A New Constitutive Model for Cyclic Plasticity

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1 INTRODUCTION

High temperature components as of fast breeder reactors often exhibit plastic deformation under repeated thermal loadings, and as a result, accurate cyclic plastic analysis methods are needed to establish the rational design methodology. Constitutive models play a substantial role in predicting inelastic structural behaviors.

The conventional linear kinematic hardening model using a bilinear idealization of stress-strain curve has some limitations in the capability of representing realistic stress-strain curve as well as cyclic hardening behavior as typical of austenitic stainless steels. In order to model the plastic behavior more accurately, Mroz(1967) proposed the concept of multi-surface model. Since then, a number of two-surface models have been proposed. Krieg(1975) and Dafalias and Popov(1976) presented their two-surface theories. Ouno and Kochi(1983) introduced the non-hardening strain surface into the Krieg's two-surface theory to represent cyclic hardening behavior. McDowell(1985) proposed an elaborate two-surface model. On the other hand, motivated by the experimental work by Phillips(1979), the two-surface model incorporating the concept of stress memory surface has been explored by Tseng and Lee(1983), Chang and Lee(1984), and Lu and Mohamed(1987).

This paper proposes a simple but accurate constitutive model which can represent primary features of plastic behaviors of cyclically loaded metallic materials under isothermal conditions. The model is developed within the framework of the two-surface plasticity model with a yield surface and a memory surface, incorporating a special plastic hardening modulus function defined in conformity with fundamental stress-strain property equations, such as monotonic and cyclic stress-strain curves under various strain ranges. The model is validated through its application to simulation of uniaxial cyclic and biaxial behaviors of cyclic hardening materials. Then, the model is implemented into the finite element computer program and applied, for demonstration, to a notched plate subjected to cyclic loading.

2 CYCLIC PLASTICITY CONSTITUTIVE MODEL.

The cyclic plasticity constitutive model proposed in this paper is a refined version of the two-surface model with a yield surface and a memory surface, which is proposed by Tseng and Lee(1983) and Lu and Mohamed(1987) based on the experimental observation by Phillips(1979). Features of the model exist in adopting a special analytic function of plastic hardening modulus for describing cyclic hardening behavior and in keeping complete equivalency between the model and the Ludwik type stress-strain property expressions. The formulation below is given for cyclic hardening materials under isothermal conditions.
2.1 Basic two-surface model

The yield and the memory surfaces are expressed using the von Mises yield function as:

\[ f = \frac{3}{2} (S - \sigma) : (S - \sigma) - Y^2 = 0 , \quad F = \frac{3}{2} S : S - Y^2 = 0 \] \hspace{1cm} (1), (2)

where \( S, \sigma \) and \( S' \) are, respectively, the deviatoric stress tensor, the back stress tensor defining the yield surface center and the deviatoric stress tensor corresponding to a stress state on the memory surface, and \( Y \) and \( Y' \) are the radii of the yield and memory surfaces, respectively. In this paper, the colon represents the scalar product of two tensors, e.g., \( S : S = S_{ij} S_{ij} \), where repeated indices represent summation. The yield surface separating the elastic and plastic regions of the material moves kinematically in the stress space and is always enclosed by the memory surface as shown in Fig. 1. It is assumed in this paper that the size of the yield surface remains constant, i.e., \( Y = Y_0 \). The memory surface, representing the maximum state of stress that ever existed in the stress space, grows in an isotropic fashion whenever the yield surface comes into contact with it. The radius \( Y' \) is defined as the maximum effective stress value in the previous loading history, i.e., \( Y' = \text{Max} \sqrt{\frac{2}{3} Y^2} \). Two surfaces coincide initially for virgin annealed materials.

The model structure is completed by specifying the translation rule of the yield surface. Here, the kinematic hardening rule proposed by Tseng and Lee (1983) is chosen. The center of the yield surface is assumed to translate in such a way that the two surfaces come into contact tangentially at the point of intersection of the direction of \( S/n \) with the memory surface (see Fig. 1). The Tseng-Lee kinematic rule is represented by

\[ \dot{\sigma} = \mu_T (\sqrt{Y' - Y} n - \dot{\alpha}) \] \hspace{1cm} \( \mu_T = \frac{(S - \sigma) : \dot{S}}{(S - \sigma) : \sqrt{\frac{2}{3}(Y' - Y)n - \dot{\alpha}}} \] \hspace{1cm} (3), (4)

where \( n \) is the unit normal vector to the memory surface at the imaged contact point.

