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A Constitutive Equation of Creep, Swelling and Damage under Irradiation Applicable to Multiaxial and Variable States of Stress

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ABSTRACT

A constitutive equation of creep, swelling and damage under neutron irradiation applicable to general conditions of stress, including stress change in direction as well as in magnitude, is developed. The creep related to irradiation is divided into irradiation-induced creep and irradiation-affected thermal creep. The irradiation-induced creep is formulated by taking account of the stress induced preferential absorption (SIPA) mechanism and climb-controlled glide (CCG) mechanism and by postulating an isotropic tensor function of stress of order zero and one which is also a function of neutron flux and neutron fluence. The irradiation-affected thermal creep, on the other hand, is modeled by extending the creep-hardening surface model of variable stress creep to include the effects of neutron irradiation and damage. Validity and utility of the proposed constitutive equation are discussed by applying it to some stress histories including reversed loading and non-proportional loading.

1. INTRODUCTION

Irradiation of high energy neutron usually induces significant effects on the mechanical behavior of structural materials by creating interstitial atoms, vacancies, dislocation loops and other microstructural change in the materials. Thus creep and creep damage under neutron irradiation are one of the most crucial phenomena in nuclear structure components operating at elevated temperature under intensive flux of high energy fission or fusion particles. The present authors (Murakami, Mizuno and Okamoto, 1991; Murakami and Mizuno 1991) recently developed a constitutive equation of creep, swelling and damage of polycrystalline metals in order to facilitate rational and accurate analyses of these components by incorporating it to computer algorithms.

By taking account of the physical mechanisms of creep under neutron irradiation and ordinary thermal creep, the constitutive equation was formulated by dividing the creep into irradiation-induced creep and irradiation-affected thermal creep. The irradiation-affected thermal creep, in particular, was formulated by extending the creep damage theory of Kachanov-Rabotnov to include the effects of neutron irradiation. However, because of the classical time-hardening and strain-hardening theory postulated in the equation, the applicability of the constitutive equation of irradiation-affected thermal creep was limited to rather insignificant change of stress state.

In the first wall of fusion reactors, for example, the direction and the magnitude of stress are subject to salient change with burn cycles of plasma. This change in the stress induces significant transient increase of creep rates, and has large influence on the initiation and the growth of creep cracks in the materials. Thus, the constitutive equations of creep under irradiation are required to describe this transient increase of creep rate brought about by the salient change of stress.

The present paper is concerned with the elaboration of the constitutive equation of creep, swelling and damage to conform to significant change in magnitude and direction of stresses. The creep-hardening surface model of non-steady creep proposed by the present authors (Murakami and Ohno 1982) is extended to include the effects of irradiation and material damage to give a constitutive equation of irradiation-affected thermal creep.

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2. CONSTITUTIVE EQUATION OF CREEP, SWELLING AND DAMAGE UNDER IRRADIATION

In the previous papers (Murakami, Mizuno and Okamoto, 1991; Murakami and Mizuno 1991), the constitutive equation of creep, swelling and damage under irradiation was formulated by dividing the total creep strain under irradiation ε^{C}_{ij} into irradiation–affected thermal creep ε^{ITC}_{ij} and irradiation–induced creep ε^{IIC}_{ij}; the former is thermal creep affected by irradiation and the latter occurs only under irradiation even under the condition of vanishing stress. We will first review the formulation of the constitutive equation of irradiation–induced creep. Then, we will discuss the elaboration of the constitutive equation of irradiation–affected thermal creep by extending the creep–hardening surface model of non–steady creep to include the effects of irradiation and material damage.

2.1 Irradiation–induced creep

Irradiated fast neutrons knock off the constituent atoms, and produce pairs of interstitial atoms and vacancies in the metals. A part of these Frenkel pairs may annihilate by the coalescence of themselves. However, since the interstitial atoms have large mobility, a part of them are absorbed by dislocations, and facilitate their climb (Gittus, 1978). The absorption of interstitial atoms into dislocations and the climb of dislocations induce creep deformation in metals. The vacancies, on the other hand, may coalesce to form voids, or may be absorbed by the existing voids. The coalescence of vacancies into voids and the increase in the number of lattice sites as a result of the absorption of interstitial atoms into the existing dislocations give rise to the dilatation of materials; i.e., swelling.

