibrations vary considerably thus rejecting the possibility of an unstable fluid mode. On the other hand, an unstable solid would most likely vibrate at one of the modes of its natural thus precluding the occurrence of threshold of instability, associated with any particular nondimensional velocity (vortex shedding is an example of stable fluid/ unstable solid mode of vibrations).

The only remaining possibility is a combination of otherwise stable modes of both fluid and solid parts of the unstable structure. How is that possible? One can imagine the tubes in the fluid as connected to the walls by springs and dampers. In addition to structural damping there is also a strong damping caused by the surrounding fluid. These two effects compete with each other. Moreover, the fluid damping depends on the ratio of streaming and ordinary Reynolds number. If the flow conditions result in equal but opposite damping of both, fluid and solid, then

- growing amplitude of the structural motion is possible as there is no net damping effect.

- there will be a threshold of the nondimensional velocity above which all the velocities result in unstable motion of the structure (instability and overstability of the structure).

The problem statement could be formulated as follows: given the ratio of ordinary and streaming Reynolds numbers and the geometry of the structure, find such combinations of frequencies and amplitudes of the solid parts of the structure that the damping of a given tube is equal to the (velocity flow field dependant) fluid damping. This will yield a critical velocity where increase have a profound effect on the amplitude of structural motion. Both the fluid damping and structural damping are cast in terms of damping parameters. Also, one has to establish a criterion to recognize what the "fluid damping parameter" entails. In order to answer this question we need to utilize some assumptions about the nature of the phenomena.

3 ASSUMPTIONS

The flow field is assumed to be solenoidal (incompressible). This is a good assumption as long as we are dealing with liquid water (as in lower part of steam generator).The velocity and pressure fields can be decomposed into mean and perturbed quantities. Mean quantities represent the zero'th order solution to the mean flow equations (i.e. the foreign objects in the flow are assumed to be rigid bodies) around which the perturbation equations can be constructed. The analysis performed will be a linearized perturbation analysis. This means that all terms of the order of the perturbation squared are considered as small. The mean velocity of the flow will be assumed to be constant with respect to space and time. This assumption was first made by Lever and Weaver (1984) and was argued in the referenced article. The effect of the assumption is to visualize the mean velocity flow field as a flexible streamtube meandering through the cylinders. The mean flow field is thus not controlled by visocus forces and forces on the cylinders are the result of bending the streamtubes. The secondary flow (perturbed quantities) is due solely to the cylinder motion and viscous forces cannot be neglected. These disturbances cause the fluidelastic instabilities. The flow can be assumed to be quasistatic. This assumption quantifies the fluid flow as an orderly rather than chaotic motion with repsect to time and enables one to prescribe a harmonic character to the motion. The fluid damping is part of the force exerted on the tube which is associated with order of ratio between frequency of the motion and natural frequency of the solid part of structure to the first power. Volume forces are neglected.

4 SIMPLIFICATIONS

The assumptions outlined above lead to a linear stability problem. Further, periodicity of the geometry of the rod bundles suggests that the secondary flow field posess periodicity in one of the spatial directions. This leads to the conclusion that two types of simplified analysis should be possible:

- Assuming the motion of the cylinders to be a harmonic function of time, determined by linear combination of sine and cosine functions, would suggest the use of a flexible streamtube approach as well as one dimensional spatial dependance of the secondary flow. One of the most useful properties of the streamtube is the preservation of the mass flux within the its limits. As the motion of the streamtube is already prescribed, the width of streamtube is small compared to its length and the flow field is assumed to be solenoidal. This suggests that there are no additional secondary movements of fluid lumps in directions perpendicular to the flow direction and as such eliminating the need for a second dimension.

- Using a standard Eulerian description of the flowfield the values for perturbed quantities could be analyzed into normal modes. This gives an additional dimension to the possible solutions, namely by not prescribing the motion of the cylinders one can analyze an impact of such motion on occurrence of the threshold of instability.

5 ONE DIMENSIONAL INTEGRAL APPROACH

The notion of an unsteady control volume is customarily not utilized when describing the flowfield. The idea for its use for calculation of secondary flows was first published by Lever and Weaver, 1984. The derivation of the governing equations is straightforward (Marn and Catton, 1990):

- continuity

$$\frac{\partial}{\partial t}[\rho V(t)]+\nabla\cdot[\rho V(t)\vec{u}]=0 \qquad (1)$$

- momentum

$$\rho\frac{\partial}{\partial t}[\vec{u}V(t)]+\rho\nabla[|\vec{u}|\vec{u}V(t)]=\sum\vec{f} \qquad (2)$$

Notice that V(t) stands for the variable volume of the streamtube as a function of time. For multiple tubes, each of which can move in somewhat arbitrary way, the volume will be a function of location as well.

