Application of Constitutive Models for the Prediction of Multiaxial Inelastic Behaviour

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1 INTRODUCTION

In the last years a very strong effort in research was made to develop new constitutive theories for the inelastic behaviour of metals. Recent publications have shown that the comparison of constitutive equations needs special attention in the future to determine advantages and disadvantages of each model. The procedure for the identification of material constants have to be studied and extended to a sufficient data base.

Some work in this field has already been done by e.g. James et. al. (1987), who compared the theories of Bodner, Krieg, Miller (Schmidt's version) and Walker. For the evaluation of the material constants they performed uniaxial experiments. Using a more complex uniaxial history test, the prediction capabilties of each model were shown. Within a NASA-project Chan, Lindholm and Bodner (1988) did some investigations for the application of constitutive models in the structural analysis of hot section components of gas turbine engines. They selected the models of Bodner-Partom and Walker. Finally Hartmann (1988) examined the models of Hart, Miller (again Schmidt's version) and Walker. He used published parameter sets for uniaxial simulations of monotonic and cyclic experiments and compared the results with experimental data from literature.

The aim of the presented work within this scope is the comparison of the mean value formulation of Steck's stochastic model (1985) with the material model of Miller (MATMOD) (1976). Both models have been implemented in a finite-element code and multiaxial model predictions were compared with experimental findings for a plate with a central hole.

2 CONSTITUTIVE MODELS

2.1 Stochastic Model in the Mean Value Formulation

The stochastic model used in this paper is a mean value formulation of the complete model by Steck (1985,1989). The basis of Steck's model is the description of the deformation process by the use of flow-units and internal barriers or obstacles, where flow-units can be dislocations or dislocation packages. Internal barriers are formed by solute atoms, grain boundaries, dispersoid particles or entangled dislocations. The flow-units can overcome the obstacles when their activation is high enough. For the high temperature range the Arrhenius-function gives a good description for the kinetic equation driven by diffusion

SMiRT 11 Transactions Vol. L (August 1991) Tokyo, Japan, © 1991

processes. The activation energy in this function includes the activation by self-diffusion, the energy of the internal barrier, and the part of the external applied stress.

With this basis the plastic strain rate is given as a function of stress, temperature and the internal barrier-energy. The flow-units are assumed to be distributed in a certain number of classes of discrete values of the barrier-energy. Over time they can move in a class of a barrier-energy, which is higher than the one before, which leads to hardening, or they can move in a lower energy class, resulting in recovery. It exists also the possibility that they stay in the original class. Each of these processes is described by a transition probability. Hardening is activated as a function of stress and temperature whereas recovery is a function of temperature. The process over time with a discrete number of classes of internal barriers is described by a Markov-chain. Looking at the development of the internal energy in the classes, it is obvious that the mean value plays an important role. A reduction of all the classes of internal barrier-energy to one, describing the mean value, seems plausible. Such a formulation of the model leads to a set of differential equations, describing the uniaxial behaviour of the model used in this paper (Schettler-Koehler (1985); Steck, Kublik (1990)). For the use in a multiaxial loading case the extension to three dimensions is necessary. This is done by using the second invariant of the deviatoric stress S_{ij} , the von Mises equivalent stress σ_{eq}, and the equivalent plastic strain rate $\dot{\varepsilon}^{pl}_{eq}$ as follows:

$$\dot{\varepsilon}^{pl}_{ij} = \frac{3}{2} \cdot \frac{\dot{\varepsilon}^{pl}_{eq}}{\sigma_{eq}} \cdot S_{ij} \tag{1}$$

The equivalent von Mises stress σ_{eq} is given by:

$$\sigma_{eq} = \sqrt{\frac{3}{2} \cdot S_{ij} \cdot S_{ij}} \tag{2}$$

The internal variable in the stochastic mean value model describes isotropic hardening, which is independent of the loading direction, so that it remains a scalar variable in this formulation. Finally the set of equations for the three dimensional case is given by:

