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# COMPUTER SIMULATION OF ACCELERATED FLUIDS: BEHAVIOUR IN ELASTIC CONTAINERS

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## ABSTRACT

The paper presents a static and dynamic comparative analysis of an elastic spherical tank, by FEM and BEM. We pointed out the stresses concentration in the support area and from due to dynamic action it results that the more dangerous situation occurs when the spherical tank is filled with the fluid at 80% pourcentage. Under the variable loads the fluid amplify the stress and the displacement state.

## 1. HYDRODYNAMIC FLUID FREE SURFACE IN CONTAINER

Consider the general two-dimensional hydrodynamic problem shown in figure 1. Under the assumptions of invisced and irrotational flow of an incompressible fluid, the flow field can be defined by means of a velocity potential

$$V = \nabla \Gamma \qquad (1)$$

where V is the velocity vector. Using the compatibility deformation equation from the same plane, it follows that the potential must be harmonic

$$\nabla^2 \Gamma = 0 \qquad (2)$$

The boundary conditions on the walls are written as

$$\Gamma_{,n} = q(x,z,t) \qquad (3)$$

where q and n are outward-directed flow velocity and unit vector, both normaal to walls. Radiation boundary conditions are not included in the analysis.
On the free surface there are two conditions that must be satisfied. The first is the kinematic boundary condition derived from the assumption that the surface is a material line. Hence if the surface is given by

$$z = \eta(x,t) \qquad (4)$$

the assumptions translate into

$$\eta_{,t} + \Gamma_{,x}\,\eta_{,x} = \Gamma_{,z} \qquad (5)$$

The dynamic boundary condition establishes continuity of the normal stresses at the surface and expressed is by Bernoulli's equation

$$\Gamma_{,t} + 0.5\left[ (\Gamma_{,x})^2 + (\Gamma_{,z})^2 \right] + g\eta + \frac{\sigma}{\rho}\,\frac{1}{R} = 0 \qquad (6)$$

where $\rho$ is the density of the fluid, $\sigma$ - the surface tension, $g$ is the gravitational acceleration and $R$ is the radius of curvature given by Dohner, 1987

$$R^{-1} = - \frac{(\eta_x)^2}{\left[(\eta_x)^2 + 1\right]^{3/2}}$$

Choosing $L$ as a characteristic length and defining the dimensionless variables

$$x^* = x/L, \quad z^* = z/L, \quad \eta^* = \eta/L$$
$$t^* = t(g/L)^{1/2}, \quad v^* = v/(g/L)^{1/2}, \quad \Gamma^* = \Gamma/(gL)^{1/2} \qquad (7)$$

the governing equation and the boundary conditions reduce to

$$v^* = \nabla\,\Gamma^*, \quad \nabla^2\Gamma^* = 0, \quad \Gamma^*_{,n} = q^*(x^*, y^*, z^*) \qquad (8)$$

$$\eta^*_{,t^*} = \frac{1}{\cos\beta}\,\Gamma^*_{,n}, \quad \Gamma^*_{,n} + \frac{1}{2}\left[(\Gamma^*_{,n^*})^2 + (\Gamma^*_{,z^*})^2\right] \qquad (9)$$

where the Weber number appears and is defined as

$$We^2 = \rho g L^2/\sigma \qquad (10)$$

The angle $\beta$ denotes the slope of the free surface measured from the horizontal and is positive in the counterclockwise direction. In the subsequent discussion the asterisks will be dropped for simplicity.

The equations can be further modified by the inclusion of a preferred direction along which the surface nodes are allowed to move. Figure 2 shows the surface at time level $k$ and a node that is to be moved along the z'-axis to a position on the surface at time level $k+1$. The choice of the angle $\gamma$ depends on the problem to be solved. For nodes in contact with solid boundaries the angle $\gamma$ is determined by the geometry of the wall so that the node remains attached to the boundary. With the notation of Figure 2, equations (9) become

$$\eta'_{,t} = \frac{1}{\sin(\gamma-\beta)}\,\Gamma_{,n} \qquad \text{on } z' = \eta' \qquad (11)$$