In contrast to the irradiation–affected thermal creep, the stress–dependence of the irradiation–induced creep and swelling is small, and the anisotropy of material properties caused by the irradiation may be insignificant (Ehrlich, 1981). Then, by taking account of the stress dependence of creep rate due to SIPA and CCG mechanisms, the irradiation–induced creep rate $\dot{\varepsilon}^{IIC}_{ij}$ can be expressed as an isotropic function of stress of order zero and order one. By using representation theorem for the isotropic tensor function, $\dot{\varepsilon}^{IIC}_{ij}$ is described as follow (Murakami, Mizuno and Okamoto, 1991):

$$\dot{\varepsilon}^{IIC}_{ij} = \eta\delta_{ij} + \xi\sigma_{kk}\delta_{ij} + \zeta\sigma_{ij} \tag{1}$$

where η, ξ and ζ are material constants. Since irradiation–induced creep describes dimensional change of material under irradiation, the volumetric part of the irradiation–induced creep may be identified with the swelling. While its deviatoric part may be interpreted as the irradiation creep. Dividing equation (1) into isotropic and deviatoric parts and rearranging the coefficients of equation (1), we finally have the constitutive equation of irradiation–induced creep and evolution equation of swelling as follows (Murakami, Mizuno and Okamoto, 1991; Murakami and Mizuno, 1991):

$$\dot{\varepsilon}^{IIC}_{ij} = (1/3)\dot{S}\delta_{ij} + (3/2)P\phi\sigma_{Dij} \tag{2a}$$

$$\dot{S} = C <1 + Q\sigma_{kk}> \phi \; [1 - e^{R(\chi-\Phi)} / \{1 + e^{R(\chi-\Phi)}\}] \tag{2b}$$

$$<x> = \begin{cases} 0 & x < 0 \\ x & x \geq 0 \end{cases}$$

where $\dot{S}$ and δ_{ij} denote the swelling rate and the Kronecker delta, while C, Q, R, χ and P are material constants. In equation (2b), Macauley bracket is employed because the swelling rate decreases under hydrostatic pressure, but it never becomes negative.

2.2 Creep–hardening surface model under unirradiated and undamaged condition

Creep at elevated temperature usually shows salient dependence on stress or strain history. Thus, in the case of classical strain–hardening theory, this feature is expressed as a function of stress and strain as follows:

$$\dot{\varepsilon}^{TC}_{ij} = (3/2)\; f(\sigma_{EQ};\; q)\; (\sigma_{Dij} / \sigma_{EQ}) \tag{3a}$$

$$\dot{q} = [(2/3)\; \dot{\varepsilon}^{TC}_{ij}\; \dot{\varepsilon}^{TC}_{ij}]^{1/2} \tag{3b}$$

where σ_{EQ} is equivalent stress in current state of stress, while q is a creep–hardening variable (i.e., an internal state variable) representing the state of creep–hardening of the material. However, a conventional theory of equation

(3) employs the current time t or the current equivalent creep strain ε^{C}_{EQ} as q, and can not describe transient increase of creep rate which is observed after stress reversal, because these creep-hardening variable q can not describe adequately the recovery of material hardening after stress change.

