Analysis of Random Response of Structures with Uncertain Parameters (Combinatiaon of Hierarchy Method and Substructure Synthesis Method)

Takuzo IWATSUBO, Shuzo KAWAMURA, Hiroyuki MORI
Kobe University, Kobe, Japan

1 INTRODUCTION

In the structures, mass, stiffness and damping coefficients of the structures have deviations in the design process or manufacturing process. And some structures, such as the piping system, are also exposed to the danger of earthquake excitation.

The authers have already reported about the reliability design of the structures against earthquake excitation [(1)-(3)]. In that papers, the dynamic responses of the structure with uncertain parameters against earthquake excitation are first obtained by using hierarchy method, which is first applied to the reliability problem by one of the authors [(1)]. Next, in the case of regarding earthquake excitation as a nonstationary narrow band random process, the dynamic responses against earthquake excitation are presumed from the dynamic responses against stationary white noise. As a numerical example, the simple piping system is considered. It is concluded that hierarchy method is effective for the response analysis and the statistical properties of earthquake have more influence on the reliability than those of the characteristic values in the structure.

The structures considered previously are so simple that the degrees of freedom are relatively small. Actual structures, however, are more complex and have a large degrees of freedom, so that hierarchy method becomes not to be effective since it takes much time for calculation and much memory space of computer. In this paper, the substructure synthesis method [(4)] is applied to calculate the random responses of structures. It is required that the impulsive response function is obtained accurately, since the mean square values of responses, that are necessary for the reliability analysis, are obtained from the auto-covariance function of the responses by using the impulsive response function. As a numerical example, it is applied to the piping system. First the accuracy of calculation is investigated. Next the random responses against earthquake and the auto-covariance functions are calculated.

2 EQUATIONS OF MOTION AND RESPONSES OF STRUCTURE WITH UNCERTAINTY

The method for obtaining the response and its auto-covariance function of the structure with uncertain paremeters against the random force is explained. The equations of motion of a structure with n degrees of freedom system are

$$[M]\{\ddot{v}(t)\} + [C]\{\dot{v}(t)\} + [K]\{v(t)\} = \{f(t)\} \tag{1}$$

where $[M]$, $[C]$ and $[K]$ are mass, damping, and stiffness matrices which have uncertain paremeters and are assumed to be symmetric, respectively. $\{v(t)\}$ is the response vector, $\{f(t)\}$ is the external random force vector. If the system is deterministic, the response $\{v(t)\}$ can be obtained in integral form by using the impulsive response function matrix $[H(t)]$ as follows;

$$\{v(t)\} = \int_0^t [H(t-\tau)]\{f(\tau)\}d\tau \tag{2}$$

where $H(t)$ is derived in the next section.

When the mean value of the external force is zero, the mean value of the response becomes zero. The auto-covariance function of the responses, $[R_v(t_1, t_2)]$, is described as follows;

$$[R_v(t_1,t_2)] = \int_0^{t_2}\int_0^{t_1} [H(t_1-\tau_1)][R_f(\tau_1,\tau_2)][H(t_2-\tau_2)]^T d\tau_1 d\tau_2 \tag{3}$$

where $[R_f(t_1, t_2])$ is the auto-covariance function matrix of $\{f(t)\}$.

If the external ramdom force $\{f(t)\}$ is regarded as a seismic force which is assumed to be identical at any excitating points, $\{f(t)\}$ is expressed as follows;

$$\{f(t)\} = [M]\{L\}g(t) \tag{4}$$

where $\{L\}$ is the vector expressed the location where the seismic wave is applied, and $g(t)$ is the acceleration of the seismic wave.

Using Eq.(4), the response $\{v(t)\}$ shown in Eq.(2) is expressed as follows;

$$\{v(t)\} = \int_0^t \{h(t-\tau)\}g(\tau)d\tau \tag{5}$$

where $\{h(t)\}$ is the impulsive response function vector expressed as follows:

$$\{h(t)\} = [H(t)][M]\{L\} \tag{6}$$

Similarly Eq.(3) is rewritten as

$$[R_v(t_1,t_2)] = \int_0^{t_2}\int_0^{t_1} \{h(t_1-\tau_1)\}\{h(t_2-\tau_2)\}^T R_g(\tau_1,\tau_2)d\tau_1 d\tau_2 \tag{7}$$

where $R_g(t_1,t_2)$ is the auto-covariance function of $g(t)$. The mean square value of response can be obtained by setting $t_1 = t_2 = t$ in Eq.(3) or (7). Therefore it is required that the impulsive response function matrix $[H(t)]$ (or impulsive response function vector $\{h(t)\}$) considered the uncertainty is obtained accurately.