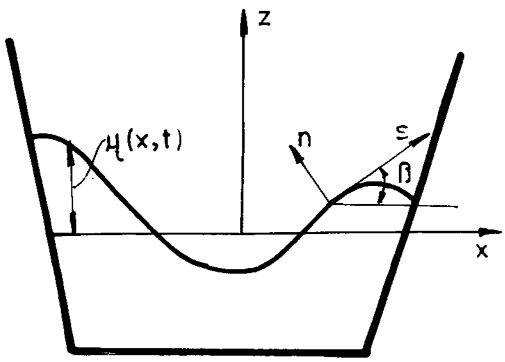


Fig. 1

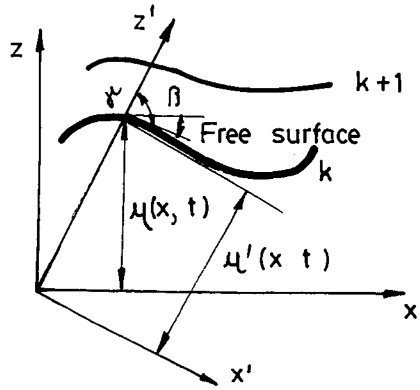


Fig. 2

$$(\Gamma_{,t})_{x'} = -\eta - \frac{1}{We^2}\frac{1}{R} - \frac{1}{2}\left[(\Gamma_{,s})^2 - \frac{2}{\tan(\gamma-\beta)}\Gamma_{,n}\Gamma_{,s} - (\Gamma_{,n})^2\right] \quad (12)$$

where the subscript on the time derivative indicates that $x'$ is to be held fixed.

## 2. ANALYSIS OF FLUID-SOLID SYSTEM BY FEM

The fluid structure interaction implies that the presure acting on the shell is calculated in terms of the fluid displacement potential. Considering wet surface of the container

$$p = \rho_0 \ddot{\Phi} \qquad \text{on } S_n \qquad (13)$$

the variational expressions describing fluid and structural motion yields to the 3-field $(p, \Phi, u_i)$ functional

$$\delta \int_{t_1}^{t_2} dt \left\{ \int_{D_L} \left[ -\frac{1}{2}\frac{p}{\rho_0 c^2} + \frac{p\ddot{\Phi}}{c^2} + \frac{1}{2}\rho_0 \dot{\Phi}_{,i}{}^2 \right] dD - \int_{D_S} \left[ \frac{1}{2}\mu \dot{u}_i \dot{u}_i - W_{(\varepsilon)} + X_i u_i \right] dD - \int_{S_\sigma} t_i u_i dS + \int_{S_W} \rho_0 \ddot{\Phi} u_i n_i dS \right\} dt = 0 \qquad (14)$$

Taking its variational derivatives restores the fluid equations the structural equations and the coupling conditions are presented in Gerardin, 1984, and the discretization of the various terms of (14) yield the following matrix expressions

$$\begin{bmatrix} K & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} q \\ r \\ f \end{Bmatrix} + \begin{bmatrix} M & 0 & B \\ 0 & 0 & -A \\ B^T & -A^T & -N \end{bmatrix} \begin{Bmatrix} \ddot{q} \\ \ddot{r} \\ \ddot{f} \end{Bmatrix} = \begin{Bmatrix} g_{(t)} \\ 0 \\ 0 \end{Bmatrix} \qquad (15)$$

Its show that separate assumptions on the displacement potential and on the fluid pressure lead to a symmetric but very sparse form of the system, with symmetric matrices.

## 3. INTERACTION OF THE FLUID-SOLID STUDIED BY BEM.

The boundaary value problem can be transformed by means of Green's second identity into an integral equation as shown in Medina, 1991 as

$$\alpha\, \Gamma_{(P)} = \int_S \left[\Gamma_{(Q)} (\ln r)_{,n} - \ln r\ \Gamma_{(Q),n}\right] dS \qquad (16)$$

where r is the distance from point P to a point Q located on the boundary.
Once the boundary is discretized, the integral equation (16) applied to a base point i located on the boundary results in

$$\alpha_i^k\, \Gamma_i^k = \sum_{j=1}^{N_e} \left[\Gamma_j^k\, T_j^a + \Gamma_{j+1}^k\, T_j^b + (\Gamma_{,n})_j^k\, T_j^c + (\Gamma_{,n})_{j+1}^k\, T_j^d\right] \qquad (17)$$

where $N_e$ is the number of the elements on the boundary of the flow surface and $T^q$, (q = a,b,c,d), are the shape functions determined from Medina, 1991.
For to derive the detalied terms of equations (17) for the case of the hydrodynamic problem, this equations can be written for the advanced time step where the incremental Δ-terms reflect the change in the geometry and the flow variables with respect to the previous time level.
The governing equations consist of the boundary element equation (17) and the dynamic and kinematic free surface boundary conditions. Thus a system of linear algebraic equations results for the unknown flow variables on the boundary and the location of the free surface at the advanced time step.