Murakami and Ohno (1982) hypothesized that a part of immobilized dislocations formulated during creep process will recover their mobility by the stress reversal or by the change of stress direction; i.e., this remobilization of dislocations induces the recovery of material hardening and transient increase of creep rate. The remobilized dislocations induced by a stress change will be immobilized again if the creep after stress change proceeds beyond a certain range, and they again start to form irreversible dislocation networks and induce the hardening of material. In order to prescribe this range of creep hardening, Murakami and Ohno introduced a creep-hardening surface in the creep strain space where immobilized dislocations recover their mobility as mentioned above. The creep-hardening surface g = 0 is assumed to be expressed in multiaxial state of strain by a hyper-sphere of center α_{ij} and radius ρ as follows:

$$g = (2/3)(\varepsilon^{TC}_{ij} - \alpha_{ij})(\varepsilon^{TC}_{ij} - \alpha_{ij}) - \rho^2 \leq 0 \tag{4a}$$

$$\dot{\alpha}_{ij} = \begin{cases} (1 - \lambda_0)(\dot{\varepsilon}^{TC}_{kl}\, n_{kl})\, n_{ij} & g = 0 \text{ and } (\partial g / \partial \varepsilon^{TC}_{ij})\, \dot{\varepsilon}^{TC}_{ij} > 0 \\ 0 & g < 0 \text{ or } (\partial g / \partial \varepsilon^{TC}_{ij})\, \dot{\varepsilon}^{TC}_{ij} \leq 0 \end{cases} \tag{4b}$$

$$\dot{\rho} = \begin{cases} \surd(2/3)\lambda_0(\dot{\varepsilon}^{TC}_{ij}\, n_{ij}) & g = 0 \text{ and } (\partial g / \partial \varepsilon^{TC}_{ij})\, \dot{\varepsilon}^{TC}_{ij} > 0 \\ 0 & g < 0 \text{ or } (\partial g / \partial \varepsilon^{TC}_{ij})\, \dot{\varepsilon}^{TC}_{ij} \leq 0 \end{cases} \tag{4c}$$

$$n_{ij} = (\varepsilon^{TC}_{ij} - \alpha_{ij}) / [(\varepsilon^{TC}_{kl} - \alpha_{kl})\, (\varepsilon^{TC}_{kl} - \alpha_{kl})]^{1/2} \tag{4d}$$

where λ_0 is a material constant specifying the rate of increase of ρ. Then, the creep-hardening variable q of equation (3b) is modified by using the above creep-hardening surface in order to describe the recovery of creep-hardening of the material after stress reversal as follow (Murakami and Ohno, 1982):

$$q = (1/2\lambda_0)[\rho + (\varepsilon^{TC}_{ij} - \alpha_{ij})\, (\sigma_{Dij} / \sigma_{EQ})] \tag{5}$$

2.3 Irradiation-affected thermal creep

The glide of dislocations is usually restrained by piling up of dislocations to various obstacles. Some restrained dislocations start to glide again by overcoming the obstacles due to climb of dislocations. The balance between glide and restraint of dislocations will determine the creep rate. Beside these transgranuluar glide of dislocations, the slides of grain boundaries produces cavities on the grain boundaries and the damage of material proceeds.

The effects of neutron irradiation on thermal creep are expressed both by neutron flux ϕ and by neutron fluence $\Phi = \int\phi dt$. Creep deformation is enhanced and creep damage is suppressed with neutron flux ϕ which is related to the production rate of interstitial atoms. On the other hand, creep deformation is suppressed and creep damage is enhanced with neutron fluence Φ which may represent the internal structure formed by previous neutron irradiation.

By means of the concept of continuum damage mechanics, the damage state of material by neutron irradiation as well as creep may be represented by a damage variable D. As discussed above, irradiation-affected thermal creep may be expressed as follow:

$$\dot{\varepsilon}^{ITC}_{ij} = (3/2)\ f(\sigma_{EQ}, \phi;\ q, D,\ \Phi)\ (\sigma_{Dij} / \sigma_{EQ}) \tag{6}$$

