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Development of Constitutive Models for Fast Reactor Design
- Strategy of the study and results in the first half stage -

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1 ABSTRACT

R&D to enable a practical fast breeder reactor plant is proceeding in Japan, which is called “FaCT(Fast reactor Cycle Technology development). One of the key issues of R&D is to realize a reasonably small reactor vessel by eliminating the thermal liner which is installed inside the vessel in order to reduce thermal loading in the conventional design. Most important concern is the amount of the inelastic strain of the vessel accumulated around the liquid sodium surface which moves up and downward cyclically with start-up and shut-down. The aim of this study is to develop rational constitutive models that enable prediction of this kind of complex inelastic behaviors precisely and to prepare the design guide based on inelastic analysis. In this paper, the framework and strategy of the R&D and the results in the first half stage are introduced.

2 INTRODUCTION

R&D to enable a practical fast breeder reactor plant is proceeding in Japan. One of the key issues of R&D is to realize a reasonably small reactor vessel by eliminating the thermal liner which is installed inside the vessel in order to reduce thermal loading in the conventional design. Most important concern is the amount of the inelastic strain of the vessel accumulated around the liquid sodium surface which moves up and downward cyclically with start-up and shut-down. If we apply the existing design method, there is the possibility that the estimation of the strain amount becomes too conservative and exceeds the strain limit of the rule. The aim of this study is to develop rational constitutive models that enable prediction of this kind of complex inelastic behaviors precisely and to prepare the design guide based on inelastic analysis.

The requirements for the target constitutive models are as follows.

(1) expression of the nonlinearity of stress-strain relation of the material (316FR stainless steel in this study)

(2) applicability to cyclic loading conditions in the elastic plastic region

(3) prediction of stress- strain behavior properly under the temperature changing conditions (~ 600 deg. C)

(4) applicability to non-proportional multi-axial behaviors

(5) conservativeness of the analysis results for design evaluation

In order to develop the constitutive models that satisfy these requirements, we are proceeding a project consisting of the following five parts.

(i) Simulation analyses of the inelastic stress-strain behavior of the reactor vessel made of 316FR by using the existing analysis methods ; The aim is to understand the mechanism of the strain accumulation in the liquid surface traveling region including the effect of the primary stress and grasp the characteristic stress-strain-temperature behaviors and their ranges to be covered by the constitutive models roughly.

(ii) Development of candidate constitutive models ; The formulation of some constitutive models was implemented and basic check was done partially by comparing with the existing studies and the verification tests mentioned later. We are preparing two kinds of models considering practical use, one is a simple and convenient model and the other is a detailed and precise model. Concerning the expression of the nonlinearity of stress-strain relation, two methods are tried, i.e., the multi-linear approximation based on the multilayer back stress model(Ohno and Wang, 1993) and the smooth curve approximation based on the two-surface model(Iwata, 1993).

(iii)Incorporation of candidate constitutive models into the general purpose structural analysis system ; Since large scale analyses are needed in the practical design, the incorporation of the constitutive models and the study of effective numerical procedures are proceeded simultaneously.

(iv) Material property tests : These tests were conducted to get the basic property data of the target material and to determine the constants of constitutive models.

(v) Verification tests for the validity of the constitutive models ; Uniaxial tests with strain and temperature controlled under monotonic / cyclic loadings, biaxial tests with non-proportional loadings and structural element tests are conducted. The uniaxial and the biaxial tests were composed by introducing the characteristic behaviors obtained from the above simulation analyses of the reactor vessel. The structural element tests aim at verification of the constitutive models, focusing on the ratcheting phenomena.

This R&D started from late 2006FY and will finish at the end of 2009FY. Figure 1 shows the main items and their rough schedule. Based on the preliminary analysis results, the detailed test conditions were determined. In this paper the preliminary analysis of the reactor vessel, the simple constitutive model(MK-SRR), the basic material tests and some verification tests are mainly introduced as the results in the first half stage of the project.