3 IMPULSIVE RESPONSE CONSIDERED UNCERTAINTY BY USING HIERARCHY METHOD

The method of obtaining the mean value of the impulsive response function considered the uncertainty by using hierarchy method is explained. Hence the structure has uncertain parameters, but the external force $\{f\}$ is the deterministic impulsive force. Here it is assumed that the uncertainties of the mass m_{ij} included in $[M]$, the damping c_{ij} included in $[C]$ and the stiffness k_{ij} included in $[K]$ are considered. First taking the ensemble averages of Eq.(1), the following equation is obtained :

$$\overline{[M]\{\ddot{v}^{(e)}_{imp.}\}} + \overline{[C]\{\dot{v}^{(e)}_{imp.}\}} + \overline{[K]\{v^{(e)}_{imp.}\}} = \{f^{(e)}_{imp.}\} \tag{8}$$

where $\{f^{(e)}_{imp.}\}$ is the impulsive force at the e-th nodal point($e = 1,\cdots,n$) and $\{v^{(e)}_{imp.}\}$ is the impulsive response against $\{f^{(e)}_{imp.}\}$. The $\overline{(\circ)}$ denotes a statistical average of $(\circ)$.

Further, multiplying Eq.(1) by m_{ij}, c_{ij}, k_{ij}, respectively, and taking the ensemble averages, the following equation is obtained:

$$\begin{aligned}
\overline{m_{ij}[M]\{\ddot{v}^{(e)}_{imp.}\}} + \overline{m_{ij}[C]\{\dot{v}^{(e)}_{imp.}\}} + \overline{m_{ij}[K]\{v^{(e)}_{imp.}\}} &= \overline{m_{ij}}\{f^{(e)}_{imp.}\} \\
\overline{c_{ij}[M]\{\ddot{v}^{(e)}_{imp.}\}} + \overline{c_{ij}[C]\{\dot{v}^{(e)}_{imp.}\}} + \overline{c_{ij}[K]\{v^{(e)}_{imp.}\}} &= \overline{c_{ij}}\{f^{(e)}_{imp.}\} \\
\overline{k_{ij}[M]\{\ddot{v}^{(e)}_{imp.}\}} + \overline{k_{ij}[C]\{\dot{v}^{(e)}_{imp.}\}} + \overline{k_{ij}[K]\{v^{(e)}_{imp.}\}} &= \overline{k_{ij}}\{f^{(e)}_{imp.}\}
\end{aligned} \tag{9}$$

Since the simultaneous equations (8) and (9) have unknown variables $\overline{\{v^{(e)}_{imp.}\}}$, $\overline{m_{ij}\{v^{(e)}_{imp.}\}}$, $\overline{c_{ij}\{v^{(e)}_{imp.}\}}$, $\overline{k_{ij}\{v^{(e)}_{imp.}\}}$, $\overline{c_{ij}k_{kl}\{v^{(e)}_{imp.}\}}$, $\overline{k_{ij}m_{kl}\{v^{(e)}_{imp.}\}}$, $\overline{m_{ij}c_{kl}\{v^{(e)}_{imp.}\}}$, $\overline{m_{ij}m_{kl}\{v^{(e)}_{imp.}\}}$, $\overline{c_{ij}c_{kl}\{v^{(e)}_{imp.}\}}$ and $\overline{k_{ij}k_{kl}\{v^{(e)}_{imp.}\}}$ $(i,j,k,l = 1,\cdots,n)$, it is impossible to solve the equations. Here the following approximate equation, which is accurate within the second order of small deviations, is used:

