1 INTRODUCTION

The creep-fatigue life prediction of components used at elevated temperatures is one of the important issues in structural design, and it requires that the inelastic stress-strain response be evaluated. A number of unified constitutive models, which do not require a separation of creep and plastic deformation, have been developed for predicting the inelastic behavior of engineering steels. Some of these unified models are based on the overstress concept (Kreplin, 1967), and utilize back stress as an internal state variable. An experimental approach to determining back stress has been proposed and applied to several engineering steels (Ashby et al., 1987, Nakamura et al., 1989, Taguchi et al., 1990). A unified constitutive model, in which the internal state variables coincide with the experimentally determined back stress and overstress, has been proposed for Modified 9Cr-1Mo steel which shows typical behavior in cyclic strain softening (Taguchi et al., 1990).

In the present study, the back stress and overstress are determined experimentally for TYPE 316 stainless steel which exhibits typical cyclic strain hardening behavior. By modeling the back stress and overstress behavior, a unified constitutive model is proposed.

2 EXPERIMENTAL PROCEDURE

The tested TYPE 316 stainless steel was 22 mm-thick hot-rolled plate. The chemical composition is 0.015% C, 0.48% Si, 0.05% Mn, 0.022% P, 0.0023% S, 10.65% Ni, 16.54% Cr, 2.15% Mo and 0.100% N. The material was solution-treated at 1050°C for half an hour.

Push-pull strain-controlled continuous cycle tests were carried out at 600°C. The applied strain waveforms were a symmetric continuous cycle with a strain rate of $10^{-3}$ s$^{-1}$ and a holdtime cycle with a 500s or 3600s holdtime period in tension at strain ranges of 0.005 through 0.02.

3 DETERMINATION OF INTERNAL STATE VARIABLES

A smooth bowing-out shape was usually observed in the unloading part of a stress-strain hysteresis loop in continuous cycle tests, as illustrated in Fig. 1. An inelastic strain develops in the direction of the prior loading, just before unloading. Thereafter, the inelastic strain reaches an extreme.
value and then a reversal of its direction takes place. The overstress concept (Kreml, 1987) explains the bowing-out shape that the applied stress coincides with the back stress at the moment when the inelastic strain takes an extremity. This means that the back stress value can be determined by analyzing the stress value corresponding to this extreme (Asada et al., 1987, Nakamura et al., 1989). In order to determine this stress value, a quadratic curve-fitting technique was applied to the numerical stress-strain response data during unloading. All unloading curves in symmetric continuous cycles with a strain rate of $10^{-3}$ s$^{-1}$ and strain ranges of 0.005 through 0.02, were analyzed to generate values for the back stress $\sigma_b$ and overstress $\sigma_o$, defined as follows:

$$\sigma_o \text{peak} = \sigma \text{peak} - \sigma_b \text{peak}$$

where, superscript "peak" means "max" at a tensile peak or "min" at a compressive peak.

The tested steel showed typical cyclic strain hardening behavior, as indicated in Fig. 2. By investigating the cycle dependence of back stress and overstress it was found that both back stress and overstress showed cyclic strain hardening behavior. The monotonic and cyclic stress-, back stress- and overstress-inelastic strain relations are shown in Figs. 3(a) and (b), respectively. It is seen in Figs. 3(a) and (b) that the back stress is strongly dependent on the inelastic strain range.

If the back stress is assumed to be constant during short holdtime period, the relationship between overstress ratio and inelastic strain rate is as given in Fig. 4. The overstress ratio was defined as the overstress during holdtime period divided by that at the beginning of the period. A unique relationship is observed, regardless of strain range, though the scatter in data points is fairly large. The same relationship was also seen in early cycles. The inelastic strain rate was very sensitive to overstress at inelastic strain rates exceeding about $10^{-5}$ s$^{-1}$, and the overstress dependence of inelastic strain rate gradually fell when the inelastic strain rate was lower.

4 UNIFIED CONSTITUTIVE MODELING

Modeling of unified constitutive equations, in which the internal state variables coincide with the experimentally determined back stress and overstress, was carried out on the basis of the properties of the back stress and overstress.

Based on the overstress concept, it is assumed that the inelastic strain rate is a function of overstress $(\sigma - \sigma_b)$ and drag stress $D$. The drag stress is applied as a variable for describing the cycle dependence of overstress. Cyclic strain hardening is caused by the hardening of the drag stress and back stress. In order to describe the stress-strain hysteresis relation, the back stress was divided into two terms, $R_1$ and $R_2$. The term $R_1$ becomes constant rapidly, when an inelastic strain develops. The cyclic hardening is caused by the hardening of $R_2$. The strain hardening behavior of the stress-strain hysteresis relation is simulated by giving $R_2$ the saturated value $B$. The evolution equations for $R_1$ and $R_2$ are of Bailey-Orwan type. In order to simulate cyclic hardening behavior, the cyclic nonhardening region concept (Ohno and Kachi, 1986) was applied to the drag stress $D$ and the cyclic hardening function $B$.

