Numerical studies on pre-stressed impact loaded concrete walls

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1 ABSTRACT

The various protective concrete barrier walls of nuclear power plants are required to withstand the effects of impacts by accidental or intentional missiles. Structural systems and solutions are under development both in building framework and in detail level, which require more sophisticated tools for different design phases. For example detailing of shear reinforcement is under development. Therefore numerical methods have been developed and taken in use for predicting the response of pre-stressed shear reinforced concrete structures subjected to impacts by hard projectiles. The impact load function on reinforced concrete wall caused by a hard missile is studied. Predicted impact loads are further used in structural analyses. Alternatively, the dynamical contact between the projectile and the target plate is modelled with nonlinear FEM.

Structural behaviour of the impact loaded pre-stressed walls has been predicted both by analytical methods and by involved non-linear FE-models. Analysis methods to predict associated damage mechanisms like crater formation, penetration, shear cone formation and perforation are examined. Experimental data is needed in order to verify the accuracy of numerical models. In this paper, numerical results obtained using different kinds of methods are compared with experimental data and observations on impact loaded pre-stressed reinforced concrete walls with shear reinforcement and with pre-stress levels of practical applications.

An experimental set-up has been constructed at VTT for medium scale impact tests. The main objective of this effort is to provide data for the calibration and verification of numerical models intended to be used in full scale practical applications.

2 INTRODUCTION

Local response to projectile impact can be described in terms of spalling, penetration, scabbing and perforation. In spalling concrete is removed from the impacted side of target slab and in scabbing from the rear side, respectively. Penetration may lead to perforation, if the contact force surpasses the remaining local load bearing capacity of the slab. Depending on the energy content of impact and the ductility of target plate concrete may be broken into smaller pieces or a shear cone is forming round the impact site. Hard and soft missile impacts lead to different kind of responses. Global behaviour of target slab is more important in the latter case.

Hard missile impact has been studied experimentally extensively within military applications, but with a limited range of parameters, and formulae for assessing penetration, perforation and scabbing have been derived based on experiments. Large number of tests is required when regression type analyses are carried out in developing design formulae. The modified Petry formula for assessing the penetration depth was developed as early as in 1910. The Army Corps of Engineers developed a penetration formula in 1946 and perforation and scabbing formulae were derived based on experiments. In 1946, the National Defence Committee developed a theory for penetration enabling also the calculation of contact force. In this theory the perforation and scabbing thicknesses are determined based on the penetration depth. The penetration theory accounts for the projectile nose shape and concrete strength, but the reinforcement is not explicitly included in the formulation. The range of parameters in nuclear industry is different from that in military applications, and therefore Chelepati and Kennedy, Kennedy (1976), derived a new formula for smaller values of the ratio of penetration depth to missile diameter. Also the penetrability value of concrete was
connected to the ultimate compressive strength of concrete. Degen (1980) derived perforation formulae based on new experiments within nuclear industry. At about the same time Chang (1981) proposed a perforation formula and a scabbing formula, which have been used widely in nuclear industry, similarly as Degen’s perforation formula. Berriaud et al. (1976) developed a perforation formula, the CEA-EDF formula, in which the parameters are the density and the compression strength of concrete, the mass, the diameter and the impact speed of missile. The UKAEA formulation is similar to the NDRC methodology, but also the perforation velocity is predicted by this method, as described in Barr (1990). Also the amount of bending reinforcement is included as a parameter in the perforation velocity formula.

Forrestal et al. (2003) have derived a new penetration theory primarily for high velocity impacts of hard missiles. This method is based on a cavity expansion theory. The first stage of penetration is the crater formation phase. If the projectile has kinetic energy enough it may proceed into a tunnel phase. For a relatively slender target the formation of a shear cone may lead to perforation. This theory does not take reinforcement into account, but the effect of concrete cracking or shear strength and the effects of bending and shear reinforcements can be included in assessing the possible shear cone separation. In none of the models so far is the pre-stress considered. In the theory of Forrestal et al. the contact force is dependent on compression strength of concrete and a parameter in which the pre-stress could be taken into account. Prestressed beams and slabs have been considered e.g. in Ito et al. (1987) for smaller impact velocities of about 20 m/s.