$$\begin{aligned}
\overline{\alpha\beta\{v^{(e)}_{imp.}\}} &\simeq \bar{\alpha}\overline{\beta\{v^{(e)}_{imp.}\}} + \bar{\beta}\overline{\alpha\{v^{(e)}_{imp.}\}} - \bar{\alpha}\bar{\beta}\overline{\{v^{(e)}_{imp.}\}} \\
\overline{\alpha^2\{v^{(e)}_{imp.}\}} &\simeq (\sigma_\alpha^2 - \bar{\alpha}^2)\overline{\{v^{(e)}_{imp.}\}} + 2\bar{\alpha}\overline{\alpha\{v^{(e)}_{imp.}\}}
\end{aligned} \tag{10}$$

where α, β express m_{ij}, c_{ij}, k_{ij} and σ_α is the standard deviation of α. By using the approximate equation (10), Eqs.(8) and (9) may be combined as follows:

$$[P]\{\ddot{y}^{(e)}_{imp.}(t)\} + [Q]\{\dot{y}^{(e)}_{imp.}(t)\} + [R]\{y^{(e)}_{imp.}(t)\} = \{F^{(e)}_{imp.}(t)\} \tag{11}$$

where $[P]$,$[Q]$ and $[R]$ are the deterministic matrices which consist of $\overline{m_{ij}}$, $\overline{c_{ij}}$, $\overline{k_{ij}}$, $\sigma_{m_{ij}}$, $\sigma_{c_{ij}}$ and $\sigma_{k_{ij}}$. $\{y(t)\}, \{F(t)\}$ are vectors expressed as

$$\begin{aligned}
\{y^{(e)}_{imp.}(t)\} &= \{\overline{\{v^{(e)}_{imp.}\}}^T, \overline{m_{ij}\{v^{(e)}_{imp.}\}}^T, \overline{c_{ij}\{v^{(e)}_{imp.}\}}^T, \overline{k_{ij}\{v^{(e)}_{imp.}\}}^T\}^T, \\
\{F^{(e)}_{imp.}(t)\} &= \{\{f^{(e)}_{imp.}\}^T, \overline{m_{ij}}\{f^{(e)}_{imp.}\}^T, \overline{c_{ij}}\{f^{(e)}_{imp.}\}^T, \overline{k_{ij}}\{f^{(e)}_{imp.}\}^T\}^T
\end{aligned} \tag{12}$$

Here $\overline{\{v^{(e)}_{imp.}\}}$ is the mean value of the impulsive response. $\overline{\{v^{(1)}_{imp.}\}},\cdots,\overline{\{v^{(n)}_{imp.}\}}$ can be calculated against $\overline{\{f^{(1)}_{imp.}\}},\cdots,$

$\overline{\{f_{imp.}^{(n)}\}}$, respectively. The matrix which consists of $\overline{\{v_{imp.}^{(1)}\}}, \cdots, \overline{\{v_{imp.}^{(n)}\}}$ is the mean value of the impulsive response function matrix $\overline{[H(t)]}$ shown as follow:

$$\overline{[H(t)]} = [\overline{\{v_{imp.}^{(1)}\}}, \cdots, \overline{\{v_{imp.}^{(n)}\}}] \tag{13}$$

By using $\overline{[H(t)]}$, the mean value of the random response and its auto-covariance function can be obtained as

$$\overline{\{v(t)\}} = \int_0^t \overline{[H(t-\tau)]}\{f(\tau)\}d\tau \tag{14}$$

$$\overline{[R_v(t_1,t_2)]} = \int_0^{t_2}\int_0^{t_1} \overline{[H(t_1-\tau_1)][R_f(\tau_1,\tau_2)][H(t_2-\tau_2)]^T} d\tau_1 d\tau_2 \tag{15}$$

If the external force $\{f(t)\}$ can be expressed as Eq.(14), Eqs.(5) and (7) are rewritten as

$$\overline{\{v(t)\}} = \int_0^t \overline{\{h(t-\tau)\}}g(\tau)d\tau \tag{16}$$

$$\overline{[R_v(t_1,t_2)]} = \int_0^{t_2}\int_0^{t_1} \overline{\{h(t_1-\tau_1)\}\{h(t_2-\tau_2)\}^T} R_g(\tau_1,\tau_2)d\tau_1 d\tau_2 \tag{17}$$