The proposed constitutive equations are expressed in the following form under uniaxial stress conditions:

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\[ \dot{\sigma} = E (\dot{\varepsilon}_t - \dot{\varepsilon}_{in}) \]  
(1)

\[ \dot{\varepsilon}_{in} = F(\sigma - \sigma_b, D) \operatorname{sgn}(\sigma - \sigma_b) \]  
(2)

\[ \dot{\sigma}_b = \dot{\varepsilon}_1 + \dot{\varepsilon}_2 \]  
(3)

\[ \dot{\varepsilon}_1 = C_1 (B_0 \dot{\varepsilon}_{in} - R_1 |\dot{\varepsilon}_{in}|) \]  
(4)

\[ \dot{\varepsilon}_2 = C_2 (B \dot{\varepsilon}_{in} - R_2 |\dot{\varepsilon}_{in}|) \]  
(5)

\[ F(\sigma - \sigma_b, D) = A_1 (|\sigma - \sigma_b|/D)^{n_1} + A_2 (|\sigma - \sigma_b|/D)^{n_2} \]  
(6)

\[ D = D_0 (1 + L_1 q_1/(B_1 D_0))^{m_1} \]  
(7)

\[ B = B_0 ((1 + L_2 q_2/(B_2 B_0))^{m_2} - 1) \]  
(8)

\[ q_k = \Gamma_k |\dot{\varepsilon}_{in}| \quad (k=1, 2) \]  
(9)

\[ s_k = |\varepsilon_{in} - \alpha_k| - \rho_k \]  
(10)

\[ s_k = \Gamma_k r_k |\dot{\varepsilon}_{in}| \]  
(11)

\[ \alpha_k = \Gamma_k (1 - r_k) \dot{\varepsilon}_{in} \]  
(12)

\[ \begin{cases} 
\Gamma_k = 1 : s_k = 0 \text{ and } (\partial \varepsilon_k / \partial \varepsilon_{in}) \dot{\varepsilon}_{in} > 0 \\
\Gamma_k = 0 : s_k < 0 \text{ or } (\partial \varepsilon_k / \partial \varepsilon_{in}) \dot{\varepsilon}_{in} \leq 0
\end{cases} \]  
(13)

where \( \sigma, \sigma_b, \varepsilon_t, \varepsilon_{in} \) and \( E \) denote stress, back stress, overstress, total strain, inelastic strain and Young's modulus, respectively, and \( (\cdot) \) represents the partial derivative with respect to time. \( s_k \) represents the cyclic nonhardening region, and \( \alpha_k \) and \( \rho_k \) denote the center and size of the region, respectively.

The function \( F(\sigma - \sigma_b, D) \) was determined from the relationship of overstress to inelastic strain rate, as shown in Fig. 4. The parameter \( q_k \) is equal to an inelastic strain in the monotonic stress-strain relation, and is approximately equal to \( (\Delta \varepsilon_{in}/2\Gamma k) \) in the cyclic stress-strain relation at a constant strain range. Therefore, the material constants for eqns. (7) through (13) can be determined from the relationships given in Figs. 3(a) and (b). The material constants of \( C_1 \) and \( C_2 \) were determined from the monotonic stress-strain relation and the steady-state stress-strain hysteresis relation, respectively. Consequently, all material constants were determined, as listed in Table 1, based on the properties of the back stress and overstress as determined experimentally.

An example of the back stress behavior simulated by the proposed constitutive model is indicated in Fig. 5. A comparison between the simulated and experimental results is shown in Figs. 6 and 7. The proposed constitutive equations can simulate stress-strain behavior and cyclic stress relaxation behavior during cyclic strain hardening.

CONCLUDING REMARKS

For TYPE 316 stainless steel, the back stress and overstress were experimentally determined by investigating the unloading curve for the stress-strain
response. A unified constitutive model, in which the internal state variables coincide with the back stress and overstress, was proposed and proved able to simulate inelastic behavior during cyclic strain hardening.

Table 1. Material constants

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<td>$A_1$</td>
<td>$10^{-3}$ (s$^{-1}$)</td>
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<td>$C_1$</td>
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<td>$L_1$</td>
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<td>$B_0$</td>
<td>63.0 (MPa)</td>
<td>$D_0$</td>
<td>69.6 (MPa)</td>
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![Stress-strain response during unloading.](image)

**Fig. 1** Stress-strain response during unloading.

![Cyclic strain hardening behavior for peak stress, back stress and overstress.](image)

**Fig. 2** Cyclic strain hardening behavior for peak stress, back stress and overstress.
Fig. 3 (a) Monotonic and (b) cyclic stress–strain relations, back stress– and overstress–inelastic strain relations.

Fig. 4 Dependence of overstress on inelastic strain rate during hold time period.

Fig. 5 Back stress behavior simulated by proposed constitutive model.
Fig. 6 Comparison between simulated and experimental stress-strain curves:
(a) for 0.01 strain range at first and half-life cycles;
(b) for several strain ranges at half-life cycle.

Fig. 7 Comparison between simulated and experimental stress relaxation curves
during holdtime period at first and half-life cycles.

REFERENCES

Damage Evaluation of 304 Stainless Steel and 2.25Cr-1Mo Steel Based on the
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