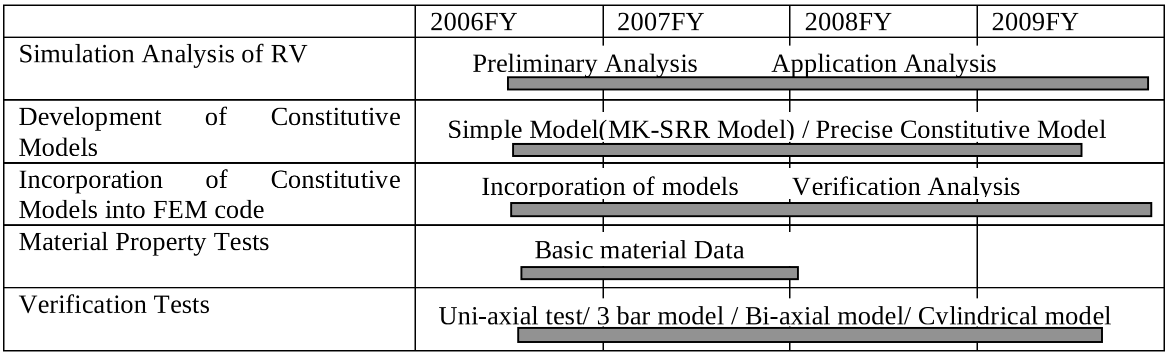


Figure 1. Main issues and the schedule of the project

3 PRELIMINARY ANALYSIS OF RATCHETING IN REACTOR VESSEL

3.1 Reactor Vessel Structure in scope

Figure 2 shows the reactor vessel structure of the current demonstration FBR plant design in Japan and the thermal transient behaviours at start-up and manual trip. At start-up, the temperature of sodium goes up from 200 C to 550 C and liquid surface goes up more than 1000 mm, simultaneously. At manual trip, the temperature distribution change is more complicated. The liquid temperature starts to descend from lower position, but the liquid near the surface cannot follow quickly as shown in Figure 2. Consequently, during the thermal cycle, the maximum stress is generated at the position of the vessel near the liquid surface (NSL-50mm) as pointed out in Figure 2. In the design this event is supposed to occur 640 times during the life, and the ratcheting phenomenon near the liquid surface is concerned. It is one of the key issues of R&D how to control the excessive strains accumulated due to this phenomenon.

3.2 Specification of Analysis

FEM code used is FINAS(FInite element Non-linear Analysis System). Firstly, thermal conduction analysis was carried out. Then, structural analyses were carried out by using temperature distribution and its history obtained by the thermal conduction analysis.

The straight portion of the reactor vessel from NsL-3000mm to NsL+1600mm was modelled by axis-symmetric elements. Inner radius is 5350mm. Material is 316FR stainless steel. The bi-linear kinematic hardening model with the α reset procedure (Corum, 1981) is used as a reference constitutive model. Primary loads considered are internal pressure and dead weight.

Parametric analyses were performed with the following conditions.

Thickness: 30/ 40/ 50 mm, Internal pressure: 0/ 0.147 MPa, Weight: with/ without weight

3.3 Analysis Results

Two graphs in Figure 2 describe the temperature distribution changes in the axial direction of R/V during the start-up and the manual trip respectively, that are obtained by thermal conduction analyses. The change of the temperature distribution profiles is very complicated due to the liquid surface movements and the sodium temperature changes. Especially, at the boundary between liquid sodium and gas, i.e. at the liquid surface, the temperature distribution changes abruptly, since gas cannot follow the temperature change of liquid sodium quickly.

Consequently, the response of structure is also complicated. Figure 3 describes the behaviors of stress and strain with temperature at the center of thickness near the liquid surface (Z=NsL-50mm). Circumferential and axial components of stress and strain are dominant but the ratio of the two components is not constant. In other words, the relation between circumferential and axial stresses is non-proportional.

Parametric analyses proved that the relation between primary and secondary stresses was affected by the thickness of the vessel, the internal pressure and the weight. Ratcheting behaviour was observed from the analysis results. The axial strain increases with cycles, while the circumferential strain decreases. And this behaviour is accelerated by largeness of thickness and weight, while it is decelerated by internal pressure.

The requirements for the constitutive models in scope and the following verification tests were constructed considering these preliminary analysis results.