4 INTRODUCTION OF SUBSTRUCTURE SYNTHESIS METHOD

When Eq.(1) which governing the system is n degrees of freedom system, Eq.(11) obtained by using hierarchy method is $\{3n^2(n+1)/2+n\}$ degrees of freedom system. Hence we must calculate the eigenvalues and eigenvectors for the huge dimensional matrix. Since actual structures, however, have a large degrees of freedom, it takes much time and much memory space of the computer by using hierarchy method. In this chapter, substructure synthesis method is applied to calculate the random response with uncertain parameters in the structure.

4.1 Equation of motion of each substructure and application of hierarchy method

For the simplicity of the explanation, the structure which is composed of two substructures shown in Fig.1 is considered. It is assumed that the parameter α and β are uncertain in the substructure 1 and 2, respectively. The over all system is n degrees of freedom system. The equation of motion for the substructure 1 which is (m_1+l) degrees of freedom system (the inside domain I_1 is m_1 degrees of freedom and the boundary domain B is l degrees of freedom) is expressed as follows:

$$[M_1]\{\ddot{v}_1\} + [C_1]\{\dot{v}_1\} + [K_1]\{v_1\} = \{f_1\} \tag{18}$$

where $\{v_1\}$ is a vector expressed as

$$\{v_1\} = \{\{v_{I1}\}^T, \{v_B\}^T\}^T \tag{19}$$

Here the subscript I denotes the inside domain and B denotes the boundary domain. Hence $\{v_{I1}\}$ is a m_1 dimensional vector and $\{v_B\}$ is a l dimensional vector.

By applying hierarchy method to Eq.(18), the following equation is obtained.

$$[P_1]\{\ddot{y}_1\} + [Q_1]\{\dot{y}_1\} + [R_1]\{y_1\} = \{F_1\} \tag{20}$$

where

$$[P_1] = \begin{bmatrix} P_{I1} & P_{IB1} \\ P_{BI1} & P_{B1} \end{bmatrix}, [Q_1] = \begin{bmatrix} Q_{I1} & Q_{IB1} \\ Q_{BI1} & Q_{B1} \end{bmatrix}, [R_1] = \begin{bmatrix} R_{I1} & R_{IB1} \\ R_{BI1} & R_{B1} \end{bmatrix},$$
$$\{y_1\} = \{\{y_{I1}\}^T, \{y_{B1}\}^T\}^T, \{y_{I1}\} = \{\{\overline{v_{I1}}\}^T, \overline{\alpha\{v_{I1}\}}^T, \overline{\beta\{v_{I1}\}}^T\}^T, \{y_{B1}\} = \{\{\overline{v_B}\}^T, \overline{\alpha\{v_B\}}^T, \overline{\beta\{v_B\}}^T\}^T \tag{21}$$

Here $[P_1], [Q_1]$ and $[R_1]$ are the deterministic matrices and $\{\overline{v_{I1}}\}$ and $\{\overline{v_B}\}$ included in $\{y_1\}$ express the mean value of the response.

Similarly the equation of motion for the substructure 2 which is (m_2+l) degrees of freedom system (the inside domain I_2 is m_2 degrees of freedom) is expressed as follows:

$$[M_2]\{\ddot{v}_2\} + [C_2]\{\dot{v}_2\} + [K_2]\{v_2\} = \{f_2\} \tag{22}$$

where $\{v_2\}$ is a vector expressed as

$$\{v_2\} = \{\{v_{I2}\}^T, \{v_B\}^T\}^T \tag{23}$$

Applying hierarchy method to Eq.(22), the following equation is obtained:

$$[P_2]\{\ddot{y_2}\} + [Q_2]\{\dot{y_2}\} + [R_2]\{y_2\} = \{F_2\} \tag{24}$$

where

$$\{y_2\} = \{\{y_{I2}\}^T, \{y_{B2}\}^T\}^T \tag{25}$$

4.2 Synthesis of substructures

Two substructures are synthesized by using substructure synthesis method. The method is that the response of the inside domain of each substructure is approximated by the summation of the linear combination of the response of the rigidly connected inside domain and the elastic deformations.