$$\begin{aligned} \dot{\varepsilon}^{pl}_{ij} &= \frac{3}{2} \cdot \left[\lambda' \cdot c_1' \cdot \exp\left[-\left(1 - \frac{\alpha - 1}{\kappa} \right) \cdot \frac{U_0}{RT} \right] \cdot \right. \\ &\quad \left. \left[2 \cdot \sinh\left(\frac{\Delta V \cdot \sigma_{eq}}{RT} \right) \right]^{1+1/\kappa} \cdot \exp\left[-\frac{F}{RT} \right] \right] \cdot \frac{1}{\sigma_{eq}} \cdot S_{ij} \end{aligned} \tag{3}$$

$$\dot{F} = \frac{1}{\lambda'} \cdot \dot{\varepsilon}^{pl}_{eq} - c_2' \cdot \exp\left[\frac{-\alpha \cdot U_0 + \beta \cdot F}{RT} \right] \tag{4}$$

with $\lambda', c_1', c_2', \alpha, \beta, \kappa, \Delta V$ - material parameters
F_0 - initial value for the internal variable F
U_0 - activation energy for self-diffusion
R - gas constant
F - internal variable - isotropic hardening

With this constitutive equation the inelastic behaviour of material including isotropic hardening can be modelled. There is one internal variable F which is correlated to the mean internal barrier-energy, and seven material parameters plus one initial value for the internal variable that have to be determined for each material by experiment. The main advantage of this model is the description of microscopic physical mechanisms in the material leading to a macroscopic set of equations that are applicable to engineering problems.

From the numerical point of view these equations are not stiff, so that they can be integrated using the explicit or implicit Euler integration method with a time step control.

2.2. Material Model of Miller (MATMOD)

The material model of Miller also belongs to the class of unified models, which means that there is no distinction between plasticity, creep and relaxation. The basis for the development of Miller's equations are the experimental investigations of Garofalo (1963). Garofalo showed that steady-state creep for low and high steady-state stresses can be described in a wide temperature range using the following function:

$$\dot{\varepsilon}_{ss}^{pl} = B' \left[\sinh(A \cdot \sigma_{ss}) \right]^n \tag{5}$$

Starting with this equation, one of Millers main assumptions is that this expression can also be used for the description of primary creep. In addition, Miller introduces two internal state variables for the description of isotropic and kinematic hardening. Using the steady-state condition in equation (5) he develops two evolution equations with a hardening term governed by the plastic strain rate and a recovery term dependent on the internal variable and temperature. For the description of cyclic processes Miller introduces an additional term in the evolution equation for the isotropic hardening variable to distinguish between isotropic and kinematic hardening, keeping the steady-state conditions unaltered.

Miller develops his equations from experimental findings in creep processes in the high temperature range. He extends the equations for the application in the low temperature area using the experimental observations that the activation energy decreases nearly linear for temperatures below 0.5 T_m (T_m: melting temperature in Kelvin). But for simulations in this area the correlation between model prediction and experimental results was not satisfactory. In the following, Miller's model is only used for high temperatures.

The model is given as a set of differential equations for the uniaxial case by Miller (1976). The generalization to three dimensions is done similar to the stochastic model. In Miller's model the internal variable D corresponds to F in the stochastic model as both describe isotropic hardening. This keeps D to a scalar quantity whereas the internal backstress G depends on the loading direction and describes kinematic hardening. To account for this a tensorial magnitude G_{ij} has to be introduced in the formulation. G_{ij} is already a deviatoric tensor so that the equivalent back stress is defined as:

$$G_{eq} = \sqrt{\frac{3}{2} \cdot G_{ij} \cdot G_{ij}} \tag{6}$$

In the kinetic equation the difference of the applied stress and the backstress results in an effective stress which is responsible for the activation of the dislocations. In the general

three dimensional formulation $S_{ij} - G_{ij}$ can be combined to an effective stress tensor σ_{ij}^{eff} as follows:

$$\sigma_{ij}^{eff} = S_{ij} - G_{ij} \tag{7}$$

$$\sigma_{eq}^{eff} = \sqrt{\frac{3}{2} \cdot \sigma_{ij}^{eff} \cdot \sigma_{ij}^{eff}} \tag{8}$$

