

A Method to Develop Seismic Fragility Functions of Piping Systems

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1 INTRODUCTION

Recently, the method of *probabilistic reliability assessment* (PRA) is being developed to quantitatively estimate the reliability of structures based on probability theory. The method is also being applied to seismic reliability evaluation, which plays an important role in safety assessment of nuclear power plants (Schüeller & Ang, 1989). For this purpose, it is essential to develop the seismic fragility of the components of a system.

Seismic fragility is expressed by a fragility curve, which gives graphically the conditional failure probability of the component for various maximum input accelerations. Usually, fragility curves are assumed to have a certain probability distribution (e.g. lognormal) and are defined with estimated distribution parameters. These estimates are often based on opinions of experts, results of actual accidents or results of experiments. It is difficult, however, to apply such methods to systems in which conditions vary greatly (e.g. piping systems). Therefore, the evaluation of fragility curves by analytical or computational methods is desirable. For this purpose, specific studies have been conducted. For example, Schüeller et al. (1987) conducted a fragility study for a piping system based on time history analysis with a specific earthquake record.

In this paper, a method is proposed to construct fragility curves of piping systems. Assuming that the failure of a piping system is initiated by an initial crack existing on the pipe surface, a formula for the probability of failure during a given earthquake is derived through random vibration analysis with random crack parameters.

2 RANDOM VIBRATION ANALYSIS OF PIPING SYSTEMS

It is assumed that there is an initial crack with a semi-elliptic shape (Fig. 1) on the pipe surface and the failure of the pipe section occurs when the stress intensity factor at the crack exceeds the toughness of the pipe material. The stress intensity factor for a semi-elliptic shape, K_I , can be approximately given as a function of the normal far-field stress, σ_∞ . Therefore, the performance function of a pipe section is,

$$g = K_{IC} - K_I \approx K_{IC} - \frac{1.12}{E_2(a/b)} \sigma_\infty \sqrt{\pi a}, \quad (1)$$

where K_{IC} is the toughness of the material, a is crack depth, b is half of the crack width and E_2 is the complete elliptic integral of the second kind with the argument $k = \sqrt{1 - (a/b)^2}$.

Because the normal far-field stress can be expressed as a function of the displacement vector of the considered element, w^e , the performance function can be rewritten as follows,

$$g = K_{IC} - CR S K^e T^e w^e, \quad (2)$$

where C is the coefficient in Eq. (1), which is a function of a and b , R is the matrix converting the stress vector to far-field stress, which is a function of the direction angle ψ (see Fig. 1), S is the matrix converting the internal force vector to the stress vector, which is a function of position angle θ (see also Fig.1), K^e is the stiffness matrix of the element and T^e is the matrix converting the global coordinates to the local coordinates. It should be emphasized that this performance function is for a specific cross section in the piping system. It is possible, however, to identify the most critical cross section by analyzing the distribution of initial cracks and/or determining the stress distribution caused by earthquakes.

Kako et al. (1985) have analyzed the response of structures excited by base accelerations through random vibration. Piping systems, however, are multiply-excited systems induced by motions at more than one support; these exciting motions are generally not identical to each other. The method, therefore, was expanded to multiply-excited systems.

An example of a piping system is shown in Fig. 2. By modeling the system with the finite element method, the equation of motion of an n -degree-of-freedom system with l support points can be written with an absolute displacement vector w , as follows,

$$M\ddot{w} + C\dot{w} + Kw = \begin{bmatrix} M_u & M_{uk} \\ M_{uk}^T & M_k \end{bmatrix} \begin{Bmatrix} \ddot{w}_u \\ \ddot{w}_k \end{Bmatrix} + \begin{bmatrix} C_u & C_{uk} \\ C_{uk}^T & C_k \end{bmatrix} \begin{Bmatrix} \dot{w}_u \\ \dot{w}_k \end{Bmatrix} + \begin{bmatrix} K_u & K_{uk} \\ K_{uk}^T & K_k \end{bmatrix} \begin{Bmatrix} w_u \\ w_k \end{Bmatrix} = \begin{Bmatrix} L_u \\ L_k \end{Bmatrix}, \quad (3)$$

where M , C and K are the mass, damping and stiffness matrices of the system, respectively, and L is an applied force vector. The vector w is divided into an unknown part, w_u , (n components) and a known part, w_k , (l components) corresponding to the support motions.