## 4. STATIC AND DYNAMIC ANALYSIS OF THE SPHERICAL TANK

The stress and displacement study was been made for a spherical tank with the radiuc r = 4m., bearing on the ring beam under the angle $\varphi = 135^o$. The thick of the wall is 1 cm. (t=1cm.). The spherical tank is filled with a liquid having the density $\rho$=1000 daN/cm$^3$ on the 80% from it height, Fig. 3a. In Fig. 3b is presented the statical hypothesis symmetrically with respect to the y axis and in the Fig. 3c is given the BEM discretization. The spherical tank with its load and the bearing considered is an axial-symmetric problem. For the numerical study we asumme a strip with unitary thick loaded with a distributed force after the law presented in Fig. 3b. The discretization of the spherical tank by B.E.M. is made by 34 nodal points and 32 linear elements. The normal stresses σ1 and σ2 and the displacements was been calculated in the discretization on points. The principal stresses diagrams σ1, σ2 and the tank deformation are given in the Fig.4a and Fig.4b. Its pointed out the high normal stress value σ2 in the point 22 where the tank is bearing on the ring beam, in comparison with the field values, Table 1.
The stress study was been made also by FEM for the same discretization. The comparative stresses and displacements values obtained by the static analysis by BEM and FEM are

presented in the Table 1. The shear stress on the thick wall are null. It remarks the near values for the stresses and the displacements obtained by these both methods.

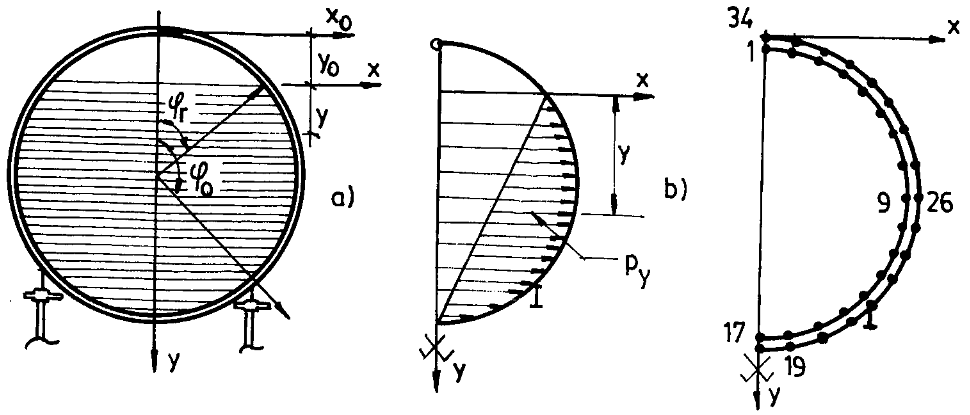


Fig 3

Table 1

| knots | B. E. M. | | | | F. E. M. | | | |
|---|---|---|---|---|---|---|---|---|
| | $\sigma_1$ [ daN/cm$^2$ ] | $\sigma_2$ | $u_1$ [ mm ] | $u_2$ | $\sigma_1$ [ daN/cm$^2$ ] | $\sigma_2$ | $u_1$ [ mm ] | $u_2$ |
| 18 | 36.50 | 1.22 | 0.0 | 0.33 | 33.2 | 1.61 | 0.0 | 0.35 |
| 20 | 71.80 | 12.55 | 0.028 | 0.171 | 70.1 | 13.06 | 0.031 | 0.176 |
| 22 | -23.30 | 41.70 | 0.0 | 0.0 | -20.6 | 42.01 | 0.0 | 0.0 |
| 24 | -1.03 | 37.9 | 0.213 | 0.089 | -1.22 | 39.82 | 0.221 | 0.092 |
| 26 | 0.63 | 28.7 | 0.403 | 0.105 | 0.98 | 28.92 | 0.41 | 0.011 |
| 28 | 4.5 | 27.6 | 0.353 | 0.093 | 5.9 | 29.8 | 0.36 | 0.098 |
| 30 | 25.2 | 23.6 | 0.177 | 0.173 | 27.3 | 25.6 | 0.178 | 0.183 |
| 32 | 48.7 | 7.38 | 0.056 | -0.292 | 50.3 | 8.32 | 0.062 | 0.299 |
| 34 | 6.52 | 0.41 | 0.0 | 0.344 | 7.27 | 0.56 | 0.0 | 0.351 |

The deformation of the spherical tank, Fig. 4c, is sketched by the simplified values ( 1000 ) of the displacements in two directions, Table 1.