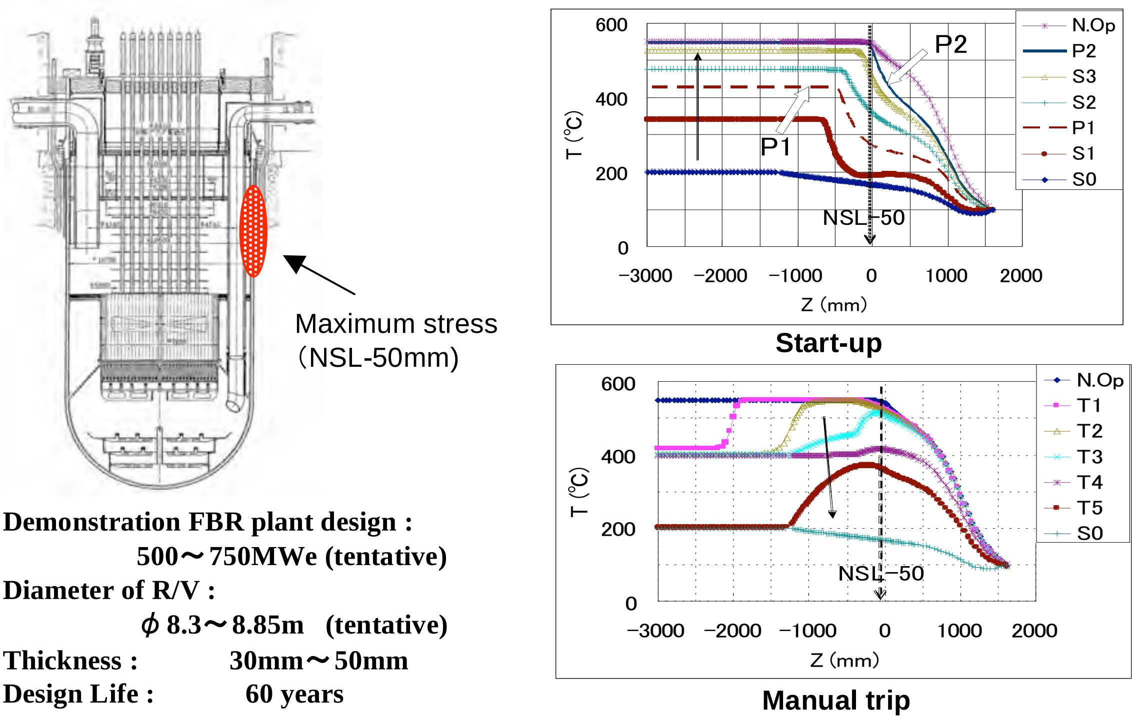
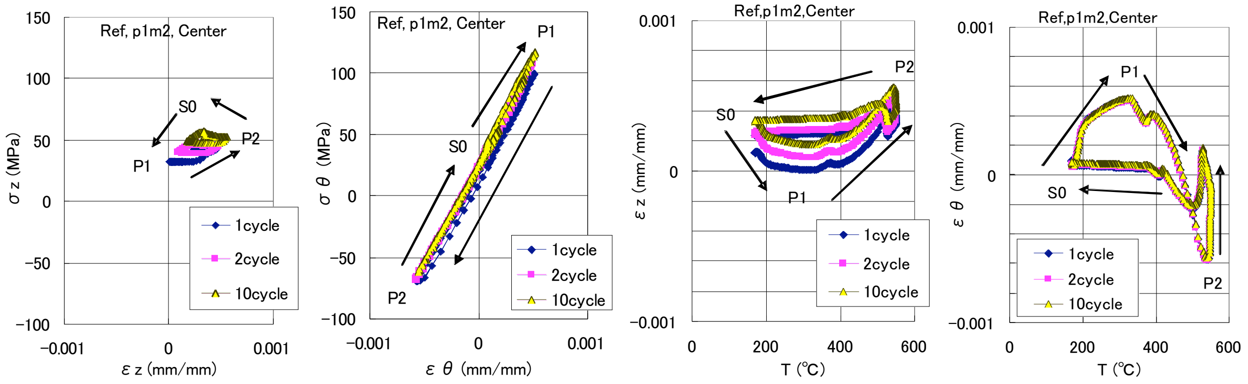


Figure 2. Reactor Vessel structure of current demonstration FBR design and thermal transient behaviours



(a) stress vs strain

(b) strain vs temperature

Figure 3. Stress, strain and temperature hysteresis at the center of thickness (Z=NsL-50mm)

4 DEVELOPMENT OF ADVACED CONSTITUTIVE EQUATIONS FOR DESIGN

Development of advanced plasticity models is a major task in the present study, because time-dependent creep phenomena of 316FR is not so important at temperatures of interest below 550 ℃. A number of plasticity models are currently under development in two categories: a simplified model and a high-accuracy model. A simplified plasticity model tries to represent idealised conservative behaviours by introducing simple rules, while a high-accuracy model tries to represent realistic behaviours as precisely as possible.

4.1 Simplified plasticity model

The Multi-linear Kinematic hardening model with Stress Reversal-on Resetting (MK-SRR model) was newly developed by incorporating a hardening reset procedure into the multi-linear kinematic hardening model at every stress reversal event (Iwata et al., 2008). The basic idea comes from the hardening reset procedure, or α-reset procedure, employed in the improved bilinear kinematic hardening model by ORNL (Corum, 1981). The MK-SRR model employing multi-linear stress-strain relations can represent stress-strain behaviour more precisely than the bilinear kinematic hardening model for which one need to assume the maximum strain in advance. The cyclic hardening is not taken into account here from the viewpoint of conservative prediction of strain. The essence of the MK-SRR model (Iwata et al., 2008) is given in this section.

4.1.1 Concept of stress reversal-on resetting

The classical bilinear kinematic hardening model predicts endless movement of the yield surface and stress increase, when plastic strain continues to increase under repeated loading. To overcome the drawback of this prediction, the α-reset procedure which restores hardening to initial condition every time stress reversal occurs, was proposed by ORNL (Corum, 1981). This model is a kind of bilinear elastic-plastic model with a constant hardening coefficient and equal yield stresses both in tension and compression, and is considered as an extension of the well-known elastic-perfectly plastic model. The MK-SRR model is a simplified nonlinear elastic-plastic model which incorporates this concept in a nonlinear stress-strain relation. Figure 4 shows a typical stress-plastic strain behaviour obtained by nonlinear kinematic hardening combined with stress reversal-on resetting. During the process of unloading, the yield surface continues to translate toward the initial position where the full hardening reset is completed.

