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THE COMPREHENSIVE ERROR INVESTIGATION OF DYNAMIC EQUATIONS OF SHELLS WITH LOCAL IMPERFECTIONS

Li Lijuan[1], Mei Zhanxin[2], Wan Hong[2] and Liu Feng[2]

[1]Rubber New Techniques Research Institute, Qingdao Institute of Chemical Technology, Qingdao, 266042, P.R. China
[2]Xi'an Institute of Metallurgy and Construction Engineering, Xi'an, 710055, P.R. China

Abstract--By using the concept of norms of vectors and matrices, this paper has put forward a new method to the comprehensive error analysis of dynamic equations of shells with local imperfections, and has given out quantification norm of error caused by imperfections. The imperfections mentioned here can be geometric ones and material property ones. This method is very important to the dynamic analysis of practical engineering structures with local imperfections.

1. INTRODUCTION

Dynamic analysis of plates and shells with local imperfections needs much study in choosing model and calculation method, it also needs overall analysis in considering influence factors of imperfections on models. Because the practical structures are very complex, the finite element category methods with no neglect and simplification will perplex the problems. On the other hand, if chosen models much differ from practical structures, they cannot reflect real problems in spite of much improvement in calculation method and precision. It is a main task for us to study at present how to make the models approach to real structures and equations easy to solve. Up to now, many papers deal with only one kind of imperfections, for example, the effects of local geometric imperfection on vibration mode and frequency of shell structure are considered in reference [1], the dynamic property of plates and shells with material imperfections is considered in reference [2]. They deal with only one kind of imperfections, and cannot do the comprehensive analysis, especially can't objectively analyze practical structures with two or more kinds of imperfections in engineering. By using the concept of norms of vectors and matrices, considering imperfections overall, this paper puts forward a comprehensive error influence quantification norm caused by local geometric imperfections and material property imperfections of structure, and provides dependable basis for model in accordence with precision requirement and coinciding with reality. The calculation method provided by this paper is very convenient.

2. DYNAMIC EQUATION

In the finite freedom dynamic analysis of plates and shells, the basic dynamic equation is

$$[M]\left\{\ddot{V}\right\} + [C]\left\{\dot{V}\right\} + [K]\{V\} = \{P\} \tag{1}$$

where V is generalized displacement, $\dot{V}$ is generalized velocities, and $\ddot{V}$ is generalized accelerations, [M], [C], and [K] are mass matrix, damping matrix, and stiffness matrix respectively.

let us use $[M_0]$, $[C_0]$, $[K_0]$ and $[P_0]$ denote mass matrix, damping matrix, stiffness matrix and external force matrix of perfect structure respectively, in the meantime, $[M_0]+[\triangle M]$, $[C_0]+[\triangle C]$ and $[K_0]+[\triangle K]$ denote imperfect structure's. Then we can add addtional matrix $[\triangle M]$, $[\triangle C]$ and $[\triangle K]$ to basic matrices $[M_0]$, $[C_0]$ and $[K_0]$ respectively if we are required to consider some imperfections. The calculation of additional matrices can refer to reference [5].

3. COMPREHENSIVE ERROR NORM

Effects of imperfections lead original axisymmetric structures to lose their axisymmetric properities and the solution of the problems to more complex to problems. But not all imperfections must be considered, i.e. some of them are permissible in practical engineering. If we can prejudge by some simple method whether the error caused by imperfections is inside the error range permitted by engineering before doing any calculations, then we can omit much unnecessary calculation work, all of these will be benifical to engineering design and imperfection analysis of engineering problems.

Definition 1: If any vectors x and y and number α in space R^n satisfy the following relations

(1) $\| x \| > 0$, $\| x \| = 0$ only while $x = 0$;

(2) $\| \alpha x \| = |\alpha| \, \| x \|$;

(3) $\| x+y \| < \| x \| + \| y \|$

then $\| x \|$ is defined as a norm of vector x in space R^n.