Also, we make a dynamic analysis of the spherical tank, for ( n = 0 ) the axial-symmetric motions of the liquid, by means of the Laplace transform. The normal stress value $\sigma_1$ the displacement ux and their variations with respect to the Laplace's, Poterasu, 1988, parameter are presented in Fig. 5.

If we consider the dynamic action of the liquid on the tank wall, as a shock, the stress values in the initial time are great and consecutively these values diminish with respect to the Laplace's parameter k following an exponential law.

## 5. CONCLUSIONS

The static and dynamic analysis by BEM for the axial symmetric problems is more advantageous comparatively with FEM.

The computer programs for the axial-symmetric problems by BEM are more simple that FEM and the computer time is the smaller. For the shock forces in the dynamic regime, the BEM is also more advantageous to study in comparison with FEM

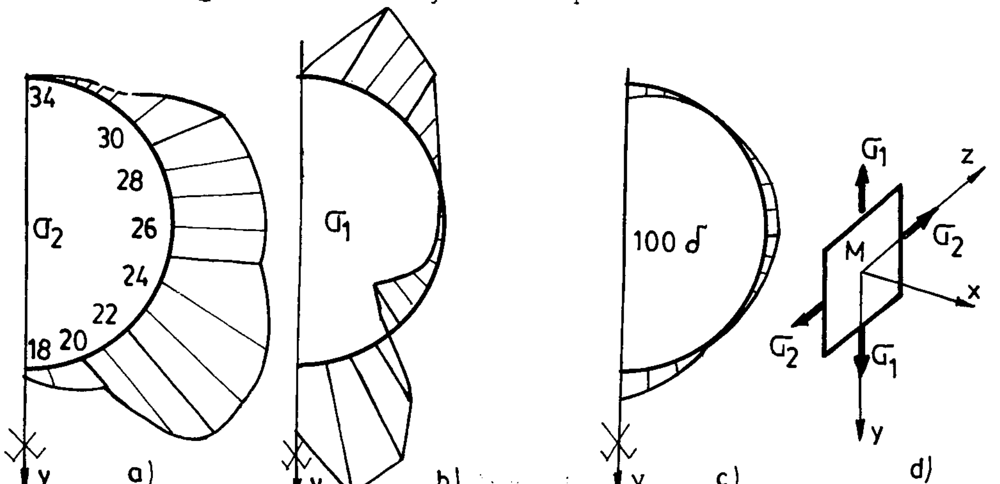


Fig. 4

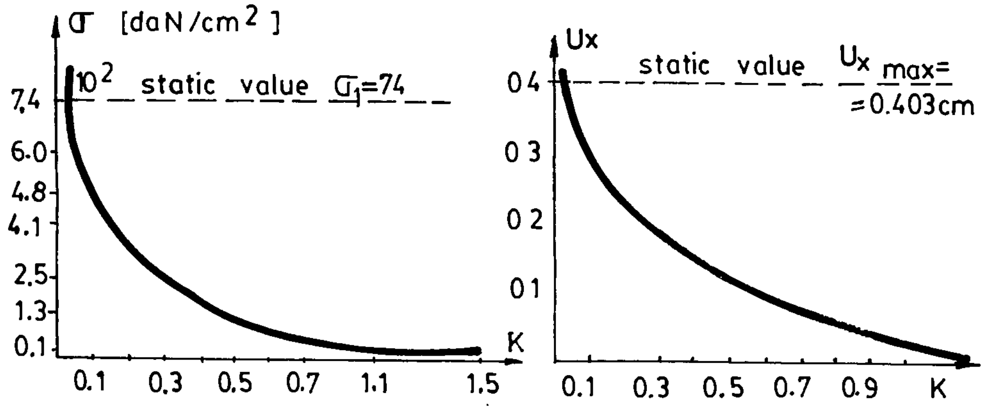


Fig. 5

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