A Practical Model for Base-Isolated Buildings

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ABSTRACT

A mathematical model for the earthquake response of base isolated buildings is developed as a function of the superstructure properties when this is assumed fixed to the ground, and of the same superstructure assumed to behave as a rigid body with a single degree of freedom when mounted on base isolators.

The resulting expressions furnish the period and damping of the isolated building, the magnitude of the seismic forces acting upon it and its base displacement, in a simple manner that could readily be incorporated into codes of practice which are appropriate for these kind of structures.

THEORETICAL DEVELOPMENT

Figure 1.a shows a base isolated building assimilated to the conceptual model shown in figure 1.b, wherein the superstructure is characterized by its effective mass M_s, its generalized stiffness K_s, and its generalized damping C_s. It is assumed that the superstructure would respond in its first mode when fixed to the ground, and that the above values relate to this mode.

The base isolators are considered to be axially inextensible. Therefore, the base will be characterized by its mass M_b, the total horizontal stiffness of the isolators K_b, and the total damping provided by the isolation system C_b.

Despite its simplicity, this conceptual model has also been used to study systems with soil-structure interaction, with results that have found their way into earthquake regulations [1].

The equation of motion for the model depicted in figure 1.b when excited by a horizontal earthquake with acceleration $a(t)$ is:

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = -M\{1\}a(t) \quad (1)$$

where $x(t)$ is the vector of relative displacements. The base and superstructure components are x_b and x_s respectively.

$\mathbb{M}$, $\mathbb{C}$, and $\mathbb{K}$ are the mass, damping and stiffness matrices respectively. For classically, low damped systems the frequencies and mode shapes would be obtained by solving the well-known eigen-problem:

$$[\mathbb{K} - \omega^2 \mathbb{M}]\, \mathbb{X} = \mathbb{0} \tag{2}$$

where ω is the circular frequency of the system and $\mathbb{X}$ is the corresponding mode shape. Since, for all practical purposes, the first mode is of main interest, we will use Rayleigh's principle to solve equation (2) and hence to obtain the fundamental frequency. Further, to obtain results with the desired format, let us define the following variables:

$$\omega_i^2 = \frac{\mathbb{K}_b}{\mathbb{M}_b + \mathbb{M}_s} \tag{3.a}$$

$$\omega_f^2 = \frac{\mathbb{K}_s}{\mathbb{M}_s} \tag{3.b}$$

$$\Omega = \frac{\omega_i}{\omega_f} \tag{3.c}$$

$$\mu = \frac{\mathbb{M}_b}{\mathbb{M}_s} \tag{3.d}$$

Equation (3.a) provides the circular frequency for a system in which the superstructure is perfectly rigid, mounted on isolators and behaving as a single degree of freedom system, as schematized by figure 2b. Equation (3.b) provides the circular frequency for the superstructure in its fixed base state as schematized by figure 2a. Equation (3.c) represents a frequential ratio, usually small as isolated buildings will typically have high values of fixed base frequency in relation to their isolation frequency. Equation (3.d) is simply a mass ratio between the base and the superstructure.

From Rayleigh's principle, using as trial shape the deflected shape under lateral forces equal to the story weights, we get the following approximation:

$$\omega^2 = \omega_i^2 \frac{\mu + (1 + \Omega^2)}{\mu + (1 + \Omega^2)^2} \quad (4)$$

Equation (4) indicates that the frequency of the base isolated system is very close to that of the single degree of freedom system constituted by a perfectly rigid superstructure mounted on base isolators. To find the corresponding mode shape, equation (2) is used, and the first mode is normalized such that the amplitude at the base level is one. From equation (2), and using the expression of equation (4), a quadratic equation is obtained, that allows to write the mode shape as:

$$x = [\ 1 \qquad 1 + \Omega^2\ ]^T \quad (5)$$

Equation (5) indicates that the isolated system tends to behave globally as a single degree of freedom system undergoing a uniform translational motion with reduced relative displacement between stories.

Next, the modal participation factor is calculated. This is written:

$$\gamma = \frac{x^T M\{1\}}{x^T Mx}$$

which after replacement of the appropriate values yields:

$$\gamma = \frac{\mu + (1 + \Omega^2)}{\mu + (1 + \Omega^2)^2} \quad (6)$$

Equation (6) shows that in the case of base isolated buildings, the participation factor for the first mode tends to be one, which would indicate, once more, that a base isolated structure behaves more like a single degree of freedom system.

Another step in this development is to determine the overall damping ξ for the fundamental mode of a base isolated building. Using the expression for generalized damping and the terms derived above it can be shown that the damping for the isolated building is given by:

$$\xi = \xi_b + \xi_s \frac{\Omega^3}{1 + \mu} \quad (7)$$

Where ξ_b and ξ_s are the isolation system damping, and the superstructure damping respectively. Equation (7) indicates that the overall damping is provided mostly by the base isolation system, as the cube of the frequential ratio is very small. Physically this would correspond to a rigid building suffering almost no distortions but mounted on isolators that provide the bulk of the deformation of the system. It therefore, indicates that adding energy dissipators to the superstructure of an isolated building is redundant.

Basically the entire damping is provided by the isolators. As said above, mathematical models, similar to the one that has been considered here, have been used to study systems with soil-structure interaction. Not surprisingly, equation (7) when particularized by making μ equal to zero, results in the same equation proposed by reference (1).

