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Application of Continuum Damage Mechanics to Predict Low Cycle Fatigue Life under Nonproportional Loading

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ABSTRACT

A multiaxial fatigue life assessment was developed by application of Continuum Damage Mechanics. A kind of cyclic soften materical 42CrMo was used as low cycle fatigue tests in order to verify this life prediction and damage assessment. Tests and analysis shown that the new approach could successfully applicate to fatigue life prediction under complex multiaxial loadings.

1 INTRODUCTION

Many structural members and machine components are subjected to repeated loading which may lead to their failure due to fatigue. In many practical situation the loading are complex, i.e. the corresponding principal stress are nonproportional or the principal directions chang during a cycle of the loading. Fatigue under such loading is generally reffered to as multiaxial fatigue. Life prediction of parts subjected to fatigue is of prime importance for their reliable design and safe in service inspection. Such a prediction is often based on simple laboratory tests under uniaxial loading and an appropriate multiaxial fatigue rule or criterion. The latter is essentially a prescription for reducing the complex multiaxial to an "equivalent" uniaxial loading.

Cumulative fatigue damage analysis using continuum damage mechanics is a valuable approach to prediction fatigue life. In the last decade, Although continuum damage Mechanics have made great advance, it only explained uniaxial fatigue successful under simple loading history. Chaboche developed a Non-Linear-Continuous-Damage model to describe nonlinear accumulative fatigue damage (chaboche et al. 1974). This model was based on the physical idea that a crack was preceded by a progressive internal deteriovation of the material.The present paper research nonlinear accumulative fatigue damage under multiaxial loading , and develop a Non-Linear-Continuous-Damage model under nonproportional loading.

2.EXPERIMENTS

The definition of the axial torsional subspace follows as a subspace of Ilyushin's five-dimensional deviatoric vector space, $\varepsilon_1=\varepsilon$, $\varepsilon_3=\gamma/\sqrt{3}$, $\sigma_1=\sigma$, $\sigma_3=\sqrt{3}\tau$. The equivalent strain is expressed as $\varepsilon_e=|\vec{\varepsilon}|=(\varepsilon^2+\gamma^2/3)^{1/2}$

All tests in this investigation were performed on computer controlled MTS809 tension-torsion electronic hydraulic-severo material testing system. the material

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examined in this study was 42CrMo, A high-alloy steel. Nominal chemical composition is C:0.43, Si:0.34, Mn:0.64, S:0.011, P:0.022, Cr:1.08, Mo:0.22. Specimen is a thin-walled tube with a uniform gage length of 25 mm and outerdiameter of 25mm and interdiameter 21mm.

2.1 cyclic stable stress-strain solution

Stable hysteresis loop under circle and square strain path was shown in Fig.1. Test shown that 42CrMo was cyclic soften material (Fig.2.) This material performed cyclic soften characteristics under various strain path. Fig.3 shown cyclic stable stress-strain characteristics under torsion, circle, square strain path in which there were not obvious additional harden behaviour observed under nonproportional loading Because of cyclic soften. This characteristics of cyclic soften material were different with that of cyclic harden material under nonproportional loading. So, unified Ramberg-Osgood type constitutive relationship in terms of cyclic maximum shear stress and maximum shear strain

$$\Delta\tau_{max}/2 = K\ (\Delta\gamma^{p}_{max}/2)^{n} \qquad (1)$$

For the material tested, K and n had been found to be 422 and 0.131 respectively.

2.2 Low cycle fatigue test

Strain controlled low cycle fatigue were performed fully reversed tensile strain range and shear strain range of triangular wave and equivalent strain range of 0.4743% were examined. Effect strain rate was maintained at 0.04 s^{-1} Failure life was defined as the number of cycles that resulted in a 25% drop from the maximum value in either tensile stress or shear stress.

The test results of different strain path shown that fatigue life was obvious distinct when equivalent strain range was equal each other. Fatigue life was dependent on loading path.

3 NON-LINEAR-CONTINUOUS DAMAGE MODEL

3.1 Damage evolution equation

Chaboche developed a differential damage equation (Chaboche et al. 1974),

$$dD = f(\sigma_M,\ \bar{\bar{\sigma}},\ D)\ dN \qquad (2)$$

where σ_M and $\bar{\sigma}$ was the maximum stress and mean stress in each cycle respectively. if non-linear accumulative effects were modeled, these equations must have unseparable variable in terms of damage and chosen forcing parameter.

Many experiments had shown multiaxial fatigue damage was controlled by maximum shear stress τ_{max} or maximum shear strain γ_{max} and strain ε_n to normal maximum shear strain plane (Kanazawa et al. 1977). Characteristics of fatigue damage process involve slip bands, localized strain, microcrack initiation and propagration until some macroscopic crack initiation. Chaboche also proposed at least two parameter maximum shear stress and hydrostatic stress, was included in damage evolution equation under multiaxial loading (Choboche et al. 1988).

