Application of Continuum Damage Mechanics to Predict Low Cycle Fatigue Life under Nonproportional Loading

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ABSTRACT

A multiaxial fatigue life assessment was developed by application of Continuum Damage Mechanics. A kind of cyclic softening material, 42CrMo, was used as low cycle fatigue tests in order to verify this life prediction and damage assessment. Tests and analysis show that the new approach could successfully apply to fatigue life prediction under complex multiaxial loadings.

1 INTRODUCTION

Many structural members and machine components are subjected to repeated loading which may lead to their failure due to fatigue. In many practical situations the loading are complex, i.e. the corresponding principal stress are nonproportional or the principal directions change during a cycle of the loading. Fatigue under such loading is generally referred to as multiaxial fatigue. Life prediction of parts subjected to fatigue is of prime importance for their reliable design and safe in service inspection. Such a prediction is often based on simple laboratory tests under uniaxial loading and an appropriate multiaxial fatigue rule or criterion. The latter is essentially a prescription for reducing the complex multiaxial to an "equivalent" uniaxial loading.

Cumulative fatigue damage analysis using continuum damage mechanics is a valuable approach to prediction fatigue life. In the last decade, although continuum damage mechanics have made great advance, it only explained uniaxial fatigue successfully under simple loading history. Chaboche developed a Non-Linear-Continuous-Damage model to describe nonlinear accumulative fatigue damage (Chaboche et al. 1974). This model was based on the physical idea that a crack was preceded by a progressive internal deterioration of the material. The present paper research nonlinear accumulative fatigue damage under multiaxial loading, and develop a Non-Linear-Continuous-Damage model under nonproportional loading.

2 EXPERIMENT

The definition of the axial torsional subspace follows as a subspace of Il'yushin's five-dimensional deviatoric vector space, \( \varepsilon_1 = \varepsilon, \varepsilon_2 = \varepsilon^2/S^3, \sigma = \sigma, \sigma_5 = \sqrt{3} \varepsilon \). The equivalent strain is expressed as \( \varepsilon_e = \sqrt{\varepsilon^2 + \varepsilon^2 / S^3} \).

All tests in this investigation were performed on computer controlled HT800 tension-torsion electronic hydraulic-severa material testing system. The material
examined in this study was 42CrMo, a high-alloy steel. Nominal chemical composition is C:0.43, Si:0.34, Mn:0.64, P:0.011, Pt:0.23, Cr:1.88, Mo:0.22. Specimens is a thin-walled tube with a uniform gage length of 26 mm and outer diameter of 35 mm and inner diameter 21 mm.

2.1 Cyclic Stable Stress-Strain Solution

Stable hysteresis loop under circles and square strain path was shown in Fig.1. Test showed that 42CrMo was cyclic softening material (Fig.2). This material performed cyclic softening characteristics under various strain path. Fig.3 shows cyclic stable stress-strain characteristics under torsion, circles, square strain path in which there were not obvious additional harden behaviour observed under nonproportional loading because of cyclic softening. This characteristics of cyclic softening material were different with that of cyclic harden material under nonproportional loading. So, unified Ramberg-Osgood-type constitutive relationship in terms of cyclic maximum shear stress and maximum shear strain

\[ \Delta \tau_{max}/2 = \kappa \left( \Delta \tau_{max}/2 \right)^n \]

For the materials tested, \( \kappa \) and \( n \) had been found to be 422 and 0.131 respectively.

2.2 Low-Cycle Fatigue Test

Strain-controlled low-cycle fatigue were performed fully reversed tensile strain range and shear strain range of triangular wave and equivalent strain range of 0.474% were examined. Effect strain rate was maintained at 0.04 s\(^{-1}\). Failure life was defined as the number of cycles that resulted in a 50% drop from the maximum value in either tensile stress or shear stress.

The test results of different strain path show that fatigue life was obvious distinct when equivalent strain range was equal each other. Fatigue life was dependent on loading path.

3 Non-Linear-Continuous Damage Model

3.1 Damage Evolution Equation

Chaboche developed a differential damage equation (Chaboche et al., 1974).

\[ \frac{d\phi}{d\Delta} = f(\Delta_{eq}, \overline{C}, \overline{D}) \Delta \]

where \( \Delta_{eq} \) and \( \overline{C} \) was the maximum stress and mean stress in each cycle respectively. If non-linear kinematic hardening was modeled, these equations must have unseparable variable in terms of damage and chosen forcing parameter.