The general set of equation for Miller's model is finally given by:

$$\dot{\varepsilon}_{ij}^{pl} = \frac{3}{2} \cdot B' \left[\sinh \left[\left[\frac{\sigma_{eq}^{eff}}{D} \right]^{1.5} \right] \right]^{n} \cdot \frac{1}{\sigma_{eq}^{eff}} \cdot \sigma_{ij}^{eff} \tag{9}$$

$$\dot{G}_{ij} = \frac{2}{3} \cdot \left[H_1 \cdot \dot{\varepsilon}_{ij}^{pl} - \left[H_1 \cdot B' \cdot \left[\sinh(A_1 \cdot G_{eq}) \right]^{n} \right] \cdot \frac{1}{G_{eq}} \cdot G_{ij} \right] \tag{10}$$

$$\dot{D} = H_2 \cdot \dot{\varepsilon}_{eq}^{pl} \cdot \left(C_2 + G_{eq} - \frac{A_2}{A_1} \cdot D^3 \right) - H_2 \cdot C_2 \cdot B' \cdot \left[\sinh \left(A_2 \cdot D^3 \right) \right]^{n} \tag{11}$$

with $B' = B \cdot \exp\left[-Q_0/RT\right]$

$A_1, A_2, B, C_2, H_1, H_2, n$	- material parameters
Q_0	- activation energy
D_0	- initial value for D
$G_{ij,t=0} = 0$	- initial value for G_{ij}
R	- gas constant
G_{ij}	- internal variable - kinematic hardening
D	- internal variable - isotropic hardening

Miller's model is able to simulate the inelastic behaviour of metals that show considerable hardening like aluminum and stainless steel. Isotropic and kinematic hardening and cylic phenomena like the Bauschinger effect and the cyclic saturation state can also be simulated using this model. In comparison to the stochastic model the background is more phenomenological while the resulting kinetic equation is in both cases the hyperbolic sine function leading to the same steady state behaviour. The model includes two internal variables (D and G_{ij}) and eight material parameters (inclusive the activation energy Q_0) plus one initial value for the internal variable D.

The equations by Miller are in some regions numerically stiff so that Miller (1976) had to use a Gear-multistep integration method to solve the initial value problem. A comparison of different integration techniques has shown that an implicit Euler integration method using a timestep control (Kublik 1991) is also applicable.

3 PARAMETER IDENTIFICATION

In the literature up to now the importance of the methods for the identification of the material parameters for a constitutive equation has been underestimated and the techniques used are quite often just trial and error. In addition, details of the experiments are not sufficiently available when the constitutive theory is applied to a different material. In the majority of the applications the models are only used with the materials for which the author has determined the parameters. First problems will occur when the stress or temperature field is beyond the range for which the parameters were identified. Further difficulties have been observed when the nominal same material leads to different material response.

For these reasons, in this study the attempt is described to identify the material parameters with methods that are generally valid. Numerical optimization techniques have been applied for both models using mainly uniaxial creep experiments for the determination of the constants. In addition both, uniaxial as well as multiaxial, experiments were carried out with the austentic stainless steel AISI 316L from the same charge (Steck,Kublik 1990). The goal of the work was to get a good result in the parameter finding for the uniaxial case so that in the multiaxial case an adequate prediction of the material response can be given.

The numerical technique employed for the parameter estimation is the evolution strategy (Kublik 1989). It is a stochastic method which has the advantage that a local minimum can be overcome, any boundary condition can be included without restricting the method too much, and it can be employed easily. The disadvantage is the amount of computer time needed for the optimization. Comparing it to the Monte-Carlo-method it has the advantage that it is a method with a memory directed towards the optimum. The method works like biologic evolution. A parents configuration (the initial parameter set) comes up with new parameter sets (children) by random generation. These new parameter sets will be included in the population (restricted to a certain number of offspring) if they are satisfactory or they are refused if they are worse than any of the actual population. The quality of each parameter set is determined by comparison with uniaxial experimental results using weight functions for the plastic strain and the plastic strain rate. In the case of Miller's model an additional weight function for the internal variables was introduced to ensure that these variables will come to a steady state.

The uniaxial creep experiments were performed at several temperatures up to 750°C (1023 K) using the equipment of the Ecole Polytechnique Federal de Lausanne (EPFL). The melting temperature T_m of the austenitic stainless steel is 1810 K so that 1023 K is 0.57 T_m. Creep tests have been carried out for five stresses at this temperature. As the other temperatures were lower than 750°C which is already close to 0.5 T_m no other experimental tests were included in the parameter finding. A good result in fitting the parameters could be achieved for both models as shown in figure 1 and 2. The primary creep region is small but the amount of primary creep is given quite correctly by both models. In the parameter identification for another material, aluminum, it could be shown (Kublik 1991), that the stochastic model gives a better simulation of the primary creep region than Miller's model, while the steady state is given equally good in both models.

