

Simplified Method for Elasto-Plastic Analysis of Structures under Variable Cyclic Loading

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ABSTRACT

A simplified method is proposed based on a cyclic elasto-plastic constitutive model of the metal for elasto-plastic analysis of mechanical structures under cyclic loading. The results from this method show a satisfactory consistency with those from detailed analyses by the elasto-plastic finite element method.

1 INTRODUCTION

Many engineering structures in nuclear reactors, thermal power stations, chemical plants and aerospace vehicles are subjected to cyclic thermo-mechanical loading, which is the main cause of structural fatigue failure. Over the past twenty years, designers and researchers have paid great attention to the research on elastoplastic analysis and life prediction of structures under cyclic loading.

A prerequisite for elastoplastic analysis is to construct a reasonable constitutive model for cyclic plasticity. In practical application, the model should have the ability to predict accurately various characteristic behaviors of materials, the number of material constants should be fewer and they can be determined conveniently, and the mathematical expression of the model should be simple enough to be incorporated easily into computer code and be implemented efficiently. Based on this viewpoint, the authors have proposed a simple model which has the same expression as the mixed hardening theory (Lei, 1988). The model has only five material constants, excluding those of elasticity, which can be determined by a strain-controlled cycling test. Good agreement has been found between theoretical prediction and experiment data under both strain-controlled and stress-controlled cyclic loading.

Although the whole history of stress and strain variation for every point in the structure can be obtained with the cyclic elasto-plastic finite element analysis program, this detailed analysis is very expensive and often is impractical for a routine analysis and design of structure. The most important data for a design are the mean strain and the amplitude of cyclic strain at some critical points, therefore it is possible to develop some simplified method to obtain directly these necessary data. In spite of that a lot of simplified methods have been proposed in the past ten years or more, it is difficult to say they are completely successful, as some of them lack a sound foundation of theory, or some of them only can be used in few special cases. The present paper

proposes a simplified method, termed an equivalent tracking method. The results from this method is in fair agreement with those from the detailed finite element analysis. Therefore the simplified method and the constitutive model provide a practicable way to the cyclic elasto-plastic analysis for engineering structures.

2 CONSTITUTIVE EQUATIONS

The most essential features of material behavior have been demonstrated in the former papers (Lei, 1988)

(1) The characteristic of monotonic stress-strain curve is independent from that of cyclic stress-strain curve.

(2) Strain-controlled cycles always tend to converge to a stable hysteresis loop after cyclic hardening or softening. The stable loop usually has zero mean stress.

(3) Stress-controlled cycles may be considered as a particular form of strain controlled cycles. Strain-controlled cycles are therefore considered as the essential pattern of cyclic loading.

The feature of material behavior mentioned above is an important basis for construction of constitutive model. Under cyclic loading, stress-strain curves at different loading phase may have a common expression:

$$\bar{\sigma} = E_0^p \bar{\varepsilon}^p + \gamma g(\bar{\varepsilon}^p) \quad (1)$$

$$E^p = \frac{d\bar{\sigma}}{d\bar{\varepsilon}^p} = E_0^p + \frac{dg(\bar{\varepsilon}^p)}{d\bar{\varepsilon}^p} \gamma \quad (2)$$

where $\bar{\sigma}$ and $\bar{\varepsilon}^p$ are effective stress and effective strain measured from the current yield point, γ is a discrete memory parameter. It remains constant on each loading branch. When cycles become stable, $\gamma=1$. E_0^p is the asymptotic slope of cyclic stress-strain curve. The proposed form of $g(\bar{\varepsilon}^p)$ is:

$$g(\bar{\varepsilon}^p) = \frac{\bar{\varepsilon}^p}{a\bar{\varepsilon}^p + b} \quad (3)$$

where a and b are material constants determined from a stable hysteresis loop.