3 SEMIEmpirical Formulae AND SIMPLIFIED METHODS

The ACE penetration formula is (in SI units, as are all the following formulae)

\[
\frac{x}{d} = 0.3506 \times 10^{-3} \frac{Mv_0^{1.5}}{d^{2.785}} \sqrt{f_c} + 0.5,
\]

where \(M\) is the projectile mass, \(v_0\) is the impact velocity, \(d\) is the diameter of projectile, \(f_c\) is the compressive strength of concrete and \(x\) is the penetration depth. The penetration depth according to the modified NDRC formulation is calculated from

\[
\frac{x}{d} = 2\sqrt{G}, \text{ if } G \leq 1 \text{ or } \frac{x}{d} = G + 1, \text{ if } G > 1, \text{ where}
\]

\[
G = 3.81 \times 10^{-5} \frac{NMv_0^{1.8}}{\sqrt{f_c}d^{2.8}},\text{ in which } N \text{ is a nose shape factor. For flat nose } N = 0.72. \text{ The perforation thickness, } e, \text{ is}
\]

\[
\frac{e}{d} = 3.19\left(\frac{x}{d}\right) - 0.718\left(\frac{x}{d}\right)^2, \text{ when } \left(\frac{x}{d}\right) < 1.35 \text{ or (by the ACE formula)}
\]

\[
\frac{e}{d} = 1.32\left(\frac{x}{d}\right) + 1.24\left(\frac{x}{d}\right)^2, \text{ when } 1.35 \leq \left(\frac{x}{d}\right) \leq 13.5.
\]

The scabbing thickness, \(s\), is

\[
\frac{s}{d} = 7.91\left(\frac{x}{d}\right) - 5.06\left(\frac{x}{d}\right)^2, \text{ when } \left(\frac{x}{d}\right) < 0.65 \text{ or}
\]

\[
\frac{s}{d} = 2.12 + 1.36\left(\frac{x}{d}\right), \text{ when } 0.65 \leq \left(\frac{x}{d}\right) \leq 11.67.
\]

Degen's modification of the ACE perforation formulae are

\[
\frac{e}{d} = 2.2\left(\frac{x}{d}\right) - 0.3\left(\frac{x}{d}\right)^2, \text{ when } \left(\frac{x}{d}\right) < 1.52 \text{ or}
\]
Figure 1. Penetration depth of test slab by various formulae. RM denotes Reinhardt and Meyer formula, FL refers to formulae in Forrestal et al. (2003) and Li et al. (2003), FLS is obtained by assuming shallow penetration, as proposed in Li et al. (2003). The lower point set referring to test results, Test-T, with T-bars included, is in order AT, BT, CT from bottom. The upper point set, Test, is in order C1, A1, B1 from bottom.

\[
\frac{e}{d} = 0.69 + 1.29 \left( \frac{x}{d} \right), \quad \text{when} \quad 1.52 \leq \left( \frac{x}{d} \right) \leq 13.42,
\]

where the penetration depth, \( x \), is obtained from the NDRC formula.

\[
\frac{e}{d} = \left( 60.96 \right)^{0.25} \left( \frac{Mv_0^2}{v_0^3 f_c} \right)^{0.5} \quad \text{and} \quad \frac{s}{d} = 1.84 \left( 60.96 \right)^{0.13} \left( \frac{Mv_0^2}{d^3 f_c} \right)^{0.4}.
\]

The CEA-EDF formula is, Berriaud (1978),

\[
\frac{e}{d} = 0.82 \rho_c^{2/3} f_c^{3/8} \frac{M}{d^{1.5}} v_0^{3/4},
\]

where \( \rho_c \) is the density of concrete.

The U.K. Atomic Energy Authority has proposed the following formulae for obtaining the penetration depth, Barr (1990)

\[
\frac{x}{d} = 0.275 - \sqrt{0.0756 - G}, \quad \text{if} \quad G < 0.0726 \quad \text{or}
\]

\[
\frac{x}{d} = \sqrt{4G - 0.242}, \quad \text{if} \quad 0.0726 \leq G \leq 1.0605 \quad \text{or} \quad \frac{x}{d} = G + 0.9395, \quad \text{if} \quad G > 1.0605, \quad \text{where}
\]

\[
G = 3.81 \times 10^{-5} \frac{NMv_0^{1.8}}{\sqrt{f_c d^{2.8}}} \quad \text{as in the NDRC formulation.}
\]
Figure 2. Perforation thickness of test slab with a thickness of 0.25 m calculated by various formulae.