For the substructure 1, first, the response of the rigidly connected inside domain $\{y_{I1}\}_{con.}$ are obtained as follows. The equation of motion of the free vibration without damping for the inside domain is obtained from Eq.(20) as follows:

$$[P_{I1}]\{\ddot{y}_{I1}\}_{con.} + [R_{I1}]\{y_{I1}\}_{con.} = \{0\} \tag{26}$$

Here the uncertainty is treated as to be deterministic because the $\{y_{I1}\}_{con.}$ expresses the mean value of the response. The vector $\{y_{I1}\}_{con.}$ is transformed into the modal displacement vector $\{\xi_1\}$ by using following equation:

$$\{y_{I1}\}_{con.} = [\Psi_{I1}]\{\xi_1\} \tag{27}$$

where the matrix $[\Psi_{I1}]$ is the $(3m_1 \times 3m_1)$ modal matrix obtained by solving the eigenvalue problem of Eq.(26).

Next the elastic deformation $\{y_{I1}\}_{ela.}$ are obtained from Eqs.(20) and (21) as follows:

$$\begin{bmatrix} R_{I1} & R_{IB1} \\ R_{BI1} & R_{B1} \end{bmatrix} \begin{Bmatrix} \{y_{I1}\}_{ela.} \\ \{y_{B1}\} \end{Bmatrix} = \begin{Bmatrix} 0 \\ F_{B1} \end{Bmatrix} \tag{28}$$

$$\{y_{I1}\}_{ela.} = -[R_{I1}]^{-1}[R_{IB1}]\{y_{B1}\} = [T_1]\{y_{B1}\} \tag{29}$$

where $[T_1]$ is $(3m_1 \times 3l)$ matrix. Hence the response of the inside domain of the substructure 1 $\{y_{I1}\}$ is assumed as follows:

$$\{y_{I1}\} = \{y_{I1}\}_{con.} + \{y_{I1}\}_{ela.} = [\Psi_{I1}]\{\xi_1\} + [T_1]\{y_{B1}\} \tag{30}$$

Similarly, for the substructure 2, the response of the inside domain of the substructure 2 $\{y_{I2}\}$ is assumed as follows:

$$\{y_{I2}\} = [\Psi_{I2}]\{\xi_2\} + [T_2]\{y_{B2}\} \tag{31}$$

Finally the synthesis method of two substructures is explained as follows. The equation that combines Eqs.(20) and (24) is

$$[P_a]\{\ddot{v_a}\} + [Q_a]\{\dot{v_a}\} + [R_a]\{v_a\} = \{F_a\} \tag{32}$$

where

$$[P_a] = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}, [Q_a] = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix}, [R_a] = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix},$$
$$\{v_a\} = \{\{y_{I1}\}^T, \{y_{B1}\}^T, \{y_{I2}\}^T, \{y_{B2}\}^T\}^T, \{F_a\} = \{\{F_1\}^T, \{F_2\}^T\}^T \tag{33}$$

Considering the transformation of the vectors $\{y_{I1}\}, \{y_{I2}\}$ into the modal displacements $\{\xi_1\}, \{\xi_2\}$, the new displacement vector $\{y\}$ is obtained as follows:

$$\{y\} = \{\{\xi_1\}^T, \{y_{B1}\}^T, \{\xi_2\}^T, \{y_{B2}\}^T\}^T \tag{34}$$

Further, when it is assumed that two substructures are rigidly connected by the following condition

$$\{y_B\} = \{y_{B1}\} = \{y_{B2}\} \tag{35}$$

Eq.(34) is transformed into

$$\{y\} = \{\{\xi_1\}^T, \{y_B\}^T, \{\xi_2\}^T\}^T \tag{36}$$

Therefore the synthesis of two substructures is performed by the coordinate transformation of Eq.(32) by using the following equation:

$$\{v_a\} = \begin{bmatrix} \Psi_{I1} & T_1 & 0 \\ 0 & I & 0 \\ 0 & T_2 & \Psi_{I2} \\ 0 & I & 0 \end{bmatrix} \{y\} \equiv [T_p]\{y\} \tag{37}$$

where $[T_p]$ is $(3(m_1 + n_1 + 2l) \times 3(m_1 + n_1 + l))$ matrix.