The vector w_u can be expressed as the summation of the dynamic or relative displacement vector, w_d , and the pseudostatic displacement vector, w_s , that is, $w_u = w_d + w_s$. The vectors w_s and w_d satisfy the following equations.

$$w_s = -K_u^{-1}K_{uk}w_k = R_I w_k, \quad (4)$$

$$M_u \ddot{w}_d + C_u \dot{w}_d + K_u w_d = L_u - (M_u R_I + M_{uk}) \ddot{w}_k - (C_u R_I + C_{uk}) \dot{w}_k \approx -M_u R_I \ddot{w}_k. \quad (5)$$

It is noted that generally $L_u = 0$ and $M_{uk} = 0$ and the term relating to velocity is usually negligible. Applying modal analysis to Eq. (5) and assuming that the damping matrix can be diagonalized by the modal matrix, the equation becomes,

$$\ddot{q} + \Delta \dot{q} + \Omega^2 q = P \ddot{w}_k, \quad (6)$$

where $u = \Phi q$, Δ is the modal damping matrix, Ω^2 is the natural frequency matrix, P is the coefficient vector, and Φ is the modal matrix.

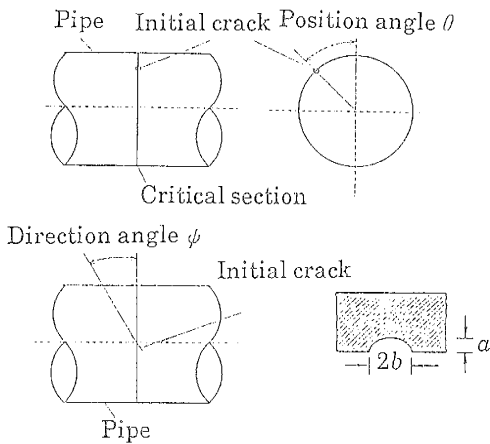


Fig. 1 Crack parameters

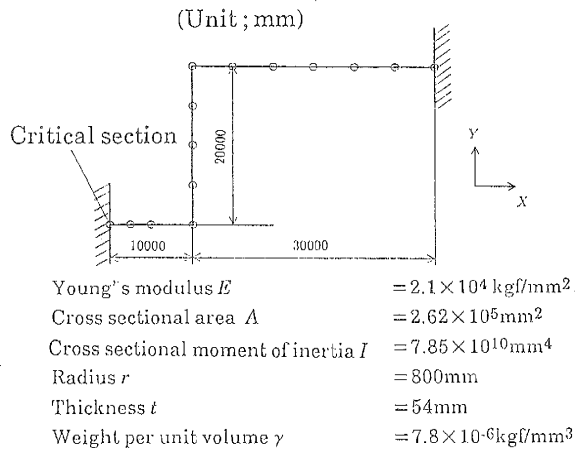


Fig. 2 Example of piping system

With the above expressions, the vector \mathbf{w}_u can be expressed, as follows,

$$\mathbf{w}_u = \Phi \mathbf{q} + \mathbf{R}_I \mathbf{w}_k = \mathbf{D} \mathbf{f} , \quad (7)$$

where $\mathbf{D} = \{ \Phi, \mathbf{R}_I \}$, $\mathbf{f}^T = \{ \mathbf{q}^T, \mathbf{w}_k^T \}$.