The scabbing thickness is calculated from, Barr (1990),

\[ \frac{s}{d} = 5.3G^{0.3} \]

The perforation velocity of projectile is calculated from

\[ v_p = 1.3 \rho_c \sqrt[16]{f_c} \left( \frac{dh^2}{M} \right)^{2/3} (\rho_p + 0.3)^{1/2}, \]  

(1)

where \( \rho_p \) is the amount of reinforcement, [%], (each face, each way) and \( h \) is the plate thickness.

In Reference Forrestal et al. (2003) a penetration theory is developed. In the crater formation or spalling phase the contact force on the projectile nose is

\[ F = cx, \]

in which the penetration depth is denoted by \( x \) and \( 0 < x < kd \), \( d \) is the projectile diameter, \( k \) is a factor and \( c \) is obtained from continuity conditions between the crater phase and the tunnel phase of penetration. For deep penetration \( k = 2 \) is proposed in Forrestal et al. (2003). For small to medium penetration depths, \( x/d < 5 \), \( k = 0.707 \) is suggested for a flat nosed projectile in Li et al. (2003). In the tunnel phase, according to the cavity expansion theory,

\[ F = 0.25\pi d^2 (Sf_c + N\rho_c v^2), \]

where \( N = 1 \) for a projectile with a flat nose, \( \rho_c \) is the density of concrete and \( S = 721/\sqrt{f_c} \).

The equation of motion of the projectile becomes

\[ M \frac{d^2 x}{dt^2} + F = 0 \]
Figure 3. Scabbing thickness of test slab with a thickness of 0.25 m calculated by various formulae.

Figure 4. Penetration and perforation model according to Forrestal et al. (2003) and Li et al.

with the initial conditions \( x(0) = 0 \) and \( \dot{x}(0) = v_0 \), and the mass of the projectile is denoted by \( M \). The equation of motion can be integrated analytically or numerically e.g. by the central difference (CD) method. Continuity conditions at the penetration value of \( x = kd \) determine the coefficient \( c \).
Figure 5. Contact force obtained by the formulation of Forrestal et al. (2003): solid line F and curve F-DT, which refers to deflecting target slab. Curve SDOF is obtained by the formulation in Reinhardt and Meyer (1998), in case SDOFR the strain rate dependency of concrete is taken into account. Curve, conc, is calculated with the concrete plasticity model of Abaqus/Explicit, and solution, cap, is obtained with the Drucker-Prager cap model. Curves Hughes and Kar are calculated with the formulae of Hughes (1984) and Kar (1978).

Defining an impact function \( I = \frac{1}{S} \frac{Mv_0^2}{d^3 f_c} \) and \( \tilde{N} = \frac{M}{N \rho d^2} \) the dimensionless penetration depth of the model of Forrestal et al. (2003) can be written in the form, Li et al. (2003)

\[
\frac{x}{d} = \sqrt{\frac{(1 + k \pi / 4 \tilde{N})}{(1 + I / \tilde{N})}} \frac{4kI}{\pi}, \quad \text{if } \frac{x}{d} \leq k \quad \text{and} \quad \frac{x}{d} = \frac{2}{\pi} \tilde{N} \ln \left[ \frac{1 + I / \tilde{N}}{1 + k \pi / 4 \tilde{N}} \right] + k, \quad \text{if } \frac{x}{d} > k. \tag{2}
\]

For shallow penetration the calculated penetration depth is in Li et al. (2003) proposed to be modified by the formula

\[
\frac{x}{d} = \left( 1.628 \frac{x_{sc}}{d} \right)^{2.789}, \tag{3}
\]

where penetration \( x_{sc} \) is calculated first by Eqn. (2), (in Li et al. (2003) the factor 1.628 is outside the brackets in formula corresponding to Eqn. (3)).