Definition 2: If any matrices A and B and number C in space $R^{n\times n}$ satisfy the following relations

(1) $\| A \| > 0$, $\| A \| = 0$ only while $A = 0$;

(2) $\| CA \| = |C| \, \| A \|$;

(3) $\| A+B \| < \| A \| + \| B \|$;

(4) $\| AB \| < \| A \| \, \| B \|$

then $\| A \|$ is defined as a norm of matrix A in space $R^{n\times n}$.

Both vector norm and matrix norm are needed in the comprehensive error analysis of dynamic equation, therefore they must satisfy compatibility condition as follows

$$\|Ax\| \leqslant \|A\|\|x\|$$

For the convenience of dealing with problems we use ∞ norm definition in error evaluation, that is

$$\|x\|_{\infty} = \max_{1 \leqslant i \leqslant n} |x_i| \quad (i = 1,2,\cdots n)$$

$$\|A\|_{\infty} = \max_{1 \leqslant i \leqslant n} \sum_{j=1}^{n} |a_{ij}| \quad (j = 1,2,\cdots n)$$

Definition 3: The comprehensive error norm E of calculation model is defined as ratio of $\|\triangle V\|$ to $\|V\|$.

$$E = \frac{\|\triangle V\|}{\|V\|}$$

4. DETERMINATION OF COMPREHENSIVE ERROR NORM

Substitute mass matrix, damping matrix and stiffness matrix, in which the effects of imperfections have been considered, into equation(1), at the same time, decompose generalized displacement V of imperfect structure into $V_0$and $\triangle$V:

$$V = V_0 + \triangle V$$

here V_0 is the generalized displacement of perfect plate and shell structure, $\triangle$V is variation of V. We have dynamic equation of imperfect structure as follows:

$$([M_0] + [\triangle M])\left(\left\{\ddot{V}_0\right\} + \left\{\triangle \ddot{V}\right\}\right) + ([C_0] + [\triangle C])\left(\left\{\dot{V}_0\right\} + \left\{\triangle \dot{V}\right\}\right)$$
$$+ ([K_0] + [\triangle K])(\{V_0\} + \{\triangle V\}) = \{P_0\}$$

After suitable reordering, we get:

$$[M_0]\left\{\ddot{V}_0\right\} + [C_0]\left\{\dot{V}_0\right\} + [K_0]\{V_0\} + [M_0]\left\{\triangle \ddot{V}\right\} + [C_0]\left\{\triangle \dot{V}\right\}$$
$$+ [K_0]\{\triangle V\} = \{P_0\} - [\triangle M]\left\{\ddot{V}_0\right\} - [\triangle C]\left\{\dot{V}_0\right\} - [\triangle K]\{V_0\}$$
$$- [\triangle M]\left\{\triangle \ddot{V}\right\} - [\triangle C]\left\{\triangle \dot{V}\right\} - [\triangle K]\{\triangle V\} \quad (2)$$

It is obivious that

$$[M_0]\left\{\ddot{V}_0\right\} + [C_0]\left\{\dot{V}_0\right\} + [K_0]\{V_0\} = \{P_0\} \quad (3)$$

Equation (3) is the basic dynamic equation of perfect structure, the last three terms of equation (2) are high order small quantity, therefore they can be omited.

Let

$$\{\triangle P\} = -[\triangle M]\left\{\ddot{V}_0\right\} - [\triangle C]\left\{\dot{V}_0\right\} - [\triangle K]\{V_0\}$$

considering equation (3), we simplify equation (2) as:

$$[M_0]\left\{\triangle \ddot{V}\right\} + [C_0]\left\{\triangle \dot{V}\right\} + [K_0]\{\triangle V\} = \{\triangle P\} \quad (4)$$

It can be seen here that after considering imperfections, the variation of generalized displacement $\triangle V$ is the acting result of equivalent external force {P} caused by additional matrices [$\triangle$M], [$\triangle$C] and [$\triangle$K]. Equation (3) is a basic dynamic equation of perfect structure and can be solved easily. The soluation of equation (3) and concept of norm of matrix are used by this paper to determine comprehensive error norm E, consenquently whether the variation $\triangle V$ caused by imperfections is within the range permissible for engineering can be decided. By considering additional matrices respectively, we can further analyze which factor is most important of all the influence factors.