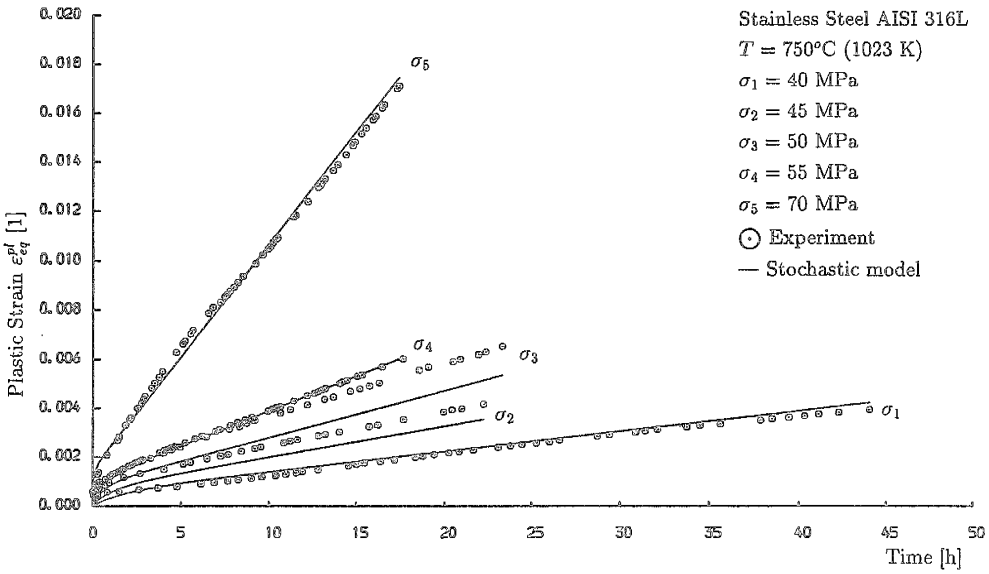

Figure 1: Representation of creep experiments by the stochastic mean value model for AISI 316L

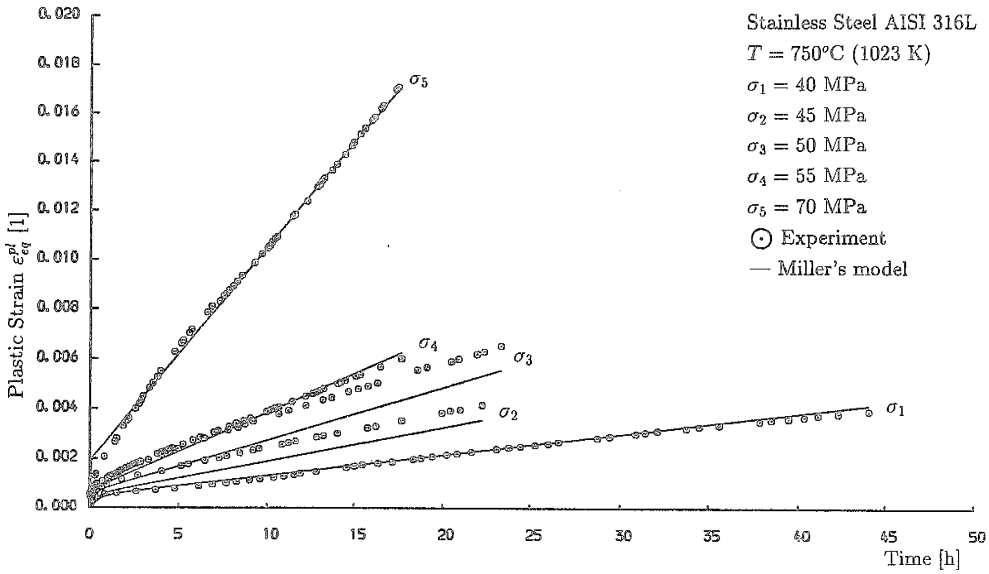

Figure 2: Representation of creep experiments by Miller's model for AISI 316L

In addition to the creep experiments some strain controlled relaxation tests carried out at the EPFL were used for identification purposes. In figure 3 it is shown that the relaxation behaviour after loading is well described by both models leading to approximately the same final stress value as found in the experiment.