When the memory surface is touched by the yield surface and the two surfaces move together in contact, the memory surface starts to expand isotropically. More precise treatments are needed to apply the above rules to the transition state of contact of the two surfaces. As another choice, the Mroz (1967) kinematic rule may also be used.

The flow rule yields the stress-strain relation in the rate form expressed as

\[ \dot{\sigma} = E^{ep} \dot{\varepsilon} \] \hspace{1cm} \( E^{ep} = E^e - \frac{9G^2}{(H' + 3G)Y^2} (S - \sigma)(S - \sigma)^T \] \hspace{1cm} (5), (6)

where \( \dot{\varepsilon} \) is the stress rate tensor, \( \dot{\varepsilon} \) is the total (elastic-plastic) strain rate tensor, \( E^e \) and \( E^{ep} \) are the elastic moduli and the elastic-plastic tangent moduli tensors, respectively, \( G \) is the elastic shear modulus, and \( H' \) is the plastic hardening modulus described below.

2.2 Fundamental stress-strain property equations

As the fundamental material properties to represent monotonic and cyclic behaviors, three kinds of stress-strain property equations illustrated in Fig. 1 are used: (a) monotonic stress-strain curve from tensile tests, (b) cyclic steady state stress range-strain range curve from cyclic tests under different levels of constant strain range and (c) cyclic steady state stress-strain hysteresis loops from the same tests of (b). It is well known that the Ludwik type equation fits well those experimental curves (Mada et al., 1966). The respective equations in the form of plastic strain and stress for three fundamental stress-strain curves (a), (b) and (c) are expressed as

\[ \varepsilon^p = \left( \frac{\sigma - Y_0}{K} \right)^m, \quad \Delta \varepsilon^p = \left( \frac{\Delta \sigma - 2Y_0}{K_1} \right)^n, \quad \varepsilon^p = \left( \frac{\sigma - 2Y_0}{K_2} \right)^{m_2} \] \hspace{1cm} (7), (8), (9)

where \( Y_0 \) is the proportional limit of materials, and \( K, m, K_1, m_1, K_2 \) and \( m_2 \) are material constants. In Eq. (9), \( \sigma' \) and \( \varepsilon^p \) are measured from the tip (in compression) of the steady state hysteresis loop, \( K_2 \) and \( m_2 \) depends on specified strain range.
2.3 Plastic hardening modulus function

There are several possible forms for analytic function of plastic hardening modulus \( H' \). The one proposed for the present two-surface model is given by

\[
H' = H''(Y')h(Y', \overline{\varepsilon}) \left( \frac{\delta_{in}}{\delta_{in} - \delta} \right)^{n}
\]  \hspace{1cm} (10)

where \( \overline{\varepsilon} \) is the accumulated equivalent plastic strain, \( \delta \) represents the current distance between the load point and the memory surface in the stress space, and \( \delta_{in} \) is the value of \( \delta \) at the most recent unloading event. The definition of \( \delta \) follows that of Tsang and Lee (1983) as

\[
\delta = -S : \nu + \sqrt{(S : \nu)^2 - S : \mathcal{S} + \frac{2}{3} \mathcal{S} : \mathcal{S}}
\]  \hspace{1cm} (11)

where \( \nu \) is the unit normal vector in the direction of \( \mathcal{S} \). In Eq. (10), the function \( H''(Y') \) represents the plastic hardening modulus of the memory surface and is given by

\[
H''(Y') = Km \left( \frac{K}{Y' - Y_0} \right)^{\frac{n}{m}}
\]  \hspace{1cm} (12)

The function \( h(Y', \overline{\varepsilon}) \) is the variable coefficient for describing transient hardening process. The condition necessary to generate a stable hardening after cyclic straining leads to a function of the form:

\[
h(Y', \overline{\varepsilon}) = h_a(Y') + (1 - h_a(Y')) \exp \left[ -\left( \frac{\overline{\varepsilon} - \overline{\varepsilon}_s}{Q} \right)^n \right]
\]  \hspace{1cm} (13)

where \( Q \) and \( n \) are the material constants representing the rate of cyclic hardening, \( h_a(Y') \) is the saturation value of \( h \), and \( \overline{\varepsilon}_s \) is the equivalent plastic strain accumulated until the first reverse loading occurs. Note that \( h(Y', \overline{\varepsilon}) = 1 \) during initial monotonic loading, and that Eq. (13) is used after the first reverse loading.

