A Cumulative Damage Method in Non Proportional Loading to Predict Crack Initiation

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ABSTRACT

For a sequence of cyclic loadings of constant amplitude and containing overloads, we propose a method for damage cumulation in non proportional loading. This method uses as data cyclic stabilized states at non proportional loading and initiation or fatigue curve in uniaxial case.

For that, we take account of mean cell size and we define a stabilized uniaxial state cyclically equivalent to a non proportional stabilized state through a family of cyclic strain stress curves (C-S-S-C). Although simple assumptions like linear damage function and linear cumulation is used we obtain a sequence effect for difficult cross slip materials as 316 stainless steel, but the Miner rule for easy cross-slip materials. We show then a difference between a load-controlled test and a strain controlled test: For a 316 stainless steel in a load controled test, the non proportional loading at each cycle is less damaging than the uniaxial one for the same equivalent stress, while the result is opposite in a strain controlled test.

1 INTRODUCTION

During the last decade a lot of works have been concerned with non proportional loadings. Substantial progress has been obtained in constitutive laws, in crack propagation, and in the study of endurance limit. But the progress has been limited in calculus of damage cumulation before propagation in the elastoplastic case. In this paper we are interested with the damage cumulation in non proportional loading in view of crack initiation prediction. The initiation phase is just important when the loading amplitude is small, but even in high amplitude cycling the non initiation may be considered as a condition of integrity of the structure. The most important law used in damage cumulation is the Miner law. In uniaxial case, and for a sequence of constant amplitude loadings, the damage cumulation is given by:$D=\Sigma_i (n_i/N_f^i)$. N_f^i is the number of cycles to failure at amplitude $\Delta\varepsilon_i$ (resp. $\Delta\sigma_i$) and n_i is the number of cycles at this amplitude. Failure is obtained when D = 1. If we consider only initiation, N_f^i is the number of cycles to initiation obtained by the French's method. In this case D = 1 corresponds to initiation of a crack. In the following, N_f is the number of cycles to initiation(in a strain or load controled test), but for small amplitudes it may be replaced by the number of cycles to failure without an important error. The Miner law has been essentially modified to get a sequence effect (a high law cycling damage differently from a law-high cycling). For this non

SMiRT 11 Transactions Vol. L (August 1991) Tokyo, Japan, © 1991

linear damage functions and a non linear damage cumulation are used. This seems complicated[1] without ensuring us a better response. So practically the Miner law is usually used in design of fatigue. In fact as we are going to show, with a cyclic stress strain curve (C-S-S-C) depending on load history a linear damage function and a linear damage cumulation will give us a cumulation damage method which takes account of the sequence effect.

2 UNIAXIAL MODEL OF DAMAGE CUMULATION

2.1 Comments on cyclic-strain-stress-curve (C-S-S-C)

Different studies show that for materials with easy cross-slip as pure copper or pure aluminium C-S-S-C is independent of load history (for copper when $\Delta\varepsilon > 0.002$[2]), while for a 316 stainless steel it depends on monotonic or cyclic prehardening. Fig. 1 shows the result of an experiment on a 316 stainless steel at 20 degrees. The curve A shows a C-S-S-C obtained by an increasing incremental test on a single sample (pushpull test). The result is the same when we use for each experience a different sample. The curve B is obtained by a decreasing pushpull test from point M, and the curve C is an increasing one obtained from point N. C and B are practically superposed. This experience shows that the loading history of a small cyclic amplitude is cleared by a greater amplitude, and that the memory of the great amplitude is perfectly conserved for smaller ones. The experiment of fig. 1 is a strain controlled test. Experimental results show that [3] we obtain the same curve A in a stress-controlled test. We are not in possession of similar experiments to curves B and C in a load controlled test, but we show using Masing rule and a microscopical analysis that we will get probably the same results in a load controlled test.

2.2 A damage cumulation method using a linear damage fonction and a linear cumulation and taking account of the sequence effect.