So, damage rate equation is proposed

$$dD = f(\tau_{max},\ \bar{\bar{\sigma}}_H,\ D,\ \phi)\ dN \qquad (3)$$

where $\bar{\sigma}_M$ is a mean hydrostatic pressure in a cycle, and ϕ is a internal variable, i.e. nonproportionality considered plastic strain path history and is defined as

$$\phi = A_{sw} / A_{mem} \qquad (4)$$

where A_{sw} is area swept by the plastic strain vector on the plastic strain plane, and A_{mem} is the plastic strain memory surface area on the plastic strain π plane (Ellyin et al. 1989).

3.2 Damage definition

The quantitutive measurement of area or density change by conventional experimental techniques are not easy. However damage is a microscopic internal variable and definition of it can be used as long as it is a good indicator of the degration mechanism. therefore, for a strain controlled fatigue test where saturation occurs it is also possible to determine the amount of damage according to

$$D = 1 - \Delta\sigma / \Delta\sigma^{*} \qquad (5)$$

where $\Delta\sigma$ is the stress range response at any particular number of cycle, and $\Delta\sigma^{*}$ is the stress range at satuation (Lemaitre et al. 1979).

3.3 Fatigue damage and fatigue life assessment

In terms of our experiments, considering symmetric cyclic loading, omitted effects of $\bar{\sigma}_M$, the damage evolution equation may be written as

$$dD = [1 - (1 - D)^{\beta+1}]^{\alpha} \left[\frac{(\tau_{max})}{M(\phi)(1-D)} \right]^{\beta} dN \qquad (7)$$

where β is material constant, α is damage coefficient depended on loading amplitude and loading path, and $M(\phi)$ express effects of nonproportionality.

Under strain controlled, we rewrite equation (7) by simple substitution of equation (1) as

$$dD = [1 - (1 - D)^{\beta+1}]^{\alpha} \left[\frac{K(\Delta\gamma^{p}_{max}/2)^{n}}{M(\phi)(1-D)} \right]^{\beta} dN \qquad (8)$$

where α is strain dependent, expressed by

$$\alpha = 1 - a\left[1 + K_1 Y(\phi) \left(\frac{\gamma_{max} - \gamma_S}{\gamma_S} \right) \right] \left(\frac{\gamma_{max} - \gamma_S}{\gamma_S} \right) \qquad (9)$$

and experimentally assume

$$M(\phi) = M\,[\,1 + K_2/Y(\phi)\,] \qquad (10)$$

where γ_S=.3464% was shear strain corresponding to yielding shear stress, a, M, k_1, K_2, was material constant.

Under proportional loading $Y(\phi)=0$, and under out of phase loading (circle path) $Y(\phi)=1$. A simple form of function $Y(\phi)$ is specified in the following formulation

$$Y(\phi) = (\phi)^{1/8} \qquad (11)$$

Integrating equation (8) for constant $\Delta\gamma^{p}_{max}$ between D=0 and D=1, leads to (D=1, for $N=N_f$)

$$D = 1 - [1 - (\frac{N}{N_f})^{1/1-\alpha}]^{1/\beta+1} \quad (12)$$

$$N_f = \frac{1}{(\beta+1)(1-\alpha)} [\frac{K(\Delta\gamma^p_{max}/2)^n}{M(\phi)}]^{-\beta} \quad (13)$$

3.4 Identification of material coefficients

Low-cycle fatigue life assessment in torsion was obtained (Leese et al. 1985)

$$\Delta\gamma^p_{max}/2 = \gamma_f'(2 \cdot N_f)^c \quad (14)$$

Comparision to equation (13), we obtained

$$\beta = -\frac{1}{nc} \quad (15)$$

$$M = \frac{K\gamma_f'^n}{[\frac{1}{2}(1+\beta)(1-\alpha)]^{-1/\beta}} = M_0(1-\alpha)^{1/\beta} \quad (16)$$

For circle strain path, nonproportionality $\phi=1$, material coefficient K_1, K_2 is obtained by equation (12) and (13). All material coefficient is shown in Table 1. Tested damage coefficients α_T and predicted damage coefficients α_p is shown in Table 2.

Table 1. Material coefficient.

a	K_1	K_2	M_0(Mpa)	β	γ_f'	c	K(MPa)	n
0.376	0.902	0.665	469	11.428	0.6597	-0.668	422	0.131

3.5 Fatigue life prediction

Damage evolution curve for square strain path which nonproportionality is =0.637is shown in Fig.4. Predicted fatigue life by equation (13) is 760, and experimental life is 735. This results is very satisified. All results is shown in Table 2.

Table 2. Fatigue life prediction

Spec.	γ_{max} (%)	γ^p_{max} (%)	Tested N_f	Predicted N_f	Nonproportionality	$1-\alpha_T$	$1-\alpha_p$
#38	0.721	0.308	2074	/	0	0.406	/
#5	0.827	0.389	375	/	1 (Circle)	1.173	/
#10	0.724	0.289	738	760	0.637 (Square)	0.765	0.778
#6	0.652	0.281	1120	902	0.51 (Rectangle)	0.58	0.574
#28*	0.575	0.183	1434	1503	0.637 (Square)	/	0.388

* note #28 is subjected ε_e =0.3794%.