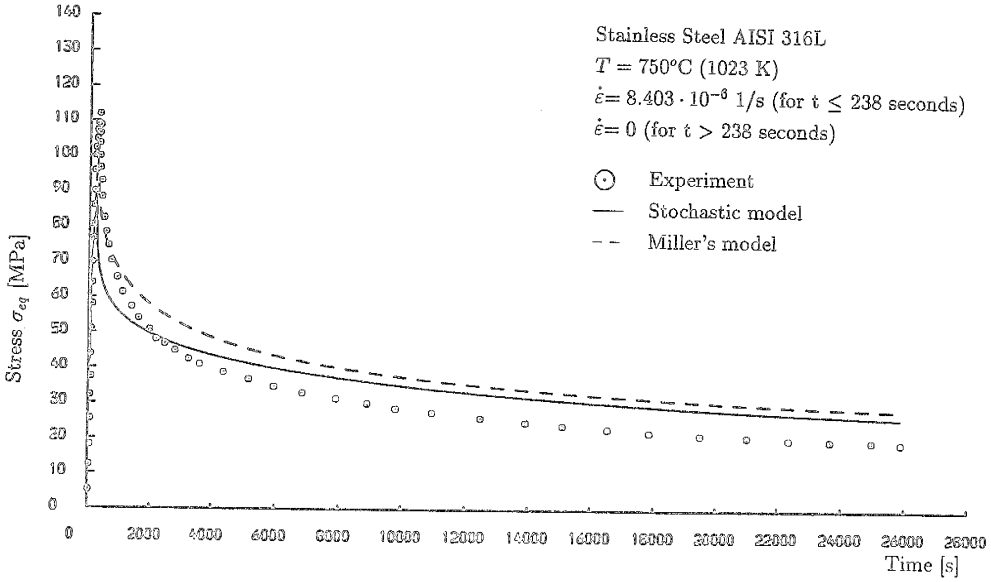

Figure 3: Representation of relaxation experiments by both models for AISI 316L using the determined parameter set

4 MULTIAXIAL EXPERIMENTS

4.1 Experimental Setup

The use of constitutive equations in the high temperature area was up to now mainly connected with uniaxial experimental findings. The measurement of multiaxial strain fields was almost impossible as strain gauges have a limited temperature range and piezoelectric measurement techniques need a large specimen which is very complicated to heat up to the high temperature.

In the area of fracture mechanics and within a special research program at the Technical University Braunschweig (SFB 319) a method could be developed that uses a photographical technique for the determination of the displacements on the specimen and the different components of the strain field (Andresen, Ritter and Steck 1989).

The specimen chosen for the investigation was a plate with a central hole. On top of the plate surface in the area of the hole a grid of 5 or 13 lines per millimeter out of titaniumdioxide was applied. This layer material has a high capability of resistance to high temperatures. The specimen was mounted in a force-controlled testing machine. The initial load was kept constant and the specimen started to creep. At the beginning of the test a photograph was taken as reference and later at different stages of the creep process further photographs were taken to fix the actual displacement field. The experimental setup is shown schematically in figure 4.