For a sufficiently high velocity perforation takes place after the penetration phase. Assuming that a conical plug is formed in front of the projectile nose the resistant force of the shear surface and rear face bending reinforcements can be calculated as in Dancygier (1997) and Chen et al. (2008)

\[
F_{sc} = \tau_c h_p \pi (d + h_p \tan \alpha), \quad F_{sb} = \pi (d + h_p \tan \alpha) A_j f_y \sin \alpha,
\]

where \( h_p = h - x \) is the remaining target thickness in front of the projectile, \( \tau_c = f_c / \sqrt{3} \) is the shear strength of concrete according to Tresca's hypothesis, \( \alpha \) is the angle between the meridional direction of the conical surface (of the shear plug) and the projectile axial direction, Fig. 4, \( f_y \) is the yield strength of reinforcement, \( A_j \) is the area of reinforcement per unit width, [m²/m]. Also the effect of shear reinforcement
can be evaluated similarly, \( F_{st} = \pi h_p (d + h_p \tan \alpha) \tan \alpha A_{st} f_{st} \), where \( A_{st} \) is the area of shear reinforcement per unit area, \([m^2/m^2]\). The total resisting force is \( F_r = F_{st} + F_{sb} + F_{st} \) and perforation is initiated when \( F_r = F \).

The effect of global or overall response of the target structure can be taken into account in a simple manner by using a single degree of freedom model (SDOF) for the target structure, as in Li et al. (2007). The equation of motion for the missile becomes, see Fig. 4, \( M(\ddot{x} + \ddot{w}) + F = 0 \) and for the target structure \( m_e \ddot{w} + R - F = 0 \), where \( w \) is the displacement of the target and \( x \) is the penetration, as before, \( m_e \) is the effective mass of the SDOF-model, and \( R \) is the support reaction or internal force of the structural spring. For an elastic structural spring the internal force is \( R = K w \), where \( K \) is the elastic spring stiffness. The structural spring plastifies when \( w = w_e = R_p / K \), where \( R_p \) is the plastic limit load of the target structure.

In the case of a simply supported plate loaded with a central point load, the plastic limit load is \( R_p = 8m_p \), where \( m_p \) is the plastic bending moment. The elastic stiffness of the simply supported plate, in the case of point load, is \( K = 86.2 D_c / L^2 \), where \( D_c = E_c h^3 / (12(1-v^2)) \) and \( L \) is the span length of the plate. For a simply supported elastic plate the effective mass is \( m_e = 0.31M \), where \( M \) is the mass of the plate. Similarly, for a clamped elastic plate \( m_e = 0.21M \). In Li et al. (2007) the resistant force on missile nose was found necessary to be calculated as \( F = \min\{F_1, F_2\} \), where \( F_1 \) is the resisting force in the crater phase and \( F_2 \) is the corresponding force in the tunnel phase of penetration.

In Reinhardt and Meyer (1998) a normalized stress-displacement curve is determined based on static penetration tests and this relationship is used in calculating the penetration depth due to projectile impact. The static curve was constructed by the few given data point values in the above reference and by using third order interpolation polynomials. The normalized displacement and contact stress in this theory are

\[
\xi = \frac{x}{d} \left( \frac{f_e}{40} \right) \quad \text{and} \quad \eta = \frac{\sigma}{f_e} \left( \frac{f_e}{40} \right)^{1/2}
\]

yielding for the initial incremental contact force-penetration relation an equation \( dF = k_0 A_m \left( \frac{f_e}{40} \right)^{1/2} \frac{f_e}{d} \ dx \), where \( k_0 \) is the initial stiffness of the normalized stress-displacement curve and \( A_m = 0.25 \pi d^2 \) is the cross-sectional area of a round projectile. This relation is used also in an unloading situation. The theory allows the calculation of the load-time function of penetration. In Reinhardt and Meyer (1998), the normalised penetration depth as a function of impact velocity in the form

\[
\frac{x}{d} = 24.058 \left( \frac{Mv_0^2}{d} \right)^{1/2}
\]

is also determined.

When the contact force is known as a function of penetration, \( F = F(x) \), the penetration time history can be integrated numerically, e.g. by the CD-method from the equation of motion \( M\ddot{x} + F(x) = 0 \).

As a result, the contact stress and contact force besides the penetration depth are determined as functions of time. The strain rate sensitivity of concrete in compression can be taken into account in the equations of \( \xi \) and \( \eta \) above by the formula, Reinhardt and Meyer (1998) \( \frac{f_e}{f_{c0}} = \left( \frac{\dot{\sigma}}{\dot{\sigma}_0} \right)^{\alpha} \), where \( \alpha = 1/(5 + 0.75 f_{cm}) \).