The effects of irradiation on thermal creep may be introduced into the constitutive equation by replacing coefficients with material functions of ϕ and Φ. By employing Bailey-Norton equation of thermal creep and Kachanov-Rabotnov creep damage theory, we have the constitutive equations of irradiation-affected thermal creep and evolution equations of damage as follows:

$$\dot{\varepsilon}^{ITC}_{ij} = (3/2)\ m_0 A(\phi,\Phi)^{1/m_0} q^{(m_0-1)/m_0}\ [\sigma_{EQ}/(1 - D)]^{(n_0-m_0)/m_0}\ [\sigma_{Dij}/(1 - D)] \tag{7a}$$

$$q = [1/2\lambda(\phi,\Phi)][\rho + (\varepsilon^{ITC}_{ij} - \alpha_{ij})\, (\sigma_{Dij} / \sigma_{EQ})] \tag{7b}$$

$$\dot{\alpha}_{ij} = \begin{cases} [1 - \lambda(\phi,\Phi)](\dot{\varepsilon}^{ITC}_{kl}\ n_{kl})\, n_{ij} & g = 0 \text{ and } (\partial g / \partial \varepsilon^{ITC}_{ij})\, \dot{\varepsilon}^{ITC}_{ij} > 0 \\ 0 & g < 0 \text{ or } (\partial g / \partial \varepsilon^{ITC}_{ij})\, \dot{\varepsilon}^{ITC}_{ij} \leq 0 \end{cases} \tag{7c}$$

$$\dot{\rho} = \begin{cases} \sqrt{(2/3)}\lambda(\phi,\Phi)(\dot{\varepsilon}^{ITC}_{ij}\ n_{ij}) & g = 0 \text{ and } (\partial g / \partial \varepsilon^{ITC}_{ij})\ \dot{\varepsilon}^{ITC}_{ij} > 0 \\ 0 & g < 0 \text{ or } (\partial g / \partial \varepsilon^{ITC}_{ij})\ \dot{\varepsilon}^{ITC}_{ij} \leq 0 \end{cases} \tag{7d}$$

$$n_{ij} = (\varepsilon^{ITC}_{ij} - \alpha_{ij}) / [(\varepsilon^{ITC}_{kl} - \alpha_{kl})(\varepsilon^{ITC}_{kl} - \alpha_{kl})]^{1/2} \tag{7e}$$

$$\dot{D} = B(\phi,\Phi)\ [\sigma^{(1)}/(1 - D)]^{k_0} \tag{7f}$$

where $\sigma^{(1)}$ represents the maximum principal stress. $A(\phi,\Phi)$ and $B(\phi,\Phi)$ are material functions which prescribe the effects of irradiation on creep rate and damage rate, while m_0, n_0, k_0, A_0, a_1, a_2, a_3, a_4, B_0, b_1, b_2, b_3 and b_4 are material constants. As regards the material functions $A(\phi,\Phi)$ and $B(\phi,\Phi)$, the identical functions to those of the previous papers are employed as follows:

$$A(\phi,\Phi) = A_0[1 + a_1(1 - e^{-a_2\phi})][1 + a_3(1 - e^{-a_4\Phi})] \tag{8a}$$

$$B(\phi,\Phi) = B_0[1 + b_1(1 - e^{-b_2\phi})][1 + b_3(1 - e^{-b_4\Phi})] \tag{8b}$$

Since no sufficient experimental data are available to specify material function $\lambda(\phi,\Phi)$, $\lambda(\phi,\Phi)$ is considered as constant λ_0 in this paper.

2.4 The constitutive equations of creep, swelling and damage under irradiation

The constitutive equations of creep, swelling and damage under irradiation formulated above are summarized as follows:

$$\begin{aligned} \dot{\varepsilon}^{C}_{ij} &= \dot{\varepsilon}^{ITC}_{ij} + \dot{\varepsilon}^{IIC}_{ij} \\ &= (3/2)\ m_0 A(\phi,\Phi)^{1/m_0} q^{(m_0-1)/m_0}\ [\sigma_{EQ}/(1 - D)]^{(n_0-m_0)/m_0}\ [\sigma_{Dij}/(1 - D)] \\ &\quad + (1/3)\dot{S}\delta_{ij} + (3/2)P\phi\sigma_{Dij} \end{aligned} \tag{9a}$$

