Analytical Evaluation Method of Creep-Fatigue Crack Propagation in Surface Cracked Plate

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INTRODUCTION

This paper describes the results of the detailed numerical evaluations of a creep-fatigue crack growth test which was run at 550°C using a type 304 stainless steel plate with a surface crack. For the purpose of predicting the fatigue crack growth rate, the stress intensity factor range $\Delta K$ is often used in the small scale yielding condition and the $J_f$ proposed by Dowling (1975) is used without regard to this condition. For the purpose of predicting the creep crack growth rate, the $J'$ proposed by Landes and Begley (1974) is used. $J'$ is sometimes written as $C_J$. There is no established method for predicting the creep-fatigue crack growth rate. The combination of $\Delta J$, and $J'$ is attractive and tried for two dimensional problems. There is, however, only a little knowledge about the creep-fatigue crack growth rate of a surface crack, which is very important to practical problems. The purpose of our analyses is to know whether the combination of $\Delta J$, and $J'$ is applicable or not to the creep-fatigue crack growth rate of a surface crack.

2. EXPERIMENT

One creep-fatigue crack growth test is selected as a target of analyses. The details of this test are presented by Kitagawa et al. (1991) in another paper. The test is done at 550°C using type 304 austenitic stainless steel plate with a semi-elliptical surface crack. This specimen is subjected to bending moment as shown in Fig.1. The rotation angle which corresponds to the bending moment is controlled throughout the experiment. The loading diagram is shown in Fig.2(A). The maximum rotation angle is 0.035 radian and the minimum is -0.035 radian. The holding time is five hours and the number of cycles is 395 cycles. The shape of crack is monitored by a magnifying glass and the electric potential method. The shape of crack is also measured by the beach mark method. The creep-fatigue crack growth rates at the deepest point and the surface point are obtained from the dimensions of the crack at a lot of loading cycles.

The fatigue crack growth rate of this 304 stainless steel plate at 550°C is as follows:

$$
\frac{da}{dn} = \begin{cases} 
6.999 \times 10^{-9} \Delta J_f^{1.4435} & \text{(maximum)} \\
2.081 \times 10^{-9} \Delta J_f^{1.4435} & \text{(mean)} \\
6.185 \times 10^{-10} \Delta J_f^{1.4435} & \text{(minimum)}
\end{cases}
$$

(1)
where $\frac{da}{dN}$ and $\Delta J_c$ are the fatigue crack growth per cycle (mm/cycle) and $\Delta J$ proposed by Dowling (N/m) respectively.

The creep crack growth rate of this 304 stainless steel plate at 550°C is as follows.

\[
\frac{da}{dN} = \begin{cases} 
8.435 \times 10^{-5} \cdot \Delta J_c^{0.8770} & \text{(maximum)} \\
4.023 \times 10^{-5} \cdot \Delta J_c^{0.8770} & \text{(mean)} \\
1.918 \times 10^{-5} \cdot \Delta J_c^{0.8770} & \text{(minimum)}
\end{cases}
\]  

(2)

$\Delta J_c = J(t = t_H) \cdot t_H$

where $t$, $t_H$, $\frac{da}{dN}$ and $J'$ are time (sec), the holding time (sec), the creep crack growth for $t_H$ (min/cycle) and $J'$ integral (N-m^-1.-sec^-1) proposed by Landes and Begley.

3. ANALYSIS

3.1 Method of analysis

The basic idea for the analysis of the creep-fatigue crack growth rate is the combination of $\Delta J_c$ and $J'$. The fatigue crack growth rate is calculated by $\Delta J_c$ and the creep crack growth rate is calculated by $J'$, then these values are added to obtain the creep-fatigue crack growth rate.

The fatigue crack growth rate is calculated by these equations:

\[
(\frac{da}{dN})_f = k_f \cdot \Delta J_f^s
\]  

(3)

\[
\Delta J_f = 4 \cdot J(\theta = 0.035 \text{ rad})
\]  

(4)

where $k_f$ and $n$ are the material constants (see Eq.(1)), $a$, $(\frac{da}{dN})_f$, $\Delta J_f$, $J$ and $\theta$ are the depth of the crack (see Fig.1), the crack growth rate due to fatigue at the deepest point, $\Delta J_f$ which corresponds to rotation angle range of 0.070 radian, $J$ proposed by Rice (1968) and the rotation angle (see Fig.1) respectively. Eq.(3) is proposed by Dowling (1976). Eq.(4) is discussed by Asada et al. (1989).

Fig.2(B) shows the loading diagram for analysis. Only the loading and holding parts are analyzed. The loading range of the rotation angle is 0.055 radian, which is a half of the range of rotation angle applied in the experiment.