Many experiments had shown multiaxial fatigue damage was controlled by maximum shear stress \( \tau_{max} \) or maximum shear strain \( \gamma_{max} \) and strain \( \Delta \), to normal maximum shear strain plane (Shaw, 1977). Characteristics of fatigue damage process involve slip bands, localized strain, microcrack initiation and propagation until some macroscopic crack initiation. Chaboche also proposed at least two parameter maximum shear stress and hydrostatic stress, was included in damage evolution equation under multiaxial loading (Chaboche et al., 1986).

So, damage rate equation is proposed

\[ \dot{\phi} = f(\Delta_{eq}, \overline{C}, \overline{D}, \phi) \Delta \]

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where $\sigma_m$ is the mean hydrostatic pressure in a cycle, and $\phi$ is an internal variable, i.e., nonproporionality considered plastic strain path history and is defined as

$$\phi = \frac{h_{sw}}{h_{wmax}}$$

(4)

where $h_{sw}$ is area swept by the plastic strain vector on the plastic strain plane, and $h_{wmax}$ is the plastic strain memory surface area on the plastic strain $\varepsilon$ plane (Elgin et al. 1989).

3.2 Damage definition

The quantitative measurement of area or density change by conventional experimental techniques are not easy. However damage is a microscopic internal variable and definition of it can be used as long as it is a good indicator of the degradation mechanism. Therefore, for a strain controlled fatigue test where saturation occurs it is also possible to determine the amount of damage according to

$$D = 1 - \frac{\Delta \sigma}{\Delta \sigma^o}$$

(5)

where $\Delta \sigma$ is the stress range response at any particular number of cycle, and $\Delta \sigma^o$ is the stress range at saturation (Lemaître et al. 1979).

3.3 Fatigue damage and fatigue life assessment

In terms of our experiments, considering symmetric cyclic loading, elated effects of $\sigma_m$, the damage evolution equation may be written as

$$dD = \left[1 - \left(1 - D\right)^{b_{el}}\right]^c \left[\frac{h_{wmax} - h_{sw}}{h_{wmax} - h_{sw}}\right]^{\frac{1}{D}} d\varepsilon$$

(6)

where $b_{el}$ is material constant, $c$ is damage coefficient depending on loading amplitude and loading path, and $n$ expresses effects of nonproporionality.

Under strain controlled, we rewrite equation (6) by simple substitution of equation (4) as

$$dD = \left[1 - \left(1 - D\right)^{b_{el}}\right]^c \left[\frac{K_{1}(\phi) - K_{2}}{K_{1}(\phi) - K_{2}}\right]^{\frac{1}{D}} d\varepsilon$$

(7)

where $b_{el}$ is strain dependent, expressed by

$$b_{el} = 1 - \alpha(1 + K_{1}Y(\phi))\left(\frac{K_{1}(\phi) - K_{2}}{K_{1}}\right)\left(\frac{K_{1}(\phi) - K_{2}}{K_{1}}\right)$$

(8)

and experimentally assume

$$n = \alpha(1 + K_{1}Y(\phi))$$

(9)

where $K_{1} = 0.0144$ was shear strain corresponding to yielding shear stress, $a$, $n$, $K_{1}$, $K_{2}$, $Y(\phi)$ was material constant.

Under proportional loading $Y(\phi) = 0$, and under out of phase loading, i.e., path $Y(\phi) = 1$. A simple form of function $Y(\phi)$ is specified in the following formulation

$$Y(\phi) = (\phi)^{\alpha}$$

(10)

Integrating equation (8) for constant $\Delta \sigma_{max}$ between $D = 0$ and $D = 1$, leads to

$$D = \left[1 - \left(1 - \frac{h_{sw}}{h_{wmax}}\right)^{b_{el}}\right]^c \left[\frac{h_{wmax} - h_{sw}}{h_{wmax} - h_{sw}}\right]^{\frac{1}{D}} d\varepsilon$$

(11)
\[ D = 1 - \left[ 1 - \left( \frac{N}{N_F} \right)^{K_{\phi\alpha}} \right]^{\beta_{\phi\alpha}} \]  

\[ N_{F}^{\phi\alpha} = \frac{1}{(\beta_{\phi\alpha})^{(1 - \gamma_{\phi})}} \left( \frac{K_{\phi\alpha}(N_{F_{\text{max}}})^{\gamma_{\phi}}}{N_{\phi}} \right)^{1 - \beta_{\phi\alpha}} \]  

3.4 Identification of material coefficients

Low-cycle fatigue life assessment in torsion was obtained (Leese et al. 1985)

\[ \frac{N_{\text{max}}}{2} = N_{F}^{\phi\alpha} \left( 2 - N_{F}^{\phi\alpha} \right)^{C} \]  