4.1.2 Translation rule for stress reversal-on resetting in a multi-axial stress field

The MK-SRR model in a multi-axial stress field specifies the translation rule of the yield surface as illustrated in Figure 5. When stress-reversal occurs at time (i), the yield surface starts to return toward its initial position and moves up to the position where the current deviatoric stress $\boldsymbol{s}$ lies just on the yield surface. The translation, or back stress, increment tensor in this process can be represented by

$$\Delta\boldsymbol{\alpha} = -h\boldsymbol{\alpha} \tag{1}$$

where $\boldsymbol{\alpha}$ is the current back stress tensor and h is the parameter obtained by solving the following von Mises type yield condition at time (i).

$$f = \frac{3}{2}\{\boldsymbol{s} - (1-h)\boldsymbol{\alpha}\} : \{\boldsymbol{s} - (1-h)\boldsymbol{\alpha}\} - \sigma_Y^2 = 0 \tag{2}$$

In the above equation, σ_Y is the yield stress, and the colon indicates the scalar product of two tensors.

After the yield surface returns to the initial position, it remains at rest as long as the stress stays inside the yield surface. The translation rule up until re-yielding occurs after unloading, which is called the stress reversal-on resetting translation rule, is expressed as follows:

$$\Delta\boldsymbol{\alpha} = \begin{cases} -h\boldsymbol{\alpha} & (for \;\; h < 1) \\ -\boldsymbol{\alpha} & (for \;\; h \geq 1) \end{cases} \tag{3}$$

4.1.3 Temperature-dependent MK-SRR model

To represent nonlinear stress-strain relation, the multi-linear kinematic hardening model proposed by Ohno and Wang (1993) is employed. This model was extended to allow application to a varying temperature field (MK model) and to facilitate the stress reversal-on resetting translation rule mentioned in 4.1.2 (MK-SRR model). The MK model decomposes back stress into a number of components and the evolution equation of each component is represented as follows:

$$\boldsymbol{\alpha} = \sum_{m=1}^{M} \boldsymbol{\alpha}_{(m)} \tag{4}$$

$$\dot{\boldsymbol{\alpha}}_{(m)} = \frac{2}{3}H_{(m)}\dot{\boldsymbol{\varepsilon}}^{p} + \frac{1}{H_{(m)}}\frac{dH_{(m)}}{dT}\dot{T}\boldsymbol{\alpha}_{(m)} - U(g_{(m)})\frac{H_{(m)}}{r_{(m)}^{2}} < \dot{\boldsymbol{\varepsilon}}^{p} : \boldsymbol{\alpha}_{(m)} > \boldsymbol{\alpha}_{(m)} \tag{5}$$

where $H_{(m)}$ and $r_{(m)}$ are respectively the kinematic hardening coefficient and the back stress limit for back stress component (m) and are temperature dependent. $\dot{\boldsymbol{\varepsilon}}^{p}$ is the plastic strain rate tensor, T is the temperature, and $U(\ )$ and $< >$ indicate the unit step function and Macauley's bracket, respectively.

The limit surface for back stress component (m) is expressed as

$$g_{(m)} = \frac{3}{2}(\boldsymbol{\alpha}_{(m)} : \boldsymbol{\alpha}_{(m)}) - r_{(m)}^{2} \quad . \tag{6}$$

The MK-SRR model employs the translation rule of eqn (5) in the process of loading, and the stress reversal-on resetting translation rule of eqn (3) in the process up until re-yielding occurs after unloading

Making use of the von Mises type yield criterion and its associated flow rule, the constitutive equation of the MK-SRR model is represented in the rate form as

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{D}^{ep} : \dot{\boldsymbol{\varepsilon}} + \dot{\boldsymbol{\sigma}}_{a} \ , \tag{7}$$

where $\dot{\boldsymbol{\sigma}}$ is the stress rate tensor, $\dot{\boldsymbol{\varepsilon}}$ is the strain rate tensor, $\dot{\boldsymbol{\sigma}}_{a}$ is the apparent stress rate tensor, ant the elasto-plastic modulus tensor is defined as

$$\boldsymbol{D}^{ep} = \boldsymbol{D}^{e} - \frac{18G^{2}}{S_{0}\sigma_{Y}}(\boldsymbol{s} - \boldsymbol{\alpha}) \otimes (\boldsymbol{s} - \boldsymbol{\alpha}) \ , \tag{8}$$

with

$$S_{0} = 2\big(3G + H\big)\sigma_{Y} \ . \tag{9}$$

In eqns (8) and (9), $\boldsymbol{D}^{e}$ is the elastic modulus tensor, G is the elastic shear modulus, $\otimes$ indicates the tensor product, and H is the kinematic hardening modulus represented by a sum of components as