For general strain path life assessment may be rewritten as

$$\Delta\gamma_{max}^{p}/2 = H(\phi)\cdot\gamma_f'\cdot(2\cdot N_f)^{c} \tag{17}$$

where

$$H(\phi) = [1 + K_2/Y(\phi)]^{1/n} \tag{18}$$

$H(\phi)$ is additional damage function depend on nonproportional loading path. For proportional loading $\phi=0$, then $H(0)=1$. Equation (17) returns to equation (14), the law of Manson-Coffin under torsion.

4 CONCLUSIONS

Based on the experimental results and Non-Linear-Continuous-Damage model, the following conclusions can be made.

1. For combined tension and torsion loading the maximum plastic shear strain may correlate low cycle fatigue life.
2. This model may discribe low cycle fatigue life under various nonproportional loading including proportional loading, and obtain satisfied results.
3. Comparision with uniaxial-based fatigue model, this model contain multiaxial information, in which additional low cycle fatigue under nonproportional loading was used as identify material coefficient.
4. From equation (17) we consider difinition of nonproportionality and function $Y(\phi)$ is very important. More experiments will be needed.

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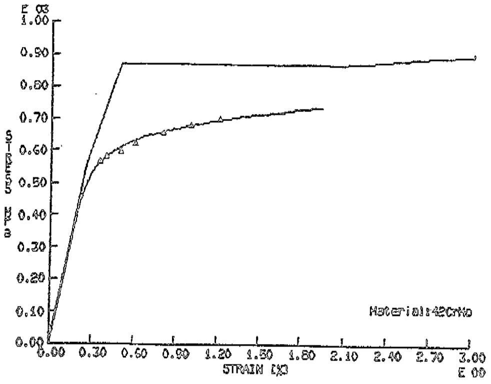


Fig.2. Monotonic and cyclic stable stress-strain curves under tension.

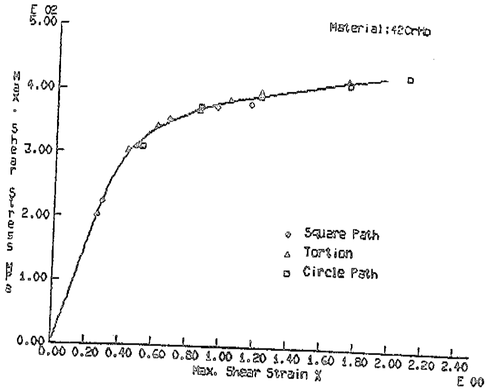
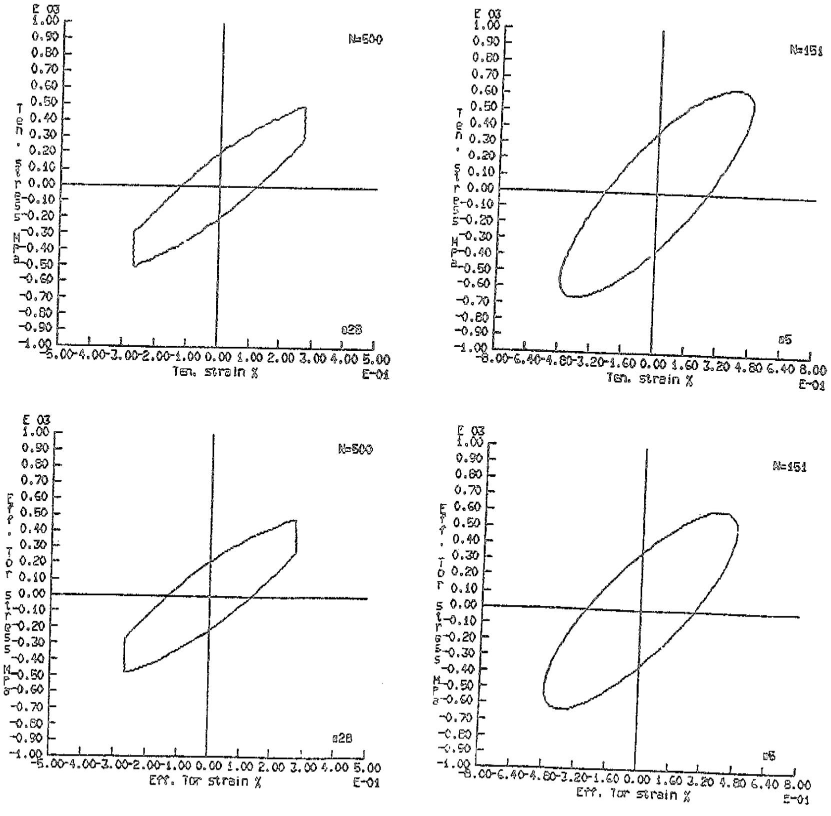


Fig.3. Cyclic stable stress-strain curves under torsion, circle, square strain path.



(b) (a)

Fig.1. Stable hysteresis loop (a) circle (b) square.