$$q = (1/2\lambda_0)[\rho + (\varepsilon^{ITC}_{ij} - \alpha_{ij})(\sigma_{Dij} / \sigma_{EQ})] \tag{9b}$$

$$\dot{\alpha}_{ij} = \begin{cases} (1 - \lambda_0)(\dot{\varepsilon}^{ITC}_{kl}\ n_{kl})\ n_{ij} & g = 0 \text{ and } (\partial g / \partial \varepsilon^{ITC}_{ij})\ \dot{\varepsilon}^{ITC}_{ij} > 0 \\ 0 & g < 0 \text{ or } (\partial g / \partial \varepsilon^{ITC}_{ij})\ \dot{\varepsilon}^{ITC}_{ij} \leq 0 \end{cases} \tag{9c}$$

$$\dot{\rho} = \begin{cases} \sqrt{(2/3)}\lambda_0(\dot{\varepsilon}^{ITC}_{ij}\ n_{ij}) & g = 0 \text{ and } (\partial g / \partial \varepsilon^{ITC}_{ij})\ \dot{\varepsilon}^{ITC}_{ij} > 0 \\ 0 & g < 0 \text{ or } (\partial g / \partial \varepsilon^{ITC}_{ij})\ \dot{\varepsilon}^{ITC}_{ij} \leq 0 \end{cases} \tag{9d}$$

$$n_{ij} = (\varepsilon^{ITC}_{ij} - \alpha_{ij}) / [(\varepsilon^{ITC}_{kl} - \alpha_{kl})(\varepsilon^{ITC}_{kl} - \alpha_{kl})]^{1/2} \tag{9e}$$

$$\dot{D} = B(\phi,\Phi)\ [\sigma^{(1)}/(1 - D)]^{k_0} \tag{9f}$$

$$\dot{S} = C\langle 1 + Q\sigma_{kk}\rangle\ \phi\ [1 - e^{R(\chi-\Phi)} / \{1 + e^{R(\chi-\Phi)}\}] \tag{9g}$$

3. ANALYSIS OF CREEP UNDER IRRADIATION

Cyclic burn of plasma in fusion reactors induces significant changes of stress in the first wall both in magnitude and in direction. Therefore, the constitutive equations applied to the analysis of nuclear reactor components are required to express these creep behavior under general state of stress as well as irradiation. The validity and the utility of the constitutive equations (9) elaborated in the present paper are demonstrated below under unirradiated, post-irradiation and irradiation conditions subjected to some nonsteady stress conditions; e.g., uniaxial stress reversals and stress change in direction. Material constants are determined for 20% cold-worked type 316 stainless steel at 650 °C according to the similar manner discussed in previous papers (Murakami, Mizuno and Okamoto, 1991; Murakami and Mizuno 1991), as follows:

$$m_0 = 0.55,\ n_0 = 3.50,\ k_0 = 3.00,\ A_0 = 1.25\times10^{-11}\ \text{hr}^{-m_0}\cdot\text{MPa}^{-n_0},\ B_0 = 2.50\times10^{-10}\ (\text{MPa}^{k_0}\cdot\text{hr})^{-1},$$

$$\lambda_0 = 0.50,\ a_1 = 0.05,\ a_2 = 2.60\times10^{-19}\ (\text{n/cm}^2\cdot\text{hr})^{-1},\ a_3 = -0.09,\ a_4 = 2.60\times10^{-21}\ (\text{n/cm}^2)^{-1},\ b_1 = -0.95,$$

$$b_2 = 4.50\times10^{-19}\ (\text{n/cm}^2\cdot\text{hr})^{-1},\ b_3 = 1.30,\ b_4 = 2.60\times10^{-21}\ (\text{n/cm}^2)^{-1},\ C = 4.00\times10^{-25}\ (\text{n/cm}^2)^{-1},$$

$$Q = 4.75\times10^{-3}\ \text{MPa}^{-1},\ R = 1.25\times10^{-22}\ (\text{n/cm}^2)^{-1},\ \chi = 5.00\times10^{22}\ \text{n/cm}^2,$$