The creep crack growth rate is calculated by these equations:

\[
(\frac{da}{dN})_c = k_c \cdot (\Delta J_c)^s
\]  

(5)

\[
\Delta J_c = J'(t = t_H) \cdot t_H
\]  

(6)

where $k_c$ and $s$ are the material constants and $(\frac{da}{dN})_c$ is the creep crack growth per cycle. Eqs.(5) and (6) are an approximation of the following equation:

\[
(\frac{da}{dN})_c = \int_0^y (k_c t_H^{-1}) \cdot (J'(t))^s dt
\]  

(7)

The creep-fatigue crack growth rate is calculated by adding two components:
\[ \frac{da}{dN} = \left( \frac{da}{dN} \right)_p + \left( \frac{da}{dN} \right)_e \]  

(8)

The method mentioned above is concerned only with the depth of the crack. The same methods are used for the crack length:

\[ \frac{dc}{dN} = \left( \frac{dc}{dN} \right)_p + \left( \frac{dc}{dN} \right)_e \]  

(9)

where \( c \) is the half crack length of the surface crack as shown in Fig.1.

3.2 Cases of analysis
Five crack sizes which appear during one creep-fatigue crack growth test are chosen as the objects of analysis as listed in Table 1. These five cases are analyzed by four analyst groups. The boundary element method is used to analyze the stress and strain of case 1. The finite element method is used for the other cases. The path integral method is used to evaluate \( J \) of cases 1 and 3. The virtual crack extension method is used for the other cases. To calculate \( J' \) the path integral is used for all cases.

3.3 Models of analysis
As for the material data, the values of type 304 stainless steel at 550°C are used in the analyses as follows:

\[ E = 153.88 \text{GPa} \]  

(10), \[ \nu = 0.306 \]  

(11)

\[ \varepsilon = \frac{\sigma}{E} + \left( \left( \frac{\sigma - \sigma_p}{k} \right)^{\frac{1}{n}} \right) \]  

(12), \[ \sigma_p = 92.08 \text{MPa} \]  

(13)

\[ k = 2032.0 \text{MPa} \]  

(14), \[ m = 0.427 \]  

(15)

\[ \dot{\varepsilon} = 2.1595 \times 10^{-18} \sigma^{0.8123} \text{mm} \cdot \text{mm}^{-1} \cdot \text{hr}^{-1} \]  

(16)

where \( E, \nu, \varepsilon, \sigma \) and \( \dot{\varepsilon} \) are Young's modulus, Poisson's ratio, elastic and plastic strain, stress (MPa) and creep strain respectively. Eq.(12) is the special stress-strain relation where \( 2\varepsilon \) and \( 2\sigma \) correspond to strain range and stress range of uniaxial cyclic loading test. Eq.(16) is an approximation for five hour hold creep test at 550°C.

Fig.3 shows the mesh subdivision used in the finite element analysis of case 4. Thanks to symmetry one fourth of the specimen is analyzed. The broken lines in Fig.4 show the typical integral paths at the deepest point and the surface point.

3.4 Results of analysis
The crack opening displacement ranges are shown in Fig.5. The circles show the calculated results. Those values are double to compare with the experimental result shown by the solid line. The calculated COD ranges agree well with the measured COD range during the experiment.

The changes of \( J' \) of each path at the deepest point and the surface point for case 4 are shown in Figs.6 and 7 respectively. The most of \( J' \) values converge at 5 hours. The average values of all \( J' \) or the average values of \( J' \) except the maximum and the minimum are used to calculate the creep-fatigue crack growth rate. The results of the creep-fatigue crack growth analyses at the deepest point and the surface point are shown in Table 2 and Table 3 respectively. The results of experiment are also listed in these tables. The comparisons between the calculated and measured crack growth rates at the deepest point and the surface point are shown in Figs.8 and 9 respectively. The ranges
of the calculated values are due to the variation of the material data shown in Eqs.(1) and (2). In spite of the complexity of the problem the calculated creep-fatigue crack growth rates agree well with the experimental results.

4. CONCLUSIONS

The predicted creep-fatigue crack growth rates by detailed numerical analyses using material data show good agreement with the experimental data. It means that the combination of $\Delta J_f$ and $J'$ is able to evaluate the creep-fatigue crack growth rates of the surface crack in the plate tested in our study.

With the present techniques the combination of $\Delta J_f$ and $J'$ is considered to be the best method to predict the creep-fatigue crack growth rate of a surface crack.

REFERENCES


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<th>Calculation of $J$</th>
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Note: *: Number of Boundary Elements, **: Number of Internal Cells
+: Number of Boundary Nodes, ++: Number of Internal Points

Table 2 Results of Creep-Fatigue Crack Growth Analyses at Deepest Point

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<tr>
<th>Case</th>
<th>Crack depth (mm)</th>
<th>Crack length (mm)</th>
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<th>$\Delta A$ (mm)</th>
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Table 3  Results of Creep-Fatigue Crack Growth Analyses at Surface Point

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Fig. 1  Plate with a Surface Crack Subjected to Bending Moment

Fig. 2  Loading Diagram of Creep-Fatigue Crack Growth Experiment and Analysis

Fig. 3  Example of Mesh Subdivision of Cracked Plate

Fig. 4  Paths of Integral
Fig. 5  Crack Opening Displacement versus Crack Depth

Fig. 6  $J$ versus Time at Deepest Point for Case 4

Fig. 7  $J$ versus Time at Surface Point for Case 4

Fig. 8  Comparison between Calculated and Measured Crack Growth Rate at Deepest Point

Fig. 9  Comparison between Calculated and Measured Crack Growth Rate at Surface Point