We consider a sequence of constant amplitude loading, 1 and 2 (fig. 2) where for each amplitude we suppose that the number of cycles n_1 or n_2 is sufficient to get a stabilized state. For each amplitude we suppose also that the number of cycles to get an stabilisation is negligible with respect to the number of cycles during stabilized state[4].

Strain-controlled-test. Stabilization at load 1(fig. 2) corresponds to the point 1 on this figure. The damage, using a linear damage function is: $d_1=n_1/N_f(\Delta\varepsilon_1)$. Applying then the load 2, the stabilized point is 2. The damage is $d_2=n_2/N_f(\Delta\varepsilon_2)$ so the total damage with a linear cumulation is $D^{\varepsilon}_{(1-2)} = n_1/N_f(\Delta\varepsilon_1)+n_2/N_f(\Delta\varepsilon_2)$. This linear cumulation seems here logical because the memory of loading 1 is cleared at loading 2. Now we apply first 2 and then 1. The points of stabilization on fig. 2 are 2 and 1'. The damage cumulation is: $D^{\varepsilon}_{(2-1)} = n_1/(\tilde{N}_f(\Delta\varepsilon_1) + n_2/N_f(\Delta\varepsilon_2)$

$\tilde{N}_f(\Delta\varepsilon_1)$ is the number of cycles to initiation for the amplitude $\Delta\varepsilon_1$ but with cyclic prehardening as far as stabilization at point 2. (This makes sense because we supposed that the number of cycles to stabilization is negligible with respect to the number of cycles to initiation). We compare points 1 and 1', strain amplitudes are identical while the stress amplitude is greater in 1' than in 1 so obviously : $D^{\varepsilon}_{2-1} > D^{\varepsilon}_{1-2}$. This result is usually accepted in literature[1], a high low cycling is more damaging than a low-high cycling. But we will see that for a load controlled test we obtain an opposite result.

Stress-controlled-test. As before the sequence 1-2 gives the points 1 and 2 while the sequence 2-1 gives the points 2 and 1". The damages are respectively:

$D^{\sigma}{}_{(1-2)} = n_1/N_f(\Delta\sigma_1)+n_2/N_f(\Delta\sigma_2)$ and $D^{\sigma}{}_{(2-1)} = n_1/(\tilde{N}_f(\Delta\sigma_1)+n_2/N_f(\Delta\sigma_2)$
If we compare 1 and 1", the stress amplitudes are the same when the strain amplitude is greater in 1 than in 1". So we have obviously: $D^{\sigma}{}_{(2-1)} < D^{\sigma}{}_{(1-2)}$

We may remark that $D^{\varepsilon}{}_{(2-1)}$ is an upper bound value of $D^{\varepsilon}{}_{(1-2)}$ and $D^{\sigma}{}_{(1-2)}$ an upper bound value of $D^{\sigma}{}_{(2-1)}$. In the literature, it is not easy to find initiation or fatigue curves with cyclic prehardening. So we use the precedent remark, and we propose these overestimatings as a simplified method. In this method, for a stress-controlled test, we just need the initiation curve without prehardening (Wohler curve), In a strain-controlled-test we need just one initiation curve with a cyclic prehardening at a maximal amplitude which we may impose before any calculation.

Another remark is that when C-S-S-C is independent of load history as for pure aluminium, through the analysis made here there is not any sequence effect and so there is no difference between strain-controlled test and load-controlled test for damage cumulation before propagation.

3 MICROSTRUCTURAL ANALYSIS.

3.1 Uniaxial case.

Fig. 1 may be interpreted by a microstrucrural analysis[4]: the difference between curves A and B comes from the difference of the mean cell sizes due to different amplitudes at which they have been created. The superposition of B and C comes from the stability of the mean cell size obtained at a great amplitude for smaller one (Fig. 2). This remark helps us, through one hypothesis described below to explain why for some materials as pure copper or pure aluminium, the C-S-S-C is independent of prehardening.