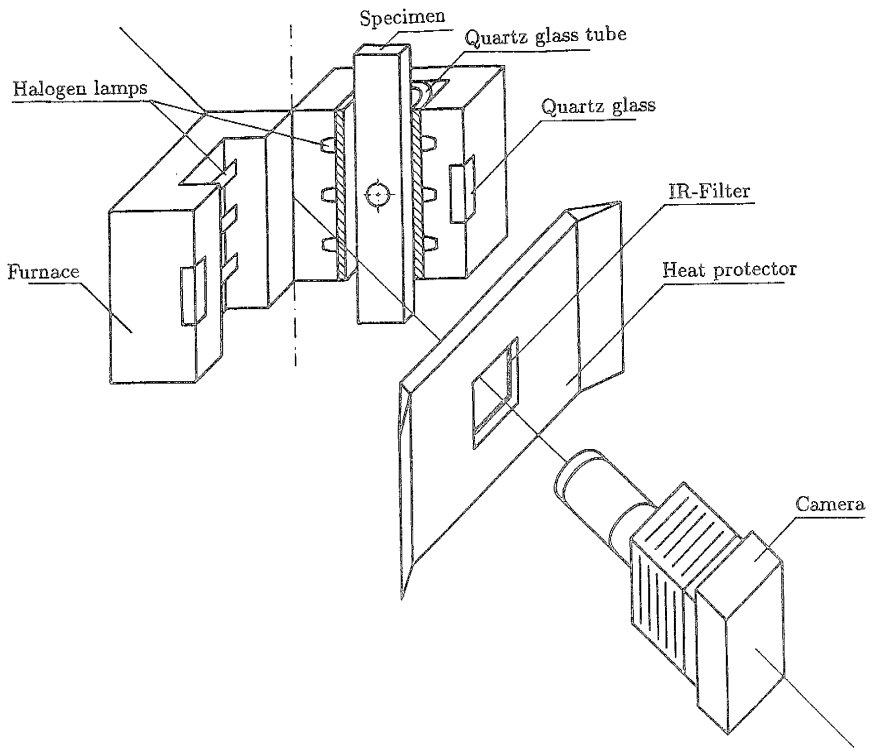

Figure 4: Experimental setup for the multiaxial experiments (open furnace)

At the end of the experiment a computer controlled analysis of the photographs is performed. The comparison of each of the photographs with the reference is used for the determination of the displacements at each point of the grid, followed by a differentiation by taking the mean value over several lines to obtain the strain field.

The method allows strain measurements of $\varepsilon \geq 0.003$. Small elastic strains cannot be resolved. The best results can be achieved with considerable total strains above one percent.

4.2 Comparison of Model Prediction and Experimental Result

The main issue of the multiaxial experiments is the comparison with the model prediction. In the multiaxial case, as given by the plate with the central hole, a numerical solution with finite elements was determined.

The nonlinear material behaviour is introduced into the finite element formulation by incremental pseudo loads on the right hand side. The finite element code developed for the simulation is the program FEMI (Finite Element Method for Inelastic deformations) that provides eight-noded rectangular isoparametric elements for geometric linear and physical nonlinear problems (Kublik 1991). Both sets of constitutive equations have been introduced in the program and for the numerical calculations a mesh of 112 elements as shown in figure 5 was used.