$$H = \sum_{m=1}^{M} H_{(m)} \left\{ 1 - U(g_{(m)}) \left(\frac{3}{2\sigma_{Y} r_{(m)}} \right)^{2} \langle B_{(m)} \rangle B_{(m)} \right\} \tag{10}$$

where

$$B_{(m)} = (\boldsymbol{s} - \alpha) : \alpha_{(m)} \tag{11}$$

4.2 High-accuracy plasticity model

In this category, we are trying to develop temperature dependent cyclic plasticity models with high-accuracy. As bases for development, the multi-linear kinematic hardening model originally proposed by Ohno and Wang (1993) and the two-surface cyclic plasticity model proposed by Iwata (1993) were judged to be most promising, because they already meet some of the important requirements for the target constitutive model mentioned earlier. Efforts to extend the two models focusing upon the method for representing temperature dependent cyclic hardening are currently under way. The basic direction of the development is briefly outlined.

4.2.1 Multi-linear kinematic hardening model

The multi-linear kinematic hardening model which was extended to enable temperature dependent analysis is called the MK model, and the resulting constitutive equations are given by eqns (4)-(11), which are common to the MK model and the MK-SRR model. The multi-linear cyclic plasticity model could be established as an extension of the MK model in much the same way as the two-surface cyclic plasticity model by Iwata (1993). Possible ways of extension of the MK model will be pursued in this direction, though we are well aware that recently Takahashi et al. (2008) along similar lines extended the multi-linear kinematic hardening model to simulate cyclic hardening by introducing a plastic hardening index surface as was done by Ohno (1982).

4.2.2 Two-surface model

The two-surface model which can simulate nonlinear stress-strain curves as they are would be an ultimate multi-linear kinematic hardening model in a way. A two-surface cyclic plasticity model under isothermal condition was proposed by Iwata (1993). The model is constructed using a yield surface which moves kinematically in the deviatoric stress space, a limit surface which grows in an isotropic fashion whenever the yield surface comes into contact with it, and a plastic hardening modulus function which is defined consistently with fundamental stress-strain characteristic equations under monotonic and cyclic loadings. It is planned that the two-surface cyclic plasticity model will be extended to allow for temperature dependent analysis. It would potentially be positioned closest to the target constitutive model of this project.

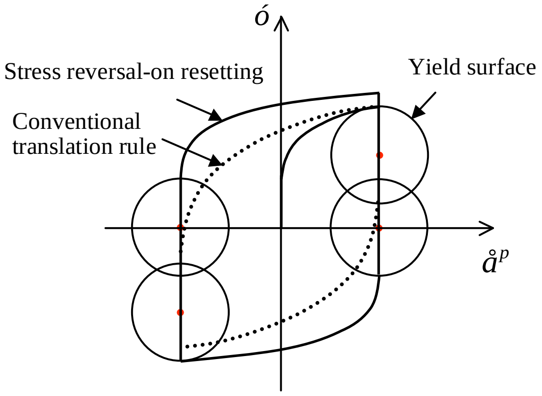


Figure 4 Concept of stress reversal-on resetting

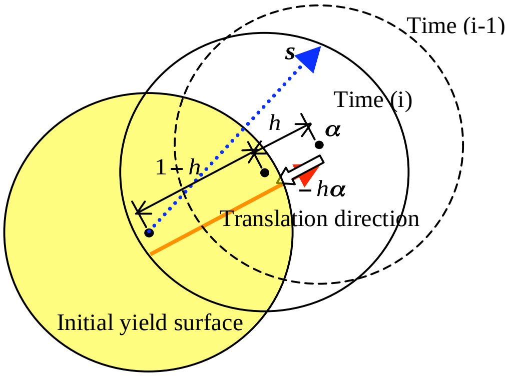


Figure 5 Translation of a yield surface in the stress reversal-on resetting process

5 EXPERIMENTS

Experiments consist of three categories, i.e., uni-axial tests, basic structure element tests, and structure model tests. The uni-axial tests consist of two series of tests, i.e., material property test and temperature dependency test. The former is used to mainly determine the parameters of constitutive models, and latter is used for basic verification of temperature dependency performance of constitutive models. The basic structure element tests consist of two kinds of simple ratcheting tests, i.e., three bar ratcheting test and bi-axial ratcheting test. The former is based on the famous two bar ratcheting theory, while parallel three bar structures are used for symmetry. The latter was proposed considering the results of the preliminary analysis. The axial and circumferencial components of stress and strain are dominant and they describe non-proportional hysteresis. In the structure model tests, similar phenomenon as the ratcheting in the reactor vessel is realized. Small cylinders are used as test specimens and the movement of temperature distribution is generated by heater and water level control.

5.1 Uni-axial tests

5.1.1 Basic material properties

Thermal expansion test (RT~650C), tensile tests and cyclic tests under constant temperature (RT~650C) were carried out to obtain basic material properties.