Hypothesis: we suppose that a minimal mean cell size exists. We have already used this to give a description of the ratcheting phenomena [5]. We may suppose so that for the amplitude of loading greater than that for creating the minimal cell size, the C-S-S-C is single. With this hypothesis it is obvious that if the minimal cell size is reached at a very low amplitude, C-S-S-C is independent of prehardening for in service loadings. This explains the uniquness of C-S-S-C for pure Cu and mild steel.

3.2 Extension to the non proportional loading.

In non proportional loading we suppose that[4] for the same equivalent stress or strain (in Mises sense) the cell size is smaller in a non proportional loading than in a uniaxial one We may suppose that the limit cell size is obtained for smaller equivalent amplitudes in non proportional loading .

C-S-S-C in non porportional loading. For easy cross slip material as pure copper and pure aluminium or mild steel, C-S-S-C in uniaxial case and non proportional case are superposed. It has been shown for pure Cu, Al [6] and for mildsteel[7] that the microscopic structure in non proportional and uniaxial loadings are similar. With the hypothesis of existence of a minimal cell size we can state that for the usual amplitude of loading ($\Delta\varepsilon^p > 0.002$ for pure Cu) the cell size is stabilized at its minimal size. So in the uniaxial and non proportional loading we have the same cell size and so that the same C-S-S-C. Finally:

easy cross slip ⇔ C-S-S-C independent of prehardening ⇔ cell size stabilized (at minimal size) ⇔ {(non proportional C-S-S-C ⇔ uniaxial C-S-S-C)}

For a 316 stainless steel, the limit cell size is not reached at usual amplitudes, the microstructure and C-S-S-C depend on prehardening. But as shown on fig. 2 (curve B), the cell size is stable on curve B for all amplitudes smaller than the amplitude of point M. So for a metal defined by curve B we are in the same situation as for pure copper. We may so suppose that for all states on the curve B, the uniaxial and the non proportional C-S-S-C are superposed. This brings us to the following definition. We define a non proportional stabilized state as cyclically equivalent to an uniaxial stabilized state when the mean size of cell structure are identical in both cases. Practically this means that for a 316 stainless steel a non proportional stabilized state defined by $(\Delta\varepsilon^1_{eq}, \Delta\sigma^1_{eq})$, (point N fig. 3), is equivalent to a uniaxial stabilized state defined by $\Delta\varepsilon_{11} = \Delta\varepsilon^1_{eq}$ and $\Delta\sigma_{11} = \Delta\sigma^1_{eq}$ with a cyclic prehardening as far as stabilization at the point m_1. It is obvious that with such a definition for the case of a single C-S-S-C, the cyclically equivalent state is the same as the equivalent state previously defined by the Mises formula.