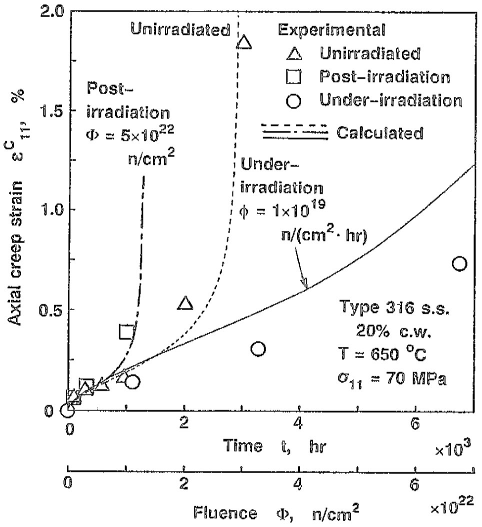


Figure 1 Axial creep strain ε^C_{11} under uniaxial constant stress σ_{11} = 70 MPa

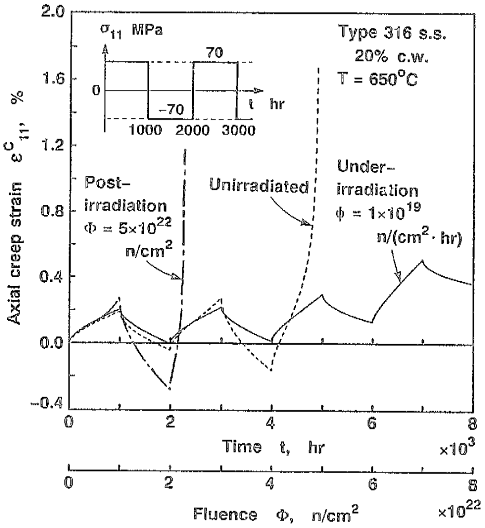
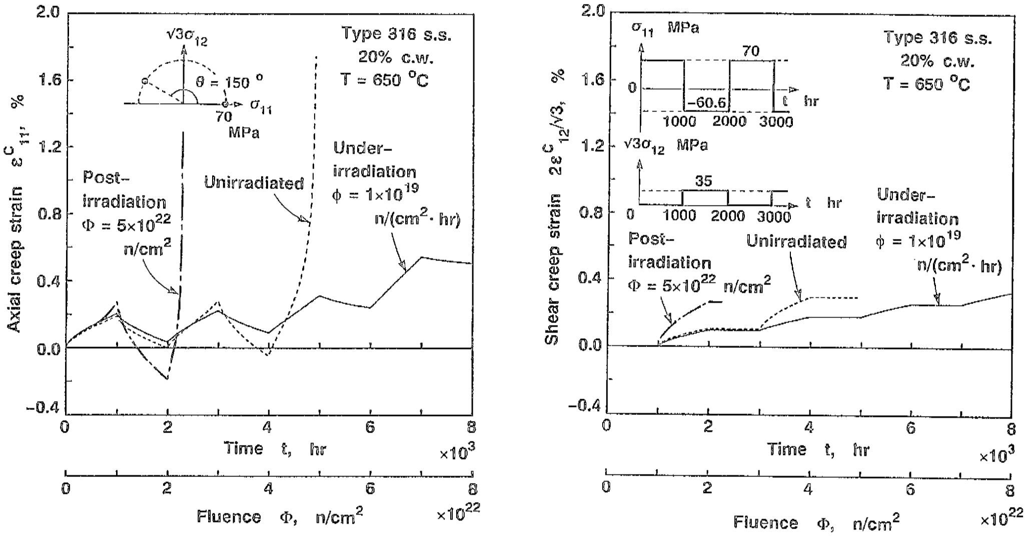


Figure 2 Axial creep strain ε^C_{11} under uniaxial stress reversals betwee σ_{11} = 70 MPa and −70 MPa with the intervals of 1000 hours.