5.1.2 Temperature dependency tests

Figure 6 and Figure 7 show the diagrams of the temperature and the strain hysteresis. Some results will be described in Chapter 6 with calculation results.

5.2 Basic structure element tests

5.2.1 Three bar ratcheting test

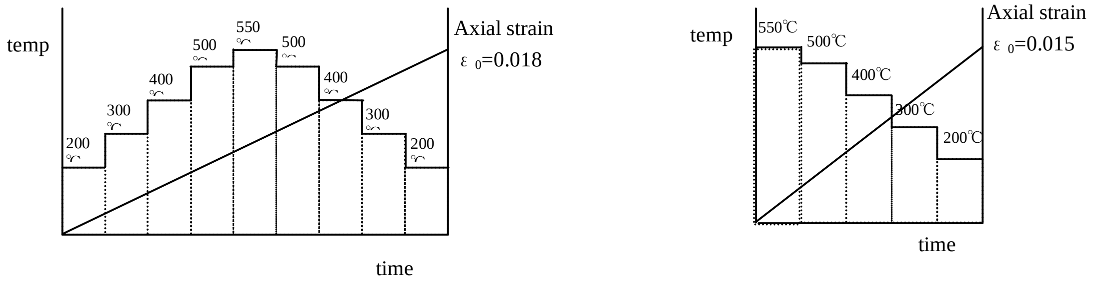
Figure 8 shows the test specimen and test conditions indicated by primary and secondary stress parameters. The center bar is subjected to cyclic thermal loading by heater and primary load can be also added by actuator. Four test cases are set in the ratchet region with or without primary load as shown in the figure.

5.2.2 Bi-axial ratcheting tests

Figure 9 shows the concept of the test. The axial strain and circumferential stress are induced respectively by the actuator and internal pressure controlled independently.

5.3 Structure model tests

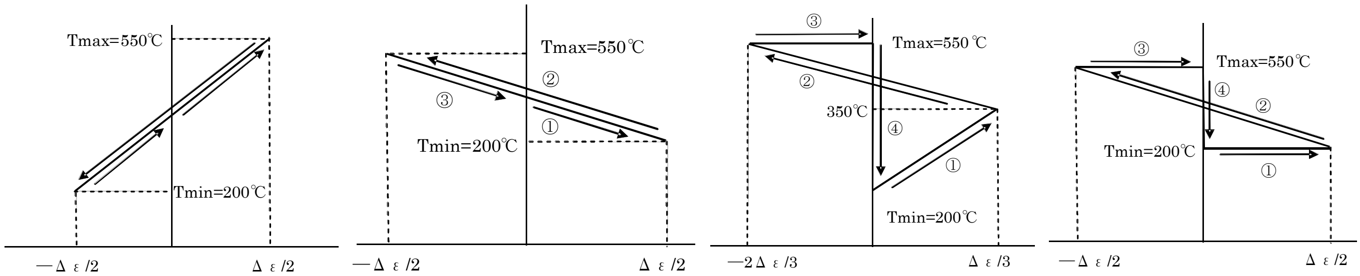
Figure 10 shows the test apparatus of the test. The coil heater and the water surface level are fixed. The movement of temperature distribution subjected to cylindrical specimen can be realized by moving the specimen up and down. Figure 11 shows one example of measured temperature distribution profiles and their movement.