Applying the requirement that \( H' \) should be perfectly consistent with that of the hysteresis loop equation in the stabilized cycle, the final analytical form of \( h_a(Y') \) can be obtained as

\[
h_a(Y') = \left( \frac{1}{2} \right)^{\frac{n}{m}} (\frac{m}{m})^n K_1 \frac{s}{K} (2Y' - 2Y_0)^{\frac{n}{m} - \frac{1}{2}}
\]  \hspace{1cm} (14)

in which the parameter \( m' \) is chosen to vary from \( m_c \) to \( m_c' \) with accumulation of equivalent plastic strain as

\[
m' = m_c' + (m_c - m_c) \exp \left[ -\left( \frac{\overline{\varepsilon} - \overline{\varepsilon}_s}{Q} \right)^n \right]
\]  \hspace{1cm} (15)

and

\[
m_c = (1 - D) \exp \left( \frac{Y' - Y_0}{C} \right) + D, \quad m_c = m \exp \left( -\frac{Y_1 - Y_0}{R} \right)
\]  \hspace{1cm} (16)

The expression of \( m_c' \) is close approximation to \( m_c \) in Eq. (9) obtained under various strain ranges. The parameter \( m_c \) is the value of \( m' \) for the second loading after the first stress reversal, \( Y_1 \) is the size of the memory surface at the first stress reversal, and \( C \) and \( D \) are material constants. Eqs. (15)-(17) are applied only in case \( \overline{\varepsilon} > \overline{\varepsilon}_s \).

The present two-surface model needs 10 material parameters: \( K, m, Y_c, K_1, m_c, C, D, Q, n \) and \( R \). Among them, \( K, m, Y_0, K_1 \) and \( m_c \) are constants in fundamental stress-strain equations (7) and (8), and \( C \) and \( D \) in Eq. (16) are determined easily so as to fit the plot of \( m_c \) versus saturated \((Y' - Y_0)\) obtained from Eq. (9). \( Q \) and \( n \) can be determined easily from such cyclic hardening characteristics as will be seen later in Fig. 4. The parameter \( R \) is determined from the first reverse loading curve obtained at various strain ranges. If we do not need a high level of accuracy in early stage of cyclic behavior, \( m_c \) can be taken equal to \( m \), by putting any infinitely large value to \( R \). The proposed model, however, guarantees that the hysteresis loop eventually obtained in the steady state coincides almost exactly with that of the original equation used (Eq. (9)).
3 COMPARISON WITH MATERIAL BEHAVIOR TESTS

Uniaxial Cyclic Problems: The cyclic stress-strain behaviors for type 304 stainless steel at 550°C are represented by the present model. Fundamental average stress-strain properties due to Wada et al. (1986) are used with slight modifications. The model parameters are: \( E = 15681 \text{ Kg/mm}^2 \), \( Y_0 = 9.387 \text{ Kg/mm}^2 \), \( K = 34.47 \text{ Kg/mm}^2 \), \( m = 0.322 \), \( K_1 = 393.6 \text{ Kg/mm}^2 \), \( m_1 = 0.417 \), \( C = 10.04 \), \( D = 0.0352 \), \( Q = 0.3333 \), \( n = 1.0 \), \( R = 20.08 = 2C \), where \( E \) is Young's modulus. Steady state hysteresis loops using a Ludwik approximation are given in Fig.2. The simulated cyclic responses are shown in Fig.3. The comparison of the saturated loops with experimental data of Fig.2 shows a complete agreement. Cyclic hardening behaviors are shown in Fig.4, comparing with experimental data. The predicted results agree with the experimental data with adequate accuracy. This simulation confirmed that the present constitutive model was developed in consistently with essential features of cyclic stress-strain behaviors under various levels of constant strain range.