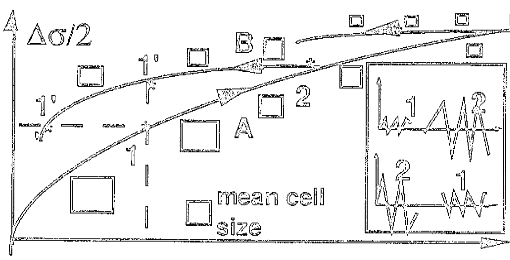

Fig. 2

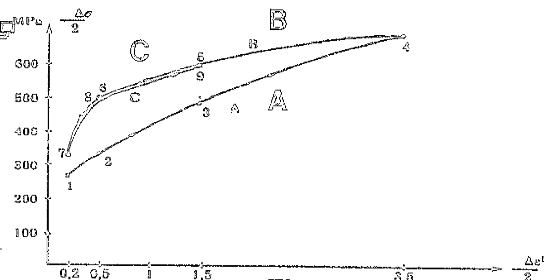

Fig. 1

4 A DAMAGE CUMULATION MODEL IN NON PROPORTIONAL LOADING.

For metals with a C-S-S-C independent of load history, for which uniaxial and non proportional C-S-S-C are superposed, a stabilized limit state $(\Delta\varepsilon_{eq}, \Delta\sigma_{eq})$ corresponds to the uniaxial point $\Delta\varepsilon_{11} = \Delta\varepsilon_{eq}$, $\Delta\sigma_{11} = \Delta\sigma_{eq}$ on the uniaxial C-S-S-C. For these metals, we suppose that the damage created at each cycle at stabilized state are identical in uniaxial case and in non proportional case (this doesn't take account of the anisotropics detected in some experiments). For these metals, fatigue curves are not very different in uniaxial and non proportional experiments[7]. For a 316 stainless steel fatigue curves are very different in uniaxial and non proportional loadings. The analysis of chapter 3 suggests us to suppose that the damage created in non proportional loading for each cycle at stabilized state $(\Delta\varepsilon_{eq}, \Delta\sigma_{eq})$ is identical to the one obtained at each cycle of a cyclically equivalent uniaxial stabilized state. Through this we give a method of damage cumulation calculus analogous to the uniaxial one through the following example.

4.1 Example of damage calculus in non proportional loading

We take a sequence of cyclic loading at constant amplitude. The stabilized states are designed successively by $(\Delta\varepsilon^1_{eq}, \Delta\sigma^1_{eq})$, $(\Delta\varepsilon^2_{eq}, \Delta\sigma^2_{eq})$, $(\Delta\varepsilon^3_{eq}, \Delta\sigma^3_{eq})$. For each state the number of cycles are n_1, n_2, n_3. We bring the above points on an uniaxial C-S-S-C diagram (fig. 3). These points are on the curves B_1, B_2, B_3 so the damage is:

$$D = n_1/\widetilde{N^{m_1}_{f_1}} + n_2/\widetilde{N^{m_2}_{f_2}} + n_3/\widetilde{N^{m_3}_{f_3}}$$

where $\widetilde{N^{m_1}_{f_1}}$ is the number of cycles to initiation in an uniaxial case for the amplitude $\Delta\varepsilon_{11} = \Delta\varepsilon_{eq}$ (in a strain controlled test) or $\Delta\sigma_{11} = \Delta\sigma_{eq}$ (in a stress controlled test), prehardened as far as stabilization at m_1. If the curves B_1, B_2, B_3 are not in the relative positions shown in the fig. 5, some of them are superposed[4]. Through this analysis, as explained in 2.2, during strain controlled test a non proportional loading is more damaging than an equivalent uniaxial loading. This is verified for a 316 [8]. But the result is opposite for a stress controlled test. A non proportional loading is less damaging than the uniaxial equivalent one. The experiments comparing non proportional and uniaxial fatigue curve in stress controlled test are seldom, but the only one we found[9] seems to confirm our proposition.

For the same reason as explained in 2.2 a simplified method may be proposed.

In a stress controlled test $\widetilde{N_f^m}$ may be replaced by N_f where N_f is the number of cycles to initiation without any prehardening. It means that N_f is the usual Wöhler curve used in literature. In this case a conservative response is obtained. In a strain controlled test $\widetilde{N_f^m}$ may be replaced by N_f^{max} where N_f^{max} is Manson-Coffin curve obtained after a cyclique prehardening at a maximal strain amplitude, here $\Delta\varepsilon_3$ for Fig .3.