Under the assumption that the motions of the supports are stationary zero-mean Gaussian random processes, the vector \mathbf{f} also becomes a stationary zero-mean Gaussian random vector with the cross-spectral density function matrix \mathbf{S}_{ff} , which can be expressed as follows,

$$\mathbf{S}_{ff} = \begin{bmatrix} \mathbf{S}_{qq} & \mathbf{S}_{q\mathbf{w}_k} \\ \mathbf{S}_{\mathbf{w}_k q} & \mathbf{S}_{\mathbf{w}_k \mathbf{w}_k} \end{bmatrix} , \quad (8)$$

where the submatrices are $\mathbf{S}_{qq} = \mathbf{H}(\omega) \mathbf{P} \mathbf{S}_{\ddot{\mathbf{w}}_k \ddot{\mathbf{w}}_k} \mathbf{P}^T \mathbf{H}^*(\omega)$ and $\mathbf{S}_{\mathbf{w}_k q} = \mathbf{S}_{q\mathbf{w}_k}^* = \mathbf{H}(\omega) \mathbf{P} \mathbf{S}_{\ddot{\mathbf{w}}_k \ddot{\mathbf{w}}_k}$ in which $\mathbf{H}(\omega)$ is the modal frequency response function matrix and the superscript * indicates the complex conjugate.

Therefore the matrices $\mathbf{S}_{\ddot{\mathbf{w}}_k \ddot{\mathbf{w}}_k}$, $\mathbf{S}_{\ddot{\mathbf{w}}_k \mathbf{w}_k}$ and $\mathbf{S}_{\mathbf{w}_k \mathbf{w}_k}$ are submatrices of the matrix \mathbf{S}_{ff} . From random vibration analysis, the covariance matrix for the vector \mathbf{f} , \mathbf{V}_{ff} , can be described as follows,

$$\mathbf{V}_{ff} = \int_{-\infty}^{\infty} \mathbf{S}_{ff}(\omega) d\omega = \mathbf{L}_f \mathbf{L}_f^T , \quad (9)$$

where \mathbf{L}_f is a lower triangular matrix.

The matrix \mathbf{L}_f transforms the vector \mathbf{f} into the vector \mathbf{v} , where the components are Gaussian and are independent of each other, that is,

$$\mathbf{v} = \mathbf{L}_f^{-1} \mathbf{f} . \quad (10)$$

The performance function, thus, can be expressed as follows.

$$g = r - \mathbf{n}^T \mathbf{v} , \quad (11)$$

where $r = K_{IC} / |\mathbf{A}|$, $\mathbf{n} = \mathbf{A} / |\mathbf{A}|$, and $\mathbf{A}^T = \mathbf{C} \mathbf{R} \mathbf{S} \mathbf{K} e^T e \mathbf{D} e \mathbf{L}_f$.

3 FAILURE PROBABILITY WITH DETERMINISTIC CRACK PARAMETERS

The parameter $\mathbf{n}^T \mathbf{v}$ in Eq. (11) is a zero-mean narrow band Gaussian random process. For this case, the failure probability during the period t , $L(t)$ can be given by the following equation (Vanmarcke, 1975),

$$L(t) = 1 - (1 - e^{-r^2/2}) \exp\{-\nu_0 t (e^{-r^2/2}) (1 - e^{-\sqrt{2\pi} q r}) / (1 - e^{-r^2/2})\} , \quad (12)$$

where $\nu_0 = (2\pi)^{-1} \sqrt{\lambda_2/\lambda_0}$, $q = \sqrt{1 - \lambda_1^2/\lambda_0\lambda_2}$ and $\lambda_i = \int_0^\infty \omega^i G(\omega) d\omega$ which is the i -th moment of $G(\omega)$, the one-sided spectral density function of the process. It is noted that $G(\omega)$ can be written as follows,

$$G(\omega) = \mathbf{n}^T \mathbf{L}_f^{-1} (2R e \mathbf{S}_{ff}) \mathbf{L}_f^{-1T} \mathbf{n} . \quad (13)$$

Therefore, the failure probability for one crack can be calculated from the cross spectral matrix \mathbf{S}_{ff} .

Because an actual piping system may have more than one initial crack on the critical surface, the failure probability of the section should be calculated by a combination of the failure probabilities of all the cracks.