The equation of motion of the synthesized system is obtained as follows by substituting Expression(37) into Eq.(32) and multiplying $[T_p]^T$

$$[P_r]\{\ddot{y}\} + [Q_r]\{\dot{y}\} + [R_r]\{y\} = \{F_r\} \tag{38}$$

where

$$[P_r] = [T_p]^T[P_a][T_p],\ \ [Q_r] = [T_p]^T[Q_a][T_p],\ \ [R_r] = [T_p]^T[R_a][T_p],\ \ \{F_r\} = [T_p]^T\{F_a\} \tag{39}$$

and this equation is $3(m_1 + n_1 + l)$ degrees of freedom system, which is the same degrees of freedom system as applying the hierarchy method for the over all system. However, when the numbers of the mode in $\{\xi_1\}$ and $\{\xi_2\}$ are reduced, the degree of freedom of the synthesized system can be reduced.

5 NUMERICAL EXAMPLES

In order to check the accuracy of the presented method, it is applied to the piping system shown in Fig.2. The piping system is fixed at both ends, and these supports are modeled as a stiffness and damping system. In order to demonstrate the presented method, this system is separated into two substructures at the center. The dimensions of the piping system are shown in Table 1. It is assumed that the damping coefficients of the substructure 1 c_1 is uncertain, i.e. its mean value and the standard deviation are given as $\overline{c_1}$ and σ_{c_1}, and the other parameters are deterministic.

The equations of motion of the substructure 1 and 2 are shown as Eqs.(18) and (22). The vectors $\{v_1\},\{v_2\}$ are expressed as follows:

$$\{v_1\} = \{x_1,\ l\theta_1,\ x_2,\ l\theta_2,\ x_3,\ l\theta_3\}^T, \{v_2\} = \{x_4,\ l\theta_4,\ x_5,\ l\theta_5,\ x_3,\ l\theta_3\}^T \tag{40}$$

The elements $x_3, l\theta_3$ are the boundary domain so that $m_1 = 4, m_2 = 4, l = 2$. Since the over all system has 10 variables, i.e. $n = 10$, and the uncertain parameter is only c_1, the equation obtained by using hierarchy method is 20 degrees of freedom system.

When the piping system is formulated by using the presented method, the impulsive response functions are obtained. The random responses and the auto-covariance functions can be calculated by using Eqs.(16) and (17).

5.1 Comparison of the impulsive response

The impulsive responses for the following four cases are compared.

CASE1 the deterministic case, i.e. $\sigma_{c_1} = 0$, obtained from Eq.(1) which is 10 degrees of freedom system.

CASE2 the stochastic case by using hierarchy method obtained from Eq.(11) which is 20 degrees of freedom system. The substructure synthesis method is not used.

CASE3 the stochastic case by using the presented method obtained from Eq.(38) which is 20 degrees of freedom system, i.e. the all eight modes in $\{\xi_1\}$ and $\{\xi_2\}$ are used.

CASE4 the stochastic case by using the presented method obtained from Eq.(38). However the first and second modes in $\{\xi_1\}$ and $\{\xi_2\}$ are used, so that the equation(38) is 8 degrees of freedom system.

Fig.3 shows the mean value of the impulsive response $\overline{x_2}$ when the impulsive force subjects at the center. The impulsive response of CASE1 is smaller than the others since the uncertainty is ignored. The impulsive response of CASE2 is different a little from the ones of CASE3 and CASE4. The impulsive responses of CASE3 and CASE4 agree well since the system is small degree of freedom system and the first mode has much influence on the impulsive response. On the point of the calculating time, the one of CASE 4 is 90 % of the one of CASE2 since we must solve the eigenvalue problem of Eq.(26) for the substructure 1 and the similar equation for the substructure 2. However, on the point of the memory space of the computer, the one of CASE 4 is 30 % of the one of CASE4.