\( f_{cm} \) is the cube compressive strength and \( \dot{\sigma}_0 \) is of the order of 1 MPa/s.
4 PRE-STRESSED SLAB

Consider a reinforced concrete square slab with a span of 2 m and thickness of 0.25 m. The bending reinforcement, A500HW, consists of bars with a diameter of 10 mm and a spacing of 90 mm on both faces and in each direction. The pre-stressing is done with Dywidag bars, St1080/1230, which have a diameter of 26.5 mm and a spacing of 180 mm. The shear reinforcement is made with T-headed bars with a diameter of 12 mm and a spacing of 90 mm. The strength of concrete, nominally K50/C40, was determined for each test. For instance, in test case AT the compression strength was 54 MPa. More details are given in Orbovic et al. (2009).

![Deformed shape showing penetration in three cases for impact velocity of 100 m/s.](image)

**Figure 6.** Deformed shape showing penetration in three cases for impact velocity of 100 m/s.

In hard missile impact tests, the mass of the projectile was about 47 kg with an impact velocity of 100 m/s. Fig. 1 shows the penetration depth for this test plate as a function of impact velocity calculated with the various methods considered above. In Figs. 2 and 3, the perforation and scabbing thicknesses are depicted. In the considered tests no perforation occurred, and the scabbing values are considered in detail in Orbovic et al. (2009).

The perforation velocity evaluated by Eqn. (1) increases from 71 m/s to 109 m/s by including a reinforcement with $\rho_p = 0.4$, as in the present case.

Solutions for contact force are shown in Fig. 5. The rigid target (solid line) and deflecting target (DT) solutions are virtually the same in the solutions adopting the formulation in Forrestal et al. (2003). However,
the simplified solutions and nonlinear FEM solutions for contact force differ considerably. The maximum penetration in the solution SDOF is about 2.5 cm while the simplified penetration formula of Reinhardt and Meyer (1998), Eqn. (4), would give about 7 cm, shown in Fig. 1.

By taking into account the strain rate sensitivity of concrete in the formulation by Reinhardt and Meyer (1998) the maximum value of contact stress increased from 580 to 695 MPa.

The capacity of the shear plug, due to concrete shear strength and bending and shear reinforcement was large enough to prevent the severance of the shear cone at the considered impact velocity. Pre-stress could be included in the material parameter $S$ in the model in Forrestal et al. (2003). The larger pre-stress of 10 MPa, in test cases C, would mean a 20 % increase in the compressive stress in relation to compression strength, and this would reduce the penetration from 70 to 67 mm using the simplified analytical tool.

Nonlinear FEM solutions were calculated with Abaqus/Explicit, Abaqus Theory Manual (2007). One symmetric quarter of the simply supported slab was divided into eight noded solid elements. In the tentative analyses 10 elements were used in the thickness direction and 40 elements in the span wise direction. Bending and shear reinforcement and pre-stressing bars were modelled with truss elements.

The damaged concrete plasticity model and the extended Drucker-Prager and cap models were tried. In the analyses of Fig. 6 the cap model was used. The lower part of the slab was modelled with a dynamic failure model because of the limitations of available material modelling. The used material models appear not to work completely satisfactorily in the present impact situation.

Fig. 6 shows the deformed shapes of the slab in three cases, without shear reinforcement but with a pre-stress of 5 MPa in case B1 and 10 MPa in case C1. The calculated penetration depth values are 90 mm, 85 mm and 80 mm in cases A1, B1 and C1, respectively.

The penetration depth values were 110 mm, 120 mm and 130 mm in tests C1, A1 and B1, respectively. With T-bars added the penetration values were 37 mm for AT, 46 mm for BT and 73 mm for CT. In the numerical solutions, the pre-stress has much a bigger influence on penetration and on deflection, especially.

5 CONCLUSION

Semi-analytical and numerical methods have been applied to pre-stressed, shear reinforced and simply supported slabs subjected to hard missile impact. Reasonable agreement between the simplified solutions and numerical and test results was obtained. Material modelling of concrete in numerical models seems to be the key issue in high kinetic energy applications leading to large deformations, crushing and cracking. No completely satisfactory formulation could be found with the used code. More extensive numerical analyses are under way in order to meet requirements for structural design and analyses also in future.

REFERENCES


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