Let E_i be the event in which crack i causes the failure of the system. If there are n cracks, the total failure probability, P_n , then, is expressed as follows,

$$P_n = P[E_1 \cup E_2 \cup \dots \cup E_n] . \quad (14)$$

The probability, however, is difficult to obtain because the events E_i may not be independent. Instead of the exact probability, the upper and lower bounds can be obtained from the following expression (Ang & Tang, 1984),

$$p_1 + \max_{i=2}^n \left[\sum_{j=1}^{i-1} \{ p_i - q_{ij} \}; 0 \right] \leq P_n \leq \sum_{i=1}^n p_i - \sum_{i=2}^n \max_{j < i} (q_{ij}) . \quad (15)$$

where $p_i = P(E_i)$ and $q_{ij} = P(E_i \cap E_j)$.

The joint failure probability, which is required in the calculation, can be obtained as follows. Consider two initial cracks on the critical section. Let the failure of crack i be $g_i < 0$, where $i = 1$ or 2

(see Eq. (11)).

The two equations, $g_i=0$, can be expressed as two hyper-planes in the v space. The joint failure here means the process v outcrosses the planes at least once during the period t . Calculation of the joint probability is difficult because the two failure events are correlated. As an approximation, the upper and lower bounds of the joint probability are available. Let $x_i = \mathbf{n}_i^T v$ ($i=1$ or 2). The value x_2 can be written with x_1 and x_1' as follows,

$$x_2 = \rho x_1 + \sqrt{1-\rho^2} x_1', \quad (16)$$

where $x_1' = \mathbf{n}_1'^T v$, in which \mathbf{n}_1' is a vector normal to \mathbf{n}_1 and on a plane defined by \mathbf{n}_1 and \mathbf{n}_2 , $\rho = \mathbf{n}_1^T \mathbf{n}_2$, which is the correlation coefficient.

The event E_i can also be expressed as $r_i - x_{i\max} \leq 0$, and

$$x_{2\max} = \max(\rho x_1 + \sqrt{1-\rho^2} x_1') \leq \rho x_{1\max} + \sqrt{1-\rho^2} x_{1\max}' = x_{2U} \quad (17)$$

$$x_{2\max} = \max(\rho x_1 + \sqrt{1-\rho^2} x_1') \geq \rho x_1 + \sqrt{1-\rho^2} x_1' = x_{2L} \quad (18)$$

For the upper bound, q' , the following expression is obtained.

$$q' = P[r_1 - x_{1\max} \leq 0, r_2 - x_{2U} \leq 0] \geq P[E_1 \cap E_2] \quad (19)$$

This probability can be calculated by integrating the domain shown in Fig. 3. The integral is approximated as shown also in Fig. 3, which is easy to calculate since x_1 and x_1' are statistically independent.

For the lower bound, q'' , the following expression is derived.

$$q'' = P[r_1 - x_{1\max} \leq 0, r_2 - x_{2L} \leq 0] \leq P[E_1 \cap E_2] \quad (20)$$

Integrating the domain shown in Fig. 4 gives the required probability. A similar approximation is also necessary for this integral.

Combining the inequalities (15), (19) and (20) gives the upper and lower bounds of the total failure probability.

4 FAILURE PROBABILITY WITH RANDOM CRACK PARAMETERS

In Section 3 above, the parameters of the cracks were assumed to be known, and only the motions of the supports were assumed to be random. The crack parameters, however, should be also considered as random because it is difficult to know the real parameter values before an earthquake occurs.

The number of cracks is assumed to be known. The duration of earthquake t is, of course, the same for every initial crack. The toughness K_{IC} is assumed to be the same for all cracks. The probability for crack i is also a function of crack depth a_i , crack width b_i , crack angle θ_i and crack direction ψ_i . Therefore, if there are n cracks, the bounds of the failure probability are functions of the following independent parameters, $t, K_{IC}, a_1, \dots, a_n, b_1, \dots, b_n, \theta_1, \dots, \theta_n, \psi_1, \dots, \psi_n$. It should be noted, however, that the probability depends largely on θ_i 's and ψ_i 's. Therefore, the integral in terms of these parameters may be evaluated numerically by Monte Carlo method.