(1) Pattern T-1

(2) Pattern T-2

Figure 6. Tensile test with temperature change



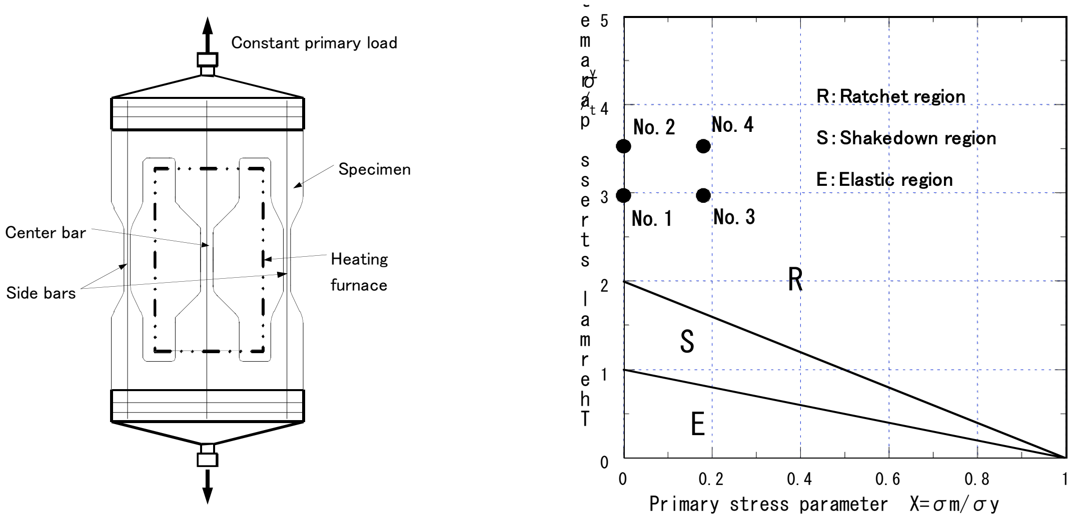
(1) Pattern C-1

(2) Pattern C-2

(3) Pattern C-3

(4) Pattern C-4

Figure 7. Cyclic tests with temperature and strain hysteresis



(1) test specimen

(2) test conditions

Figure 8. Three bar ratcheting test

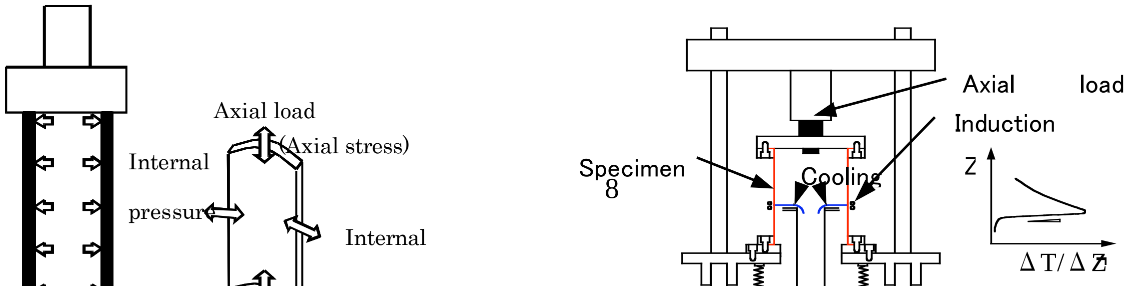


Figure 9. Bi-axial ratcheting test

Figure 10. Structure model tests

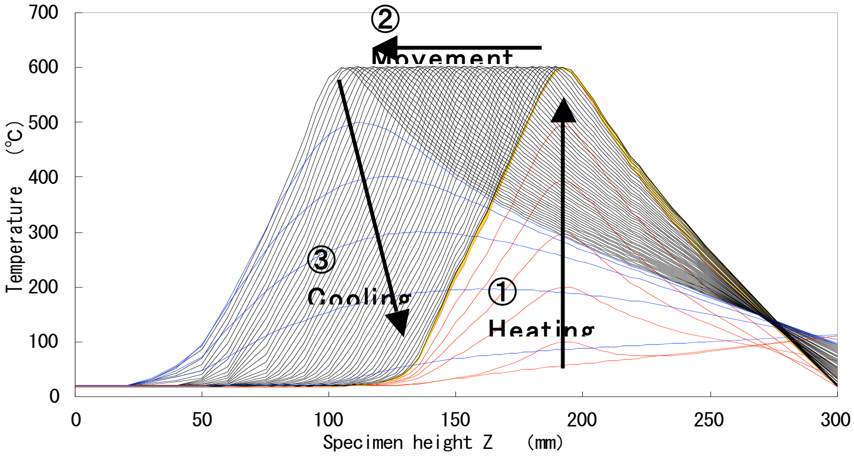


Figure 11. Measured temperature distribution (No.1 test specimen, N=20)

6 APPLICATION OF MK MODEL AND MK-SRR MODEL

6.1 Analysis of Experiments

6.1.1 Temperature dependency tests

Two tests, an in-phase test and a sodium-level simulating test, from a series of temperature-strain cyclic tests mentioned in 5.1.2 were analyzed with both the MK and the MK –SRR models. Applied strain range is 0.5%. As cyclic hardening is not considered in both models by nature, only the 1st-cycle behaviours were calculated and compared with the test results. Material properties used were taken from the test material. Figure 12 compares the predictions with the test results for the in-phase test. Apart from the discrepancies due to cyclic hardening appearing even in the 1st cycle of the test, the calculated results express pretty well the hysteresis loop shape (sharp on the underside, and round on the upside) which can typically be seen in temperature-rising and -falling process of the tests. As expected, the MK-SRR model exhibited a somewhat harder hysteresis loop compared with the MK model. Figure 13 shows the results for the sodium-level simulating test where the hysteresis loop features a comb like projection appearing due to a temperature step change under a zero strain condition. Generally, reasonable predictions are obtained, although not so small discrepancies are found in the temperature step change process.

6.2 Analysis of Reactor Vessel Model

MK-SRR model was applied to the reactor vessel analysis. The case with 30mm in thickness, internal pressure and weight was picked up. Figure 14 shows the stress / strain relation used in the constitutive models. Number of segments is the parameter of multi-linear modelling. The strain range from 0 to 1.0 % is divided into 2 to 13 segments for each model. In MK-SRR model the first break point is the proportional limit and all break points are on the original monotonic stress – strain curve. The reference denotes the bi-linear model used in the preliminary analyses. The approximation of stress- strain curve adopted in MK-SRR model is always conservative in evaluating strains, since the segments of MK-SRR model never exceed the original curve. The larger number of segments makes the stress- strain relation approximation of MK-SRR model close to the original curve. The analysis results shown in Figure 15 prove this tendency. The ratcheting strain decreases with the increase of segments and almost converges at 9 segments. The ratcheting strain is hardly observed in reference analysis. The reason seems that the first break point of the bi-linear model of reference analysis exceeds the stress-strain curve as shown in figure 14.