In order to make the equations dimensionless, time is scaled with the most dangerous frequency to underline the quasi-stationary character of the instabilities, whereas the velocity and pressure are scaled with natural frequency. The flowfield and forcing functions have to relate to the properties of the geometry and incoming flow leading to the use of natural frequency and reduced velocity as scaling parameters for velocity and pressure. Using the assumption that the flowfield is basically one dimensional, the variable volume can be translated into a variable area assuming a unit depth. The area is further decomposed into a mean and a perturbed part, similarly to the pressure and velocity fields. Using the above assumptions while integrating the momentum equation along a streamtube and using the integrated form of the continuity equation, one arrives at the following expression for pressure anywhere along the cylinder (Marn and Catton, 1990). The forces on cylinder are evaluated by integrating the pressure around the cylinder and decomposing the resultant force into x and y components (subscript T stands for transversal, or y-, component). Using these in the equations for cylinder motion (Yetisir and Weaver, 1988), one finds the following expression for the threshold of instability,

$$C_L=i\left[\frac{\delta}{\pi}\frac{m}{\rho d^2}-4\sin\alpha\frac{U_r}{A_0^*}\int_{s_a^*}^{s_b^*}\sin\left(\frac{2s^*}{P/d}\right)\int_{s_i^*}^{s^*}a^*(\xi)\,d\xi\,ds^*\right] \qquad (3)$$

The structure becomes unstable when value for C_L changes the signs its value 0 is assumed to be a criterion for the threshold of instability occurrence. In addition to evaluation of the threshold of dynamic instability this approach gives us some answers to quality of our assumptions.

5.1 Quasi stationary approach and unimportance of logarithmic decrement

If a quasi stationary approach is justified, then varying the ratio of the most dangerous versus natural frequency as a function of the ratio of the streaming versus ordinary Reynolds numbers should not change significantly when calculated as a function of time. Figure 1 (Marn and Catton, 1990) shows this assumption, in fact, justified. In addition, experimental results (Blevins, 1990) show that the logarithmic decrement does not influence the threshold of instability unless very large. Our calculations agree very well with this (see Figure 2).

6 ANALYSIS INTO NORMAL MODES

This analysis assumes that the equations can be transformed into ordinary differential equations by assuming a form the functions that will represent the secondary flow velocity and pressure. A standard Eulerian approach is used to describe the flowfield. The advantage of this approach is that one doesn't have to prescribe the way the streamtube will change its volume. Rather one has to limit oneself to prescribing the ratios of frequencies and amplitudes of neighboring cylinders.

As an first approximation, we had examined a row of flexible oscillating cylinders, assembled in rotated square array with rigid cylinders. The setup is shown in Figure 3. It is evident that due to the periodic geometrical arrangement, the natural numerical approach is to break the flowfield into small cells and try to solve the perturbation equation within each cell. The governing equations, after utilizing the standard linearized perturbation assumptions, using the same scaling procedure as before and assuming the normal modes of perturbed variables, are

$$\dot{U}+ikV=0 \tag{4}$$

$$\left(\frac{\omega}{\omega_n}\right)U+\frac{Re_s}{Re}\dot{U}=-\dot{P}+\frac{Re_s}{Re^2}(\ddot{U}-k^2U) \tag{5}$$

$$\left(\frac{\omega}{\omega_n}\right)V+\frac{Re_s}{Re}\dot{V}=-ikP+\frac{Re_s}{Re^2}(\ddot{U}-k^2U) \tag{6}$$

$$\left(\frac{\omega}{\omega_n}\right)^2\frac{m}{\rho d^2}X+\left(\frac{\omega}{\omega_n}\right)\frac{m}{\rho d^2}\frac{\delta}{\pi}X+\frac{m}{\rho d^2}X=\Sigma F_x \tag{7}$$

$$\left(\frac{\omega}{\omega_n}\right)^2\frac{m}{\rho d^2}X+\left(\frac{\omega}{\omega_n}\right)\frac{m}{\rho d^2}\frac{\delta}{\pi}X+\frac{m}{\rho d^2}X=\Sigma F_x \tag{8}$$

The boundary conditions need to be specified at least at one point in the flowfield (Schlichting, 1979) and the most natural point seems to be on the cylinder. The boundary conditions require that the fluid close to the cylinder interface move with the same velocity and frequency as a cylinder and that velocity between the cells remains continuous.

The stability analysis is carried out by solving for the pressure field utilizing the given equations, integrating around the cylinder, decomposing into streamwise and perpendicular components of the force and calculating the most dangerous flow damping parameter.