SEISMIC FORCES AND BASE DISPLACEMENTS

It is simple to demonstrate that the effective mass for the superstructure of a properly base isolated building, is approximately equal to its total actual mass. Then, for a spectral acceleration A, the inertia force applied to the base of the building F_b is:

$$F_b = \frac{\mu + (1 + \Omega^2)}{\mu + (1 + \Omega^2)^2} * A * M_b \qquad (8.a)$$

The inertia force F_s applied to the superstructure is:

$$F_s = \frac{\mu(1 + \Omega^2) + (1 + \Omega^2)^2}{\mu + (1 + \Omega^2)^2} * A * M_s \qquad (8.b)$$

And the shear at ground level, V, is simply:

$$V = F_b + F_s \qquad (8.c)$$

The inertia forces acting on the superstructure may still be decomposed in two parts, so equation (8.b) may be written:

$$F_s = \frac{\mu + (1 + \Omega^2)}{\mu + (1 + \Omega^2)^2} * A * M_s + \Omega^2 \frac{\mu + (1 + \Omega^2)}{\mu + (1 + \Omega^2)^2} * A * M_s$$

The first term of the last equation represents a "rigid" component, and the second would represent an assumed "triangular" component. If called F_R and F_T respectively, it is possible to write:

$$F_S = F_R + F_T \qquad (8.d)$$

The distribution along the height of the building is made in proportion to the masses for the "rigid" part, and in proportion to the familiar mass-height product for the "triangular" part.

Therefore, for mass M_i at height H_i above the base, it may be written:

$$F_R = F_R \frac{M_i}{M_s} + F_T \frac{M_i \; H_i}{\sum_{i=1}^{N} M_i \; H_i} \qquad (8.e)$$

Finally, the displacement across the isolation interface may be written:

$$X_b = \frac{V}{K_b} \qquad (8.f)$$

This last equation provides a link between the procedures developed in this paper, which attempt to explicitly calculate the seismic forces as heretofore has been the practice, and the procedures proposed by the SEAONC [2] in which base displacements are prescribed.

CONCLUSIONS

The study presented in this paper provides simple and readily usable expressions for the period, mode shape, participation factor and damping for a base-isolated building. It also furnishes equations to assess the seismic forces acting on the building as well as its relative base displacement. The expressions are presented in a simple manner and can be used, with modest effort, without and in-depth knowledge requirement of structural dynamics.

The procedure shown in this paper, has been successfully used in the design of several structures using the GAPEC Base Isolation System, [3].

BIBLIOGRAPHY

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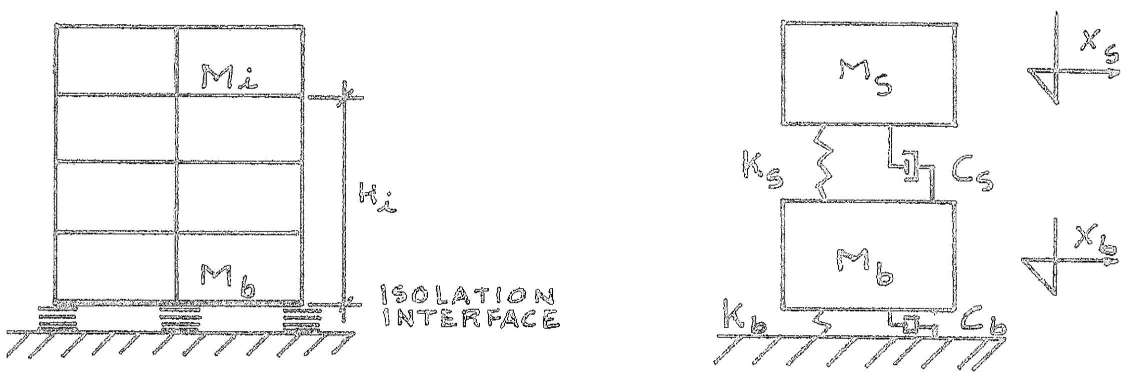
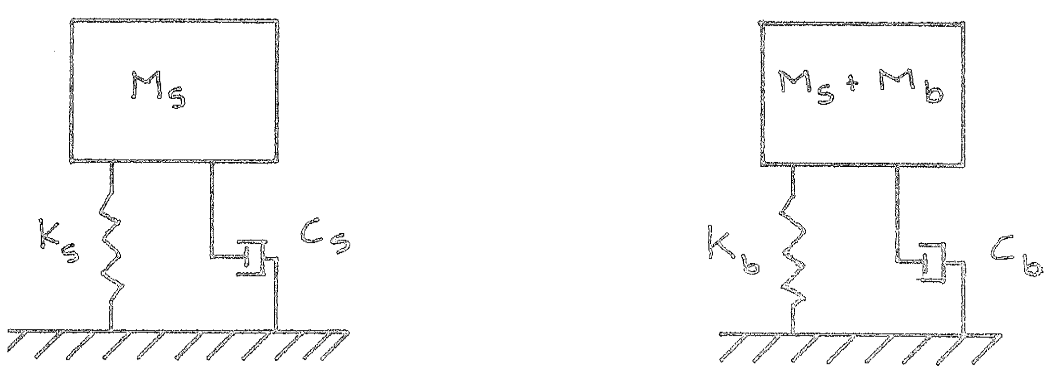
[3] Earthquake Safety, A New Approach. The GAPEC Base Isolation System and Its Applications. New Dynamics/Edudes et Recherches Appliquees, 1991.



(1.a) (1.b)

FIGURE 1



(2.a) (2.b)

FIGURE 2