Comparison to equation (13), we obtained

\[ \beta = -\frac{1}{mc} \]  

\[ N_{F}^{\phi_{\alpha}^{\phi\alpha}} = \frac{K_{\phi\alpha}(N_{F_{\text{max}}})^{\gamma_{\phi}}}{N_{\phi}^{\phi_{\alpha}^{\phi\alpha}}} (1 - \gamma_{\phi})^{\beta_{\phi\alpha}} \]  

For cyclic strain path, nonproportionality \( \phi = 1 \), material coefficient \( K_{\phi} \), \( K_{\phi} \) is obtained by equation (12) and (13). All material coefficient is shown in Table 1. Tested damage coefficients \( \varepsilon_{\text{t}} \) and predicted damage coefficients \( \varepsilon_{\text{p}} \) is shown in Table 2.

<p>| Table 1. Material coefficient. |</p>
<table>
<thead>
<tr>
<th>a</th>
<th>( K_{\phi} )</th>
<th>( K_{\phi} )</th>
<th>( N_{F}^{\phi\alpha} ) (Hpa)</th>
<th>( \beta_{\phi\alpha} )</th>
<th>( \gamma_{\phi\alpha} )</th>
<th>c</th>
<th>( K_{\phi\alpha} )</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.376</td>
<td>0.902</td>
<td>0.665</td>
<td>459</td>
<td>11,428</td>
<td>0.6597</td>
<td>-0.668</td>
<td>422</td>
<td>0.131</td>
</tr>
</tbody>
</table>

3.5 Fatigue life prediction

Damage evolution curve for square strain path which nonproportionality is \( \phi = 0.637 \) is shown in Fig.4. Predicted fatigue life by equation (13) is 760, and experimental life is 735. This results is very satisfied. All results is shown in Table 2.

| Table 2. Fatigue life prediction |
| Spec: \( \tau_{\text{max}}^{\phi\alpha} \): \( \tau_{\text{max}}^{\phi_{\alpha}^{\phi\alpha}} \): Tested \( N_{F}^{\phi\alpha} \): Predicted \( N_{F}^{\phi_{\alpha}^{\phi\alpha}} \): Nonproportionality: \( 1 - \varepsilon_{\phi}^{\phi\alpha} \); \( 1 - \varepsilon_{\phi}^{\phi_{\alpha}^{\phi\alpha}} \) |
|---|---|---|---|---|---|---|---|---|---|
| #38 | 0.721 | 0.308 | 2074 | / | 0 | 0.485 | / |
| #5 | 0.827 | 0.389 | 375 | / | 1 (Circle) | 1.173 | / |
| #9 | 0.724 | 0.289 | 2074 | / | 0 | 0.485 | / |
| #6 | 0.652 | 0.281 | 1128 | 768 | 0.637 (Square) | 0.765 | 0.778 |
| #20 | 0.575 | 0.183 | 1434 | 1503 | 0.637 (Square) | / | 0.388 |
| #28 | 0.575 | 0.183 | 1434 | 1503 | 0.637 (Square) | / | 0.388 |

\text{note} \#28 \text{ is subjected } \varepsilon_{\phi} = 0.3794\%.
For general strain path life assessment may be rewritten as

$$\Delta K_{eq}^{\phi} = H(\phi) \cdot \Delta K_{eq}^{\phi} (2 \cdot N)^{1/n}$$  \hspace{1cm} (17)

where

$$H(\phi) = 1 + K_0/(\phi)^{1/n}$$  \hspace{1cm} (18)

$H(\phi)$ is additional damage function depend on nonproportional loading path. For proportional loading $\phi = 0$, then $H(0) = 1$. Equation (17) returns to equation (14), the law of Hanson-Coffin under torsion.

4 CONCLUSIONS

Based on the experimental results and Non-linear-Continuous-Damage model, the following conclusions can be made:

1. For combined tension and torsion loading the maximum plastic shear strain may correlate low cycle fatigue life.
2. This model may describe low cycle fatigue life under various nonproportional loading including proportional loading, and obtain satisfied results.
3. Comparison with uniaxial-based fatigue model, this model contain multiaxial information, in which additional low cycle fatigue under nonproportional loading was used as identify material coefficient.
4. From equation (17) we consider definition of nonproportionality and function $H(\phi)$ is very important. More experiments will be needed.

REFERENCE


Fig. 2. Monotonic and cyclic stable stress-strain curves under tension.

Fig. 3. Cyclic stable stress-strain curves under torsion, circle, square strain path.

Fig. 1. Stable hysteresis loop (a) circle (b) square.