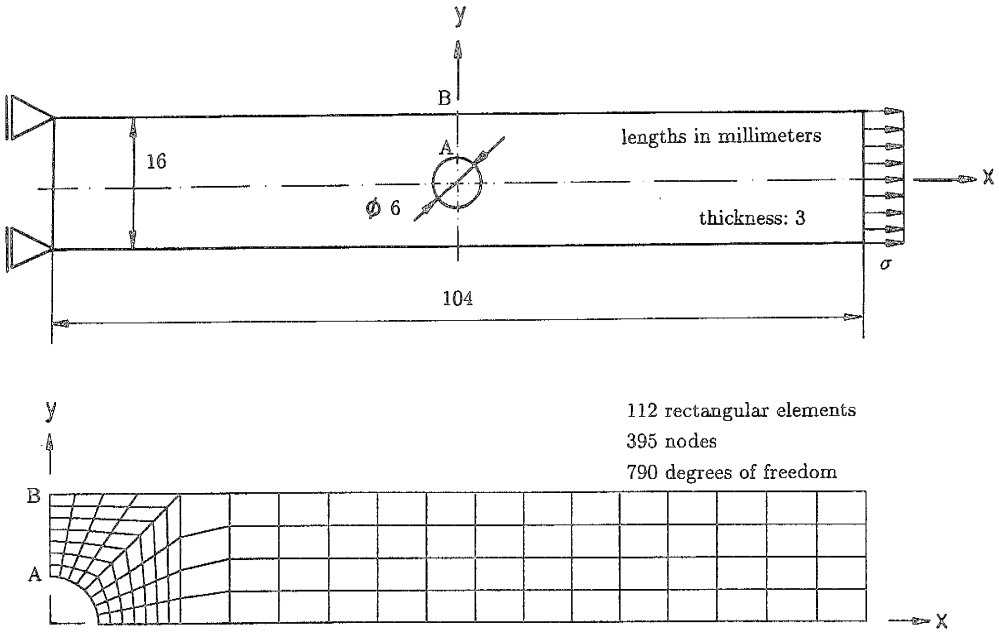

Figure 5: Finite Element mesh used for the prediction by the constitutive models

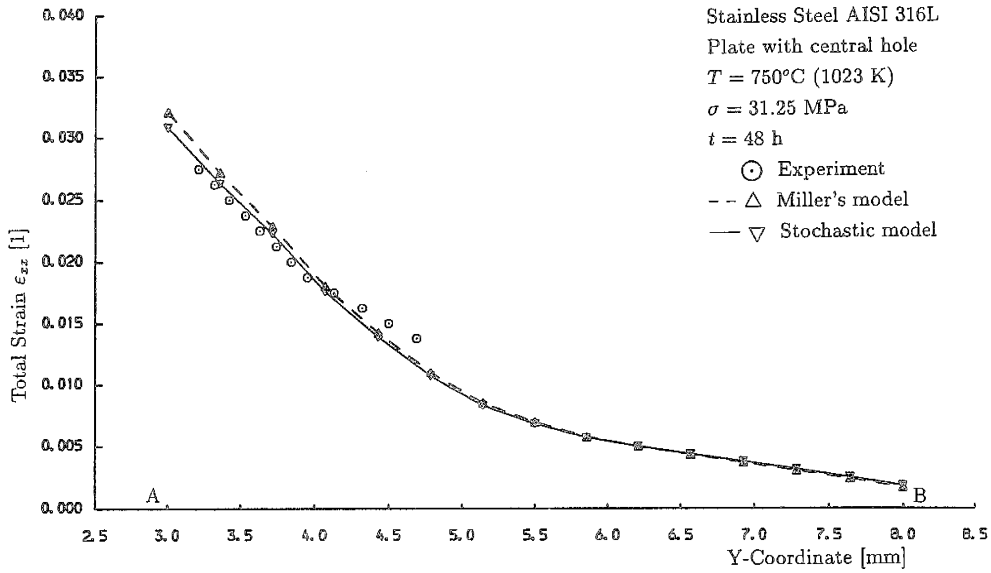

Figure 6: Comparison of the multiaxial experimental result and the stochastic mean value model and Miller's model (Section A-B / AISI 316L / 750°C)

In figure 6 the total strain given by the multiaxial experiment after 48 hours of creep is shown with the predictions of the stochastic mean value model and Miller's model. In the experiment a grid of 5 lines / mm was used, in the analysis of the photographs an average of 5 lines was taken and only strains above one percent were plotted.

The model predictions agree excellent with the experimental results in this case. It must be admitted that the experiment was designed so that the stresses occurring in the specimen in the section A-B are in the region of the stresses of the uniaxial creep curves used for the parameter determination.

Some additional multiaxial creep experiments were carried out with the same experimental setup at a temperature of 850°C (1123 K). The total strain predicted by the stochastic model in section A-B was in the region of the experiment while Miller's model was giving strains which were higher (Kublik 1991). A comparison of model predictions and experimental findings for uniaxial creep curves performed at the same temperature showed that again the strains predicted by the stochastic model were close to the experiment whereas the strains predicted by Miller's model were much higher.

5 CONCLUSIONS

The considered constitutive models are both based on high temperature creep processes leading to a kinetic equation with the hyperbolic sine function. While Miller's model has its origin in macroscopic experimental findings, the stochastic model describes physical mechanisms in the material.

A comparison with multiaxial experimental results shows the good prediction capabilities of both models. Most promising is the stochastic model which seems to describe the physical background of the inelastic behaviour properly.

More uniaxial and multiaxial experiments in the future will support the parameter identification in a wide temperature range and further interpretation of the model predictions, with the aim of a quantitative connection between macroscopic phenomena and processes on the microscale.

ACKNOWLEDGEMENT

The presented work was supported by the Deutsche Forschungsgemeinschaft within the Sonderforschungsbereich 319 (SFB 319): 'Stoffgesetze fuer das inelastische Verhalten metallischer Werkstoffe - Entwicklung und technische Anwendung'

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