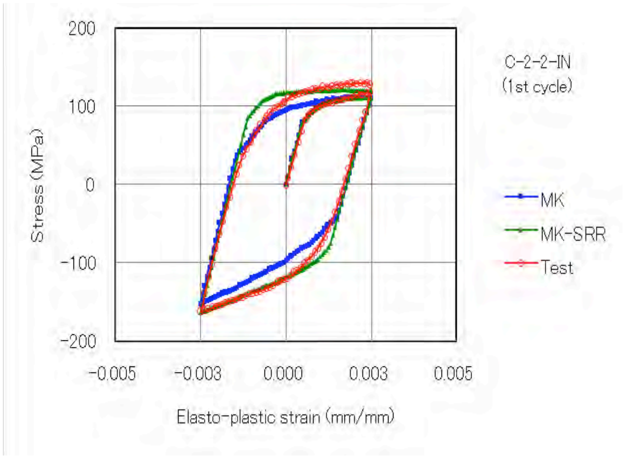


Figure 12. In-phase temperature-strain cyclic test and predictions (for the 1st cycle)

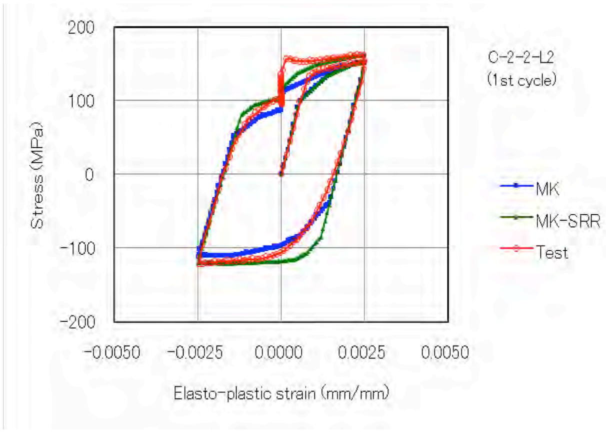


Figure 13. Sodium level simulating temperature-strain cyclic test and predictions (for the 1st cycle)

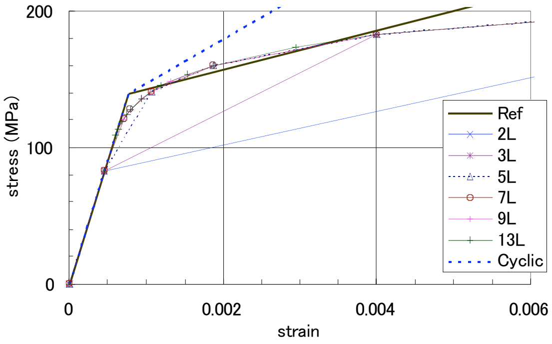


Figure 14. Multi-linear approximation of stress vs strain

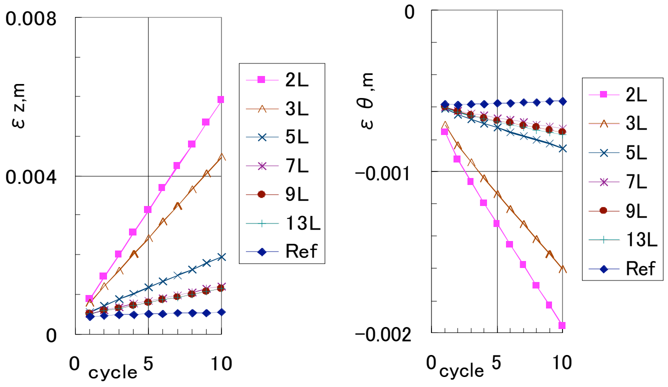


Figure 15. Strain behaviour by MK-SRR model

7 CONCLUSION

The aim of this study is to develop rational constitutive models that enable prediction of complex inelastic behaviors of reactor vessel precisely and to prepare the design guide based on inelastic analysis. In this paper, the R&D program which is going on and the results in the first half stage are introduced.

R&D program consists of five items, i.e., simulation analysis of RV, development of constitutive models, incorporation of constitutive models into FEM code, material property tests and verification tests. Preliminary analysis of reactor vessel and material property tests were almost finished. Some of verification tests were conducted. The simple constitutive model (MK-SRR model) was developed and was evaluated by compared with some verification test results.

Accurate constitutive models will be developed and evaluated through the analyses of test results and their effectiveness will be assessed through its application to the simulation analysis of RV in the second half stage.

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