7 COMPARISON OF RESULTS

Figure 4 depicts the comparison of results for one dimensional integral approach and for analysis into normal modes. It is evident that the results for a one dimensional integral approach hold only for small values of the ratio of streaming and ordinary Reynolds numbers and mass damping parameter. The results based on analysis into normal modes match those obtained through experiments for higher values of both parameters. The range of applicability of the one dimensional integral approach is between values 1 and 10 for nondimensional velocity whereas analysis into normal modes gives the best results between values 10 and 100. Why is this so?

The answer can be found if different approaches are examined from physical point of view. One dimensional integral approach was developed for a single cylinder, vibrating in an array of rigid cylinders. This setup would represent a combination of a solid part of the structure with small mass damping. In square arrays the effect of perturbation dies out rapidly and there is no coupling mechanism present although the structure is experiencing an instability due to the fluidelastic forces. Analysis into normal modes on the other hand treats the coupling mechanism, which manifests itself in different values of frequency and amplitude ratios.

8 FUTURE WORK

Most structures experiencing fluidelastic instabilities fail due to fatigue and repeated clashes, which suggests that coupling mechanism is more important than single mode fluidelastic instability. In the future we would have to use a more general approach to analysis into normal modes. As a first approximation we will try to relax the predefined spatial forms of the perturbation velocity and limit ourselves to timewise normal modes of the perturbed variables. Also, we would have to formulate the problem statement differently, we will have to analyze the optimum combination of different frequencies and amplitudes to yield the criterion for threshold of instability.

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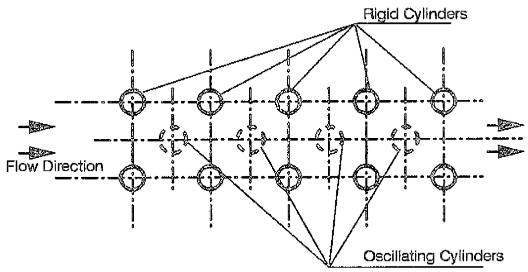

Figure 3: Geometrical Arrangement.

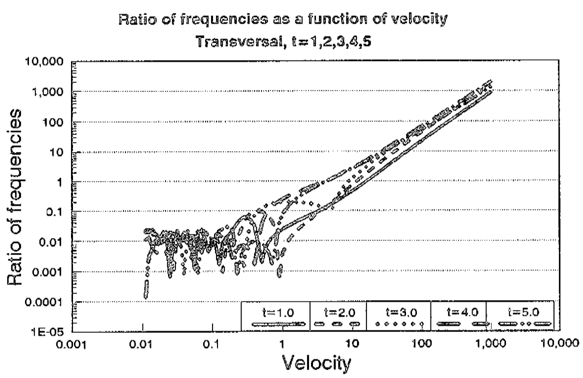

Figure 1: Time Dependance of Frequency Ratio.

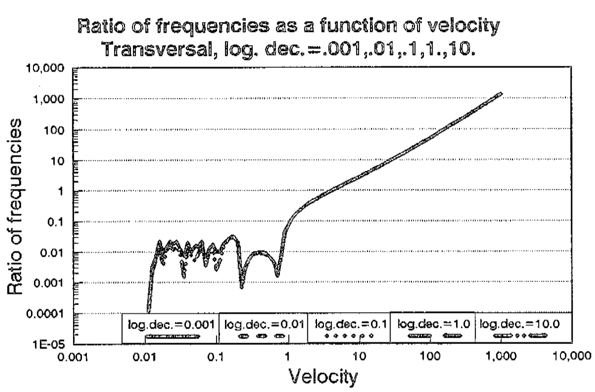

Figure 2: Log. Decrement Dependance of Frequency Ratio.

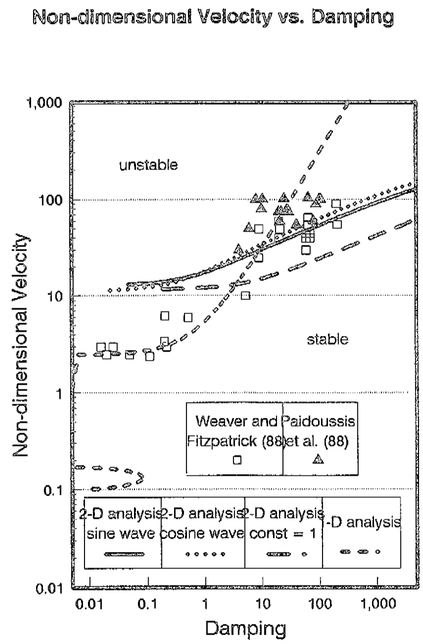

Figure 4: Comparison 1-D vs. 2-D Results.