5.2 Random response against earthquake excitation

The random responses against El Centro 1940 NS earthquake excitation for the previous four cases are calculated by using Eq.(16). Fig.4 shows the responses at the nodal point 2. As seen in Fig.4, the responses for four cases almost agree.

5.3 Auto-covariance function

The auto-covariance function for the stationary narrow band process are calculated by using Eq.(17). The process is expressed as follows:

$$\begin{gathered} R_g(t_1, t_2) = S\exp(-\beta\tau)\cos(\omega\tau) \\ (\tau = | t_1 - t_2 |) \end{gathered} \tag{41}$$

Fig.5 shows the root mean square values at the nodal point 2. The root mean square value of CASE1 is smaller than the others since the uncertainty is ignored. The cases of CASE3 and CASE4 agree well, but they are amaller than the one of CASE2.

6 CONCLUSION

In this paper, the hierarchy method is applied to the substructure synthesis method to analyze the random response of complicated structures. As a result, it is shown that the presented method is useful to analyze the random response and effective specially from the point of view of saving the memory space of the computer.

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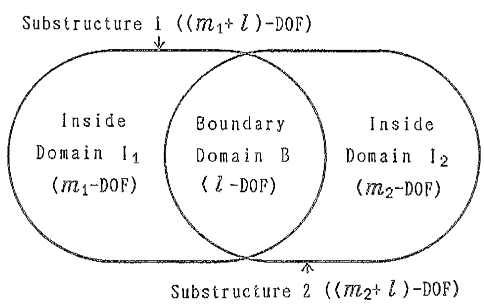

Fig. 1 Two substructurs

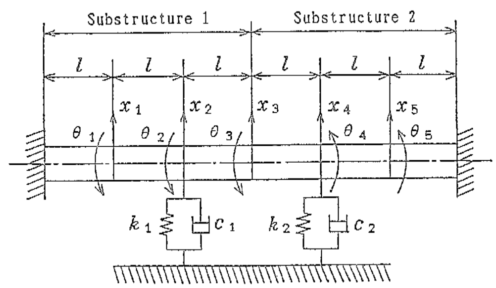

Fig. 2 Piping system (c_1 is uncertain.)

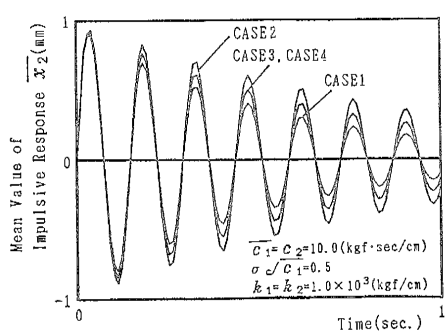

Fig. 3 Comparison of mean value of impulsive response $\overline{x_2}$

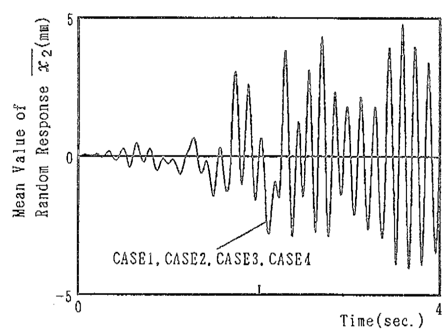

Fig. 4 Comparison of mean value of random response $\overline{x_2}$ against earthquake

Table 1 Dimensions of piping system

Total Length	3.00×10^4 [mm]
Density	7.70×10^{-6} [kg/mm^3]
Modulus of Elasticity	2.10×10^4 [kgf/mm^2]
Outside Diameter	1.00×10^3 [mm]
Inside Diameter	8.66×10^2 [mm]

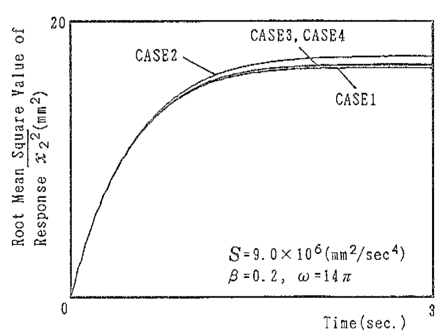

Fig. 5 Comparison of root mean square value of response $x_2{}^2$ against stationary narrow band process