Figure 1 illustrates how to determine γ . L and N denote the current and the last stress reversal points. The strain range between L and N would uniquely determine a stable hysteresis loop $L'N'$ ($\gamma=1$). Starting from N , it is assumed that the subsequent loading would go through D and pass across LL' at point F , so that

$$\overline{LF} / \overline{LL'} = C_v \quad (4)$$

where C_v is a material constant describing the rate of cyclic stabilization. It is determined by the relation between stress peaks from experimental data of a strain-controlled test. Finally γ is determined.

$$\gamma = (\sigma_{DF} - E_o^P \varepsilon_{DF}^P) / g(\varepsilon_{DF}^P) \quad (5)$$

γ does not change until the subsequent stress reversal at G. The next γ should be determined in a similar way.

The model has been incorporated in a finite element code to predict the responses of cyclic loaded structural components (Wang, 1989). The comparison with results from other theories and experiments demonstrated the effectiveness of the model.

Three structural components are considered here, a double-edge-notched plate, an axisymmetric notched bar and an axisymmetric pressure vessel component. These components are subjected to repeated or reversed loading.

Figure 2, Fig.3 and Fig.4 show the finite element models of the grooved bar, the notched plate and the nozzle-shell component respectively. Numerical results of the notched components are shown in Fig.5 and Fig.6. Figure 7 shows response of nozzle-shell connecting under repeated internal pressure.

3 SIMPLIFIED METHOD

For most structures, the history of load variation may be expressed as

$$Q(x, y, z, t) = q(t) \hat{Q}(x, y, z) \quad (6)$$

where $\hat{Q}(x, y, z)$ and $q(t)$ are the mode and the amplitude of load respectively. For the amplitude $\Delta\varepsilon$ of the strain variation at the critical point of structure, corresponding to the amplitude Δq of the load variation between two load reversals, this paper suggests following formula

$$\Delta\varepsilon = A K_1 \Delta e \quad A = (K_1 / K_o)^m \quad (7)$$

where Δe is the amplitude of the strain variation in the uniform stress-strain region of the structure, K_1 and A are respectively the elastic strain concentration coefficient and the plastic strain amplification coefficient at the critical point. $K_o = \frac{\Delta\sigma}{\Delta S}$ is the stress concentration coefficient, $\Delta\sigma$, ΔS are the amplitudes of stress variations respectively at the critical point and in the uniform stress-strain region. It can be seen that the coefficient A depends through K_1 on the configuration of structure and the mode of load, and through K_o on the amplitude of load and the hardening property of material. Moreover, the exponent m is varying with the amplitude of load Δq . When the structure is entirely in elastic state, $m=0$. With increase of Δq , the critical zone of structure enters into plasticity and it enlarges gradually. When $m=1$, equation (7) is equivalent to Neuber's formula.

For a given structure and a prescribed mode of load, function $m(\Delta q)$ can be derived from the finite element calculations twice under nonotonic loading respectively based on the virgin stress-strain curves and the stable elasto-plastic hysteresis loop of material, in order to consider the difference between them.

At the first load reversals, from equation (7) and constitutive equation (1) following equations can be derived to determine ε_1^p and σ_1

$$(E\varepsilon_1^p + \sigma_1)\sigma_1^m = K_i^{m+1} S_1^{m+1} \quad (8)$$

$$\sigma_1 = \sigma_s + E_o^p \varepsilon_1^p + \frac{\varepsilon_1^p}{a\varepsilon_1^p + b} \quad (9)$$

where S_1 is the stress in uniform stress-strain region of structure corresponding to the amplitude of load q_1 at the first load reversal and σ_s is yield stress. The values of E_o^p , a , b , and m are adopted from the case of virgin monotonic loading.