Non-proportional Path Problems: This example deals with the non-proportional path experiment performed by Kimura et al. (1981) and taken as benchmark by Lu and Mohanad (1987). The strain path is shown in Fig.5. A thin-walled brass tube is loaded firstly by tensile load up to a prescribed strain of \( \varepsilon = 1.0 \% \). This is followed by a non-proportional combined tension-torsion load. The loading path is characterized by a bilinear path in strain space. Two cases with angles of 60° and 120° between the first and second linear paths were predicted by the present model. The model parameters determined from the uniaxial stress-strain curve of Tokuda et al. (1981) are: \( E = 1.03 \times 10^6 \text{ MPa} \), \( \nu = 0.339 \), \( Y_0 = 110.0 \text{ MPa} \), \( K = 376.3 \text{ MPa} \), \( m = 0.294 \) , where \( \nu \) is Poisson's ratio. The value of \( R \) was taken infinitely large, for simplicity, that is, \( m_1 = m \). The value of \( Q \) was also taken infinitely large, since the cyclic hardening effects can be neglected in this problem. The simulated stress trajectories corresponding to the prescribed strain path are compared with the experimental ones in Fig.6. A fairly good agreement was obtained, though some differences were observed. It supports the appropriateness of the kinematic hardening rule adopted.

4 APPLICATION TO STRUCTURAL ANALYSIS

The proposed constitutive model was implemented in the general purpose finite element computer program FINAS (Iwata 1990) developed at Power Reactor and Nuclear Fuel Development Corporation. As an application example to structural analysis, a notched plate under uniaxial cyclic tension-compression loading was taken. The finite element idealization with 82 quadrilateral 8-node plate elements is shown in Fig.7. The plate was assumed to be made of type 304 stainless steel, and kept at uniform room temperature. The model parameters were determined from uniaxial monotonic and cyclic material data at room temperature. The model parameters are: \( E = 19800 \text{ Kg/mm}^2 \), \( \nu = 0.286 \), \( Y_0 = 16.0 \text{ Kg/mm}^2 \), \( K = 48.61 \text{ Kg/mm}^2 \), \( m = 0.3535 \), \( K_1 = 1382.2 \text{ Kg/mm}^2 \), \( m_1 = 0.808 \), \( C = 0.782 \), \( D = 0.1984 \), \( Q = 0.3333 \), \( n = 1.0 \), \( R = 13.564 = 2C \). The hysteresis loops shown in Fig.7 are obtained at the notch root by the finite element calculation using the present two-surface model. Significant strain hardening occurs at the root. The stress and strain distributions ahead of the notch root in the ligament are shown in Fig.8. It is seen that the stress peak only around the notch root increases with increasing number of cycles due to cyclic hardening. After about 20 cycles, a saturation stage is reached. It should be emphasized that such seemingly reasonable stress and strain distributions as shown in Fig.8 could not be expected with the conventionally used linear kinematic hardening model employing one kind of bilinear stress-strain relation over the plate.

5 CONCLUSIONS

A simple two-surface model has been presented. All the model constants can be determined easily from uniaxial tests. With an elaborate representation of plastic hardening modulus, accurate cyclic hardening prediction as well as complete consistency between the model and the fundamental stress-strain characteristics
represented by the Ludwik type equations were achieved. The accuracy and the
applicability of the model were demonstrated through uniaxial, biaxial and structural
behavior simulations. Further study is needed on the predictability for cyclic non-
proportional behaviors.

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REFERENCES

ics of Solids, Vol. 15, pp. 163-175.
41-646.
Dafalias, Y.F. and Popov, E.P. (1976). Plastic Internal Variables Formalism of Cyclic Plastic-
McDowell, D.L. (1985). A Two Surface Model for Transient Nonproportional Cyclic Plastic-
Engng Mech. 109, pp. 795-810.
proportional loading, ASCE J. Engng Mech. 112(8), pp. 806-820.
Wada, Y., Iwata, K., Aoto, K. and Kawakami, Y. (1986). Analytical Representation for the Cy-
clic Stress-Strain Hysteresis Loops of Type 304 Austenitic Stainless Steel, Computa-
tional Mechanics '86 (ed. C. Yagawa and S. N. Atluri), Springer-Verlag.

Fig.1 Two-surface model and three fundamental
stress-strain properties

Fig.2 Steady state hysteresis loops
by the fundamental property
equations for 304SS at 500 °C
Fig. 3 Simulated stress-strain responses for constant strain range cyclic tests (304SS, 550°C)

Fig. 4 Simulation of cyclic hardening behavior for constant strain range cyclic tests

Fig. 5 Bilinear strain path problem

Fig. 6 Experimental and predicted stress responses for bilinear strain path problem

Fig. 7 Finite element mesh for a notched plate and calculated stress-strain responses at the notch root

Fig. 8 Peak stress and peak strain distributions in the ligament of a notched plate