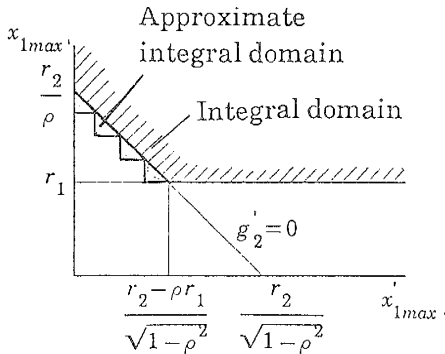


Fig.3 Integral domain for upper bound

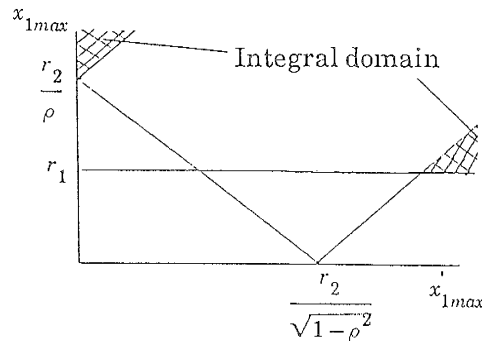


Fig. 4 Integral domain for lower bound

Then the mean of the lower bound, P_n' , can be written as follows,

$$E(P_n') = \int \cdots \int P f(\mathbf{y}) d\mathbf{y}, \quad (21)$$

where \mathbf{y} is a vector of the above random variables except for the θ_i 's and ϕ_i 's, and $f(\mathbf{y})$ is the probability distribution function of \mathbf{y} . In addition,

$$P = \sum_{i=1}^n \hat{p}_i - \sum_{i=2}^n \sum_{j=1}^{i-1} \hat{q}_{ij}, \quad (22)$$

where $\hat{p}_i = \iint p_i f(\theta_i) f(\phi_i) d\theta_i d\phi_i$,

and $\hat{q}_{ij} = \iiint q_{ij} f(\theta_i) f(\phi_i) f(\theta_j) f(\phi_j) d\theta_i d\phi_i d\theta_j d\phi_j$

Thus, the mean of the lower bound, P_n' , becomes a function of the number of initial cracks, n , as follows,

$$E(P_n') = n\bar{p} - (1/2)n(n-1)\bar{q}. \quad (23)$$

Similarly, the mean of the upper bound, P_n'' , is

$$E(P_n'') = n\bar{p} - (n-1)\bar{q}, \quad (24)$$

where $\bar{p} = E(\hat{p}_i)$ and $\bar{q} = E(\hat{q}_{ij})$.

Because it is difficult to calculate the integrals necessary for obtaining p and q , first order second moment approximation (Ang & Tang, 1984) is applied, that is,

$$E(g) \approx g(\mu_{X_1}, \mu_{X_2}, \dots, \mu_{X_n}) + \frac{1}{2} \left(\sum_{i=1}^n \left(\frac{\partial^2 g}{\partial X_i^2} \right) \text{Var}(X_i) \right). \quad (25)$$

As shown in Fig. 5, the lower bound has a maximum at $n_{cr} = p/q + 1/2$. Therefore, the probability for $n > n_{cr}$ is defined to be equal to p_{cr} , which is the probability with $n = n_{cr}$.

Next, the number of initial cracks is considered to be a Poisson random variable. Then, the mean for the bounds of the total failure probability, P_F , becomes

$$E(P_F) = \sum_{n=1}^{\infty} \frac{(\nu x)^n}{n!} e^{-\nu x} E(P_n'). \quad (26)$$

where νx is the mean number of initial cracks at the critical section.

5 CONSTRUCTING FRAGILITY CURVES AND EXAMPLES

The failure probability can be calculated by the method discussed so far for a given input earthquake described by a spectral density function S_{ff} .