At the second and subsequent stress reversals the equations to determine $\Delta\varepsilon_p$ and $\Delta\sigma$ are following

$$(E\varepsilon^p + \Delta\sigma)\Delta\sigma^m = K_i^{m+1} \Delta S^{m+1} \quad (10)$$

$$\Delta\sigma = 2\sigma_c + E_p^o \Delta\varepsilon^p + \frac{\gamma\Delta\varepsilon^p}{a\Delta\varepsilon^p + b} \quad (11)$$

where Δs is the uniform stress amplitude corresponding to load amplitude Δq , and $2\sigma_c$ is the stress amplitude of elastic region of stable cyclic loop. the values of E_o^p , a , b , and m are adopted according to the case of cyclic loading.

By solving equations (8) and (9) or equation (10) and (11) the values of $\Delta\sigma$ and $\Delta\varepsilon_p$ at every load reversal can be determined in turn. Finally the mean strain ε_m and the strain amplitude $\Delta\varepsilon$ in the cyclic stable state can be obtained. Further the damage and remaining life of structure can be evaluated. The proposed method is applied to calculate equivalent stress-strain responses at critical points of a variety of structures. Examples, nozzle calculated previously by the detailed finite element analysis are calculated again. The results are given in Fig.8-10. Comparison between two methods shows a satisfactory consistency.

4 CONCLUSION

The simplified method presented in this paper has extensive applicability to all kinds of structures, and is proved to be highly effective. The results from this method contain the history of σ_m and ε_m as well as $\Delta\sigma$ and $\Delta\varepsilon$, therefore it provides a useful and practical way to calculate the damage and the fatigue life of structures.

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- Wang, X.-c. and Lei, Y.-T. (1989). Constitutive Model and Elastoplastic Analysis of Structures under Cyclic Loading, SMiRT-10, Vol.L, pp.13-18, Aug. 1989, Anaheim, California.

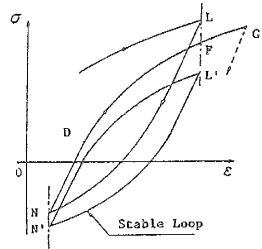


Fig. 1 The determination of γ

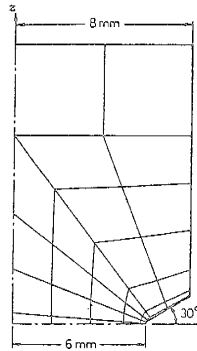


Fig. 2. Finite element model of notched bar

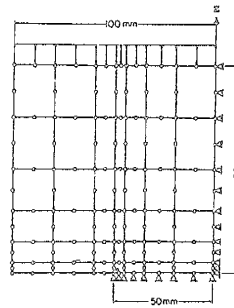


Fig. 3 Finite element model of notched plate

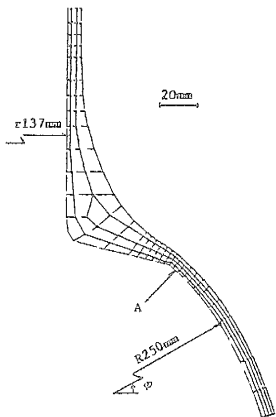


Fig. 4. Finite element model of nozzle-spherical shell

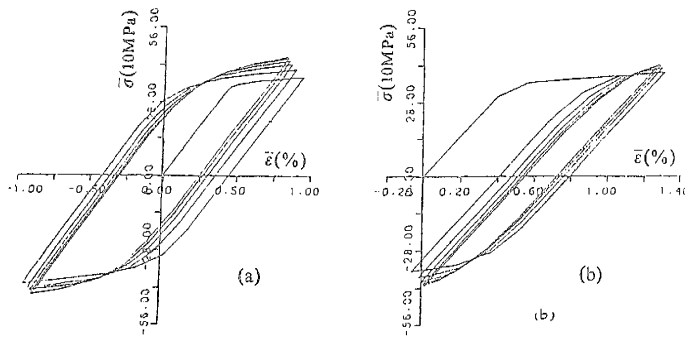


Fig. 5. Results of grooved bar from detailed finite element analysis
(a) reversed loading (80MPa—80MPa)
(b) partially reversed loading (100MPa—30MPa)