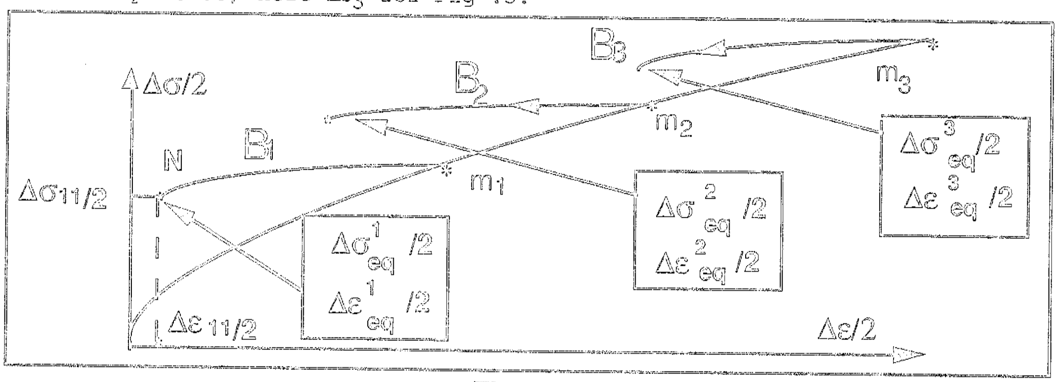

Fig. 3

5 A SIMPLIFIED METHOD OF DAMAGE CUMULATION TAKING ACCOUNT OF OVERLOADS.

Some uniaxial traction-compression tests show an increase in life after an overload in traction. This has been explained by the presence of a residual compressive state of stress. But there is some controversy on this analysis. Here we give an explanation based on initiation mechanism. Our analysis is in relation with precedent chapters. We suppose that the cell size in fact is determined by the maximal stress supported in the load history[4]: the greater the value, the smaller the cell size. With this hypothesis the C-S-S-C after an overload (Fig. 4) at σ_{max} or ε_{max} will be placed (Fig. 5) between the C-S-S-C without any prehardening and the C-S-S-C prehardened cyclically as far as stabilization for $\Delta\sigma = 2\ \sigma_{max}$ or $\Delta\varepsilon = 2\ \varepsilon_{max}$ [4].

To evaluate the effect of a single overload, using fig. 5 we may remark that in a load controlled test damaging at each cycle is less important for a prehardened material (point p") than for a virgin material (point A), while in a strain controlled test we get an opposite result (points p' and A).

When C-S-S-C is single, the overload effect may be neglected for damage cumulation because it doesn't modify C-S-S-C.

Whe C-S-S-C depends on the load history as for a 316, an upper bound of damage may be given. According to the precedent analysis in a load controlled test we can say that the damage at each cycle at point p" may be overestimated by each cycle at point A fig. 5 , while for a strain controlled test damage at point p' may be overestimated by the point A'.

That means for a stress controlled test, if we use the initiation curves without prehardening, a conservative result is obtained if we neglect the overload while that is not the case for a strain controlled test where we need an initiation curve with cyclic prehardening at $\Delta\varepsilon = 2\ \varepsilon_{max}$.

In the case of a stress controlled test for a periodic overload [4], we show that, when the period is great (great number of normal cycles between two overloads), the life time increases, while for a small period it decreases. This result is in agreement with some results of crack propagation[10], even if here we talk about initiation, a crack propagation may be considered as a set of initiations at the tip of the crack.

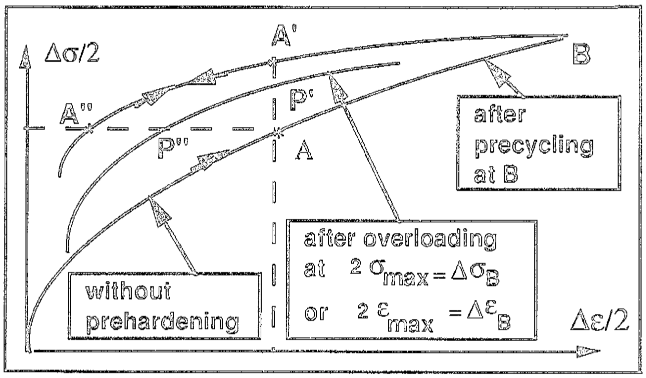

Fig. 5

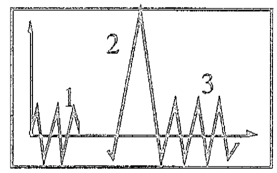

Fig. 4

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