If the piping system is installed at a specific site, the matrix S_{ff} can be described as follows,

$$S_{ff} = S_0 S_{ff0}, \quad (27)$$

where S_{ff0} is a reference cross spectral density function and S_0 is a parameter specifying the intensity of the earthquake. Using the proposed method, the bounds of the failure probability can be calculated for various values of S_0 .

At the same time, the seismic input parameter, R , (for example, maximum input acceleration) can be related to the parameter, S_0 ; that is,

$$R = f(S_0). \quad (28)$$

Plotting the bounds of the failure probability, p' and p'' versus the seismic input parameter, R , yields the required fragility curves.

An example piping system and its parameters are shown in Fig 2. The lowest five natural frequencies, which were used in the calculations,

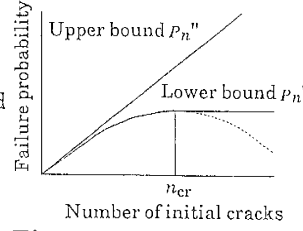


Fig. 5 Failure probability for number of initial cracks

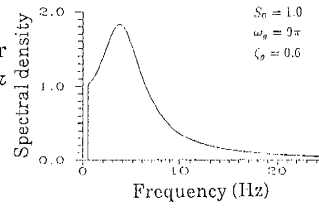


Fig. 6 Spectral density function of input acceleration

Table 1 Natural frequencies of example model

Mode No.	Frequency (Hz)
1	2.840
2	4.576
3	8.725
4	14.213
5	23.825

Table 2 Random parameters

Parameter	Mean	Standard deviation
Duration of Earthquake t (sec)	10.0	2.0
Toughness K_{IC} (kgfmm ^{-3/2})	350.0	35.0
Crack depth a (mm)	3.65	1.80
Crack geometry factor a/b	0.55	0.182

are summarized in Table. 1. The statistics of the parameters are tabulated in Table. 2. The spectral density function used in this example, S_{gg} , is shown in Fig. 6. The integrals were evaluated by Monte Carlo method using one thousand samples. The maximum acceleration, A_{max} , was defined by $3\sigma_{gg}$, in which σ_{gg} is the standard deviation of the input acceleration.

The fragility curves for various correlation coefficients, ρ , between the two supports were calculated. The curve for $\rho=1.0$ is shown in Fig. 7. The upper bounds for various correlation coefficients are shown in Fig. 8, for a relatively flexible systems. For this system, larger ρ 's tend to give higher failure probabilities. On the other hand, for a more rigid model, which has the same material and section parameters as the original, but the dimensions are half of the original; larger ρ 's give lower failure probabilities, as shown in Fig. 9. This is because the dynamic part of the response is dominant for the former case, whereas the pseudostatic part becomes dominant in the latter case.

6 CONCLUSIONS

An analytical method for constructing seismic fragilities of piping systems is proposed. The input earthquake can be described by its spectral density matrix, which is more general than a time history. The presence of multiple cracks at a critical section is considered in the calculation of the failure probability. The effectiveness of the method is demonstrated with examples, showing also that the influence of the correlation coefficients between the support motions depends on the rigidity of the piping system.

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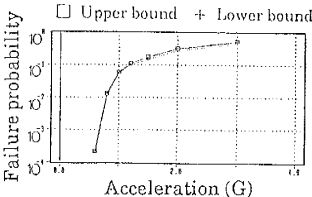


Fig. 7 Upper and Lower bound P_F for maximum input acceleration ($\rho=1.0$)

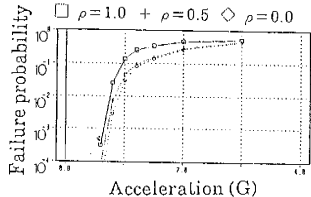


Fig. 8 Upper bound P_F for various correlation coefficient (Flexible model)

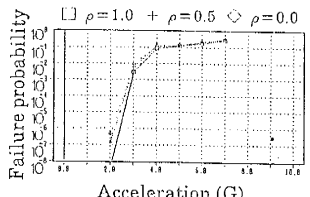


Fig. 9 Upper bound P_F for various correlation coefficient (Rigid model)