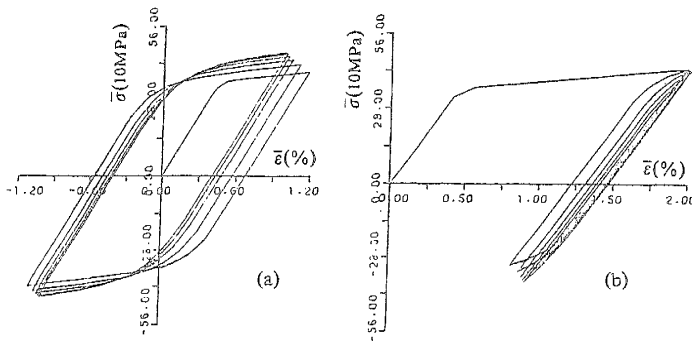


Fig. 6. Results of notched plate from detailed finite element analysis
(a) reversed loading
(b) repeated loading

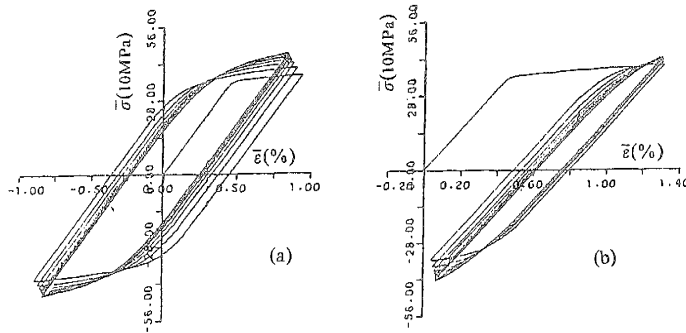


Fig.8. Results of grooved bar from proposed simplified method
 (a) reversed loading (80MPa—80MPa)
 (b) partially reversed loading (100MPa—30MPa)

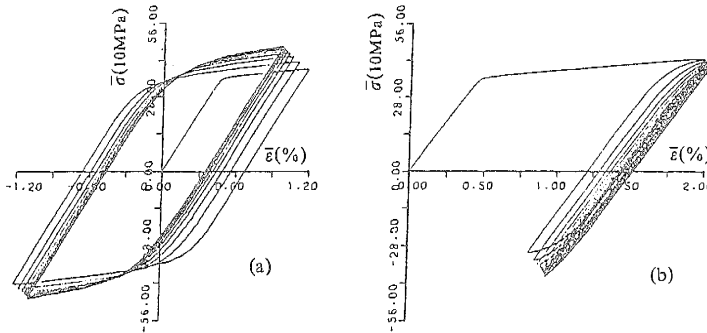


Fig.9. Results of notched plate from proposed simplified method
 (a) reversed loading
 (b) repeated loading

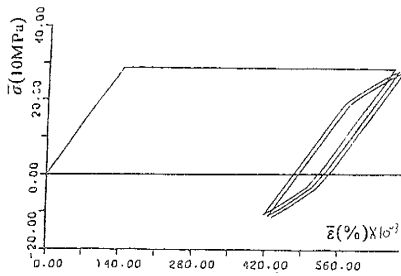


Fig.7. Results of nozzle-shell under repeated pressure (16Mpa—0) from detailed finite element analysis

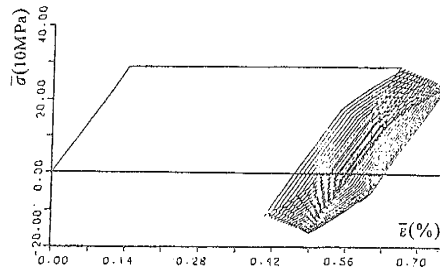


Fig.10. Results of nozzle-shell under repeated pressure (160MPa—0) from proposed simplified method