Do some simplified calculation to comprehensive error norm as follows.

Firstly we do free vibration analysis to equation (3):

$$\left([K_0] - \omega_n^2 [M_0]\right)\{\varphi_n\} = 0 \tag{5}$$

frequency ω_n and vibration mode vector $\{\varphi_n\}$ can be obtained from above. Then we calculate generalized mass and force of each vibration mode.

$$M_n = \{\varphi_n\}^T [M_0]\{\varphi_n\}$$

$$\triangle P_n(t) = \{\varphi_n\}^T \{\triangle P(t)\}$$

uncoupling equation of equation (4) is then given by

$$\ddot{Y}_n + 2\zeta_n \omega_n \dot{Y}_n + \omega_n^2 Y_n = \frac{\triangle P_n(t)}{M_n} \tag{6}$$

here $\{\varphi_n\}^T [C_0]\{\varphi_n\} = 2\zeta_n \omega_n M_n$

Let plate and shell structure be zero initial state, and do Duhamel intergration to equation (6) as

$$Y_n(t) = \frac{1}{M_n \omega_{Dn}} \int_0^t \triangle P_n(t) e^{-\zeta_n \omega_n (t-\tau)} \sin\omega_{Dn}(t-\tau) d\tau$$

where $\omega_{Dn} = \omega_n \sqrt{1-\zeta_n^2}$

Therefore

$$\{\triangle V(t)\} = \sum_{n=1}^{N} \{\varphi_n\} Y_n(t)$$

When norms of matrices are given to two sides of above equation, we have

$$\|\{\triangle V(t)\}\| = \|\sum_{n=1}^{N} \{\varphi_n\} Y_n(t)\| \leqslant \sum_{n=1}^{N} \|\{\varphi_n\} Y_n(t)\| \leqslant \sum_{n=1}^{N} |Y_n(t)| \|\{\varphi_n\}\|$$

It should be noticed

$$|Y_n(t)| = \frac{1}{M_n \omega_{Dn}} \left| \int_0^t \triangle P_n(\tau) e^{-\zeta_n \omega_n (t-\tau)} \sin\omega_{Dn}(t-\tau) d\tau \right| \leqslant \frac{1}{M_n \omega_{Dn}} \int_0^t |\triangle P_n(\tau)| d\tau$$

Then we get

$$E=\frac{\|\triangle V\|}{\|V\|}=\frac{\|\triangle V\|_{\infty}}{\|V\|_{\infty}}\leqslant\frac{\sum_{n=1}^{N}\frac{1}{M_{n}\omega_{Dn}}\int_{0}^{t}\left|\{\triangle P_{n}(\tau)\}\right|d\tau\|\varphi_{n}\|_{\infty}}{\|\{V_{0}\}\|_{\infty}}$$

It can be seen here that the soluation of comprehensive error norm E is only based on the soluation of basic vibration equation (3) of perfect structure. When $V, \dot{V} and \ddot{V}$, as well as vibration vector and frequency are determined by equation (3) of perfect structure, then $\{\triangle P\}$, $\{\varphi_n\}$, M_n and ω_{Dn} can be calculated, this calculation is easy to realize, the final calculation is to do some basic matrix operation, which is not complex. According to engineering requirement, if E is not large than comprehensive error norm limit δ, the effects of imperfections need not considering, otherwise the effects of imperfections shoud be considered in vibration model. The range of E also can be determined according to the characteristics of practical problems in special condition.

5. CONCLUSION

The comprehensive error norm provided in this paper can be used to analyze imperfection error of plate and shell structures with initial imperfections. It is made possible to consider imperfections from qualitative analysis to quantitative analysis. Calculation model can be simplified to some extent on the condition of contenting with requirement of precision. This paper provided the essential condition for the practical use of various models in engineering analysis. The determination of value δ needs further analysis.

6. REFERENCES

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