(a) Axial strain ε^C_{11}

(b) Shear strain $2\varepsilon^C_{12}/\sqrt{3}$

Figure 3 Creep strains under change of stress direction between 0° and 150° at the intervals of 1000 hours.

$$P = 7.00\times10^{-28}\ (\mathrm{MPa\cdot n/cm^2})^{-1}. \tag{10}$$

Figure 1 shows the comparison of the predictions of equations (9) with the experimental result of creep by

Gilbert and Chin (1981) under unirradiated, post-irradiation and irradiation conditions under constant uniaxial stress σ_{11} = 70 MPa. As shown in Figure 1, the equations (9) can describe both brittle behavior of post-irradiation creep and ductile behavior of creep under irradiation with taking account of the effects of irradiation on creep curves; e.g., variations of rupture time and rupture strain and delay of tertiary creep. Figure 1 shows the validity of the introduction of the effects of irradiation and damage into the creep-hardening surface model.

Figure 2 shows the creep curves under three different conditions of irradiation subjected to uniaxial stress reversals between σ_{11} = 70 MPa and −70 MPa with the intervals of 1000 hours as shown in the figure. It will be observed that equations (9) describe well transient increase of creep rate after stress reversals. Conventional strain-hardening theory expressed in equations (3), in general, can not describe this increase of creep rate, accordingly underestimate relaxation of stress. It means that conventional strain-hardening theory predicts little redistribution of stress in the materials.

The effects of irradiation on creep show the similar tendencies under uniaxial constant stress condition; brittle behavior of post-irradiation creep and ductile behavior of creep under irradiation concerning rupture time and rupture strain. Rupture time under stress reversals condition in Figure 2, however, become about two times as long as rupture time under constant stress condition in Figure 1. It is because the maximum principal stress, which govern progress of creep damage, is zero when stress is negative $\sigma_{11} = -70$ MPa. Noting the behavior of creep under irradiation, the absolute value of creep rate under positive stress σ_{11} = 70 MPa is found to be higher than that under negative stress $\sigma_{11} = -70$ MPa beyond neutron fluence about $\Phi = 5\times10^{22}$ n/cm^2, which is caused by swelling at a constant rate after incubation period. Thus, creep strain under irradiation is found to increase with stress cycles in contrast with creep behavior under another irradiation conditions.

Finally, Figures 3 shows the creep under different irradiation conditions brought about by the multiaxial and nonproportional stress conditions, where the direction of stress change between 0° and 150° alternatively in σ_{11}–$\sqrt{3}\sigma_{12}$ stress plane under constant equivalent stress 70 MPa as shown in Figure 3(a) at the intervals of 1000 hours. Individual stress change, σ_{11} and $\sqrt{3}\sigma_{12}$, with time are shown in Figure 3 (b). Figures 3 (a) and (b) show axial creep strain ε^C_{11} and shear creep strain $2\varepsilon^C_{12}/\sqrt{3}$, respectively. Unfortunately, there is not experimental data corresponding to Figure 3 under post-irradiation and irradiation to confirm the validity of the constitutive equations (9). However, the constitutive equations (9) under unirradiated conditions is confirmed to describe adequately the transient increase of creep rate after salient stress change by comparing the experimental data with the equations (9) under unirradiated conditions. The effects of irradiation on creep show the similar tendencies under uniaxial stress conditions.

4. CONCLUSION

The constitutive equations which can describe creep and creep damage process precisely under general conditions of loading and irradiation, is prerequisite for accurate structural analyses of nuclear reactor components. In the present paper, the constitutive equation of creep, swelling and damage under unirradiated, post-irradiation and irradiation conditions formulated by the present authors previously was elaborated by extending the creep-hardening surface model to take account of the effects of irradiation. The results of the elaboration were demonstrated by showing that the constitutive equation could describe well the creep under various irradiation condition under multiaxial variable state of stress.

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